Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.2-c-x-^m-a+b-x^2-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [697]. This is test number [6].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (697)	0.00 (0)
Mathematica	100.00 (697)	0.00 (0)
Fricas	96.70 (674)	3.30 (23)
Sympy	95.27 (664)	% 4.73 (33)
Maple	92.97 (648)	7.03 (49)
Maxima	90.67 (632)	9.33 (65)
Mupad	89.96 (627)	10.04 (70)
Giac	88.38 (616)	11.62 (81)
IntegrateAlgebraic	59.54 (415)	40.46 (282)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

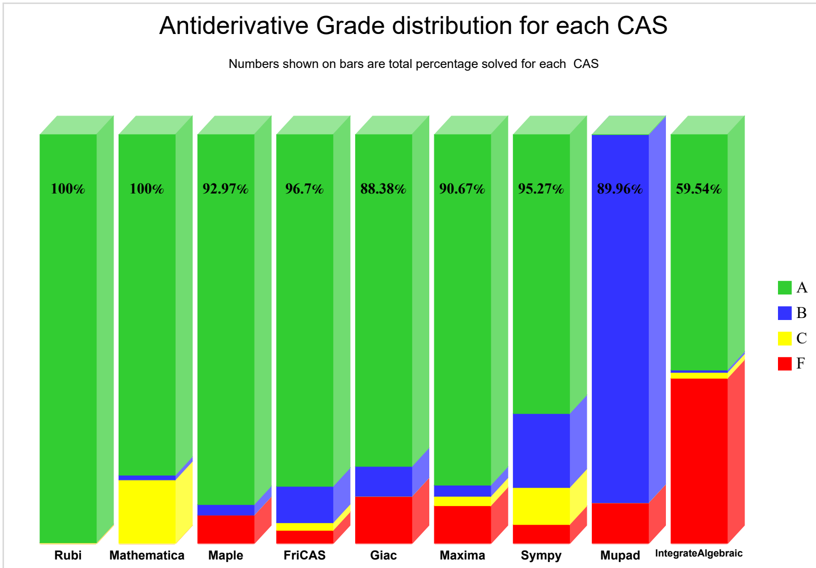
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

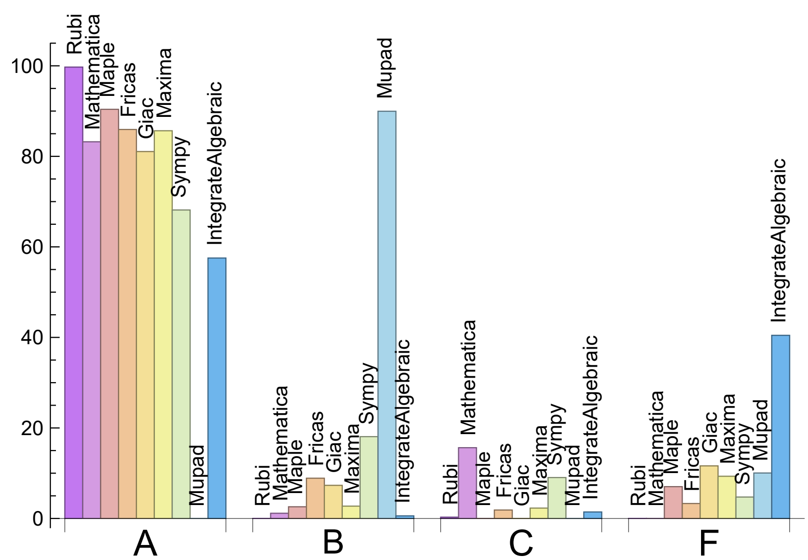
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.71	0.00	0.29	0.00
Maple	90.39	2.58	0.00	7.03
Fricas	85.94	8.90	1.87	3.30
Maxima	85.65	2.73	2.30	9.33
Mathematica	83.21	1.15	15.64	0.00
Giac	81.06	7.32	0.00	11.62
Sympy	68.15	18.08	9.04	4.73
IntegrateAlgebraic	57.53	0.57	1.43	40.46
Mupad	N/A	89.96	0.00	10.04

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	49	100.00 %	0.00 %	0.00 %
Fricas	23	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	282	100.00 %	0.00 %	0.00 %
Giac	81	100.00 %	0.00 %	0.00 %
Maxima	65	92.31 %	0.00 %	7.69 %
Sympy	33	0.00 %	100.00 %	0.00 %
Mupad	70	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

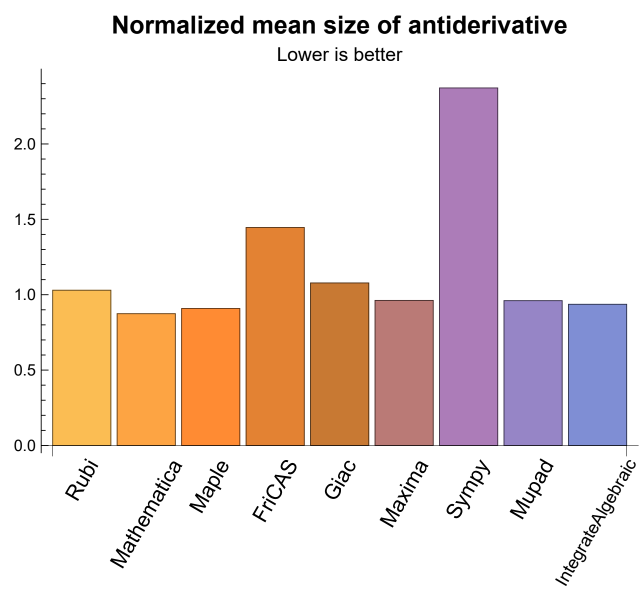
1.3 Performance

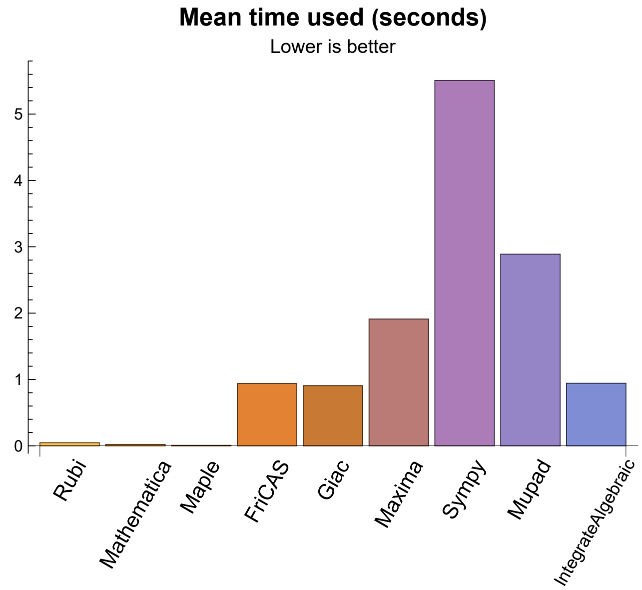
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.05	78.11	1.03	60.00	1.00
Mathematica	0.02	54.56	0.87	42.00	0.89
Maple	0.01	60.17	0.91	44.50	0.84
Maxima	1.91	69.75	0.96	55.00	0.88
Fricas	0.94	108.34	1.45	67.00	1.11
Sympy	5.51	175.37	2.37	76.00	1.28
Giac	0.91	74.24	1.08	57.00	0.88
Mupad	2.89	62.53	0.96	48.00	0.85
IntegrateAlgebraic	0.94	69.72	0.94	50.00	0.88

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

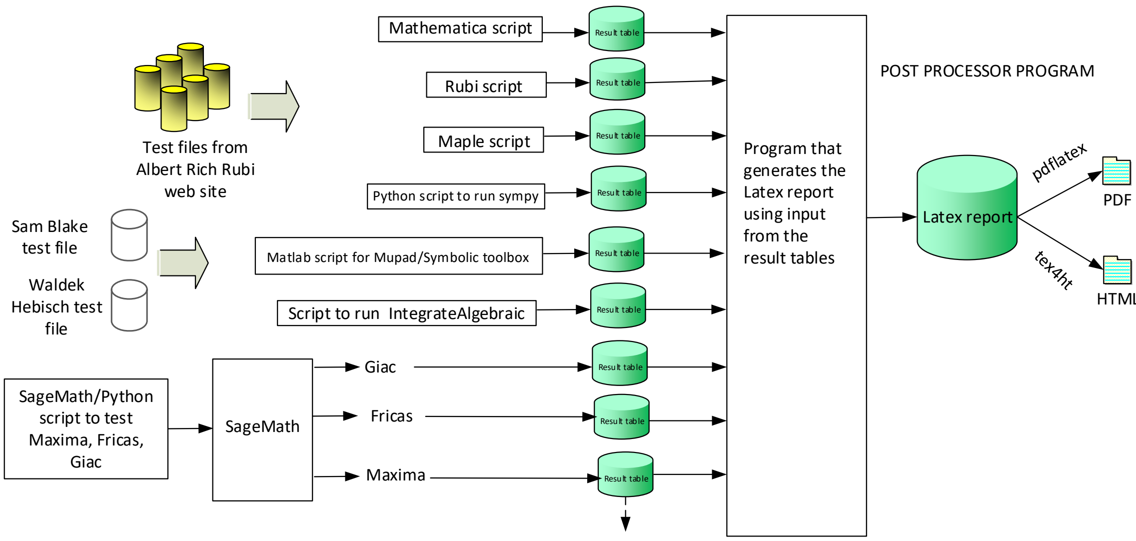
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697 }

B grade: { }

C grade: { 581, 583 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 298, 300, 304, 306, 308, 312, 313, 315, 317, 321, 323, 325, 329, 331, 333, 337, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 369, 370, 371, 374, 375, 376, 377, 378, 379, 380, 381, 382, 385, 388, 389, 390, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 413, 416, 417, 418, 419, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 487, 488, 489, 490, 491, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 505, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 520, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 545, 546, 547, 548, 549, 551, 552, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 584, 585, 586, 587, 588, 591, 592, 593, 594, 595, 598, 599, 600, 601, 602, 605, 606, 607, 608, 609, 610, 613, 614, 615, 616, 617, 620, 621, 622, 623, 631, 632, 633, 634, 640, 641, 642, 647, 648, 649, 650, 654, 655, 656, 657, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 687, 688, 689, 690, 691, 692, 693, 694, 697 }

B grade: { 38, 65, 90, 101, 102, 196, 197, 550 }

C grade: { 293, 294, 295, 297, 299, 301, 302, 303, 305, 307, 309, 310, 311, 314, 316, 318, 319, 320, 322, 324, 326, 327, 328, 330, 332, 334, 335, 336, 338, 350, 351, 365, 367, 368, 372, 373, 383, 384, 386, 387, 391, 392, 393, 409, 410, 411, 412, 414, 415, 420, 421, 422, 423, 424, 442, 453, 464, 475, 486, 493, 495, 504, 506, 519, 521, 533, 544, 555, 566, 580, 581, 582, 583, 589, 590, 596, 597, 603, 604, 611, 612, 618, 619, 624, 625, 626, 627, 628, 629, 630, 635, 636, 637, 638, 639, 643, 644, 645, 646, 651, 652, 653, 658, 659, 660, 685, 686, 695, 696 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 584, 585, 586, 587, 591, 592, 593, 594, 598, 599, 600, 601, 605, 606, 607, 608, 609, 613, 614, 615, 616, 620, 621, 622, 623, 631, 632, 633, 634, 640, 641, 642, 647, 648, 649, 650, 654, 655, 656, 657, 661, 662, 663, 664, 667, 668, 669, 672, 673, 674, 677, 678, 679, 682, 683, 684, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697 }

B grade: { 33, 38, 58, 65, 90, 91, 101, 102, 196, 197, 198, 337, 339, 340, 341, 342, 413, 424 }

C grade: { }

F grade: { 581, 583, 588, 589, 590, 595, 596, 597, 602, 603, 604, 610, 611, 612, 617, 618, 619, 624, 625, 626, 627, 628, 629, 630, 635, 636, 637, 638, 639, 643, 644, 645, 646, 651, 652, 653, 658, 659, 660, 665, 666, 670, 671, 675, 676, 680, 681, 685, 686 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 509, 511, 513, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 560, 563, 565, 567, 568, 569, 571, 572, 573, 575, 576, 577, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 631, 634, 647, 650, 691, 692, 693, 694, 695, 696, 697 }

B grade: { 38, 65, 90, 101, 102, 174, 196, 197, 198, 199, 201, 202, 203, 312, 337, 508, 510, 512, 514 }

C grade: { 466, 468, 470, 471, 472, 473, 475, 557, 559, 561, 562, 564, 566, 570, 574, 578 }

F grade: { 257, 258, 261, 262, 263, 627, 628, 629, 630, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 503, 504, 505, 506, 507, 508, 509, 510, 511, 513, 514, 516, 518, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 560, 563, 565, 569, 570, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 619, 620, 621, 622, 623, 624, 626, 631, 632, 633, 634, 640, 641, 642, 647, 648, 649, 650, 654, 655, 656, 657, 661, 662, 663, 664, 667, 668, 669, 670, 671, 672, 673, 674, 677, 678, 679, 682, 683, 684, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697 }

B grade: { 33, 38, 58, 65, 90, 91, 101, 102, 174, 196, 197, 198, 199, 201, 202, 203, 204, 205, 206, 207, 208, 221, 248, 250, 252, 263, 312, 335, 337, 338, 339, 340, 341, 363, 374, 381, 394, 406, 407, 425, 426, 501, 502, 512, 515, 517, 519, 528, 529, 539, 567, 568, 571, 601, 605, 617, 618, 625, 665, 666, 685, 686 }

C grade: { 466, 468, 470, 471, 472, 473, 475, 557, 559, 561, 562, 564, 566 }

F grade: { 627, 628, 629, 630, 635, 636, 637, 638, 639, 643, 644, 645, 646, 651, 652, 653, 658, 659, 660, 675, 676, 680, 681 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 136, 138, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 231, 232, 233, 234, 235, 236, 237, 238, 242, 243, 244, 245, 247, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 312, 313, 314, 315, 316, 317, 318, 319, 320, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 432, 433, 434, 435, 437, 438, 439, 440, 442, 443, 444, 445, 446, 448, 449, 450, 451, 453, 454, 455, 456, 457, 459, 461, 462, 464, 465, 467, 470, 472, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 498, 500, 502, 509, 511, 513, 515, 517, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 567, 568, 569, 571, 572, 573, 575, 576, 577, 579, 587, 593, 594, 598, 599, 600, 601, 609, 616, 622, 623, 647, 648, 667, 668, 672, 673, 677, 678, 682, 683, 687, 688, 691, 692, 693, 694 }

B grade: { 17, 33, 38, 57, 58, 65, 89, 90, 91, 101, 102, 131, 133, 135, 137, 139, 141, 156, 158, 174, 186, 196, 197, 198, 199, 201, 202, 203, 204, 228, 230, 239, 240, 241, 246, 248, 257, 258, 261, 262, 263, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 356, 358, 359, 374, 375, 376, 377, 394, 395, 396, 397, 398, 399, 425, 426, 427, 428, 429, 430, 431, 436, 441, 447, 452, 458, 463, 469, 493, 496, 497, 499, 501, 503, 504, 505, 506, 507, 508, 510, 512, 514, 516, 518, 519, 520, 521, 522, 584, 585, 586, 591, 592, 605, 606, 607, 608, 613, 614, 615, 620, 621, 631, 654, 655, 661, 662, 679, 689 }

C grade: { 460, 466, 468, 471, 473, 474, 475, 562, 563, 564, 565, 566, 570, 574, 578, 580, 581, 582, 583, 588, 589, 590, 595, 596, 597, 602, 603, 604, 610, 611, 612, 617, 618, 619, 624, 625, 626, 628, 629, 630, 636, 637, 638, 639, 645, 646, 651, 652, 653, 658, 659, 660, 665, 666, 670, 671, 675, 676, 680, 681, 684, 685, 686 }

F grade: { 296, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 627, 632, 633, 634, 635, 640, 641, 642, 643, 644, 649, 650, 656, 657, 663, 664, 669, 674, 690, 695, 696, 697 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 432, 433, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 464, 465, 467, 469, 471, 473, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 542, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 560, 562, 564, 566, 567, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 694 }

B grade: { 38, 65, 90, 101, 102, 196, 197, 312, 337, 339, 340, 341, 342, 356, 357, 358, 359, 373, 374, 375, 376, 377, 393, 394, 395, 396, 397, 398, 399, 423, 424, 425, 426, 427, 428, 429, 430, 431, 441, 452, 463, 528, 529, 540, 541, 543, 568, 572, 691, 692, 693 }

C grade: { }

F grade: { 466, 468, 470, 472, 474, 557, 559, 561, 563, 565, 580, 581, 582, 583, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 695, 696, 697 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 371, 372, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 390, 391, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 419, 420, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 498, 500, 501, 502, 503, 504, 505, 506, 507, 509, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 654, 655, 656, 657, 661, 662, 663, 664, 667, 668, 669, 672, 673, 674, 677, 678, 679, 682, 683, 684, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697 }

C grade: { }

F grade: { 352, 353, 369, 370, 373, 388, 389, 392, 393, 416, 417, 418, 421, 422, 423, 424, 455, 457, 466, 468, 477, 488, 497, 499, 508, 510, 546, 557, 581, 583, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 658, 659, 660, 665, 666, 670, 671, 675, 676, 680, 681, 685, 686 }

2.1.9 IntegrateAlgebraic

A grade: { 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 471, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 560, 562, 563, 564, 565, 566, 567, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690 }

B grade: { 394, 406, 425, 568 }

C grade: { 338, 466, 468, 470, 472, 539, 557, 559, 561, 579 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 339, 340, 341, 342, 343, 580, 581, 582, 583, 642, 657, 664, 691, 692, 693, 694, 695, 696, 697 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.005	0.001	0.000	1.315	1.045	0.060	1.094	0.029	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.005	0.001	0.000	1.353	0.832	0.059	0.915	0.021	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.005	0.001	0.000	1.344	0.772	0.058	1.114	0.020	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	14	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.82	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.005	0.001	0.000	1.336	0.992	0.059	1.026	0.021	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	0
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	0.00
time (sec)	N/A	0.002	0.000	0.000	1.344	0.987	0.056	0.943	0.016	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	14	11	10	14	11	0
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	1.08	0.85	0.00
time (sec)	N/A	0.004	0.001	0.004	1.345	1.171	0.083	1.024	0.021	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	13	5	10	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.00	0.00
time (sec)	N/A	0.004	0.001	0.004	1.333	1.058	0.079	0.984	0.023	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	14	17	10	20	11	0
N.S.	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	0.85	0.00
time (sec)	N/A	0.004	0.003	0.005	1.363	1.277	0.107	1.063	4.925	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	14	13	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87	0.00
time (sec)	N/A	0.005	0.002	0.006	1.287	0.795	0.115	1.107	0.026	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	14	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76	0.00
time (sec)	N/A	0.005	0.002	0.004	1.333	0.818	0.119	0.992	0.024	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	15	15	15	15	15	0
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.00
time (sec)	N/A	0.005	0.002	0.006	1.303	0.910	0.133	1.085	0.027	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	15	15	15	15	15	0
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.00
time (sec)	N/A	0.005	0.002	0.006	1.418	1.155	0.135	1.034	0.026	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	24	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.018	0.001	0.001	1.341	0.943	0.065	1.071	0.037	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.011	0.001	0.001	1.312	1.087	0.065	1.292	0.031	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	24	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.018	0.001	0.002	1.354	0.844	0.064	1.044	0.031	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.010	0.001	0.001	1.347	0.920	0.066	1.129	0.034	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	25	14	24	24	14	24	0
N.S.	1	1.00	1.00	1.56	0.88	1.50	1.50	0.88	1.50	0.00
time (sec)	N/A	0.002	0.002	0.000	1.349	0.486	0.065	1.052	0.031	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	22	21	21	0
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84	0.00
time (sec)	N/A	0.007	0.001	0.001	1.311	0.693	0.064	1.052	0.029	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	24	21	20	24	21	0
N.S.	1	1.00	1.00	0.96	1.04	0.91	0.87	1.04	0.91	0.00
time (sec)	N/A	0.013	0.001	0.003	1.393	0.875	0.098	1.067	0.033	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	22	25	19	22	22	0
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92	0.00
time (sec)	N/A	0.010	0.001	0.004	1.356	2.055	0.097	1.072	0.032	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	24	24	27	24	32	23	0
N.S.	1	1.00	1.00	0.89	0.89	1.00	0.89	1.19	0.85	0.00
time (sec)	N/A	0.014	0.001	0.007	1.387	1.559	0.131	1.139	4.937	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	22	26	22	22	24	0
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.04	0.00
time (sec)	N/A	0.009	0.001	0.006	1.319	0.779	0.139	0.877	0.026	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	26	28	24	34	24	0
N.S.	1	1.00	1.00	0.96	1.08	1.17	1.00	1.42	1.00	0.00
time (sec)	N/A	0.013	0.001	0.007	1.277	1.035	0.172	1.145	0.043	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	26	26	27	26	25	0
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89	0.00
time (sec)	N/A	0.010	0.001	0.004	1.304	1.032	0.175	1.050	0.034	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	30	25	24	24	26	24	26	0
N.S.	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37	0.00
time (sec)	N/A	0.003	0.001	0.006	1.376	1.189	0.187	1.107	0.034	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	26	26	27	26	26	0
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.00
time (sec)	N/A	0.010	0.001	0.004	1.381	0.794	0.197	1.060	0.036	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	26	26	27	26	26	0
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.00
time (sec)	N/A	0.014	0.001	0.006	1.379	1.132	0.209	1.086	0.035	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	26	26	27	26	26	0
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.00
time (sec)	N/A	0.010	0.001	0.004	1.337	1.281	0.220	1.048	0.035	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	37	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.00
time (sec)	N/A	0.028	0.002	0.000	1.362	1.221	0.069	1.167	0.042	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	37	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.00
time (sec)	N/A	0.027	0.002	0.001	1.315	0.984	0.070	1.180	0.040	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	39	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81	0.00
time (sec)	N/A	0.025	0.002	0.001	1.351	0.466	0.068	0.967	0.044	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	43	36	35	35	37	35	35	0
N.S.	1	1.00	1.26	1.06	1.03	1.03	1.09	1.03	1.03	0.00
time (sec)	N/A	0.032	0.002	0.002	1.372	0.949	0.069	0.951	0.043	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	36	14	35	37	14	35	0
N.S.	1	1.00	1.00	2.25	0.88	2.19	2.31	0.88	2.19	0.00
time (sec)	N/A	0.002	0.002	0.000	1.335	0.887	0.068	0.995	0.057	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	34	36	33	37	36	33	0
N.S.	1	1.00	1.00	0.87	0.92	0.85	0.95	0.92	0.85	0.00
time (sec)	N/A	0.018	0.004	0.002	1.307	0.758	0.109	0.961	0.036	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	35	36	38	37	46	34	0
N.S.	1	1.00	1.00	0.88	0.90	0.95	0.92	1.15	0.85	0.00
time (sec)	N/A	0.021	0.007	0.008	1.379	0.931	0.140	1.078	0.036	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	35	37	39	37	46	37	0
N.S.	1	1.00	1.00	0.88	0.92	0.98	0.92	1.15	0.92	0.00
time (sec)	N/A	0.019	0.005	0.007	1.358	1.104	0.190	1.144	4.902	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	34	39	39	37	47	36	0
N.S.	1	1.00	1.00	0.87	1.00	1.00	0.95	1.21	0.92	0.00
time (sec)	N/A	0.021	0.004	0.007	1.286	1.016	0.241	1.054	0.045	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	43	36	35	35	37	35	37	0
N.S.	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95	0.00
time (sec)	N/A	0.003	0.007	0.004	1.320	1.739	0.266	1.042	0.027	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	43	36	37	37	39	37	37	0
N.S.	1	1.00	1.08	0.90	0.92	0.92	0.98	0.92	0.92	0.00
time (sec)	N/A	0.019	0.004	0.005	1.368	0.991	0.292	1.017	0.057	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	37	37	39	37	37	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.019	0.004	0.005	1.286	0.780	0.319	0.835	0.047	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	37	37	39	37	37	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.019	0.007	0.006	1.322	0.711	0.346	1.041	0.031	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	37	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.00
time (sec)	N/A	0.014	0.002	0.001	1.346	0.836	0.068	0.933	0.041	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	37	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.00
time (sec)	N/A	0.013	0.002	0.000	1.375	0.962	0.068	1.026	0.040	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	39	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81	0.00
time (sec)	N/A	0.013	0.002	0.000	1.322	1.007	0.067	1.027	0.041	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	32	31	31	32	31	31	0
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89	0.00
time (sec)	N/A	0.011	0.001	0.000	1.360	1.020	0.065	0.722	0.038	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	33	32	36	29	32	32	0
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.85	0.94	0.94	0.00
time (sec)	N/A	0.013	0.004	0.003	1.383	0.731	0.105	1.042	0.041	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	34	34	36	36	34	36	0
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.97	0.92	0.97	0.00
time (sec)	N/A	0.013	0.004	0.006	1.336	1.243	0.148	1.087	4.802	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	33	33	37	34	33	34	0
N.S.	1	1.00	1.00	0.97	0.97	1.09	1.00	0.97	1.00	0.00
time (sec)	N/A	0.013	0.005	0.005	1.343	1.330	0.197	1.078	0.029	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	36	37	37	39	37	35	0
N.S.	1	1.00	1.00	0.92	0.95	0.95	1.00	0.95	0.90	0.00
time (sec)	N/A	0.013	0.004	0.005	1.247	1.278	0.256	1.086	0.027	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	37	37	39	37	37	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.013	0.004	0.006	1.487	1.198	0.273	1.196	0.032	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	37	37	39	37	37	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.015	0.006	0.005	1.424	1.004	0.303	1.027	0.029	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	65	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83	0.00
time (sec)	N/A	0.049	0.003	0.001	1.432	0.427	0.078	0.873	0.026	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	65	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83	0.00
time (sec)	N/A	0.044	0.002	0.000	1.359	1.134	0.078	1.011	0.023	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	66	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83	0.00
time (sec)	N/A	0.043	0.002	0.001	1.375	0.921	0.080	1.157	0.024	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	69	58	57	57	65	57	57	0
N.S.	1	1.00	0.96	0.81	0.79	0.79	0.90	0.79	0.79	0.00
time (sec)	N/A	0.091	0.002	0.000	1.369	0.943	0.093	1.021	0.024	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	66	57	56	56	63	56	56	0
N.S.	1	1.00	1.25	1.08	1.06	1.06	1.19	1.06	1.06	0.00
time (sec)	N/A	0.065	0.002	0.002	1.269	1.042	0.077	1.056	0.024	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	66	57	56	56	63	56	56	0
N.S.	1	1.00	1.94	1.68	1.65	1.65	1.85	1.65	1.65	0.00
time (sec)	N/A	0.036	0.002	0.002	1.363	0.813	0.077	1.185	0.025	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	58	14	57	65	14	57	0
N.S.	1	1.00	1.00	3.62	0.88	3.56	4.06	0.88	3.56	0.00
time (sec)	N/A	0.002	0.002	0.002	1.399	0.789	0.077	1.057	0.023	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	56	58	55	65	58	55	0
N.S.	1	1.00	1.00	0.86	0.89	0.85	1.00	0.89	0.85	0.00
time (sec)	N/A	0.034	0.004	0.003	1.373	0.898	0.136	0.923	0.028	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	64	57	58	61	63	68	56	0
N.S.	1	1.00	1.00	0.89	0.91	0.95	0.98	1.06	0.88	0.00
time (sec)	N/A	0.037	0.005	0.005	1.369	0.928	0.169	1.056	0.030	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	64	57	59	61	63	70	59	0
N.S.	1	1.00	1.00	0.89	0.92	0.95	0.98	1.09	0.92	0.00
time (sec)	N/A	0.036	0.007	0.005	1.329	1.320	0.223	0.881	0.031	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	64	57	61	61	65	72	59	0
N.S.	1	1.00	1.00	0.89	0.95	0.95	1.02	1.12	0.92	0.00
time (sec)	N/A	0.034	0.005	0.006	1.398	1.147	0.283	1.189	0.040	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	64	57	61	61	63	70	59	0
N.S.	1	1.00	1.00	0.89	0.95	0.95	0.98	1.09	0.92	0.00
time (sec)	N/A	0.033	0.005	0.007	1.410	0.844	0.362	1.049	0.042	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	56	61	61	61	69	58	0
N.S.	1	1.00	1.00	0.86	0.94	0.94	0.94	1.06	0.89	0.00
time (sec)	N/A	0.031	0.005	0.007	1.351	1.084	0.440	1.060	4.772	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	69	58	57	57	61	57	59	0
N.S.	1	1.00	3.63	3.05	3.00	3.00	3.21	3.00	3.11	0.00
time (sec)	N/A	0.004	0.004	0.007	1.370	0.604	0.474	1.069	4.751	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	67	58	59	59	63	59	58	0
N.S.	1	1.00	1.68	1.45	1.48	1.48	1.58	1.48	1.45	0.00
time (sec)	N/A	0.018	0.006	0.004	1.344	0.881	0.506	1.064	4.736	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	67	58	59	59	63	59	58	0
N.S.	1	1.00	1.08	0.94	0.95	0.95	1.02	0.95	0.94	0.00
time (sec)	N/A	0.028	0.004	0.007	1.376	1.013	0.550	0.979	0.041	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	59	59	63	59	59	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.032	0.004	0.006	1.356	1.123	0.591	0.945	0.040	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	59	59	63	59	59	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.031	0.004	0.005	1.346	0.667	0.628	1.018	0.041	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	66	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83	0.00
time (sec)	N/A	0.025	0.002	0.001	1.320	0.499	0.077	1.045	0.024	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	65	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83	0.00
time (sec)	N/A	0.024	0.002	0.002	1.310	1.123	0.077	0.983	0.024	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	66	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83	0.00
time (sec)	N/A	0.022	0.002	0.001	1.311	0.905	0.076	1.008	0.025	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	66	57	56	56	63	56	56	0
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.85	0.00
time (sec)	N/A	0.022	0.002	0.002	1.382	0.835	0.076	1.094	0.024	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	62	55	54	54	61	54	54	0
N.S.	1	1.00	1.00	0.89	0.87	0.87	0.98	0.87	0.87	0.00
time (sec)	N/A	0.019	0.001	0.001	1.380	0.814	0.072	1.144	0.022	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	61	56	55	59	58	55	55	0
N.S.	1	1.00	1.00	0.92	0.90	0.97	0.95	0.90	0.90	0.00
time (sec)	N/A	0.021	0.004	0.005	1.388	0.712	0.132	1.126	0.026	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	55	55	59	60	55	57	0
N.S.	1	1.00	1.00	0.92	0.92	0.98	1.00	0.92	0.95	0.00
time (sec)	N/A	0.022	0.004	0.005	1.364	1.075	0.174	1.133	0.025	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	56	58	59	63	58	58	0
N.S.	1	1.00	1.00	0.89	0.92	0.94	1.00	0.92	0.92	0.00
time (sec)	N/A	0.022	0.004	0.005	1.378	0.996	0.227	0.938	0.047	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	61	56	58	59	61	58	59	0
N.S.	1	1.00	1.00	0.92	0.95	0.97	1.00	0.95	0.97	0.00
time (sec)	N/A	0.022	0.004	0.007	1.435	1.037	0.294	1.118	4.789	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	55	57	59	60	57	57	0
N.S.	1	1.00	1.00	0.92	0.95	0.98	1.00	0.95	0.95	0.00
time (sec)	N/A	0.022	0.005	0.008	1.443	1.060	0.362	1.035	0.039	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	58	59	59	63	59	58	0
N.S.	1	1.00	1.00	0.89	0.91	0.91	0.97	0.91	0.89	0.00
time (sec)	N/A	0.022	0.004	0.006	1.362	0.728	0.431	1.039	0.038	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	67	58	59	59	63	59	58	0
N.S.	1	1.00	1.00	0.87	0.88	0.88	0.94	0.88	0.87	0.00
time (sec)	N/A	0.023	0.004	0.006	1.359	0.866	0.469	1.091	0.038	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	59	59	63	59	59	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.023	0.004	0.006	1.368	0.659	0.505	1.080	4.747	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	59	59	63	59	59	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.022	0.004	0.004	1.324	1.098	0.536	1.099	0.040	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	59	59	63	59	59	0
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.00
time (sec)	N/A	0.023	0.006	0.007	1.394	1.076	0.567	1.129	0.040	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	108	91	90	90	105	90	90	0
N.S.	1	1.00	0.84	0.71	0.70	0.70	0.81	0.70	0.70	0.00
time (sec)	N/A	0.210	0.003	0.002	1.296	0.710	0.088	1.055	0.103	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	108	91	90	90	107	90	90	0
N.S.	1	1.00	0.98	0.83	0.82	0.82	0.97	0.82	0.82	0.00
time (sec)	N/A	0.171	0.003	0.001	1.345	1.079	0.088	1.005	4.566	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	106	91	90	90	104	90	90	0
N.S.	1	1.00	1.16	1.00	0.99	0.99	1.14	0.99	0.99	0.00
time (sec)	N/A	0.142	0.003	0.001	1.283	0.926	0.087	0.996	4.593	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	106	91	90	90	105	90	90	0
N.S.	1	1.00	1.47	1.26	1.25	1.25	1.46	1.25	1.25	0.00
time (sec)	N/A	0.116	0.003	0.001	1.289	0.875	0.087	1.079	0.090	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	103	90	89	89	102	89	89	0
N.S.	1	1.00	1.94	1.70	1.68	1.68	1.92	1.68	1.68	0.00
time (sec)	N/A	0.084	0.003	0.001	1.354	0.984	0.087	1.127	0.092	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	106	91	90	90	105	90	90	0
N.S.	1	1.00	3.12	2.68	2.65	2.65	3.09	2.65	2.65	0.00
time (sec)	N/A	0.046	0.003	0.000	1.433	0.940	0.085	1.183	0.092	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	91	14	90	99	14	14	0
N.S.	1	1.00	1.00	5.69	0.88	5.62	6.19	0.88	0.88	0.00
time (sec)	N/A	0.002	0.002	0.001	1.276	1.147	0.084	1.124	4.612	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	100	89	91	88	102	91	88	0
N.S.	1	1.00	1.00	0.89	0.91	0.88	1.02	0.91	0.88	0.00
time (sec)	N/A	0.059	0.004	0.003	1.299	1.102	0.183	0.900	4.618	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	99	90	91	94	100	101	89	0
N.S.	1	1.00	1.00	0.91	0.92	0.95	1.01	1.02	0.90	0.00
time (sec)	N/A	0.060	0.005	0.007	1.378	1.174	0.217	1.059	0.059	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	101	90	92	94	104	103	92	0
N.S.	1	1.00	1.00	0.89	0.91	0.93	1.03	1.02	0.91	0.00
time (sec)	N/A	0.061	0.005	0.007	1.325	1.393	0.271	1.121	0.055	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	94	89	91	94	97	102	91	0
N.S.	1	1.00	1.00	0.95	0.97	1.00	1.03	1.09	0.97	0.00
time (sec)	N/A	0.057	0.005	0.007	1.345	1.295	0.334	1.051	0.053	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	90	94	94	100	105	92	0
N.S.	1	1.00	1.00	0.93	0.97	0.97	1.03	1.08	0.95	0.00
time (sec)	N/A	0.057	0.005	0.008	1.386	1.444	0.416	0.997	0.052	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	95	90	94	94	99	105	91	0
N.S.	1	1.00	1.00	0.95	0.99	0.99	1.04	1.11	0.96	0.00
time (sec)	N/A	0.058	0.005	0.007	1.330	1.378	0.507	1.128	5.110	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	101	90	94	94	99	105	92	0
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.98	1.04	0.91	0.00
time (sec)	N/A	0.054	0.005	0.008	1.380	0.814	0.618	1.215	0.061	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	99	90	94	94	99	103	94	0
N.S.	1	1.00	1.00	0.91	0.95	0.95	1.00	1.04	0.95	0.00
time (sec)	N/A	0.051	0.005	0.009	1.348	1.537	0.733	1.076	5.148	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	100	89	94	94	97	102	91	0
N.S.	1	1.00	1.00	0.89	0.94	0.94	0.97	1.02	0.91	0.00
time (sec)	N/A	0.054	0.005	0.009	1.360	0.898	0.833	1.056	5.089	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	100	91	90	90	97	90	92	0
N.S.	1	1.00	5.26	4.79	4.74	4.74	5.11	4.74	4.84	0.00
time (sec)	N/A	0.003	0.005	0.006	1.324	0.773	0.896	1.068	0.076	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	106	91	92	92	99	92	92	0
N.S.	1	1.00	2.65	2.28	2.30	2.30	2.48	2.30	2.30	0.00
time (sec)	N/A	0.017	0.004	0.007	1.347	0.645	0.937	1.002	0.077	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	104	91	92	92	99	92	91	0
N.S.	1	1.00	1.68	1.47	1.48	1.48	1.60	1.48	1.47	0.00
time (sec)	N/A	0.029	0.004	0.007	1.363	1.079	0.992	1.061	0.079	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	106	91	92	92	99	92	92	0
N.S.	1	1.00	1.26	1.08	1.10	1.10	1.18	1.10	1.10	0.00
time (sec)	N/A	0.041	0.004	0.006	1.394	1.132	1.078	1.202	4.967	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	106	91	92	92	99	92	92	0
N.S.	1	1.00	1.00	0.86	0.87	0.87	0.93	0.87	0.87	0.00
time (sec)	N/A	0.055	0.005	0.007	1.304	1.117	1.141	1.057	0.080	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	108	91	92	92	99	92	92	0
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.92	0.85	0.85	0.00
time (sec)	N/A	0.054	0.004	0.006	1.415	0.597	1.205	1.051	4.893	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	108	91	92	92	99	92	92	0
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.92	0.85	0.85	0.00
time (sec)	N/A	0.052	0.004	0.006	1.314	0.592	1.281	1.219	4.862	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	106	91	92	92	99	92	91	0
N.S.	1	1.00	1.00	0.86	0.87	0.87	0.93	0.87	0.86	0.00
time (sec)	N/A	0.052	0.004	0.007	1.337	0.774	1.336	1.053	0.084	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	108	91	90	90	107	90	90	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83	0.00
time (sec)	N/A	0.048	0.003	0.003	1.362	0.550	0.088	1.096	4.945	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	108	91	90	90	107	90	90	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83	0.00
time (sec)	N/A	0.039	0.003	0.002	1.324	0.531	0.087	1.140	0.097	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	108	91	90	90	107	90	90	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83	0.00
time (sec)	N/A	0.039	0.003	0.001	1.335	0.527	0.086	1.097	0.096	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	106	91	90	90	105	90	90	0
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.85	0.00
time (sec)	N/A	0.040	0.003	0.002	1.372	0.407	0.085	0.912	4.970	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	101	88	87	87	102	87	87	0
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.01	0.86	0.86	0.00
time (sec)	N/A	0.037	0.001	0.000	1.304	0.773	0.083	1.178	0.053	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	100	89	88	92	99	88	88	0
N.S.	1	1.00	1.00	0.89	0.88	0.92	0.99	0.88	0.88	0.00
time (sec)	N/A	0.039	0.017	0.004	1.301	1.002	0.179	1.071	0.057	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	98	89	89	92	100	89	91	0
N.S.	1	1.00	1.00	0.91	0.91	0.94	1.02	0.91	0.93	0.00
time (sec)	N/A	0.038	0.008	0.006	1.382	1.025	0.223	1.099	0.053	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	100	89	91	92	102	91	91	0
N.S.	1	1.00	1.00	0.89	0.91	0.92	1.02	0.91	0.91	0.00
time (sec)	N/A	0.039	0.012	0.006	1.213	1.114	0.275	0.974	4.959	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	102	89	91	92	102	91	91	0
N.S.	1	1.00	1.00	0.87	0.89	0.90	1.00	0.89	0.89	0.00
time (sec)	N/A	0.037	0.006	0.005	1.356	1.140	0.333	0.934	4.798	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	102	89	91	92	100	91	91	0
N.S.	1	1.00	1.00	0.87	0.89	0.90	0.98	0.89	0.89	0.00
time (sec)	N/A	0.039	0.012	0.007	1.360	0.959	0.417	0.939	0.047	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	100	89	91	92	99	91	91	0
N.S.	1	1.00	1.00	0.89	0.91	0.92	0.99	0.91	0.91	0.00
time (sec)	N/A	0.040	0.014	0.007	1.396	1.157	0.499	0.925	4.578	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	98	89	91	92	97	91	92	0
N.S.	1	1.00	1.00	0.91	0.93	0.94	0.99	0.93	0.94	0.00
time (sec)	N/A	0.038	0.013	0.007	1.331	1.291	0.571	1.105	0.074	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	99	88	90	92	95	90	90	0
N.S.	1	1.00	1.00	0.89	0.91	0.93	0.96	0.91	0.91	0.00
time (sec)	N/A	0.039	0.006	0.008	1.407	1.023	0.694	0.875	4.525	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	104	91	92	92	99	92	91	0
N.S.	1	1.00	1.00	0.88	0.88	0.88	0.95	0.88	0.88	0.00
time (sec)	N/A	0.039	0.011	0.006	1.360	1.075	0.788	0.817	0.074	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	106	91	92	92	99	92	92	0
N.S.	1	1.00	1.00	0.86	0.87	0.87	0.93	0.87	0.87	0.00
time (sec)	N/A	0.040	0.014	0.006	1.360	1.099	0.836	1.137	0.076	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	79	68	68	67	68	69	67	0
N.S.	1	1.00	1.00	0.86	0.86	0.85	0.86	0.87	0.85	0.00
time (sec)	N/A	0.056	0.006	0.005	1.358	1.164	0.175	0.988	0.064	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	81	71	72	170	119	77	65	0
N.S.	1	1.00	1.00	0.88	0.89	2.10	1.47	0.95	0.80	0.00
time (sec)	N/A	0.035	0.035	0.005	2.921	1.004	0.200	1.078	0.056	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	66	57	57	56	56	58	56	0
N.S.	1	1.00	1.00	0.86	0.86	0.85	0.85	0.88	0.85	0.00
time (sec)	N/A	0.044	0.006	0.005	1.347	0.978	0.166	1.075	0.083	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	60	60	148	107	65	54	0
N.S.	1	1.00	1.00	0.88	0.88	2.18	1.57	0.96	0.79	0.00
time (sec)	N/A	0.030	0.026	0.004	2.936	0.970	0.191	1.099	0.052	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	53	46	46	45	44	47	45	0
N.S.	1	1.00	1.00	0.87	0.87	0.85	0.83	0.89	0.85	0.00
time (sec)	N/A	0.035	0.005	0.003	1.363	0.929	0.157	1.048	4.727	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	49	50	126	95	55	43	0
N.S.	1	1.00	1.00	0.89	0.91	2.29	1.73	1.00	0.78	0.00
time (sec)	N/A	0.024	0.023	0.003	2.839	1.257	0.180	0.995	0.070	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	35	34	33	32	35	33	0
N.S.	1	1.00	1.00	0.88	0.85	0.82	0.80	0.88	0.82	0.00
time (sec)	N/A	0.027	0.005	0.002	1.307	0.921	0.150	1.121	4.646	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	38	37	99	80	40	32	0
N.S.	1	1.00	1.00	0.90	0.88	2.36	1.90	0.95	0.76	0.00
time (sec)	N/A	0.020	0.019	0.004	2.803	0.668	0.168	1.012	0.069	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	24	23	22	20	24	22	0
N.S.	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81	0.00
time (sec)	N/A	0.018	0.004	0.003	1.323	0.956	0.135	1.091	0.037	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	27	26	82	56	26	23	0
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74	0.00
time (sec)	N/A	0.012	0.008	0.002	2.972	0.831	0.146	1.119	0.034	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	10	14	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.00
time (sec)	N/A	0.003	0.002	0.001	1.338	0.865	0.108	1.068	4.632	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	16	15	67	53	15	16	0
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67	0.00
time (sec)	N/A	0.005	0.004	0.003	2.903	0.895	0.134	0.974	4.695	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	23	18	15	24	18	0
N.S.	1	1.00	1.00	0.95	1.05	0.82	0.68	1.09	0.82	0.00
time (sec)	N/A	0.011	0.005	0.005	1.332	1.371	0.200	1.093	0.081	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	30	29	82	65	29	26	0
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76	0.00
time (sec)	N/A	0.012	0.012	0.006	2.891	1.256	0.178	1.126	4.620	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	32	33	33	31	43	31	0
N.S.	1	1.00	1.00	0.91	0.94	0.94	0.89	1.23	0.89	0.00
time (sec)	N/A	0.022	0.007	0.006	1.371	1.238	0.263	1.148	0.074	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	39	40	106	87	40	37	0
N.S.	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86	0.00
time (sec)	N/A	0.017	0.020	0.007	2.884	0.985	0.218	1.119	4.672	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	44	47	45	42	57	46	0
N.S.	1	1.00	1.00	0.90	0.96	0.92	0.86	1.16	0.94	0.00
time (sec)	N/A	0.028	0.007	0.007	1.359	1.037	0.304	1.192	0.078	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	58	52	52	132	100	52	48	0
N.S.	1	1.00	1.00	0.90	0.90	2.28	1.72	0.90	0.83	0.00
time (sec)	N/A	0.025	0.023	0.008	2.886	0.911	0.257	0.617	0.063	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	56	58	58	56	70	58	0
N.S.	1	1.00	1.00	0.89	0.92	0.92	0.89	1.11	0.92	0.00
time (sec)	N/A	0.035	0.007	0.008	1.358	1.086	0.347	0.628	4.645	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	61	62	154	112	62	59	0
N.S.	1	1.00	1.00	0.88	0.90	2.23	1.62	0.90	0.86	0.00
time (sec)	N/A	0.035	0.025	0.007	2.925	1.130	0.296	0.637	0.065	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	75	66	69	69	68	81	68	0
N.S.	1	1.00	1.00	0.88	0.92	0.92	0.91	1.08	0.91	0.00
time (sec)	N/A	0.041	0.007	0.007	1.331	0.742	0.386	0.626	4.667	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	83	85	88	104	88	103	90	0
N.S.	1	1.00	0.88	0.90	0.94	1.11	0.94	1.10	0.96	0.00
time (sec)	N/A	0.076	0.032	0.011	1.335	0.849	0.313	0.606	0.085	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	93	90	93	234	151	95	88	0
N.S.	1	1.00	0.89	0.86	0.89	2.23	1.44	0.90	0.84	0.00
time (sec)	N/A	0.046	0.057	0.008	2.916	0.970	0.340	0.645	0.071	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	72	74	77	93	80	92	79	0
N.S.	1	1.00	0.87	0.89	0.93	1.12	0.96	1.11	0.95	0.00
time (sec)	N/A	0.066	0.025	0.010	1.360	0.986	0.295	0.626	4.485	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	82	78	82	212	134	84	77	0
N.S.	1	1.00	0.89	0.85	0.89	2.30	1.46	0.91	0.84	0.00
time (sec)	N/A	0.037	0.057	0.010	3.009	0.780	0.325	0.642	4.555	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	60	63	65	81	66	80	68	0
N.S.	1	1.00	0.86	0.90	0.93	1.16	0.94	1.14	0.97	0.00
time (sec)	N/A	0.054	0.025	0.011	1.346	1.305	0.274	0.654	0.069	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	71	68	71	190	124	73	66	0
N.S.	1	1.00	0.90	0.86	0.90	2.41	1.57	0.92	0.84	0.00
time (sec)	N/A	0.032	0.053	0.009	2.967	1.431	0.308	0.631	4.586	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	49	52	54	70	53	67	57	0
N.S.	1	1.00	0.86	0.91	0.95	1.23	0.93	1.18	1.00	0.00
time (sec)	N/A	0.042	0.020	0.010	1.373	0.677	0.260	0.619	0.077	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	60	57	59	164	107	61	56	0
N.S.	1	1.00	0.91	0.86	0.89	2.48	1.62	0.92	0.85	0.00
time (sec)	N/A	0.027	0.043	0.009	2.972	0.996	0.287	0.639	0.089	0.001

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	38	41	43	56	39	49	45	0
N.S.	1	1.00	0.86	0.93	0.98	1.27	0.89	1.11	1.02	0.00
time (sec)	N/A	0.032	0.023	0.010	1.385	0.880	0.233	0.618	0.046	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	51	43	45	136	83	42	43	0
N.S.	1	1.00	0.93	0.78	0.82	2.47	1.51	0.76	0.78	0.00
time (sec)	N/A	0.017	0.034	0.010	2.944	1.007	0.255	0.629	4.591	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	27	30	32	35	29	48	29	0
N.S.	1	1.00	0.82	0.91	0.97	1.06	0.88	1.45	0.88	0.00
time (sec)	N/A	0.024	0.008	0.009	1.318	0.544	0.193	0.639	0.046	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	36	36	120	78	35	33	0
N.S.	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	0.73	0.00
time (sec)	N/A	0.012	0.021	0.007	2.964	1.454	0.209	0.638	4.756	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	15	15	14	14	0
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88	0.00
time (sec)	N/A	0.003	0.002	0.000	1.298	0.920	0.158	0.642	0.031	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	36	35	120	78	35	33	0
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73	0.00
time (sec)	N/A	0.010	0.023	0.005	2.962	0.755	0.211	0.653	4.740	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	33	35	37	47	34	47	34	0
N.S.	1	1.00	0.87	0.92	0.97	1.24	0.89	1.24	0.89	0.00
time (sec)	N/A	0.027	0.015	0.010	1.391	1.045	0.295	0.632	4.701	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	54	46	49	136	92	47	44	0
N.S.	1	1.00	0.95	0.81	0.86	2.39	1.61	0.82	0.77	0.00
time (sec)	N/A	0.017	0.037	0.010	2.968	0.890	0.290	0.646	0.073	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	41	46	52	73	51	51	51	0
N.S.	1	1.00	0.84	0.94	1.06	1.49	1.04	1.04	1.04	0.00
time (sec)	N/A	0.036	0.040	0.011	1.366	1.045	0.363	0.603	0.078	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	67	59	64	172	114	59	58	0
N.S.	1	1.00	0.99	0.87	0.94	2.53	1.68	0.87	0.85	0.00
time (sec)	N/A	0.026	0.043	0.011	3.014	1.093	0.347	0.614	4.729	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	57	61	70	90	68	86	67	0
N.S.	1	1.00	0.86	0.92	1.06	1.36	1.03	1.30	1.02	0.00
time (sec)	N/A	0.044	0.052	0.014	1.320	1.084	0.441	0.633	4.804	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	80	70	75	198	126	70	70	0
N.S.	1	1.00	0.99	0.86	0.93	2.44	1.56	0.86	0.86	0.00
time (sec)	N/A	0.033	0.044	0.012	2.914	0.886	0.397	0.647	4.850	0.001

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	68	73	79	99	78	99	78	0
N.S.	1	1.00	0.85	0.91	0.99	1.24	0.98	1.24	0.98	0.00
time (sec)	N/A	0.054	0.057	0.014	1.320	0.590	0.467	0.639	0.119	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	91	81	86	220	138	81	80	0
N.S.	1	1.00	0.97	0.86	0.91	2.34	1.47	0.86	0.85	0.00
time (sec)	N/A	0.043	0.056	0.014	2.974	0.639	0.446	0.620	4.560	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	79	84	92	112	94	110	89	0
N.S.	1	1.00	0.85	0.90	0.99	1.20	1.01	1.18	0.96	0.00
time (sec)	N/A	0.065	0.076	0.013	1.305	0.752	0.523	0.628	4.731	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	97	101	111	137	119	114	111	0
N.S.	1	1.00	0.85	0.89	0.97	1.20	1.04	1.00	0.97	0.00
time (sec)	N/A	0.103	0.030	0.011	1.363	0.445	0.482	0.628	4.725	0.001

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	85	91	99	125	104	102	100	0
N.S.	1	1.00	0.85	0.91	0.99	1.25	1.04	1.02	1.00	0.00
time (sec)	N/A	0.082	0.028	0.012	1.293	0.948	0.466	0.615	0.077	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	75	80	89	115	92	92	90	0
N.S.	1	1.00	0.86	0.92	1.02	1.32	1.06	1.06	1.03	0.00
time (sec)	N/A	0.072	0.025	0.010	1.344	0.804	0.436	0.625	4.493	0.001

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	63	69	77	103	78	80	78	0
N.S.	1	1.00	0.85	0.93	1.04	1.39	1.05	1.08	1.05	0.00
time (sec)	N/A	0.058	0.021	0.011	1.340	1.037	0.416	0.609	0.083	0.001

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	48	58	66	91	68	62	68	0
N.S.	1	1.00	0.74	0.89	1.02	1.40	1.05	0.95	1.05	0.00
time (sec)	N/A	0.046	0.057	0.010	1.279	0.840	0.380	0.661	4.749	0.001

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	39	46	55	69	53	42	52	0
N.S.	1	1.00	0.80	0.94	1.12	1.41	1.08	0.86	1.06	0.00
time (sec)	N/A	0.038	0.018	0.008	1.299	0.729	0.319	0.623	0.059	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	24	31	36	36	36	22	37	0
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95	0.00
time (sec)	N/A	0.004	0.007	0.007	1.346	0.852	0.265	0.618	0.032	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	26	27	14	28	0
N.S.	1	1.00	1.00	0.94	0.88	1.62	1.69	0.88	1.75	0.00
time (sec)	N/A	0.003	0.002	0.002	1.326	0.688	0.235	0.627	4.623	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	43	49	60	90	56	59	56	0
N.S.	1	1.00	0.80	0.91	1.11	1.67	1.04	1.09	1.04	0.00
time (sec)	N/A	0.038	0.038	0.012	1.325	0.880	0.407	0.624	4.675	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	59	62	77	119	80	82	75	0
N.S.	1	1.00	0.88	0.93	1.15	1.78	1.19	1.22	1.12	0.00
time (sec)	N/A	0.047	0.059	0.013	1.339	0.791	0.508	0.619	0.080	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	74	79	92	134	90	80	88	0
N.S.	1	1.00	0.86	0.92	1.07	1.56	1.05	0.93	1.02	0.00
time (sec)	N/A	0.060	0.051	0.015	1.424	0.549	0.546	0.646	4.672	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	85	90	103	145	104	110	101	0
N.S.	1	1.00	0.89	0.95	1.08	1.53	1.09	1.16	1.06	0.00
time (sec)	N/A	0.068	0.071	0.013	1.362	0.730	0.594	0.654	4.692	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	96	101	114	156	116	119	111	0
N.S.	1	1.00	0.86	0.90	1.02	1.39	1.04	1.06	0.99	0.00
time (sec)	N/A	0.080	0.061	0.014	1.337	1.185	0.653	0.634	4.855	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	99	99	105	278	162	96	99	0
N.S.	1	1.00	0.89	0.89	0.95	2.50	1.46	0.86	0.89	0.00
time (sec)	N/A	0.048	0.062	0.010	2.954	0.715	0.489	0.613	0.076	0.001

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	88	88	93	256	144	84	87	0
N.S.	1	1.00	0.90	0.90	0.95	2.61	1.47	0.86	0.89	0.00
time (sec)	N/A	0.041	0.049	0.010	2.858	0.926	0.465	0.641	0.070	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	77	77	82	230	133	73	77	0
N.S.	1	1.00	0.91	0.91	0.96	2.71	1.56	0.86	0.91	0.00
time (sec)	N/A	0.035	0.046	0.011	2.970	0.879	0.437	0.630	4.725	0.001

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	66	63	68	202	107	54	64	0
N.S.	1	1.00	0.89	0.85	0.92	2.73	1.45	0.73	0.86	0.00
time (sec)	N/A	0.025	0.049	0.010	2.900	1.023	0.398	0.636	4.746	0.001

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	55	47	59	188	110	45	56	0
N.S.	1	1.00	0.86	0.73	0.92	2.94	1.72	0.70	0.88	0.00
time (sec)	N/A	0.020	0.041	0.008	2.996	0.861	0.332	0.598	4.771	0.001

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	58	49	62	190	110	50	55	0
N.S.	1	1.00	0.89	0.75	0.95	2.92	1.69	0.77	0.85	0.00
time (sec)	N/A	0.018	0.029	0.008	3.039	0.746	0.306	0.615	4.738	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	55	51	58	188	105	45	55	0
N.S.	1	1.00	0.89	0.82	0.94	3.03	1.69	0.73	0.89	0.00
time (sec)	N/A	0.016	0.038	0.005	3.026	0.807	0.319	0.633	4.656	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	68	66	71	202	116	57	66	0
N.S.	1	1.00	0.89	0.87	0.93	2.66	1.53	0.75	0.87	0.00
time (sec)	N/A	0.026	0.040	0.012	2.981	0.812	0.418	0.637	4.673	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	79	79	86	238	138	71	80	0
N.S.	1	1.00	0.91	0.91	0.99	2.74	1.59	0.82	0.92	0.00
time (sec)	N/A	0.034	0.050	0.015	2.956	0.807	0.476	0.640	4.672	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	90	89	97	264	150	80	92	0
N.S.	1	1.00	0.90	0.89	0.97	2.64	1.50	0.80	0.92	0.00
time (sec)	N/A	0.042	0.054	0.014	2.911	0.807	0.529	0.638	5.021	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	101	101	108	286	162	93	102	0
N.S.	1	1.00	0.89	0.89	0.96	2.53	1.43	0.82	0.90	0.00
time (sec)	N/A	0.056	0.056	0.014	3.024	0.886	0.579	0.607	4.985	0.001

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	216	216	169	199	242	346	260	168	242	0
N.S.	1	1.00	0.78	0.92	1.12	1.60	1.20	0.78	1.12	0.00
time (sec)	N/A	0.255	0.048	0.018	1.615	0.655	2.151	0.635	5.295	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	158	188	231	335	245	157	230	0
N.S.	1	1.00	0.77	0.92	1.13	1.63	1.20	0.77	1.12	0.00
time (sec)	N/A	0.214	0.030	0.018	1.578	0.886	2.060	0.634	0.401	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	145	177	220	322	233	139	220	0
N.S.	1	1.00	0.77	0.94	1.17	1.71	1.24	0.74	1.17	0.00
time (sec)	N/A	0.189	0.039	0.019	1.595	0.706	2.025	0.634	0.425	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	116	166	209	300	219	119	207	0
N.S.	1	1.00	0.65	0.93	1.17	1.68	1.22	0.66	1.16	0.00
time (sec)	N/A	0.169	0.033	0.012	1.525	1.301	1.835	0.615	5.362	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	101	150	190	190	202	99	192	0
N.S.	1	1.00	5.32	7.89	10.00	10.00	10.63	5.21	10.11	0.00
time (sec)	N/A	0.003	0.021	0.010	1.496	1.350	1.699	0.640	0.115	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	90	133	179	179	190	88	181	0
N.S.	1	1.00	2.31	3.41	4.59	4.59	4.87	2.26	4.64	0.00
time (sec)	N/A	0.017	0.017	0.009	1.468	0.857	1.419	0.627	4.886	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	79	116	168	168	178	77	170	0
N.S.	1	1.00	1.36	2.00	2.90	2.90	3.07	1.33	2.93	0.00
time (sec)	N/A	0.027	0.018	0.010	1.457	0.869	1.327	0.626	0.105	0.001

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	68	99	157	157	167	66	159	0
N.S.	1	1.00	0.88	1.29	2.04	2.04	2.17	0.86	2.06	0.00
time (sec)	N/A	0.039	0.017	0.009	1.429	1.843	1.182	0.633	0.097	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	57	82	146	146	155	55	148	0
N.S.	1	1.00	0.63	0.90	1.60	1.60	1.70	0.60	1.63	0.00
time (sec)	N/A	0.068	0.016	0.010	1.445	1.114	1.079	0.635	4.819	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	46	65	135	135	143	44	136	0
N.S.	1	1.00	0.64	0.90	1.88	1.88	1.99	0.61	1.89	0.00
time (sec)	N/A	0.053	0.011	0.007	1.446	0.859	1.007	0.644	0.099	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	35	48	124	124	131	33	125	0
N.S.	1	1.00	0.66	0.91	2.34	2.34	2.47	0.62	2.36	0.00
time (sec)	N/A	0.039	0.014	0.009	1.506	0.892	0.945	0.639	4.828	0.001

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	31	113	113	119	22	114	0
N.S.	1	1.00	0.71	0.91	3.32	3.32	3.50	0.65	3.35	0.00
time (sec)	N/A	0.025	0.008	0.008	1.392	0.569	0.926	0.623	0.108	0.001

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	103	110	14	14	0
N.S.	1	1.00	1.00	0.94	0.88	6.44	6.88	0.88	0.88	0.00
time (sec)	N/A	0.003	0.003	0.001	1.297	0.733	0.893	0.587	0.126	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	120	147	214	398	223	136	210	0
N.S.	1	1.00	0.72	0.89	1.29	2.40	1.34	0.82	1.27	0.00
time (sec)	N/A	0.128	0.104	0.018	1.648	1.005	1.308	0.632	5.464	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	136	167	231	427	245	159	229	0
N.S.	1	1.00	0.74	0.91	1.26	2.32	1.33	0.86	1.24	0.00
time (sec)	N/A	0.186	0.130	0.020	1.670	0.682	1.464	0.592	0.522	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	217	151	198	246	442	260	174	243	0
N.S.	1	1.00	0.70	0.91	1.13	2.04	1.20	0.80	1.12	0.00
time (sec)	N/A	0.220	0.103	0.022	1.708	0.612	1.549	0.607	5.775	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	226	226	162	209	257	453	270	187	255	0
N.S.	1	1.00	0.72	0.92	1.14	2.00	1.19	0.83	1.13	0.00
time (sec)	N/A	0.233	0.127	0.022	1.707	1.077	1.604	0.644	1.088	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	166	228	248	718	314	162	241	0
N.S.	1	1.00	0.72	0.99	1.07	3.11	1.36	0.70	1.04	0.00
time (sec)	N/A	0.167	0.094	0.022	3.391	1.092	2.040	0.594	4.898	0.001

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	155	217	236	692	299	150	231	0
N.S.	1	1.00	0.71	1.00	1.08	3.17	1.37	0.69	1.06	0.00
time (sec)	N/A	0.141	0.080	0.023	3.247	0.812	1.919	0.646	0.402	0.001

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	207	144	203	222	664	274	131	218	0
N.S.	1	1.00	0.70	0.98	1.07	3.21	1.32	0.63	1.05	0.00
time (sec)	N/A	0.124	0.076	0.024	3.173	0.761	1.816	0.635	0.444	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	197	134	124	213	650	277	122	210	0
N.S.	1	1.00	0.68	0.63	1.08	3.30	1.41	0.62	1.07	0.00
time (sec)	N/A	0.113	0.078	0.018	3.165	0.588	1.550	0.617	4.964	0.001

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	138	124	219	654	289	128	207	0
N.S.	1	1.00	0.70	0.63	1.11	3.30	1.46	0.65	1.05	0.00
time (sec)	N/A	0.122	0.073	0.017	3.156	0.976	1.456	0.631	4.757	0.001

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	138	122	221	654	291	128	205	0
N.S.	1	1.00	0.69	0.61	1.11	3.29	1.46	0.64	1.03	0.00
time (sec)	N/A	0.123	0.060	0.018	3.155	0.805	1.353	0.636	4.753	0.001

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	138	122	221	654	291	128	205	0
N.S.	1	1.00	0.69	0.61	1.10	3.27	1.46	0.64	1.02	0.00
time (sec)	N/A	0.120	0.073	0.019	3.090	0.857	1.274	0.634	0.146	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	138	122	221	654	291	128	205	0
N.S.	1	1.00	0.69	0.61	1.10	3.25	1.45	0.64	1.02	0.00
time (sec)	N/A	0.119	0.072	0.016	3.075	0.901	1.190	0.631	4.770	0.001

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	138	122	221	654	291	128	204	0
N.S.	1	1.00	0.68	0.60	1.09	3.24	1.44	0.63	1.01	0.00
time (sec)	N/A	0.116	0.062	0.015	3.078	0.850	1.154	0.602	4.736	0.001

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	138	122	221	654	291	128	204	0
N.S.	1	1.00	0.68	0.60	1.09	3.22	1.43	0.63	1.00	0.00
time (sec)	N/A	0.111	0.066	0.015	3.141	0.563	1.084	0.609	4.794	0.001

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	138	122	221	654	291	128	204	0
N.S.	1	1.00	0.68	0.60	1.08	3.21	1.43	0.63	1.00	0.00
time (sec)	N/A	0.108	0.064	0.016	3.187	0.806	1.036	0.632	4.738	0.001

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	138	124	219	654	286	128	206	0
N.S.	1	1.00	0.67	0.60	1.07	3.19	1.40	0.62	1.00	0.00
time (sec)	N/A	0.106	0.058	0.016	3.095	0.590	1.027	0.617	0.167	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	181	181	131	156	212	650	272	122	209	0
N.S.	1	1.00	0.72	0.86	1.17	3.59	1.50	0.67	1.15	0.00
time (sec)	N/A	0.100	0.105	0.006	3.110	0.820	1.036	0.601	4.743	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	147	206	225	664	282	134	220	0
N.S.	1	1.00	0.70	0.99	1.08	3.18	1.35	0.64	1.05	0.00
time (sec)	N/A	0.131	0.099	0.023	3.131	0.975	1.348	0.634	5.089	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	220	220	157	219	240	700	304	148	234	0
N.S.	1	1.00	0.71	1.00	1.09	3.18	1.38	0.67	1.06	0.00
time (sec)	N/A	0.141	0.088	0.024	3.297	0.870	1.454	0.632	5.106	0.001

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	233	233	169	230	251	726	316	159	246	0
N.S.	1	1.00	0.73	0.99	1.08	3.12	1.36	0.68	1.06	0.00
time (sec)	N/A	0.158	0.093	0.026	3.351	0.853	1.500	0.598	5.892	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	26	25	23	22	26	23	0
N.S.	1	1.00	1.00	0.93	0.89	0.82	0.79	0.93	0.82	0.00
time (sec)	N/A	0.022	0.006	0.003	1.356	0.731	0.141	0.620	0.049	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	27	42	80	49	29	23	0
N.S.	1	1.00	1.00	0.87	1.35	2.58	1.58	0.94	0.74	0.00
time (sec)	N/A	0.012	0.010	0.004	2.969	0.659	0.152	0.628	4.567	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	16	15	15	12	16	15	0
N.S.	1	1.00	1.00	1.00	0.94	0.94	0.75	1.00	0.94	0.00
time (sec)	N/A	0.003	0.003	0.002	1.293	0.504	0.115	0.611	0.033	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	16	31	68	46	18	16	0
N.S.	1	1.00	1.00	0.67	1.29	2.83	1.92	0.75	0.67	0.00
time (sec)	N/A	0.007	0.004	0.003	2.996	0.756	0.140	0.588	0.177	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	23	25	20	15	26	21	0
N.S.	1	1.00	1.00	1.00	1.09	0.87	0.65	1.13	0.91	0.00
time (sec)	N/A	0.012	0.006	0.006	1.324	0.856	0.214	0.578	4.537	0.001

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	29	44	82	58	31	25	0
N.S.	1	1.00	1.00	0.88	1.33	2.48	1.76	0.94	0.76	0.00
time (sec)	N/A	0.012	0.012	0.005	2.888	0.547	0.186	0.622	4.610	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	33	35	33	31	43	31	0
N.S.	1	1.00	1.00	0.94	1.00	0.94	0.89	1.23	0.89	0.00
time (sec)	N/A	0.024	0.008	0.006	1.388	1.072	0.273	0.583	0.073	0.001

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	29	34	35	42	29	53	32	0
N.S.	1	1.00	0.83	0.97	1.00	1.20	0.83	1.51	0.91	0.00
time (sec)	N/A	0.027	0.013	0.008	1.348	0.402	0.196	0.630	0.044	0.001

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	47	38	52	127	71	39	34	0
N.S.	1	1.00	1.02	0.83	1.13	2.76	1.54	0.85	0.74	0.00
time (sec)	N/A	0.014	0.030	0.007	2.997	0.533	0.214	0.608	4.684	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	17	16	16	15	16	15	0
N.S.	1	1.00	1.00	1.00	0.94	0.94	0.88	0.94	0.88	0.00
time (sec)	N/A	0.003	0.002	0.000	1.377	0.446	0.156	0.605	0.021	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	47	38	52	126	71	39	34	0
N.S.	1	1.00	1.02	0.83	1.13	2.74	1.54	0.85	0.74	0.00
time (sec)	N/A	0.012	0.017	0.004	2.958	0.813	0.220	0.582	4.514	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	35	39	41	53	34	51	36	0
N.S.	1	1.00	0.88	0.98	1.02	1.32	0.85	1.28	0.90	0.00
time (sec)	N/A	0.028	0.020	0.011	1.365	0.498	0.299	0.626	0.055	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	56	47	65	140	83	50	45	0
N.S.	1	1.00	0.97	0.81	1.12	2.41	1.43	0.86	0.78	0.00
time (sec)	N/A	0.018	0.036	0.010	2.977	0.652	0.298	0.585	4.628	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	44	51	57	80	49	56	55	0
N.S.	1	1.00	0.85	0.98	1.10	1.54	0.94	1.08	1.06	0.00
time (sec)	N/A	0.038	0.033	0.013	1.310	0.552	0.367	0.600	4.591	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	25	35	38	38	36	26	37	0
N.S.	1	1.00	1.25	1.75	1.90	1.90	1.80	1.30	1.85	0.00
time (sec)	N/A	0.004	0.009	0.006	1.315	0.491	0.275	0.623	0.038	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	56	52	76	188	105	53	54	0
N.S.	1	1.00	0.84	0.78	1.13	2.81	1.57	0.79	0.81	0.00
time (sec)	N/A	0.020	0.031	0.009	2.894	0.584	0.315	0.634	4.616	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	17	16	26	26	16	26	0
N.S.	1	1.00	1.00	1.00	0.94	1.53	1.53	0.94	1.53	0.00
time (sec)	N/A	0.003	0.002	0.000	1.341	0.654	0.241	0.613	0.026	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	56	61	73	188	99	49	55	0
N.S.	1	1.00	0.88	0.95	1.14	2.94	1.55	0.77	0.86	0.00
time (sec)	N/A	0.018	0.041	0.005	2.993	0.505	0.327	0.619	4.600	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	45	55	62	92	56	63	57	0
N.S.	1	1.00	0.79	0.96	1.09	1.61	0.98	1.11	1.00	0.00
time (sec)	N/A	0.038	0.037	0.012	1.366	0.485	0.419	0.625	0.065	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	69	56	86	202	107	61	66	0
N.S.	1	1.00	0.88	0.72	1.10	2.59	1.37	0.78	0.85	0.00
time (sec)	N/A	0.026	0.047	0.012	2.906	0.578	0.427	0.630	4.605	0.001

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	60	68	79	121	78	84	76	0
N.S.	1	1.00	0.87	0.99	1.14	1.75	1.13	1.22	1.10	0.00
time (sec)	N/A	0.051	0.056	0.013	1.306	0.610	0.515	0.631	4.610	0.001

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	25	35	60	60	60	39	59	0
N.S.	1	1.00	0.69	0.97	1.67	1.67	1.67	1.08	1.64	0.00
time (sec)	N/A	0.030	0.010	0.007	1.361	0.632	0.435	0.592	4.595	0.001

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	81	72	124	324	160	77	96	0
N.S.	1	1.00	0.74	0.66	1.14	2.97	1.47	0.71	0.88	0.00
time (sec)	N/A	0.037	0.051	0.010	2.982	0.540	0.498	0.626	4.759	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	17	16	48	49	16	48	0
N.S.	1	1.00	1.00	1.00	0.94	2.82	2.88	0.94	2.82	0.00
time (sec)	N/A	0.003	0.004	0.000	1.308	0.530	0.404	0.623	4.690	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	79	107	117	320	146	71	99	0
N.S.	1	1.00	0.79	1.07	1.17	3.20	1.46	0.71	0.99	0.00
time (sec)	N/A	0.034	0.044	0.004	2.901	0.559	0.518	0.621	4.795	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	67	87	106	180	104	85	101	0
N.S.	1	1.00	0.74	0.96	1.16	1.98	1.14	0.93	1.11	0.00
time (sec)	N/A	0.064	0.035	0.013	1.394	0.542	0.640	0.606	5.238	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	92	78	130	334	155	83	110	0
N.S.	1	1.00	0.78	0.66	1.10	2.83	1.31	0.70	0.93	0.00
time (sec)	N/A	0.049	0.055	0.016	2.904	0.563	0.657	0.631	5.175	0.001

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	83	102	123	209	126	106	120	0
N.S.	1	1.00	0.78	0.96	1.16	1.97	1.19	1.00	1.13	0.00
time (sec)	N/A	0.087	0.068	0.016	1.353	0.688	0.763	0.643	5.351	0.001

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	17	13	12	18	14	0
N.S.	1	1.00	1.00	0.93	1.13	0.87	0.80	1.20	0.93	0.00
time (sec)	N/A	0.009	0.004	0.004	1.345	0.548	0.135	0.623	5.108	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	16	17	15	12	18	16	0
N.S.	1	1.00	1.00	0.89	0.94	0.83	0.67	1.00	0.89	0.00
time (sec)	N/A	0.010	0.005	0.004	1.366	0.448	0.137	0.620	0.056	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	23	24	28	22	32	22	0
N.S.	1	1.00	1.00	0.88	0.92	1.08	0.85	1.23	0.85	0.00
time (sec)	N/A	0.017	0.006	0.007	1.342	0.642	0.205	0.626	0.059	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	23	24	28	22	32	22	0
N.S.	1	1.00	1.00	0.85	0.89	1.04	0.81	1.19	0.81	0.00
time (sec)	N/A	0.018	0.004	0.005	1.338	0.570	0.210	0.593	0.059	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	28	20	0	82	83	23	23	0
N.S.	1	1.00	0.93	0.67	0.00	2.73	2.77	0.77	0.77	0.00
time (sec)	N/A	0.027	0.011	0.006	0.000	0.530	0.166	0.581	0.167	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	44	33	0	102	87	33	29	0
N.S.	1	1.00	1.19	0.89	0.00	2.76	2.35	0.89	0.78	0.00
time (sec)	N/A	0.033	0.016	0.006	0.000	0.523	0.227	0.586	0.293	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	25	25	28	40	22	36	24	0
N.S.	1	1.00	0.89	0.89	1.00	1.43	0.79	1.29	0.86	0.00
time (sec)	N/A	0.020	0.013	0.011	1.349	0.452	0.204	0.629	0.035	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	26	25	28	40	22	36	26	0
N.S.	1	1.00	0.87	0.83	0.93	1.33	0.73	1.20	0.87	0.00
time (sec)	N/A	0.019	0.014	0.010	1.355	0.470	0.206	0.635	0.036	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	36	34	0	106	60	37	38	0
N.S.	1	1.00	1.06	1.00	0.00	3.12	1.76	1.09	1.12	0.00
time (sec)	N/A	0.025	0.016	0.007	0.000	0.600	0.249	0.630	5.138	0.001

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	36	34	0	105	66	36	38	0
N.S.	1	1.00	1.06	1.00	0.00	3.09	1.94	1.06	1.12	0.00
time (sec)	N/A	0.011	0.011	0.003	0.000	0.837	0.232	0.617	4.638	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	38	0	182	104	58	46	0
N.S.	1	1.00	1.00	0.76	0.00	3.64	2.08	1.16	0.92	0.00
time (sec)	N/A	0.053	0.022	0.007	0.000	0.514	0.319	0.639	4.876	0.001

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	13	18	19	13	15	21
N.S.	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71	1.00
time (sec)	N/A	0.004	0.005	0.003	1.325	0.472	5.450	0.628	4.505	0.012

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	13	18	19	13	15	21
N.S.	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71	1.00
time (sec)	N/A	0.004	0.005	0.003	1.293	0.659	2.406	0.603	0.026	0.011

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	13	18	19	13	15	21
N.S.	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71	1.00
time (sec)	N/A	0.004	0.005	0.001	1.333	0.640	0.952	0.631	0.025	0.011

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	13	16	19	13	15	21
N.S.	1	1.00	1.00	0.76	0.62	0.76	0.90	0.62	0.71	1.00
time (sec)	N/A	0.004	0.005	0.003	1.315	0.532	1.303	0.612	0.025	0.010

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	15	13	14	17	13	14	20
N.S.	1	1.00	1.00	0.79	0.68	0.74	0.89	0.68	0.74	1.05
time (sec)	N/A	0.004	0.005	0.004	1.296	0.571	0.227	0.628	0.026	0.011

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	13	14	17	13	15	18
N.S.	1	1.00	1.00	0.84	0.68	0.74	0.89	0.68	0.79	0.95
time (sec)	N/A	0.004	0.005	0.004	1.353	0.555	0.384	0.593	0.026	0.013

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	14	13	15	17	13	15	19
N.S.	1	1.00	1.00	0.74	0.68	0.79	0.89	0.68	0.79	1.00
time (sec)	N/A	0.004	0.006	0.003	1.321	0.455	0.560	0.597	0.027	0.012

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	14	13	13	19	13	15	17
N.S.	1	1.00	1.00	0.74	0.68	0.68	1.00	0.68	0.79	0.89
time (sec)	N/A	0.004	0.006	0.003	1.312	0.530	1.239	0.586	0.027	0.014

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	29	34	24	25	34
N.S.	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69	0.94
time (sec)	N/A	0.009	0.009	0.004	1.360	0.650	10.605	0.621	0.045	0.016

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	29	34	24	26	34
N.S.	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.72	0.94
time (sec)	N/A	0.008	0.008	0.005	1.262	0.636	5.506	0.625	0.037	0.015

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	29	34	24	26	34
N.S.	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.72	0.94
time (sec)	N/A	0.009	0.008	0.006	1.362	0.500	2.546	0.629	0.038	0.016

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	30	27	24	27	34	24	26	34
N.S.	1	1.00	0.83	0.75	0.67	0.75	0.94	0.67	0.72	0.94
time (sec)	N/A	0.008	0.008	0.005	1.338	0.639	1.705	0.582	0.037	0.014

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	30	27	24	26	32	24	26	34
N.S.	1	1.00	0.88	0.79	0.71	0.76	0.94	0.71	0.76	1.00
time (sec)	N/A	0.009	0.008	0.005	1.303	0.625	0.748	0.628	0.035	0.014

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	30	27	24	26	32	24	26	30
N.S.	1	1.00	0.88	0.79	0.71	0.76	0.94	0.71	0.76	0.88
time (sec)	N/A	0.008	0.009	0.005	1.333	0.557	0.942	0.632	0.038	0.018

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	30	27	24	26	32	24	26	30
N.S.	1	1.00	0.88	0.79	0.71	0.76	0.94	0.71	0.76	0.88
time (sec)	N/A	0.008	0.009	0.005	1.352	0.619	1.111	0.636	0.037	0.018

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	30	27	25	26	32	25	26	30
N.S.	1	1.00	0.88	0.79	0.74	0.76	0.94	0.74	0.76	0.88
time (sec)	N/A	0.008	0.009	0.004	1.308	0.479	1.710	0.601	0.032	0.019

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	41	38	35	40	49	35	35	47
N.S.	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.013	0.012	0.006	1.340	0.635	19.691	0.608	0.044	0.018

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	41	38	35	40	49	35	35	47
N.S.	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.012	0.010	0.004	1.336	0.527	10.623	0.626	0.044	0.017

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	41	38	35	40	49	35	35	47
N.S.	1	1.00	0.80	0.75	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.013	0.010	0.005	1.366	0.799	5.734	0.626	0.042	0.018

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	41	38	35	38	49	35	35	47
N.S.	1	1.00	0.80	0.75	0.69	0.75	0.96	0.69	0.69	0.92
time (sec)	N/A	0.013	0.010	0.006	1.338	0.817	2.175	0.580	0.042	0.017

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	41	38	35	37	48	35	35	47
N.S.	1	1.00	0.84	0.78	0.71	0.76	0.98	0.71	0.71	0.96
time (sec)	N/A	0.012	0.010	0.007	1.335	0.557	2.101	0.586	0.044	0.017

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	41	38	35	37	46	35	35	41
N.S.	1	1.00	0.87	0.81	0.74	0.79	0.98	0.74	0.74	0.87
time (sec)	N/A	0.012	0.011	0.006	1.386	0.456	2.312	0.578	0.044	0.020

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	41	38	35	37	48	35	35	41
N.S.	1	1.00	0.84	0.78	0.71	0.76	0.98	0.71	0.71	0.84
time (sec)	N/A	0.012	0.012	0.006	1.349	0.669	2.802	0.579	0.045	0.019

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	41	38	36	37	46	36	37	41
N.S.	1	1.00	0.87	0.81	0.77	0.79	0.98	0.77	0.79	0.87
time (sec)	N/A	0.012	0.011	0.004	1.347	0.702	3.855	0.610	0.042	0.023

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	203	152	194	170	192	196	67	134
N.S.	1	1.00	0.94	0.71	0.90	0.79	0.89	0.91	0.31	0.62
time (sec)	N/A	0.198	0.068	0.011	3.075	0.598	50.315	0.623	4.486	0.187

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	78	143	186	165	180	178	54	124
N.S.	1	1.00	0.38	0.70	0.91	0.81	0.88	0.87	0.26	0.61
time (sec)	N/A	0.152	0.027	0.005	3.035	0.634	15.293	0.648	0.087	0.177

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	189	140	185	124	172	178	55	123
N.S.	1	1.00	0.94	0.69	0.92	0.61	0.85	0.88	0.27	0.61
time (sec)	N/A	0.163	0.039	0.007	2.900	0.495	5.892	0.648	0.094	0.174

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	54	132	172	126	165	182	38	114
N.S.	1	1.00	0.28	0.69	0.90	0.66	0.86	0.95	0.20	0.59
time (sec)	N/A	0.144	0.025	0.007	2.978	0.528	3.167	0.601	0.071	0.146

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	146	132	172	126	160	182	37	113
N.S.	1	1.00	0.76	0.69	0.90	0.66	0.83	0.95	0.19	0.59
time (sec)	N/A	0.146	0.034	0.004	3.042	0.640	5.942	0.631	0.084	0.146

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	27	140	186	142	170	190	54	122
N.S.	1	1.00	0.13	0.69	0.92	0.70	0.84	0.94	0.27	0.60
time (sec)	N/A	0.167	0.006	0.009	3.042	0.555	12.160	0.624	4.501	0.187

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	29	143	187	167	178	178	53	125
N.S.	1	1.00	0.14	0.70	0.92	0.82	0.87	0.87	0.26	0.61
time (sec)	N/A	0.161	0.007	0.009	3.131	0.571	30.395	0.643	0.090	0.188

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	29	152	198	193	190	200	66	134
N.S.	1	1.00	0.13	0.71	0.92	0.90	0.88	0.93	0.31	0.62
time (sec)	N/A	0.175	0.006	0.011	3.044	0.693	109.164	0.621	4.481	0.193

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	221	158	206	192	0	196	80	151
N.S.	1	1.00	0.96	0.69	0.90	0.83	0.00	0.85	0.35	0.66
time (sec)	N/A	0.166	0.103	0.013	3.025	0.777	0.000	0.653	4.515	0.333

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	43	149	195	185	595	199	64	139
N.S.	1	1.00	0.20	0.68	0.89	0.85	2.73	0.91	0.29	0.64
time (sec)	N/A	0.148	0.013	0.012	2.994	0.589	153.056	0.617	0.081	0.327

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	198	158	195	187	440	199	64	139
N.S.	1	1.00	0.91	0.72	0.89	0.86	2.02	0.91	0.29	0.64
time (sec)	N/A	0.148	0.101	0.010	2.884	0.636	78.161	0.647	4.651	0.315

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	29	158	194	182	578	199	64	139
N.S.	1	1.00	0.13	0.72	0.89	0.83	2.65	0.91	0.29	0.64
time (sec)	N/A	0.147	0.006	0.008	3.022	0.679	50.142	0.616	0.086	0.307

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	199	149	194	179	434	199	64	139
N.S.	1	1.00	0.91	0.68	0.89	0.82	1.99	0.91	0.29	0.64
time (sec)	N/A	0.151	0.102	0.009	3.051	0.781	75.795	0.651	0.090	0.299

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	27	158	208	208	700	210	77	149
N.S.	1	1.00	0.12	0.69	0.90	0.90	3.04	0.91	0.33	0.65
time (sec)	N/A	0.188	0.006	0.016	2.983	0.570	150.184	0.618	0.083	0.342

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	29	161	209	228	0	196	77	149
N.S.	1	1.00	0.13	0.70	0.91	0.99	0.00	0.85	0.33	0.65
time (sec)	N/A	0.177	0.007	0.015	3.063	0.525	0.000	0.642	4.683	0.340

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	243	29	172	221	251	0	220	87	160
N.S.	1	1.00	0.12	0.71	0.91	1.03	0.00	0.91	0.36	0.66
time (sec)	N/A	0.191	0.007	0.017	3.035	0.634	0.000	0.658	4.690	0.411

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	242	170	218	254	0	209	87	151
N.S.	1	1.00	1.01	0.71	0.91	1.06	0.00	0.87	0.36	0.63
time (sec)	N/A	0.177	0.113	0.014	3.091	0.643	0.000	0.654	4.688	0.467

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	45	169	222	260	0	212	85	154
N.S.	1	1.00	0.19	0.70	0.92	1.07	0.00	0.88	0.35	0.64
time (sec)	N/A	0.170	0.017	0.016	2.971	0.682	0.000	0.659	0.083	0.447

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	223	169	221	257	0	211	85	153
N.S.	1	1.00	0.92	0.70	0.91	1.06	0.00	0.87	0.35	0.63
time (sec)	N/A	0.164	0.109	0.016	2.894	0.770	0.000	0.658	4.669	0.436

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	29	175	217	250	0	209	86	149
N.S.	1	1.00	0.12	0.73	0.91	1.05	0.00	0.87	0.36	0.62
time (sec)	N/A	0.164	0.006	0.010	2.995	0.637	0.000	0.647	0.083	0.274

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	220	166	217	241	0	209	86	149
N.S.	1	1.00	0.92	0.69	0.91	1.01	0.00	0.87	0.36	0.62
time (sec)	N/A	0.172	0.083	0.010	3.023	0.544	0.000	0.651	4.667	0.274

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	27	178	230	263	0	220	99	160
N.S.	1	1.00	0.11	0.71	0.92	1.05	0.00	0.88	0.39	0.64
time (sec)	N/A	0.191	0.006	0.020	3.002	0.460	0.000	0.652	0.102	0.480

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	29	181	231	283	0	208	99	160
N.S.	1	1.00	0.12	0.72	0.92	1.13	0.00	0.83	0.39	0.64
time (sec)	N/A	0.185	0.006	0.020	2.967	0.616	0.000	0.653	4.657	0.466

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	264	264	29	192	243	306	0	232	109	171
N.S.	1	1.00	0.11	0.73	0.92	1.16	0.00	0.88	0.41	0.65
time (sec)	N/A	0.212	0.007	0.022	3.050	0.550	0.000	0.626	4.648	0.482

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	48	66	86	117	122	194	33	58
N.S.	1	1.00	0.83	1.14	1.48	2.02	2.10	3.34	0.57	1.00
time (sec)	N/A	0.034	0.016	0.007	2.967	0.653	3.102	0.634	0.083	0.046

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	108	72	84	117	105	84	47	66
N.S.	1	1.00	1.00	0.67	0.78	1.08	0.97	0.78	0.44	0.61
time (sec)	N/A	0.061	0.025	0.008	3.046	0.624	4.146	0.635	0.079	0.079

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	24	67	79	110	99	79	42	62
N.S.	1	1.00	0.24	0.66	0.78	1.09	0.98	0.78	0.42	0.61
time (sec)	N/A	0.055	0.007	0.007	2.967	0.544	1.749	0.631	0.044	0.073

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	99	67	79	110	97	79	42	61
N.S.	1	1.00	1.00	0.68	0.80	1.11	0.98	0.80	0.42	0.62
time (sec)	N/A	0.055	0.016	0.006	2.959	0.623	0.843	0.603	0.041	0.076

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	22	62	74	107	90	74	37	53
N.S.	1	1.00	0.24	0.67	0.80	1.16	0.98	0.80	0.40	0.58
time (sec)	N/A	0.055	0.005	0.006	2.870	0.664	0.547	0.631	0.037	0.062

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	76	62	74	107	90	74	37	52
N.S.	1	1.00	0.83	0.67	0.80	1.16	0.98	0.80	0.40	0.57
time (sec)	N/A	0.053	0.013	0.005	2.893	0.535	0.734	0.594	0.027	0.060

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	20	67	79	120	97	79	42	60
N.S.	1	1.00	0.20	0.68	0.80	1.21	0.98	0.80	0.42	0.61
time (sec)	N/A	0.056	0.005	0.007	2.988	0.574	1.351	0.634	0.037	0.075

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	22	67	79	129	99	79	42	63
N.S.	1	1.00	0.22	0.66	0.78	1.28	0.98	0.78	0.42	0.62
time (sec)	N/A	0.054	0.006	0.008	2.932	0.691	2.435	0.628	0.039	0.085

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	22	72	86	136	105	86	48	69
N.S.	1	1.00	0.20	0.67	0.80	1.26	0.97	0.80	0.44	0.64
time (sec)	N/A	0.057	0.005	0.010	3.048	0.555	5.465	0.626	0.040	0.089

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	121	79	91	148	277	91	55	84
N.S.	1	1.00	0.99	0.65	0.75	1.21	2.27	0.75	0.45	0.69
time (sec)	N/A	0.061	0.050	0.011	2.997	0.557	8.345	0.622	4.617	0.156

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	30	74	86	141	264	86	51	74
N.S.	1	1.00	0.27	0.65	0.76	1.25	2.34	0.76	0.45	0.65
time (sec)	N/A	0.059	0.011	0.011	2.996	0.708	5.308	0.602	4.623	0.163

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	106	74	86	140	257	86	51	74
N.S.	1	1.00	0.94	0.65	0.76	1.24	2.27	0.76	0.45	0.65
time (sec)	N/A	0.059	0.045	0.010	2.959	0.773	3.387	0.629	4.677	0.157

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	22	74	86	140	257	86	50	74
N.S.	1	1.00	0.19	0.65	0.76	1.24	2.27	0.76	0.44	0.65
time (sec)	N/A	0.060	0.005	0.006	2.936	0.616	2.166	0.631	0.035	0.153

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	107	74	86	141	264	86	50	74
N.S.	1	1.00	0.95	0.65	0.76	1.25	2.34	0.76	0.44	0.65
time (sec)	N/A	0.059	0.045	0.006	2.838	0.549	2.989	0.604	4.663	0.137

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	20	79	92	148	366	92	55	81
N.S.	1	1.00	0.16	0.65	0.75	1.21	3.00	0.75	0.45	0.66
time (sec)	N/A	0.065	0.005	0.014	3.037	0.745	4.563	0.635	4.677	0.198

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	22	79	92	158	366	91	55	81
N.S.	1	1.00	0.18	0.65	0.75	1.30	3.00	0.75	0.45	0.66
time (sec)	N/A	0.061	0.005	0.012	2.944	0.631	8.330	0.634	0.077	0.173

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	22	84	97	163	384	98	59	86
N.S.	1	1.00	0.17	0.64	0.74	1.24	2.93	0.75	0.45	0.66
time (sec)	N/A	0.065	0.007	0.016	3.042	0.703	18.732	0.639	0.070	0.178

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	135	82	99	173	481	94	62	81
N.S.	1	1.00	1.05	0.64	0.77	1.34	3.73	0.73	0.48	0.63
time (sec)	N/A	0.068	0.046	0.012	3.014	0.742	23.354	0.685	4.733	0.245

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	32	82	99	175	481	94	62	81
N.S.	1	1.00	0.25	0.64	0.77	1.36	3.73	0.73	0.48	0.63
time (sec)	N/A	0.070	0.012	0.010	2.911	0.931	15.486	0.647	0.066	0.209

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	121	82	97	171	481	92	62	79
N.S.	1	1.00	0.94	0.64	0.75	1.33	3.73	0.71	0.48	0.61
time (sec)	N/A	0.068	0.031	0.012	2.969	0.537	9.810	0.640	4.687	0.206

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	22	86	99	175	481	94	61	81
N.S.	1	1.00	0.17	0.67	0.77	1.36	3.73	0.73	0.47	0.63
time (sec)	N/A	0.069	0.005	0.008	3.020	0.658	6.379	0.589	0.037	0.120

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	121	86	99	173	481	94	61	81
N.S.	1	1.00	0.94	0.67	0.77	1.34	3.73	0.73	0.47	0.63
time (sec)	N/A	0.068	0.033	0.007	2.896	0.986	8.459	0.610	4.750	0.123

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	20	87	102	178	653	99	65	86
N.S.	1	1.00	0.14	0.63	0.74	1.29	4.73	0.72	0.47	0.62
time (sec)	N/A	0.070	0.005	0.013	2.976	0.744	12.635	0.628	4.718	0.270

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	22	87	102	188	653	99	65	86
N.S.	1	1.00	0.16	0.63	0.74	1.36	4.73	0.72	0.47	0.62
time (sec)	N/A	0.071	0.005	0.015	2.853	0.708	22.417	0.625	4.721	0.263

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	22	92	107	193	678	106	69	91
N.S.	1	1.00	0.15	0.63	0.73	1.31	4.61	0.72	0.47	0.62
time (sec)	N/A	0.075	0.006	0.014	2.951	0.607	51.156	0.638	0.075	0.238

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	24	23	23	26	24	11	15
N.S.	1	1.00	1.00	1.60	1.53	1.53	1.73	1.60	0.73	1.00
time (sec)	N/A	0.007	0.004	0.009	2.969	0.814	0.291	0.624	0.032	0.024

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	100	22	69	68	108	94	68	57	79
N.S.	1	1.37	0.30	0.95	0.93	1.48	1.29	0.93	0.78	1.08
time (sec)	N/A	0.257	0.005	0.050	2.971	0.833	1.781	0.630	4.767	0.133

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	88	432	97	367	1999	540	389	0
N.S.	1	1.00	0.91	4.45	1.00	3.78	20.61	5.57	4.01	0.00
time (sec)	N/A	0.043	0.055	0.008	1.330	0.841	4.497	0.690	5.113	0.098

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	72	291	79	251	1221	365	272	0
N.S.	1	1.00	0.91	3.68	1.00	3.18	15.46	4.62	3.44	0.00
time (sec)	N/A	0.032	0.035	0.006	1.339	0.689	2.847	0.610	4.988	0.052

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	56	178	61	157	683	224	167	0
N.S.	1	1.00	0.92	2.92	1.00	2.57	11.20	3.67	2.74	0.00
time (sec)	N/A	0.022	0.031	0.007	1.340	0.907	1.665	0.658	4.793	0.033

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	40	93	43	85	306	117	93	0
N.S.	1	1.00	0.93	2.16	1.00	1.98	7.12	2.72	2.16	0.00
time (sec)	N/A	0.015	0.023	0.006	1.315	0.927	0.943	0.654	4.751	0.025

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	35	25	33	94	43	34	0
N.S.	1	1.00	1.00	1.40	1.00	1.32	3.76	1.72	1.36	0.00
time (sec)	N/A	0.007	0.014	0.003	1.297	0.866	0.412	0.689	4.801	0.017

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	50	47	73	57	110	57	55	61
N.S.	1	1.00	0.62	0.59	0.91	0.71	1.38	0.71	0.69	0.76
time (sec)	N/A	0.045	0.026	0.007	1.356	1.014	1.418	0.605	4.587	0.029

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	36	53	46	87	43	44	50
N.S.	1	1.00	0.66	0.61	0.90	0.78	1.47	0.73	0.75	0.85
time (sec)	N/A	0.033	0.019	0.006	1.323	1.346	0.704	0.592	4.659	0.027

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	25	33	34	63	29	33	38
N.S.	1	1.00	0.74	0.66	0.87	0.89	1.66	0.76	0.87	1.00
time (sec)	N/A	0.022	0.014	0.008	1.337	1.190	0.318	0.595	4.650	0.024

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	39	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	0.78	1.00
time (sec)	N/A	0.003	0.002	0.001	1.334	0.961	0.168	0.604	4.627	0.012

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	39	27	77	56	33	29	37
N.S.	1	1.00	1.00	1.05	0.73	2.08	1.51	0.89	0.78	1.00
time (sec)	N/A	0.024	0.008	0.003	1.363	0.990	1.409	0.592	4.658	0.029

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	59	63	51	106	42	46	35	47
N.S.	1	1.00	1.26	1.34	1.09	2.26	0.89	0.98	0.74	1.00
time (sec)	N/A	0.025	0.033	0.005	1.369	1.702	1.926	0.611	4.795	0.066

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	39	85	73	131	92	72	54	62
N.S.	1	1.00	0.55	1.20	1.03	1.85	1.30	1.01	0.76	0.87
time (sec)	N/A	0.039	0.009	0.006	1.324	0.635	3.735	0.596	4.910	0.090

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	39	105	93	157	117	92	74	73
N.S.	1	1.00	0.41	1.11	0.98	1.65	1.23	0.97	0.78	0.77
time (sec)	N/A	0.052	0.008	0.009	1.340	1.516	5.907	0.677	4.942	0.119

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	77	77	69	146	117	64	-1	74
N.S.	1	1.00	0.82	0.82	0.73	1.55	1.24	0.68	-0.01	0.79
time (sec)	N/A	0.033	0.034	0.008	1.305	1.096	5.773	0.691	0.000	0.080

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	64	57	49	119	92	50	-1	62
N.S.	1	1.00	0.91	0.81	0.70	1.70	1.31	0.71	-0.01	0.89
time (sec)	N/A	0.021	0.024	0.005	1.359	0.908	3.527	0.715	0.000	0.051

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	49	36	28	94	41	37	35	48
N.S.	1	1.00	1.07	0.78	0.61	2.04	0.89	0.80	0.76	1.04
time (sec)	N/A	0.010	0.014	0.003	1.328	0.730	1.831	0.661	4.673	0.042

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	63	54	28	88	56	57	56	45
N.S.	1	1.00	1.50	1.29	0.67	2.10	1.33	1.36	1.33	1.07
time (sec)	N/A	0.012	0.072	0.004	1.373	0.580	1.417	0.652	4.816	0.056

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	42	59	17	31
N.S.	1	1.00	1.00	0.86	0.81	0.81	2.00	2.81	0.81	1.48
time (sec)	N/A	0.004	0.005	0.003	1.315	0.888	0.706	0.681	4.567	0.058

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	31	28	36	38	68	112	37	42
N.S.	1	1.00	0.70	0.64	0.82	0.86	1.55	2.55	0.84	0.95
time (sec)	N/A	0.011	0.009	0.004	1.283	0.940	0.912	0.684	4.767	0.071

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	42	39	56	49	359	138	73	53
N.S.	1	1.00	0.62	0.57	0.82	0.72	5.28	2.03	1.07	0.78
time (sec)	N/A	0.018	0.010	0.006	1.315	1.122	1.298	0.653	4.707	0.084

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	53	50	76	60	575	166	93	64
N.S.	1	1.00	0.58	0.54	0.83	0.65	6.25	1.80	1.01	0.70
time (sec)	N/A	0.029	0.012	0.006	1.337	0.975	1.742	0.697	4.946	0.085

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	50	47	73	68	133	57	64	72
N.S.	1	1.00	0.62	0.59	0.91	0.85	1.66	0.71	0.80	0.90
time (sec)	N/A	0.048	0.026	0.006	1.328	1.040	3.813	0.682	4.604	0.032

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	36	53	57	109	43	53	61
N.S.	1	1.00	0.66	0.61	0.90	0.97	1.85	0.73	0.90	1.03
time (sec)	N/A	0.035	0.019	0.005	1.317	1.063	2.239	0.642	4.659	0.029

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	25	33	45	85	29	42	49
N.S.	1	1.00	0.74	0.66	0.87	1.18	2.24	0.76	1.11	1.29
time (sec)	N/A	0.024	0.014	0.005	1.328	0.864	1.133	0.645	4.637	0.026

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	32	61	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	1.78	3.39	0.78	0.78	1.00
time (sec)	N/A	0.004	0.003	0.003	1.325	0.777	0.592	0.672	4.567	0.012

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	50	52	40	100	78	48	42	50
N.S.	1	1.00	0.93	0.96	0.74	1.85	1.44	0.89	0.78	0.93
time (sec)	N/A	0.036	0.021	0.006	1.278	0.743	1.924	0.631	4.696	0.037

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	37	75	63	119	88	63	47	57
N.S.	1	1.00	0.59	1.19	1.00	1.89	1.40	1.00	0.75	0.90
time (sec)	N/A	0.037	0.009	0.006	1.342	0.618	2.333	0.677	4.816	0.080

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	76	102	90	136	71	70	52	59
N.S.	1	1.00	1.12	1.50	1.32	2.00	1.04	1.03	0.76	0.87
time (sec)	N/A	0.040	0.037	0.006	1.370	0.817	2.984	0.647	4.905	0.087

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	39	122	110	157	119	92	72	73
N.S.	1	1.00	0.42	1.33	1.20	1.71	1.29	1.00	0.78	0.79
time (sec)	N/A	0.054	0.010	0.010	1.333	0.714	5.331	0.644	4.941	0.096

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	39	142	130	179	148	109	89	84
N.S.	1	1.00	0.34	1.22	1.12	1.54	1.28	0.94	0.77	0.72
time (sec)	N/A	0.069	0.010	0.016	1.384	0.909	8.264	0.650	5.212	0.135

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	94	95	87	168	148	76	-1	85
N.S.	1	1.00	0.82	0.83	0.76	1.46	1.29	0.66	-0.01	0.74
time (sec)	N/A	0.042	0.136	0.006	1.356	0.929	8.195	0.662	0.000	0.077

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	83	75	67	145	119	63	-1	74
N.S.	1	1.00	0.91	0.82	0.74	1.59	1.31	0.69	-0.01	0.81
time (sec)	N/A	0.031	0.118	0.005	1.334	0.569	5.206	0.640	0.000	0.081

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	51	43	124	70	49	37	60
N.S.	1	1.00	1.00	0.78	0.66	1.91	1.08	0.75	0.57	0.92
time (sec)	N/A	0.015	0.089	0.003	1.314	0.859	2.905	0.628	4.608	0.067

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	50	69	43	112	88	73	40	59
N.S.	1	1.00	0.79	1.10	0.68	1.78	1.40	1.16	0.63	0.94
time (sec)	N/A	0.018	0.008	0.004	1.344	0.917	2.369	0.670	5.153	0.109

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	52	92	66	112	78	114	-1	57
N.S.	1	1.00	0.85	1.51	1.08	1.84	1.28	1.87	-0.02	0.93
time (sec)	N/A	0.018	0.008	0.004	1.317	0.949	2.026	0.709	0.000	0.099

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	35	68	86	17	42
N.S.	1	1.00	1.00	0.86	0.81	1.67	3.24	4.10	0.81	2.00
time (sec)	N/A	0.005	0.006	0.004	1.336	0.816	0.911	0.642	5.295	0.083

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	31	28	36	49	94	166	71	53
N.S.	1	1.00	0.70	0.64	0.82	1.11	2.14	3.77	1.61	1.20
time (sec)	N/A	0.012	0.010	0.003	1.386	0.840	1.179	0.687	5.655	0.091

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	42	39	56	60	420	192	91	64
N.S.	1	1.00	0.62	0.57	0.82	0.88	6.18	2.82	1.34	0.94
time (sec)	N/A	0.021	0.011	0.004	1.375	0.718	1.613	0.638	5.710	0.102

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	53	50	76	71	648	220	111	75
N.S.	1	1.00	0.58	0.54	0.83	0.77	7.04	2.39	1.21	0.82
time (sec)	N/A	0.029	0.013	0.006	1.424	0.858	2.124	0.686	5.781	0.113

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	50	47	73	79	158	57	75	50
N.S.	1	1.00	0.62	0.59	0.91	0.99	1.98	0.71	0.94	0.62
time (sec)	N/A	0.047	0.028	0.006	1.326	0.813	9.131	0.660	4.863	0.035

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	36	53	68	133	43	64	39
N.S.	1	1.00	0.66	0.61	0.90	1.15	2.25	0.73	1.08	0.66
time (sec)	N/A	0.034	0.021	0.005	1.357	0.841	6.275	0.922	4.752	0.030

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	25	33	56	109	29	53	60
N.S.	1	1.00	0.74	0.66	0.87	1.47	2.87	0.76	1.39	1.58
time (sec)	N/A	0.024	0.017	0.004	1.370	0.911	3.832	0.992	4.744	0.028

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	43	85	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	2.39	4.72	0.78	0.78	1.00
time (sec)	N/A	0.003	0.004	0.003	1.341	0.964	2.100	1.199	4.595	0.012

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	62	66	54	126	105	62	59	62
N.S.	1	1.00	0.86	0.92	0.75	1.75	1.46	0.86	0.82	0.86
time (sec)	N/A	0.043	0.027	0.005	1.326	0.956	3.556	1.187	4.682	0.045

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	37	88	76	142	112	82	66	68
N.S.	1	1.00	0.46	1.10	0.95	1.78	1.40	1.02	0.82	0.85
time (sec)	N/A	0.045	0.010	0.006	1.379	0.698	3.223	1.130	4.948	0.093

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	39	116	104	145	117	88	71	70
N.S.	1	1.00	0.45	1.35	1.21	1.69	1.36	1.02	0.83	0.81
time (sec)	N/A	0.048	0.010	0.006	1.349	0.907	3.727	1.164	5.007	0.105

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	87	139	127	158	99	87	72	70
N.S.	1	1.00	0.98	1.56	1.43	1.78	1.11	0.98	0.81	0.79
time (sec)	N/A	0.053	0.042	0.010	1.469	0.992	4.528	1.218	5.201	0.124

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	39	159	147	179	150	109	89	84
N.S.	1	1.00	0.35	1.41	1.30	1.58	1.33	0.96	0.79	0.74
time (sec)	N/A	0.067	0.011	0.018	1.381	0.979	7.576	1.113	5.430	0.129

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	39	179	167	201	175	126	106	95
N.S.	1	1.00	0.28	1.31	1.22	1.47	1.28	0.92	0.77	0.69
time (sec)	N/A	0.083	0.011	0.036	1.439	1.037	11.535	1.160	5.678	0.156

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	105	113	105	190	175	91	-1	96
N.S.	1	1.00	0.77	0.83	0.77	1.40	1.29	0.67	-0.01	0.71
time (sec)	N/A	0.054	0.154	0.007	1.264	0.971	11.108	1.090	0.000	0.125

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	94	93	85	167	150	77	-1	85
N.S.	1	1.00	0.84	0.83	0.76	1.49	1.34	0.69	-0.01	0.76
time (sec)	N/A	0.042	0.137	0.008	1.342	0.886	7.223	1.080	0.000	0.102

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	76	66	58	146	97	63	37	71
N.S.	1	1.00	0.90	0.79	0.69	1.74	1.15	0.75	0.44	0.85
time (sec)	N/A	0.022	0.110	0.003	1.322	1.015	4.214	1.226	4.460	0.086

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	52	85	59	140	117	87	40	73
N.S.	1	1.00	0.63	1.02	0.71	1.69	1.41	1.05	0.48	0.88
time (sec)	N/A	0.025	0.008	0.005	1.323	0.775	3.631	1.113	5.050	0.114

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	54	110	84	141	112	132	-1	71
N.S.	1	1.00	0.63	1.28	0.98	1.64	1.30	1.53	-0.01	0.83
time (sec)	N/A	0.027	0.008	0.006	1.321	0.935	3.165	1.125	0.000	0.134

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	54	130	104	140	105	168	-1	68
N.S.	1	1.00	0.66	1.59	1.27	1.71	1.28	2.05	-0.01	0.83
time (sec)	N/A	0.028	0.010	0.010	1.401	0.853	3.662	1.139	0.000	0.135

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	46	95	113	71	53
N.S.	1	1.00	1.00	0.86	0.81	2.19	4.52	5.38	3.38	2.52
time (sec)	N/A	0.005	0.008	0.003	1.409	0.564	1.260	0.993	5.073	0.107

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	31	28	36	60	121	220	91	64
N.S.	1	1.00	0.70	0.64	0.82	1.36	2.75	5.00	2.07	1.45
time (sec)	N/A	0.011	0.011	0.006	1.528	0.960	1.591	1.142	5.369	0.119

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	42	39	56	71	481	246	111	75
N.S.	1	1.00	0.62	0.57	0.82	1.04	7.07	3.62	1.63	1.10
time (sec)	N/A	0.019	0.012	0.006	1.414	0.662	2.160	1.156	5.618	0.131

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	53	50	76	82	721	274	131	86
N.S.	1	1.00	0.58	0.54	0.83	0.89	7.84	2.98	1.42	0.93
time (sec)	N/A	0.028	0.015	0.007	1.390	1.205	2.793	1.183	5.981	0.146

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	64	61	96	93	1012	300	151	97
N.S.	1	1.00	0.55	0.53	0.83	0.80	8.72	2.59	1.30	0.84
time (sec)	N/A	0.042	0.016	0.007	1.447	1.438	3.488	1.230	6.365	0.158

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	75	72	116	104	1346	328	171	108
N.S.	1	1.00	0.54	0.51	0.83	0.74	9.61	2.34	1.22	0.77
time (sec)	N/A	0.056	0.019	0.008	1.536	1.470	4.423	1.101	6.869	0.172

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	94	91	153	145	301	113	141	149
N.S.	1	1.00	0.58	0.57	0.95	0.90	1.87	0.70	0.88	0.93
time (sec)	N/A	0.100	0.058	0.008	1.470	1.044	103.129	1.068	4.837	0.057

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	83	80	133	134	277	99	130	138
N.S.	1	1.00	0.59	0.57	0.95	0.96	1.98	0.71	0.93	0.99
time (sec)	N/A	0.080	0.044	0.008	1.453	1.284	79.370	1.096	4.836	0.051

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	72	69	113	123	253	85	119	72
N.S.	1	1.00	0.59	0.57	0.93	1.01	2.07	0.70	0.98	0.59
time (sec)	N/A	0.071	0.041	0.006	1.482	0.841	62.850	1.010	4.809	0.040

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	61	58	93	112	230	71	108	61
N.S.	1	1.00	0.60	0.57	0.92	1.11	2.28	0.70	1.07	0.60
time (sec)	N/A	0.058	0.035	0.006	1.429	0.887	48.285	1.077	4.741	0.037

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	50	47	73	101	204	57	97	50
N.S.	1	1.00	0.62	0.59	0.91	1.26	2.55	0.71	1.21	0.62
time (sec)	N/A	0.046	0.030	0.006	1.419	0.726	37.198	1.146	4.735	0.035

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	36	53	90	180	43	86	39
N.S.	1	1.00	0.66	0.61	0.90	1.53	3.05	0.73	1.46	0.66
time (sec)	N/A	0.035	0.023	0.005	1.388	0.966	27.022	0.960	4.600	0.030

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	25	33	78	156	29	29	82
N.S.	1	1.00	0.74	0.66	0.87	2.05	4.11	0.76	0.76	2.16
time (sec)	N/A	0.024	0.019	0.005	1.294	1.310	18.496	1.058	4.739	0.029

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	65	133	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	3.61	7.39	0.78	0.78	1.00
time (sec)	N/A	0.003	0.006	0.003	1.321	0.936	12.960	1.188	4.813	0.013

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	84	94	82	170	160	90	87	84
N.S.	1	1.00	0.78	0.87	0.76	1.57	1.48	0.83	0.81	0.78
time (sec)	N/A	0.070	0.040	0.005	1.402	1.080	10.477	1.168	5.309	0.053

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	37	118	106	188	167	116	95	90
N.S.	1	1.00	0.31	1.00	0.90	1.59	1.42	0.98	0.81	0.76
time (sec)	N/A	0.071	0.012	0.007	1.432	1.224	9.267	1.144	5.445	0.102

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	39	148	136	192	175	124	132	92
N.S.	1	1.00	0.31	1.17	1.08	1.52	1.39	0.98	1.05	0.73
time (sec)	N/A	0.076	0.013	0.007	1.456	0.997	8.727	1.145	5.651	0.111

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	39	168	156	190	175	124	149	92
N.S.	1	1.00	0.31	1.33	1.24	1.51	1.39	0.98	1.18	0.73
time (sec)	N/A	0.076	0.013	0.012	1.471	1.040	7.118	1.096	5.992	0.128

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	39	190	178	189	173	122	105	92
N.S.	1	1.00	0.30	1.48	1.39	1.48	1.35	0.95	0.82	0.72
time (sec)	N/A	0.079	0.012	0.023	1.452	1.337	7.741	1.127	6.197	0.152

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	109	213	201	202	153	121	106	92
N.S.	1	1.00	0.83	1.63	1.53	1.54	1.17	0.92	0.81	0.70
time (sec)	N/A	0.083	0.050	0.051	1.480	1.128	9.314	1.087	6.621	0.183

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	39	233	221	223	204	143	123	106
N.S.	1	1.00	0.25	1.50	1.43	1.44	1.32	0.92	0.79	0.68
time (sec)	N/A	0.101	0.013	0.128	1.507	1.250	14.979	1.102	6.678	0.182

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	39	253	241	245	231	160	140	117
N.S.	1	1.00	0.22	1.41	1.35	1.37	1.29	0.89	0.78	0.65
time (sec)	N/A	0.122	0.013	0.335	1.505	1.164	21.819	1.092	7.225	0.195

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	138	169	161	255	258	133	-1	129
N.S.	1	1.00	0.68	0.84	0.80	1.26	1.28	0.66	-0.00	0.64
time (sec)	N/A	0.114	0.219	0.013	1.461	1.316	30.595	1.209	0.000	0.193

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	127	149	141	234	231	119	-1	118
N.S.	1	1.00	0.71	0.84	0.79	1.31	1.30	0.67	-0.01	0.66
time (sec)	N/A	0.086	0.193	0.009	1.400	1.528	20.001	1.166	0.000	0.193

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	116	129	121	211	204	105	-1	107
N.S.	1	1.00	0.75	0.84	0.79	1.37	1.32	0.68	-0.01	0.69
time (sec)	N/A	0.070	0.182	0.007	1.375	1.490	13.568	1.244	0.000	0.168

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	98	96	88	190	151	91	37	93
N.S.	1	1.00	0.80	0.79	0.72	1.56	1.24	0.75	0.30	0.76
time (sec)	N/A	0.042	0.140	0.002	1.323	1.567	8.348	1.123	4.620	0.151

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	52	117	91	184	173	115	40	95
N.S.	1	1.00	0.42	0.95	0.74	1.50	1.41	0.93	0.33	0.77
time (sec)	N/A	0.044	0.009	0.005	1.458	1.199	7.465	1.217	6.031	0.164

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	54	146	120	189	175	160	-1	95
N.S.	1	1.00	0.42	1.14	0.94	1.48	1.37	1.25	-0.01	0.74
time (sec)	N/A	0.049	0.010	0.008	1.425	1.242	6.843	1.146	0.000	0.182

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	54	166	140	191	175	200	-1	95
N.S.	1	1.00	0.42	1.29	1.09	1.48	1.36	1.55	-0.01	0.74
time (sec)	N/A	0.050	0.011	0.010	1.448	0.706	7.823	1.079	0.000	0.206

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	54	186	160	187	167	240	-1	93
N.S.	1	1.00	0.43	1.48	1.27	1.48	1.33	1.90	-0.01	0.74
time (sec)	N/A	0.051	0.011	0.018	1.474	0.540	8.788	1.201	0.000	0.225

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	54	206	180	184	160	276	-1	90
N.S.	1	1.00	0.44	1.66	1.45	1.48	1.29	2.23	-0.01	0.73
time (sec)	N/A	0.052	0.011	0.034	1.509	1.344	9.863	1.119	0.000	0.237

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	68	150	167	111	75
N.S.	1	1.00	1.00	0.86	0.81	3.24	7.14	7.95	5.29	3.57
time (sec)	N/A	0.005	0.008	0.003	1.495	0.635	2.450	1.062	6.277	0.205

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	31	28	36	82	175	328	131	86
N.S.	1	1.00	0.70	0.64	0.82	1.86	3.98	7.45	2.98	1.95
time (sec)	N/A	0.011	0.014	0.006	1.566	1.086	2.978	1.092	6.849	0.191

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	42	39	56	93	604	354	151	97
N.S.	1	1.00	0.62	0.57	0.82	1.37	8.88	5.21	2.22	1.43
time (sec)	N/A	0.021	0.015	0.005	1.513	1.282	3.928	1.224	7.416	0.229

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	53	50	76	104	867	382	171	108
N.S.	1	1.00	0.58	0.54	0.83	1.13	9.42	4.15	1.86	1.17
time (sec)	N/A	0.030	0.017	0.006	1.551	1.884	4.969	1.295	8.173	0.234

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	64	61	96	115	1182	408	191	119
N.S.	1	1.00	0.55	0.53	0.83	0.99	10.19	3.52	1.65	1.03
time (sec)	N/A	0.045	0.018	0.006	1.565	1.599	6.262	1.170	8.840	0.255

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	75	72	116	126	1540	436	211	130
N.S.	1	1.00	0.54	0.51	0.83	0.90	11.00	3.11	1.51	0.93
time (sec)	N/A	0.056	0.021	0.007	1.532	2.719	7.799	1.246	9.629	0.258

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	86	83	136	137	1950	462	231	141
N.S.	1	1.00	0.52	0.51	0.83	0.84	11.89	2.82	1.41	0.86
time (sec)	N/A	0.072	0.022	0.008	1.589	3.171	9.066	1.233	10.465	0.273

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	27	24	40	28	61	34	25	27
N.S.	1	1.00	0.59	0.52	0.87	0.61	1.33	0.74	0.54	0.59
time (sec)	N/A	0.020	0.011	0.003	2.975	1.328	1.972	1.031	4.547	0.017

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	39	46	45	42	75	43	30	50
N.S.	1	1.00	0.62	0.73	0.71	0.67	1.19	0.68	0.48	0.79
time (sec)	N/A	0.016	0.013	0.006	2.935	1.120	4.564	1.172	0.028	0.048

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	22	19	26	23	44	23	20	22
N.S.	1	1.00	0.71	0.61	0.84	0.74	1.42	0.74	0.65	0.71
time (sec)	N/A	0.015	0.007	0.002	2.964	0.623	0.629	1.014	0.021	0.015

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	36	32	31	37	54	36	23	45
N.S.	1	1.00	0.80	0.71	0.69	0.82	1.20	0.80	0.51	1.00
time (sec)	N/A	0.010	0.015	0.004	2.967	1.154	2.707	1.045	0.034	0.029

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	12	11	11	27	11	16	15
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.80	0.73	1.07	1.00
time (sec)	N/A	0.002	0.002	0.003	1.315	1.280	0.200	1.042	0.019	0.009

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	20	19	29	22	29	16	37
N.S.	1	1.00	1.00	0.74	0.70	1.07	0.81	1.07	0.59	1.37
time (sec)	N/A	0.004	0.006	0.003	2.980	0.775	0.214	0.992	0.032	0.029

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	19	44	39	38	22	30
N.S.	1	1.00	1.00	0.83	0.63	1.47	1.30	1.27	0.73	1.00
time (sec)	N/A	0.016	0.005	0.005	2.939	1.424	1.245	1.029	0.028	0.023

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	34	21	35	19	40	19	35
N.S.	1	1.00	1.00	1.36	0.84	1.40	0.76	1.60	0.76	1.40
time (sec)	N/A	0.005	0.006	0.005	2.938	1.180	0.249	1.148	0.028	0.036

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	37	41	35	57	24	43	25	39
N.S.	1	1.00	0.95	1.05	0.90	1.46	0.62	1.10	0.64	1.00
time (sec)	N/A	0.016	0.018	0.006	2.810	0.831	1.670	1.206	0.030	0.036

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	20	34	42	27	25
N.S.	1	1.00	1.00	0.83	0.78	1.11	1.89	2.33	1.50	1.39
time (sec)	N/A	0.003	0.003	0.003	2.937	1.088	0.928	1.007	0.033	0.038

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	32	55	49	64	63	55	45	46
N.S.	1	1.00	0.56	0.96	0.86	1.12	1.11	0.96	0.79	0.81
time (sec)	N/A	0.022	0.005	0.006	3.012	1.743	3.197	1.052	0.031	0.050

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	27	34	40	28	61	52	28	32
N.S.	1	1.00	0.59	0.74	0.87	0.61	1.33	1.13	0.61	0.70
time (sec)	N/A	0.021	0.013	0.004	2.912	0.968	2.008	1.017	4.532	0.018

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	39	46	45	45	167	33	32	53
N.S.	1	1.00	0.62	0.73	0.71	0.71	2.65	0.52	0.51	0.84
time (sec)	N/A	0.016	0.013	0.006	3.000	1.156	4.655	1.077	4.563	0.145

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	22	29	26	23	44	32	23	22
N.S.	1	1.00	0.71	0.94	0.84	0.74	1.42	1.03	0.74	0.71
time (sec)	N/A	0.015	0.009	0.005	2.968	1.329	0.671	1.140	0.022	0.014

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	36	32	31	40	124	26	27	48
N.S.	1	1.00	0.80	0.71	0.69	0.89	2.76	0.58	0.60	1.07
time (sec)	N/A	0.010	0.016	0.006	2.952	1.139	2.684	1.088	0.035	0.090

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	22	11	18	27	11	18	15
N.S.	1	1.00	1.00	1.47	0.73	1.20	1.80	0.73	1.20	1.00
time (sec)	N/A	0.002	0.002	0.003	1.277	1.509	0.203	1.081	0.020	0.009

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	20	19	32	22	19	18	41
N.S.	1	1.00	1.00	0.74	0.70	1.19	0.81	0.70	0.67	1.52
time (sec)	N/A	0.003	0.006	0.003	2.943	0.637	0.215	1.000	0.024	0.053

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	35	28	76	40	32	30
N.S.	1	1.00	1.00	0.83	1.17	0.93	2.53	1.33	1.07	1.00
time (sec)	N/A	0.015	0.004	0.006	2.865	1.341	1.320	1.176	4.552	0.021

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	34	21	36	20	39	21	39
N.S.	1	1.00	1.00	1.36	0.84	1.44	0.80	1.56	0.84	1.56
time (sec)	N/A	0.005	0.007	0.005	2.923	0.728	0.245	1.144	0.027	0.047

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	37	41	51	38	97	45	35	39
N.S.	1	1.00	0.95	1.05	1.31	0.97	2.49	1.15	0.90	1.00
time (sec)	N/A	0.016	0.027	0.006	2.993	1.145	1.733	1.072	4.655	0.038

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	25	14	21	76	73	31	25
N.S.	1	1.00	1.00	1.39	0.78	1.17	4.22	4.06	1.72	1.39
time (sec)	N/A	0.003	0.003	0.003	2.897	1.312	0.966	1.050	0.033	0.032

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	32	55	65	45	139	57	49	46
N.S.	1	1.00	0.56	0.96	1.14	0.79	2.44	1.00	0.86	0.81
time (sec)	N/A	0.022	0.006	0.005	3.018	1.006	3.275	1.055	0.030	0.041

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	27	34	40	28	61	34	28	27
N.S.	1	1.00	0.59	0.74	0.87	0.61	1.33	0.74	0.61	0.59
time (sec)	N/A	0.020	0.010	0.006	2.941	1.289	1.918	0.967	5.314	0.016

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	49	61	57	42	167	44	-1	50
N.S.	1	1.00	0.68	0.85	0.79	0.58	2.32	0.61	-0.01	0.69
time (sec)	N/A	0.019	0.012	0.007	2.916	1.218	4.563	1.186	0.000	0.045

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	22	29	26	23	44	23	23	22
N.S.	1	1.00	0.71	0.94	0.84	0.74	1.42	0.74	0.74	0.71
time (sec)	N/A	0.014	0.007	0.005	2.911	1.039	0.656	1.065	5.511	0.015

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	46	47	43	37	124	37	-1	45
N.S.	1	1.00	0.85	0.87	0.80	0.69	2.30	0.69	-0.02	0.83
time (sec)	N/A	0.013	0.014	0.006	2.894	0.951	2.799	1.156	0.000	0.030

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	22	11	11	27	11	11	15
N.S.	1	1.00	1.00	1.47	0.73	0.73	1.80	0.73	0.73	1.00
time (sec)	N/A	0.002	0.002	0.004	1.303	0.766	0.202	1.033	0.060	0.009

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	37	35	31	29	22	30	29	37
N.S.	1	1.00	1.03	0.97	0.86	0.81	0.61	0.83	0.81	1.03
time (sec)	N/A	0.006	0.006	0.003	2.974	0.782	0.215	1.112	5.032	0.028

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	19	28	82	24	24	30
N.S.	1	1.00	1.00	0.83	0.63	0.93	2.73	0.80	0.80	1.00
time (sec)	N/A	0.017	0.005	0.005	3.009	0.902	1.371	1.013	5.338	0.019

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	48	48	33	35	19	44	39	35
N.S.	1	1.00	1.41	1.41	0.97	1.03	0.56	1.29	1.15	1.03
time (sec)	N/A	0.008	0.010	0.003	2.930	1.274	0.249	1.231	5.548	0.037

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	55	41	35	38	97	29	29	39
N.S.	1	1.00	1.41	1.05	0.90	0.97	2.49	0.74	0.74	1.00
time (sec)	N/A	0.016	0.008	0.004	2.959	0.815	1.738	1.071	5.330	0.031

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	25	14	20	76	42	31	18
N.S.	1	1.00	1.00	1.39	0.78	1.11	4.22	2.33	1.72	1.00
time (sec)	N/A	0.003	0.003	0.003	3.014	0.920	0.975	1.096	5.098	0.035

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	32	55	49	45	139	41	43	46
N.S.	1	1.00	0.56	0.96	0.86	0.79	2.44	0.72	0.75	0.81
time (sec)	N/A	0.021	0.005	0.005	2.955	1.057	3.360	0.981	5.190	0.042

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	27	24	40	28	68	37	27	27
N.S.	1	1.00	0.59	0.52	0.87	0.61	1.48	0.80	0.59	0.59
time (sec)	N/A	0.022	0.014	0.004	2.948	0.989	2.081	1.060	5.127	0.016

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	48	55	45	72	83	0	-1	54
N.S.	1	1.00	0.67	0.76	0.62	1.00	1.15	0.00	-0.01	0.75
time (sec)	N/A	0.021	0.016	0.008	2.939	0.993	4.672	0.000	0.000	0.051

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	22	19	26	23	49	25	22	22
N.S.	1	1.00	0.71	0.61	0.84	0.74	1.58	0.81	0.71	0.71
time (sec)	N/A	0.014	0.009	0.004	3.032	1.237	0.686	0.978	5.145	0.014

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	43	41	31	67	61	0	-1	49
N.S.	1	1.00	0.80	0.76	0.57	1.24	1.13	0.00	-0.02	0.91
time (sec)	N/A	0.014	0.016	0.005	2.982	0.904	2.759	0.000	0.000	0.035

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	12	11	18	31	12	11	15
N.S.	1	1.00	1.00	0.80	0.73	1.20	2.07	0.80	0.73	1.00
time (sec)	N/A	0.002	0.002	0.002	1.308	0.764	0.218	1.083	0.070	0.009

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	29	19	59	34	0	28	41
N.S.	1	1.00	1.00	0.81	0.53	1.64	0.94	0.00	0.78	1.14
time (sec)	N/A	0.006	0.008	0.003	2.992	0.738	0.385	0.000	5.027	0.029

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	35	52	44	24	24	30
N.S.	1	1.00	1.00	0.83	1.17	1.73	1.47	0.80	0.80	1.00
time (sec)	N/A	0.015	0.004	0.006	2.929	1.102	1.264	1.171	4.732	0.018

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	49	43	21	64	32	0	41	39
N.S.	1	1.00	1.44	1.26	0.62	1.88	0.94	0.00	1.21	1.15
time (sec)	N/A	0.008	0.007	0.005	2.915	0.805	0.417	0.000	4.775	0.038

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	55	41	51	65	27	29	29	39
N.S.	1	1.00	1.41	1.05	1.31	1.67	0.69	0.74	0.74	1.00
time (sec)	N/A	0.016	0.014	0.004	2.897	1.012	1.701	1.212	4.792	0.026

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	37	0	31	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.06	0.00	1.72	1.00
time (sec)	N/A	0.003	0.003	0.004	2.892	0.694	1.000	0.000	4.740	0.039

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	C	C	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	32	55	65	72	68	43	43	46
N.S.	1	1.00	0.56	0.96	1.14	1.26	1.19	0.75	0.75	0.81
time (sec)	N/A	0.021	0.005	0.007	2.985	1.056	3.367	1.112	4.695	0.029

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	39	36	53	35	68	46	36	39
N.S.	1	1.00	0.70	0.64	0.95	0.62	1.21	0.82	0.64	0.70
time (sec)	N/A	0.031	0.019	0.006	1.322	0.801	0.827	1.145	4.674	0.025

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	62	59	51	124	95	54	-1	63
N.S.	1	1.00	0.85	0.81	0.70	1.70	1.30	0.74	-0.01	0.86
time (sec)	N/A	0.020	0.026	0.006	1.351	0.678	4.030	1.037	0.000	0.069

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	27	25	33	23	44	30	24	27
N.S.	1	1.00	0.75	0.69	0.92	0.64	1.22	0.83	0.67	0.75
time (sec)	N/A	0.022	0.013	0.005	1.345	1.104	0.488	1.092	4.729	0.023

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	39	31	93	42	40	56	51
N.S.	1	1.00	1.00	0.80	0.63	1.90	0.86	0.82	1.14	1.04
time (sec)	N/A	0.012	0.018	0.005	1.262	1.253	2.253	1.182	4.818	0.050

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	20	13	13	15
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87	1.00
time (sec)	N/A	0.003	0.002	0.002	1.397	1.038	0.382	0.998	4.584	0.012

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	21	13	59	17	23	20	28
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	0.92	0.80	1.12
time (sec)	N/A	0.006	0.004	0.003	1.356	1.342	1.023	1.242	0.123	0.023

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	17	60	19	22	19	25
N.S.	1	1.00	1.00	1.16	0.68	2.40	0.76	0.88	0.76	1.00
time (sec)	N/A	0.017	0.005	0.005	1.380	0.896	1.061	0.993	4.873	0.023

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	18	17	17	19	30	17	19
N.S.	1	1.00	1.00	0.95	0.89	0.89	1.00	1.58	0.89	1.00
time (sec)	N/A	0.005	0.004	0.005	1.354	1.162	0.686	1.156	4.600	0.040

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	61	48	36	105	42	51	38	50
N.S.	1	1.00	1.22	0.96	0.72	2.10	0.84	1.02	0.76	1.00
time (sec)	N/A	0.026	0.053	0.005	1.347	1.381	2.251	1.086	4.753	0.066

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	29	26	36	27	46	55	25	31
N.S.	1	1.00	0.66	0.59	0.82	0.61	1.05	1.25	0.57	0.70
time (sec)	N/A	0.010	0.006	0.003	1.265	1.661	0.903	1.147	4.622	0.055

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	37	68	56	135	97	75	57	62
N.S.	1	1.00	0.50	0.92	0.76	1.82	1.31	1.01	0.77	0.84
time (sec)	N/A	0.038	0.007	0.005	1.290	0.899	4.249	1.113	4.732	0.074

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	38	36	53	46	68	52	41	38
N.S.	1	1.00	0.69	0.65	0.96	0.84	1.24	0.95	0.75	0.69
time (sec)	N/A	0.033	0.017	0.007	1.313	0.999	0.942	0.985	4.721	0.029

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	71	57	49	159	71	51	-1	60
N.S.	1	1.00	1.04	0.84	0.72	2.34	1.04	0.75	-0.01	0.88
time (sec)	N/A	0.023	0.034	0.008	1.325	1.284	3.257	1.150	0.000	0.090

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	24	23	32	34	41	32	22	24
N.S.	1	1.00	0.75	0.72	1.00	1.06	1.28	1.00	0.69	0.75
time (sec)	N/A	0.022	0.011	0.004	1.377	0.920	0.575	1.062	4.698	0.027

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	59	37	29	130	37	39	36	46
N.S.	1	1.00	1.37	0.86	0.67	3.02	0.86	0.91	0.84	1.07
time (sec)	N/A	0.013	0.052	0.006	1.391	0.756	1.706	1.230	0.091	0.064

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	24	24	14	14	16
N.S.	1	1.00	1.00	0.94	0.88	1.50	1.50	0.88	0.88	1.00
time (sec)	N/A	0.003	0.003	0.003	1.324	0.968	0.540	1.046	0.040	0.014

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	23	17	14	14	16
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88	1.00
time (sec)	N/A	0.002	0.004	0.003	1.332	1.040	0.629	1.117	0.029	0.038

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	33	43	31	126	184	39	33	41
N.S.	1	1.00	0.80	1.05	0.76	3.07	4.49	0.95	0.80	1.00
time (sec)	N/A	0.027	0.006	0.005	1.360	0.611	1.731	1.117	4.776	0.037

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	26	34	35	46	50	35	28
N.S.	1	1.00	0.71	0.68	0.89	0.92	1.21	1.32	0.92	0.74
time (sec)	N/A	0.008	0.006	0.005	1.294	1.037	0.856	1.150	4.635	0.051

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	68	35	63	51	171	73	72	53	60
N.S.	1	0.99	0.51	0.91	0.74	2.48	1.06	1.04	0.77	0.87
time (sec)	N/A	0.039	0.007	0.006	1.311	1.116	3.386	1.119	4.937	0.065

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	40	37	54	50	233	106	38	42
N.S.	1	1.00	0.61	0.56	0.82	0.76	3.53	1.61	0.58	0.64
time (sec)	N/A	0.016	0.008	0.007	1.313	0.739	1.233	1.160	5.125	0.080

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	90	75	89	227	367	65	-1	72
N.S.	1	1.00	0.99	0.82	0.98	2.49	4.03	0.71	-0.01	0.79
time (sec)	N/A	0.029	0.138	0.008	1.415	1.171	5.122	1.209	0.000	0.132

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	39	36	52	58	138	44	38	39
N.S.	1	1.00	0.72	0.67	0.96	1.07	2.56	0.81	0.70	0.72
time (sec)	N/A	0.031	0.018	0.005	1.351	1.139	1.092	1.021	5.201	0.031

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	80	54	65	199	303	51	-1	58
N.S.	1	1.00	1.25	0.84	1.02	3.11	4.73	0.80	-0.02	0.91
time (sec)	N/A	0.020	0.117	0.008	1.372	0.719	2.992	1.051	0.000	0.107

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	28	25	33	47	92	24	24	28
N.S.	1	1.00	0.78	0.69	0.92	1.31	2.56	0.67	0.67	0.78
time (sec)	N/A	0.022	0.012	0.005	1.265	0.877	1.073	1.100	5.174	0.026

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	34	37	44	17	17	21
N.S.	1	1.00	1.00	0.86	1.62	1.76	2.10	0.81	0.81	1.00
time (sec)	N/A	0.005	0.005	0.006	1.309	1.108	0.756	1.076	5.128	0.063

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	35	46	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	1.94	2.56	0.78	0.78	1.00
time (sec)	N/A	0.003	0.004	0.003	1.332	0.494	1.002	1.018	4.944	0.014

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	29	26	31	47	95	27	28	29
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72	0.74
time (sec)	N/A	0.005	0.008	0.003	1.342	1.061	0.828	1.038	4.885	0.053

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	36	57	45	197	740	50	47	54
N.S.	1	1.00	0.61	0.97	0.76	3.34	12.54	0.85	0.80	0.92
time (sec)	N/A	0.035	0.007	0.005	1.338	1.022	2.889	1.145	5.199	0.053

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	42	39	50	59	165	64	42	42
N.S.	1	1.00	0.70	0.65	0.83	0.98	2.75	1.07	0.70	0.70
time (sec)	N/A	0.014	0.009	0.004	1.343	0.629	1.287	1.203	5.181	0.078

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	92	37	78	66	241	864	73	73	71
N.S.	1	1.05	0.42	0.89	0.75	2.74	9.82	0.83	0.83	0.81
time (sec)	N/A	0.050	0.008	0.006	1.354	0.676	5.169	1.075	5.253	0.090

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	53	48	72	72	354	121	78	53
N.S.	1	1.00	0.62	0.56	0.84	0.84	4.12	1.41	0.91	0.62
time (sec)	N/A	0.022	0.011	0.006	1.398	1.163	1.731	1.261	5.000	0.088

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	114	111	285	359	3181	91	-1	94
N.S.	1	1.00	0.87	0.85	2.18	2.74	24.28	0.69	-0.01	0.72
time (sec)	N/A	0.054	0.195	0.033	1.686	2.110	13.422	1.191	0.000	0.231

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	61	58	92	102	454	72	80	61
N.S.	1	1.00	0.65	0.62	0.98	1.09	4.83	0.77	0.85	0.65
time (sec)	N/A	0.053	0.031	0.007	1.638	1.293	6.389	1.126	4.945	0.037

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	101	88	255	331	2980	78	-1	80
N.S.	1	1.00	0.95	0.83	2.41	3.12	28.11	0.74	-0.01	0.75
time (sec)	N/A	0.042	0.132	0.015	1.673	2.086	9.036	1.200	0.000	0.196

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	50	47	73	91	364	55	63	50
N.S.	1	1.00	0.67	0.63	0.97	1.21	4.85	0.73	0.84	0.67
time (sec)	N/A	0.045	0.025	0.008	1.364	0.904	6.081	0.980	4.880	0.033

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	103	59	95	17	68	21
N.S.	1	1.00	1.00	0.86	4.90	2.81	4.52	0.81	3.24	1.00
time (sec)	N/A	0.005	0.006	0.005	1.334	1.522	1.472	1.223	4.764	0.110

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	36	53	80	272	41	41	39
N.S.	1	1.00	0.66	0.61	0.90	1.36	4.61	0.69	0.69	0.66
time (sec)	N/A	0.033	0.019	0.007	1.407	0.755	5.911	0.946	4.805	0.030

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	31	28	85	71	199	29	68	31
N.S.	1	1.00	0.70	0.64	1.93	1.61	4.52	0.66	1.55	0.70
time (sec)	N/A	0.011	0.012	0.005	1.380	0.830	1.599	0.986	4.876	0.103

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	25	33	69	180	24	24	28
N.S.	1	1.00	0.74	0.66	0.87	1.82	4.74	0.63	0.63	0.74
time (sec)	N/A	0.023	0.014	0.006	1.301	0.938	5.861	0.826	4.824	0.027

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	42	39	70	82	517	43	70	42
N.S.	1	1.00	0.62	0.57	1.03	1.21	7.60	0.63	1.03	0.62
time (sec)	N/A	0.021	0.013	0.007	1.388	0.777	1.870	1.081	4.761	0.098

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	57	90	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	3.17	5.00	0.78	0.78	1.00
time (sec)	N/A	0.004	0.005	0.004	1.332	0.945	5.728	0.970	4.597	0.014

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	51	48	61	91	1265	55	61	51
N.S.	1	1.00	0.66	0.62	0.79	1.18	16.43	0.71	0.79	0.66
time (sec)	N/A	0.016	0.013	0.004	1.367	1.078	2.192	1.117	4.602	0.081

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	36	85	73	329	5250	81	75	76
N.S.	1	1.00	0.38	0.89	0.77	3.46	55.26	0.85	0.79	0.80
time (sec)	N/A	0.059	0.006	0.008	1.356	1.006	7.904	1.084	4.853	0.072

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	64	61	82	103	400	90	76	64
N.S.	1	1.00	0.64	0.61	0.82	1.03	4.00	0.90	0.76	0.64
time (sec)	N/A	0.026	0.013	0.007	1.433	1.612	2.928	1.122	4.715	0.116

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	132	37	108	96	373	5540	104	113	93
N.S.	1	1.05	0.29	0.86	0.76	2.96	43.97	0.83	0.90	0.74
time (sec)	N/A	0.078	0.009	0.007	1.390	0.995	12.428	1.130	4.959	0.110

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	75	72	108	116	668	147	97	75
N.S.	1	1.00	0.57	0.55	0.82	0.88	5.06	1.11	0.73	0.57
time (sec)	N/A	0.041	0.014	0.008	1.422	1.158	3.837	1.159	4.803	0.131

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	27	24	40	23	44	34	21	27
N.S.	1	1.00	0.59	0.52	0.87	0.50	0.96	0.74	0.46	0.59
time (sec)	N/A	0.019	0.009	0.003	2.886	0.974	1.287	1.127	0.025	0.017

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	34	34	33	37	39	36	25	45
N.S.	1	1.00	0.76	0.76	0.73	0.82	0.87	0.80	0.56	1.00
time (sec)	N/A	0.009	0.012	0.006	2.906	0.816	0.731	1.088	0.031	0.056

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	22	19	26	18	27	23	15	22
N.S.	1	1.00	0.71	0.61	0.84	0.58	0.87	0.74	0.48	0.71
time (sec)	N/A	0.014	0.006	0.003	2.914	0.910	0.378	1.133	0.018	0.014

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	20	19	29	22	29	17	37
N.S.	1	1.00	1.00	0.74	0.70	1.07	0.81	1.07	0.63	1.37
time (sec)	N/A	0.006	0.005	0.006	2.923	0.907	0.226	1.072	0.033	0.043

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	12	11	11	10	11	9	15
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.67	0.73	0.60	1.00
time (sec)	N/A	0.002	0.001	0.003	1.318	0.909	0.151	1.118	0.015	0.009

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	7	6	16	7	16	6	20
N.S.	1	1.00	1.00	0.70	0.60	1.60	0.70	1.60	0.60	2.00
time (sec)	N/A	0.001	0.004	0.002	2.964	0.883	0.153	1.168	0.027	0.017

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	15	9	35	8	29	12	20
N.S.	1	1.00	1.00	0.75	0.45	1.75	0.40	1.45	0.60	1.00
time (sec)	N/A	0.010	0.003	0.003	2.961	1.044	1.018	1.064	0.037	0.018

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	18	15	23	12	18
N.S.	1	1.00	1.00	0.83	0.78	1.00	0.83	1.28	0.67	1.00
time (sec)	N/A	0.003	0.003	0.003	2.979	0.903	0.759	1.117	0.022	0.030

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	37	30	24	57	44	43	25	39
N.S.	1	1.00	0.95	0.77	0.62	1.46	1.13	1.10	0.64	1.00
time (sec)	N/A	0.017	0.011	0.005	2.939	0.968	2.101	1.105	0.028	0.038

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	27	22	29	28	32	42	19	25
N.S.	1	1.00	0.73	0.59	0.78	0.76	0.86	1.14	0.51	0.68
time (sec)	N/A	0.007	0.005	0.004	2.863	1.079	1.294	1.220	0.021	0.040

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	32	44	38	64	63	55	33	46
N.S.	1	1.00	0.56	0.77	0.67	1.12	1.11	0.96	0.58	0.81
time (sec)	N/A	0.023	0.005	0.005	2.949	0.914	3.890	1.180	0.029	0.043

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	27	34	40	23	46	43	23	27
N.S.	1	1.00	0.59	0.74	0.87	0.50	1.00	0.93	0.50	0.59
time (sec)	N/A	0.019	0.009	0.004	2.951	1.104	1.288	1.241	0.041	0.017

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	34	34	33	40	39	26	27	48
N.S.	1	1.00	0.76	0.76	0.73	0.89	0.87	0.58	0.60	1.07
time (sec)	N/A	0.010	0.012	0.007	2.930	0.759	0.731	1.138	0.030	0.091

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	22	29	26	18	29	23	18	22
N.S.	1	1.00	0.71	0.94	0.84	0.58	0.94	0.74	0.58	0.71
time (sec)	N/A	0.014	0.006	0.006	2.899	1.171	0.383	0.979	0.017	0.014

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	20	19	32	22	19	19	40
N.S.	1	1.00	1.00	0.74	0.70	1.19	0.81	0.70	0.70	1.48
time (sec)	N/A	0.005	0.006	0.006	2.879	0.946	0.226	1.178	0.022	0.059

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	22	11	11	12	11	11	15
N.S.	1	1.00	1.00	1.47	0.73	0.73	0.80	0.73	0.73	1.00
time (sec)	N/A	0.002	0.002	0.003	1.334	1.053	0.150	1.128	4.556	0.009

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	7	6	19	7	6	6	24
N.S.	1	1.00	1.00	0.70	0.60	1.90	0.70	0.60	0.60	2.40
time (sec)	N/A	0.001	0.004	0.003	2.978	0.720	0.154	1.121	0.008	0.027

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	15	25	18	26	31	20	20
N.S.	1	1.00	1.00	0.75	1.25	0.90	1.30	1.55	1.00	1.00
time (sec)	N/A	0.010	0.003	0.004	2.989	0.847	1.064	0.938	0.116	0.019

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	25	14	14	41	33	14	18
N.S.	1	1.00	1.00	1.39	0.78	0.78	2.28	1.83	0.78	1.00
time (sec)	N/A	0.003	0.003	0.003	2.902	1.090	0.795	1.115	0.022	0.042

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	37	30	40	38	99	45	35	39
N.S.	1	1.00	0.95	0.77	1.03	0.97	2.54	1.15	0.90	1.00
time (sec)	N/A	0.016	0.011	0.006	2.932	1.359	2.172	1.206	0.026	0.036

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	27	32	29	21	80	73	22	25
N.S.	1	1.00	0.73	0.86	0.78	0.57	2.16	1.97	0.59	0.68
time (sec)	N/A	0.007	0.004	0.004	2.972	1.200	1.329	1.132	0.021	0.056

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	32	44	54	45	136	57	49	46
N.S.	1	1.00	0.56	0.77	0.95	0.79	2.39	1.00	0.86	0.81
time (sec)	N/A	0.024	0.005	0.005	2.926	0.918	3.963	0.977	4.505	0.035

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	27	34	40	23	44	34	22	27
N.S.	1	1.00	0.59	0.74	0.87	0.50	0.96	0.74	0.48	0.59
time (sec)	N/A	0.019	0.009	0.005	2.932	1.074	1.252	1.122	4.740	0.017

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	43	49	45	37	39	37	-1	45
N.S.	1	1.00	0.80	0.91	0.83	0.69	0.72	0.69	-0.02	0.83
time (sec)	N/A	0.013	0.013	0.006	2.809	1.092	0.718	1.236	0.000	0.055

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	22	29	26	18	27	23	18	22
N.S.	1	1.00	0.71	0.94	0.84	0.58	0.87	0.74	0.58	0.71
time (sec)	N/A	0.013	0.006	0.004	2.920	0.897	0.366	0.966	4.861	0.015

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	35	31	29	22	30	29	37
N.S.	1	1.00	1.00	0.97	0.86	0.81	0.61	0.83	0.81	1.03
time (sec)	N/A	0.008	0.006	0.006	2.908	0.961	0.224	1.183	0.101	0.044

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	22	11	11	10	11	11	15
N.S.	1	1.00	1.00	1.47	0.73	0.73	0.67	0.73	0.73	1.00
time (sec)	N/A	0.002	0.002	0.003	1.272	1.037	0.152	1.071	0.145	0.010

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	43	22	18	16	7	17	16	20
N.S.	1	1.00	2.26	1.16	0.95	0.84	0.37	0.89	0.84	1.05
time (sec)	N/A	0.003	0.003	0.002	2.958	0.708	0.156	0.985	4.746	0.018

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	15	9	18	26	14	20	20
N.S.	1	1.00	1.00	0.75	0.45	0.90	1.30	0.70	1.00	1.00
time (sec)	N/A	0.010	0.003	0.003	2.909	0.929	1.071	1.062	0.122	0.016

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	25	14	18	37	23	14	18
N.S.	1	1.00	1.00	1.39	0.78	1.00	2.06	1.28	0.78	1.00
time (sec)	N/A	0.003	0.002	0.004	2.842	0.888	0.805	1.062	4.747	0.029

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	54	30	24	38	99	29	29	39
N.S.	1	1.00	1.38	0.77	0.62	0.97	2.54	0.74	0.74	1.00
time (sec)	N/A	0.016	0.021	0.006	2.930	1.391	2.133	1.099	4.881	0.028

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	25	32	29	28	76	42	31	25
N.S.	1	1.00	0.68	0.86	0.78	0.76	2.05	1.14	0.84	0.68
time (sec)	N/A	0.007	0.004	0.004	2.961	0.715	1.349	1.132	4.874	0.037

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	32	44	38	45	136	41	57	46
N.S.	1	1.00	0.56	0.77	0.67	0.79	2.39	0.72	1.00	0.81
time (sec)	N/A	0.023	0.005	0.007	2.921	1.112	3.821	1.043	5.319	0.036

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	27	24	40	23	49	36	23	27
N.S.	1	1.00	0.59	0.52	0.87	0.50	1.07	0.78	0.50	0.59
time (sec)	N/A	0.019	0.009	0.005	2.912	0.626	1.213	0.995	5.295	0.016

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	A	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	43	43	33	67	53	0	-1	49
N.S.	1	1.00	0.80	0.80	0.61	1.24	0.98	0.00	-0.02	0.91
time (sec)	N/A	0.013	0.014	0.005	2.946	0.642	0.888	0.000	0.000	0.044

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	22	19	26	18	31	24	18	22
N.S.	1	1.00	0.71	0.61	0.84	0.58	1.00	0.77	0.58	0.71
time (sec)	N/A	0.014	0.008	0.004	2.963	0.950	0.375	1.084	5.067	0.014

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	29	19	59	36	0	31	41
N.S.	1	1.00	1.00	0.81	0.53	1.64	1.00	0.00	0.86	1.14
time (sec)	N/A	0.007	0.006	0.006	2.927	0.680	0.392	0.000	0.124	0.033

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	12	11	11	14	11	11	15
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.93	0.73	0.73	1.00
time (sec)	N/A	0.002	0.002	0.003	1.324	1.041	0.164	1.135	4.986	0.009

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	6	47	17	0	15	24
N.S.	1	1.00	1.00	0.84	0.32	2.47	0.89	0.00	0.79	1.26
time (sec)	N/A	0.003	0.003	0.003	2.979	0.852	0.320	0.000	0.107	0.024

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	15	25	43	8	14	14	20
N.S.	1	1.00	1.00	0.75	1.25	2.15	0.40	0.70	0.70	1.00
time (sec)	N/A	0.010	0.003	0.004	2.945	0.735	1.019	1.132	5.272	0.016

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	15	0	14	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.83	0.00	0.78	1.00
time (sec)	N/A	0.003	0.002	0.003	2.951	0.768	0.765	0.000	5.038	0.041

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	C	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	54	30	40	65	46	29	29	39
N.S.	1	1.00	1.38	0.77	1.03	1.67	1.18	0.74	0.74	1.00
time (sec)	N/A	0.016	0.022	0.005	2.924	0.730	2.062	1.074	5.179	0.024

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	25	22	29	21	36	0	31	25
N.S.	1	1.00	0.68	0.59	0.78	0.57	0.97	0.00	0.84	0.68
time (sec)	N/A	0.007	0.006	0.004	2.963	0.890	1.286	0.000	5.082	0.055

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	C	C	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	32	44	54	72	65	43	60	46
N.S.	1	1.00	0.56	0.77	0.95	1.26	1.14	0.75	1.05	0.81
time (sec)	N/A	0.023	0.005	0.006	2.888	0.872	3.876	1.107	5.070	0.025

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	21	11	65	14	22	11	28
N.S.	1	1.00	1.00	1.24	0.65	3.82	0.82	1.29	0.65	1.65
time (sec)	N/A	0.002	0.007	0.003	1.304	0.704	0.948	1.144	0.036	0.023

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	21	11	58	39	27	15	35
N.S.	1	1.00	1.00	1.24	0.65	3.41	2.29	1.59	0.88	2.06
time (sec)	N/A	0.003	0.008	0.004	2.826	0.897	1.014	1.110	0.036	0.040

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	21	24	57	39	23	20	28
N.S.	1	1.00	1.00	0.84	0.96	2.28	1.56	0.92	0.80	1.12
time (sec)	N/A	0.006	0.007	0.003	1.329	1.133	1.026	1.138	0.082	0.023

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	21	12	68	17	28	25	35
N.S.	1	1.00	1.00	0.81	0.46	2.62	0.65	1.08	0.96	1.35
time (sec)	N/A	0.005	0.007	0.003	1.382	0.916	0.968	1.203	0.092	0.038

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	21	13	59	17	22	20	28
N.S.	1	1.00	1.00	1.11	0.68	3.11	0.89	1.16	1.05	1.47
time (sec)	N/A	0.006	0.008	0.005	1.329	0.985	0.981	1.119	5.119	0.025

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	21	13	62	46	28	25	35
N.S.	1	1.00	1.00	1.11	0.68	3.26	2.42	1.47	1.32	1.84
time (sec)	N/A	0.003	0.008	0.007	2.858	0.934	1.040	1.054	0.101	0.029

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	23	26	74	46	25	22	30
N.S.	1	1.00	1.00	0.85	0.96	2.74	1.70	0.93	0.81	1.11
time (sec)	N/A	0.006	0.007	0.005	1.387	0.923	1.051	1.142	0.129	0.039

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	23	14	74	20	30	27	37
N.S.	1	1.00	1.00	0.82	0.50	2.64	0.71	1.07	0.96	1.32
time (sec)	N/A	0.005	0.006	0.005	1.347	0.861	0.997	1.159	5.095	0.042

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	21	13	59	17	23	20	28
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	0.92	0.80	1.12
time (sec)	N/A	0.005	0.006	0.001	1.355	0.900	1.015	1.084	0.002	0.001

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	21	13	72	46	28	25	35
N.S.	1	1.00	1.00	0.81	0.50	2.77	1.77	1.08	0.96	1.35
time (sec)	N/A	0.005	0.006	0.005	2.956	0.693	1.074	1.146	0.114	0.042

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	23	26	63	46	25	22	30
N.S.	1	1.00	1.00	0.85	0.96	2.33	1.70	0.93	0.81	1.11
time (sec)	N/A	0.006	0.006	0.003	1.320	0.973	1.091	1.053	0.125	0.026

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	23	14	74	20	30	27	37
N.S.	1	1.00	1.00	0.82	0.50	2.64	0.71	1.07	0.96	1.32
time (sec)	N/A	0.005	0.007	0.005	1.322	0.925	1.023	1.274	5.125	0.041

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	6	23	19	9	14	24
N.S.	1	1.00	1.00	0.94	0.38	1.44	1.19	0.56	0.88	1.50
time (sec)	N/A	0.002	0.004	0.004	2.891	0.832	1.021	1.074	4.738	0.033

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	104	16	16	16	202	0	24	0
N.S.	1	1.00	6.12	0.94	0.94	0.94	11.88	0.00	1.41	0.00
time (sec)	N/A	0.011	0.108	0.013	1.881	0.890	10.061	0.000	5.193	0.369

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	127	104	0	16	18	105	0	-1	0
N.S.	1	7.47	6.12	0.00	0.94	1.06	6.18	0.00	-0.06	0.00
time (sec)	N/A	0.066	0.069	0.278	1.890	0.862	5.288	0.000	0.000	1.894

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	103	14	13	16	97	0	13	0
N.S.	1	1.00	6.87	0.93	0.87	1.07	6.47	0.00	0.87	0.00
time (sec)	N/A	0.012	0.109	0.007	1.906	1.188	52.957	0.000	5.431	0.494

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	123	103	0	13	26	94	0	-1	0
N.S.	1	8.20	6.87	0.00	0.87	1.73	6.27	0.00	-0.07	0.00
time (sec)	N/A	0.065	0.065	0.327	1.915	0.943	5.586	0.000	0.000	0.695

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	50	47	64	57	1795	57	55	50
N.S.	1	1.00	0.62	0.59	0.80	0.71	22.44	0.71	0.69	0.62
time (sec)	N/A	0.049	0.025	0.006	1.374	0.797	2.848	0.922	5.254	0.031

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	36	47	46	700	43	44	39
N.S.	1	1.00	0.66	0.61	0.80	0.78	11.86	0.73	0.75	0.66
time (sec)	N/A	0.035	0.018	0.006	1.342	0.740	1.877	1.028	4.803	0.026

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	25	30	34	223	29	33	39
N.S.	1	1.00	0.74	0.66	0.79	0.89	5.87	0.76	0.87	1.03
time (sec)	N/A	0.023	0.013	0.007	1.318	0.702	1.216	1.204	4.720	0.025

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	42	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.33	0.78	0.78	1.00
time (sec)	N/A	0.003	0.003	0.005	1.370	0.630	0.195	1.119	4.657	0.013

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	126	0	97	102	46	98	115	133
N.S.	1	1.00	1.25	0.00	0.96	1.01	0.46	0.97	1.14	1.32
time (sec)	N/A	0.079	0.056	0.291	3.035	1.416	1.066	2.406	4.735	0.103

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	37	0	103	155	42	115	125	139
N.S.	1	1.00	0.35	0.00	0.96	1.45	0.39	1.07	1.17	1.30
time (sec)	N/A	0.071	0.009	0.316	2.912	0.829	1.195	2.482	4.880	0.194

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	39	0	155	199	42	140	217	158
N.S.	1	1.00	0.29	0.00	1.15	1.47	0.31	1.04	1.61	1.17
time (sec)	N/A	0.094	0.008	0.282	2.914	0.822	1.387	2.369	5.109	0.199

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	50	47	64	57	1795	57	55	50
N.S.	1	1.00	0.62	0.59	0.80	0.71	22.44	0.71	0.69	0.62
time (sec)	N/A	0.047	0.026	0.006	1.303	0.793	2.941	0.982	4.629	0.031

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	36	47	46	700	43	44	39
N.S.	1	1.00	0.66	0.61	0.80	0.78	11.86	0.73	0.75	0.66
time (sec)	N/A	0.036	0.018	0.006	1.354	0.774	1.996	0.966	4.663	0.030

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	25	30	35	66	29	33	39
N.S.	1	1.00	0.74	0.66	0.79	0.92	1.74	0.76	0.87	1.03
time (sec)	N/A	0.023	0.014	0.006	1.351	0.827	0.751	1.084	4.715	0.026

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	42	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.33	0.78	0.78	1.00
time (sec)	N/A	0.004	0.004	0.003	1.327	0.772	0.374	1.056	4.583	0.012

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	93	0	97	122	46	98	125	133
N.S.	1	1.00	0.92	0.00	0.96	1.21	0.46	0.97	1.24	1.32
time (sec)	N/A	0.066	0.037	0.284	2.971	0.944	1.116	2.358	4.666	0.090

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	37	0	103	290	42	116	136	136
N.S.	1	1.00	0.36	0.00	0.99	2.79	0.40	1.12	1.31	1.31
time (sec)	N/A	0.066	0.008	0.287	2.971	0.670	1.219	2.366	4.889	0.176

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	39	0	155	380	42	141	212	158
N.S.	1	1.00	0.29	0.00	1.15	2.81	0.31	1.04	1.57	1.17
time (sec)	N/A	0.088	0.009	0.278	3.019	0.790	1.412	2.551	5.161	0.219

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	50	47	64	68	136	57	64	50
N.S.	1	1.00	0.62	0.59	0.80	0.85	1.70	0.71	0.80	0.62
time (sec)	N/A	0.049	0.027	0.005	1.384	0.792	6.578	0.648	5.188	0.034

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	36	47	57	112	43	53	39
N.S.	1	1.00	0.66	0.61	0.80	0.97	1.90	0.73	0.90	0.66
time (sec)	N/A	0.038	0.019	0.007	1.335	0.794	3.979	0.646	5.100	0.031

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	25	30	45	88	29	42	28
N.S.	1	1.00	0.74	0.66	0.79	1.18	2.32	0.76	1.11	0.74
time (sec)	N/A	0.024	0.015	0.006	1.324	0.978	2.541	0.694	5.054	0.027

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	32	65	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	1.78	3.61	0.78	0.78	1.00
time (sec)	N/A	0.004	0.005	0.002	1.333	1.140	1.369	0.626	4.990	0.013

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	144	0	109	111	49	110	133	142
N.S.	1	1.00	1.23	0.00	0.93	0.95	0.42	0.94	1.14	1.21
time (sec)	N/A	0.082	0.045	0.297	3.057	1.399	1.303	1.464	5.001	0.100

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	37	0	116	129	46	131	141	147
N.S.	1	1.00	0.32	0.00	1.00	1.11	0.40	1.13	1.22	1.27
time (sec)	N/A	0.084	0.009	0.295	2.901	1.569	1.414	1.423	5.416	0.134

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	39	0	152	174	42	139	191	155
N.S.	1	1.00	0.30	0.00	1.15	1.32	0.32	1.05	1.45	1.17
time (sec)	N/A	0.087	0.010	0.303	3.085	0.894	1.536	1.155	5.491	0.177

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	16	9	24	56	9	25	13
N.S.	1	1.00	1.00	1.23	0.69	1.85	4.31	0.69	1.92	1.00
time (sec)	N/A	0.002	0.005	0.003	1.319	1.371	2.908	0.561	5.131	0.009

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	50	47	64	46	1690	61	48	50
N.S.	1	1.00	0.62	0.59	0.80	0.58	21.12	0.76	0.60	0.62
time (sec)	N/A	0.046	0.029	0.006	1.355	0.875	2.771	0.585	5.316	0.034

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	36	47	35	631	47	36	39
N.S.	1	1.00	0.66	0.61	0.80	0.59	10.69	0.80	0.61	0.66
time (sec)	N/A	0.034	0.021	0.006	1.297	1.576	1.784	0.573	5.207	0.029

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	25	30	24	178	30	24	28
N.S.	1	1.00	0.74	0.66	0.79	0.63	4.68	0.79	0.63	0.74
time (sec)	N/A	0.023	0.013	0.005	1.400	1.049	1.142	0.573	5.006	0.025

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	24	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.33	0.78	0.78	1.00
time (sec)	N/A	0.003	0.003	0.003	1.296	0.629	0.396	0.572	4.671	0.013

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	70	0	86	235	41	87	106	118
N.S.	1	1.00	0.81	0.00	1.00	2.73	0.48	1.01	1.23	1.37
time (sec)	N/A	0.051	0.030	0.274	2.975	1.014	1.015	1.079	4.828	0.073

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	37	0	118	344	41	119	138	142
N.S.	1	1.00	0.34	0.00	1.07	3.13	0.37	1.08	1.25	1.29
time (sec)	N/A	0.065	0.007	0.277	2.946	1.021	1.189	1.137	5.011	0.127

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	39	0	158	326	41	142	201	158
N.S.	1	1.00	0.28	0.00	1.14	2.36	0.30	1.03	1.46	1.14
time (sec)	N/A	0.093	0.008	0.273	2.973	1.724	1.372	1.133	5.063	0.148

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	50	47	64	46	1690	61	48	50
N.S.	1	1.00	0.62	0.59	0.80	0.58	21.12	0.76	0.60	0.62
time (sec)	N/A	0.046	0.024	0.006	1.337	0.959	2.744	0.575	4.749	0.035

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	36	47	35	631	47	36	39
N.S.	1	1.00	0.66	0.61	0.80	0.59	10.69	0.80	0.61	0.66
time (sec)	N/A	0.034	0.019	0.007	1.329	1.117	1.801	0.573	4.766	0.029

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	25	30	23	178	30	24	28
N.S.	1	1.00	0.71	0.66	0.79	0.61	4.68	0.79	0.63	0.74
time (sec)	N/A	0.024	0.014	0.003	1.339	0.846	1.140	0.586	4.786	0.024

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	24	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.33	0.78	0.78	1.00
time (sec)	N/A	0.004	0.003	0.003	1.239	0.937	0.414	0.568	4.695	0.012

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	101	0	86	123	41	87	102	118
N.S.	1	1.00	1.17	0.00	1.00	1.43	0.48	1.01	1.19	1.37
time (sec)	N/A	0.052	0.031	0.280	3.093	1.433	1.059	1.126	4.843	0.076

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	37	0	118	182	41	118	130	139
N.S.	1	1.00	0.35	0.00	1.10	1.70	0.38	1.10	1.21	1.30
time (sec)	N/A	0.066	0.008	0.284	2.958	0.918	1.237	1.107	5.052	0.114

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	39	0	158	174	41	142	193	158
N.S.	1	1.00	0.28	0.00	1.14	1.26	0.30	1.03	1.40	1.14
time (sec)	N/A	0.088	0.008	0.286	2.980	1.414	1.447	1.132	5.144	0.143

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	50	47	64	58	1584	70	55	50
N.S.	1	1.00	0.62	0.59	0.80	0.72	19.80	0.88	0.69	0.62
time (sec)	N/A	0.044	0.023	0.007	1.338	0.880	2.841	0.587	5.440	0.033

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	38	36	47	46	561	52	41	39
N.S.	1	1.00	0.64	0.61	0.80	0.78	9.51	0.88	0.69	0.66
time (sec)	N/A	0.033	0.017	0.006	1.283	0.941	1.823	0.584	5.361	0.031

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	24	30	35	46	34	24	27
N.S.	1	1.00	0.71	0.63	0.79	0.92	1.21	0.89	0.63	0.71
time (sec)	N/A	0.023	0.012	0.005	1.353	1.095	0.704	0.573	5.591	0.027

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	24	26	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	1.33	1.44	0.78	0.78	1.00
time (sec)	N/A	0.004	0.003	0.003	1.316	1.017	0.675	0.580	5.393	0.014

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	36	0	100	327	41	101	123	136
N.S.	1	1.00	0.35	0.00	0.96	3.14	0.39	0.97	1.18	1.31
time (sec)	N/A	0.065	0.007	0.287	3.020	1.730	1.135	1.103	5.590	0.097

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	125	37	0	136	453	41	134	178	150
N.S.	1	1.02	0.30	0.00	1.11	3.68	0.33	1.09	1.45	1.22
time (sec)	N/A	0.080	0.008	0.302	3.010	1.185	1.350	1.113	5.645	0.175

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	39	0	176	437	41	154	224	169
N.S.	1	1.00	0.25	0.00	1.11	2.75	0.26	0.97	1.41	1.06
time (sec)	N/A	0.106	0.008	0.304	2.972	1.583	1.622	1.103	5.672	0.186

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	275	102	0	0	0	0	0	-1	267
N.S.	1	1.41	0.52	0.00	0.00	0.00	0.00	0.00	-0.01	1.37
time (sec)	N/A	0.391	0.074	0.079	0.000	0.000	0.000	0.000	0.000	16.551

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	244	85	0	0	0	46	0	-1	248
N.S.	1	1.49	0.52	0.00	0.00	0.00	0.28	0.00	-0.01	1.51
time (sec)	N/A	0.302	0.050	0.041	0.000	0.000	33.290	0.000	0.000	11.268

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	211	56	0	0	0	46	0	-1	223
N.S.	1	1.59	0.42	0.00	0.00	0.00	0.35	0.00	-0.01	1.68
time (sec)	N/A	0.274	0.014	0.040	0.000	0.000	1.522	0.000	0.000	1.921

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	208	56	0	0	0	49	0	-1	220
N.S.	1	1.59	0.43	0.00	0.00	0.00	0.37	0.00	-0.01	1.68
time (sec)	N/A	0.274	0.014	0.041	0.000	0.000	3.052	0.000	0.000	1.316

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	21	35	25	78	0	-1	28
N.S.	1	1.00	0.93	0.75	1.25	0.89	2.79	0.00	-0.04	1.00
time (sec)	N/A	0.006	0.009	0.003	1.453	1.257	56.586	0.000	0.000	6.187

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	41	31	0	46	0	0	-1	52
N.S.	1	1.00	0.72	0.54	0.00	0.81	0.00	0.00	-0.02	0.91
time (sec)	N/A	0.015	0.016	0.006	0.000	1.296	0.000	0.000	0.000	17.270

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	52	42	0	57	0	0	-1	52
N.S.	1	1.00	0.61	0.49	0.00	0.67	0.00	0.00	-0.01	0.61
time (sec)	N/A	0.025	0.019	0.006	0.000	1.267	0.000	0.000	0.000	18.123

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	63	53	64	68	0	0	-1	63
N.S.	1	1.00	0.56	0.47	0.57	0.60	0.00	0.00	-0.01	0.56
time (sec)	N/A	0.039	0.018	0.013	1.474	1.218	0.000	0.000	0.000	21.079

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	303	102	0	0	0	0	0	-1	285
N.S.	1	1.36	0.46	0.00	0.00	0.00	0.00	0.00	-0.00	1.28
time (sec)	N/A	0.366	0.081	0.039	0.000	0.000	0.000	0.000	0.000	16.783

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	272	89	0	0	0	46	0	-1	267
N.S.	1	1.42	0.46	0.00	0.00	0.00	0.24	0.00	-0.01	1.39
time (sec)	N/A	0.323	0.066	0.042	0.000	0.000	64.864	0.000	0.000	16.375

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	243	57	0	0	0	46	0	-1	246
N.S.	1	1.49	0.35	0.00	0.00	0.00	0.28	0.00	-0.01	1.51
time (sec)	N/A	0.290	0.015	0.039	0.000	0.000	7.090	0.000	0.000	4.757

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	233	57	0	0	0	49	0	-1	236
N.S.	1	1.52	0.37	0.00	0.00	0.00	0.32	0.00	-0.01	1.54
time (sec)	N/A	0.287	0.012	0.046	0.000	0.000	7.232	0.000	0.000	1.530

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	234	57	0	0	0	53	0	-1	235
N.S.	1	1.49	0.36	0.00	0.00	0.00	0.34	0.00	-0.01	1.50
time (sec)	N/A	0.291	0.016	0.051	0.000	0.000	56.366	0.000	0.000	1.907

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	21	0	43	0	0	-1	31
N.S.	1	1.00	0.93	0.75	0.00	1.54	0.00	0.00	-0.04	1.11
time (sec)	N/A	0.006	0.011	0.004	0.000	1.827	0.000	0.000	0.000	12.454

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	41	31	0	57	0	0	-1	41
N.S.	1	1.00	0.72	0.54	0.00	1.00	0.00	0.00	-0.02	0.72
time (sec)	N/A	0.015	0.019	0.006	0.000	1.761	0.000	0.000	0.000	18.849

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	52	42	0	68	0	0	-1	0
N.S.	1	1.00	0.61	0.49	0.00	0.80	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.027	0.016	0.005	0.000	1.725	0.000	0.000	0.000	9.622

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	278	87	0	0	0	0	0	-1	243
N.S.	1	1.40	0.44	0.00	0.00	0.00	0.00	0.00	-0.01	1.23
time (sec)	N/A	0.338	0.029	0.045	0.000	0.000	0.000	0.000	0.000	14.159

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	247	76	0	0	0	0	0	-1	247
N.S.	1	1.48	0.46	0.00	0.00	0.00	0.00	0.00	-0.01	1.48
time (sec)	N/A	0.290	0.023	0.042	0.000	0.000	0.000	0.000	0.000	16.848

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	209	58	0	0	0	44	0	-1	221
N.S.	1	1.60	0.44	0.00	0.00	0.00	0.34	0.00	-0.01	1.69
time (sec)	N/A	0.262	0.020	0.039	0.000	0.000	30.011	0.000	0.000	6.523

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	183	45	0	0	0	44	0	-1	195
N.S.	1	1.73	0.42	0.00	0.00	0.00	0.42	0.00	-0.01	1.84
time (sec)	N/A	0.245	0.010	0.273	0.000	0.000	1.525	0.000	0.000	1.749

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	21	35	25	36	0	-1	28
N.S.	1	1.00	0.93	0.75	1.25	0.89	1.29	0.00	-0.04	1.00
time (sec)	N/A	0.006	0.006	0.003	1.473	1.642	4.166	0.000	0.000	2.118

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	34	29	0	35	78	0	-1	44
N.S.	1	1.00	0.60	0.51	0.00	0.61	1.37	0.00	-0.02	0.77
time (sec)	N/A	0.015	0.019	0.004	0.000	1.549	109.454	0.000	0.000	9.375

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	52	42	0	46	0	0	-1	52
N.S.	1	1.00	0.61	0.49	0.00	0.54	0.00	0.00	-0.01	0.61
time (sec)	N/A	0.025	0.025	0.006	0.000	1.518	0.000	0.000	0.000	31.481

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	63	53	64	57	0	0	-1	63
N.S.	1	1.00	0.56	0.47	0.57	0.50	0.00	0.00	-0.01	0.56
time (sec)	N/A	0.039	0.031	0.004	1.536	1.698	0.000	0.000	0.000	33.125

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	85	0	0	0	46	0	-1	171
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.31	0.00	-0.01	1.16
time (sec)	N/A	0.092	0.046	0.058	0.000	0.000	9.471	0.000	0.000	0.628

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	56	0	0	0	46	0	-1	148
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.40	0.00	-0.01	1.28
time (sec)	N/A	0.070	0.012	0.052	0.000	0.000	1.806	0.000	0.000	0.417

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	54	0	0	0	49	0	-1	139
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.46	0.00	-0.01	1.30
time (sec)	N/A	0.069	0.013	0.056	0.000	0.000	2.315	0.000	0.000	0.401

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	21	0	25	78	0	37	28
N.S.	1	1.00	0.93	0.75	0.00	0.89	2.79	0.00	1.32	1.00
time (sec)	N/A	0.006	0.008	0.004	0.000	0.853	9.132	0.000	4.916	0.238

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	41	31	0	46	124	0	51	58
N.S.	1	1.00	0.72	0.54	0.00	0.81	2.18	0.00	0.89	1.02
time (sec)	N/A	0.015	0.015	0.006	0.000	0.987	69.581	0.000	4.976	0.298

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	52	42	0	57	0	0	65	58
N.S.	1	1.00	0.61	0.49	0.00	0.67	0.00	0.00	0.76	0.68
time (sec)	N/A	0.024	0.018	0.007	0.000	1.094	0.000	0.000	5.002	0.347

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	63	53	0	68	0	0	79	0
N.S.	1	1.00	0.56	0.47	0.00	0.60	0.00	0.00	0.70	0.00
time (sec)	N/A	0.038	0.016	0.008	0.000	1.264	0.000	0.000	5.022	6.426

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	88	0	0	0	48	0	-1	278
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.14	0.00	-0.00	0.81
time (sec)	N/A	0.368	0.050	0.064	0.000	0.000	9.288	0.000	0.000	8.884

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	307	307	57	0	0	0	48	0	-1	254
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.16	0.00	-0.00	0.83
time (sec)	N/A	0.268	0.012	0.058	0.000	0.000	1.782	0.000	0.000	8.226

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	296	296	55	0	0	0	51	0	-1	245
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.17	0.00	-0.00	0.83
time (sec)	N/A	0.275	0.013	0.058	0.000	0.000	2.324	0.000	0.000	12.558

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	27	22	0	35	178	0	38	29
N.S.	1	1.00	0.93	0.76	0.00	1.21	6.14	0.00	1.31	1.00
time (sec)	N/A	0.006	0.009	0.003	0.000	1.104	8.763	0.000	4.847	0.234

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	42	32	0	46	462	0	51	58
N.S.	1	1.00	0.71	0.54	0.00	0.78	7.83	0.00	0.86	0.98
time (sec)	N/A	0.016	0.017	0.005	0.000	1.465	68.247	0.000	4.892	0.295

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	53	43	0	58	0	0	65	59
N.S.	1	1.00	0.60	0.49	0.00	0.66	0.00	0.00	0.74	0.67
time (sec)	N/A	0.028	0.019	0.005	0.000	1.490	0.000	0.000	4.911	0.347

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	64	54	0	69	0	0	79	0
N.S.	1	1.00	0.55	0.46	0.00	0.59	0.00	0.00	0.68	0.00
time (sec)	N/A	0.040	0.017	0.006	0.000	0.826	0.000	0.000	4.930	6.442

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	97	0	0	314	44	0	-1	149
N.S.	1	1.00	0.83	0.00	0.00	2.68	0.38	0.00	-0.01	1.27
time (sec)	N/A	0.065	0.037	0.321	0.000	1.200	2.175	0.000	0.000	0.577

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	65	0	0	241	44	0	-1	115
N.S.	1	1.00	0.78	0.00	0.00	2.90	0.53	0.00	-0.01	1.39
time (sec)	N/A	0.050	0.011	0.283	0.000	1.059	1.505	0.000	0.000	0.401

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	21	0	25	36	0	25	28
N.S.	1	1.00	0.93	0.75	0.00	0.89	1.29	0.00	0.89	1.00
time (sec)	N/A	0.006	0.007	0.005	0.000	1.116	3.583	0.000	4.946	0.314

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	41	31	0	35	80	0	40	44
N.S.	1	1.00	0.72	0.54	0.00	0.61	1.40	0.00	0.70	0.77
time (sec)	N/A	0.016	0.018	0.006	0.000	0.958	34.720	0.000	4.997	0.391

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	52	42	0	46	0	0	54	58
N.S.	1	1.00	0.61	0.49	0.00	0.54	0.00	0.00	0.64	0.68
time (sec)	N/A	0.026	0.024	0.006	0.000	1.141	0.000	0.000	5.005	0.648

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	308	308	241	0	0	340	46	0	-1	257
N.S.	1	1.00	0.78	0.00	0.00	1.10	0.15	0.00	-0.00	0.83
time (sec)	N/A	0.265	0.152	0.323	0.000	0.910	2.205	0.000	0.000	9.047

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	272	272	197	0	0	267	46	0	-1	163
N.S.	1	1.00	0.72	0.00	0.00	0.98	0.17	0.00	-0.00	0.60
time (sec)	N/A	0.228	0.043	0.325	0.000	1.052	1.523	0.000	0.000	2.804

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	27	22	0	26	88	0	26	29
N.S.	1	1.00	0.93	0.76	0.00	0.90	3.03	0.00	0.90	1.00
time (sec)	N/A	0.006	0.007	0.004	0.000	0.827	3.617	0.000	5.124	0.315

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	42	32	0	36	343	0	41	45
N.S.	1	1.00	0.71	0.54	0.00	0.61	5.81	0.00	0.69	0.76
time (sec)	N/A	0.015	0.019	0.004	0.000	1.022	34.343	0.000	5.145	0.393

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	53	43	0	47	0	0	55	59
N.S.	1	1.00	0.60	0.49	0.00	0.53	0.00	0.00	0.62	0.67
time (sec)	N/A	0.027	0.026	0.006	0.000	0.885	0.000	0.000	5.251	0.640

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	97	0	0	0	44	0	-1	149
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.38	0.00	-0.01	1.27
time (sec)	N/A	0.067	0.039	0.058	0.000	0.000	7.779	0.000	0.000	0.902

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	67	0	0	0	44	0	-1	116
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.52	0.00	-0.01	1.38
time (sec)	N/A	0.056	0.014	0.278	0.000	0.000	1.501	0.000	0.000	0.543

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	24	21	0	25	36	0	22	26
N.S.	1	1.00	0.92	0.81	0.00	0.96	1.38	0.00	0.85	1.00
time (sec)	N/A	0.006	0.006	0.005	0.000	1.007	2.924	0.000	4.903	0.388

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	34	29	0	35	78	0	40	44
N.S.	1	1.00	0.62	0.53	0.00	0.64	1.42	0.00	0.73	0.80
time (sec)	N/A	0.014	0.018	0.006	0.000	1.503	24.762	0.000	4.982	0.555

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	52	42	0	46	483	0	54	58
N.S.	1	1.00	0.63	0.51	0.00	0.55	5.82	0.00	0.65	0.70
time (sec)	N/A	0.025	0.024	0.005	0.000	1.542	172.060	0.000	5.172	0.988

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	308	308	112	0	0	0	46	0	-1	257
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.15	0.00	-0.00	0.83
time (sec)	N/A	0.262	0.057	0.061	0.000	0.000	7.817	0.000	0.000	9.603

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	272	272	75	0	0	0	46	0	-1	164
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.17	0.00	-0.00	0.60
time (sec)	N/A	0.232	0.024	0.280	0.000	0.000	1.564	0.000	0.000	5.601

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	25	22	0	26	90	0	23	27
N.S.	1	1.00	0.93	0.81	0.00	0.96	3.33	0.00	0.85	1.00
time (sec)	N/A	0.007	0.006	0.004	0.000	1.460	3.001	0.000	5.076	0.394

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	35	30	0	34	352	0	41	44
N.S.	1	1.00	0.61	0.53	0.00	0.60	6.18	0.00	0.72	0.77
time (sec)	N/A	0.014	0.018	0.006	0.000	1.681	25.445	0.000	5.096	0.563

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	53	43	0	47	1263	0	55	59
N.S.	1	1.00	0.62	0.50	0.00	0.55	14.69	0.00	0.64	0.69
time (sec)	N/A	0.025	0.024	0.006	0.000	0.852	171.234	0.000	5.153	1.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	63	0	0	389	44	0	-1	183
N.S.	1	1.00	0.43	0.00	0.00	2.66	0.30	0.00	-0.01	1.25
time (sec)	N/A	0.076	0.036	0.328	0.000	1.485	24.494	0.000	0.000	2.710

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	59	0	0	319	44	0	-1	157
N.S.	1	1.00	0.55	0.00	0.00	2.98	0.41	0.00	-0.01	1.47
time (sec)	N/A	0.059	0.013	0.280	0.000	1.226	4.257	0.000	0.000	1.265

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	24	21	0	31	34	0	29	40
N.S.	1	1.00	0.92	0.81	0.00	1.19	1.31	0.00	1.12	1.54
time (sec)	N/A	0.006	0.006	0.005	0.000	0.928	3.141	0.000	4.968	0.565

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	34	29	0	48	78	0	57	59
N.S.	1	1.00	0.62	0.53	0.00	0.87	1.42	0.00	1.04	1.07
time (sec)	N/A	0.015	0.010	0.006	0.000	0.743	14.989	0.000	5.097	0.733

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	47	42	0	61	384	0	70	74
N.S.	1	1.00	0.57	0.51	0.00	0.73	4.63	0.00	0.84	0.89
time (sec)	N/A	0.024	0.011	0.007	0.000	0.934	109.044	0.000	5.154	1.366

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	58	53	0	72	0	0	85	88
N.S.	1	1.00	0.53	0.49	0.00	0.66	0.00	0.00	0.78	0.81
time (sec)	N/A	0.037	0.013	0.007	0.000	0.851	0.000	0.000	5.197	3.494

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	95	132	106	148	2025	410	183	0
N.S.	1	1.00	0.95	1.32	1.06	1.48	20.25	4.10	1.83	0.00
time (sec)	N/A	0.064	0.055	0.006	1.445	0.913	11.065	0.585	4.966	0.041

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	64	80	73	98	981	231	117	0
N.S.	1	1.00	0.89	1.11	1.01	1.36	13.62	3.21	1.62	0.00
time (sec)	N/A	0.043	0.031	0.006	1.412	0.945	5.045	0.585	4.902	0.039

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	40	42	47	58	364	94	68	0
N.S.	1	1.00	0.83	0.88	0.98	1.21	7.58	1.96	1.42	0.00
time (sec)	N/A	0.029	0.019	0.004	1.288	0.808	2.003	0.579	4.887	0.036

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	22	22	21	25	97	21	21	0
N.S.	1	1.00	0.96	0.96	0.91	1.09	4.22	0.91	0.91	0.00
time (sec)	N/A	0.005	0.003	0.002	1.381	0.753	0.732	0.571	4.892	0.026

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	62	81	84	106	0	0	154	0
N.S.	1	1.00	0.59	0.77	0.80	1.01	0.00	0.00	1.47	0.00
time (sec)	N/A	0.057	0.014	0.006	1.458	0.687	0.000	0.000	5.089	0.027

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	62	45	59	67	0	0	96	0
N.S.	1	1.00	0.93	0.67	0.88	1.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.020	0.015	0.004	1.405	0.794	0.000	0.000	5.020	0.026

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	29	29	37	34	0	0	52	0
N.S.	1	1.00	0.97	0.97	1.23	1.13	0.00	0.00	1.73	0.00
time (sec)	N/A	0.006	0.012	0.003	1.478	0.891	0.000	0.000	5.053	0.026

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [321] had the largest ratio of [.6923]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	11	0.091
2	A	2	1	1.00	11	0.091
3	A	2	1	1.00	11	0.091
4	A	2	1	1.00	9	0.111
5	A	1	0	1.00	7	0.000
6	A	2	1	1.00	11	0.091
7	A	2	1	1.00	11	0.091
8	A	2	1	1.00	11	0.091
9	A	2	1	1.00	11	0.091
10	A	2	1	1.00	11	0.091
11	A	2	1	1.00	11	0.091
12	A	2	1	1.00	11	0.091
13	A	3	2	1.00	13	0.154
14	A	2	1	1.00	13	0.077
15	A	3	2	1.00	13	0.154
16	A	2	1	1.00	13	0.077
17	A	1	1	1.00	11	0.091
18	A	2	1	1.00	9	0.111
19	A	3	2	1.00	13	0.154
20	A	2	1	1.00	13	0.077
21	A	3	2	1.00	13	0.154
22	A	2	1	1.00	13	0.077
23	A	3	2	1.00	13	0.154
24	A	2	1	1.00	13	0.077
25	A	1	1	1.00	13	0.077
26	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	3	2	1.00	13	0.154
28	A	2	1	1.00	13	0.077
29	A	3	2	1.00	13	0.154
30	A	3	2	1.00	13	0.154
31	A	3	2	1.00	13	0.154
32	A	3	2	1.00	13	0.154
33	A	1	1	1.00	11	0.091
34	A	3	2	1.00	13	0.154
35	A	3	2	1.00	13	0.154
36	A	3	2	1.00	13	0.154
37	A	3	2	1.00	13	0.154
38	A	1	1	1.00	13	0.077
39	A	3	3	1.00	13	0.231
40	A	3	2	1.00	13	0.154
41	A	3	2	1.00	13	0.154
42	A	2	1	1.00	13	0.077
43	A	2	1	1.00	13	0.077
44	A	2	1	1.00	13	0.077
45	A	2	1	1.00	9	0.111
46	A	2	1	1.00	13	0.077
47	A	2	1	1.00	13	0.077
48	A	2	1	1.00	13	0.077
49	A	2	1	1.00	13	0.077
50	A	2	1	1.00	13	0.077
51	A	2	1	1.00	13	0.077
52	A	3	2	1.00	13	0.154
53	A	3	2	1.00	13	0.154
54	A	3	2	1.00	13	0.154
55	A	3	2	1.00	13	0.154
56	A	3	2	1.00	13	0.154
57	A	3	2	1.00	13	0.154
58	A	1	1	1.00	11	0.091
59	A	3	2	1.00	13	0.154
60	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	13	0.154
62	A	3	2	1.00	13	0.154
63	A	3	2	1.00	13	0.154
64	A	3	2	1.00	13	0.154
65	A	1	1	1.00	13	0.077
66	A	3	3	1.00	13	0.231
67	A	4	3	1.00	13	0.231
68	A	3	2	1.00	13	0.154
69	A	3	2	1.00	13	0.154
70	A	2	1	1.00	13	0.077
71	A	2	1	1.00	13	0.077
72	A	2	1	1.00	13	0.077
73	A	2	1	1.00	13	0.077
74	A	2	1	1.00	9	0.111
75	A	2	1	1.00	13	0.077
76	A	2	1	1.00	13	0.077
77	A	2	1	1.00	13	0.077
78	A	2	1	1.00	13	0.077
79	A	2	1	1.00	13	0.077
80	A	2	1	1.00	13	0.077
81	A	2	1	1.00	13	0.077
82	A	2	1	1.00	13	0.077
83	A	2	1	1.00	13	0.077
84	A	2	1	1.00	13	0.077
85	A	3	2	1.00	13	0.154
86	A	3	2	1.00	13	0.154
87	A	3	2	1.00	13	0.154
88	A	3	2	1.00	13	0.154
89	A	3	2	1.00	13	0.154
90	A	3	2	1.00	13	0.154
91	A	1	1	1.00	11	0.091
92	A	3	2	1.00	13	0.154
93	A	3	2	1.00	13	0.154
94	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
95	A	3	2	1.00	13	0.154
96	A	3	2	1.00	13	0.154
97	A	3	2	1.00	13	0.154
98	A	3	2	1.00	13	0.154
99	A	3	2	1.00	13	0.154
100	A	3	2	1.00	13	0.154
101	A	1	1	1.00	13	0.077
102	A	3	3	1.00	13	0.231
103	A	4	3	1.00	13	0.231
104	A	5	3	1.00	13	0.231
105	A	6	3	1.00	13	0.231
106	A	3	2	1.00	13	0.154
107	A	3	2	1.00	13	0.154
108	A	3	2	1.00	13	0.154
109	A	2	1	1.00	13	0.077
110	A	2	1	1.00	13	0.077
111	A	2	1	1.00	13	0.077
112	A	2	1	1.00	13	0.077
113	A	2	1	1.00	9	0.111
114	A	2	1	1.00	13	0.077
115	A	2	1	1.00	13	0.077
116	A	2	1	1.00	13	0.077
117	A	2	1	1.00	13	0.077
118	A	2	1	1.00	13	0.077
119	A	2	1	1.00	13	0.077
120	A	2	1	1.00	13	0.077
121	A	2	1	1.00	13	0.077
122	A	2	1	1.00	13	0.077
123	A	2	1	1.00	13	0.077
124	A	3	2	1.00	13	0.154
125	A	3	2	1.00	13	0.154
126	A	3	2	1.00	13	0.154
127	A	3	2	1.00	13	0.154
128	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	3	2	1.00	13	0.154
130	A	3	2	1.00	13	0.154
131	A	3	2	1.00	13	0.154
132	A	3	2	1.00	13	0.154
133	A	2	2	1.00	13	0.154
134	A	1	1	1.00	11	0.091
135	A	1	1	1.00	9	0.111
136	A	4	4	1.00	13	0.308
137	A	2	2	1.00	13	0.154
138	A	3	2	1.00	13	0.154
139	A	3	2	1.00	13	0.154
140	A	3	2	1.00	13	0.154
141	A	4	2	1.00	13	0.154
142	A	3	2	1.00	13	0.154
143	A	5	2	1.00	13	0.154
144	A	3	2	1.00	13	0.154
145	A	3	2	1.00	13	0.154
146	A	4	3	1.00	13	0.231
147	A	3	2	1.00	13	0.154
148	A	4	3	1.00	13	0.231
149	A	3	2	1.00	13	0.154
150	A	4	3	1.00	13	0.231
151	A	3	2	1.00	13	0.154
152	A	4	3	1.00	13	0.231
153	A	3	2	1.00	13	0.154
154	A	3	3	1.00	13	0.231
155	A	3	2	1.00	13	0.154
156	A	2	2	1.00	13	0.154
157	A	1	1	1.00	11	0.091
158	A	2	2	1.00	9	0.222
159	A	3	2	1.00	13	0.154
160	A	3	3	1.00	13	0.231
161	A	3	2	1.00	13	0.154
162	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
163	A	3	2	1.00	13	0.154
164	A	5	3	1.00	13	0.231
165	A	3	2	1.00	13	0.154
166	A	6	3	1.00	13	0.231
167	A	3	2	1.00	13	0.154
168	A	3	2	1.00	13	0.154
169	A	3	2	1.00	13	0.154
170	A	3	2	1.00	13	0.154
171	A	3	2	1.00	13	0.154
172	A	3	2	1.00	13	0.154
173	A	3	2	1.00	13	0.154
174	A	1	1	1.00	13	0.077
175	A	1	1	1.00	11	0.091
176	A	3	2	1.00	13	0.154
177	A	3	2	1.00	13	0.154
178	A	3	2	1.00	13	0.154
179	A	3	2	1.00	13	0.154
180	A	3	2	1.00	13	0.154
181	A	5	3	1.00	13	0.231
182	A	5	3	1.00	13	0.231
183	A	5	3	1.00	13	0.231
184	A	4	3	1.00	13	0.231
185	A	3	2	1.00	13	0.154
186	A	3	3	1.00	13	0.231
187	A	3	2	1.00	9	0.222
188	A	4	3	1.00	13	0.231
189	A	5	3	1.00	13	0.231
190	A	6	3	1.00	13	0.231
191	A	7	3	1.00	13	0.231
192	A	3	2	1.00	13	0.154
193	A	3	2	1.00	13	0.154
194	A	3	2	1.00	13	0.154
195	A	3	2	1.00	13	0.154
196	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	3	3	1.00	13	0.231
198	A	4	3	1.00	13	0.231
199	A	5	3	1.00	13	0.231
200	A	3	2	1.00	13	0.154
201	A	3	2	1.00	13	0.154
202	A	3	2	1.00	13	0.154
203	A	3	2	1.00	13	0.154
204	A	1	1	1.00	11	0.091
205	A	3	2	1.00	13	0.154
206	A	3	2	1.00	13	0.154
207	A	3	2	1.00	13	0.154
208	A	3	2	1.00	13	0.154
209	A	12	3	1.00	13	0.231
210	A	12	3	1.00	13	0.231
211	A	11	3	1.00	13	0.231
212	A	10	2	1.00	13	0.154
213	A	10	3	1.00	13	0.231
214	A	10	3	1.00	13	0.231
215	A	10	3	1.00	13	0.231
216	A	10	3	1.00	13	0.231
217	A	10	3	1.00	13	0.231
218	A	10	3	1.00	13	0.231
219	A	10	3	1.00	13	0.231
220	A	10	3	1.00	13	0.231
221	A	10	2	1.00	9	0.222
222	A	11	3	1.00	13	0.231
223	A	12	3	1.00	13	0.231
224	A	13	3	1.00	13	0.231
225	A	3	2	1.00	14	0.143
226	A	2	2	1.00	14	0.143
227	A	1	1	1.00	12	0.083
228	A	1	1	1.00	10	0.100
229	A	4	4	1.00	14	0.286
230	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
231	A	3	2	1.00	14	0.143
232	A	3	2	1.00	14	0.143
233	A	2	2	1.00	14	0.143
234	A	1	1	1.00	12	0.083
235	A	2	2	1.00	10	0.200
236	A	3	2	1.00	14	0.143
237	A	3	3	1.00	14	0.214
238	A	3	2	1.00	14	0.143
239	A	1	1	1.00	14	0.071
240	A	3	3	1.00	14	0.214
241	A	1	1	1.00	12	0.083
242	A	3	2	1.00	10	0.200
243	A	3	2	1.00	14	0.143
244	A	4	3	1.00	14	0.214
245	A	3	2	1.00	14	0.143
246	A	3	2	1.00	14	0.143
247	A	5	3	1.00	14	0.214
248	A	1	1	1.00	12	0.083
249	A	5	2	1.00	10	0.200
250	A	3	2	1.00	14	0.143
251	A	6	3	1.00	14	0.214
252	A	3	2	1.00	14	0.143
253	A	4	4	1.00	13	0.308
254	A	4	4	1.00	13	0.308
255	A	3	2	1.00	13	0.154
256	A	3	2	1.00	13	0.154
257	A	1	1	1.00	10	0.100
258	A	1	1	1.00	18	0.056
259	A	3	2	1.00	13	0.154
260	A	3	2	1.00	13	0.154
261	A	1	1	1.00	14	0.071
262	A	1	1	1.00	15	0.067
263	A	1	1	1.00	20	0.050
264	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	2	1	1.00	13	0.077
266	A	2	1	1.00	13	0.077
267	A	2	1	1.00	13	0.077
268	A	2	1	1.00	13	0.077
269	A	2	1	1.00	13	0.077
270	A	2	1	1.00	13	0.077
271	A	2	1	1.00	13	0.077
272	A	2	1	1.00	15	0.067
273	A	2	1	1.00	15	0.067
274	A	2	1	1.00	15	0.067
275	A	2	1	1.00	15	0.067
276	A	2	1	1.00	15	0.067
277	A	2	1	1.00	15	0.067
278	A	2	1	1.00	15	0.067
279	A	2	1	1.00	15	0.067
280	A	2	1	1.00	15	0.067
281	A	2	1	1.00	15	0.067
282	A	2	1	1.00	15	0.067
283	A	2	1	1.00	15	0.067
284	A	2	1	1.00	15	0.067
285	A	2	1	1.00	15	0.067
286	A	2	1	1.00	15	0.067
287	A	2	1	1.00	15	0.067
288	A	12	8	1.00	15	0.533
289	A	11	8	1.00	15	0.533
290	A	11	8	1.00	15	0.533
291	A	10	7	1.00	15	0.467
292	A	10	7	1.00	15	0.467
293	A	11	8	1.00	15	0.533
294	A	11	8	1.00	15	0.533
295	A	12	8	1.00	15	0.533
296	A	12	9	1.00	15	0.600
297	A	11	8	1.00	15	0.533
298	A	11	8	1.00	15	0.533

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
299	A	11	8	1.00	15	0.533
300	A	11	8	1.00	15	0.533
301	A	12	9	1.00	15	0.600
302	A	12	9	1.00	15	0.600
303	A	13	9	1.00	15	0.600
304	A	12	8	1.00	15	0.533
305	A	12	9	1.00	15	0.600
306	A	12	9	1.00	15	0.600
307	A	12	8	1.00	15	0.533
308	A	12	8	1.00	15	0.533
309	A	13	9	1.00	15	0.600
310	A	13	9	1.00	15	0.600
311	A	14	9	1.00	15	0.600
312	A	4	4	1.00	16	0.250
313	A	12	8	1.00	13	0.615
314	A	11	8	1.00	13	0.615
315	A	11	8	1.00	13	0.615
316	A	10	7	1.00	13	0.538
317	A	10	7	1.00	13	0.538
318	A	11	8	1.00	13	0.615
319	A	11	8	1.00	13	0.615
320	A	12	8	1.00	13	0.615
321	A	12	9	1.00	13	0.692
322	A	11	8	1.00	13	0.615
323	A	11	8	1.00	13	0.615
324	A	11	8	1.00	13	0.615
325	A	11	8	1.00	13	0.615
326	A	12	9	1.00	13	0.692
327	A	12	9	1.00	13	0.692
328	A	13	9	1.00	13	0.692
329	A	12	8	1.00	13	0.615
330	A	12	9	1.00	13	0.692
331	A	12	9	1.00	13	0.692
332	A	12	8	1.00	13	0.615

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
333	A	12	8	1.00	13	0.615
334	A	13	9	1.00	13	0.692
335	A	13	9	1.00	13	0.692
336	A	14	9	1.00	13	0.692
337	A	4	4	1.00	15	0.267
338	A	11	7	1.37	13	0.538
339	A	2	1	1.00	13	0.077
340	A	2	1	1.00	13	0.077
341	A	2	1	1.00	13	0.077
342	A	2	1	1.00	13	0.077
343	A	2	1	1.00	11	0.091
344	A	3	2	1.00	15	0.133
345	A	3	2	1.00	15	0.133
346	A	3	2	1.00	15	0.133
347	A	1	1	1.00	13	0.077
348	A	4	4	1.00	15	0.267
349	A	4	4	1.00	15	0.267
350	A	5	5	1.00	15	0.333
351	A	6	5	1.00	15	0.333
352	A	5	4	1.00	15	0.267
353	A	4	4	1.00	15	0.267
354	A	3	3	1.00	11	0.273
355	A	3	3	1.00	15	0.200
356	A	1	1	1.00	15	0.067
357	A	2	2	1.00	15	0.133
358	A	3	2	1.00	15	0.133
359	A	4	2	1.00	15	0.133
360	A	3	2	1.00	15	0.133
361	A	3	2	1.00	15	0.133
362	A	3	2	1.00	15	0.133
363	A	1	1	1.00	13	0.077
364	A	5	4	1.00	15	0.267
365	A	5	5	1.00	15	0.333
366	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
367	A	6	5	1.00	15	0.333
368	A	7	5	1.00	15	0.333
369	A	6	4	1.00	15	0.267
370	A	5	4	1.00	15	0.267
371	A	4	3	1.00	11	0.273
372	A	4	4	1.00	15	0.267
373	A	4	3	1.00	15	0.200
374	A	1	1	1.00	15	0.067
375	A	2	2	1.00	15	0.133
376	A	3	2	1.00	15	0.133
377	A	4	2	1.00	15	0.133
378	A	3	2	1.00	15	0.133
379	A	3	2	1.00	15	0.133
380	A	3	2	1.00	15	0.133
381	A	1	1	1.00	13	0.077
382	A	6	4	1.00	15	0.267
383	A	6	5	1.00	15	0.333
384	A	6	5	1.00	15	0.333
385	A	6	4	1.00	15	0.267
386	A	7	5	1.00	15	0.333
387	A	8	5	1.00	15	0.333
388	A	7	4	1.00	15	0.267
389	A	6	4	1.00	15	0.267
390	A	5	3	1.00	11	0.273
391	A	5	4	1.00	15	0.267
392	A	5	4	1.00	15	0.267
393	A	5	3	1.00	15	0.200
394	A	1	1	1.00	15	0.067
395	A	2	2	1.00	15	0.133
396	A	3	2	1.00	15	0.133
397	A	4	2	1.00	15	0.133
398	A	5	2	1.00	15	0.133
399	A	6	2	1.00	15	0.133
400	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	3	2	1.00	15	0.133
402	A	3	2	1.00	15	0.133
403	A	3	2	1.00	15	0.133
404	A	3	2	1.00	15	0.133
405	A	3	2	1.00	15	0.133
406	A	3	2	1.00	15	0.133
407	A	1	1	1.00	13	0.077
408	A	8	4	1.00	15	0.267
409	A	8	5	1.00	15	0.333
410	A	8	5	1.00	15	0.333
411	A	8	5	1.00	15	0.333
412	A	8	5	1.00	15	0.333
413	A	8	4	1.00	15	0.267
414	A	9	5	1.00	15	0.333
415	A	10	5	1.00	15	0.333
416	A	10	4	1.00	15	0.267
417	A	9	4	1.00	15	0.267
418	A	8	4	1.00	15	0.267
419	A	7	3	1.00	11	0.273
420	A	7	4	1.00	15	0.267
421	A	7	4	1.00	15	0.267
422	A	7	4	1.00	15	0.267
423	A	7	4	1.00	15	0.267
424	A	7	3	1.00	15	0.200
425	A	1	1	1.00	15	0.067
426	A	2	2	1.00	15	0.133
427	A	3	2	1.00	15	0.133
428	A	4	2	1.00	15	0.133
429	A	5	2	1.00	15	0.133
430	A	6	2	1.00	15	0.133
431	A	7	2	1.00	15	0.133
432	A	3	2	1.00	15	0.133
433	A	4	3	1.00	15	0.200
434	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
435	A	3	3	1.00	15	0.200
436	A	1	1	1.00	13	0.077
437	A	2	2	1.00	11	0.182
438	A	4	4	1.00	15	0.267
439	A	2	2	1.00	15	0.133
440	A	4	4	1.00	15	0.267
441	A	1	1	1.00	15	0.067
442	A	5	5	1.00	15	0.333
443	A	3	2	1.00	15	0.133
444	A	4	3	1.00	15	0.200
445	A	3	2	1.00	15	0.133
446	A	3	3	1.00	15	0.200
447	A	1	1	1.00	13	0.077
448	A	2	2	1.00	11	0.182
449	A	4	4	1.00	15	0.267
450	A	2	2	1.00	15	0.133
451	A	4	4	1.00	15	0.267
452	A	1	1	1.00	15	0.067
453	A	5	5	1.00	15	0.333
454	A	3	2	1.00	15	0.133
455	A	5	4	1.00	15	0.267
456	A	3	2	1.00	15	0.133
457	A	4	4	1.00	15	0.267
458	A	1	1	1.00	13	0.077
459	A	3	3	1.00	11	0.273
460	A	4	4	1.00	15	0.267
461	A	3	3	1.00	15	0.200
462	A	4	4	1.00	15	0.267
463	A	1	1	1.00	15	0.067
464	A	5	5	1.00	15	0.333
465	A	3	2	1.00	15	0.133
466	A	5	4	1.00	15	0.267
467	A	3	2	1.00	15	0.133
468	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
469	A	1	1	1.00	13	0.077
470	A	3	3	1.00	11	0.273
471	A	4	4	1.00	15	0.267
472	A	3	3	1.00	15	0.200
473	A	4	4	1.00	15	0.267
474	A	1	1	1.00	15	0.067
475	A	5	5	1.00	15	0.333
476	A	3	2	1.00	15	0.133
477	A	4	3	1.00	15	0.200
478	A	3	2	1.00	15	0.133
479	A	3	3	1.00	15	0.200
480	A	1	1	1.00	13	0.077
481	A	2	2	1.00	11	0.182
482	A	3	3	1.00	15	0.200
483	A	1	1	1.00	15	0.067
484	A	4	4	1.00	15	0.267
485	A	2	2	1.00	15	0.133
486	A	5	4	1.00	15	0.267
487	A	3	2	1.00	15	0.133
488	A	4	4	1.00	15	0.267
489	A	3	2	1.00	15	0.133
490	A	3	3	1.00	15	0.200
491	A	1	1	1.00	13	0.077
492	A	1	1	1.00	11	0.091
493	A	4	4	1.00	15	0.267
494	A	2	2	1.00	15	0.133
495	A	5	4	0.99	15	0.267
496	A	3	2	1.00	15	0.133
497	A	5	4	1.00	15	0.267
498	A	3	2	1.00	15	0.133
499	A	4	3	1.00	15	0.200
500	A	3	2	1.00	15	0.133
501	A	1	1	1.00	15	0.067
502	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
503	A	2	2	1.00	11	0.182
504	A	5	4	1.00	15	0.267
505	A	3	3	1.00	15	0.200
506	A	6	4	1.05	15	0.267
507	A	4	3	1.00	15	0.200
508	A	7	4	1.00	15	0.267
509	A	3	2	1.00	15	0.133
510	A	6	3	1.00	15	0.200
511	A	3	2	1.00	15	0.133
512	A	1	1	1.00	15	0.067
513	A	3	2	1.00	15	0.133
514	A	2	2	1.00	15	0.133
515	A	3	2	1.00	15	0.133
516	A	3	2	1.00	15	0.133
517	A	1	1	1.00	13	0.077
518	A	4	2	1.00	11	0.182
519	A	7	4	1.00	15	0.267
520	A	5	3	1.00	15	0.200
521	A	8	4	1.05	15	0.267
522	A	6	3	1.00	15	0.200
523	A	3	2	1.00	15	0.133
524	A	3	2	1.00	15	0.133
525	A	3	2	1.00	15	0.133
526	A	2	2	1.00	15	0.133
527	A	1	1	1.00	13	0.077
528	A	1	1	1.00	11	0.091
529	A	3	3	1.00	15	0.200
530	A	1	1	1.00	15	0.067
531	A	4	4	1.00	15	0.267
532	A	2	2	1.00	15	0.133
533	A	5	4	1.00	15	0.267
534	A	3	2	1.00	15	0.133
535	A	3	2	1.00	15	0.133
536	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
537	A	2	2	1.00	15	0.133
538	A	1	1	1.00	13	0.077
539	A	1	1	1.00	11	0.091
540	A	3	3	1.00	15	0.200
541	A	1	1	1.00	15	0.067
542	A	4	4	1.00	15	0.267
543	A	2	2	1.00	15	0.133
544	A	5	4	1.00	15	0.267
545	A	3	2	1.00	15	0.133
546	A	4	3	1.00	15	0.200
547	A	3	2	1.00	15	0.133
548	A	3	3	1.00	15	0.200
549	A	1	1	1.00	13	0.077
550	A	2	2	1.00	11	0.182
551	A	3	3	1.00	15	0.200
552	A	1	1	1.00	15	0.067
553	A	4	4	1.00	15	0.267
554	A	2	2	1.00	15	0.133
555	A	5	4	1.00	15	0.267
556	A	3	2	1.00	15	0.133
557	A	4	3	1.00	15	0.200
558	A	3	2	1.00	15	0.133
559	A	3	3	1.00	15	0.200
560	A	1	1	1.00	13	0.077
561	A	2	2	1.00	11	0.182
562	A	3	3	1.00	15	0.200
563	A	1	1	1.00	15	0.067
564	A	4	4	1.00	15	0.267
565	A	2	2	1.00	15	0.133
566	A	5	4	1.00	15	0.267
567	A	1	1	1.00	11	0.091
568	A	1	1	1.00	12	0.083
569	A	2	2	1.00	11	0.182
570	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
571	A	1	1	1.00	11	0.091
572	A	1	1	1.00	12	0.083
573	A	2	2	1.00	13	0.154
574	A	2	2	1.00	14	0.143
575	A	2	2	1.00	11	0.182
576	A	2	2	1.00	12	0.167
577	A	2	2	1.00	13	0.154
578	A	2	2	1.00	14	0.143
579	A	2	2	1.00	13	0.154
580	A	1	1	1.00	31	0.032
581	C	5	2	7.47	43	0.047
582	A	1	1	1.00	29	0.034
583	C	5	2	8.20	38	0.053
584	A	3	2	1.00	15	0.133
585	A	3	2	1.00	15	0.133
586	A	3	2	1.00	15	0.133
587	A	1	1	1.00	13	0.077
588	A	6	6	1.00	15	0.400
589	A	6	6	1.00	15	0.400
590	A	7	7	1.00	15	0.467
591	A	3	2	1.00	15	0.133
592	A	3	2	1.00	15	0.133
593	A	3	2	1.00	15	0.133
594	A	1	1	1.00	13	0.077
595	A	6	6	1.00	15	0.400
596	A	6	6	1.00	15	0.400
597	A	7	7	1.00	15	0.467
598	A	3	2	1.00	15	0.133
599	A	3	2	1.00	15	0.133
600	A	3	2	1.00	15	0.133
601	A	1	1	1.00	13	0.077
602	A	7	6	1.00	15	0.400
603	A	7	7	1.00	15	0.467
604	A	7	6	1.00	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
605	A	1	1	1.00	11	0.091
606	A	3	2	1.00	15	0.133
607	A	3	2	1.00	15	0.133
608	A	3	2	1.00	15	0.133
609	A	1	1	1.00	13	0.077
610	A	5	5	1.00	15	0.333
611	A	6	6	1.00	15	0.400
612	A	7	6	1.00	15	0.400
613	A	3	2	1.00	15	0.133
614	A	3	2	1.00	15	0.133
615	A	3	2	1.00	15	0.133
616	A	1	1	1.00	13	0.077
617	A	5	5	1.00	15	0.333
618	A	6	6	1.00	15	0.400
619	A	7	6	1.00	15	0.400
620	A	3	2	1.00	15	0.133
621	A	3	2	1.00	15	0.133
622	A	3	2	1.00	15	0.133
623	A	1	1	1.00	13	0.077
624	A	6	6	1.00	15	0.400
625	A	7	6	1.02	15	0.400
626	A	8	6	1.00	15	0.400
627	A	12	11	1.41	19	0.579
628	A	11	11	1.49	19	0.579
629	A	10	10	1.59	19	0.526
630	A	10	10	1.59	19	0.526
631	A	1	1	1.00	19	0.053
632	A	2	2	1.00	19	0.105
633	A	3	2	1.00	19	0.105
634	A	4	2	1.00	19	0.105
635	A	13	11	1.36	19	0.579
636	A	12	11	1.42	19	0.579
637	A	11	10	1.49	19	0.526
638	A	11	11	1.52	19	0.579

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
639	A	11	10	1.49	19	0.526
640	A	1	1	1.00	19	0.053
641	A	2	2	1.00	19	0.105
642	A	3	2	1.00	19	0.105
643	A	12	10	1.40	19	0.526
644	A	11	10	1.48	19	0.526
645	A	10	10	1.60	19	0.526
646	A	9	9	1.73	19	0.474
647	A	1	1	1.00	19	0.053
648	A	2	2	1.00	19	0.105
649	A	3	2	1.00	19	0.105
650	A	4	2	1.00	19	0.105
651	A	7	7	1.00	19	0.368
652	A	6	6	1.00	19	0.316
653	A	6	6	1.00	19	0.316
654	A	1	1	1.00	19	0.053
655	A	2	2	1.00	19	0.105
656	A	3	2	1.00	19	0.105
657	A	4	2	1.00	19	0.105
658	A	13	10	1.00	20	0.500
659	A	12	9	1.00	20	0.450
660	A	12	9	1.00	20	0.450
661	A	1	1	1.00	20	0.050
662	A	2	2	1.00	20	0.100
663	A	3	2	1.00	20	0.100
664	A	4	2	1.00	20	0.100
665	A	6	6	1.00	19	0.316
666	A	5	5	1.00	19	0.263
667	A	1	1	1.00	19	0.053
668	A	2	2	1.00	19	0.105
669	A	3	2	1.00	19	0.105
670	A	12	9	1.00	20	0.450
671	A	11	8	1.00	20	0.400
672	A	1	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
673	A	2	2	1.00	20	0.100
674	A	3	2	1.00	20	0.100
675	A	6	6	1.00	19	0.316
676	A	5	5	1.00	19	0.263
677	A	1	1	1.00	19	0.053
678	A	2	2	1.00	19	0.105
679	A	3	2	1.00	19	0.105
680	A	12	9	1.00	20	0.450
681	A	11	8	1.00	20	0.400
682	A	1	1	1.00	20	0.050
683	A	2	2	1.00	20	0.100
684	A	3	2	1.00	20	0.100
685	A	7	7	1.00	19	0.368
686	A	6	6	1.00	19	0.316
687	A	1	1	1.00	19	0.053
688	A	2	2	1.00	19	0.105
689	A	3	2	1.00	19	0.105
690	A	4	2	1.00	19	0.105
691	A	3	2	1.00	13	0.154
692	A	3	2	1.00	13	0.154
693	A	3	2	1.00	13	0.154
694	A	1	1	1.00	11	0.091
695	A	3	2	1.00	17	0.118
696	A	2	2	1.00	17	0.118
697	A	1	1	1.00	17	0.059

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^4 \left(a + bx^2\right) dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2),x]
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^4 \left(a + bx^2\right) dx &= \int \left(ax^4 + bx^6\right) dx \\ &= \frac{ax^5}{5} + \frac{bx^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2),x]
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2),x]
[Out] IntegrateAlgebraic[x^4*(a + b*x^2), x]
```

fricas [A] time = 1.05, size = 13, normalized size = 0.76

$$\frac{1}{7}x^7b + \frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="fricas")
[Out] 1/7*x^7*b + 1/5*x^5*a
```

giac [A] time = 1.09, size = 13, normalized size = 0.76

$$\frac{1}{7}\,bx^7 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="giac")
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{7}b\,x^7 + \frac{1}{5}a\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a),x)
[Out] 1/5*a*x^5+1/7*b*x^7
```

maxima [A] time = 1.31, size = 13, normalized size = 0.76

$$\frac{1}{7}\,bx^7 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$\frac{b\,x^7}{7}+\frac{a\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2),x)
```

```
[Out] (a*x^5)/5 + (b*x^7)/7
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^5}{5}+\frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a),x)
```

```
[Out] a*x**5/5 + b*x**7/7
```

3.2 $\int x^3 \left(a + bx^2\right) dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a + bx^2\right) dx &= \int \left(ax^3 + bx^5\right) dx \\ &= \frac{ax^4}{4} + \frac{bx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2),x]
[Out] IntegrateAlgebraic[x^3*(a + b*x^2), x]
```

fricas [A] time = 0.83, size = 13, normalized size = 0.76

$$\frac{1}{6}x^6b+\frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="fricas")
[Out] 1/6*x^6*b + 1/4*x^4*a
```

giac [A] time = 0.92, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="giac")
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{6}b\,x^6+\frac{1}{4}a\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a),x)
[Out] 1/4*a*x^4+1/6*b*x^6
```

maxima [A] time = 1.35, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^6}{6}+\frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2),x)
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

3.3 $\int x^2 \left(a+bx^2\right) dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right) dx &= \int \left(ax^2+bx^4\right) dx\\ &= \frac{ax^3}{3}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2),x]
[Out] IntegrateAlgebraic[x^2*(a + b*x^2), x]
```

fricas [A] time = 0.77, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="fricas")
[Out] 1/5*x^5*b + 1/3*x^3*a
```

giac [A] time = 1.11, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="giac")
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{5}b\,x^5 + \frac{1}{3}a\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a),x)
[Out] 1/3*a*x^3+1/5*b*x^5
```

maxima [A] time = 1.34, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^5}{5}+\frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

3.4 $\int x\left(a+bx^2\right)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\,dx &= \int \left(ax+bx^3\right)\,dx\\ &= \frac{ax^2}{2}+\frac{bx^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2), x]
```

fricas [A] time = 0.99, size = 13, normalized size = 0.76

$$\frac{1}{4}x^4b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b + 1/2*x^2*a
```

giac [A] time = 1.03, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{4}b\,x^4+\frac{1}{2}a\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a),x)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4
```

maxima [A] time = 1.34, size = 14, normalized size = 0.82

$$\frac{\left(bx^2+a\right)^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^2/b
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^4}{4}+\frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2),x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a),x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

3.5 $\int (a + bx^2) \, dx$

Optimal. Leaf size=12

$$ax + \frac{bx^3}{3}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^2,x]
```

```
[Out] a*x + (b*x^3)/3
```

Rubi steps

$$\int (a + bx^2) \, dx = ax + \frac{bx^3}{3}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^2,x]
```

```
[Out] a*x + (b*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx^2) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + b*x^2,x]
```

```
[Out] IntegrateAlgebraic[a + b*x^2, x]
```

fricas [A] time = 0.99, size = 10, normalized size = 0.83

$$\frac{1}{3}x^3b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="fricas")
[Out] 1/3*x^3*b + x*a
```

giac [A] time = 0.94, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="giac")
[Out] 1/3*b*x^3 + a*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{3}b\,x^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^2+a,x)
[Out] a*x+1/3*b*x^3
```

maxima [A] time = 1.34, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^2+a,x, algorithm="maxima")
[Out] 1/3*b*x^3 + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^3}{3} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^2,x)
```

```
[Out] a*x + (b*x^3)/3
```

sympy **[A]** time = 0.06, size = 8, normalized size = 0.67

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**2+a,x)
```

```
[Out] a*x + b*x**3/3
```

3.6 $\int \frac{a+bx^2}{x}\,dx$

Optimal. Leaf size=13

$$a\log(x)+\frac{bx^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$a\log(x)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x,x]
```

```
[Out] (b*x^2)/2 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x}\,dx &= \int \left(\frac{a}{x}+bx\right)dx\\ &= \frac{bx^2}{2}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$a\log(x)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x,x]
```

```
[Out] (b*x^2)/2 + a*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)/x, x]
```

fricas [A] time = 1.17, size = 11, normalized size = 0.85

$$\frac{1}{2}\,bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

giac [A] time = 1.02, size = 14, normalized size = 1.08

$$\frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{b\,x^2}{2} + a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x,x)
```

```
[Out] 1/2*b*x^2+a*ln(x)
```

maxima [A] time = 1.35, size = 14, normalized size = 1.08

$$\frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$\frac{b\,x^2}{2}+a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x,x)
```

```
[Out] (b*x^2)/2 + a*log(x)
```

sympy [A] time = 0.08, size = 10, normalized size = 0.77

$$a\log{(x)}+\frac{bx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x,x)
```

```
[Out] a*log(x) + b*x**2/2
```

3.7 $\int \frac{a+bx^2}{x^2} \, dx$

Optimal. Leaf size=10

$$bx - \frac{a}{x}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$bx - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^2,x]
```

```
[Out] -(a/x) + b*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^2} \, dx &= \int \left(b+\frac{a}{x^2}\right) \, dx\\ &= -\frac{a}{x}+bx\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$bx - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^2,x]
```

```
[Out] -(a/x) + b*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/x^2,x]
[Out] IntegrateAlgebraic[(a + b*x^2)/x^2, x]
```

fricas [A] time = 1.06, size = 13, normalized size = 1.30

$$\frac{bx^2-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="fricas")
[Out] (b*x^2 - a)/x
```

giac [A] time = 0.98, size = 10, normalized size = 1.00

$$bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="giac")
[Out] b*x - a/x
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2,x)
[Out] -a/x+b*x
```

maxima [A] time = 1.33, size = 10, normalized size = 1.00

$$bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] b*x - a/x
```

mupad [B] time = 0.02, size = 10, normalized size = 1.00

$$b x - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^2,x)
```

```
[Out] b*x - a/x
```

sympy [A] time = 0.08, size = 5, normalized size = 0.50

$$-\frac{a}{x} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2,x)
```

```
[Out] -a/x + b*x
```

3.8 $\int \frac{a+bx^2}{x^3} dx$

Optimal. Leaf size=13

$$b\log(x) - \frac{a}{2x^2}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$b\log(x) - \frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^3,x]
```

```
[Out] -a/(2*x^2) + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^3}\,dx &= \int \left(\frac{a}{x^3}+\frac{b}{x}\right) dx\\ &= -\frac{a}{2x^2}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$b\log(x) - \frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^3,x]
```

```
[Out] -1/2*a/x^2 + b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)/x^3, x]
```

fricas [A] time = 1.28, size = 17, normalized size = 1.31

$$\frac{2\,bx^2\log(x)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*log(x) - a)/x^2
```

giac [A] time = 1.06, size = 20, normalized size = 1.54

$$\frac{1}{2}\,b\log\left(x^2\right)-\frac{bx^2+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$b\ln(x)-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3,x)
```

```
[Out] -1/2*a/x^2+b*ln(x)
```

maxima [A] time = 1.36, size = 14, normalized size = 1.08

$$\frac{1}{2}\,b\log\left(x^2\right)-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3,x, algorithm="maxima")
[Out] 1/2*b*log(x^2) - 1/2*a/x^2
```

mupad [B] time = 4.93, size = 11, normalized size = 0.85

$$b \ln(x) - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^3,x)
[Out] b*log(x) - a/(2*x^2)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$-\frac{a}{2x^2} + b \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3,x)
[Out] -a/(2*x**2) + b*log(x)
```

3.9 $\int \frac{a+bx^2}{x^4} \, dx$

Optimal. Leaf size=15

$$-\frac{a}{3x^3}-\frac{b}{x}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{3x^3}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^4,x]
```

```
[Out] -a/(3*x^3) - b/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^4} \, dx &= \int \left(\frac{a}{x^4}+\frac{b}{x^2}\right) dx\\ &= -\frac{a}{3x^3}-\frac{b}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/x^4,x]
[Out] IntegrateAlgebraic[(a + b*x^2)/x^4, x]
```

fricas [A] time = 0.80, size = 13, normalized size = 0.87

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="fricas")
[Out] -1/3*(3*b*x^2 + a)/x^3
```

giac [A] time = 1.11, size = 13, normalized size = 0.87

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="giac")
[Out] -1/3*(3*b*x^2 + a)/x^3
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$-\frac{b}{x}-\frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4,x)
[Out] -1/3*a/x^3-b/x
```

maxima [A] time = 1.29, size = 13, normalized size = 0.87

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$-\frac{3\,b\,x^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^4,x)
```

```
[Out] -(a + 3*b*x^2)/(3*x^3)
```

sympy [A] time = 0.11, size = 14, normalized size = 0.93

$$\frac{-a-3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4,x)
```

```
[Out] (-a - 3*b*x**2)/(3*x**3)
```

3.10 $\int \frac{a+bx^2}{x^5} \, dx$

Optimal. Leaf size=17

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^5} \, dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^3}\right) \, dx \\ &= -\frac{a}{4x^4}-\frac{b}{2x^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/x^5,x]
[Out] IntegrateAlgebraic[(a + b*x^2)/x^5, x]
```

fricas [A] time = 0.82, size = 13, normalized size = 0.76

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="fricas")
[Out] -1/4*(2*b*x^2 + a)/x^4
```

giac [A] time = 0.99, size = 13, normalized size = 0.76

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="giac")
[Out] -1/4*(2*b*x^2 + a)/x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{2x^2}-\frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5,x)
[Out] -1/4*a/x^4-1/2*b/x^2
```

maxima [A] time = 1.33, size = 13, normalized size = 0.76

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 + a)/x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$-\frac{2\,b\,x^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^5,x)
```

```
[Out] -(a + 2*b*x^2)/(4*x^4)
```

sympy [A] time = 0.12, size = 14, normalized size = 0.82

$$\frac{-a-2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5,x)
```

```
[Out] (-a - 2*b*x**2)/(4*x**4)
```

3.11 $\int \frac{a+bx^2}{x^6} dx$

Optimal. Leaf size=17

$$-\frac{a}{5x^5}-\frac{b}{3x^3}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{5x^5}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^6}\,dx &= \int \left(\frac{a}{x^6}+\frac{b}{x^4}\right) dx\\ &= -\frac{a}{5x^5}-\frac{b}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{5x^5}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^6,x]
```

```
[Out] -1/5*a/x^5 - b/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/x^6,x]
[Out] IntegrateAlgebraic[(a + b*x^2)/x^6, x]
```

fricas [A] time = 0.91, size = 15, normalized size = 0.88

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="fricas")
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

giac [A] time = 1.09, size = 15, normalized size = 0.88

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="giac")
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$-\frac{b}{3x^3}-\frac{a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^6,x)
[Out] -1/5*a/x^5-1/3*b/x^3
```

maxima [A] time = 1.30, size = 15, normalized size = 0.88

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{5\,b\,x^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^6,x)
```

```
[Out] -(3*a + 5*b*x^2)/(15*x^5)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$\frac{-3a-5bx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**6,x)
```

```
[Out] (-3*a - 5*b*x**2)/(15*x**5)
```

3.12 $\int \frac{a+bx^2}{x^7} \, dx$

Optimal. Leaf size=17

$$-\frac{a}{6x^6}-\frac{b}{4x^4}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$-\frac{a}{6x^6}-\frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^7,x]
[Out] -a/(6*x^6) - b/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^7} \, dx &= \int \left(\frac{a}{x^7}+\frac{b}{x^5}\right) \, dx\\ &= -\frac{a}{6x^6}-\frac{b}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{6x^6}-\frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^7,x]
[Out] -1/6*a/x^6 - b/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/x^7,x]
[Out] IntegrateAlgebraic[(a + b*x^2)/x^7, x]
```

fricas [A] time = 1.16, size = 15, normalized size = 0.88

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="fricas")
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

giac [A] time = 1.03, size = 15, normalized size = 0.88

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="giac")
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$-\frac{b}{4x^4}-\frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^7,x)
[Out] -1/6*a/x^6-1/4*b/x^4
```

maxima [A] time = 1.42, size = 15, normalized size = 0.88

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{3\,b\,x^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^7,x)
```

```
[Out] -(2*a + 3*b*x^2)/(12*x^6)
```

sympy [A] time = 0.14, size = 15, normalized size = 0.88

$$\frac{-2a-3bx^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**7,x)
```

```
[Out] (-2*a - 3*b*x**2)/(12*x**6)
```

3.13 $\int x^5 \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5 \left(a+bx^2\right)^2 \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^2 \, dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(a^2x^2+2abx^3+b^2x^4\right) \, dx, x, x^2\right)\\ &= \frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^2,x]
[Out] (a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+bx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^2,x]
[Out] IntegrateAlgebraic[x^5*(a + b*x^2)^2, x]
```

fricas [A] time = 0.94, size = 24, normalized size = 0.80

$$\frac{1}{10}x^{10}b^2+\frac{1}{4}x^8ba+\frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/10*x^10*b^2 + 1/4*x^8*b*a + 1/6*x^6*a^2
```

giac [A] time = 1.07, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10}+\frac{1}{4}\,abx^8+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="giac")
[Out] 1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{10}b^2x^{10}+\frac{1}{4}ab\,x^8+\frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^2,x)
```

```
[Out] 1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^10
```

maxima [A] time = 1.34, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10} + \frac{1}{4}\,abx^8 + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^6}{6} + \frac{a\,b\,x^8}{4} + \frac{b^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^6)/6 + (b^2*x^10)/10 + (a*b*x^8)/4
```

sympy [A] time = 0.06, size = 24, normalized size = 0.80

$$\frac{a^2x^6}{6} + \frac{abx^8}{4} + \frac{b^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**6/6 + a*b*x**8/4 + b**2*x**10/10
```

3.14 $\int x^4 \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^2\right)^2 \, dx &= \int \left(a^2x^4+2abx^6+b^2x^8\right) \, dx\\ &= \frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + bx^2\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^2,x]
[Out] IntegrateAlgebraic[x^4*(a + b*x^2)^2, x]
```

fricas [A] time = 1.09, size = 24, normalized size = 0.80

$$\frac{1}{9}x^9b^2 + \frac{2}{7}x^7ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/9*x^9*b^2 + 2/7*x^7*b*a + 1/5*x^5*a^2
```

giac [A] time = 1.29, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^2x^9 + \frac{2}{7}\,abx^7 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="giac")
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{9}b^2x^9 + \frac{2}{7}ab\,x^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2,x)
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

maxima [A] time = 1.31, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^2x^9 + \frac{2}{7}\,abx^7 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5}+\frac{2\,a\,b\,x^7}{7}+\frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

sympy [A] time = 0.06, size = 26, normalized size = 0.87

$$\frac{a^{2} x^{5}}{5} + \frac{2 a b x^{7}}{7} + \frac{b^{2} x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

3.15 $\int x^3 \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^2\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(a^2x+2abx^2+b^2x^3\right)\,dx,x,x^2\right)\\ &= \frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2,x]
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^2,x]
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)^2, x]
```

fricas [A] time = 0.84, size = 24, normalized size = 0.80

$$\frac{1}{8}x^8b^2+\frac{1}{3}x^6ba+\frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2
```

giac [A] time = 1.04, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="giac")
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{8}b^2x^8+\frac{1}{3}ab\,x^6+\frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2,x)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

sympy [A] time = 0.06, size = 24, normalized size = 0.80

$$\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

3.16 $\int x^2 \left(a + bx^2\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a + bx^2\right)^2 \, dx &= \int \left(a^2x^2 + 2abx^4 + b^2x^6\right) dx \\ &= \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^2, x]
```

fricas [A] time = 0.92, size = 24, normalized size = 0.80

$$\frac{1}{7}x^7b^2 + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2
```

giac [A] time = 1.13, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{7}b^2x^7 + \frac{2}{5}ab\,x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2,x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

3.17 $\int x\left(a+bx^2\right)^2\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2,x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^2\,dx=\frac{\left(a+bx^2\right)^3}{6b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2,x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^2,x]
[Out] IntegrateAlgebraic[x*(a + b*x^2)^2, x]
```

fricas [A] time = 0.49, size = 24, normalized size = 1.50

$$\frac{1}{6}x^6b^2+\frac{1}{2}x^4ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/6*x^6*b^2 + 1/2*x^4*b*a + 1/2*x^2*a^2
```

giac [A] time = 1.05, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="giac")
[Out] 1/6*(b*x^2 + a)^3/b
```

maple [A] time = 0.00, size = 25, normalized size = 1.56

$$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2,x)
[Out] 1/6*b^2*x^6+1/2*a*b*x^4+1/2*a^2*x^2
```

maxima [A] time = 1.35, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^3/b
```

mupad [B] time = 0.03, size = 24, normalized size = 1.50

$$\frac{a^2\,x^2}{2}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

sympy [B] time = 0.06, size = 24, normalized size = 1.50

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

3.18 $\int \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2 \, dx &= \int \left(a^2+2abx^2+b^2x^4\right) \, dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2, x]
```

fricas [A] time = 0.69, size = 21, normalized size = 0.84

$$\frac{1}{5}x^5b^2+\frac{2}{3}x^3ba+xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^2 + 2/3*x^3*b*a + x*a^2
```

giac [A] time = 1.05, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}b^2x^5+\frac{2}{3}ab\,x^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

maxima [A] time = 1.31, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x+\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2,x)
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

sympy [A] time = 0.06, size = 22, normalized size = 0.88

$$a^{2} x + \frac{2 a b x^{3}}{3} + \frac{b^{2} x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2,x)
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

3.19 $\int \frac{(a+bx^2)^2}{x} \, dx$

Optimal. Leaf size=23

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^2}{x} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(2ab + \frac{a^2}{x} + b^2x\right) \, dx, x, x^2\right) \\
&= abx^2 + \frac{b^2x^4}{4} + a^2 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^2}{x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/x, x]
```

fricas [A] time = 0.88, size = 21, normalized size = 0.91

$$\frac{1}{4} b^2x^4 + abx^2 + a^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

giac [A] time = 1.07, size = 24, normalized size = 1.04

$$\frac{1}{4} b^2x^4 + abx^2 + \frac{1}{2} a^2 \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{b^2x^4}{4} + ab\,x^2 + a^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x,x)
```

```
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

maxima [A] time = 1.39, size = 24, normalized size = 1.04

$$\frac{1}{4}\,b^2x^4+abx^2+\frac{1}{2}\,a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.91

$$a^2\,\ln(x)+\frac{b^2\,x^4}{4}+a\,b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x,x)
```

```
[Out] a^2*log(x) + (b^2*x^4)/4 + a*b*x^2
```

sympy [A] time = 0.10, size = 20, normalized size = 0.87

$$a^2\log\left(x\right)+abx^2+\frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

3.20 $\int \frac{(a+bx^2)^2}{x^2} \, dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^2} \, dx &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right) dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^2,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^2/x^2, x]
```

fricas [A] time = 2.06, size = 25, normalized size = 1.04

$$\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="fricas")
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

giac [A] time = 1.07, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="giac")
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{b^2x^3}{3}+2abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2,x)
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

maxima [A] time = 1.36, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

mupad [B] time = 0.03, size = 22, normalized size = 0.92

$$\frac{b^2\,x^3}{3}-\frac{a^2}{x}+2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^2,x)
```

```
[Out] (b^2*x^3)/3 - a^2/x + 2*a*b*x
```

sympy [A] time = 0.10, size = 19, normalized size = 0.79

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

3.21 $\int \frac{(a+bx^2)^2}{x^3}\,dx$

Optimal. Leaf size=27

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^3,x]
[Out] -1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^3,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^2/x^3, x]
```

fricas [A] time = 1.56, size = 27, normalized size = 1.00

$$\frac{b^2x^4+4\,abx^2\log(x)-a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="fricas")
[Out] 1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

giac [A] time = 1.14, size = 32, normalized size = 1.19

$$\frac{1}{2}\,b^2x^2+ab\log\left(x^2\right)-\frac{2\,abx^2+a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="giac")
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{b^2x^2}{2}+2ab\ln(x)-\frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

maxima [A] time = 1.39, size = 24, normalized size = 0.89

$$\frac{1}{2}\,b^2x^2 + ab\log\left(x^2\right) - \frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

mupad [B] time = 4.94, size = 23, normalized size = 0.85

$$\frac{b^2\,x^2}{2} - \frac{a^2}{2\,x^2} + 2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^3,x)
```

```
[Out] (b^2*x^2)/2 - a^2/(2*x^2) + 2*a*b*log(x)
```

sympy [A] time = 0.13, size = 24, normalized size = 0.89

$$-\frac{a^2}{2x^2} + 2ab\log\left(x\right) + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

3.22 $\int \frac{(a+bx^2)^2}{x^4} \, dx$

Optimal. Leaf size=23

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^2}{x^4} \, dx &= \int \left(b^2+\frac{a^2}{x^4}+\frac{2ab}{x^2}\right) dx \\ &= -\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x \end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^4,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^2/x^4, x]
```

fricas [A] time = 0.78, size = 26, normalized size = 1.13

$$\frac{3\,b^2x^4 - 6\,abx^2 - a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="fricas")
[Out] 1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

giac [A] time = 0.88, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="giac")
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

maple [A] time = 0.01, size = 22, normalized size = 0.96

$$b^2x - \frac{2ab}{x} - \frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4,x)
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

maxima [A] time = 1.32, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 1.04

$$b^2\,x-\frac{\frac{a^2}{3}+2\,b\,a\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^4,x)
```

```
[Out] b^2*x - (a^2/3 + 2*a*b*x^2)/x^3
```

sympy [A] time = 0.14, size = 22, normalized size = 0.96

$$b^2x+\frac{-a^2-6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4,x)
```

```
[Out] b**2*x + (-a**2 - 6*a*b*x**2)/(3*x**3)
```

3.23 $\int \frac{(a+bx^2)^2}{x^5} \, dx$

Optimal. Leaf size=24

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^5} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^2}{x^3} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^2}{x^3} + \frac{2ab}{x^2} + \frac{b^2}{x}\right) dx, x, x^2\right) \\
&= -\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (a*b)/x^2 + b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/x^5, x]
```

fricas [A] time = 1.04, size = 28, normalized size = 1.17

$$\frac{4\,b^2x^4\log(x)-4\,abx^2-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

giac [A] time = 1.14, size = 34, normalized size = 1.42

$$\frac{1}{2}\,b^2\log\left(x^2\right)-\frac{3\,b^2x^4+4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

maple [A] time = 0.01, size = 23, normalized size = 0.96

$$b^2\ln(x)-\frac{ab}{x^2}-\frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

maxima [A] time = 1.28, size = 26, normalized size = 1.08

$$\frac{1}{2}\,b^2\log\left(x^2\right)-\frac{4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

mupad [B] time = 0.04, size = 24, normalized size = 1.00

$$b^2\,\ln(x)-\frac{\frac{a^2}{4}+b\,a\,x^2}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^5,x)
```

```
[Out] b^2*log(x) - (a^2/4 + a*b*x^2)/x^4
```

sympy [A] time = 0.17, size = 24, normalized size = 1.00

$$b^2\log\left(x\right)+\frac{-a^2-4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5,x)
```

```
[Out] b**2*log(x) + (-a**2 - 4*a*b*x**2)/(4*x**4)
```

3.24 $\int \frac{(a+bx^2)^2}{x^6}\,dx$

Optimal. Leaf size=28

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^6}\,dx &= \int \left(\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2}{x^2}\right)dx\\ &= -\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^6,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^2/x^6, x]
```

fricas [A] time = 1.03, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="fricas")
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

giac [A] time = 1.05, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="giac")
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$-\frac{b^2}{x}-\frac{2ab}{3x^3}-\frac{a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6,x)
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

maxima [A] time = 1.30, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

mupad [B] time = 0.03, size = 25, normalized size = 0.89

$$-\frac{\frac{a^2}{5}+\frac{2\,a\,b\,x^2}{3}+b^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^6,x)
```

```
[Out] -(a^2/5 + b^2*x^4 + (2*a*b*x^2)/3)/x^5
```

sympy [A] time = 0.17, size = 27, normalized size = 0.96

$$\frac{-3a^2-10abx^2-15b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6,x)
```

```
[Out] (-3*a**2 - 10*a*b*x**2 - 15*b**2*x**4)/(15*x**5)
```

3.25 $\int \frac{(a+bx^2)^2}{x^7} dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^3}{6ax^6}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^3}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^7,x]
```

```
[Out] -(a + b*x^2)^3/(6*a*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2}{x^7} dx = -\frac{(a+bx^2)^3}{6ax^6}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.58

$$-\frac{a^2}{6x^6} - \frac{ab}{2x^4} - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^7,x]
```

```
[Out] -1/6*a^2/x^6 - (a*b)/(2*x^4) - b^2/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^7,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^2/x^7, x]
```

fricas [A] time = 1.19, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="fricas")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

giac [A] time = 1.11, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="giac")
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

maple [A] time = 0.01, size = 25, normalized size = 1.32

$$-\frac{b^2}{2x^2}-\frac{ab}{2x^4}-\frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7,x)
[Out] -1/6*a^2/x^6-1/2*b^2/x^2-1/2*a*b/x^4
```

maxima [A] time = 1.38, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

mupad [**B**] time = 0.03, size = 26, normalized size = 1.37

$$-\frac{\frac{a^2}{6}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^7,x)
```

```
[Out] -(a^2/6 + (b^2*x^4)/2 + (a*b*x^2)/2)/x^6
```

sympy [**A**] time = 0.19, size = 26, normalized size = 1.37

$$\frac{-a^2-3abx^2-3b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7,x)
```

```
[Out] (-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*x**6)
```

3.26 $\int \frac{(a+bx^2)^2}{x^8}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^8}\,dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2}{x^4}\right)dx\\ &= -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^2}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/x^8, x]
```

fricas [A] time = 0.79, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

giac [A] time = 1.06, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{3x^3} - \frac{2ab}{5x^5} - \frac{a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^8,x)
```

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

maxima [A] time = 1.38, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{7}+\frac{2\,a\,b\,x^2}{5}+\frac{b^2\,x^4}{3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^8,x)
```

```
[Out] -(a^2/7 + (b^2*x^4)/3 + (2*a*b*x^2)/5)/x^7
```

sympy [A] time = 0.20, size = 27, normalized size = 0.90

$$\frac{-15a^2-42abx^2-35b^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**8,x)
```

```
[Out] (-15*a**2 - 42*a*b*x**2 - 35*b**2*x**4)/(105*x**7)
```

3.27 $\int \frac{\left(a+bx^2\right)^2}{x^9}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - b^2/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^9}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^5}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^9,x]
```

```
[Out] -1/8*a^2/x^8 - (a*b)/(3*x^6) - b^2/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/x^9, x]
```

fricas [A] time = 1.13, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

giac [A] time = 1.09, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="giac")
```

```
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{b^2}{4x^4}-\frac{ab}{3x^6}-\frac{a^2}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^9,x)
```

```
[Out] -1/8*a^2/x^8-1/3*a*b/x^6-1/4*b^2/x^4
```

maxima [A] time = 1.38, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^4 + 8\,abx^2 + 3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^9,x, algorithm="maxima")
```

```
[Out] -1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{8} + \frac{a\,b\,x^2}{3} + \frac{b^2\,x^4}{4}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^9,x)
```

```
[Out] -(a^2/8 + (b^2*x^4)/4 + (a*b*x^2)/3)/x^8
```

sympy [A] time = 0.21, size = 27, normalized size = 0.90

$$\frac{-3a^2 - 8abx^2 - 6b^2x^4}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**9,x)
```

```
[Out] (-3*a**2 - 8*a*b*x**2 - 6*b**2*x**4)/(24*x**8)
```

3.28 $\int \frac{(a+bx^2)^2}{x^{10}} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^10,x]
```

```
[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - b^2/(5*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{10}} dx &= \int \left(\frac{a^2}{x^{10}} + \frac{2ab}{x^8} + \frac{b^2}{x^6}\right) dx\\ &= -\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^10,x]
```

```
[Out] -1/9*a^2/x^9 - (2*a*b)/(7*x^7) - b^2/(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^10,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^2/x^10, x]
```

fricas [A] time = 1.28, size = 26, normalized size = 0.87

$$-\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="fricas")
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

giac [A] time = 1.05, size = 26, normalized size = 0.87

$$-\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="giac")
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{a^2}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^10,x)
[Out] -1/9*a^2/x^9-2/7*a*b/x^7-1/5*b^2/x^5
```

maxima [A] time = 1.34, size = 26, normalized size = 0.87

$$-\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

mupad [B] time = 0.03, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{9}+\frac{2\,a\,b\,x^2}{7}+\frac{b^2\,x^4}{5}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^10,x)
```

```
[Out] -(a^2/9 + (b^2*x^4)/5 + (2*a*b*x^2)/7)/x^9
```

sympy [A] time = 0.22, size = 27, normalized size = 0.90

$$\frac{-35a^2-90abx^2-63b^2x^4}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**10,x)
```

```
[Out] (-35*a**2 - 90*a*b*x**2 - 63*b**2*x**4)/(315*x**9)
```

3.29 $\int x^9 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{4}a^2bx^{12}+\frac{a^3x^{10}}{10}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^10)/10 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14 + (b^3*x^16)/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^3\,dx&=\frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^3\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^3x^4+3a^2bx^5+3ab^2x^6+b^3x^7\right)\,dx,x,x^2\right)\\&=\frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^3,x]
[Out] (a^3*x^10)/10 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14 + (b^3*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9\left(a+bx^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9*(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[x^9*(a + b*x^2)^3, x]
```

fricas [A] time = 1.22, size = 35, normalized size = 0.81

$$\frac{1}{16}x^{16}b^3+\frac{3}{14}x^{14}b^2a+\frac{1}{4}x^{12}ba^2+\frac{1}{10}x^{10}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/16*x^16*b^3 + 3/14*x^14*b^2*a + 1/4*x^12*b*a^2 + 1/10*x^10*a^3
```

giac [A] time = 1.17, size = 35, normalized size = 0.81

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{16}b^3x^{16}+\frac{3}{14}a\,b^2x^{14}+\frac{1}{4}a^2b\,x^{12}+\frac{1}{10}a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^3,x)
```

```
[Out] 1/10*a^3*x^10+1/4*a^2*b*x^12+3/14*a*b^2*x^14+1/16*b^3*x^16
```

maxima [A] time = 1.36, size = 35, normalized size = 0.81

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^{10}}{10}+\frac{a^2\,b\,x^{12}}{4}+\frac{3\,a\,b^2\,x^{14}}{14}+\frac{b^3\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^10)/10 + (b^3*x^16)/16 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14
```

sympy [A] time = 0.07, size = 37, normalized size = 0.86

$$\frac{a^3x^{10}}{10}+\frac{a^2bx^{12}}{4}+\frac{3ab^2x^{14}}{14}+\frac{b^3x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**10/10 + a**2*b*x**12/4 + 3*a*b**2*x**14/14 + b**3*x**16/16
```

3.30 $\int x^7 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{10}a^2bx^{10}+\frac{a^3x^8}{8}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^8)/8 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4 + (b^3*x^14)/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^3 \, dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^3 \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right) dx, x, x^2\right)\\
&= \frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^8)/8 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4 + (b^3*x^14)/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\left(a+bx^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^7*(a + b*x^2)^3, x]
```

fricas [A] time = 0.98, size = 35, normalized size = 0.81

$$\frac{1}{14}x^{14}b^3+\frac{1}{4}x^{12}b^2a+\frac{3}{10}x^{10}ba^2+\frac{1}{8}x^8a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^3 + 1/4*x^12*b^2*a + 3/10*x^10*b*a^2 + 1/8*x^8*a^3
```

giac [A] time = 1.18, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{14}b^3x^{14}+\frac{1}{4}a\,b^2x^{12}+\frac{3}{10}a^2b\,x^{10}+\frac{1}{8}a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^3,x)
```

```
[Out] 1/8*a^3*x^8+3/10*a^2*b*x^10+1/4*a*b^2*x^12+1/14*b^3*x^14
```

maxima [**A**] time = 1.32, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

mupad [**B**] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^8}{8}+\frac{3\,a^2\,b\,x^{10}}{10}+\frac{a\,b^2\,x^{12}}{4}+\frac{b^3\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^8)/8 + (b^3*x^14)/14 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4
```

sympy [**A**] time = 0.07, size = 37, normalized size = 0.86

$$\frac{a^3x^8}{8}+\frac{3a^2bx^{10}}{10}+\frac{ab^2x^{12}}{4}+\frac{b^3x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**8/8 + 3*a**2*b*x**10/10 + a*b**2*x**12/4 + b**3*x**14/14
```

3.31 $\int x^5 \left(a+bx^2\right)^3 dx$

Optimal. Leaf size=43

$$\frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{8}a^2bx^8+\frac{a^3x^6}{6}+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^6)/6 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10 + (b^3*x^12)/12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^3 dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^3\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)dx, x, x^2\right)\\ &= \frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^6}{6} + \frac{3}{8}a^2bx^8 + \frac{3}{10}ab^2x^{10} + \frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^3,x]
[Out] (a^3*x^6)/6 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10 + (b^3*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a + bx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[x^5*(a + b*x^2)^3, x]
```

fricas [A] time = 0.47, size = 35, normalized size = 0.81

$$\frac{1}{12}x^{12}b^3 + \frac{3}{10}x^{10}b^2a + \frac{3}{8}x^8ba^2 + \frac{1}{6}x^6a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/12*x^12*b^3 + 3/10*x^10*b^2*a + 3/8*x^8*b*a^2 + 1/6*x^6*a^3
```

giac [A] time = 0.97, size = 35, normalized size = 0.81

$$\frac{1}{12}\,b^3x^{12} + \frac{3}{10}\,ab^2x^{10} + \frac{3}{8}\,a^2bx^8 + \frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{12}b^3x^{12} + \frac{3}{10}a\,b^2x^{10} + \frac{3}{8}a^2b\,x^8 + \frac{1}{6}a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^3,x)
```

```
[Out] 1/6*a^3*x^6+3/8*a^2*b*x^8+3/10*a*b^2*x^10+1/12*b^3*x^12
```

maxima [A] time = 1.35, size = 35, normalized size = 0.81

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^6}{6}+\frac{3\,a^2\,b\,x^8}{8}+\frac{3\,a\,b^2\,x^{10}}{10}+\frac{b^3\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^6)/6 + (b^3*x^12)/12 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10
```

sympy [A] time = 0.07, size = 39, normalized size = 0.91

$$\frac{a^3x^6}{6}+\frac{3a^2bx^8}{8}+\frac{3ab^2x^{10}}{10}+\frac{b^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**6/6 + 3*a**2*b*x**8/8 + 3*a*b**2*x**10/10 + b**3*x**12/12
```

3.32 $\int x^3 \left(a+bx^2\right)^3 dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^5}{10b^2}-\frac{a\left(a+bx^2\right)^4}{8b^2}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^5}{10b^2}-\frac{a\left(a+bx^2\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^3,x]
```

```
[Out] -(a*(a + b*x^2)^4)/(8*b^2) + (a + b*x^2)^5/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+bx^2\right)^3 dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^3\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^4}{8b^2}+\frac{\left(a+bx^2\right)^5}{10b^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.26

$$\frac{a^3x^4}{4} + \frac{1}{2}a^2bx^6 + \frac{3}{8}ab^2x^8 + \frac{b^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^3,x]
[Out] (a^3*x^4)/4 + (a^2*b*x^6)/2 + (3*a*b^2*x^8)/8 + (b^3*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + bx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)^3, x]
```

fricas [A] time = 0.95, size = 35, normalized size = 1.03

$$\frac{1}{10}x^{10}b^3 + \frac{3}{8}x^8b^2a + \frac{1}{2}x^6ba^2 + \frac{1}{4}x^4a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/10*x^10*b^3 + 3/8*x^8*b^2*a + 1/2*x^6*b*a^2 + 1/4*x^4*a^3
```

giac [A] time = 0.95, size = 35, normalized size = 1.03

$$\frac{1}{10}\,b^3x^{10} + \frac{3}{8}\,ab^2x^8 + \frac{1}{2}\,a^2bx^6 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

maple [A] time = 0.00, size = 36, normalized size = 1.06

$$\frac{1}{10}b^3x^{10} + \frac{3}{8}a\,b^2x^8 + \frac{1}{2}a^2b\,x^6 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^3,x)
```

```
[Out] 1/10*b^3*x^10+3/8*a*b^2*x^8+1/2*a^2*b*x^6+1/4*a^3*x^4
```

maxima [A] time = 1.37, size = 35, normalized size = 1.03

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{2}\,a^2bx^6+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

mupad [B] time = 0.04, size = 35, normalized size = 1.03

$$\frac{a^3\,x^4}{4}+\frac{a^2\,b\,x^6}{2}+\frac{3\,a\,b^2\,x^8}{8}+\frac{b^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^10)/10 + (a^2*b*x^6)/2 + (3*a*b^2*x^8)/8
```

sympy [A] time = 0.07, size = 37, normalized size = 1.09

$$\frac{a^3x^4}{4}+\frac{a^2bx^6}{2}+\frac{3ab^2x^8}{8}+\frac{b^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**4/4 + a**2*b*x**6/2 + 3*a*b**2*x**8/8 + b**3*x**10/10
```

3.33 $\int x\left(a+bx^2\right)^3\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^4}{8b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^4}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^3,x]
```

```
[Out] (a + b*x^2)^4/(8*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^3\,dx=\frac{\left(a+bx^2\right)^4}{8b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^4}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^3,x]
```

```
[Out] (a + b*x^2)^4/(8*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[x*(a + b*x^2)^3, x]
```

fricas [B] time = 0.89, size = 35, normalized size = 2.19

$$\frac{1}{8}x^8b^3+\frac{1}{2}x^6b^2a+\frac{3}{4}x^4ba^2+\frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/8*x^8*b^3 + 1/2*x^6*b^2*a + 3/4*x^4*b*a^2 + 1/2*x^2*a^3
```

giac [A] time = 1.00, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^4}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/8*(b*x^2 + a)^4/b
```

maple [B] time = 0.00, size = 36, normalized size = 2.25

$$\frac{1}{8}b^3x^8+\frac{1}{2}a\,b^2x^6+\frac{3}{4}a^2b\,x^4+\frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^3,x)
[Out] 1/8*b^3*x^8+1/2*a*b^2*x^6+3/4*a^2*b*x^4+1/2*a^3*x^2
```

maxima [A] time = 1.34, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^4}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^4/b
```

mupad [B] time = 0.06, size = 35, normalized size = 2.19

$$\frac{a^3\,x^2}{2}+\frac{3\,a^2\,b\,x^4}{4}+\frac{a\,b^2\,x^6}{2}+\frac{b^3\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^2)/2 + (b^3*x^8)/8 + (3*a^2*b*x^4)/4 + (a*b^2*x^6)/2
```

sympy [B] time = 0.07, size = 37, normalized size = 2.31

$$\frac{a^3x^2}{2}+\frac{3a^2bx^4}{4}+\frac{ab^2x^6}{2}+\frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**2/2 + 3*a**2*b*x**4/4 + a*b**2*x**6/2 + b**3*x**8/8
```

3.34 $\int \frac{(a+bx^2)^3}{x} dx$

Optimal. Leaf size=39

$$a^3\log(x)+\frac{3}{2}a^2bx^2+\frac{3}{4}ab^2x^4+\frac{b^3x^6}{6}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{2}a^2bx^2+a^3\log(x)+\frac{3}{4}ab^2x^4+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3ab^2x+b^3x^2\right) dx, x, x^2\right)\\ &= \frac{3}{2}a^2bx^2+\frac{3}{4}ab^2x^4+\frac{b^3x^6}{6}+a^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$a^3 \log(x) + \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x,x]
[Out] (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^3}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x, x]
```

fricas [A] time = 0.76, size = 33, normalized size = 0.85

$$\frac{1}{6}\,b^3x^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{2}\,a^2bx^2 + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="fricas")
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

giac [A] time = 0.96, size = 36, normalized size = 0.92

$$\frac{1}{6}\,b^3x^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{2}\,a^2bx^2 + \frac{1}{2}\,a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="giac")
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*log(x^2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$\frac{b^3x^6}{6} + \frac{3a\,b^2x^4}{4} + \frac{3a^2b\,x^2}{2} + a^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x,x)
```

```
[Out] 3/2*a^2*b*x^2+3/4*a*b^2*x^4+1/6*b^3*x^6+a^3*ln(x)
```

maxima [A] time = 1.31, size = 36, normalized size = 0.92

$$\frac{1}{6}\,b^3x^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{2}\,a^2bx^2 + \frac{1}{2}\,a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*log(x^2)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.85

$$a^3\,\ln(x) + \frac{b^3\,x^6}{6} + \frac{3\,a^2\,b\,x^2}{2} + \frac{3\,a\,b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x,x)
```

```
[Out] a^3*log(x) + (b^3*x^6)/6 + (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4
```

sympy [A] time = 0.11, size = 37, normalized size = 0.95

$$a^3\log\left(x\right) + \frac{3a^2bx^2}{2} + \frac{3ab^2x^4}{4} + \frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x,x)
```

```
[Out] a**3*log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x**4/4 + b**3*x**6/6
```

3.35 $\int \frac{(a+bx^2)^3}{x^3} \, dx$

Optimal. Leaf size=40

$$-\frac{a^3}{2x^2} + 3a^2 b \log(x) + \frac{3}{2} a b^2 x^2 + \frac{b^3 x^4}{4}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$3a^2 b \log(x) - \frac{a^3}{2x^2} + \frac{3}{2} a b^2 x^2 + \frac{b^3 x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^3} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^3}{x^2} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(3ab^2 + \frac{a^3}{x^2} + \frac{3a^2 b}{x} + b^3 x\right) dx, x, x^2\right) \\
&= -\frac{a^3}{2x^2} + \frac{3}{2} a b^2 x^2 + \frac{b^3 x^4}{4} + 3a^2 b \log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^3,x]
[Out] -1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2\right)^3}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^3,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^3, x]
```

fricas [A] time = 0.93, size = 38, normalized size = 0.95

$$\frac{b^3x^6+6\,ab^2x^4+12\,a^2bx^2\log(x)-2\,a^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="fricas")
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

giac [A] time = 1.08, size = 46, normalized size = 1.15

$$\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+\frac{3}{2}\,a^2b\log\left(x^2\right)-\frac{3\,a^2bx^2+a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="giac")
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/
x^2
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{b^3x^4}{4}+\frac{3a\,b^2x^2}{2}+3a^2b\ln(x)-\frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^3,x)
```

```
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

maxima [A] time = 1.38, size = 36, normalized size = 0.90

$$\frac{1}{4}\,b^3x^4 + \frac{3}{2}\,ab^2x^2 + \frac{3}{2}\,a^2b\log\left(x^2\right) - \frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*a^3/x^2
```

mupad [B] time = 0.04, size = 34, normalized size = 0.85

$$\frac{b^3\,x^4}{4} - \frac{a^3}{2\,x^2} + \frac{3\,a\,b^2\,x^2}{2} + 3\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^3,x)
```

```
[Out] (b^3*x^4)/4 - a^3/(2*x^2) + (3*a*b^2*x^2)/2 + 3*a^2*b*log(x)
```

sympy [A] time = 0.14, size = 37, normalized size = 0.92

$$-\frac{a^3}{2x^2} + 3a^2b\log\left(x\right) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

3.36 $\int \frac{\left(a+bx^2\right)^3}{x^5} \, dx$

Optimal. Leaf size=40

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{2x^2}+3ab^2\log(x)+\frac{b^3x^2}{2}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{3a^2b}{2x^2}-\frac{a^3}{4x^4}+3ab^2\log(x)+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^5,x]
```

```
[Out] -a^3/(4*x^4) - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^3}{x^5} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^3} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right) dx, x, x^2\right)\\
&= -\frac{a^3}{4x^4}-\frac{3a^2b}{2x^2}+\frac{b^3x^2}{2}+3ab^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 40, normalized size = 1.00

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{2x^2}+3ab^2\log(x)+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^5,x]
```

```
[Out] -1/4*a^3/x^4 - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^5, x]
```

fricas [A] time = 1.10, size = 39, normalized size = 0.98

$$\frac{2\,b^3x^6+12\,ab^2x^4\log(x)-6\,a^2bx^2-a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

giac [A] time = 1.14, size = 46, normalized size = 1.15

$$\frac{1}{2}\,b^3x^2+\frac{3}{2}\,ab^2\log\left(x^2\right)-\frac{9\,ab^2x^4+6\,a^2bx^2+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2 + 3/2*a*b^2*log(x^2) - 1/4*(9*a*b^2*x^4 + 6*a^2*b*x^2 + a^3)/x^
4
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{b^3x^2}{2}+3a\,b^2\ln(x)-\frac{3a^2b}{2x^2}-\frac{a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^5,x)
```

```
[Out] -1/4*a^3/x^4-3/2*a^2*b/x^2+1/2*b^3*x^2+3*a*b^2*ln(x)
```

maxima [A] time = 1.36, size = 37, normalized size = 0.92

$$\frac{1}{2}\,b^{3}x^{2}+\frac{3}{2}\,ab^{2}\log\left(x^{2}\right)-\frac{6\,a^{2}bx^{2}+a^{3}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2 + 3/2*a*b^2*log(x^2) - 1/4*(6*a^2*b*x^2 + a^3)/x^4
```

mupad [B] time = 4.90, size = 37, normalized size = 0.92

$$\frac{b^{3}\,x^{2}}{2}-\frac{\frac{a^{3}}{4}+\frac{3\,b\,a^{2}\,x^{2}}{2}}{x^{4}}+3\,a\,b^{2}\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^5,x)
```

```
[Out] (b^3*x^2)/2 - (a^3/4 + (3*a^2*b*x^2)/2)/x^4 + 3*a*b^2*log(x)
```

sympy [A] time = 0.19, size = 37, normalized size = 0.92

$$3ab^{2}\log\left(x\right)+\frac{b^{3}x^{2}}{2}+\frac{-a^{3}-6a^{2}bx^{2}}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**5,x)
```

```
[Out] 3*a*b**2*log(x) + b**3*x**2/2 + (-a**3 - 6*a**2*b*x**2)/(4*x**4)
```

3.37 $\int \frac{(a+bx^2)^3}{x^7} \, dx$

Optimal. Leaf size=39

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{3a^2b}{4x^4}-\frac{a^3}{6x^6}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^7} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^4} \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right) dx, x, x^2\right)\\ &= -\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^7,x]
[Out] -1/6*a^3/x^6 - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^3}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^7,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^7, x]
```

fricas [A] time = 1.02, size = 39, normalized size = 1.00

$$\frac{12\,b^3x^6\log(x)-18\,ab^2x^4-9\,a^2bx^2-2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="fricas")
[Out] 1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

giac [A] time = 1.05, size = 47, normalized size = 1.21

$$\frac{1}{2}\,b^3\log\left(x^2\right)-\frac{11\,b^3x^6+18\,ab^2x^4+9\,a^2bx^2+2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^7,x, algorithm="giac")
[Out] 1/2*b^3*log(x^2) - 1/12*(11*b^3*x^6 + 18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x
^6
```

maple [A] time = 0.01, size = 34, normalized size = 0.87

$$b^3\ln(x)-\frac{3a\,b^2}{2x^2}-\frac{3a^2b}{4x^4}-\frac{a^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^3/x^7,x)

[Out] -1/6*a^3/x^6-3/4*a^2*b/x^4-3/2*a*b^2/x^2+b^3*ln(x)

maxima [A] time = 1.29, size = 39, normalized size = 1.00

$$\frac{1}{2}\,b^3\log\left(x^2\right)-\frac{18\,ab^2x^4+9\,a^2bx^2+2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^3/x^7,x, algorithm="maxima")

[Out] 1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x^6

mupad [B] time = 0.05, size = 36, normalized size = 0.92

$$b^3\,\ln(x)-\frac{\frac{a^3}{6}+\frac{3\,a^2\,b\,x^2}{4}+\frac{3\,a\,b^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^3/x^7,x)

[Out] b^3*log(x) - (a^3/6 + (3*a^2*b*x^2)/4 + (3*a*b^2*x^4)/2)/x^6

sympy [A] time = 0.24, size = 37, normalized size = 0.95

$$b^3\log\left(x\right)+\frac{-2a^3-9a^2bx^2-18ab^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**3/x**7,x)

[Out] b**3*log(x) + (-2*a**3 - 9*a**2*b*x**2 - 18*a*b**2*x**4)/(12*x**6)

3.38 $\int \frac{(a+bx^2)^3}{x^9}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^4}{8ax^8}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^4}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^9,x]
```

```
[Out] -(a + b*x^2)^4/(8*a*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^3}{x^9}\,dx = -\frac{(a+bx^2)^4}{8ax^8}$$

Mathematica [B] time = 0.01, size = 43, normalized size = 2.26

$$-\frac{a^3}{8x^8}-\frac{a^2b}{2x^6}-\frac{3ab^2}{4x^4}-\frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^9,x]
```

```
[Out] -1/8*a^3/x^8 - (a^2*b)/(2*x^6) - (3*a*b^2)/(4*x^4) - b^3/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^9,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^9, x]
```

fricas [B] time = 1.74, size = 35, normalized size = 1.84

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="fricas")
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

giac [B] time = 1.04, size = 35, normalized size = 1.84

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="giac")
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

maple [B] time = 0.00, size = 36, normalized size = 1.89

$$-\frac{b^3}{2x^2}-\frac{3a\,b^2}{4x^4}-\frac{a^2b}{2x^6}-\frac{a^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^9,x)
[Out] -1/2*a^2*b/x^6-1/2*b^3/x^2-3/4*a*b^2/x^4-1/8*a^3/x^8
```

maxima [B] time = 1.32, size = 35, normalized size = 1.84

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^9,x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

mupad [B] time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{a^3}{8}+\frac{a^2\,b\,x^2}{2}+\frac{3\,a\,b^2\,x^4}{4}+\frac{b^3\,x^6}{2}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^9,x)
```

```
[Out] -(a^3/8 + (b^3*x^6)/2 + (a^2*b*x^2)/2 + (3*a*b^2*x^4)/4)/x^8
```

sympy [B] time = 0.27, size = 37, normalized size = 1.95

$$\frac{-a^3-4a^2bx^2-6ab^2x^4-4b^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**9,x)
```

```
[Out] (-a**3 - 4*a**2*b*x**2 - 6*a*b**2*x**4 - 4*b**3*x**6)/(8*x**8)
```

3.39 $\int \frac{(a+bx^2)^3}{x^{11}} \, dx$

Optimal. Leaf size=40

$$\frac{b(a+bx^2)^4}{40a^2x^8} - \frac{(a+bx^2)^4}{10ax^{10}}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b(a+bx^2)^4}{40a^2x^8} - \frac{(a+bx^2)^4}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^11,x]
```

```
[Out] -(a + b*x^2)^4/(10*a*x^10) + (b*(a + b*x^2)^4)/(40*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^3}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^6}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^4}{10ax^{10}} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^5}\,dx,x,x^2\right)}{10a}$$
$$= -\frac{\left(a+bx^2\right)^4}{10ax^{10}} + \frac{b\left(a+bx^2\right)^4}{40a^2x^8}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.08

$$-\frac{a^3}{10x^{10}} - \frac{3a^2b}{8x^8} - \frac{ab^2}{2x^6} - \frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^11,x]
[Out] -1/10*a^3/x^10 - (3*a^2*b)/(8*x^8) - (a*b^2)/(2*x^6) - b^3/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^11,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^11, x]
```

fricas [A] time = 0.99, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^6 + 20\,ab^2x^4 + 15\,a^2bx^2 + 4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="fricas")
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

giac [A] time = 1.02, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^6 + 20\,ab^2x^4 + 15\,a^2bx^2 + 4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="giac")
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.90

$$-\frac{b^3}{4x^4}-\frac{a\,b^2}{2x^6}-\frac{3a^2b}{8x^8}-\frac{a^3}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^11,x)
[Out] -1/2*a*b^2/x^6-1/4*b^3/x^4-1/10*a^3/x^10-3/8*a^2*b/x^8
```

maxima [A] time = 1.37, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^11,x, algorithm="maxima")
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

mupad [B] time = 0.06, size = 37, normalized size = 0.92

$$-\frac{\frac{a^3}{10}+\frac{3\,a^2\,b\,x^2}{8}+\frac{a\,b^2\,x^4}{2}+\frac{b^3\,x^6}{4}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^11,x)
[Out] -(a^3/10 + (b^3*x^6)/4 + (3*a^2*b*x^2)/8 + (a*b^2*x^4)/2)/x^10
```

sympy [A] time = 0.29, size = 39, normalized size = 0.98

$$\frac{-4a^3-15a^2bx^2-20ab^2x^4-10b^3x^6}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**11,x)
[Out] (-4*a**3 - 15*a**2*b*x**2 - 20*a*b**2*x**4 - 10*b**3*x**6)/(40*x**10)
```

3.40 $\int \frac{\left(a+bx^2\right)^3}{x^{13}} \, dx$

Optimal. Leaf size=43

$$-\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{3a^2b}{10x^{10}}-\frac{a^3}{12x^{12}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^13,x]
```

```
[Out] -a^3/(12*x^12) - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^3}{x^{13}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^7} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right) dx, x, x^2\right)\\
&= -\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{12x^{12}}-\frac{3a^2b}{10x^{10}}-\frac{3ab^2}{8x^8}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^13,x]
[Out] -1/12*a^3/x^12 - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^13,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^13, x]
```

fricas [A] time = 0.78, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="fricas")
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

giac [A] time = 0.83, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="giac")
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{b^3}{6x^6}-\frac{3a\,b^2}{8x^8}-\frac{3a^2b}{10x^{10}}-\frac{a^3}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^13,x)
```

```
[Out] -1/12*a^3/x^12-3/10*a^2*b/x^10-3/8*a*b^2/x^8-1/6*b^3/x^6
```

maxima [A] time = 1.29, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^13,x, algorithm="maxima")
```

```
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

mupad [B] time = 0.05, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{12}+\frac{3\,a^2\,b\,x^2}{10}+\frac{3\,a\,b^2\,x^4}{8}+\frac{b^3\,x^6}{6}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^13,x)
```

```
[Out] -(a^3/12 + (b^3*x^6)/6 + (3*a^2*b*x^2)/10 + (3*a*b^2*x^4)/8)/x^12
```

sympy [A] time = 0.32, size = 39, normalized size = 0.91

$$\frac{-10a^3-36a^2bx^2-45ab^2x^4-20b^3x^6}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**13,x)
```

```
[Out] (-10*a**3 - 36*a**2*b*x**2 - 45*a*b**2*x**4 - 20*b**3*x**6)/(120*x**12)
```

3.41 $\int \frac{(a+bx^2)^3}{x^{15}} \, dx$

Optimal. Leaf size=43

$$-\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2b}{4x^{12}}-\frac{a^3}{14x^{14}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^15,x]
```

```
[Out] -a^3/(14*x^14) - (a^2*b)/(4*x^12) - (3*a*b^2)/(10*x^10) - b^3/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^3}{x^{15}} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^3}{x^8} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right) dx, x, x^2\right) \\
&= -\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$-\frac{a^3}{14x^{14}}-\frac{a^2b}{4x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^15,x]
[Out] -1/14*a^3/x^14 - (a^2*b)/(4*x^12) - (3*a*b^2)/(10*x^10) - b^3/(8*x^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^3}{x^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^15,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^15, x]
```

fricas [A] time = 0.71, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="fricas")
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

giac [A] time = 1.04, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="giac")
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

maple [A] time = 0.01, size = 36, normalized size = 0.84

$$-\frac{b^3}{8x^8}-\frac{3a\,b^2}{10x^{10}}-\frac{a^2b}{4x^{12}}-\frac{a^3}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^15,x)
```

```
[Out] -1/14*a^3/x^14-1/4*a^2*b/x^12-3/10*a*b^2/x^10-1/8*b^3/x^8
```

maxima [A] time = 1.32, size = 37, normalized size = 0.86

$$-\frac{35\,b^{3}x^{6}+84\,ab^{2}x^{4}+70\,a^{2}bx^{2}+20\,a^{3}}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^15,x, algorithm="maxima")
```

```
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^{3}}{14}+\frac{a^{2}\,b\,x^{2}}{4}+\frac{3\,a\,b^{2}\,x^{4}}{10}+\frac{b^{3}\,x^{6}}{8}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^15,x)
```

```
[Out] -(a^3/14 + (b^3*x^6)/8 + (a^2*b*x^2)/4 + (3*a*b^2*x^4)/10)/x^14
```

sympy [A] time = 0.35, size = 39, normalized size = 0.91

$$\frac{-20a^{3}-70a^{2}bx^{2}-84ab^{2}x^{4}-35b^{3}x^{6}}{280x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**15,x)
```

```
[Out] (-20*a**3 - 70*a**2*b*x**2 - 84*a*b**2*x**4 - 35*b**3*x**6)/(280*x**14)
```

3.42 $\int x^6 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{1}{3}a^2bx^9+\frac{a^3x^7}{7}+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a+bx^2\right)^3 \, dx &= \int \left(a^3x^6+3a^2bx^8+3ab^2x^{10}+b^3x^{12}\right) dx\\ &= \frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left(a+bx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[x^6*(a + b*x^2)^3, x]
```

fricas [A] time = 0.84, size = 35, normalized size = 0.81

$$\frac{1}{13}x^{13}b^3+\frac{3}{11}x^{11}b^2a+\frac{1}{3}x^9ba^2+\frac{1}{7}x^7a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/13*x^13*b^3 + 3/11*x^11*b^2*a + 1/3*x^9*b*a^2 + 1/7*x^7*a^3
```

giac [A] time = 0.93, size = 35, normalized size = 0.81

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{13}b^3x^{13}+\frac{3}{11}a\,b^2x^{11}+\frac{1}{3}a^2b\,x^9+\frac{1}{7}a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^3,x)
[Out] 1/7*a^3*x^7+1/3*a^2*b*x^9+3/11*a*b^2*x^11+1/13*b^3*x^13
```

maxima [A] time = 1.35, size = 35, normalized size = 0.81

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^7}{7}+\frac{a^2\,b\,x^9}{3}+\frac{3\,a\,b^2\,x^{11}}{11}+\frac{b^3\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^7)/7 + (b^3*x^13)/13 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11
```

sympy [A] time = 0.07, size = 37, normalized size = 0.86

$$\frac{a^{3}x^{7}}{7}+\frac{a^{2}bx^{9}}{3}+\frac{3ab^{2}x^{11}}{11}+\frac{b^{3}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**7/7 + a**2*b*x**9/3 + 3*a*b**2*x**11/11 + b**3*x**13/13
```

3.43 $\int x^4 \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3}{7}a^2bx^7+\frac{a^3x^5}{5}+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^2\right)^3 \, dx &= \int \left(a^3x^4+3a^2bx^6+3ab^2x^8+b^3x^{10}\right) \, dx\\ &= \frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + bx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[x^4*(a + b*x^2)^3, x]
```

fricas [A] time = 0.96, size = 35, normalized size = 0.81

$$\frac{1}{11}x^{11}b^3 + \frac{1}{3}x^9b^2a + \frac{3}{7}x^7ba^2 + \frac{1}{5}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/11*x^11*b^3 + 1/3*x^9*b^2*a + 3/7*x^7*b*a^2 + 1/5*x^5*a^3
```

giac [A] time = 1.03, size = 35, normalized size = 0.81

$$\frac{1}{11} b^3x^{11} + \frac{1}{3} ab^2x^9 + \frac{3}{7} a^2bx^7 + \frac{1}{5} a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{11}b^3x^{11} + \frac{1}{3}a\,b^2x^9 + \frac{3}{7}a^2b\,x^7 + \frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^3,x)
[Out] 1/5*a^3*x^5+3/7*a^2*b*x^7+1/3*a*b^2*x^9+1/11*b^3*x^11
```

maxima [A] time = 1.38, size = 35, normalized size = 0.81

$$\frac{1}{11} b^3x^{11} + \frac{1}{3} ab^2x^9 + \frac{3}{7} a^2bx^7 + \frac{1}{5} a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^5}{5}+\frac{3\,a^2\,b\,x^7}{7}+\frac{a\,b^2\,x^9}{3}+\frac{b^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^5)/5 + (b^3*x^11)/11 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3
```

sympy [A] time = 0.07, size = 37, normalized size = 0.86

$$\frac{a^{3}x^{5}}{5}+\frac{3a^{2}bx^{7}}{7}+\frac{ab^{2}x^{9}}{3}+\frac{b^{3}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**5/5 + 3*a**2*b*x**7/7 + a*b**2*x**9/3 + b**3*x**11/11
```

3.44 $\int x^2 \left(a+bx^2\right)^3 dx$

Optimal. Leaf size=43

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^3}{3}+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right)^3 dx &= \int \left(a^3x^2+3a^2bx^4+3ab^2x^6+b^3x^8\right) dx\\ &= \frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^3, x]
```

fricas [A] time = 1.01, size = 35, normalized size = 0.81

$$\frac{1}{9}x^9b^3 + \frac{3}{7}x^7b^2a + \frac{3}{5}x^5ba^2 + \frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/9*x^9*b^3 + 3/7*x^7*b^2*a + 3/5*x^5*b*a^2 + 1/3*x^3*a^3
```

giac [A] time = 1.03, size = 35, normalized size = 0.81

$$\frac{1}{9}\,b^3x^9 + \frac{3}{7}\,ab^2x^7 + \frac{3}{5}\,a^2bx^5 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{9}b^3x^9 + \frac{3}{7}a\,b^2x^7 + \frac{3}{5}a^2b\,x^5 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^3,x)
[Out] 1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*b^2*x^7+1/9*b^3*x^9
```

maxima [A] time = 1.32, size = 35, normalized size = 0.81

$$\frac{1}{9}\,b^3x^9 + \frac{3}{7}\,ab^2x^7 + \frac{3}{5}\,a^2bx^5 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^3}{3}+\frac{3\,a^2\,b\,x^5}{5}+\frac{3\,a\,b^2\,x^7}{7}+\frac{b^3\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^9)/9 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7
```

sympy [A] time = 0.07, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3}+\frac{3a^2bx^5}{5}+\frac{3ab^2x^7}{7}+\frac{b^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*a*b**2*x**7/7 + b**3*x**9/9
```

3.45 $\int \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=35

$$a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2bx^3 + a^3x + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^3 \, dx &= \int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right) dx\\ &= a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3, x]
```

fricas [A] time = 1.02, size = 31, normalized size = 0.89

$$\frac{1}{7}x^7b^3 + \frac{3}{5}x^5b^2a + x^3ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/7*x^7*b^3 + 3/5*x^5*b^2*a + x^3*b*a^2 + x*a^3
```

giac [A] time = 0.72, size = 31, normalized size = 0.89

$$\frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="giac")
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{1}{7}b^3x^7 + \frac{3}{5}a\,b^2x^5 + a^2b\,x^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3,x)
[Out] a^3*x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7
```

maxima [A] time = 1.36, size = 31, normalized size = 0.89

$$\frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$a^3\,x+a^2\,b\,x^3+\frac{3\,a\,b^2\,x^5}{5}+\frac{b^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3,x)
```

```
[Out] a^3*x + (b^3*x^7)/7 + a^2*b*x^3 + (3*a*b^2*x^5)/5
```

sympy [A] time = 0.06, size = 32, normalized size = 0.91

$$a^3x+a^2bx^3+\frac{3ab^2x^5}{5}+\frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*b*x**3 + 3*a*b**2*x**5/5 + b**3*x**7/7
```

3.46 $\int \frac{(a+bx^2)^3}{x^2}\,dx$

Optimal. Leaf size=34

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$3a^2bx-\frac{a^3}{x}+ab^2x^3+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^2}\,dx &= \int \left(3a^2b+\frac{a^3}{x^2}+3ab^2x^2+b^3x^4\right)dx\\ &= -\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^2,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^2, x]
```

fricas [A] time = 0.73, size = 36, normalized size = 1.06

$$\frac{b^3x^6+5\,ab^2x^4+15\,a^2bx^2-5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="fricas")
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

giac [A] time = 1.04, size = 32, normalized size = 0.94

$$\frac{1}{5}\,b^3x^5+ab^2x^3+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="giac")
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

maple [A] time = 0.00, size = 33, normalized size = 0.97

$$\frac{b^3x^5}{5}+a\,b^2x^3+3a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^2,x)
[Out] -a^3/x+3*a^2*b*x+a*b^2*x^3+1/5*b^3*x^5
```

maxima [A] time = 1.38, size = 32, normalized size = 0.94

$$\frac{1}{5}\,b^3x^5+ab^2x^3+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

mupad [B] time = 0.04, size = 32, normalized size = 0.94

$$\frac{b^3\,x^5}{5}-\frac{a^3}{x}+a\,b^2\,x^3+3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^2,x)
```

```
[Out] (b^3*x^5)/5 - a^3/x + a*b^2*x^3 + 3*a^2*b*x
```

sympy [A] time = 0.11, size = 29, normalized size = 0.85

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*x + a*b**2*x**3 + b**3*x**5/5
```

3.47 $\int \frac{(a+bx^2)^3}{x^4} \, dx$

Optimal. Leaf size=37

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3ab^2x+\frac{b^3x^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{3a^2b}{x}-\frac{a^3}{3x^3}+3ab^2x+\frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^4} \, dx &= \int \left(3ab^2+\frac{a^3}{x^4}+\frac{3a^2b}{x^2}+b^3x^2\right) dx\\ &= -\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3ab^2x+\frac{b^3x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3ab^2x+\frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^4, x]
```

fricas [A] time = 1.24, size = 36, normalized size = 0.97

$$\frac{b^3x^6+9\,ab^2x^4-9\,a^2bx^2-a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

giac [A] time = 1.09, size = 34, normalized size = 0.92

$$\frac{1}{3}\,b^3x^3+3\,ab^2x-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

maple [A] time = 0.01, size = 34, normalized size = 0.92

$$\frac{b^3x^3}{3}+3a\,b^2x-\frac{3a^2b}{x}-\frac{a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^4,x)
```

```
[Out] -1/3*a^3/x^3-3*a^2*b/x+3*a*b^2*x+1/3*b^3*x^3
```

maxima [A] time = 1.34, size = 34, normalized size = 0.92

$$\frac{1}{3}\,b^3x^3+3\,ab^2x-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

mupad [B] time = 4.80, size = 36, normalized size = 0.97

$$\frac{b^3\,x^3}{3}-\frac{\frac{a^3}{3}+3\,b\,a^2\,x^2}{x^3}+3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^4,x)
```

```
[Out] (b^3*x^3)/3 - (a^3/3 + 3*a^2*b*x^2)/x^3 + 3*a*b^2*x
```

sympy [A] time = 0.15, size = 36, normalized size = 0.97

$$3ab^2x+\frac{b^3x^3}{3}+\frac{-a^3-9a^2bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**4,x)
```

```
[Out] 3*a*b**2*x + b**3*x**3/3 + (-a**3 - 9*a**2*b*x**2)/(3*x**3)
```

3.48 $\int \frac{\left(a+bx^2\right)^3}{x^6}\,dx$

Optimal. Leaf size=34

$$-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2b}{x^3}-\frac{a^3}{5x^5}-\frac{3ab^2}{x}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^6,x]
```

```
[Out] -a^3/(5*x^5) - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3}{x^6}\,dx &= \int \left(b^3+\frac{a^3}{x^6}+\frac{3a^2b}{x^4}+\frac{3ab^2}{x^2}\right)dx\\ &= -\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^6,x]
```

```
[Out] -1/5*a^3/x^5 - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^3}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^6,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^6, x]
```

fricas [A] time = 1.33, size = 37, normalized size = 1.09

$$\frac{5\,b^3x^6-15\,ab^2x^4-5\,a^2bx^2-a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="fricas")
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

giac [A] time = 1.08, size = 33, normalized size = 0.97

$$b^3x-\frac{15\,ab^2x^4+5\,a^2bx^2+a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="giac")
[Out] b^3*x - 1/5*(15*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/x^5
```

maple [A] time = 0.00, size = 33, normalized size = 0.97

$$b^3x-\frac{3a\,b^2}{x}-\frac{a^2b}{x^3}-\frac{a^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^6,x)
[Out] -1/5*a^3/x^5-a^2*b/x^3-3*a*b^2/x+b^3*x
```

maxima [A] time = 1.34, size = 33, normalized size = 0.97

$$b^3x-\frac{15\,ab^2x^4+5\,a^2bx^2+a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^6,x, algorithm="maxima")
```

```
[Out] b^3*x - 1/5*(15*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/x^5
```

mupad [B] time = 0.03, size = 34, normalized size = 1.00

$$b^3\,x-\frac{\frac{a^3}{5}+a^2\,b\,x^2+3\,a\,b^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^6,x)
```

```
[Out] b^3*x - (a^3/5 + a^2*b*x^2 + 3*a*b^2*x^4)/x^5
```

sympy [A] time = 0.20, size = 34, normalized size = 1.00

$$b^{3}x+\frac{-a^{3}-5a^{2}bx^{2}-15ab^{2}x^{4}}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**6,x)
```

```
[Out] b**3*x + (-a**3 - 5*a**2*b*x**2 - 15*a*b**2*x**4)/(5*x**5)
```

3.49 $\int \frac{(a+bx^2)^3}{x^8}\,dx$

Optimal. Leaf size=39

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{7x^7}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^8,x]
[Out] -a^3/(7*x^7) - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^8}\,dx &= \int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^4}+\frac{b^3}{x^2}\right)dx\\ &= -\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^8,x]
[Out] -1/7*a^3/x^7 - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^3}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^8, x]
```

fricas [A] time = 1.28, size = 37, normalized size = 0.95

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

giac [A] time = 1.09, size = 37, normalized size = 0.95

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

maple [A] time = 0.00, size = 36, normalized size = 0.92

$$-\frac{b^3}{x}-\frac{a\,b^2}{x^3}-\frac{3a^2b}{5x^5}-\frac{a^3}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^8,x)
```

```
[Out] -1/7*a^3/x^7-3/5*a^2*b/x^5-a*b^2/x^3-b^3/x
```

maxima [A] time = 1.25, size = 37, normalized size = 0.95

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^8,x, algorithm="maxima")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

mupad [B] time = 0.03, size = 35, normalized size = 0.90

$$-\frac{\frac{a^3}{7}+\frac{3\,a^2\,b\,x^2}{5}+a\,b^2\,x^4+b^3\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^8,x)
```

```
[Out] -(a^3/7 + b^3*x^6 + (3*a^2*b*x^2)/5 + a*b^2*x^4)/x^7
```

sympy [A] time = 0.26, size = 39, normalized size = 1.00

$$\frac{-5a^3-21a^2bx^2-35ab^2x^4-35b^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**8,x)
```

```
[Out] (-5*a**3 - 21*a**2*b*x**2 - 35*a*b**2*x**4 - 35*b**3*x**6)/(35*x**7)
```

3.50 $\int \frac{\left(a+bx^2\right)^3}{x^{10}}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{3a^2b}{7x^7}-\frac{a^3}{9x^9}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^10,x]
```

```
[Out] -a^3/(9*x^9) - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3}{x^{10}}\,dx &= \int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^8}+\frac{3ab^2}{x^6}+\frac{b^3}{x^4}\right)dx\\ &= -\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^10,x]
```

```
[Out] -1/9*a^3/x^9 - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^3}{x^{10}}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^10,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^10, x]
```

fricas [A] time = 1.20, size = 37, normalized size = 0.86

$$-\frac{105\, b^3 x^6 + 189\, ab^2 x^4 + 135\, a^2 bx^2 + 35\, a^3}{315\, x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="fricas")
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

giac [A] time = 1.20, size = 37, normalized size = 0.86

$$-\frac{105\, b^3 x^6 + 189\, ab^2 x^4 + 135\, a^2 bx^2 + 35\, a^3}{315\, x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="giac")
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

maple [A] time = 0.01, size = 36, normalized size = 0.84

$$-\frac{b^3}{3x^3} - \frac{3a\, b^2}{5x^5} - \frac{3a^2 b}{7x^7} - \frac{a^3}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^10,x)
[Out] -1/9*a^3/x^9-3/7*a^2*b/x^7-3/5*a*b^2/x^5-1/3*b^3/x^3
```

maxima [A] time = 1.49, size = 37, normalized size = 0.86

$$-\frac{105\, b^3 x^6 + 189\, ab^2 x^4 + 135\, a^2 bx^2 + 35\, a^3}{315\, x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{9}+\frac{3\,a^2\,b\,x^2}{7}+\frac{3\,a\,b^2\,x^4}{5}+\frac{b^3\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^10,x)
```

```
[Out] -(a^3/9 + (b^3*x^6)/3 + (3*a^2*b*x^2)/7 + (3*a*b^2*x^4)/5)/x^9
```

sympy [A] time = 0.27, size = 39, normalized size = 0.91

$$\frac{-35a^3-135a^2bx^2-189ab^2x^4-105b^3x^6}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**10,x)
```

```
[Out] (-35*a**3 - 135*a**2*b*x**2 - 189*a*b**2*x**4 - 105*b**3*x**6)/(315*x**9)
```

3.51 $\int \frac{(a+bx^2)^3}{x^{12}}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{a^2b}{3x^9}-\frac{a^3}{11x^{11}}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^12,x]
```

```
[Out] -a^3/(11*x^11) - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{12}}\,dx &= \int \left(\frac{a^3}{x^{12}}+\frac{3a^2b}{x^{10}}+\frac{3ab^2}{x^8}+\frac{b^3}{x^6}\right)dx\\ &= -\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$-\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^12,x]
```

```
[Out] -1/11*a^3/x^11 - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b x^2\right)^3}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^12,x]
[Out] IntegrateAlgebraic[(a + b*x^2)^3/x^12, x]
```

fricas [A] time = 1.00, size = 37, normalized size = 0.86

$$-\frac{231\,b^3x^6 + 495\,ab^2x^4 + 385\,a^2bx^2 + 105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="fricas")
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

giac [A] time = 1.03, size = 37, normalized size = 0.86

$$-\frac{231\,b^3x^6 + 495\,ab^2x^4 + 385\,a^2bx^2 + 105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="giac")
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{b^3}{5x^5} - \frac{3a\,b^2}{7x^7} - \frac{a^2b}{3x^9} - \frac{a^3}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^12,x)
[Out] -1/11*a^3/x^11-1/3*a^2*b/x^9-3/7*a*b^2/x^7-1/5*b^3/x^5
```

maxima [A] time = 1.42, size = 37, normalized size = 0.86

$$-\frac{231\,b^3x^6 + 495\,ab^2x^4 + 385\,a^2bx^2 + 105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^12,x, algorithm="maxima")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{11}+\frac{a^2\,b\,x^2}{3}+\frac{3\,a\,b^2\,x^4}{7}+\frac{b^3\,x^6}{5}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^12,x)
```

```
[Out] -(a^3/11 + (b^3*x^6)/5 + (a^2*b*x^2)/3 + (3*a*b^2*x^4)/7)/x^11
```

sympy [A] time = 0.30, size = 39, normalized size = 0.91

$$\frac{-105a^3-385a^2bx^2-495ab^2x^4-231b^3x^6}{1155x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**12,x)
```

```
[Out] (-105*a**3 - 385*a**2*b*x**2 - 495*a*b**2*x**4 - 231*b**3*x**6)/(1155*x**11
)
```

3.52 $\int x^{13} \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{2}a^2b^3x^{20}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{16}a^4bx^{16}+\frac{a^5x^{14}}{14}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^14)/14 + (5*a^4*b*x^16)/16 + (5*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2 +
 (5*a*b^4*x^22)/22 + (b^5*x^24)/24
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{13} \left(a+bx^2\right)^5 \, dx &= \frac{1}{2}\text{Subst}\left(\int x^6(a+bx)^5 \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^5x^6+5a^4bx^7+10a^3b^2x^8+10a^2b^3x^9+5ab^4x^{10}+b^5x^{11}\right) dx, x, x^2\right)\\
&= \frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^14)/14 + (5*a^4*b*x^16)/16 + (5*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2 +
 (5*a*b^4*x^22)/22 + (b^5*x^24)/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13}\left(a+bx^2\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^13*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^13*(a + b*x^2)^5, x]
```

fricas [A] time = 0.43, size = 57, normalized size = 0.83

$$\frac{1}{24}x^{24}b^5+\frac{5}{22}x^{22}b^4a+\frac{1}{2}x^{20}b^3a^2+\frac{5}{9}x^{18}b^2a^3+\frac{5}{16}x^{16}ba^4+\frac{1}{14}x^{14}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*b^5 + 5/22*x^22*b^4*a + 1/2*x^20*b^3*a^2 + 5/9*x^18*b^2*a^3 + 5/1
6*x^16*b*a^4 + 1/14*x^14*a^5
```

giac [A] time = 0.87, size = 57, normalized size = 0.83

$$\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{24}b^5x^{24}+\frac{5}{22}a\,b^4x^{22}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{16}a^4b\,x^{16}+\frac{1}{14}a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^5,x)
```

```
[Out] 1/14*a^5*x^14+5/16*a^4*b*x^16+5/9*a^3*b^2*x^18+1/2*a^2*b^3*x^20+5/22*a*b^4*
x^22+1/24*b^5*x^24
```

maxima [A] time = 1.43, size = 57, normalized size = 0.83

$$\frac{1}{24}\,b^5x^{24} + \frac{5}{22}\,ab^4x^{22} + \frac{1}{2}\,a^2b^3x^{20} + \frac{5}{9}\,a^3b^2x^{18} + \frac{5}{16}\,a^4bx^{16} + \frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

mupad [B] time = 0.03, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{14}}{14} + \frac{5\,a^4\,b\,x^{16}}{16} + \frac{5\,a^3\,b^2\,x^{18}}{9} + \frac{a^2\,b^3\,x^{20}}{2} + \frac{5\,a\,b^4\,x^{22}}{22} + \frac{b^5\,x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^14)/14 + (b^5*x^24)/24 + (5*a^4*b*x^16)/16 + (5*a*b^4*x^22)/22 + (5*
a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2
```

sympy [A] time = 0.08, size = 65, normalized size = 0.94

$$\frac{a^5x^{14}}{14} + \frac{5a^4bx^{16}}{16} + \frac{5a^3b^2x^{18}}{9} + \frac{a^2b^3x^{20}}{2} + \frac{5ab^4x^{22}}{22} + \frac{b^5x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**14/14 + 5*a**4*b*x**16/16 + 5*a**3*b**2*x**18/9 + a**2*b**3*x**20/2
 + 5*a*b**4*x**22/22 + b**5*x**24/24
```

3.53 $\int x^{11}\left(a+bx^2\right)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{9}a^2b^3x^{18}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{14}a^4bx^{14}+\frac{a^5x^{12}}{12}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^12)/12 + (5*a^4*b*x^14)/14 + (5*a^3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
 + (a*b^4*x^20)/4 + (b^5*x^22)/22
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^2\right)^5\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x^5(a+bx)^5\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)dx,x,x^2\right)\\&=\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^12)/12 + (5*a^4*b*x^14)/14 + (5*a^3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
 + (a*b^4*x^20)/4 + (b^5*x^22)/22
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\left(a+bx^2\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^11*(a + b*x^2)^5, x]
```

fricas [A] time = 1.13, size = 57, normalized size = 0.83

$$\frac{1}{22}x^{22}b^5+\frac{1}{4}x^{20}b^4a+\frac{5}{9}x^{18}b^3a^2+\frac{5}{8}x^{16}b^2a^3+\frac{5}{14}x^{14}ba^4+\frac{1}{12}x^{12}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^5 + 1/4*x^20*b^4*a + 5/9*x^18*b^3*a^2 + 5/8*x^16*b^2*a^3 + 5/14
*x^14*b*a^4 + 1/12*x^12*a^5
```

giac [A] time = 1.01, size = 57, normalized size = 0.83

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{22}b^5x^{22}+\frac{1}{4}a\,b^4x^{20}+\frac{5}{9}a^2b^3x^{18}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{14}a^4b\,x^{14}+\frac{1}{12}a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^5,x)
```

```
[Out] 1/12*a^5*x^12+5/14*a^4*b*x^14+5/8*a^3*b^2*x^16+5/9*a^2*b^3*x^18+1/4*a*b^4*x
^20+1/22*b^5*x^22
```

maxima [A] time = 1.36, size = 57, normalized size = 0.83

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{12}}{12}+\frac{5\,a^4\,b\,x^{14}}{14}+\frac{5\,a^3\,b^2\,x^{16}}{8}+\frac{5\,a^2\,b^3\,x^{18}}{9}+\frac{a\,b^4\,x^{20}}{4}+\frac{b^5\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^12)/12 + (b^5*x^22)/22 + (5*a^4*b*x^14)/14 + (a*b^4*x^20)/4 + (5*a^3
*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
```

sympy [A] time = 0.08, size = 65, normalized size = 0.94

$$\frac{a^5x^{12}}{12}+\frac{5a^4bx^{14}}{14}+\frac{5a^3b^2x^{16}}{8}+\frac{5a^2b^3x^{18}}{9}+\frac{ab^4x^{20}}{4}+\frac{b^5x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**12/12 + 5*a**4*b*x**14/14 + 5*a**3*b**2*x**16/8 + 5*a**2*b**3*x**18
/9 + a*b**4*x**20/4 + b**5*x**22/22
```

3.54 $\int x^9 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{8}a^2b^3x^{16}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{12}a^4bx^{12}+\frac{a^5x^{10}}{10}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^10)/10 + (5*a^4*b*x^12)/12 + (5*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
 + (5*a*b^4*x^18)/18 + (b^5*x^20)/20
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^5\,dx&=\frac{1}{2}\text{Subst}\left(\int x^4(a+bx)^5\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)dx,x,x^2\right)\\&=\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^10)/10 + (5*a^4*b*x^12)/12 + (5*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
 + (5*a*b^4*x^18)/18 + (b^5*x^20)/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9\left(a+bx^2\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^9*(a + b*x^2)^5, x]
```

fricas [A] time = 0.92, size = 57, normalized size = 0.83

$$\frac{1}{20}x^{20}b^5+\frac{5}{18}x^{18}b^4a+\frac{5}{8}x^{16}b^3a^2+\frac{5}{7}x^{14}b^2a^3+\frac{5}{12}x^{12}ba^4+\frac{1}{10}x^{10}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^5 + 5/18*x^18*b^4*a + 5/8*x^16*b^3*a^2 + 5/7*x^14*b^2*a^3 + 5/1
2*x^12*b*a^4 + 1/10*x^10*a^5
```

giac [A] time = 1.16, size = 57, normalized size = 0.83

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{20}b^5x^{20}+\frac{5}{18}a\,b^4x^{18}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{12}a^4b\,x^{12}+\frac{1}{10}a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^5,x)
```

```
[Out] 1/10*a^5*x^10+5/12*a^4*b*x^12+5/7*a^3*b^2*x^14+5/8*a^2*b^3*x^16+5/18*a*b^4*
x^18+1/20*b^5*x^20
```

maxima [A] time = 1.37, size = 57, normalized size = 0.83

$$\frac{1}{20}\,b^5x^{20} + \frac{5}{18}\,ab^4x^{18} + \frac{5}{8}\,a^2b^3x^{16} + \frac{5}{7}\,a^3b^2x^{14} + \frac{5}{12}\,a^4bx^{12} + \frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{10}}{10} + \frac{5\,a^4\,b\,x^{12}}{12} + \frac{5\,a^3\,b^2\,x^{14}}{7} + \frac{5\,a^2\,b^3\,x^{16}}{8} + \frac{5\,a\,b^4\,x^{18}}{18} + \frac{b^5\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^10)/10 + (b^5*x^20)/20 + (5*a^4*b*x^12)/12 + (5*a*b^4*x^18)/18 + (5*
a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
```

sympy [A] time = 0.08, size = 66, normalized size = 0.96

$$\frac{a^5x^{10}}{10} + \frac{5a^4bx^{12}}{12} + \frac{5a^3b^2x^{14}}{7} + \frac{5a^2b^3x^{16}}{8} + \frac{5ab^4x^{18}}{18} + \frac{b^5x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**10/10 + 5*a**4*b*x**12/12 + 5*a**3*b**2*x**14/7 + 5*a**2*b**3*x**16
/8 + 5*a*b**4*x**18/18 + b**5*x**20/20
```

3.55 $\int x^7 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=72

$$-\frac{a^3 \left(a+bx^2\right)^6}{12b^4}+\frac{3a^2 \left(a+bx^2\right)^7}{14b^4}+\frac{\left(a+bx^2\right)^9}{18b^4}-\frac{3a \left(a+bx^2\right)^8}{16b^4}$$

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2 \left(a+bx^2\right)^7}{14b^4}-\frac{a^3 \left(a+bx^2\right)^6}{12b^4}+\frac{\left(a+bx^2\right)^9}{18b^4}-\frac{3a \left(a+bx^2\right)^8}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^5,x]
```

```
[Out] -(a^3*(a + b*x^2)^6)/(12*b^4) + (3*a^2*(a + b*x^2)^7)/(14*b^4) - (3*a*(a +
b*x^2)^8)/(16*b^4) + (a + b*x^2)^9/(18*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^5 \, dx &= \frac{1}{2} \text{Subst}\left(\int x^3 (a+bx)^5 \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(-\frac{a^3 (a+bx)^5}{b^3}+\frac{3a^2 (a+bx)^6}{b^3}-\frac{3a (a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right) dx, x, x^2\right)\\
&= -\frac{a^3 \left(a+bx^2\right)^6}{12b^4}+\frac{3a^2 \left(a+bx^2\right)^7}{14b^4}-\frac{3a \left(a+bx^2\right)^8}{16b^4}+\frac{\left(a+bx^2\right)^9}{18b^4}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 0.96

$$\frac{a^5x^8}{8} + \frac{1}{2}a^4bx^{10} + \frac{5}{6}a^3b^2x^{12} + \frac{5}{7}a^2b^3x^{14} + \frac{5}{16}ab^4x^{16} + \frac{b^5x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^8)/8 + (a^4*b*x^10)/2 + (5*a^3*b^2*x^12)/6 + (5*a^2*b^3*x^14)/7 + (5
*a*b^4*x^16)/16 + (b^5*x^18)/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(a + bx^2\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^7*(a + b*x^2)^5, x]
```

fricas [A] time = 0.94, size = 57, normalized size = 0.79

$$\frac{1}{18}x^{18}b^5 + \frac{5}{16}x^{16}b^4a + \frac{5}{7}x^{14}b^3a^2 + \frac{5}{6}x^{12}b^2a^3 + \frac{1}{2}x^{10}ba^4 + \frac{1}{8}x^8a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^5 + 5/16*x^16*b^4*a + 5/7*x^14*b^3*a^2 + 5/6*x^12*b^2*a^3 + 1/2
*x^10*b*a^4 + 1/8*x^8*a^5
```

giac [A] time = 1.02, size = 57, normalized size = 0.79

$$\frac{1}{18}\,b^5x^{18} + \frac{5}{16}\,ab^4x^{16} + \frac{5}{7}\,a^2b^3x^{14} + \frac{5}{6}\,a^3b^2x^{12} + \frac{1}{2}\,a^4bx^{10} + \frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

maple [A] time = 0.00, size = 58, normalized size = 0.81

$$\frac{1}{18}b^5x^{18} + \frac{5}{16}a\,b^4x^{16} + \frac{5}{7}a^2b^3x^{14} + \frac{5}{6}a^3b^2x^{12} + \frac{1}{2}a^4b\,x^{10} + \frac{1}{8}a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^5,x)
```

```
[Out] 1/18*b^5*x^18+5/16*a*b^4*x^16+5/7*a^2*b^3*x^14+5/6*a^3*b^2*x^12+1/2*a^4*b*x
^10+1/8*a^5*x^8
```

maxima [A] time = 1.37, size = 57, normalized size = 0.79

$$\frac{1}{18}\,b^5x^{18} + \frac{5}{16}\,ab^4x^{16} + \frac{5}{7}\,a^2b^3x^{14} + \frac{5}{6}\,a^3b^2x^{12} + \frac{1}{2}\,a^4bx^{10} + \frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

mupad [B] time = 0.02, size = 57, normalized size = 0.79

$$\frac{a^5\,x^8}{8} + \frac{a^4\,b\,x^{10}}{2} + \frac{5\,a^3\,b^2\,x^{12}}{6} + \frac{5\,a^2\,b^3\,x^{14}}{7} + \frac{5\,a\,b^4\,x^{16}}{16} + \frac{b^5\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^8)/8 + (b^5*x^18)/18 + (a^4*b*x^10)/2 + (5*a*b^4*x^16)/16 + (5*a^3*b
^2*x^12)/6 + (5*a^2*b^3*x^14)/7
```

sympy [A] time = 0.09, size = 65, normalized size = 0.90

$$\frac{a^5x^8}{8} + \frac{a^4bx^{10}}{2} + \frac{5a^3b^2x^{12}}{6} + \frac{5a^2b^3x^{14}}{7} + \frac{5ab^4x^{16}}{16} + \frac{b^5x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**8/8 + a**4*b*x**10/2 + 5*a**3*b**2*x**12/6 + 5*a**2*b**3*x**14/7 +
5*a*b**4*x**16/16 + b**5*x**18/18
```

3.56 $\int x^5 \left(a+bx^2\right)^5 dx$

Optimal. Leaf size=53

$$\frac{a^2 \left(a+bx^2\right)^6}{12b^3}+\frac{\left(a+bx^2\right)^8}{16b^3}-\frac{a \left(a+bx^2\right)^7}{7b^3}$$

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^6}{12b^3}+\frac{\left(a+bx^2\right)^8}{16b^3}-\frac{a \left(a+bx^2\right)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^5,x]
```

```
[Out] (a^2*(a + b*x^2)^6)/(12*b^3) - (a*(a + b*x^2)^7)/(7*b^3) + (a + b*x^2)^8/(1
6*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^5 dx &= \frac{1}{2} \text{Subst}\left(\int x^2(a+bx)^5 \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^2(a+bx)^5}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{(a+bx)^7}{b^2}\right) dx, x, x^2\right)\\
&= \frac{a^2 \left(a+bx^2\right)^6}{12b^3}-\frac{a \left(a+bx^2\right)^7}{7b^3}+\frac{\left(a+bx^2\right)^8}{16b^3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.25

$$\frac{a^5x^6}{6}+\frac{5}{8}a^4bx^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^8)/8 + a^3*b^2*x^10 + (5*a^2*b^3*x^12)/6 + (5*a*b^
4*x^14)/14 + (b^5*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+bx^2\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^5*(a + b*x^2)^5, x]
```

fricas [A] time = 1.04, size = 56, normalized size = 1.06

$$\frac{1}{16}x^{16}b^5+\frac{5}{14}x^{14}b^4a+\frac{5}{6}x^{12}b^3a^2+x^{10}b^2a^3+\frac{5}{8}x^8ba^4+\frac{1}{6}x^6a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^5 + 5/14*x^14*b^4*a + 5/6*x^12*b^3*a^2 + x^10*b^2*a^3 + 5/8*x^8
*b*a^4 + 1/6*x^6*a^5
```

giac [A] time = 1.06, size = 56, normalized size = 1.06

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

maple [A] time = 0.00, size = 57, normalized size = 1.08

$$\frac{1}{16}b^5x^{16}+\frac{5}{14}a\,b^4x^{14}+\frac{5}{6}a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}a^4b\,x^8+\frac{1}{6}a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^5,x)
```

```
[Out] 1/16*b^5*x^16+5/14*a*b^4*x^14+5/6*a^2*b^3*x^12+a^3*b^2*x^10+5/8*a^4*b*x^8+1
/6*a^5*x^6
```

maxima [A] time = 1.27, size = 56, normalized size = 1.06

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

mupad [B] time = 0.02, size = 56, normalized size = 1.06

$$\frac{a^5\,x^6}{6}+\frac{5\,a^4\,b\,x^8}{8}+a^3\,b^2\,x^{10}+\frac{5\,a^2\,b^3\,x^{12}}{6}+\frac{5\,a\,b^4\,x^{14}}{14}+\frac{b^5\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^6)/6 + (b^5*x^16)/16 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^14)/14 + a^3*b^2
*x^10 + (5*a^2*b^3*x^12)/6
```

sympy [A] time = 0.08, size = 63, normalized size = 1.19

$$\frac{a^5x^6}{6}+\frac{5a^4bx^8}{8}+a^3b^2x^{10}+\frac{5a^2b^3x^{12}}{6}+\frac{5ab^4x^{14}}{14}+\frac{b^5x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**6/6 + 5*a**4*b*x**8/8 + a**3*b**2*x**10 + 5*a**2*b**3*x**12/6 + 5*a
*b**4*x**14/14 + b**5*x**16/16
```

3.57 $\int x^3 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^7}{14b^2}-\frac{a\left(a+bx^2\right)^6}{12b^2}$$

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^7}{14b^2}-\frac{a\left(a+bx^2\right)^6}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^5,x]
```

```
[Out] -(a*(a + b*x^2)^6)/(12*b^2) + (a + b*x^2)^7/(14*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^5 \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^5 \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^5}{b}+\frac{(a+bx)^6}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^6}{12b^2}+\frac{\left(a+bx^2\right)^7}{14b^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.94

$$\frac{a^5x^4}{4}+\frac{5}{6}a^4bx^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}ab^4x^{12}+\frac{b^5x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^4)/4 + (5*a^4*b*x^6)/6 + (5*a^3*b^2*x^8)/4 + a^2*b^3*x^10 + (5*a*b^4
*x^12)/12 + (b^5*x^14)/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)^5, x]
```

fricas [A] time = 0.81, size = 56, normalized size = 1.65

$$\frac{1}{14}x^{14}b^5+\frac{5}{12}x^{12}b^4a+x^{10}b^3a^2+\frac{5}{4}x^8b^2a^3+\frac{5}{6}x^6ba^4+\frac{1}{4}x^4a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^5 + 5/12*x^12*b^4*a + x^10*b^3*a^2 + 5/4*x^8*b^2*a^3 + 5/6*x^6*
b*a^4 + 1/4*x^4*a^5
```

giac [A] time = 1.18, size = 56, normalized size = 1.65

$$\frac{1}{14}\,b^5x^{14}+\frac{5}{12}\,ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}\,a^3b^2x^8+\frac{5}{6}\,a^4bx^6+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

maple [A] time = 0.00, size = 57, normalized size = 1.68

$$\frac{1}{14}b^5x^{14}+\frac{5}{12}a\,b^4x^{12}+a^2b^3x^{10}+\frac{5}{4}a^3b^2x^8+\frac{5}{6}a^4b\,x^6+\frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^5,x)
```

```
[Out] 1/14*b^5*x^14+5/12*a*b^4*x^12+a^2*b^3*x^10+5/4*a^3*b^2*x^8+5/6*a^4*b*x^6+1/
4*a^5*x^4
```

maxima [A] time = 1.36, size = 56, normalized size = 1.65

$$\frac{1}{14}\,b^5x^{14}+\frac{5}{12}\,ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}\,a^3b^2x^8+\frac{5}{6}\,a^4bx^6+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

mupad [B] time = 0.02, size = 56, normalized size = 1.65

$$\frac{a^5\,x^4}{4}+\frac{5\,a^4\,b\,x^6}{6}+\frac{5\,a^3\,b^2\,x^8}{4}+a^2\,b^3\,x^{10}+\frac{5\,a\,b^4\,x^{12}}{12}+\frac{b^5\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^4)/4 + (b^5*x^14)/14 + (5*a^4*b*x^6)/6 + (5*a*b^4*x^12)/12 + (5*a^3*
b^2*x^8)/4 + a^2*b^3*x^10
```

sympy [B] time = 0.08, size = 63, normalized size = 1.85

$$\frac{a^5x^4}{4}+\frac{5a^4bx^6}{6}+\frac{5a^3b^2x^8}{4}+a^2b^3x^{10}+\frac{5ab^4x^{12}}{12}+\frac{b^5x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**4/4 + 5*a**4*b*x**6/6 + 5*a**3*b**2*x**8/4 + a**2*b**3*x**10 + 5*a*
b**4*x**12/12 + b**5*x**14/14
```

3.58 $\int x\left(a+bx^2\right)^5\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^6}{12b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^6}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^5,x]
```

```
[Out] (a + b*x^2)^6/(12*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^5\,dx=\frac{\left(a+bx^2\right)^6}{12b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^6}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^5,x]
```

```
[Out] (a + b*x^2)^6/(12*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2)^5, x]
```

fricas [B] time = 0.79, size = 57, normalized size = 3.56

$$\frac{1}{12}x^{12}b^5+\frac{1}{2}x^{10}b^4a+\frac{5}{4}x^8b^3a^2+\frac{5}{3}x^6b^2a^3+\frac{5}{4}x^4ba^4+\frac{1}{2}x^2a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^5 + 1/2*x^10*b^4*a + 5/4*x^8*b^3*a^2 + 5/3*x^6*b^2*a^3 + 5/4*x^
4*b*a^4 + 1/2*x^2*a^5
```

giac [A] time = 1.06, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^6}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*(b*x^2 + a)^6/b
```

maple [B] time = 0.00, size = 58, normalized size = 3.62

$$\frac{1}{12}b^5x^{12}+\frac{1}{2}a\,b^4x^{10}+\frac{5}{4}a^2b^3x^8+\frac{5}{3}a^3b^2x^6+\frac{5}{4}a^4b\,x^4+\frac{1}{2}a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^5,x)
```

```
[Out] 1/12*b^5*x^12+1/2*a*b^4*x^10+5/4*a^2*b^3*x^8+5/3*a^3*b^2*x^6+5/4*a^4*b*x^4+
1/2*a^5*x^2
```

maxima [A] time = 1.40, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^6}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^6/b
```

mupad [B] time = 0.02, size = 57, normalized size = 3.56

$$\frac{a^5\,x^2}{2}+\frac{5\,a^4\,b\,x^4}{4}+\frac{5\,a^3\,b^2\,x^6}{3}+\frac{5\,a^2\,b^3\,x^8}{4}+\frac{a\,b^4\,x^{10}}{2}+\frac{b^5\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^2)/2 + (b^5*x^12)/12 + (5*a^4*b*x^4)/4 + (a*b^4*x^10)/2 + (5*a^3*b^2
*x^6)/3 + (5*a^2*b^3*x^8)/4
```

sympy [B] time = 0.08, size = 65, normalized size = 4.06

$$\frac{a^5x^2}{2}+\frac{5a^4bx^4}{4}+\frac{5a^3b^2x^6}{3}+\frac{5a^2b^3x^8}{4}+\frac{ab^4x^{10}}{2}+\frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**2/2 + 5*a**4*b*x**4/4 + 5*a**3*b**2*x**6/3 + 5*a**2*b**3*x**8/4 + a
*b**4*x**10/2 + b**5*x**12/12
```

3.59 $\int \frac{\left(a+bx^2\right)^5}{x} \, dx$

Optimal. Leaf size=65

$$a^5 \log(x) + \frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10}$$

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{3}a^2b^3x^6 + \frac{5}{2}a^3b^2x^4 + \frac{5}{2}a^4bx^2 + a^5 \log(x) + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x,x]
```

```
[Out] (5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 +
 (b^5*x^10)/10 + a^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^5}{x} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^5}{x} \, dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left(5a^4b + \frac{a^5}{x} + 10a^3b^2x + 10a^2b^3x^2 + 5ab^4x^3 + b^5x^4\right) dx, x, x^2\right) \\
&= \frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10} + a^5 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$a^5\log(x)+\frac{5}{2}a^4bx^2+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^2b^3x^6+\frac{5}{8}ab^4x^8+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x,x]
```

```
[Out] (5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 +
 (b^5*x^10)/10 + a^5*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2\right)^5}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x, x]
```

fricas [A] time = 0.90, size = 55, normalized size = 0.85

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

giac [A] time = 0.92, size = 58, normalized size = 0.89

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+\frac{1}{2}\,a^5\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + 1/2*a^5*log(x^2)
```

maple [A] time = 0.00, size = 56, normalized size = 0.86

$$\frac{b^5x^{10}}{10}+\frac{5a\,b^4x^8}{8}+\frac{5a^2b^3x^6}{3}+\frac{5a^3b^2x^4}{2}+\frac{5a^4b\,x^2}{2}+a^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x,x)
```

```
[Out] 5/2*a^4*b*x^2+5/2*a^3*b^2*x^4+5/3*a^2*b^3*x^6+5/8*a*b^4*x^8+1/10*b^5*x^10+a
^5*ln(x)
```

maxima [A] time = 1.37, size = 58, normalized size = 0.89

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+\frac{1}{2}\,a^5\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + 1/2*a^5*log(x^2)
```

mupad [B] time = 0.03, size = 55, normalized size = 0.85

$$a^5\,\ln(x)+\frac{b^5\,x^{10}}{10}+\frac{5\,a^4\,b\,x^2}{2}+\frac{5\,a\,b^4\,x^8}{8}+\frac{5\,a^3\,b^2\,x^4}{2}+\frac{5\,a^2\,b^3\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x,x)
```

```
[Out] a^5*log(x) + (b^5*x^10)/10 + (5*a^4*b*x^2)/2 + (5*a*b^4*x^8)/8 + (5*a^3*b^2
*x^4)/2 + (5*a^2*b^3*x^6)/3
```

sympy [A] time = 0.14, size = 65, normalized size = 1.00

$$a^5\log{(x)}+\frac{5a^4bx^2}{2}+\frac{5a^3b^2x^4}{2}+\frac{5a^2b^3x^6}{3}+\frac{5ab^4x^8}{8}+\frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x,x)
```

```
[Out] a**5*log(x) + 5*a**4*b*x**2/2 + 5*a**3*b**2*x**4/2 + 5*a**2*b**3*x**6/3 + 5
*a*b**4*x**8/8 + b**5*x**10/10
```

3.60 $\int \frac{(a+bx^2)^5}{x^3}\,dx$

Optimal. Leaf size=64

$$-\frac{a^5}{2x^2}+5a^4b\log(x)+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}$$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5}{2}a^2b^3x^4+5a^3b^2x^2+5a^4b\log(x)-\frac{a^5}{2x^2}+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^3,x]
```

```
[Out] -a^5/(2*x^2) + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x
^8)/8 + 5*a^4*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(10a^3b^2+\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^2b^3x+5ab^4x^2+b^5x^3\right)dx,x,x^2\right)\\
&= -\frac{a^5}{2x^2}+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}+5a^4b\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 64, normalized size = 1.00

$$-\frac{a^5}{2x^2}+5a^4b\log(x)+5a^3b^2x^2+\frac{5}{2}a^2b^3x^4+\frac{5}{6}ab^4x^6+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x
^8)/8 + 5*a^4*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx^2)^5}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^3, x]
```

fricas [A] time = 0.93, size = 61, normalized size = 0.95

$$\frac{3\,b^5x^{10}+20\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4+120\,a^4bx^2\log(x)-12\,a^5}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a^
4*b*x^2*log(x) - 12*a^5)/x^2
```

giac [A] time = 1.06, size = 68, normalized size = 1.06

$$\frac{1}{8}\,b^5x^8+\frac{5}{6}\,ab^4x^6+\frac{5}{2}\,a^2b^3x^4+5\,a^3b^2x^2+\frac{5}{2}\,a^4b\log\left(x^2\right)-\frac{5\,a^4bx^2+a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*l
og(x^2) - 1/2*(5*a^4*b*x^2 + a^5)/x^2
```

maple [A] time = 0.00, size = 57, normalized size = 0.89

$$\frac{b^5x^8}{8}+\frac{5a\,b^4x^6}{6}+\frac{5a^2b^3x^4}{2}+5a^3b^2x^2+5a^4b\ln(x)-\frac{a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^3,x)
```

```
[Out] -1/2*a^5/x^2+5*a^3*b^2*x^2+5/2*a^2*b^3*x^4+5/6*a*b^4*x^6+1/8*b^5*x^8+5*a^4*
b*ln(x)
```

maxima [A] time = 1.37, size = 58, normalized size = 0.91

$$\frac{1}{8}\,b^5x^8+\frac{5}{6}\,ab^4x^6+\frac{5}{2}\,a^2b^3x^4+5\,a^3b^2x^2+\frac{5}{2}\,a^4b\log\left(x^2\right)-\frac{a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*l
og(x^2) - 1/2*a^5/x^2
```

mupad [B] time = 0.03, size = 56, normalized size = 0.88

$$\frac{b^5\,x^8}{8}-\frac{a^5}{2\,x^2}+\frac{5\,a\,b^4\,x^6}{6}+5\,a^4\,b\,\ln(x)+5\,a^3\,b^2\,x^2+\frac{5\,a^2\,b^3\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^3,x)
```

```
[Out] (b^5*x^8)/8 - a^5/(2*x^2) + (5*a*b^4*x^6)/6 + 5*a^4*b*log(x) + 5*a^3*b^2*x^
2 + (5*a^2*b^3*x^4)/2
```

sympy [A] time = 0.17, size = 63, normalized size = 0.98

$$-\frac{a^5}{2x^2}+5a^4b\log{(x)}+5a^3b^2x^2+\frac{5a^2b^3x^4}{2}+\frac{5ab^4x^6}{6}+\frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**3,x)
```

```
[Out] -a**5/(2*x**2) + 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 +
5*a*b**4*x**6/6 + b**5*x**8/8
```

3.61 $\int \frac{(a+bx^2)^5}{x^5}\,dx$

Optimal. Leaf size=64

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+10a^3b^2\log(x)+5a^2b^3x^2+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}$$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$5a^2b^3x^2+10a^3b^2\log(x)-\frac{5a^4b}{2x^2}-\frac{a^5}{4x^4}+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^5,x]
```

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x
^6)/6 + 10*a^3*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(10a^2b^3+\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+5ab^4x+b^5x^2\right)dx,x,x^2\right)\\ &= -\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+5a^2b^3x^2+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}+10a^3b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 1.00

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+10a^3b^2\log(x)+5a^2b^3x^2+\frac{5}{4}ab^4x^4+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x
^6)/6 + 10*a^3*b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^5}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^5, x]
```

fricas [A] time = 1.32, size = 61, normalized size = 0.95

$$\frac{2\,b^5x^{10}+15\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4\log(x)-30\,a^4bx^2-3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*log(x) -
 30*a^4*b*x^2 - 3*a^5)/x^4
```

giac [A] time = 0.88, size = 70, normalized size = 1.09

$$\frac{1}{6}\,b^5x^6+\frac{5}{4}\,ab^4x^4+5\,a^2b^3x^2+5\,a^3b^2\log\left(x^2\right)-\frac{30\,a^3b^2x^4+10\,a^4bx^2+a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*log(x^2) - 1/4*(30*
a^3*b^2*x^4 + 10*a^4*b*x^2 + a^5)/x^4
```

maple [A] time = 0.00, size = 57, normalized size = 0.89

$$\frac{b^5x^6}{6}+\frac{5a\,b^4x^4}{4}+5a^2b^3x^2+10a^3b^2\ln(x)-\frac{5a^4b}{2x^2}-\frac{a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^5,x)
```

```
[Out] -1/4*a^5/x^4-5/2*a^4*b/x^2+5*a^2*b^3*x^2+5/4*a*b^4*x^4+1/6*b^5*x^6+10*a^3*b
^2*ln(x)
```

maxima [A] time = 1.33, size = 59, normalized size = 0.92

$$\frac{1}{6}\,b^5x^6+\frac{5}{4}\,ab^4x^4+5\,a^2b^3x^2+5\,a^3b^2\log\left(x^2\right)-\frac{10\,a^4bx^2+a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*log(x^2) - 1/4*(10*
a^4*b*x^2 + a^5)/x^4
```

mupad [B] time = 0.03, size = 59, normalized size = 0.92

$$\frac{b^5\,x^6}{6}-\frac{\frac{a^5}{4}+\frac{5\,b\,a^4\,x^2}{2}}{x^4}+\frac{5\,a\,b^4\,x^4}{4}+5\,a^2\,b^3\,x^2+10\,a^3\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^5,x)
```

```
[Out] (b^5*x^6)/6 - (a^5/4 + (5*a^4*b*x^2)/2)/x^4 + (5*a*b^4*x^4)/4 + 5*a^2*b^3*x
^2 + 10*a^3*b^2*log(x)
```

sympy [A] time = 0.22, size = 63, normalized size = 0.98

$$10a^3b^2\log{(x)}+5a^2b^3x^2+\frac{5ab^4x^4}{4}+\frac{b^5x^6}{6}+\frac{-a^5-10a^4bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**5,x)
```

```
[Out] 10*a**3*b**2*log(x) + 5*a**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6 + (-
a**5 - 10*a**4*b*x**2)/(4*x**4)
```

3.62 $\int \frac{\left(a+bx^2\right)^5}{x^7} \, dx$

Optimal. Leaf size=64

$$-\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+10a^2b^3\log(x)+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{x^2}+10a^2b^3\log(x)-\frac{5a^4b}{4x^4}-\frac{a^5}{6x^6}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^7,x]
```

```
[Out] -a^5/(6*x^6) - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5
*x^4)/4 + 10*a^2*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^5}{x^7} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^4} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right) dx, x, x^2\right) \\
&= -\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}+10a^2b^3\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 64, normalized size = 1.00

$$-\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+10a^2b^3\log(x)+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^7,x]
```

```
[Out] -1/6*a^5/x^6 - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5
*x^4)/4 + 10*a^2*b^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2\right)^5}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^7, x]
```

fricas [A] time = 1.15, size = 61, normalized size = 0.95

$$\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3x^6\log(x)-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3*b^2*x^4 -
 15*a^4*b*x^2 - 2*a^5)/x^6
```

giac [A] time = 1.19, size = 72, normalized size = 1.12

$$\frac{1}{4}\,b^5x^4+\frac{5}{2}\,ab^4x^2+5\,a^2b^3\log\left(x^2\right)-\frac{110\,a^2b^3x^6+60\,a^3b^2x^4+15\,a^4bx^2+2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*log(x^2) - 1/12*(110*a^2*b^3*x^6 +
60*a^3*b^2*x^4 + 15*a^4*b*x^2 + 2*a^5)/x^6
```

maple [A] time = 0.01, size = 57, normalized size = 0.89

$$\frac{b^5x^4}{4}+\frac{5a\,b^4x^2}{2}+10a^2b^3\ln(x)-\frac{5a^3b^2}{x^2}-\frac{5a^4b}{4x^4}-\frac{a^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^7,x)
```

```
[Out] -1/6*a^5/x^6-5/4*a^4*b/x^4-5*a^3*b^2/x^2+5/2*a*b^4*x^2+1/4*b^5*x^4+10*a^2*b
^3*ln(x)
```

maxima [A] time = 1.40, size = 61, normalized size = 0.95

$$\frac{1}{4}\,b^5x^4+\frac{5}{2}\,ab^4x^2+5\,a^2b^3\log\left(x^2\right)-\frac{60\,a^3b^2x^4+15\,a^4bx^2+2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*log(x^2) - 1/12*(60*a^3*b^2*x^4 + 1
5*a^4*b*x^2 + 2*a^5)/x^6
```

mupad [B] time = 0.04, size = 59, normalized size = 0.92

$$\frac{b^5\,x^4}{4}-\frac{\frac{a^5}{6}+\frac{5\,a^4\,b\,x^2}{4}+5\,a^3\,b^2\,x^4}{x^6}+\frac{5\,a\,b^4\,x^2}{2}+10\,a^2\,b^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^7,x)
```

```
[Out] (b^5*x^4)/4 - (a^5/6 + (5*a^4*b*x^2)/4 + 5*a^3*b^2*x^4)/x^6 + (5*a*b^4*x^2)
/2 + 10*a^2*b^3*log(x)
```

sympy [A] time = 0.28, size = 65, normalized size = 1.02

$$10a^2b^3\log\left(x\right)+\frac{5ab^4x^2}{2}+\frac{b^5x^4}{4}+\frac{-2a^5-15a^4bx^2-60a^3b^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**7,x)
```

```
[Out] 10*a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4 + (-2*a**5 - 15*a**4*b*
x**2 - 60*a**3*b**2*x**4)/(12*x**6)
```

3.63 $\int \frac{\left(a+bx^2\right)^5}{x^9}\,dx$

Optimal. Leaf size=64

$$-\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+5ab^4\log(x)+\frac{b^5x^2}{2}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}-\frac{5a^4b}{6x^6}-\frac{a^5}{8x^8}+5ab^4\log(x)+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^9,x]
```

```
[Out] -a^5/(8*x^8) - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 +
(b^5*x^2)/2 + 5*a*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^5}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx,x,x^2\right)\\ &= -\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+\frac{b^5x^2}{2}+5ab^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 64, normalized size = 1.00

$$-\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+5ab^4\log(x)+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^9,x]
```

```
[Out] -1/8*a^5/x^8 - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 +
(b^5*x^2)/2 + 5*a*b^4*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^5}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^9, x]
```

fricas [A] time = 0.84, size = 61, normalized size = 0.95

$$\frac{12\,b^5x^{10}+120\,ab^4x^8\log(x)-120\,a^2b^3x^6-60\,a^3b^2x^4-20\,a^4bx^2-3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^4
 - 20*a^4*b*x^2 - 3*a^5)/x^8
```

giac [A] time = 1.05, size = 70, normalized size = 1.09

$$\frac{1}{2}\,b^5x^2+\frac{5}{2}\,ab^4\log\left(x^2\right)-\frac{125\,ab^4x^8+120\,a^2b^3x^6+60\,a^3b^2x^4+20\,a^4bx^2+3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2 + 5/2*a*b^4*log(x^2) - 1/24*(125*a*b^4*x^8 + 120*a^2*b^3*x^6 +
60*a^3*b^2*x^4 + 20*a^4*b*x^2 + 3*a^5)/x^8
```

maple [A] time = 0.01, size = 57, normalized size = 0.89

$$\frac{b^5x^2}{2}+5a\,b^4\ln(x)-\frac{5a^2b^3}{x^2}-\frac{5a^3b^2}{2x^4}-\frac{5a^4b}{6x^6}-\frac{a^5}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^9,x)
```

```
[Out] -1/8*a^5/x^8-5/6*a^4*b/x^6-5/2*a^3*b^2/x^4-5*a^2*b^3/x^2+1/2*b^5*x^2+5*a*b^
4*ln(x)
```

maxima [A] time = 1.41, size = 61, normalized size = 0.95

$$\frac{1}{2}\,b^5x^2+\frac{5}{2}\,ab^4\log\left(x^2\right)-\frac{120\,a^2b^3x^6+60\,a^3b^2x^4+20\,a^4bx^2+3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5/2*a*b^4*log(x^2) - 1/24*(120*a^2*b^3*x^6 + 60*a^3*b^2*x^4 +
 20*a^4*b*x^2 + 3*a^5)/x^8
```

mupad [B] time = 0.04, size = 59, normalized size = 0.92

$$\frac{b^5\,x^2}{2}-\frac{\frac{a^5}{8}+\frac{5\,a^4\,b\,x^2}{6}+\frac{5\,a^3\,b^2\,x^4}{2}+5\,a^2\,b^3\,x^6}{x^8}+5\,a\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^9,x)
```

```
[Out] (b^5*x^2)/2 - (a^5/8 + (5*a^4*b*x^2)/6 + (5*a^3*b^2*x^4)/2 + 5*a^2*b^3*x^6)
/x^8 + 5*a*b^4*log(x)
```

sympy [A] time = 0.36, size = 63, normalized size = 0.98

$$5ab^4\log\left(x\right)+\frac{b^5x^2}{2}+\frac{-3a^5-20a^4bx^2-60a^3b^2x^4-120a^2b^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**9,x)
```

```
[Out] 5*a*b**4*log(x) + b**5*x**2/2 + (-3*a**5 - 20*a**4*b*x**2 - 60*a**3*b**2*x*
*4 - 120*a**2*b**3*x**6)/(24*x**8)
```

3.64 $\int \frac{(a+bx^2)^5}{x^{11}}\,dx$

Optimal. Leaf size=65

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5a^4b}{8x^8}-\frac{a^5}{10x^{10}}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^11,x]
```

```
[Out] -a^5/(10*x^10) - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x
^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{11}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^6}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx,x,x^2\right)\\ &= -\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{8x^8}-\frac{5a^3b^2}{3x^6}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{2x^2}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^11,x]
```

```
[Out] -1/10*a^5/x^10 - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x
^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^5}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^11,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^11, x]
```

fricas [A] time = 1.08, size = 61, normalized size = 0.94

$$\frac{120\,b^5x^{10}\log(x)-300\,ab^4x^8-300\,a^2b^3x^6-200\,a^3b^2x^4-75\,a^4bx^2-12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2*
x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

giac [A] time = 1.06, size = 69, normalized size = 1.06

$$\frac{1}{2}\,b^5\log\left(x^2\right)-\frac{137\,b^5x^{10}+300\,ab^4x^8+300\,a^2b^3x^6+200\,a^3b^2x^4+75\,a^4bx^2+12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(x^2) - 1/120*(137*b^5*x^10 + 300*a*b^4*x^8 + 300*a^2*b^3*x^6 +
200*a^3*b^2*x^4 + 75*a^4*b*x^2 + 12*a^5)/x^10
```

maple [A] time = 0.01, size = 56, normalized size = 0.86

$$b^5 \ln(x) - \frac{5a\,b^4}{2x^2} - \frac{5a^2b^3}{2x^4} - \frac{5a^3b^2}{3x^6} - \frac{5a^4b}{8x^8} - \frac{a^5}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^11,x)
```

```
[Out] -1/10*a^5/x^10-5/8*a^4*b/x^8-5/3*a^3*b^2/x^6-5/2*a^2*b^3/x^4-5/2*a*b^4/x^2+
b^5*ln(x)
```

maxima [A] time = 1.35, size = 61, normalized size = 0.94

$$\frac{1}{2}\,b^5 \log\left(x^2\right) - \frac{300\,ab^4x^8 + 300\,a^2b^3x^6 + 200\,a^3b^2x^4 + 75\,a^4bx^2 + 12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*log(x^2) - 1/120*(300*a*b^4*x^8 + 300*a^2*b^3*x^6 + 200*a^3*b^2*x^4
 + 75*a^4*b*x^2 + 12*a^5)/x^10
```

mupad [B] time = 4.77, size = 58, normalized size = 0.89

$$b^5\,\ln(x) - \frac{\frac{a^5}{10} + \frac{5\,a^4\,b\,x^2}{8} + \frac{5\,a^3\,b^2\,x^4}{3} + \frac{5\,a^2\,b^3\,x^6}{2} + \frac{5\,a\,b^4\,x^8}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^11,x)
```

```
[Out] b^5*log(x) - (a^5/10 + (5*a^4*b*x^2)/8 + (5*a*b^4*x^8)/2 + (5*a^3*b^2*x^4)/
3 + (5*a^2*b^3*x^6)/2)/x^10
```

sympy [A] time = 0.44, size = 61, normalized size = 0.94

$$b^5 \log(x) + \frac{-12a^5 - 75a^4bx^2 - 200a^3b^2x^4 - 300a^2b^3x^6 - 300ab^4x^8}{120x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**11,x)
```

```
[Out] b**5*log(x) + (-12*a**5 - 75*a**4*b*x**2 - 200*a**3*b**2*x**4 - 300*a**2*b*
*3*x**6 - 300*a*b**4*x**8)/(120*x**10)
```

3.65 $\int \frac{(a+bx^2)^5}{x^{13}}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^13,x]
```

```
[Out] -(a + b*x^2)^6/(12*a*x^12)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx = -\frac{(a+bx^2)^6}{12ax^{12}}$$

Mathematica [B] time = 0.00, size = 69, normalized size = 3.63

$$-\frac{a^5}{12x^{12}}-\frac{a^4b}{2x^{10}}-\frac{5a^3b^2}{4x^8}-\frac{5a^2b^3}{3x^6}-\frac{5ab^4}{4x^4}-\frac{b^5}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^13,x]
```

```
[Out] -1/12*a^5/x^12 - (a^4*b)/(2*x^10) - (5*a^3*b^2)/(4*x^8) - (5*a^2*b^3)/(3*x^
6) - (5*a*b^4)/(4*x^4) - b^5/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^13,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^13, x]
```

fricas [B] time = 0.60, size = 57, normalized size = 3.00

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

giac [B] time = 1.07, size = 57, normalized size = 3.00

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

maple [B] time = 0.01, size = 58, normalized size = 3.05

$$-\frac{b^5}{2x^2}-\frac{5a\,b^4}{4x^4}-\frac{5a^2b^3}{3x^6}-\frac{5a^3b^2}{4x^8}-\frac{a^4b}{2x^{10}}-\frac{a^5}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^13,x)
```

```
[Out] -5/3*a^2*b^3/x^6-1/2*b^5/x^2-5/4*a*b^4/x^4-1/2*a^4*b/x^10-1/12*a^5/x^12-5/4
*a^3*b^2/x^8
```

maxima [B] time = 1.37, size = 57, normalized size = 3.00

$$-\frac{6 b^5 x^{10} + 15 a b^4 x^8 + 20 a^2 b^3 x^6 + 15 a^3 b^2 x^4 + 6 a^4 b x^2 + a^5}{12 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^13,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

mupad [B] time = 4.75, size = 59, normalized size = 3.11

$$-\frac{\frac{a^5}{12} + \frac{a^4 b x^2}{2} + \frac{5 a^3 b^2 x^4}{4} + \frac{5 a^2 b^3 x^6}{3} + \frac{5 a b^4 x^8}{4} + \frac{b^5 x^{10}}{2}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^13,x)
```

```
[Out] -(a^5/12 + (b^5*x^10)/2 + (a^4*b*x^2)/2 + (5*a*b^4*x^8)/4 + (5*a^3*b^2*x^4)
/4 + (5*a^2*b^3*x^6)/3)/x^12
```

sympy [B] time = 0.47, size = 61, normalized size = 3.21

$$\frac{-a^5 - 6a^4bx^2 - 15a^3b^2x^4 - 20a^2b^3x^6 - 15ab^4x^8 - 6b^5x^{10}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**13,x)
```

```
[Out] (-a**5 - 6*a**4*b*x**2 - 15*a**3*b**2*x**4 - 20*a**2*b**3*x**6 - 15*a*b**4*
x**8 - 6*b**5*x**10)/(12*x**12)
```

3.66 $\int \frac{(a+bx^2)^5}{x^{15}} dx$

Optimal. Leaf size=40

$$\frac{b(a+bx^2)^6}{84a^2x^{12}} - \frac{(a+bx^2)^6}{14ax^{14}}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b(a+bx^2)^6}{84a^2x^{12}} - \frac{(a+bx^2)^6}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^15,x]
```

```
[Out] -(a + b*x^2)^6/(14*a*x^14) + (b*(a + b*x^2)^6)/(84*a^2*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^6}{14ax^{14}} - \frac{b\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^2\right)}{14a}$$
$$= -\frac{(a+bx^2)^6}{14ax^{14}} + \frac{b(a+bx^2)^6}{84a^2x^{12}}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 1.68

$$-\frac{a^5}{14x^{14}} - \frac{5a^4b}{12x^{12}} - \frac{a^3b^2}{x^{10}} - \frac{5a^2b^3}{4x^8} - \frac{5ab^4}{6x^6} - \frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^15,x]
```

```
[Out] -1/14*a^5/x^14 - (5*a^4*b)/(12*x^12) - (a^3*b^2)/x^10 - (5*a^2*b^3)/(4*x^8)
 - (5*a*b^4)/(6*x^6) - b^5/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^15,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^15, x]
```

fricas [A] time = 0.88, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{10} + 70\,ab^4x^8 + 105\,a^2b^3x^6 + 84\,a^3b^2x^4 + 35\,a^4bx^2 + 6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

giac [A] time = 1.06, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

maple [A] time = 0.00, size = 58, normalized size = 1.45

$$-\frac{b^5}{4x^4}-\frac{5a\,b^4}{6x^6}-\frac{5a^2b^3}{4x^8}-\frac{a^3b^2}{x^{10}}-\frac{5a^4b}{12x^{12}}-\frac{a^5}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^15,x)
```

```
[Out] -5/12*a^4*b/x^12-5/6*a*b^4/x^6-1/4*b^5/x^4-a^3*b^2/x^10-5/4*a^2*b^3/x^8-1/1
4*a^5/x^14
```

maxima [A] time = 1.34, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^15,x, algorithm="maxima")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

mupad [B] time = 4.74, size = 58, normalized size = 1.45

$$-\frac{\frac{a^5}{14}+\frac{5\,a^4\,b\,x^2}{12}+a^3\,b^2\,x^4+\frac{5\,a^2\,b^3\,x^6}{4}+\frac{5\,a\,b^4\,x^8}{6}+\frac{b^5\,x^{10}}{4}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^15,x)
```

```
[Out] -(a^5/14 + (b^5*x^10)/4 + (5*a^4*b*x^2)/12 + (5*a*b^4*x^8)/6 + a^3*b^2*x^4
+ (5*a^2*b^3*x^6)/4)/x^14
```

sympy [A] time = 0.51, size = 63, normalized size = 1.58

$$\frac{-6a^5-35a^4bx^2-84a^3b^2x^4-105a^2b^3x^6-70ab^4x^8-21b^5x^{10}}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**15,x)
```

```
[Out] (-6*a**5 - 35*a**4*b*x**2 - 84*a**3*b**2*x**4 - 105*a**2*b**3*x**6 - 70*a*b
**4*x**8 - 21*b**5*x**10)/(84*x**14)
```

3.67 $\int \frac{(a+bx^2)^5}{x^{17}}\,dx$

Optimal. Leaf size=62

$$-\frac{b^2(a+bx^2)^6}{336a^3x^{12}}+\frac{b(a+bx^2)^6}{56a^2x^{14}}-\frac{(a+bx^2)^6}{16ax^{16}}$$

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$-\frac{b^2(a+bx^2)^6}{336a^3x^{12}}+\frac{b(a+bx^2)^6}{56a^2x^{14}}-\frac{(a+bx^2)^6}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^17,x]
```

```
[Out] -(a + b*x^2)^6/(16*a*x^16) + (b*(a + b*x^2)^6)/(56*a^2*x^14) - (b^2*(a + b*
x^2)^6)/(336*a^3*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^5}{x^{17}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^9}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^6}{16ax^{16}} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, x^2\right)}{8a}$$
$$= -\frac{(a+bx^2)^6}{16ax^{16}} + \frac{b(a+bx^2)^6}{56a^2x^{14}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^2\right)}{56a^2}$$
$$= -\frac{(a+bx^2)^6}{16ax^{16}} + \frac{b(a+bx^2)^6}{56a^2x^{14}} - \frac{b^2(a+bx^2)^6}{336a^3x^{12}}$$

Mathematica [**A**] time = 0.00, size = 67, normalized size = 1.08

$$-\frac{a^5}{16x^{16}} - \frac{5a^4b}{14x^{14}} - \frac{5a^3b^2}{6x^{12}} - \frac{a^2b^3}{x^{10}} - \frac{5ab^4}{8x^8} - \frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^17,x]
```

```
[Out] -1/16*a^5/x^16 - (5*a^4*b)/(14*x^14) - (5*a^3*b^2)/(6*x^12) - (a^2*b^3)/x^1
0 - (5*a*b^4)/(8*x^8) - b^5/(6*x^6)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^5}{x^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^17,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^17, x]
```

fricas [**A**] time = 1.01, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{10} + 210\,ab^4x^8 + 336\,a^2b^3x^6 + 280\,a^3b^2x^4 + 120\,a^4bx^2 + 21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

giac [A] time = 0.98, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

maple [A] time = 0.01, size = 58, normalized size = 0.94

$$-\frac{b^5}{6x^6}-\frac{5a\,b^4}{8x^8}-\frac{a^2b^3}{x^{10}}-\frac{5a^3b^2}{6x^{12}}-\frac{5a^4b}{14x^{14}}-\frac{a^5}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^17,x)
```

```
[Out] -1/6*b^5/x^6-1/16*a^5/x^16-5/6*a^3*b^2/x^12-a^2*b^3/x^10-5/8*a*b^4/x^8-5/14
*a^4*b/x^14
```

maxima [A] time = 1.38, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^17,x, algorithm="maxima")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

mupad [B] time = 0.04, size = 58, normalized size = 0.94

$$-\frac{\frac{a^5}{16}+\frac{5\,a^4\,b\,x^2}{14}+\frac{5\,a^3\,b^2\,x^4}{6}+a^2\,b^3\,x^6+\frac{5\,a\,b^4\,x^8}{8}+\frac{b^5\,x^{10}}{6}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^17,x)
```

```
[Out] -(a^5/16 + (b^5*x^10)/6 + (5*a^4*b*x^2)/14 + (5*a*b^4*x^8)/8 + (5*a^3*b^2*x
^4)/6 + a^2*b^3*x^6)/x^16
```

sympy [A] time = 0.55, size = 63, normalized size = 1.02

$$\frac{-21a^5 - 120a^4bx^2 - 280a^3b^2x^4 - 336a^2b^3x^6 - 210ab^4x^8 - 56b^5x^{10}}{336x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**17,x)
```

```
[Out] (-21*a**5 - 120*a**4*b*x**2 - 280*a**3*b**2*x**4 - 336*a**2*b**3*x**6 - 210
*a*b**4*x**8 - 56*b**5*x**10)/(336*x**16)
```

3.68 $\int \frac{(a+bx^2)^5}{x^{19}} \, dx$

Optimal. Leaf size=69

$$-\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{5a^4b}{16x^{16}}-\frac{a^5}{18x^{18}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^19,x]
```

```
[Out] -a^5/(18*x^18) - (5*a^4*b)/(16*x^16) - (5*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/(
6*x^12) - (a*b^4)/(2*x^10) - b^5/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^{19}} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{(a+bx)^5}{x^{10}} \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right) dx, x, x^2\right)\\
&= -\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^19,x]
```

```
[Out] -1/18*a^5/x^18 - (5*a^4*b)/(16*x^16) - (5*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/(
6*x^12) - (a*b^4)/(2*x^10) - b^5/(8*x^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5}{x^{19}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^19,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^19, x]
```

fricas [A] time = 1.12, size = 59, normalized size = 0.86

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

giac [A] time = 0.94, size = 59, normalized size = 0.86

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

maple [A] time = 0.01, size = 58, normalized size = 0.84

$$-\frac{b^5}{8x^8}-\frac{a\,b^4}{2x^{10}}-\frac{5a^2b^3}{6x^{12}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^4b}{16x^{16}}-\frac{a^5}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^19,x)
```

```
[Out] -1/18*a^5/x^18-5/16*a^4*b/x^16-5/7*a^3*b^2/x^14-5/6*a^2*b^3/x^12-1/2*a*b^4/
x^10-1/8*b^5/x^8
```

maxima [A] time = 1.36, size = 59, normalized size = 0.86

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^19,x, algorithm="maxima")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

mupad [B] time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{18}+\frac{5\,a^4\,b\,x^2}{16}+\frac{5\,a^3\,b^2\,x^4}{7}+\frac{5\,a^2\,b^3\,x^6}{6}+\frac{a\,b^4\,x^8}{2}+\frac{b^5\,x^{10}}{8}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^19,x)
```

```
[Out] -(a^5/18 + (b^5*x^10)/8 + (5*a^4*b*x^2)/16 + (a*b^4*x^8)/2 + (5*a^3*b^2*x^4
)/7 + (5*a^2*b^3*x^6)/6)/x^18
```

sympy [A] time = 0.59, size = 63, normalized size = 0.91

$$\frac{-56a^5-315a^4bx^2-720a^3b^2x^4-840a^2b^3x^6-504ab^4x^8-126b^5x^{10}}{1008x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**19,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x**2 - 720*a**3*b**2*x**4 - 840*a**2*b**3*x**6 - 504
*a*b**4*x**8 - 126*b**5*x**10)/(1008*x**18)
```

3.69 $\int \frac{(a+bx^2)^5}{x^{21}} \, dx$

Optimal. Leaf size=69

$$-\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5a^4b}{18x^{18}}-\frac{a^5}{20x^{20}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^21,x]
```

```
[Out] -a^5/(20*x^20) - (5*a^4*b)/(18*x^18) - (5*a^3*b^2)/(8*x^16) - (5*a^2*b^3)/(
7*x^14) - (5*a*b^4)/(12*x^12) - b^5/(10*x^10)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^5}{x^{21}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^5}{x^{11}} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right) dx, x, x^2\right) \\
&= -\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^21,x]
```

```
[Out] -1/20*a^5/x^20 - (5*a^4*b)/(18*x^18) - (5*a^3*b^2)/(8*x^16) - (5*a^2*b^3)/(
7*x^14) - (5*a*b^4)/(12*x^12) - b^5/(10*x^10)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^5}{x^{21}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^21,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^21, x]
```

fricas [A] time = 0.67, size = 59, normalized size = 0.86

$$-\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

giac [A] time = 1.02, size = 59, normalized size = 0.86

$$-\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="giac")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$-\frac{b^{5}}{10x^{10}}-\frac{5a\,b^{4}}{12x^{12}}-\frac{5a^{2}b^{3}}{7x^{14}}-\frac{5a^{3}b^{2}}{8x^{16}}-\frac{5a^{4}b}{18x^{18}}-\frac{a^{5}}{20x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^21,x)
```

```
[Out] -1/20*a^5/x^20-5/18*a^4*b/x^18-5/8*a^3*b^2/x^16-5/7*a^2*b^3/x^14-5/12*a*b^4
/x^12-1/10*b^5/x^10
```

maxima [A] time = 1.35, size = 59, normalized size = 0.86

$$-\frac{252\,b^{5}x^{10}+1050\,ab^{4}x^{8}+1800\,a^{2}b^{3}x^{6}+1575\,a^{3}b^{2}x^{4}+700\,a^{4}bx^{2}+126\,a^{5}}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^21,x, algorithm="maxima")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

mupad [B] time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^{5}}{20}+\frac{5\,a^{4}\,b\,x^{2}}{18}+\frac{5\,a^{3}\,b^{2}\,x^{4}}{8}+\frac{5\,a^{2}\,b^{3}\,x^{6}}{7}+\frac{5\,a\,b^{4}\,x^{8}}{12}+\frac{b^{5}\,x^{10}}{10}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^21,x)
```

```
[Out] -(a^5/20 + (b^5*x^10)/10 + (5*a^4*b*x^2)/18 + (5*a*b^4*x^8)/12 + (5*a^3*b^2
*x^4)/8 + (5*a^2*b^3*x^6)/7)/x^20
```

sympy [A] time = 0.63, size = 63, normalized size = 0.91

$$\frac{-126a^{5}-700a^{4}bx^{2}-1575a^{3}b^{2}x^{4}-1800a^{2}b^{3}x^{6}-1050ab^{4}x^{8}-252b^{5}x^{10}}{2520x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**21,x)
```

```
[Out] (-126*a**5 - 700*a**4*b*x**2 - 1575*a**3*b**2*x**4 - 1800*a**2*b**3*x**6 -
1050*a*b**4*x**8 - 252*b**5*x**10)/(2520*x**20)
```

3.70 $\int x^8 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{2}{3}a^2b^3x^{15}+\frac{10}{13}a^3b^2x^{13}+\frac{5}{11}a^4bx^{11}+\frac{a^5x^9}{9}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^8 \left(a+bx^2\right)^5 \, dx &= \int \left(a^5x^8+5a^4bx^{10}+10a^3b^2x^{12}+10a^2b^3x^{14}+5ab^4x^{16}+b^5x^{18}\right) dx\\ &= \frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \left(a+bx^2\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^8*(a + b*x^2)^5, x]
```

fricas [A] time = 0.50, size = 57, normalized size = 0.83

$$\frac{1}{19}x^{19}b^5 + \frac{5}{17}x^{17}b^4a + \frac{2}{3}x^{15}b^3a^2 + \frac{10}{13}x^{13}b^2a^3 + \frac{5}{11}x^{11}ba^4 + \frac{1}{9}x^9a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^5 + 5/17*x^17*b^4*a + 2/3*x^15*b^3*a^2 + 10/13*x^13*b^2*a^3 + 5
/11*x^11*b*a^4 + 1/9*x^9*a^5
```

giac [A] time = 1.05, size = 57, normalized size = 0.83

$$\frac{1}{19}\,b^5x^{19} + \frac{5}{17}\,ab^4x^{17} + \frac{2}{3}\,a^2b^3x^{15} + \frac{10}{13}\,a^3b^2x^{13} + \frac{5}{11}\,a^4bx^{11} + \frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{19}b^5x^{19} + \frac{5}{17}a\,b^4x^{17} + \frac{2}{3}a^2b^3x^{15} + \frac{10}{13}a^3b^2x^{13} + \frac{5}{11}a^4b\,x^{11} + \frac{1}{9}a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^5,x)
```

```
[Out] 1/9*a^5*x^9+5/11*a^4*b*x^11+10/13*a^3*b^2*x^13+2/3*a^2*b^3*x^15+5/17*a*b^4*
x^17+1/19*b^5*x^19
```

maxima [A] time = 1.32, size = 57, normalized size = 0.83

$$\frac{1}{19}\,b^{5}x^{19}+\frac{5}{17}\,ab^{4}x^{17}+\frac{2}{3}\,a^{2}b^{3}x^{15}+\frac{10}{13}\,a^{3}b^{2}x^{13}+\frac{5}{11}\,a^{4}bx^{11}+\frac{1}{9}\,a^{5}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^9}{9}+\frac{5\,a^4\,b\,x^{11}}{11}+\frac{10\,a^3\,b^2\,x^{13}}{13}+\frac{2\,a^2\,b^3\,x^{15}}{3}+\frac{5\,a\,b^4\,x^{17}}{17}+\frac{b^5\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^9)/9 + (b^5*x^19)/19 + (5*a^4*b*x^11)/11 + (5*a*b^4*x^17)/17 + (10*a
^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
```

sympy [A] time = 0.08, size = 66, normalized size = 0.96

$$\frac{a^{5}x^{9}}{9}+\frac{5a^{4}bx^{11}}{11}+\frac{10a^{3}b^{2}x^{13}}{13}+\frac{2a^{2}b^{3}x^{15}}{3}+\frac{5ab^{4}x^{17}}{17}+\frac{b^{5}x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**9/9 + 5*a**4*b*x**11/11 + 10*a**3*b**2*x**13/13 + 2*a**2*b**3*x**15
/3 + 5*a*b**4*x**17/17 + b**5*x**19/19
```

3.71 $\int x^6 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{13}a^2b^3x^{13}+\frac{10}{11}a^3b^2x^{11}+\frac{5}{9}a^4bx^9+\frac{a^5x^7}{7}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/13
 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a+bx^2\right)^5 \, dx &= \int \left(a^5x^6+5a^4bx^8+10a^3b^2x^{10}+10a^2b^3x^{12}+5ab^4x^{14}+b^5x^{16}\right) dx\\ &= \frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/13
 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left(a + bx^2\right)^5 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^6*(a + b*x^2)^5, x]
```

fricas [A] time = 1.12, size = 57, normalized size = 0.83

$$\frac{1}{17}x^{17}b^5 + \frac{1}{3}x^{15}b^4a + \frac{10}{13}x^{13}b^3a^2 + \frac{10}{11}x^{11}b^2a^3 + \frac{5}{9}x^9ba^4 + \frac{1}{7}x^7a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^5 + 1/3*x^15*b^4*a + 10/13*x^13*b^3*a^2 + 10/11*x^11*b^2*a^3 +
5/9*x^9*b*a^4 + 1/7*x^7*a^5
```

giac [A] time = 0.98, size = 57, normalized size = 0.83

$$\frac{1}{17}\,b^5x^{17} + \frac{1}{3}\,ab^4x^{15} + \frac{10}{13}\,a^2b^3x^{13} + \frac{10}{11}\,a^3b^2x^{11} + \frac{5}{9}\,a^4bx^9 + \frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{17}b^5x^{17} + \frac{1}{3}a\,b^4x^{15} + \frac{10}{13}a^2b^3x^{13} + \frac{10}{11}a^3b^2x^{11} + \frac{5}{9}a^4b\,x^9 + \frac{1}{7}a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^5,x)
```

```
[Out] 1/7*a^5*x^7+5/9*a^4*b*x^9+10/11*a^3*b^2*x^11+10/13*a^2*b^3*x^13+1/3*a*b^4*x
^15+1/17*b^5*x^17
```

maxima [A] time = 1.31, size = 57, normalized size = 0.83

$$\frac{1}{17}\,b^5x^{17}+\frac{1}{3}\,ab^4x^{15}+\frac{10}{13}\,a^2b^3x^{13}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{9}\,a^4bx^9+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^7}{7}+\frac{5\,a^4\,b\,x^9}{9}+\frac{10\,a^3\,b^2\,x^{11}}{11}+\frac{10\,a^2\,b^3\,x^{13}}{13}+\frac{a\,b^4\,x^{15}}{3}+\frac{b^5\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^7)/7 + (b^5*x^17)/17 + (5*a^4*b*x^9)/9 + (a*b^4*x^15)/3 + (10*a^3*b^
2*x^11)/11 + (10*a^2*b^3*x^13)/13
```

sympy [A] time = 0.08, size = 65, normalized size = 0.94

$$\frac{a^5x^7}{7}+\frac{5a^4bx^9}{9}+\frac{10a^3b^2x^{11}}{11}+\frac{10a^2b^3x^{13}}{13}+\frac{ab^4x^{15}}{3}+\frac{b^5x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**7/7 + 5*a**4*b*x**9/9 + 10*a**3*b**2*x**11/11 + 10*a**2*b**3*x**13/
13 + a*b**4*x**15/3 + b**5*x**17/17
```

3.72 $\int x^4 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{11}a^2b^3x^{11}+\frac{10}{9}a^3b^2x^9+\frac{5}{7}a^4bx^7+\frac{a^5x^5}{5}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11 +
 (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^2\right)^5 \, dx &= \int \left(a^5x^4+5a^4bx^6+10a^3b^2x^8+10a^2b^3x^{10}+5ab^4x^{12}+b^5x^{14}\right) dx\\ &= \frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11 +
 (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + bx^2\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x^2)^5, x]
```

fricas [A] time = 0.91, size = 57, normalized size = 0.83

$$\frac{1}{15}x^{15}b^5 + \frac{5}{13}x^{13}b^4a + \frac{10}{11}x^{11}b^3a^2 + \frac{10}{9}x^9b^2a^3 + \frac{5}{7}x^7ba^4 + \frac{1}{5}x^5a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^5 + 5/13*x^13*b^4*a + 10/11*x^11*b^3*a^2 + 10/9*x^9*b^2*a^3 + 5
/7*x^7*b*a^4 + 1/5*x^5*a^5
```

giac [A] time = 1.01, size = 57, normalized size = 0.83

$$\frac{1}{15}\,b^5x^{15} + \frac{5}{13}\,ab^4x^{13} + \frac{10}{11}\,a^2b^3x^{11} + \frac{10}{9}\,a^3b^2x^9 + \frac{5}{7}\,a^4bx^7 + \frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{15}b^5x^{15} + \frac{5}{13}a\,b^4x^{13} + \frac{10}{11}a^2b^3x^{11} + \frac{10}{9}a^3b^2x^9 + \frac{5}{7}a^4b\,x^7 + \frac{1}{5}a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^5,x)
```

```
[Out] 1/5*a^5*x^5+5/7*a^4*b*x^7+10/9*a^3*b^2*x^9+10/11*a^2*b^3*x^11+5/13*a*b^4*x^
13+1/15*b^5*x^15
```

maxima [A] time = 1.31, size = 57, normalized size = 0.83

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^5}{5}+\frac{5\,a^4\,b\,x^7}{7}+\frac{10\,a^3\,b^2\,x^9}{9}+\frac{10\,a^2\,b^3\,x^{11}}{11}+\frac{5\,a\,b^4\,x^{13}}{13}+\frac{b^5\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^5)/5 + (b^5*x^15)/15 + (5*a^4*b*x^7)/7 + (5*a*b^4*x^13)/13 + (10*a^3
*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11
```

sympy [A] time = 0.08, size = 66, normalized size = 0.96

$$\frac{a^5x^5}{5}+\frac{5a^4bx^7}{7}+\frac{10a^3b^2x^9}{9}+\frac{10a^2b^3x^{11}}{11}+\frac{5ab^4x^{13}}{13}+\frac{b^5x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**5/5 + 5*a**4*b*x**7/7 + 10*a**3*b**2*x**9/9 + 10*a**2*b**3*x**11/11
 + 5*a*b**4*x**13/13 + b**5*x**15/15
```

3.73 $\int x^2 \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=66

$$\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{9}a^2b^3x^9+\frac{10}{7}a^3b^2x^7+a^4bx^5+\frac{a^5x^3}{3}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b^
4*x^11)/11 + (b^5*x^13)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right)^5 \, dx &= \int \left(a^5x^2+5a^4bx^4+10a^3b^2x^6+10a^2b^3x^8+5ab^4x^{10}+b^5x^{12}\right) dx\\ &= \frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.00

$$\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b^
4*x^11)/11 + (b^5*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^5, x]
```

fricas [A] time = 0.84, size = 56, normalized size = 0.85

$$\frac{1}{13} x^{13} b^5 + \frac{5}{11} x^{11} b^4 a + \frac{10}{9} x^9 b^3 a^2 + \frac{10}{7} x^7 b^2 a^3 + x^5 b a^4 + \frac{1}{3} x^3 a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^5 + 5/11*x^11*b^4*a + 10/9*x^9*b^3*a^2 + 10/7*x^7*b^2*a^3 + x^5
*b*a^4 + 1/3*x^3*a^5
```

giac [A] time = 1.09, size = 56, normalized size = 0.85

$$\frac{1}{13} \, b^5 x^{13} + \frac{5}{11} \, a b^4 x^{11} + \frac{10}{9} \, a^2 b^3 x^9 + \frac{10}{7} \, a^3 b^2 x^7 + a^4 b x^5 + \frac{1}{3} \, a^5 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

maple [A] time = 0.00, size = 57, normalized size = 0.86

$$\frac{1}{13} b^5 x^{13} + \frac{5}{11} a\, b^4 x^{11} + \frac{10}{9} a^2 b^3 x^9 + \frac{10}{7} a^3 b^2 x^7 + a^4 b\, x^5 + \frac{1}{3} a^5 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^5,x)
```

```
[Out] 1/3*a^5*x^3+a^4*b*x^5+10/7*a^3*b^2*x^7+10/9*a^2*b^3*x^9+5/11*a*b^4*x^11+1/1
3*b^5*x^13
```

maxima [A] time = 1.38, size = 56, normalized size = 0.85

$$\frac{1}{13}\,b^{5}x^{13}+\frac{5}{11}\,ab^{4}x^{11}+\frac{10}{9}\,a^{2}b^{3}x^{9}+\frac{10}{7}\,a^{3}b^{2}x^{7}+a^{4}bx^{5}+\frac{1}{3}\,a^{5}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4
*b*x^5 + 1/3*a^5*x^3
```

mupad [B] time = 0.02, size = 56, normalized size = 0.85

$$\frac{a^5\,x^3}{3}+a^4\,b\,x^5+\frac{10\,a^3\,b^2\,x^7}{7}+\frac{10\,a^2\,b^3\,x^9}{9}+\frac{5\,a\,b^4\,x^{11}}{11}+\frac{b^5\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x^3)/3 + (b^5*x^13)/13 + a^4*b*x^5 + (5*a*b^4*x^11)/11 + (10*a^3*b^2*x
^7)/7 + (10*a^2*b^3*x^9)/9
```

sympy [A] time = 0.08, size = 63, normalized size = 0.95

$$\frac{a^{5}x^{3}}{3}+a^{4}bx^{5}+\frac{10a^{3}b^{2}x^{7}}{7}+\frac{10a^{2}b^{3}x^{9}}{9}+\frac{5ab^{4}x^{11}}{11}+\frac{b^{5}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**5,x)
```

```
[Out] a**5*x**3/3 + a**4*b*x**5 + 10*a**3*b**2*x**7/7 + 10*a**2*b**3*x**9/9 + 5*a
*b**4*x**11/11 + b**5*x**13/13
```

3.74 $\int \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=62

$$a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{10}{7}a^2b^3x^7+2a^3b^2x^5+\frac{5}{3}a^4bx^3+a^5x+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9
)/9 + (b^5*x^11)/11
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^5 \, dx &= \int \left(a^5+5a^4bx^2+10a^3b^2x^4+10a^2b^3x^6+5ab^4x^8+b^5x^{10}\right) dx\\ &= a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 62, normalized size = 1.00

$$a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9
)/9 + (b^5*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5, x]
```

fricas [A] time = 0.81, size = 54, normalized size = 0.87

$$\frac{1}{11}x^{11}b^5 + \frac{5}{9}x^9b^4a + \frac{10}{7}x^7b^3a^2 + 2x^5b^2a^3 + \frac{5}{3}x^3ba^4 + xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^5 + 5/9*x^9*b^4*a + 10/7*x^7*b^3*a^2 + 2*x^5*b^2*a^3 + 5/3*x^3*
b*a^4 + x*a^5
```

giac [A] time = 1.14, size = 54, normalized size = 0.87

$$\frac{1}{11}\,b^5x^{11} + \frac{5}{9}\,ab^4x^9 + \frac{10}{7}\,a^2b^3x^7 + 2\,a^3b^2x^5 + \frac{5}{3}\,a^4bx^3 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

maple [A] time = 0.00, size = 55, normalized size = 0.89

$$\frac{1}{11}b^5x^{11} + \frac{5}{9}a\,b^4x^9 + \frac{10}{7}a^2b^3x^7 + 2a^3b^2x^5 + \frac{5}{3}a^4b\,x^3 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5,x)
```

```
[Out] a^5*x+5/3*a^4*b*x^3+2*a^3*b^2*x^5+10/7*a^2*b^3*x^7+5/9*a*b^4*x^9+1/11*b^5*x
^11
```

maxima [A] time = 1.38, size = 54, normalized size = 0.87

$$\frac{1}{11}\,b^5x^{11} + \frac{5}{9}\,ab^4x^9 + \frac{10}{7}\,a^2b^3x^7 + 2\,a^3b^2x^5 + \frac{5}{3}\,a^4bx^3 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

mupad [B] time = 0.02, size = 54, normalized size = 0.87

$$a^5\,x+\frac{5\,a^4\,b\,x^3}{3}+2\,a^3\,b^2\,x^5+\frac{10\,a^2\,b^3\,x^7}{7}+\frac{5\,a\,b^4\,x^9}{9}+\frac{b^5\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5,x)
```

```
[Out] a^5*x + (b^5*x^11)/11 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^9)/9 + 2*a^3*b^2*x^5 +
 (10*a^2*b^3*x^7)/7
```

sympy [A] time = 0.07, size = 61, normalized size = 0.98

$$a^{5}x+\frac{5a^{4}bx^{3}}{3}+2a^{3}b^{2}x^{5}+\frac{10a^{2}b^{3}x^{7}}{7}+\frac{5ab^{4}x^{9}}{9}+\frac{b^{5}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**3/3 + 2*a**3*b**2*x**5 + 10*a**2*b**3*x**7/7 + 5*a*b**
4*x**9/9 + b**5*x**11/11
```

3.75 $\int \frac{\left(a+bx^2\right)^5}{x^2} \, dx$

Optimal. Leaf size=61

$$-\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$2a^2b^3x^5+\frac{10}{3}a^3b^2x^3+5a^4bx-\frac{a^5}{x}+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^2,x]
```

```
[Out] -(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7
 + (b^5*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5}{x^2} \, dx &= \int \left(5a^4b+\frac{a^5}{x^2}+10a^3b^2x^2+10a^2b^3x^4+5ab^4x^6+b^5x^8\right) dx\\ &= -\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^2,x]
```

```
[Out] -(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7
 + (b^5*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^5}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^2, x]
```

fricas [A] time = 0.71, size = 59, normalized size = 0.97

$$\frac{7 b^5 x^{10} + 45 ab^4 x^8 + 126 a^2 b^3 x^6 + 210 a^3 b^2 x^4 + 315 a^4 bx^2 - 63 a^5}{63 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*a
^4*b*x^2 - 63*a^5)/x
```

giac [A] time = 1.13, size = 55, normalized size = 0.90

$$\frac{1}{9} b^5 x^9 + \frac{5}{7} ab^4 x^7 + 2 a^2 b^3 x^5 + \frac{10}{3} a^3 b^2 x^3 + 5 a^4 bx - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
- a^5/x
```

maple [A] time = 0.00, size = 56, normalized size = 0.92

$$\frac{b^5 x^9}{9} + \frac{5a\, b^4 x^7}{7} + 2a^2 b^3 x^5 + \frac{10a^3 b^2 x^3}{3} + 5a^4 bx - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^2,x)
```

```
[Out] -a^5/x+5*a^4*b*x+10/3*a^3*b^2*x^3+2*a^2*b^3*x^5+5/7*a*b^4*x^7+1/9*b^5*x^9
```

maxima [A] time = 1.39, size = 55, normalized size = 0.90

$$\frac{1}{9}\,b^5x^9 + \frac{5}{7}\,ab^4x^7 + 2\,a^2b^3x^5 + \frac{10}{3}\,a^3b^2x^3 + 5\,a^4bx - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
- a^5/x
```

mupad [B] time = 0.03, size = 55, normalized size = 0.90

$$\frac{b^5\,x^9}{9} - \frac{a^5}{x} + \frac{5\,a\,b^4\,x^7}{7} + \frac{10\,a^3\,b^2\,x^3}{3} + 2\,a^2\,b^3\,x^5 + 5\,a^4\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^2,x)
```

```
[Out] (b^5*x^9)/9 - a^5/x + (5*a*b^4*x^7)/7 + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5
+ 5*a^4*b*x
```

sympy [A] time = 0.13, size = 58, normalized size = 0.95

$$-\frac{a^5}{x} + 5a^4bx + \frac{10a^3b^2x^3}{3} + 2a^2b^3x^5 + \frac{5ab^4x^7}{7} + \frac{b^5x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**2,x)
```

```
[Out] -a**5/x + 5*a**4*b*x + 10*a**3*b**2*x**3/3 + 2*a**2*b**3*x**5 + 5*a*b**4*x*
*7/7 + b**5*x**9/9
```

3.76 $\int \frac{\left(a+bx^2\right)^5}{x^4} dx$

Optimal. Leaf size=60

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10}{3}a^2b^3x^3+ab^4x^5+\frac{b^5x^7}{7}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10}{3}a^2b^3x^3+10a^3b^2x-\frac{5a^4b}{x}-\frac{a^5}{3x^3}+ab^4x^5+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^4,x]
```

```
[Out] -a^5/(3*x^3) - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
+ (b^5*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5}{x^4}\,dx &= \int \left(10a^3b^2+\frac{a^5}{x^4}+\frac{5a^4b}{x^2}+10a^2b^3x^2+5ab^4x^4+b^5x^6\right)dx\\ &= -\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10}{3}a^2b^3x^3+ab^4x^5+\frac{b^5x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10}{3}a^2b^3x^3+ab^4x^5+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^4,x]
```

```
[Out] -1/3*a^5/x^3 - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
+ (b^5*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^4, x]
```

fricas [A] time = 1.08, size = 59, normalized size = 0.98

$$\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a^
4*b*x^2 - 7*a^5)/x^3
```

giac [A] time = 1.13, size = 55, normalized size = 0.92

$$\frac{1}{7}\,b^5x^7+ab^4x^5+\frac{10}{3}\,a^2b^3x^3+10\,a^3b^2x-\frac{15\,a^4bx^2+a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*x
^2 + a^5)/x^3
```

maple [A] time = 0.00, size = 55, normalized size = 0.92

$$\frac{b^5x^7}{7}+a\,b^4x^5+\frac{10a^2b^3x^3}{3}+10a^3b^2x-\frac{5a^4b}{x}-\frac{a^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^4,x)
```

[Out] -1/3*a^5/x^3-5*a^4*b/x+10*a^3*b^2*x+10/3*a^2*b^3*x^3+a*b^4*x^5+1/7*b^5*x^7

maxima [A] time = 1.36, size = 55, normalized size = 0.92

$$\frac{1}{7}\,b^5x^7 + ab^4x^5 + \frac{10}{3}\,a^2b^3x^3 + 10\,a^3b^2x - \frac{15\,a^4bx^2 + a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^5/x^4,x, algorithm="maxima")

[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*x^2 + a^5)/x^3

mupad [B] time = 0.03, size = 57, normalized size = 0.95

$$\frac{b^5\,x^7}{7} - \frac{\frac{a^5}{3} + 5\,b\,a^4\,x^2}{x^3} + 10\,a^3\,b^2\,x + a\,b^4\,x^5 + \frac{10\,a^2\,b^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^5/x^4,x)

[Out] (b^5*x^7)/7 - (a^5/3 + 5*a^4*b*x^2)/x^3 + 10*a^3*b^2*x + a*b^4*x^5 + (10*a^2*b^3*x^3)/3

sympy [A] time = 0.17, size = 60, normalized size = 1.00

$$10a^3b^2x + \frac{10a^2b^3x^3}{3} + ab^4x^5 + \frac{b^5x^7}{7} + \frac{-a^5 - 15a^4bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**5/x**4,x)

[Out] 10*a**3*b**2*x + 10*a**2*b**3*x**3/3 + a*b**4*x**5 + b**5*x**7/7 + (-a**5 - 15*a**4*b*x**2)/(3*x**3)

3.77 $\int \frac{(a+bx^2)^5}{x^6}\,dx$

Optimal. Leaf size=63

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{x}+10a^2b^3x-\frac{5a^4b}{3x^3}-\frac{a^5}{5x^5}+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^6,x]
```

```
[Out] -a^5/(5*x^5) - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4
*x^3)/3 + (b^5*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^6}\,dx &= \int \left(10a^2b^3+\frac{a^5}{x^6}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^2}+5ab^4x^2+b^5x^4\right)dx\\ &= -\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 63, normalized size = 1.00

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4
*x^3)/3 + (b^5*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^6, x]
```

fricas [A] time = 1.00, size = 59, normalized size = 0.94

$$\frac{3\,b^5x^{10}+25\,ab^4x^8+150\,a^2b^3x^6-150\,a^3b^2x^4-25\,a^4bx^2-3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a^
4*b*x^2 - 3*a^5)/x^5
```

giac [A] time = 0.94, size = 58, normalized size = 0.92

$$\frac{1}{5}\,b^5x^5+\frac{5}{3}\,ab^4x^3+10\,a^2b^3x-\frac{150\,a^3b^2x^4+25\,a^4bx^2+3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^4
*b*x^2 + 3*a^5)/x^5
```

maple [A] time = 0.00, size = 56, normalized size = 0.89

$$\frac{b^5x^5}{5}+\frac{5a\,b^4x^3}{3}+10a^2b^3x-\frac{10a^3b^2}{x}-\frac{5a^4b}{3x^3}-\frac{a^5}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^6,x)
```

[Out] -1/5*a^5/x^5-5/3*a^4*b/x^3-10*a^3*b^2/x+10*a^2*b^3*x+5/3*a*b^4*x^3+1/5*b^5*
x^5

maxima [A] time = 1.38, size = 58, normalized size = 0.92

$$\frac{1}{5}\,b^5x^5+\frac{5}{3}\,ab^4x^3+10\,a^2b^3x-\frac{150\,a^3b^2x^4+25\,a^4bx^2+3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^5/x^6,x, algorithm="maxima")

[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^4
*b*x^2 + 3*a^5)/x^5

mupad [B] time = 0.05, size = 58, normalized size = 0.92

$$\frac{b^5\,x^5}{5}-\frac{\frac{a^5}{5}+\frac{5\,a^4\,b\,x^2}{3}+10\,a^3\,b^2\,x^4}{x^5}+10\,a^2\,b^3\,x+\frac{5\,a\,b^4\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)^5/x^6,x)

[Out] (b^5*x^5)/5 - (a^5/5 + (5*a^4*b*x^2)/3 + 10*a^3*b^2*x^4)/x^5 + 10*a^2*b^3*x
 + (5*a*b^4*x^3)/3

sympy [A] time = 0.23, size = 63, normalized size = 1.00

$$10a^2b^3x+\frac{5ab^4x^3}{3}+\frac{b^5x^5}{5}+\frac{-3a^5-25a^4bx^2-150a^3b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**5/x**6,x)

[Out] 10*a**2*b**3*x + 5*a*b**4*x**3/3 + b**5*x**5/5 + (-3*a**5 - 25*a**4*b*x**2
- 150*a**3*b**2*x**4)/(15*x**5)

3.78 $\int \frac{(a+bx^2)^5}{x^8} dx$

Optimal. Leaf size=61

$$-\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}-\frac{a^4b}{x^5}-\frac{a^5}{7x^7}+5ab^4x+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^8,x]
```

```
[Out] -a^5/(7*x^7) - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^
4*x + (b^5*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^8}\,dx &= \int \left(5ab^4+\frac{a^5}{x^8}+\frac{5a^4b}{x^6}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^2}+b^5x^2\right)dx\\ &= -\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^8,x]
```

```
[Out] -1/7*a^5/x^7 - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^
4*x + (b^5*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^8, x]
```

fricas [A] time = 1.04, size = 59, normalized size = 0.97

$$\frac{7\,b^5x^{10}+105\,ab^4x^8-210\,a^2b^3x^6-70\,a^3b^2x^4-21\,a^4bx^2-3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a^
4*b*x^2 - 3*a^5)/x^7
```

giac [A] time = 1.12, size = 58, normalized size = 0.95

$$\frac{1}{3}\,b^5x^3+5\,ab^4x-\frac{210\,a^2b^3x^6+70\,a^3b^2x^4+21\,a^4bx^2+3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*b
*x^2 + 3*a^5)/x^7
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$\frac{b^5x^3}{3}+5a\,b^4x-\frac{10a^2b^3}{x}-\frac{10a^3b^2}{3x^3}-\frac{a^4b}{x^5}-\frac{a^5}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^8,x)
```

```
[Out] -1/7*a^5/x^7-a^4*b/x^5-10/3*a^3*b^2/x^3-10*a^2*b^3/x+5*a*b^4*x+1/3*b^5*x^3
```

maxima [A] time = 1.44, size = 58, normalized size = 0.95

$$\frac{1}{3}\,b^5x^3 + 5\,ab^4x - \frac{210\,a^2b^3x^6 + 70\,a^3b^2x^4 + 21\,a^4bx^2 + 3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*b
*x^2 + 3*a^5)/x^7
```

mupad [B] time = 4.79, size = 59, normalized size = 0.97

$$-\frac{3\,a^5 + 21\,a^4\,b\,x^2 + 70\,a^3\,b^2\,x^4 + 210\,a^2\,b^3\,x^6 - 105\,a\,b^4\,x^8 - 7\,b^5\,x^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^8,x)
```

```
[Out] -(3*a^5 - 7*b^5*x^10 + 21*a^4*b*x^2 - 105*a*b^4*x^8 + 70*a^3*b^2*x^4 + 210*
a^2*b^3*x^6)/(21*x^7)
```

sympy [A] time = 0.29, size = 61, normalized size = 1.00

$$5ab^4x + \frac{b^5x^3}{3} + \frac{-3a^5 - 21a^4bx^2 - 70a^3b^2x^4 - 210a^2b^3x^6}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**8,x)
```

```
[Out] 5*a*b**4*x + b**5*x**3/3 + (-3*a**5 - 21*a**4*b*x**2 - 70*a**3*b**2*x**4 -
210*a**2*b**3*x**6)/(21*x**7)
```

3.79 $\int \frac{(a+bx^2)^5}{x^{10}}\,dx$

Optimal. Leaf size=60

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5a^4b}{7x^7}-\frac{a^5}{9x^9}-\frac{5ab^4}{x}+b^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) -
 (5*a*b^4)/x + b^5*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{10}}\,dx &= \int \left(b^5+\frac{a^5}{x^{10}}+\frac{5a^4b}{x^8}+\frac{10a^3b^2}{x^6}+\frac{10a^2b^3}{x^4}+\frac{5ab^4}{x^2}\right)dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) -
 (5*a*b^4)/x + b^5*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^10, x]
```

fricas [A] time = 1.06, size = 59, normalized size = 0.98

$$\frac{63\,b^5x^{10}-315\,ab^4x^8-210\,a^2b^3x^6-126\,a^3b^2x^4-45\,a^4bx^2-7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45*
a^4*b*x^2 - 7*a^5)/x^9
```

giac [A] time = 1.04, size = 57, normalized size = 0.95

$$b^5x-\frac{315\,ab^4x^8+210\,a^2b^3x^6+126\,a^3b^2x^4+45\,a^4bx^2+7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="giac")
```

```
[Out] b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b*
x^2 + 7*a^5)/x^9
```

maple [A] time = 0.01, size = 55, normalized size = 0.92

$$b^5x-\frac{5a\,b^4}{x}-\frac{10a^2b^3}{3x^3}-\frac{2a^3b^2}{x^5}-\frac{5a^4b}{7x^7}-\frac{a^5}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^10,x)
```

```
[Out] -1/9*a^5/x^9-5/7*a^4*b/x^7-2*a^3*b^2/x^5-10/3*a^2*b^3/x^3-5*a*b^4/x+b^5*x
```

maxima [A] time = 1.44, size = 57, normalized size = 0.95

$$b^5x - \frac{315\,ab^4x^8 + 210\,a^2b^3x^6 + 126\,a^3b^2x^4 + 45\,a^4bx^2 + 7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b*
x^2 + 7*a^5)/x^9
```

mupad [B] time = 0.04, size = 57, normalized size = 0.95

$$b^5\,x - \frac{\frac{a^5}{9} + \frac{5\,a^4\,b\,x^2}{7} + 2\,a^3\,b^2\,x^4 + \frac{10\,a^2\,b^3\,x^6}{3} + 5\,a\,b^4\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^10,x)
```

```
[Out] b^5*x - (a^5/9 + (5*a^4*b*x^2)/7 + 5*a*b^4*x^8 + 2*a^3*b^2*x^4 + (10*a^2*b^
3*x^6)/3)/x^9
```

sympy [A] time = 0.36, size = 60, normalized size = 1.00

$$b^5x + \frac{-7a^5 - 45a^4bx^2 - 126a^3b^2x^4 - 210a^2b^3x^6 - 315ab^4x^8}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**10,x)
```

```
[Out] b**5*x + (-7*a**5 - 45*a**4*b*x**2 - 126*a**3*b**2*x**4 - 210*a**2*b**3*x**
6 - 315*a*b**4*x**8)/(63*x**9)
```

3.80 $\int \frac{(a+bx^2)^5}{x^{12}}\,dx$

Optimal. Leaf size=65

$$-\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5a^4b}{9x^9}-\frac{a^5}{11x^{11}}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^12,x]
```

```
[Out] -a^5/(11*x^11) - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5
 - (5*a*b^4)/(3*x^3) - b^5/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{12}}\,dx &= \int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^6}+\frac{5ab^4}{x^4}+\frac{b^5}{x^2}\right)dx\\ &= -\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$-\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^12,x]
```

```
[Out] -1/11*a^5/x^11 - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5
 - (5*a*b^4)/(3*x^3) - b^5/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^12, x]
```

fricas [A] time = 0.73, size = 59, normalized size = 0.91

$$-\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

giac [A] time = 1.04, size = 59, normalized size = 0.91

$$-\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="giac")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

maple [A] time = 0.01, size = 58, normalized size = 0.89

$$-\frac{b^5}{x}-\frac{5a\,b^4}{3x^3}-\frac{2a^2b^3}{x^5}-\frac{10a^3b^2}{7x^7}-\frac{5a^4b}{9x^9}-\frac{a^5}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^12,x)
```

```
[Out] -1/11*a^5/x^11-5/9*a^4*b/x^9-10/7*a^3*b^2/x^7-2*a^2*b^3/x^5-5/3*a*b^4/x^3-b
^5/x
```

maxima [A] time = 1.36, size = 59, normalized size = 0.91

$$-\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^12,x, algorithm="maxima")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

mupad [B] time = 0.04, size = 58, normalized size = 0.89

$$-\frac{\frac{a^5}{11}+\frac{5\,a^4\,b\,x^2}{9}+\frac{10\,a^3\,b^2\,x^4}{7}+2\,a^2\,b^3\,x^6+\frac{5\,a\,b^4\,x^8}{3}+b^5\,x^{10}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^12,x)
```

```
[Out] -(a^5/11 + b^5*x^10 + (5*a^4*b*x^2)/9 + (5*a*b^4*x^8)/3 + (10*a^3*b^2*x^4)/
7 + 2*a^2*b^3*x^6)/x^11
```

sympy [A] time = 0.43, size = 63, normalized size = 0.97

$$\frac{-63a^5-385a^4bx^2-990a^3b^2x^4-1386a^2b^3x^6-1155ab^4x^8-693b^5x^{10}}{693x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**12,x)
```

```
[Out] (-63*a**5 - 385*a**4*b*x**2 - 990*a**3*b**2*x**4 - 1386*a**2*b**3*x**6 - 11
55*a*b**4*x**8 - 693*b**5*x**10)/(693*x**11)
```

3.81 $\int \frac{(a+bx^2)^5}{x^{14}}\,dx$

Optimal. Leaf size=67

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{13x^{13}}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^14,x]
```

```
[Out] -a^5/(13*x^13) - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/
(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{14}}\,dx &= \int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^6}+\frac{b^5}{x^4}\right)dx\\ &= -\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^14,x]
```

```
[Out] -1/13*a^5/x^13 - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/
(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^5}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^14,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^14, x]
```

fricas [A]   time = 0.87, size = 59, normalized size = 0.88

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

giac [A]   time = 1.09, size = 59, normalized size = 0.88

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

maple [A]   time = 0.01, size = 58, normalized size = 0.87

$$-\frac{b^5}{3x^3}-\frac{a\,b^4}{x^5}-\frac{10a^2b^3}{7x^7}-\frac{10a^3b^2}{9x^9}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^14,x)
```

```
[Out] -1/13*a^5/x^13-5/11*a^4*b/x^11-10/9*a^3*b^2/x^9-10/7*a^2*b^3/x^7-a*b^4/x^5-
1/3*b^5/x^3
```

maxima [A] time = 1.36, size = 59, normalized size = 0.88

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^14,x, algorithm="maxima")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

mupad [B] time = 0.04, size = 58, normalized size = 0.87

$$-\frac{\frac{a^5}{13}+\frac{5\,a^4\,b\,x^2}{11}+\frac{10\,a^3\,b^2\,x^4}{9}+\frac{10\,a^2\,b^3\,x^6}{7}+a\,b^4\,x^8+\frac{b^5\,x^{10}}{3}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^14,x)
```

```
[Out] -(a^5/13 + (b^5*x^10)/3 + (5*a^4*b*x^2)/11 + a*b^4*x^8 + (10*a^3*b^2*x^4)/9
 + (10*a^2*b^3*x^6)/7)/x^13
```

sympy [A] time = 0.47, size = 63, normalized size = 0.94

$$\frac{-693a^5-4095a^4bx^2-10010a^3b^2x^4-12870a^2b^3x^6-9009ab^4x^8-3003b^5x^{10}}{9009x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**14,x)
```

```
[Out] (-693*a**5 - 4095*a**4*b*x**2 - 10010*a**3*b**2*x**4 - 12870*a**2*b**3*x**6
 - 9009*a*b**4*x**8 - 3003*b**5*x**10)/(9009*x**13)
```

3.82 $\int \frac{(a+bx^2)^5}{x^{16}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5a^4b}{13x^{13}}-\frac{a^5}{15x^{15}}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^16,x]
```

```
[Out] -a^5/(15*x^15) - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^3
)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{16}}\,dx &= \int \left(\frac{a^5}{x^{16}}+\frac{5a^4b}{x^{14}}+\frac{10a^3b^2}{x^{12}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^8}+\frac{b^5}{x^6}\right)dx\\ &= -\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^16,x]
```

```
[Out] -1/15*a^5/x^15 - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^3
)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^5}{x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^16,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^16, x]
```

fricas [A] time = 0.66, size = 59, normalized size = 0.86

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

giac [A] time = 1.08, size = 59, normalized size = 0.86

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

maple [A] time = 0.01, size = 58, normalized size = 0.84

$$-\frac{b^5}{5x^5}-\frac{5a\,b^4}{7x^7}-\frac{10a^2b^3}{9x^9}-\frac{10a^3b^2}{11x^{11}}-\frac{5a^4b}{13x^{13}}-\frac{a^5}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^16,x)
```

```
[Out] -1/15*a^5/x^15-5/13*a^4*b/x^13-10/11*a^3*b^2/x^11-10/9*a^2*b^3/x^9-5/7*a*b^
4/x^7-1/5*b^5/x^5
```

maxima [A] time = 1.37, size = 59, normalized size = 0.86

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

mupad [B] time = 4.75, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{15}+\frac{5\,a^4\,b\,x^2}{13}+\frac{10\,a^3\,b^2\,x^4}{11}+\frac{10\,a^2\,b^3\,x^6}{9}+\frac{5\,a\,b^4\,x^8}{7}+\frac{b^5\,x^{10}}{5}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^16,x)
```

```
[Out] -(a^5/15 + (b^5*x^10)/5 + (5*a^4*b*x^2)/13 + (5*a*b^4*x^8)/7 + (10*a^3*b^2*
x^4)/11 + (10*a^2*b^3*x^6)/9)/x^15
```

sympy [A] time = 0.51, size = 63, normalized size = 0.91

$$\frac{-3003a^5-17325a^4bx^2-40950a^3b^2x^4-50050a^2b^3x^6-32175ab^4x^8-9009b^5x^{10}}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**16,x)
```

```
[Out] (-3003*a**5 - 17325*a**4*b*x**2 - 40950*a**3*b**2*x**4 - 50050*a**2*b**3*x*
*6 - 32175*a*b**4*x**8 - 9009*b**5*x**10)/(45045*x**15)
```

3.83 $\int \frac{(a+bx^2)^5}{x^{18}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{a^4b}{3x^{15}}-\frac{a^5}{17x^{17}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^18,x]
```

```
[Out] -a^5/(17*x^17) - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/(
11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{18}}\,dx &= \int \left(\frac{a^5}{x^{18}}+\frac{5a^4b}{x^{16}}+\frac{10a^3b^2}{x^{14}}+\frac{10a^2b^3}{x^{12}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^8}\right)dx\\ &= -\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^18,x]
```

```
[Out] -1/17*a^5/x^17 - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/(
11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^5}{x^{18}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^18,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^18, x]
```

fricas [A] time = 1.10, size = 59, normalized size = 0.86

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="fricas")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

giac [A] time = 1.10, size = 59, normalized size = 0.86

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="giac")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$-\frac{b^5}{7x^7}-\frac{5a\,b^4}{9x^9}-\frac{10a^2b^3}{11x^{11}}-\frac{10a^3b^2}{13x^{13}}-\frac{a^4b}{3x^{15}}-\frac{a^5}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^18,x)
```

```
[Out] -1/17*a^5/x^17-1/3*a^4*b/x^15-10/13*a^3*b^2/x^13-10/11*a^2*b^3/x^11-5/9*a*b
^4/x^9-1/7*b^5/x^7
```

maxima [A] time = 1.32, size = 59, normalized size = 0.86

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^18,x, algorithm="maxima")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a
^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

mupad [B] time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{17}+\frac{a^4\,b\,x^2}{3}+\frac{10\,a^3\,b^2\,x^4}{13}+\frac{10\,a^2\,b^3\,x^6}{11}+\frac{5\,a\,b^4\,x^8}{9}+\frac{b^5\,x^{10}}{7}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^18,x)
```

```
[Out] -(a^5/17 + (b^5*x^10)/7 + (a^4*b*x^2)/3 + (5*a*b^4*x^8)/9 + (10*a^3*b^2*x^4
)/13 + (10*a^2*b^3*x^6)/11)/x^17
```

sympy [A] time = 0.54, size = 63, normalized size = 0.91

$$\frac{-9009a^5-51051a^4bx^2-117810a^3b^2x^4-139230a^2b^3x^6-85085ab^4x^8-21879b^5x^{10}}{153153x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**18,x)
```

```
[Out] (-9009*a**5 - 51051*a**4*b*x**2 - 117810*a**3*b**2*x**4 - 139230*a**2*b**3*
x**6 - 85085*a*b**4*x**8 - 21879*b**5*x**10)/(153153*x**17)
```

3.84 $\int \frac{(a+bx^2)^5}{x^{20}} dx$

Optimal. Leaf size=69

$$-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5a^4b}{17x^{17}}-\frac{a^5}{19x^{19}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5/x^20,x]
```

```
[Out] -a^5/(19*x^19) - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)/
(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5}{x^{20}} dx &= \int \left(\frac{a^5}{x^{20}}+\frac{5a^4b}{x^{18}}+\frac{10a^3b^2}{x^{16}}+\frac{10a^2b^3}{x^{14}}+\frac{5ab^4}{x^{12}}+\frac{b^5}{x^{10}}\right) dx\\ &= -\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 69, normalized size = 1.00

$$-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5/x^20,x]
```

```
[Out] -1/19*a^5/x^19 - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)/
(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5}{x^{20}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5/x^20,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5/x^20, x]
```

fricas [A] time = 1.08, size = 59, normalized size = 0.86

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="fricas")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

giac [A] time = 1.13, size = 59, normalized size = 0.86

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="giac")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

maple [A] time = 0.01, size = 58, normalized size = 0.84

$$-\frac{b^5}{9x^9}-\frac{5a\,b^4}{11x^{11}}-\frac{10a^2b^3}{13x^{13}}-\frac{2a^3b^2}{3x^{15}}-\frac{5a^4b}{17x^{17}}-\frac{a^5}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5/x^20,x)
```

```
[Out] -1/19*a^5/x^19-5/17*a^4*b/x^17-2/3*a^3*b^2/x^15-10/13*a^2*b^3/x^13-5/11*a*b
^4/x^11-1/9*b^5/x^9
```

maxima [A] time = 1.39, size = 59, normalized size = 0.86

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5/x^20,x, algorithm="maxima")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

mupad [B] time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{19}+\frac{5\,a^4\,b\,x^2}{17}+\frac{2\,a^3\,b^2\,x^4}{3}+\frac{10\,a^2\,b^3\,x^6}{13}+\frac{5\,a\,b^4\,x^8}{11}+\frac{b^5\,x^{10}}{9}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^5/x^20,x)
```

```
[Out] -(a^5/19 + (b^5*x^10)/9 + (5*a^4*b*x^2)/17 + (5*a*b^4*x^8)/11 + (2*a^3*b^2*
x^4)/3 + (10*a^2*b^3*x^6)/13)/x^19
```

sympy [A] time = 0.57, size = 63, normalized size = 0.91

$$\frac{-21879a^5-122265a^4bx^2-277134a^3b^2x^4-319770a^2b^3x^6-188955ab^4x^8-46189b^5x^{10}}{415701x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5/x**20,x)
```

```
[Out] (-21879*a**5 - 122265*a**4*b*x**2 - 277134*a**3*b**2*x**4 - 319770*a**2*b**
3*x**6 - 188955*a*b**4*x**8 - 46189*b**5*x**10)/(415701*x**19)
```

3.85 $\int x^{13} \left(a + bx^2\right)^8 \, dx$

Optimal. Leaf size=129

$$\frac{a^6 \left(a + bx^2\right)^9}{18b^7} - \frac{3a^5 \left(a + bx^2\right)^{10}}{10b^7} + \frac{15a^4 \left(a + bx^2\right)^{11}}{22b^7} - \frac{5a^3 \left(a + bx^2\right)^{12}}{6b^7} + \frac{15a^2 \left(a + bx^2\right)^{13}}{26b^7} + \frac{\left(a + bx^2\right)^{15}}{30b^7} - \frac{3a \left(a + bx^2\right)^{1}}{14b^7}$$

Rubi [A] time = 0.21, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{15a^2 \left(a + bx^2\right)^{13}}{26b^7} - \frac{5a^3 \left(a + bx^2\right)^{12}}{6b^7} + \frac{15a^4 \left(a + bx^2\right)^{11}}{22b^7} - \frac{3a^5 \left(a + bx^2\right)^{10}}{10b^7} + \frac{a^6 \left(a + bx^2\right)^9}{18b^7} + \frac{\left(a + bx^2\right)^{15}}{30b^7} - \frac{3a \left(a + bx^2\right)^{14}}{14b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^8,x]
```

```
[Out] (a^6*(a + b*x^2)^9)/(18*b^7) - (3*a^5*(a + b*x^2)^10)/(10*b^7) + (15*a^4*(a
 + b*x^2)^11)/(22*b^7) - (5*a^3*(a + b*x^2)^12)/(6*b^7) + (15*a^2*(a + b*x^
2)^13)/(26*b^7) - (3*a*(a + b*x^2)^14)/(14*b^7) + (a + b*x^2)^15/(30*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{13} \left(a + bx^2\right)^8 \, dx &= \frac{1}{2}\text{Subst}\left(\int x^6 (a + bx)^8 \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^6 (a + bx)^8}{b^6} - \frac{6a^5 (a + bx)^9}{b^6} + \frac{15a^4 (a + bx)^{10}}{b^6} - \frac{20a^3 (a + bx)^{11}}{b^6} + \frac{15a^2 (a + b}{b^6}\right.\right. \\
&= \frac{a^6 \left(a + bx^2\right)^9}{18b^7} - \frac{3a^5 \left(a + bx^2\right)^{10}}{10b^7} + \frac{15a^4 \left(a + bx^2\right)^{11}}{22b^7} - \frac{5a^3 \left(a + bx^2\right)^{12}}{6b^7} + \frac{15a^2 \left(a + bx^2\right)^{13}}{26b^7}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 0.84

$$\frac{a^8x^{14}}{14}+\frac{1}{2}a^7bx^{16}+\frac{14}{9}a^6b^2x^{18}+\frac{14}{5}a^5b^3x^{20}+\frac{35}{11}a^4b^4x^{22}+\frac{7}{3}a^3b^5x^{24}+\frac{14}{13}a^2b^6x^{26}+\frac{2}{7}ab^7x^{28}+\frac{b^8x^{30}}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^14)/14 + (a^7*b*x^16)/2 + (14*a^6*b^2*x^18)/9 + (14*a^5*b^3*x^20)/5
+ (35*a^4*b^4*x^22)/11 + (7*a^3*b^5*x^24)/3 + (14*a^2*b^6*x^26)/13 + (2*a*b
^7*x^28)/7 + (b^8*x^30)/30
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13}\left(a+bx^2\right)^8\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^13*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x^13*(a + b*x^2)^8, x]
```

fricas [A] time = 0.71, size = 90, normalized size = 0.70

$$\frac{1}{30}x^{30}b^8+\frac{2}{7}x^{28}b^7a+\frac{14}{13}x^{26}b^6a^2+\frac{7}{3}x^{24}b^5a^3+\frac{35}{11}x^{22}b^4a^4+\frac{14}{5}x^{20}b^3a^5+\frac{14}{9}x^{18}b^2a^6+\frac{1}{2}x^{16}ba^7+\frac{1}{14}x^{14}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/30*x^30*b^8 + 2/7*x^28*b^7*a + 14/13*x^26*b^6*a^2 + 7/3*x^24*b^5*a^3 + 35
/11*x^22*b^4*a^4 + 14/5*x^20*b^3*a^5 + 14/9*x^18*b^2*a^6 + 1/2*x^16*b*a^7 +
 1/14*x^14*a^8
```

giac [A] time = 1.06, size = 90, normalized size = 0.70

$$\frac{1}{30}\,b^8x^{30}+\frac{2}{7}\,ab^7x^{28}+\frac{14}{13}\,a^2b^6x^{26}+\frac{7}{3}\,a^3b^5x^{24}+\frac{35}{11}\,a^4b^4x^{22}+\frac{14}{5}\,a^5b^3x^{20}+\frac{14}{9}\,a^6b^2x^{18}+\frac{1}{2}\,a^7bx^{16}+\frac{1}{14}\,a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 35
/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16 +
 1/14*a^8*x^14
```

maple [A] time = 0.00, size = 91, normalized size = 0.71

$$\frac{1}{30}b^8x^{30}+\frac{2}{7}a\,b^7x^{28}+\frac{14}{13}a^2b^6x^{26}+\frac{7}{3}a^3b^5x^{24}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{5}a^5b^3x^{20}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{2}a^7b\,x^{16}+\frac{1}{14}a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^8,x)
```

```
[Out] 1/30*b^8*x^30+2/7*a*b^7*x^28+14/13*a^2*b^6*x^26+7/3*a^3*b^5*x^24+35/11*a^4*
b^4*x^22+14/5*a^5*b^3*x^20+14/9*a^6*b^2*x^18+1/2*a^7*b*x^16+1/14*a^8*x^14
```

maxima [A] time = 1.30, size = 90, normalized size = 0.70

$$\frac{1}{30}\,b^8x^{30}+\frac{2}{7}\,ab^7x^{28}+\frac{14}{13}\,a^2b^6x^{26}+\frac{7}{3}\,a^3b^5x^{24}+\frac{35}{11}\,a^4b^4x^{22}+\frac{14}{5}\,a^5b^3x^{20}+\frac{14}{9}\,a^6b^2x^{18}+\frac{1}{2}\,a^7bx^{16}+\frac{1}{14}\,a^8x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 35
/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16 +
 1/14*a^8*x^14
```

mupad [B] time = 0.10, size = 90, normalized size = 0.70

$$\frac{a^8\,x^{14}}{14}+\frac{a^7\,b\,x^{16}}{2}+\frac{14\,a^6\,b^2\,x^{18}}{9}+\frac{14\,a^5\,b^3\,x^{20}}{5}+\frac{35\,a^4\,b^4\,x^{22}}{11}+\frac{7\,a^3\,b^5\,x^{24}}{3}+\frac{14\,a^2\,b^6\,x^{26}}{13}+\frac{2\,a\,b^7\,x^{28}}{7}+\frac{b^8\,x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^14)/14 + (b^8*x^30)/30 + (a^7*b*x^16)/2 + (2*a*b^7*x^28)/7 + (14*a^6
*b^2*x^18)/9 + (14*a^5*b^3*x^20)/5 + (35*a^4*b^4*x^22)/11 + (7*a^3*b^5*x^24
)/3 + (14*a^2*b^6*x^26)/13
```

sympy [A] time = 0.09, size = 105, normalized size = 0.81

$$\frac{a^8x^{14}}{14}+\frac{a^7bx^{16}}{2}+\frac{14a^6b^2x^{18}}{9}+\frac{14a^5b^3x^{20}}{5}+\frac{35a^4b^4x^{22}}{11}+\frac{7a^3b^5x^{24}}{3}+\frac{14a^2b^6x^{26}}{13}+\frac{2ab^7x^{28}}{7}+\frac{b^8x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**14/14 + a**7*b*x**16/2 + 14*a**6*b**2*x**18/9 + 14*a**5*b**3*x**20/
5 + 35*a**4*b**4*x**22/11 + 7*a**3*b**5*x**24/3 + 14*a**2*b**6*x**26/13 + 2
*a*b**7*x**28/7 + b**8*x**30/30
```

3.86 $\int x^{11} \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=110

$$-\frac{a^5\left(a+bx^2\right)^9}{18b^6}+\frac{a^4\left(a+bx^2\right)^{10}}{4b^6}-\frac{5a^3\left(a+bx^2\right)^{11}}{11b^6}+\frac{5a^2\left(a+bx^2\right)^{12}}{12b^6}+\frac{\left(a+bx^2\right)^{14}}{28b^6}-\frac{5a\left(a+bx^2\right)^{13}}{26b^6}$$

Rubi [A] time = 0.17, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{5a^2\left(a+bx^2\right)^{12}}{12b^6}-\frac{5a^3\left(a+bx^2\right)^{11}}{11b^6}+\frac{a^4\left(a+bx^2\right)^{10}}{4b^6}-\frac{a^5\left(a+bx^2\right)^9}{18b^6}+\frac{\left(a+bx^2\right)^{14}}{28b^6}-\frac{5a\left(a+bx^2\right)^{13}}{26b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^8,x]
```

```
[Out] -(a^5*(a + b*x^2)^9)/(18*b^6) + (a^4*(a + b*x^2)^10)/(4*b^6) - (5*a^3*(a +
b*x^2)^11)/(11*b^6) + (5*a^2*(a + b*x^2)^12)/(12*b^6) - (5*a*(a + b*x^2)^13
)/(26*b^6) + (a + b*x^2)^14/(28*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^2\right)^8\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^5(a+bx)^8\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^5(a+bx)^8}{b^5}+\frac{5a^4(a+bx)^9}{b^5}-\frac{10a^3(a+bx)^{10}}{b^5}+\frac{10a^2(a+bx)^{11}}{b^5}-\frac{5a(a+\right.\right.\\ &= -\frac{a^5\left(a+bx^2\right)^9}{18b^6}+\frac{a^4\left(a+bx^2\right)^{10}}{4b^6}-\frac{5a^3\left(a+bx^2\right)^{11}}{11b^6}+\frac{5a^2\left(a+bx^2\right)^{12}}{12b^6}-\frac{5a\left(a+bx^2\right)^{13}}{26b^6}+\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 0.98

$$\frac{a^8x^{12}}{12}+\frac{4}{7}a^7bx^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{28}{9}a^5b^3x^{18}+\frac{7}{2}a^4b^4x^{20}+\frac{28}{11}a^3b^5x^{22}+\frac{7}{6}a^2b^6x^{24}+\frac{4}{13}ab^7x^{26}+\frac{b^8x^{28}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^12)/12 + (4*a^7*b*x^14)/7 + (7*a^6*b^2*x^16)/4 + (28*a^5*b^3*x^18)/9
 + (7*a^4*b^4*x^20)/2 + (28*a^3*b^5*x^22)/11 + (7*a^2*b^6*x^24)/6 + (4*a*b^
7*x^26)/13 + (b^8*x^28)/28
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\left(a+bx^2\right)^8\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x^11*(a + b*x^2)^8, x]
```

fricas [A] time = 1.08, size = 90, normalized size = 0.82

$$\frac{1}{28}x^{28}b^8+\frac{4}{13}x^{26}b^7a+\frac{7}{6}x^{24}b^6a^2+\frac{28}{11}x^{22}b^5a^3+\frac{7}{2}x^{20}b^4a^4+\frac{28}{9}x^{18}b^3a^5+\frac{7}{4}x^{16}b^2a^6+\frac{4}{7}x^{14}ba^7+\frac{1}{12}x^{12}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/28*x^28*b^8 + 4/13*x^26*b^7*a + 7/6*x^24*b^6*a^2 + 28/11*x^22*b^5*a^3 + 7
/2*x^20*b^4*a^4 + 28/9*x^18*b^3*a^5 + 7/4*x^16*b^2*a^6 + 4/7*x^14*b*a^7 + 1
/12*x^12*a^8
```

giac [A] time = 1.00, size = 90, normalized size = 0.82

$$\frac{1}{28}\,b^8x^{28}+\frac{4}{13}\,ab^7x^{26}+\frac{7}{6}\,a^2b^6x^{24}+\frac{28}{11}\,a^3b^5x^{22}+\frac{7}{2}\,a^4b^4x^{20}+\frac{28}{9}\,a^5b^3x^{18}+\frac{7}{4}\,a^6b^2x^{16}+\frac{4}{7}\,a^7bx^{14}+\frac{1}{12}\,a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 + 7
/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 + 1
/12*a^8*x^12
```

maple [A] time = 0.00, size = 91, normalized size = 0.83

$$\frac{1}{28}b^{8}x^{28}+\frac{4}{13}a\,b^{7}x^{26}+\frac{7}{6}a^{2}b^{6}x^{24}+\frac{28}{11}a^{3}b^{5}x^{22}+\frac{7}{2}a^{4}b^{4}x^{20}+\frac{28}{9}a^{5}b^{3}x^{18}+\frac{7}{4}a^{6}b^{2}x^{16}+\frac{4}{7}a^{7}b\,x^{14}+\frac{1}{12}a^{8}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^8,x)
```

```
[Out] 1/28*b^8*x^28+4/13*a*b^7*x^26+7/6*a^2*b^6*x^24+28/11*a^3*b^5*x^22+7/2*a^4*b
^4*x^20+28/9*a^5*b^3*x^18+7/4*a^6*b^2*x^16+4/7*a^7*b*x^14+1/12*a^8*x^12
```

maxima [A] time = 1.35, size = 90, normalized size = 0.82

$$\frac{1}{28}\,b^{8}x^{28}+\frac{4}{13}\,ab^{7}x^{26}+\frac{7}{6}\,a^{2}b^{6}x^{24}+\frac{28}{11}\,a^{3}b^{5}x^{22}+\frac{7}{2}\,a^{4}b^{4}x^{20}+\frac{28}{9}\,a^{5}b^{3}x^{18}+\frac{7}{4}\,a^{6}b^{2}x^{16}+\frac{4}{7}\,a^{7}bx^{14}+\frac{1}{12}\,a^{8}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 + 7
/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 + 1
/12*a^8*x^12
```

mupad [B] time = 4.57, size = 90, normalized size = 0.82

$$\frac{a^8\,x^{12}}{12}+\frac{4\,a^7\,b\,x^{14}}{7}+\frac{7\,a^6\,b^2\,x^{16}}{4}+\frac{28\,a^5\,b^3\,x^{18}}{9}+\frac{7\,a^4\,b^4\,x^{20}}{2}+\frac{28\,a^3\,b^5\,x^{22}}{11}+\frac{7\,a^2\,b^6\,x^{24}}{6}+\frac{4\,a\,b^7\,x^{26}}{13}+\frac{b^8\,x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^12)/12 + (b^8*x^28)/28 + (4*a^7*b*x^14)/7 + (4*a*b^7*x^26)/13 + (7*a
^6*b^2*x^16)/4 + (28*a^5*b^3*x^18)/9 + (7*a^4*b^4*x^20)/2 + (28*a^3*b^5*x^2
2)/11 + (7*a^2*b^6*x^24)/6
```

sympy [A] time = 0.09, size = 107, normalized size = 0.97

$$\frac{a^{8}x^{12}}{12}+\frac{4a^{7}bx^{14}}{7}+\frac{7a^{6}b^{2}x^{16}}{4}+\frac{28a^{5}b^{3}x^{18}}{9}+\frac{7a^{4}b^{4}x^{20}}{2}+\frac{28a^{3}b^{5}x^{22}}{11}+\frac{7a^{2}b^{6}x^{24}}{6}+\frac{4ab^{7}x^{26}}{13}+\frac{b^{8}x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**12/12 + 4*a**7*b*x**14/7 + 7*a**6*b**2*x**16/4 + 28*a**5*b**3*x**18
/9 + 7*a**4*b**4*x**20/2 + 28*a**3*b**5*x**22/11 + 7*a**2*b**6*x**24/6 + 4*
a*b**7*x**26/13 + b**8*x**28/28
```

3.87 $\int x^9 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=91

$$\frac{a^4 \left(a+bx^2\right)^9}{18b^5}-\frac{a^3 \left(a+bx^2\right)^{10}}{5b^5}+\frac{3a^2 \left(a+bx^2\right)^{11}}{11b^5}+\frac{\left(a+bx^2\right)^{13}}{26b^5}-\frac{a \left(a+bx^2\right)^{12}}{6b^5}$$

Rubi [A] time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2 \left(a+bx^2\right)^{11}}{11b^5}-\frac{a^3 \left(a+bx^2\right)^{10}}{5b^5}+\frac{a^4 \left(a+bx^2\right)^9}{18b^5}+\frac{\left(a+bx^2\right)^{13}}{26b^5}-\frac{a \left(a+bx^2\right)^{12}}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^8,x]
```

```
[Out] (a^4*(a + b*x^2)^9)/(18*b^5) - (a^3*(a + b*x^2)^10)/(5*b^5) + (3*a^2*(a + b
*x^2)^11)/(11*b^5) - (a*(a + b*x^2)^12)/(6*b^5) + (a + b*x^2)^13/(26*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9 \left(a+bx^2\right)^8 \, dx &= \frac{1}{2} \text{Subst}\left(\int x^4 (a+bx)^8 \, dx, x, x^2\right)\\ &= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^4 (a+bx)^8}{b^4}-\frac{4a^3 (a+bx)^9}{b^4}+\frac{6a^2 (a+bx)^{10}}{b^4}-\frac{4a (a+bx)^{11}}{b^4}+\frac{(a+bx)^{12}}{b^4}\right) dx\\ &= \frac{a^4 \left(a+bx^2\right)^9}{18b^5}-\frac{a^3 \left(a+bx^2\right)^{10}}{5b^5}+\frac{3a^2 \left(a+bx^2\right)^{11}}{11b^5}-\frac{a \left(a+bx^2\right)^{12}}{6b^5}+\frac{\left(a+bx^2\right)^{13}}{26b^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.16

$$\frac{a^8x^{10}}{10} + \frac{2}{3}a^7bx^{12} + 2a^6b^2x^{14} + \frac{7}{2}a^5b^3x^{16} + \frac{35}{9}a^4b^4x^{18} + \frac{14}{5}a^3b^5x^{20} + \frac{14}{11}a^2b^6x^{22} + \frac{1}{3}ab^7x^{24} + \frac{b^8x^{26}}{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^10)/10 + (2*a^7*b*x^12)/3 + 2*a^6*b^2*x^14 + (7*a^5*b^3*x^16)/2 + (3
5*a^4*b^4*x^18)/9 + (14*a^3*b^5*x^20)/5 + (14*a^2*b^6*x^22)/11 + (a*b^7*x^2
4)/3 + (b^8*x^26)/26
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \left(a + bx^2\right)^8 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x^9*(a + b*x^2)^8, x]
```

fricas [A] time = 0.93, size = 90, normalized size = 0.99

$$\frac{1}{26}x^{26}b^8 + \frac{1}{3}x^{24}b^7a + \frac{14}{11}x^{22}b^6a^2 + \frac{14}{5}x^{20}b^5a^3 + \frac{35}{9}x^{18}b^4a^4 + \frac{7}{2}x^{16}b^3a^5 + 2x^{14}b^2a^6 + \frac{2}{3}x^{12}ba^7 + \frac{1}{10}x^{10}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*b^8 + 1/3*x^24*b^7*a + 14/11*x^22*b^6*a^2 + 14/5*x^20*b^5*a^3 + 3
5/9*x^18*b^4*a^4 + 7/2*x^16*b^3*a^5 + 2*x^14*b^2*a^6 + 2/3*x^12*b*a^7 + 1/1
0*x^10*a^8
```

giac [A] time = 1.00, size = 90, normalized size = 0.99

$$\frac{1}{26}\,b^8x^{26} + \frac{1}{3}\,ab^7x^{24} + \frac{14}{11}\,a^2b^6x^{22} + \frac{14}{5}\,a^3b^5x^{20} + \frac{35}{9}\,a^4b^4x^{18} + \frac{7}{2}\,a^5b^3x^{16} + 2\,a^6b^2x^{14} + \frac{2}{3}\,a^7bx^{12} + \frac{1}{10}\,a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 + 3
5/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1/1
0*a^8*x^10
```

maple [A] time = 0.00, size = 91, normalized size = 1.00

$$\frac{1}{26}b^8x^{26}+\frac{1}{3}a\,b^7x^{24}+\frac{14}{11}a^2b^6x^{22}+\frac{14}{5}a^3b^5x^{20}+\frac{35}{9}a^4b^4x^{18}+\frac{7}{2}a^5b^3x^{16}+2a^6b^2x^{14}+\frac{2}{3}a^7b\,x^{12}+\frac{1}{10}a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^8,x)
```

```
[Out] 1/26*b^8*x^26+1/3*a*b^7*x^24+14/11*a^2*b^6*x^22+14/5*a^3*b^5*x^20+35/9*a^4*
b^4*x^18+7/2*a^5*b^3*x^16+2*a^6*b^2*x^14+2/3*a^7*b*x^12+1/10*a^8*x^10
```

maxima [A] time = 1.28, size = 90, normalized size = 0.99

$$\frac{1}{26}\,b^8x^{26}+\frac{1}{3}\,ab^7x^{24}+\frac{14}{11}\,a^2b^6x^{22}+\frac{14}{5}\,a^3b^5x^{20}+\frac{35}{9}\,a^4b^4x^{18}+\frac{7}{2}\,a^5b^3x^{16}+2\,a^6b^2x^{14}+\frac{2}{3}\,a^7bx^{12}+\frac{1}{10}\,a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 + 3
5/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1/1
0*a^8*x^10
```

mupad [B] time = 4.59, size = 90, normalized size = 0.99

$$\frac{a^8\,x^{10}}{10}+\frac{2\,a^7\,b\,x^{12}}{3}+2\,a^6\,b^2\,x^{14}+\frac{7\,a^5\,b^3\,x^{16}}{2}+\frac{35\,a^4\,b^4\,x^{18}}{9}+\frac{14\,a^3\,b^5\,x^{20}}{5}+\frac{14\,a^2\,b^6\,x^{22}}{11}+\frac{a\,b^7\,x^{24}}{3}+\frac{b^8\,x^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^10)/10 + (b^8*x^26)/26 + (2*a^7*b*x^12)/3 + (a*b^7*x^24)/3 + 2*a^6*b
^2*x^14 + (7*a^5*b^3*x^16)/2 + (35*a^4*b^4*x^18)/9 + (14*a^3*b^5*x^20)/5 +
(14*a^2*b^6*x^22)/11
```

sympy [A] time = 0.09, size = 104, normalized size = 1.14

$$\frac{a^8x^{10}}{10}+\frac{2a^7bx^{12}}{3}+2a^6b^2x^{14}+\frac{7a^5b^3x^{16}}{2}+\frac{35a^4b^4x^{18}}{9}+\frac{14a^3b^5x^{20}}{5}+\frac{14a^2b^6x^{22}}{11}+\frac{ab^7x^{24}}{3}+\frac{b^8x^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**10/10 + 2*a**7*b*x**12/3 + 2*a**6*b**2*x**14 + 7*a**5*b**3*x**16/2
+ 35*a**4*b**4*x**18/9 + 14*a**3*b**5*x**20/5 + 14*a**2*b**6*x**22/11 + a*b
**7*x**24/3 + b**8*x**26/26
```

3.88 $\int x^7 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=72

$$-\frac{a^3 \left(a+bx^2\right)^9}{18b^4}+\frac{3a^2 \left(a+bx^2\right)^{10}}{20b^4}+\frac{\left(a+bx^2\right)^{12}}{24b^4}-\frac{3a \left(a+bx^2\right)^{11}}{22b^4}$$

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2 \left(a+bx^2\right)^{10}}{20b^4}-\frac{a^3 \left(a+bx^2\right)^9}{18b^4}+\frac{\left(a+bx^2\right)^{12}}{24b^4}-\frac{3a \left(a+bx^2\right)^{11}}{22b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^8,x]
```

```
[Out] -(a^3*(a + b*x^2)^9)/(18*b^4) + (3*a^2*(a + b*x^2)^10)/(20*b^4) - (3*a*(a +
 b*x^2)^11)/(22*b^4) + (a + b*x^2)^12/(24*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^8 \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)^8 \, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^3(a+bx)^8}{b^3}+\frac{3a^2(a+bx)^9}{b^3}-\frac{3a(a+bx)^{10}}{b^3}+\frac{(a+bx)^{11}}{b^3}\right) dx, x, x^2\right)\\
&= -\frac{a^3 \left(a+bx^2\right)^9}{18b^4}+\frac{3a^2 \left(a+bx^2\right)^{10}}{20b^4}-\frac{3a \left(a+bx^2\right)^{11}}{22b^4}+\frac{\left(a+bx^2\right)^{12}}{24b^4}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.47

$$\frac{a^8x^8}{8}+\frac{4}{5}a^7bx^{10}+\frac{7}{3}a^6b^2x^{12}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{28}{9}a^3b^5x^{18}+\frac{7}{5}a^2b^6x^{20}+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^8)/8 + (4*a^7*b*x^10)/5 + (7*a^6*b^2*x^12)/3 + 4*a^5*b^3*x^14 + (35*
a^4*b^4*x^16)/8 + (28*a^3*b^5*x^18)/9 + (7*a^2*b^6*x^20)/5 + (4*a*b^7*x^22)
/11 + (b^8*x^24)/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\left(a+bx^2\right)^8\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x^7*(a + b*x^2)^8, x]
```

fricas [A] time = 0.88, size = 90, normalized size = 1.25

$$\frac{1}{24}x^{24}b^8+\frac{4}{11}x^{22}b^7a+\frac{7}{5}x^{20}b^6a^2+\frac{28}{9}x^{18}b^5a^3+\frac{35}{8}x^{16}b^4a^4+4x^{14}b^3a^5+\frac{7}{3}x^{12}b^2a^6+\frac{4}{5}x^{10}ba^7+\frac{1}{8}x^8a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*b^8 + 4/11*x^22*b^7*a + 7/5*x^20*b^6*a^2 + 28/9*x^18*b^5*a^3 + 35
/8*x^16*b^4*a^4 + 4*x^14*b^3*a^5 + 7/3*x^12*b^2*a^6 + 4/5*x^10*b*a^7 + 1/8*
x^8*a^8
```

giac [A] time = 1.08, size = 90, normalized size = 1.25

$$\frac{1}{24}\,b^8x^{24}+\frac{4}{11}\,ab^7x^{22}+\frac{7}{5}\,a^2b^6x^{20}+\frac{28}{9}\,a^3b^5x^{18}+\frac{35}{8}\,a^4b^4x^{16}+4\,a^5b^3x^{14}+\frac{7}{3}\,a^6b^2x^{12}+\frac{4}{5}\,a^7bx^{10}+\frac{1}{8}\,a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 35
/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/8*
a^8*x^8
```

maple [A] time = 0.00, size = 91, normalized size = 1.26

$$\frac{1}{24}b^8x^{24}+\frac{4}{11}a\,b^7x^{22}+\frac{7}{5}a^2b^6x^{20}+\frac{28}{9}a^3b^5x^{18}+\frac{35}{8}a^4b^4x^{16}+4a^5b^3x^{14}+\frac{7}{3}a^6b^2x^{12}+\frac{4}{5}a^7b\,x^{10}+\frac{1}{8}a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^8,x)
```

```
[Out] 1/24*b^8*x^24+4/11*a*b^7*x^22+7/5*a^2*b^6*x^20+28/9*a^3*b^5*x^18+35/8*a^4*b
^4*x^16+4*a^5*b^3*x^14+7/3*a^6*b^2*x^12+4/5*a^7*b*x^10+1/8*a^8*x^8
```

maxima [A] time = 1.29, size = 90, normalized size = 1.25

$$\frac{1}{24}\,b^8x^{24}+\frac{4}{11}\,ab^7x^{22}+\frac{7}{5}\,a^2b^6x^{20}+\frac{28}{9}\,a^3b^5x^{18}+\frac{35}{8}\,a^4b^4x^{16}+4\,a^5b^3x^{14}+\frac{7}{3}\,a^6b^2x^{12}+\frac{4}{5}\,a^7bx^{10}+\frac{1}{8}\,a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 35
/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/8*
a^8*x^8
```

mupad [B] time = 0.09, size = 90, normalized size = 1.25

$$\frac{a^8\,x^8}{8}+\frac{4\,a^7\,b\,x^{10}}{5}+\frac{7\,a^6\,b^2\,x^{12}}{3}+4\,a^5\,b^3\,x^{14}+\frac{35\,a^4\,b^4\,x^{16}}{8}+\frac{28\,a^3\,b^5\,x^{18}}{9}+\frac{7\,a^2\,b^6\,x^{20}}{5}+\frac{4\,a\,b^7\,x^{22}}{11}+\frac{b^8\,x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^8)/8 + (b^8*x^24)/24 + (4*a^7*b*x^10)/5 + (4*a*b^7*x^22)/11 + (7*a^6
*b^2*x^12)/3 + 4*a^5*b^3*x^14 + (35*a^4*b^4*x^16)/8 + (28*a^3*b^5*x^18)/9 +
 (7*a^2*b^6*x^20)/5
```

sympy [A] time = 0.09, size = 105, normalized size = 1.46

$$\frac{a^8x^8}{8}+\frac{4a^7bx^{10}}{5}+\frac{7a^6b^2x^{12}}{3}+4a^5b^3x^{14}+\frac{35a^4b^4x^{16}}{8}+\frac{28a^3b^5x^{18}}{9}+\frac{7a^2b^6x^{20}}{5}+\frac{4ab^7x^{22}}{11}+\frac{b^8x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**8/8 + 4*a**7*b*x**10/5 + 7*a**6*b**2*x**12/3 + 4*a**5*b**3*x**14 +
35*a**4*b**4*x**16/8 + 28*a**3*b**5*x**18/9 + 7*a**2*b**6*x**20/5 + 4*a*b**
7*x**22/11 + b**8*x**24/24
```

3.89 $\int x^5 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^2\right)^9}{18b^3}+\frac{\left(a+bx^2\right)^{11}}{22b^3}-\frac{a\left(a+bx^2\right)^{10}}{10b^3}$$

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^9}{18b^3}+\frac{\left(a+bx^2\right)^{11}}{22b^3}-\frac{a\left(a+bx^2\right)^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^8,x]
```

```
[Out] (a^2*(a + b*x^2)^9)/(18*b^3) - (a*(a + b*x^2)^10)/(10*b^3) + (a + b*x^2)^11
/(22*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^8 dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^8\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^8}{b^2}-\frac{2a(a+bx)^9}{b^2}+\frac{(a+bx)^{10}}{b^2}\right)dx, x, x^2\right)\\ &= \frac{a^2\left(a+bx^2\right)^9}{18b^3}-\frac{a\left(a+bx^2\right)^{10}}{10b^3}+\frac{\left(a+bx^2\right)^{11}}{22b^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 103, normalized size = 1.94

$$\frac{a^8x^6}{6} + a^7bx^8 + \frac{14}{5}a^6b^2x^{10} + \frac{14}{3}a^5b^3x^{12} + 5a^4b^4x^{14} + \frac{7}{2}a^3b^5x^{16} + \frac{14}{9}a^2b^6x^{18} + \frac{2}{5}ab^7x^{20} + \frac{b^8x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^6)/6 + a^7*b*x^8 + (14*a^6*b^2*x^10)/5 + (14*a^5*b^3*x^12)/3 + 5*a^4
*b^4*x^14 + (7*a^3*b^5*x^16)/2 + (14*a^2*b^6*x^18)/9 + (2*a*b^7*x^20)/5 + (
b^8*x^22)/22
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a + bx^2\right)^8 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x^5*(a + b*x^2)^8, x]
```

fricas [A] time = 0.98, size = 89, normalized size = 1.68

$$\frac{1}{22}x^{22}b^8 + \frac{2}{5}x^{20}b^7a + \frac{14}{9}x^{18}b^6a^2 + \frac{7}{2}x^{16}b^5a^3 + 5x^{14}b^4a^4 + \frac{14}{3}x^{12}b^3a^5 + \frac{14}{5}x^{10}b^2a^6 + x^8ba^7 + \frac{1}{6}x^6a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^8 + 2/5*x^20*b^7*a + 14/9*x^18*b^6*a^2 + 7/2*x^16*b^5*a^3 + 5*x
^14*b^4*a^4 + 14/3*x^12*b^3*a^5 + 14/5*x^10*b^2*a^6 + x^8*b*a^7 + 1/6*x^6*a
^8
```

giac [A] time = 1.13, size = 89, normalized size = 1.68

$$\frac{1}{22}\,b^8x^{22} + \frac{2}{5}\,ab^7x^{20} + \frac{14}{9}\,a^2b^6x^{18} + \frac{7}{2}\,a^3b^5x^{16} + 5\,a^4b^4x^{14} + \frac{14}{3}\,a^5b^3x^{12} + \frac{14}{5}\,a^6b^2x^{10} + a^7bx^8 + \frac{1}{6}\,a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*a
^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8*x
^6
```

maple [A] time = 0.00, size = 90, normalized size = 1.70

$$\frac{1}{22}b^{8}x^{22}+\frac{2}{5}a\,b^{7}x^{20}+\frac{14}{9}a^{2}b^{6}x^{18}+\frac{7}{2}a^{3}b^{5}x^{16}+5a^{4}b^{4}x^{14}+\frac{14}{3}a^{5}b^{3}x^{12}+\frac{14}{5}a^{6}b^{2}x^{10}+a^{7}b\,x^{8}+\frac{1}{6}a^{8}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^8,x)
```

```
[Out] 1/22*b^8*x^22+2/5*a*b^7*x^20+14/9*a^2*b^6*x^18+7/2*a^3*b^5*x^16+5*a^4*b^4*x
^14+14/3*a^5*b^3*x^12+14/5*a^6*b^2*x^10+a^7*b*x^8+1/6*a^8*x^6
```

maxima [A] time = 1.35, size = 89, normalized size = 1.68

$$\frac{1}{22}\,b^{8}x^{22}+\frac{2}{5}\,ab^{7}x^{20}+\frac{14}{9}\,a^{2}b^{6}x^{18}+\frac{7}{2}\,a^{3}b^{5}x^{16}+5\,a^{4}b^{4}x^{14}+\frac{14}{3}\,a^{5}b^{3}x^{12}+\frac{14}{5}\,a^{6}b^{2}x^{10}+a^{7}bx^{8}+\frac{1}{6}\,a^{8}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*a
^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8*x
^6
```

mupad [B] time = 0.09, size = 89, normalized size = 1.68

$$\frac{a^8\,x^6}{6}+a^7\,b\,x^8+\frac{14\,a^6\,b^2\,x^{10}}{5}+\frac{14\,a^5\,b^3\,x^{12}}{3}+5\,a^4\,b^4\,x^{14}+\frac{7\,a^3\,b^5\,x^{16}}{2}+\frac{14\,a^2\,b^6\,x^{18}}{9}+\frac{2\,a\,b^7\,x^{20}}{5}+\frac{b^8\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^6)/6 + (b^8*x^22)/22 + a^7*b*x^8 + (2*a*b^7*x^20)/5 + (14*a^6*b^2*x^
10)/5 + (14*a^5*b^3*x^12)/3 + 5*a^4*b^4*x^14 + (7*a^3*b^5*x^16)/2 + (14*a^2
*b^6*x^18)/9
```

sympy [B] time = 0.09, size = 102, normalized size = 1.92

$$\frac{a^{8}x^{6}}{6}+a^{7}bx^{8}+\frac{14a^{6}b^{2}x^{10}}{5}+\frac{14a^{5}b^{3}x^{12}}{3}+5a^{4}b^{4}x^{14}+\frac{7a^{3}b^{5}x^{16}}{2}+\frac{14a^{2}b^{6}x^{18}}{9}+\frac{2ab^{7}x^{20}}{5}+\frac{b^{8}x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**6/6 + a**7*b*x**8 + 14*a**6*b**2*x**10/5 + 14*a**5*b**3*x**12/3 + 5
*a**4*b**4*x**14 + 7*a**3*b**5*x**16/2 + 14*a**2*b**6*x**18/9 + 2*a*b**7*x*
*20/5 + b**8*x**22/22
```

3.90 $\int x^3 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^{10}}{20b^2}-\frac{a\left(a+bx^2\right)^9}{18b^2}$$

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{10}}{20b^2}-\frac{a\left(a+bx^2\right)^9}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^8,x]
```

```
[Out] -(a*(a + b*x^2)^9)/(18*b^2) + (a + b*x^2)^10/(20*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right)^8 \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^8 \, dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^8}{b}+\frac{(a+bx)^9}{b}\right) dx, x, x^2\right)\\ &= -\frac{a\left(a+bx^2\right)^9}{18b^2}+\frac{\left(a+bx^2\right)^{10}}{20b^2}\end{aligned}$$

Mathematica [B] time = 0.00, size = 106, normalized size = 3.12

$$\frac{a^8x^4}{4}+\frac{4}{3}a^7bx^6+\frac{7}{2}a^6b^2x^8+\frac{28}{5}a^5b^3x^{10}+\frac{35}{6}a^4b^4x^{12}+4a^3b^5x^{14}+\frac{7}{4}a^2b^6x^{16}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^4)/4 + (4*a^7*b*x^6)/3 + (7*a^6*b^2*x^8)/2 + (28*a^5*b^3*x^10)/5 + (
35*a^4*b^4*x^12)/6 + 4*a^3*b^5*x^14 + (7*a^2*b^6*x^16)/4 + (4*a*b^7*x^18)/9
 + (b^8*x^20)/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)^8\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)^8, x]
```

fricas [B] time = 0.94, size = 90, normalized size = 2.65

$$\frac{1}{20}x^{20}b^8+\frac{4}{9}x^{18}b^7a+\frac{7}{4}x^{16}b^6a^2+4x^{14}b^5a^3+\frac{35}{6}x^{12}b^4a^4+\frac{28}{5}x^{10}b^3a^5+\frac{7}{2}x^8b^2a^6+\frac{4}{3}x^6ba^7+\frac{1}{4}x^4a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^8 + 4/9*x^18*b^7*a + 7/4*x^16*b^6*a^2 + 4*x^14*b^5*a^3 + 35/6*x
^12*b^4*a^4 + 28/5*x^10*b^3*a^5 + 7/2*x^8*b^2*a^6 + 4/3*x^6*b*a^7 + 1/4*x^4
*a^8
```

giac [B] time = 1.18, size = 90, normalized size = 2.65

$$\frac{1}{20}\,b^8x^{20}+\frac{4}{9}\,ab^7x^{18}+\frac{7}{4}\,a^2b^6x^{16}+4\,a^3b^5x^{14}+\frac{35}{6}\,a^4b^4x^{12}+\frac{28}{5}\,a^5b^3x^{10}+\frac{7}{2}\,a^6b^2x^8+\frac{4}{3}\,a^7bx^6+\frac{1}{4}\,a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*a
^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a^8
*x^4
```

maple [B] time = 0.00, size = 91, normalized size = 2.68

$$\frac{1}{20}b^{8}x^{20}+\frac{4}{9}a\,b^{7}x^{18}+\frac{7}{4}a^{2}b^{6}x^{16}+4a^{3}b^{5}x^{14}+\frac{35}{6}a^{4}b^{4}x^{12}+\frac{28}{5}a^{5}b^{3}x^{10}+\frac{7}{2}a^{6}b^{2}x^{8}+\frac{4}{3}a^{7}b\,x^{6}+\frac{1}{4}a^{8}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^8,x)
```

```
[Out] 1/20*b^8*x^20+4/9*a*b^7*x^18+7/4*a^2*b^6*x^16+4*a^3*b^5*x^14+35/6*a^4*b^4*x
^12+28/5*a^5*b^3*x^10+7/2*a^6*b^2*x^8+4/3*a^7*b*x^6+1/4*a^8*x^4
```

maxima [B] time = 1.43, size = 90, normalized size = 2.65

$$\frac{1}{20}\,b^{8}x^{20}+\frac{4}{9}\,ab^{7}x^{18}+\frac{7}{4}\,a^{2}b^{6}x^{16}+4\,a^{3}b^{5}x^{14}+\frac{35}{6}\,a^{4}b^{4}x^{12}+\frac{28}{5}\,a^{5}b^{3}x^{10}+\frac{7}{2}\,a^{6}b^{2}x^{8}+\frac{4}{3}\,a^{7}bx^{6}+\frac{1}{4}\,a^{8}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*a
^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a^8
*x^4
```

mupad [B] time = 0.09, size = 90, normalized size = 2.65

$$\frac{a^8\,x^4}{4}+\frac{4\,a^7\,b\,x^6}{3}+\frac{7\,a^6\,b^2\,x^8}{2}+\frac{28\,a^5\,b^3\,x^{10}}{5}+\frac{35\,a^4\,b^4\,x^{12}}{6}+4\,a^3\,b^5\,x^{14}+\frac{7\,a^2\,b^6\,x^{16}}{4}+\frac{4\,a\,b^7\,x^{18}}{9}+\frac{b^8\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^4)/4 + (b^8*x^20)/20 + (4*a^7*b*x^6)/3 + (4*a*b^7*x^18)/9 + (7*a^6*b
^2*x^8)/2 + (28*a^5*b^3*x^10)/5 + (35*a^4*b^4*x^12)/6 + 4*a^3*b^5*x^14 + (7
*a^2*b^6*x^16)/4
```

sympy [B] time = 0.08, size = 105, normalized size = 3.09

$$\frac{a^{8}x^{4}}{4}+\frac{4a^{7}bx^{6}}{3}+\frac{7a^{6}b^{2}x^{8}}{2}+\frac{28a^{5}b^{3}x^{10}}{5}+\frac{35a^{4}b^{4}x^{12}}{6}+4a^{3}b^{5}x^{14}+\frac{7a^{2}b^{6}x^{16}}{4}+\frac{4ab^{7}x^{18}}{9}+\frac{b^{8}x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**4/4 + 4*a**7*b*x**6/3 + 7*a**6*b**2*x**8/2 + 28*a**5*b**3*x**10/5 +
 35*a**4*b**4*x**12/6 + 4*a**3*b**5*x**14 + 7*a**2*b**6*x**16/4 + 4*a*b**7*
x**18/9 + b**8*x**20/20
```

3.91 $\int x\left(a+bx^2\right)^8\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^9}{18b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^9}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^8,x]
```

```
[Out] (a + b*x^2)^9/(18*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^8\,dx=\frac{\left(a+bx^2\right)^9}{18b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^9}{18b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^8,x]
```

```
[Out] (a + b*x^2)^9/(18*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^8\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2)^8, x]
```

fricas [B] time = 1.15, size = 90, normalized size = 5.62

$$\frac{1}{18}x^{18}b^8+\frac{1}{2}x^{16}b^7a+2x^{14}b^6a^2+\frac{14}{3}x^{12}b^5a^3+7x^{10}b^4a^4+7x^8b^3a^5+\frac{14}{3}x^6b^2a^6+2x^4ba^7+\frac{1}{2}x^2a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^8 + 1/2*x^16*b^7*a + 2*x^14*b^6*a^2 + 14/3*x^12*b^5*a^3 + 7*x^1
0*b^4*a^4 + 7*x^8*b^3*a^5 + 14/3*x^6*b^2*a^6 + 2*x^4*b*a^7 + 1/2*x^2*a^8
```

giac [A] time = 1.12, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/18*(b*x^2 + a)^9/b
```

maple [B] time = 0.00, size = 91, normalized size = 5.69

$$\frac{1}{18}b^8x^{18}+\frac{1}{2}a\,b^7x^{16}+2a^2b^6x^{14}+\frac{14}{3}a^3b^5x^{12}+7a^4b^4x^{10}+7a^5b^3x^8+\frac{14}{3}a^6b^2x^6+2a^7b\,x^4+\frac{1}{2}a^8x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^8,x)
```

```
[Out] 1/18*b^8*x^18+1/2*a*b^7*x^16+2*a^2*b^6*x^14+14/3*a^3*b^5*x^12+7*a^4*b^4*x^1
0+7*a^5*b^3*x^8+14/3*a^6*b^2*x^6+2*a^7*b*x^4+1/2*a^8*x^2
```

maxima [A] time = 1.28, size = 14, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/18*(b*x^2 + a)^9/b
```

mupad [B] time = 4.61, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^2+a\right)^9}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^8,x)
```

```
[Out] (a + b*x^2)^9/(18*b)
```

sympy [B] time = 0.08, size = 99, normalized size = 6.19

$$\frac{a^8x^2}{2}+2a^7bx^4+\frac{14a^6b^2x^6}{3}+7a^5b^3x^8+7a^4b^4x^{10}+\frac{14a^3b^5x^{12}}{3}+2a^2b^6x^{14}+\frac{ab^7x^{16}}{2}+\frac{b^8x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**2/2 + 2*a**7*b*x**4 + 14*a**6*b**2*x**6/3 + 7*a**5*b**3*x**8 + 7*a*
*4*b**4*x**10 + 14*a**3*b**5*x**12/3 + 2*a**2*b**6*x**14 + a*b**7*x**16/2 +
 b**8*x**18/18
```

3.92 $\int \frac{(a+bx^2)^8}{x} dx$

Optimal. Leaf size=100

$$a^8\log(x)+4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}$$

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{7}{3}a^2b^6x^{12}+\frac{28}{5}a^3b^5x^{10}+\frac{35}{4}a^4b^4x^8+\frac{28}{3}a^5b^3x^6+7a^6b^2x^4+4a^7bx^2+a^8\log(x)+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x,x]
```

```
[Out] 4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28
*a^3*b^5*x^10)/5 + (7*a^2*b^6*x^12)/3 + (4*a*b^7*x^14)/7 + (b^8*x^16)/16 +
a^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(8a^7b+\frac{a^8}{x}+28a^6b^2x+56a^5b^3x^2+70a^4b^4x^3+56a^3b^5x^4+28a^2b^6x^5+8ab^7x^6\right.\right.\\ &= 4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}+a^8\end{aligned}$$

Mathematica [A] time = 0.00, size = 100, normalized size = 1.00

$$a^8 \log(x) + 4a^7bx^2 + 7a^6b^2x^4 + \frac{28}{3}a^5b^3x^6 + \frac{35}{4}a^4b^4x^8 + \frac{28}{5}a^3b^5x^{10} + \frac{7}{3}a^2b^6x^{12} + \frac{4}{7}ab^7x^{14} + \frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x,x]
```

```
[Out] 4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28
*a^3*b^5*x^10)/5 + (7*a^2*b^6*x^12)/3 + (4*a*b^7*x^14)/7 + (b^8*x^16)/16 +
a^8*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^8}{x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x, x]
```

fricas [A] time = 1.10, size = 88, normalized size = 0.88

$$\frac{1}{16}\,b^8x^{16} + \frac{4}{7}\,ab^7x^{14} + \frac{7}{3}\,a^2b^6x^{12} + \frac{28}{5}\,a^3b^5x^{10} + \frac{35}{4}\,a^4b^4x^8 + \frac{28}{3}\,a^5b^3x^6 + 7\,a^6b^2x^4 + 4\,a^7bx^2 + a^8\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="fricas")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8*log(x)
```

giac [A] time = 0.90, size = 91, normalized size = 0.91

$$\frac{1}{16}\,b^8x^{16} + \frac{4}{7}\,ab^7x^{14} + \frac{7}{3}\,a^2b^6x^{12} + \frac{28}{5}\,a^3b^5x^{10} + \frac{35}{4}\,a^4b^4x^8 + \frac{28}{3}\,a^5b^3x^6 + 7\,a^6b^2x^4 + 4\,a^7bx^2 + \frac{1}{2}\,a^8\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="giac")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*lo
g(x^2)
```

maple [A] time = 0.00, size = 89, normalized size = 0.89

$$\frac{b^8x^{16}}{16}+\frac{4a\,b^7x^{14}}{7}+\frac{7a^2b^6x^{12}}{3}+\frac{28a^3b^5x^{10}}{5}+\frac{35a^4b^4x^8}{4}+\frac{28a^5b^3x^6}{3}+7a^6b^2x^4+4a^7b\,x^2+a^8\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x,x)
```

```
[Out] 4*a^7*b*x^2+7*a^6*b^2*x^4+28/3*a^5*b^3*x^6+35/4*a^4*b^4*x^8+28/5*a^3*b^5*x^
10+7/3*a^2*b^6*x^12+4/7*a*b^7*x^14+1/16*b^8*x^16+a^8*ln(x)
```

maxima [A] time = 1.30, size = 91, normalized size = 0.91

$$\frac{1}{16}\,b^8x^{16}+\frac{4}{7}\,ab^7x^{14}+\frac{7}{3}\,a^2b^6x^{12}+\frac{28}{5}\,a^3b^5x^{10}+\frac{35}{4}\,a^4b^4x^8+\frac{28}{3}\,a^5b^3x^6+7\,a^6b^2x^4+4\,a^7bx^2+\frac{1}{2}\,a^8\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35/
4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*lo
g(x^2)
```

mupad [B] time = 4.62, size = 88, normalized size = 0.88

$$a^8\,\ln(x)+\frac{b^8\,x^{16}}{16}+4\,a^7\,b\,x^2+\frac{4\,a\,b^7\,x^{14}}{7}+7\,a^6\,b^2\,x^4+\frac{28\,a^5\,b^3\,x^6}{3}+\frac{35\,a^4\,b^4\,x^8}{4}+\frac{28\,a^3\,b^5\,x^{10}}{5}+\frac{7\,a^2\,b^6\,x^{12}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x,x)
```

```
[Out] a^8*log(x) + (b^8*x^16)/16 + 4*a^7*b*x^2 + (4*a*b^7*x^14)/7 + 7*a^6*b^2*x^4
 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^10)/5 + (7*a^2*b
^6*x^12)/3
```

sympy [A] time = 0.18, size = 102, normalized size = 1.02

$$a^8\log{(x)}+4a^7bx^2+7a^6b^2x^4+\frac{28a^5b^3x^6}{3}+\frac{35a^4b^4x^8}{4}+\frac{28a^3b^5x^{10}}{5}+\frac{7a^2b^6x^{12}}{3}+\frac{4ab^7x^{14}}{7}+\frac{b^8x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x,x)
```

```
[Out] a**8*log(x) + 4*a**7*b*x**2 + 7*a**6*b**2*x**4 + 28*a**5*b**3*x**6/3 + 35*a
**4*b**4*x**8/4 + 28*a**3*b**5*x**10/5 + 7*a**2*b**6*x**12/3 + 4*a*b**7*x**
14/7 + b**8*x**16/16
```

3.93 $\int \frac{(a+bx^2)^8}{x^3} dx$

Optimal. Leaf size=99

$$-\frac{a^8}{2x^2}+8a^7b\log(x)+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}$$

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{14}{5}a^2b^6x^{10}+7a^3b^5x^8+\frac{35}{3}a^4b^4x^6+14a^5b^3x^4+14a^6b^2x^2+8a^7b\log(x)-\frac{a^8}{2x^2}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^3,x]
```

```
[Out] -a^8/(2*x^2) + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3
*b^5*x^8 + (14*a^2*b^6*x^10)/5 + (2*a*b^7*x^12)/3 + (b^8*x^14)/14 + 8*a^7*b
*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^2} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(28a^6b^2+\frac{a^8}{x^2}+\frac{8a^7b}{x}+56a^5b^3x+70a^4b^4x^2+56a^3b^5x^3+28a^2b^6x^4+8ab^7x^5+\right.\right.\\ &= -\frac{a^8}{2x^2}+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}+8a^7b\end{aligned}$$

Mathematica [A] time = 0.01, size = 99, normalized size = 1.00

$$-\frac{a^8}{2x^2}+8a^7b\log(x)+14a^6b^2x^2+14a^5b^3x^4+\frac{35}{3}a^4b^4x^6+7a^3b^5x^8+\frac{14}{5}a^2b^6x^{10}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^3,x]
```

```
[Out] -1/2*a^8/x^2 + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3
*b^5*x^8 + (14*a^2*b^6*x^10)/5 + (2*a*b^7*x^12)/3 + (b^8*x^14)/14 + 8*a^7*b
*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^8}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^3, x]
```

fricas [A] time = 1.17, size = 94, normalized size = 0.95

$$\frac{15\,b^8x^{16}+140\,ab^7x^{14}+588\,a^2b^6x^{12}+1470\,a^3b^5x^{10}+2450\,a^4b^4x^8+2940\,a^5b^3x^6+2940\,a^6b^2x^4+1680\,a^7bx^2\log(x)-105\,a^8}{210\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="fricas")
```

```
[Out] 1/210*(15*b^8*x^16 + 140*a*b^7*x^14 + 588*a^2*b^6*x^12 + 1470*a^3*b^5*x^10
+ 2450*a^4*b^4*x^8 + 2940*a^5*b^3*x^6 + 2940*a^6*b^2*x^4 + 1680*a^7*b*x^2*l
og(x) - 105*a^8)/x^2
```

giac [A] time = 1.06, size = 101, normalized size = 1.02

$$\frac{1}{14}\,b^8x^{14}+\frac{2}{3}\,ab^7x^{12}+\frac{14}{5}\,a^2b^6x^{10}+7\,a^3b^5x^8+\frac{35}{3}\,a^4b^4x^6+14\,a^5b^3x^4+14\,a^6b^2x^2+4\,a^7b\log\left(x^2\right)-\frac{8\,a^7bx^2+a^8}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="giac")
```

```
[Out] 1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*a
^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*log(x^2) - 1/2*(8*a^
7*b*x^2 + a^8)/x^2
```

maple [A] time = 0.01, size = 90, normalized size = 0.91

$$\frac{b^8x^{14}}{14}+\frac{2a\,b^7x^{12}}{3}+\frac{14a^2b^6x^{10}}{5}+7a^3b^5x^8+\frac{35a^4b^4x^6}{3}+14a^5b^3x^4+14a^6b^2x^2+8a^7b\ln(x)-\frac{a^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^3,x)
```

```
[Out] -1/2*a^8/x^2+14*a^6*b^2*x^2+14*a^5*b^3*x^4+35/3*a^4*b^4*x^6+7*a^3*b^5*x^8+1
4/5*a^2*b^6*x^10+2/3*a*b^7*x^12+1/14*b^8*x^14+8*a^7*b*ln(x)
```

maxima [A] time = 1.38, size = 91, normalized size = 0.92

$$\frac{1}{14}\,b^8x^{14}+\frac{2}{3}\,ab^7x^{12}+\frac{14}{5}\,a^2b^6x^{10}+7\,a^3b^5x^8+\frac{35}{3}\,a^4b^4x^6+14\,a^5b^3x^4+14\,a^6b^2x^2+4\,a^7b\log\left(x^2\right)-\frac{a^8}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*a
^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*log(x^2) - 1/2*a^8/x
^2
```

mupad [B] time = 0.06, size = 89, normalized size = 0.90

$$\frac{b^8\,x^{14}}{14}-\frac{a^8}{2\,x^2}+\frac{2\,a\,b^7\,x^{12}}{3}+8\,a^7\,b\,\ln(x)+14\,a^6\,b^2\,x^2+14\,a^5\,b^3\,x^4+\frac{35\,a^4\,b^4\,x^6}{3}+7\,a^3\,b^5\,x^8+\frac{14\,a^2\,b^6\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^3,x)
```

```
[Out] (b^8*x^14)/14 - a^8/(2*x^2) + (2*a*b^7*x^12)/3 + 8*a^7*b*log(x) + 14*a^6*b^
2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3*b^5*x^8 + (14*a^2*b^6*x
^10)/5
```

sympy [A] time = 0.22, size = 100, normalized size = 1.01

$$-\frac{a^8}{2x^2}+8a^7b\log\left(x\right)+14a^6b^2x^2+14a^5b^3x^4+\frac{35a^4b^4x^6}{3}+7a^3b^5x^8+\frac{14a^2b^6x^{10}}{5}+\frac{2ab^7x^{12}}{3}+\frac{b^8x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**3,x)
```

```
[Out] -a**8/(2*x**2) + 8*a**7*b*log(x) + 14*a**6*b**2*x**2 + 14*a**5*b**3*x**4 +
35*a**4*b**4*x**6/3 + 7*a**3*b**5*x**8 + 14*a**2*b**6*x**10/5 + 2*a*b**7*x*
*12/3 + b**8*x**14/14
```

3.94 $\int \frac{(a+bx^2)^8}{x^5}\,dx$

Optimal. Leaf size=101

$$-\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^6b^2\log(x)+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}$$

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{7}{2}a^2b^6x^8+\frac{28}{3}a^3b^5x^6+\frac{35}{2}a^4b^4x^4+28a^5b^3x^2+28a^6b^2\log(x)-\frac{4a^7b}{x^2}-\frac{a^8}{4x^4}+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^5,x]
```

```
[Out] -a^8/(4*x^4) - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^
3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^10)/5 + (b^8*x^12)/12 + 28*a^
6*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(56a^5b^3+\frac{a^8}{x^3}+\frac{8a^7b}{x^2}+\frac{28a^6b^2}{x}+70a^4b^4x+56a^3b^5x^2+28a^2b^6x^3+8ab^7x^4+\right.\right.$$

$$= -\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}+28a^6b^2$$

Mathematica [A] time = 0.01, size = 101, normalized size = 1.00

$$-\frac{a^8}{4x^4}-\frac{4a^7b}{x^2}+28a^6b^2\log(x)+28a^5b^3x^2+\frac{35}{2}a^4b^4x^4+\frac{28}{3}a^3b^5x^6+\frac{7}{2}a^2b^6x^8+\frac{4}{5}ab^7x^{10}+\frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^5,x]
```

```
[Out] -1/4*a^8/x^4 - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^
3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^10)/5 + (b^8*x^12)/12 + 28*a^
6*b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^8}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^5, x]
```

fricas [A] time = 1.39, size = 94, normalized size = 0.93

$$\frac{5\,b^8x^{16}+48\,ab^7x^{14}+210\,a^2b^6x^{12}+560\,a^3b^5x^{10}+1050\,a^4b^4x^8+1680\,a^5b^3x^6+1680\,a^6b^2x^4\log(x)-240\,a^7bx^2-15\,a^8}{60\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="fricas")
```

```
[Out] 1/60*(5*b^8*x^16 + 48*a*b^7*x^14 + 210*a^2*b^6*x^12 + 560*a^3*b^5*x^10 + 10
50*a^4*b^4*x^8 + 1680*a^5*b^3*x^6 + 1680*a^6*b^2*x^4*log(x) - 240*a^7*b*x^2
 - 15*a^8)/x^4
```

giac [A] time = 1.12, size = 103, normalized size = 1.02

$$\frac{1}{12}\,b^8x^{12}+\frac{4}{5}\,ab^7x^{10}+\frac{7}{2}\,a^2b^6x^8+\frac{28}{3}\,a^3b^5x^6+\frac{35}{2}\,a^4b^4x^4+28\,a^5b^3x^2+14\,a^6b^2\log\left(x^2\right)-\frac{84\,a^6b^2x^4+16\,a^7bx^2+a^8}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="giac")
```

```
[Out] 1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2*
a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*log(x^2) - 1/4*(84*a^6*b^2*x^4 +
16*a^7*b*x^2 + a^8)/x^4
```

maple [A] time = 0.01, size = 90, normalized size = 0.89

$$\frac{b^8x^{12}}{12}+\frac{4a\,b^7x^{10}}{5}+\frac{7a^2b^6x^8}{2}+\frac{28a^3b^5x^6}{3}+\frac{35a^4b^4x^4}{2}+28a^5b^3x^2+28a^6b^2\ln(x)-\frac{4a^7b}{x^2}-\frac{a^8}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^5,x)
```

```
[Out] -1/4*a^8/x^4-4*a^7*b/x^2+28*a^5*b^3*x^2+35/2*a^4*b^4*x^4+28/3*a^3*b^5*x^6+7
/2*a^2*b^6*x^8+4/5*a*b^7*x^10+1/12*b^8*x^12+28*a^6*b^2*ln(x)
```

maxima [A] time = 1.33, size = 92, normalized size = 0.91

$$\frac{1}{12}\,b^8x^{12}+\frac{4}{5}\,ab^7x^{10}+\frac{7}{2}\,a^2b^6x^8+\frac{28}{3}\,a^3b^5x^6+\frac{35}{2}\,a^4b^4x^4+28\,a^5b^3x^2+14\,a^6b^2\log\left(x^2\right)-\frac{16\,a^7bx^2+a^8}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2*
a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*log(x^2) - 1/4*(16*a^7*b*x^2 + a^
8)/x^4
```

mupad [B] time = 0.06, size = 92, normalized size = 0.91

$$\frac{b^8\,x^{12}}{12}-\frac{\frac{a^8}{4}+4\,b\,a^7\,x^2}{x^4}+\frac{4\,a\,b^7\,x^{10}}{5}+28\,a^5\,b^3\,x^2+\frac{35\,a^4\,b^4\,x^4}{2}+\frac{28\,a^3\,b^5\,x^6}{3}+\frac{7\,a^2\,b^6\,x^8}{2}+28\,a^6\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^5,x)
```

```
[Out] (b^8*x^12)/12 - (a^8/4 + 4*a^7*b*x^2)/x^4 + (4*a*b^7*x^10)/5 + 28*a^5*b^3*x
^2 + (35*a^4*b^4*x^4)/2 + (28*a^3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + 28*a^6*b
^2*log(x)
```

sympy [A] time = 0.27, size = 104, normalized size = 1.03

$$28a^6b^2\log(x)+28a^5b^3x^2+\frac{35a^4b^4x^4}{2}+\frac{28a^3b^5x^6}{3}+\frac{7a^2b^6x^8}{2}+\frac{4ab^7x^{10}}{5}+\frac{b^8x^{12}}{12}+\frac{-a^8-16a^7bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**5,x)
```

```
[Out] 28*a**6*b**2*log(x) + 28*a**5*b**3*x**2 + 35*a**4*b**4*x**4/2 + 28*a**3*b**
5*x**6/3 + 7*a**2*b**6*x**8/2 + 4*a*b**7*x**10/5 + b**8*x**12/12 + (-a**8 -
 16*a**7*b*x**2)/(4*x**4)
```

3.95 $\int \frac{(a+bx^2)^8}{x^7} dx$

Optimal. Leaf size=94

$$-\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+56a^5b^3\log(x)+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}$$

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{14}{3}a^2b^6x^6+14a^3b^5x^4+35a^4b^4x^2-\frac{14a^6b^2}{x^2}+56a^5b^3\log(x)-\frac{2a^7b}{x^4}-\frac{a^8}{6x^6}+ab^7x^8+\frac{b^8x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^7,x]
```

```
[Out] -a^8/(6*x^6) - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b
^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^10)/10 + 56*a^5*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^7} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^4} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(70a^4b^4+\frac{a^8}{x^4}+\frac{8a^7b}{x^3}+\frac{28a^6b^2}{x^2}+\frac{56a^5b^3}{x}+56a^3b^5x+28a^2b^6x^2+8ab^7x^3+b^8x\right.\right.\\ &= -\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}+56a^5b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 94, normalized size = 1.00

$$-\frac{a^8}{6x^6}-\frac{2a^7b}{x^4}-\frac{14a^6b^2}{x^2}+56a^5b^3\log(x)+35a^4b^4x^2+14a^3b^5x^4+\frac{14}{3}a^2b^6x^6+ab^7x^8+\frac{b^8x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^7,x]
```

```
[Out] -1/6*a^8/x^6 - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b
^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^10)/10 + 56*a^5*b^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^8}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^7, x]
```

fricas [A] time = 1.29, size = 94, normalized size = 1.00

$$\frac{3\,b^8x^{16}+30\,ab^7x^{14}+140\,a^2b^6x^{12}+420\,a^3b^5x^{10}+1050\,a^4b^4x^8+1680\,a^5b^3x^6\log(x)-420\,a^6b^2x^4-60\,a^7bx^2-5\,a^8}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="fricas")
```

```
[Out] 1/30*(3*b^8*x^16 + 30*a*b^7*x^14 + 140*a^2*b^6*x^12 + 420*a^3*b^5*x^10 + 10
50*a^4*b^4*x^8 + 1680*a^5*b^3*x^6*log(x) - 420*a^6*b^2*x^4 - 60*a^7*b*x^2 -
 5*a^8)/x^6
```

giac [A] time = 1.05, size = 102, normalized size = 1.09

$$\frac{1}{10}\,b^8x^{10}+ab^7x^8+\frac{14}{3}\,a^2b^6x^6+14\,a^3b^5x^4+35\,a^4b^4x^2+28\,a^5b^3\log\left(x^2\right)-\frac{308\,a^5b^3x^6+84\,a^6b^2x^4+12\,a^7bx^2+a^8}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="giac")
```

```
[Out] 1/10*b^8*x^10 + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4*
x^2 + 28*a^5*b^3*log(x^2) - 1/6*(308*a^5*b^3*x^6 + 84*a^6*b^2*x^4 + 12*a^7*
b*x^2 + a^8)/x^6
```

maple [A] time = 0.01, size = 89, normalized size = 0.95

$$\frac{b^8x^{10}}{10}+a\,b^7x^8+\frac{14a^2b^6x^6}{3}+14a^3b^5x^4+35a^4b^4x^2+56a^5b^3\ln(x)-\frac{14a^6b^2}{x^2}-\frac{2a^7b}{x^4}-\frac{a^8}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^7,x)
```

```
[Out] -1/6*a^8/x^6-2*a^7*b/x^4-14*a^6*b^2/x^2+35*a^4*b^4*x^2+14*a^3*b^5*x^4+14/3*
a^2*b^6*x^6+a*b^7*x^8+1/10*b^8*x^10+56*a^5*b^3*ln(x)
```

maxima [A] time = 1.35, size = 91, normalized size = 0.97

$$\frac{1}{10}\,b^8x^{10}+ab^7x^8+\frac{14}{3}\,a^2b^6x^6+14\,a^3b^5x^4+35\,a^4b^4x^2+28\,a^5b^3\log\left(x^2\right)-\frac{84\,a^6b^2x^4+12\,a^7bx^2+a^8}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^8*x^10 + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4*
x^2 + 28*a^5*b^3*log(x^2) - 1/6*(84*a^6*b^2*x^4 + 12*a^7*b*x^2 + a^8)/x^6
```

mupad [B] time = 0.05, size = 91, normalized size = 0.97

$$\frac{b^8\,x^{10}}{10}-\frac{\frac{a^8}{6}+2\,a^7\,b\,x^2+14\,a^6\,b^2\,x^4}{x^6}+a\,b^7\,x^8+35\,a^4\,b^4\,x^2+14\,a^3\,b^5\,x^4+\frac{14\,a^2\,b^6\,x^6}{3}+56\,a^5\,b^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^7,x)
```

```
[Out] (b^8*x^10)/10 - (a^8/6 + 2*a^7*b*x^2 + 14*a^6*b^2*x^4)/x^6 + a*b^7*x^8 + 35
*a^4*b^4*x^2 + 14*a^3*b^5*x^4 + (14*a^2*b^6*x^6)/3 + 56*a^5*b^3*log(x)
```

sympy [A] time = 0.33, size = 97, normalized size = 1.03

$$56a^5b^3\log(x)+35a^4b^4x^2+14a^3b^5x^4+\frac{14a^2b^6x^6}{3}+ab^7x^8+\frac{b^8x^{10}}{10}+\frac{-a^8-12a^7bx^2-84a^6b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**7,x)
```

```
[Out] 56*a**5*b**3*log(x) + 35*a**4*b**4*x**2 + 14*a**3*b**5*x**4 + 14*a**2*b**6*
x**6/3 + a*b**7*x**8 + b**8*x**10/10 + (-a**8 - 12*a**7*b*x**2 - 84*a**6*b*
*2*x**4)/(6*x**6)
```

3.96 $\int \frac{\left(a+bx^2\right)^8}{x^9} \, dx$

Optimal. Leaf size=97

$$-\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+70a^4b^4\log(x)+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}$$

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+70a^4b^4\log(x)-\frac{4a^7b}{3x^6}-\frac{a^8}{8x^8}+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^9,x]
```

```
[Out] -a^8/(8*x^8) - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*
a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Lo
g[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^8}{x^9} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^5} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(56a^3b^5+\frac{a^8}{x^5}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^3}+\frac{56a^5b^3}{x^2}+\frac{70a^4b^4}{x}+28a^2b^6x+8ab^7x^2+b^8x^3\right) dx, x, x^2\right)\\
&= -\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}+70a^4b^4\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 97, normalized size = 1.00

$$-\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+70a^4b^4\log(x)+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^9,x]
```

```
[Out] -1/8*a^8/x^8 - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*
a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Lo
g[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^9, x]
```

fricas [A] time = 1.44, size = 94, normalized size = 0.97

$$\frac{3\,b^8x^{16}+32\,ab^7x^{14}+168\,a^2b^6x^{12}+672\,a^3b^5x^{10}+1680\,a^4b^4x^8\log(x)-672\,a^5b^3x^6-168\,a^6b^2x^4-32\,a^7bx^2-3\,a^8}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^8*x^16 + 32*a*b^7*x^14 + 168*a^2*b^6*x^12 + 672*a^3*b^5*x^10 + 16
80*a^4*b^4*x^8*log(x) - 672*a^5*b^3*x^6 - 168*a^6*b^2*x^4 - 32*a^7*b*x^2 -
3*a^8)/x^8
```

giac [A] time = 1.00, size = 105, normalized size = 1.08

$$\frac{1}{8}\,b^8x^8+\frac{4}{3}\,ab^7x^6+7\,a^2b^6x^4+28\,a^3b^5x^2+35\,a^4b^4\log\left(x^2\right)-\frac{1750\,a^4b^4x^8+672\,a^5b^3x^6+168\,a^6b^2x^4+32\,a^7bx^2+3\,a^8}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="giac")
```

```
[Out] 1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*l
og(x^2) - 1/24*(1750*a^4*b^4*x^8 + 672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32*a
^7*b*x^2 + 3*a^8)/x^8
```

maple [A] time = 0.01, size = 90, normalized size = 0.93

$$\frac{b^8x^8}{8}+\frac{4a\,b^7x^6}{3}+7a^2b^6x^4+28a^3b^5x^2+70a^4b^4\ln(x)-\frac{28a^5b^3}{x^2}-\frac{7a^6b^2}{x^4}-\frac{4a^7b}{3x^6}-\frac{a^8}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^9,x)
```

```
[Out] -1/8*a^8/x^8-4/3*a^7*b/x^6-7*a^6*b^2/x^4-28*a^5*b^3/x^2+28*a^3*b^5*x^2+7*a^
2*b^6*x^4+4/3*a*b^7*x^6+1/8*b^8*x^8+70*a^4*b^4*ln(x)
```

maxima [A] time = 1.39, size = 94, normalized size = 0.97

$$\frac{1}{8}\,b^8x^8+\frac{4}{3}\,ab^7x^6+7\,a^2b^6x^4+28\,a^3b^5x^2+35\,a^4b^4\log\left(x^2\right)-\frac{672\,a^5b^3x^6+168\,a^6b^2x^4+32\,a^7bx^2+3\,a^8}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^9,x, algorithm="maxima")
```

```
[Out] 1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*l
og(x^2) - 1/24*(672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32*a^7*b*x^2 + 3*a^8)/x
^8
```

mupad [B] time = 0.05, size = 92, normalized size = 0.95

$$\frac{b^8\,x^8}{8}-\frac{\frac{a^8}{8}+\frac{4\,a^7\,b\,x^2}{3}+7\,a^6\,b^2\,x^4+28\,a^5\,b^3\,x^6}{x^8}+\frac{4\,a\,b^7\,x^6}{3}+28\,a^3\,b^5\,x^2+7\,a^2\,b^6\,x^4+70\,a^4\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^9,x)
```

```
[Out] (b^8*x^8)/8 - (a^8/8 + (4*a^7*b*x^2)/3 + 7*a^6*b^2*x^4 + 28*a^5*b^3*x^6)/x^
8 + (4*a*b^7*x^6)/3 + 28*a^3*b^5*x^2 + 7*a^2*b^6*x^4 + 70*a^4*b^4*log(x)
```

sympy [A] time = 0.42, size = 100, normalized size = 1.03

$$70a^4b^4\log{(x)}+28a^3b^5x^2+7a^2b^6x^4+\frac{4ab^7x^6}{3}+\frac{b^8x^8}{8}+\frac{-3a^8-32a^7bx^2-168a^6b^2x^4-672a^5b^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**9,x)
```

```
[Out] 70*a**4*b**4*log(x) + 28*a**3*b**5*x**2 + 7*a**2*b**6*x**4 + 4*a*b**7*x**6/
3 + b**8*x**8/8 + (-3*a**8 - 32*a**7*b*x**2 - 168*a**6*b**2*x**4 - 672*a**5
*b**3*x**6)/(24*x**8)
```

3.97 $\int \frac{(a+bx^2)^8}{x^{11}} \, dx$

Optimal. Leaf size=95

$$-\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+56a^3b^5\log(x)+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}$$

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+56a^3b^5\log(x)-\frac{a^7b}{x^8}-\frac{a^8}{10x^{10}}+2ab^7x^4+\frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^11,x]
```

```
[Out] -a^8/(10*x^10) - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (3
5*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Lo
g[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{11}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^6} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(28a^2b^6+\frac{a^8}{x^6}+\frac{8a^7b}{x^5}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^3}+\frac{70a^4b^4}{x^2}+\frac{56a^3b^5}{x}+8ab^7x+b^8x^2\right) d\right.\\
&= -\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}+56a^3b^5\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 95, normalized size = 1.00

$$-\frac{a^8}{10x^{10}}-\frac{a^7b}{x^8}-\frac{14a^6b^2}{3x^6}-\frac{14a^5b^3}{x^4}-\frac{35a^4b^4}{x^2}+56a^3b^5\log(x)+14a^2b^6x^2+2ab^7x^4+\frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^11,x]
```

```
[Out] -1/10*a^8/x^10 - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (3
5*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Lo
g[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2\right)^8}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^11,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^11, x]
```

fricas [A] time = 1.38, size = 94, normalized size = 0.99

$$\frac{5\,b^8x^{16}+60\,ab^7x^{14}+420\,a^2b^6x^{12}+1680\,a^3b^5x^{10}\log(x)-1050\,a^4b^4x^8-420\,a^5b^3x^6-140\,a^6b^2x^4-30\,a^7bx^2-3\,a^8}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="fricas")
```

```
[Out] 1/30*(5*b^8*x^16 + 60*a*b^7*x^14 + 420*a^2*b^6*x^12 + 1680*a^3*b^5*x^10*log
(x) - 1050*a^4*b^4*x^8 - 420*a^5*b^3*x^6 - 140*a^6*b^2*x^4 - 30*a^7*b*x^2 -
 3*a^8)/x^10
```

giac [A] time = 1.13, size = 105, normalized size = 1.11

$$\frac{1}{6}\,b^8x^6+2\,ab^7x^4+14\,a^2b^6x^2+28\,a^3b^5\log\left(x^2\right)-\frac{1918\,a^3b^5x^{10}+1050\,a^4b^4x^8+420\,a^5b^3x^6+140\,a^6b^2x^4+30\,a^7bx^2+3\,a^8}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="giac")
```

```
[Out] 1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*log(x^2) - 1/30*(19
18*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 + 30
*a^7*b*x^2 + 3*a^8)/x^10
```

maple [A] time = 0.01, size = 90, normalized size = 0.95

$$\frac{b^8x^6}{6} + 2a\,b^7x^4 + 14a^2b^6x^2 + 56a^3b^5\ln(x) - \frac{35a^4b^4}{x^2} - \frac{14a^5b^3}{x^4} - \frac{14a^6b^2}{3x^6} - \frac{a^7b}{x^8} - \frac{a^8}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^11,x)
```

```
[Out] -1/10*a^8/x^10-a^7*b/x^8-14/3*a^6*b^2/x^6-14*a^5*b^3/x^4-35*a^4*b^4/x^2+14*
a^2*b^6*x^2+2*a*b^7*x^4+1/6*b^8*x^6+56*a^3*b^5*ln(x)
```

maxima [A] time = 1.33, size = 94, normalized size = 0.99

$$\frac{1}{6}\,b^8x^6 + 2\,ab^7x^4 + 14\,a^2b^6x^2 + 28\,a^3b^5\log\left(x^2\right) - \frac{1050\,a^4b^4x^8 + 420\,a^5b^3x^6 + 140\,a^6b^2x^4 + 30\,a^7bx^2 + 3\,a^8}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^11,x, algorithm="maxima")
```

```
[Out] 1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*log(x^2) - 1/30*(10
50*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 + 30*a^7*b*x^2 + 3*a^8)/
x^10
```

mupad [B] time = 5.11, size = 91, normalized size = 0.96

$$\frac{b^8\,x^6}{6} - \frac{\frac{a^8}{10} + a^7\,b\,x^2 + \frac{14\,a^6\,b^2\,x^4}{3} + 14\,a^5\,b^3\,x^6 + 35\,a^4\,b^4\,x^8}{x^{10}} + 2\,a\,b^7\,x^4 + 14\,a^2\,b^6\,x^2 + 56\,a^3\,b^5\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^11,x)
```

```
[Out] (b^8*x^6)/6 - (a^8/10 + a^7*b*x^2 + (14*a^6*b^2*x^4)/3 + 14*a^5*b^3*x^6 + 3
5*a^4*b^4*x^8)/x^10 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 56*a^3*b^5*log(x)
```

sympy [A] time = 0.51, size = 99, normalized size = 1.04

$$56a^3b^5\log(x) + 14a^2b^6x^2 + 2ab^7x^4 + \frac{b^8x^6}{6} + \frac{-3a^8 - 30a^7bx^2 - 140a^6b^2x^4 - 420a^5b^3x^6 - 1050a^4b^4x^8}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**11,x)
```

```
[Out] 56*a**3*b**5*log(x) + 14*a**2*b**6*x**2 + 2*a*b**7*x**4 + b**8*x**6/6 + (-3
*a**8 - 30*a**7*b*x**2 - 140*a**6*b**2*x**4 - 420*a**5*b**3*x**6 - 1050*a**
4*b**4*x**8)/(30*x**10)
```

3.98 $\int \frac{(a+bx^2)^8}{x^{13}} \, dx$

Optimal. Leaf size=101

$$-\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+28a^2b^6\log(x)+4ab^7x^2+\frac{b^8x^4}{4}$$

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+28a^2b^6\log(x)-\frac{4a^7b}{5x^{10}}-\frac{a^8}{12x^{12}}+4ab^7x^2+\frac{b^8x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^13,x]
```

```
[Out] -a^8/(12*x^12) - (4*a^7*b)/(5*x^10) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3
*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a*b^7*x^2 + (b^8*x^4)/4
 + 28*a^2*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{13}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^7} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(8ab^7+\frac{a^8}{x^7}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^5}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^3}+\frac{56a^3b^5}{x^2}+\frac{28a^2b^6}{x}+b^8x\right) d\\
&= -\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+4ab^7x^2+\frac{b^8x^4}{4}+28a^2b^6\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 101, normalized size = 1.00

$$-\frac{a^8}{12x^{12}}-\frac{4a^7b}{5x^{10}}-\frac{7a^6b^2}{2x^8}-\frac{28a^5b^3}{3x^6}-\frac{35a^4b^4}{2x^4}-\frac{28a^3b^5}{x^2}+28a^2b^6\log(x)+4ab^7x^2+\frac{b^8x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^13,x]
```

```
[Out] -1/12*a^8/x^12 - (4*a^7*b)/(5*x^10) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3
*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a*b^7*x^2 + (b^8*x^4)/4
 + 28*a^2*b^6*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^13,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^13, x]
```

fricas [A] time = 0.81, size = 94, normalized size = 0.93

$$\frac{15\,b^8x^{16}+240\,ab^7x^{14}+1680\,a^2b^6x^{12}\log(x)-1680\,a^3b^5x^{10}-1050\,a^4b^4x^8-560\,a^5b^3x^6-210\,a^6b^2x^4-48\,a^7bx^2-5\,a^8}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="fricas")
```

```
[Out] 1/60*(15*b^8*x^16 + 240*a*b^7*x^14 + 1680*a^2*b^6*x^12*log(x) - 1680*a^3*b^
5*x^10 - 1050*a^4*b^4*x^8 - 560*a^5*b^3*x^6 - 210*a^6*b^2*x^4 - 48*a^7*b*x^
2 - 5*a^8)/x^12
```

giac [A] time = 1.21, size = 105, normalized size = 1.04

$$\frac{1}{4}\,b^8x^4+4\,ab^7x^2+14\,a^2b^6\log\left(x^2\right)-\frac{2058\,a^2b^6x^{12}+1680\,a^3b^5x^{10}+1050\,a^4b^4x^8+560\,a^5b^3x^6+210\,a^6b^2x^4+48\,a^7bx^2+5\,a^8}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="giac")
```

```
[Out] 1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*log(x^2) - 1/60*(2058*a^2*b^6*x^12 +
 1680*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 +
 48*a^7*b*x^2 + 5*a^8)/x^12
```

maple [A] time = 0.01, size = 90, normalized size = 0.89

$$\frac{b^8x^4}{4} + 4a\,b^7x^2 + 28a^2b^6\ln(x) - \frac{28a^3b^5}{x^2} - \frac{35a^4b^4}{2x^4} - \frac{28a^5b^3}{3x^6} - \frac{7a^6b^2}{2x^8} - \frac{4a^7b}{5x^{10}} - \frac{a^8}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^13,x)
```

```
[Out] -1/12*a^8/x^12-4/5*a^7*b/x^10-7/2*a^6*b^2/x^8-28/3*a^5*b^3/x^6-35/2*a^4*b^4
/x^4-28*a^3*b^5/x^2+4*a*b^7*x^2+1/4*b^8*x^4+28*a^2*b^6*ln(x)
```

maxima [A] time = 1.38, size = 94, normalized size = 0.93

$$\frac{1}{4}\,b^8x^4 + 4\,ab^7x^2 + 14\,a^2b^6\log\left(x^2\right) - \frac{1680\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 560\,a^5b^3x^6 + 210\,a^6b^2x^4 + 48\,a^7bx^2 + 5\,a^8}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^13,x, algorithm="maxima")
```

```
[Out] 1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*log(x^2) - 1/60*(1680*a^3*b^5*x^10 +
 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 + 48*a^7*b*x^2 + 5*a^
8)/x^12
```

mupad [B] time = 0.06, size = 92, normalized size = 0.91

$$\frac{b^8\,x^4}{4} - \frac{\frac{a^8}{12} + \frac{4\,a^7\,b\,x^2}{5} + \frac{7\,a^6\,b^2\,x^4}{2} + \frac{28\,a^5\,b^3\,x^6}{3} + \frac{35\,a^4\,b^4\,x^8}{2} + 28\,a^3\,b^5\,x^{10}}{x^{12}} + 4\,a\,b^7\,x^2 + 28\,a^2\,b^6\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^13,x)
```

```
[Out] (b^8*x^4)/4 - (a^8/12 + (4*a^7*b*x^2)/5 + (7*a^6*b^2*x^4)/2 + (28*a^5*b^3*x
^6)/3 + (35*a^4*b^4*x^8)/2 + 28*a^3*b^5*x^10)/x^12 + 4*a*b^7*x^2 + 28*a^2*b
^6*log(x)
```

sympy [A] time = 0.62, size = 99, normalized size = 0.98

$$28a^2b^6\log(x) + 4ab^7x^2 + \frac{b^8x^4}{4} + \frac{-5a^8 - 48a^7bx^2 - 210a^6b^2x^4 - 560a^5b^3x^6 - 1050a^4b^4x^8 - 1680a^3b^5x^{10}}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**13,x)
```

```
[Out] 28*a**2*b**6*log(x) + 4*a*b**7*x**2 + b**8*x**4/4 + (-5*a**8 - 48*a**7*b*x*
*2 - 210*a**6*b**2*x**4 - 560*a**5*b**3*x**6 - 1050*a**4*b**4*x**8 - 1680*a
**3*b**5*x**10)/(60*x**12)
```

3.99 $\int \frac{\left(a+bx^2\right)^8}{x^{15}} \, dx$

Optimal. Leaf size=99

$$-\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+8ab^7\log(x)+\frac{b^8x^2}{2}$$

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}-\frac{2a^7b}{3x^{12}}-\frac{a^8}{14x^{14}}+8ab^7\log(x)+\frac{b^8x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^15,x]
```

```
[Out] -a^8/(14*x^14) - (2*a^7*b)/(3*x^12) - (14*a^6*b^2)/(5*x^10) - (7*a^5*b^3)/x
^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)
/2 + 8*a*b^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^8}{x^{15}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^8} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^8+\frac{a^8}{x^8}+\frac{8a^7b}{x^7}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^5}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^3}+\frac{28a^2b^6}{x^2}+\frac{8ab^7}{x}\right) dx,\\
&= -\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+\frac{b^8x^2}{2}+8ab^7\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 99, normalized size = 1.00

$$-\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+8ab^7\log(x)+\frac{b^8x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^15,x]
```

```
[Out] -1/14*a^8/x^14 - (2*a^7*b)/(3*x^12) - (14*a^6*b^2)/(5*x^10) - (7*a^5*b^3)/x
^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)
/2 + 8*a*b^7*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^8}{x^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^15,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^15, x]
```

fricas [A] time = 1.54, size = 94, normalized size = 0.95

$$\frac{105\,b^8x^{16}+1680\,ab^7x^{14}\log(x)-2940\,a^2b^6x^{12}-2940\,a^3b^5x^{10}-2450\,a^4b^4x^8-1470\,a^5b^3x^6-588\,a^6b^2x^4-140\,a^7bx^2-15\,a^8}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="fricas")
```

```
[Out] 1/210*(105*b^8*x^16 + 1680*a*b^7*x^14*log(x) - 2940*a^2*b^6*x^12 - 2940*a^3
*b^5*x^10 - 2450*a^4*b^4*x^8 - 1470*a^5*b^3*x^6 - 588*a^6*b^2*x^4 - 140*a^7
*b*x^2 - 15*a^8)/x^14
```

giac [A] time = 1.08, size = 103, normalized size = 1.04

$$\frac{1}{2}\,b^8x^2+4\,ab^7\log\left(x^2\right)-\frac{2178\,ab^7x^{14}+2940\,a^2b^6x^{12}+2940\,a^3b^5x^{10}+2450\,a^4b^4x^8+1470\,a^5b^3x^6+588\,a^6b^2x^4+140\,a^7bx^2+15\,a^8}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="giac")
```

```
[Out] 1/2*b^8*x^2 + 4*a*b^7*log(x^2) - 1/210*(2178*a*b^7*x^14 + 2940*a^2*b^6*x^12
 + 2940*a^3*b^5*x^10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*x^
4 + 140*a^7*b*x^2 + 15*a^8)/x^14
```

maple [A] time = 0.01, size = 90, normalized size = 0.91

$$\frac{b^8x^2}{2}+8a\,b^7\ln(x)-\frac{14a^2b^6}{x^2}-\frac{14a^3b^5}{x^4}-\frac{35a^4b^4}{3x^6}-\frac{7a^5b^3}{x^8}-\frac{14a^6b^2}{5x^{10}}-\frac{2a^7b}{3x^{12}}-\frac{a^8}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^15,x)
```

```
[Out] -1/14*a^8/x^14-2/3*a^7*b/x^12-14/5*a^6*b^2/x^10-7*a^5*b^3/x^8-35/3*a^4*b^4/
x^6-14*a^3*b^5/x^4-14*a^2*b^6/x^2+1/2*b^8*x^2+8*a*b^7*ln(x)
```

maxima [A] time = 1.35, size = 94, normalized size = 0.95

$$\frac{1}{2}\,b^8x^2+4\,ab^7\log\left(x^2\right)-\frac{2940\,a^2b^6x^{12}+2940\,a^3b^5x^{10}+2450\,a^4b^4x^8+1470\,a^5b^3x^6+588\,a^6b^2x^4+140\,a^7bx^2+15\,a^8}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^15,x, algorithm="maxima")
```

```
[Out] 1/2*b^8*x^2 + 4*a*b^7*log(x^2) - 1/210*(2940*a^2*b^6*x^12 + 2940*a^3*b^5*x^
10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*x^4 + 140*a^7*b*x^2
+ 15*a^8)/x^14
```

mupad [B] time = 5.15, size = 94, normalized size = 0.95

$$-\frac{\frac{a^8}{14}-\frac{b^8\,x^{16}}{2}+\frac{2\,a^7\,b\,x^2}{3}+\frac{14\,a^6\,b^2\,x^4}{5}+7\,a^5\,b^3\,x^6+\frac{35\,a^4\,b^4\,x^8}{3}+14\,a^3\,b^5\,x^{10}+14\,a^2\,b^6\,x^{12}-8\,a\,b^7\,x^{14}\,\ln(x)}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^15,x)
```

```
[Out] -(a^8/14 - (b^8*x^16)/2 + (2*a^7*b*x^2)/3 + (14*a^6*b^2*x^4)/5 + 7*a^5*b^3*
x^6 + (35*a^4*b^4*x^8)/3 + 14*a^3*b^5*x^10 + 14*a^2*b^6*x^12 - 8*a*b^7*x^14
*log(x))/x^14
```

sympy [A] time = 0.73, size = 99, normalized size = 1.00

$$8ab^7\log\left(x\right)+\frac{b^8x^2}{2}+\frac{-15a^8-140a^7bx^2-588a^6b^2x^4-1470a^5b^3x^6-2450a^4b^4x^8-2940a^3b^5x^{10}-2940a^2b^6x^{12}}{210x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**15,x)
```

```
[Out] 8*a*b**7*log(x) + b**8*x**2/2 + (-15*a**8 - 140*a**7*b*x**2 - 588*a**6*b**2
*x**4 - 1470*a**5*b**3*x**6 - 2450*a**4*b**4*x**8 - 2940*a**3*b**5*x**10 -
2940*a**2*b**6*x**12)/(210*x**14)
```

3.100 $\int \frac{(a+bx^2)^8}{x^{17}}\,dx$

Optimal. Leaf size=100

$$-\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)$$

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4a^7b}{7x^{14}}-\frac{a^8}{16x^{16}}-\frac{4ab^7}{x^2}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^17,x]
```

```
[Out] -a^8/(16*x^16) - (4*a^7*b)/(7*x^14) - (7*a^6*b^2)/(3*x^12) - (28*a^5*b^3)/(
5*x^10) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (
4*a*b^7)/x^2 + b^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{17}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^9}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^9}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^7}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^5}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^3}+\frac{8ab^7}{x^2}+\frac{b^8}{x}\right) d\right.\\
&= -\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 100, normalized size = 1.00

$$-\frac{a^8}{16x^{16}}-\frac{4a^7b}{7x^{14}}-\frac{7a^6b^2}{3x^{12}}-\frac{28a^5b^3}{5x^{10}}-\frac{35a^4b^4}{4x^8}-\frac{28a^3b^5}{3x^6}-\frac{7a^2b^6}{x^4}-\frac{4ab^7}{x^2}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^17,x]
```

```
[Out] -1/16*a^8/x^16 - (4*a^7*b)/(7*x^14) - (7*a^6*b^2)/(3*x^12) - (28*a^5*b^3)/(
5*x^10) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (
4*a*b^7)/x^2 + b^8*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^17,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^17, x]
```

fricas [A] time = 0.90, size = 94, normalized size = 0.94

$$\frac{1680\,b^8x^{16}\log(x)-6720\,ab^7x^{14}-11760\,a^2b^6x^{12}-15680\,a^3b^5x^{10}-14700\,a^4b^4x^8-9408\,a^5b^3x^6-3920\,a^6b^2x^4-960\,a^7bx^2-105\,a^8}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="fricas")
```

```
[Out] 1/1680*(1680*b^8*x^16*log(x) - 6720*a*b^7*x^14 - 11760*a^2*b^6*x^12 - 15680
*a^3*b^5*x^10 - 14700*a^4*b^4*x^8 - 9408*a^5*b^3*x^6 - 3920*a^6*b^2*x^4 - 9
60*a^7*b*x^2 - 105*a^8)/x^16
```

giac [A] time = 1.06, size = 102, normalized size = 1.02

$$\frac{1}{2}\,b^8\log\left(x^2\right)-\frac{2283\,b^8x^{16}+6720\,ab^7x^{14}+11760\,a^2b^6x^{12}+15680\,a^3b^5x^{10}+14700\,a^4b^4x^8+9408\,a^5b^3x^6+3920\,a^6b^2x^4+960\,a^7bx^2+105\,a^8}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="giac")
```

```
[Out] 1/2*b^8*log(x^2) - 1/1680*(2283*b^8*x^16 + 6720*a*b^7*x^14 + 11760*a^2*b^6*
x^12 + 15680*a^3*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a^6
*b^2*x^4 + 960*a^7*b*x^2 + 105*a^8)/x^16
```

maple [A] time = 0.01, size = 89, normalized size = 0.89

$$b^8\ln(x)-\frac{4a\,b^7}{x^2}-\frac{7a^2b^6}{x^4}-\frac{28a^3b^5}{3x^6}-\frac{35a^4b^4}{4x^8}-\frac{28a^5b^3}{5x^{10}}-\frac{7a^6b^2}{3x^{12}}-\frac{4a^7b}{7x^{14}}-\frac{a^8}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^17,x)
```

```
[Out] -1/16*a^8/x^16-4/7*a^7*b/x^14-7/3*a^6*b^2/x^12-28/5*a^5*b^3/x^10-35/4*a^4*b
^4/x^8-28/3*a^3*b^5/x^6-7*a^2*b^6/x^4-4*a*b^7/x^2+b^8*ln(x)
```

maxima [A] time = 1.36, size = 94, normalized size = 0.94

$$\frac{1}{2}\,b^8\log\left(x^2\right)-\frac{6720\,ab^7x^{14}+11760\,a^2b^6x^{12}+15680\,a^3b^5x^{10}+14700\,a^4b^4x^8+9408\,a^5b^3x^6+3920\,a^6b^2x^4+960\,a^7bx^2+105\,a^8}{1680\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^17,x, algorithm="maxima")
```

```
[Out] 1/2*b^8*log(x^2) - 1/1680*(6720*a*b^7*x^14 + 11760*a^2*b^6*x^12 + 15680*a^3
*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a^6*b^2*x^4 + 960*a
^7*b*x^2 + 105*a^8)/x^16
```

mupad [B] time = 5.09, size = 91, normalized size = 0.91

$$b^8\,\ln(x)-\frac{\frac{a^8}{16}+\frac{4\,a^7\,b\,x^2}{7}+\frac{7\,a^6\,b^2\,x^4}{3}+\frac{28\,a^5\,b^3\,x^6}{5}+\frac{35\,a^4\,b^4\,x^8}{4}+\frac{28\,a^3\,b^5\,x^{10}}{3}+7\,a^2\,b^6\,x^{12}+4\,a\,b^7\,x^{14}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^17,x)
```

```
[Out] b^8*log(x) - (a^8/16 + (4*a^7*b*x^2)/7 + 4*a*b^7*x^14 + (7*a^6*b^2*x^4)/3 +
 (28*a^5*b^3*x^6)/5 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^10)/3 + 7*a^2*b^6*
x^12)/x^16
```

sympy [A] time = 0.83, size = 97, normalized size = 0.97

$$b^8\log\left(x\right)+\frac{-105a^8-960a^7bx^2-3920a^6b^2x^4-9408a^5b^3x^6-14700a^4b^4x^8-15680a^3b^5x^{10}-11760a^2b^6x^{12}-6720ab^7x^{14}}{1680x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**17,x)
```

```
[Out] b**8*log(x) + (-105*a**8 - 960*a**7*b*x**2 - 3920*a**6*b**2*x**4 - 9408*a**
5*b**3*x**6 - 14700*a**4*b**4*x**8 - 15680*a**3*b**5*x**10 - 11760*a**2*b**
6*x**12 - 6720*a*b**7*x**14)/(1680*x**16)
```

3.101 $\int \frac{(a+bx^2)^8}{x^{19}}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.077, Rules used = {264}

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^19,x]
```

```
[Out] -(a + b*x^2)^9/(18*a*x^18)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx = -\frac{(a+bx^2)^9}{18ax^{18}}$$

Mathematica [B] time = 0.00, size = 100, normalized size = 5.26

$$-\frac{a^8}{18x^{18}}-\frac{a^7b}{2x^{16}}-\frac{2a^6b^2}{x^{14}}-\frac{14a^5b^3}{3x^{12}}-\frac{7a^4b^4}{x^{10}}-\frac{7a^3b^5}{x^8}-\frac{14a^2b^6}{3x^6}-\frac{2ab^7}{x^4}-\frac{b^8}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^19,x]
```

```
[Out] -1/18*a^8/x^18 - (a^7*b)/(2*x^16) - (2*a^6*b^2)/x^14 - (14*a^5*b^3)/(3*x^12
) - (7*a^4*b^4)/x^10 - (7*a^3*b^5)/x^8 - (14*a^2*b^6)/(3*x^6) - (2*a*b^7)/x
^4 - b^8/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^19,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^19, x]
```

fricas [B] time = 0.77, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="fricas")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

giac [B] time = 1.07, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

maple [B] time = 0.01, size = 91, normalized size = 4.79

$$-\frac{b^8}{2x^2}-\frac{2a\,b^7}{x^4}-\frac{14a^2b^6}{3x^6}-\frac{7a^3b^5}{x^8}-\frac{7a^4b^4}{x^{10}}-\frac{14a^5b^3}{3x^{12}}-\frac{2a^6b^2}{x^{14}}-\frac{a^7b}{2x^{16}}-\frac{a^8}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^19,x)
```

```
[Out] -14/3*a^2*b^6/x^6-1/18*a^8/x^18-1/2*a^7*b/x^16-1/2*b^8/x^2-2*a*b^7/x^4-7*a^
4*b^4/x^10-14/3*a^5*b^3/x^12-7*a^3*b^5/x^8-2*a^6*b^2/x^14
```

maxima [B] time = 1.32, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^19,x, algorithm="maxima")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

mupad [B] time = 0.08, size = 92, normalized size = 4.84

$$-\frac{\frac{a^8}{18}+\frac{a^7\,b\,x^2}{2}+2\,a^6\,b^2\,x^4+\frac{14\,a^5\,b^3\,x^6}{3}+7\,a^4\,b^4\,x^8+7\,a^3\,b^5\,x^{10}+\frac{14\,a^2\,b^6\,x^{12}}{3}+2\,a\,b^7\,x^{14}+\frac{b^8\,x^{16}}{2}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^19,x)
```

```
[Out] -(a^8/18 + (b^8*x^16)/2 + (a^7*b*x^2)/2 + 2*a*b^7*x^14 + 2*a^6*b^2*x^4 + (1
4*a^5*b^3*x^6)/3 + 7*a^4*b^4*x^8 + 7*a^3*b^5*x^10 + (14*a^2*b^6*x^12)/3)/x^
18
```

sympy [B] time = 0.90, size = 97, normalized size = 5.11

$$\frac{-a^8-9a^7bx^2-36a^6b^2x^4-84a^5b^3x^6-126a^4b^4x^8-126a^3b^5x^{10}-84a^2b^6x^{12}-36ab^7x^{14}-9b^8x^{16}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**19,x)
```

```
[Out] (-a**8 - 9*a**7*b*x**2 - 36*a**6*b**2*x**4 - 84*a**5*b**3*x**6 - 126*a**4*b
**4*x**8 - 126*a**3*b**5*x**10 - 84*a**2*b**6*x**12 - 36*a*b**7*x**14 - 9*b
**8*x**16)/(18*x**18)
```

3.102 $\int \frac{(a+bx^2)^8}{x^{21}} \, dx$

Optimal. Leaf size=40

$$\frac{b(a+bx^2)^9}{180a^2x^{18}} - \frac{(a+bx^2)^9}{20ax^{20}}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b(a+bx^2)^9}{180a^2x^{18}} - \frac{(a+bx^2)^9}{20ax^{20}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^21,x]
```

```
[Out] -(a + b*x^2)^9/(20*a*x^20) + (b*(a + b*x^2)^9)/(180*a^2*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)^9}{20ax^{20}} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^2\right)}{20a}$$
$$= -\frac{(a+bx^2)^9}{20ax^{20}} + \frac{b\,(a+bx^2)^9}{180a^2x^{18}}$$

Mathematica [B] time = 0.00, size = 106, normalized size = 2.65

$$-\frac{a^8}{20x^{20}} - \frac{4a^7b}{9x^{18}} - \frac{7a^6b^2}{4x^{16}} - \frac{4a^5b^3}{x^{14}} - \frac{35a^4b^4}{6x^{12}} - \frac{28a^3b^5}{5x^{10}} - \frac{7a^2b^6}{2x^8} - \frac{4ab^7}{3x^6} - \frac{b^8}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^21,x]
```

```
[Out] -1/20*a^8/x^20 - (4*a^7*b)/(9*x^18) - (7*a^6*b^2)/(4*x^16) - (4*a^5*b^3)/x^
14 - (35*a^4*b^4)/(6*x^12) - (28*a^3*b^5)/(5*x^10) - (7*a^2*b^6)/(2*x^8) -
(4*a*b^7)/(3*x^6) - b^8/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^21,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^21, x]
```

fricas [B] time = 0.65, size = 92, normalized size = 2.30

$$-\frac{45\,b^8x^{16} + 240\,ab^7x^{14} + 630\,a^2b^6x^{12} + 1008\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 720\,a^5b^3x^6 + 315\,a^6b^2x^4 + 80\,a^7bx^2 + 9\,a^8}{180\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="fricas")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

giac [B] time = 1.00, size = 92, normalized size = 2.30

$$-\frac{45\,b^8x^{16}+240\,ab^7x^{14}+630\,a^2b^6x^{12}+1008\,a^3b^5x^{10}+1050\,a^4b^4x^8+720\,a^5b^3x^6+315\,a^6b^2x^4+80\,a^7bx^2+9\,a^8}{180\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="giac")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

maple [B] time = 0.01, size = 91, normalized size = 2.28

$$-\frac{b^8}{4x^4}-\frac{4a\,b^7}{3x^6}-\frac{7a^2b^6}{2x^8}-\frac{28a^3b^5}{5x^{10}}-\frac{35a^4b^4}{6x^{12}}-\frac{4a^5b^3}{x^{14}}-\frac{7a^6b^2}{4x^{16}}-\frac{4a^7b}{9x^{18}}-\frac{a^8}{20x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^21,x)
```

```
[Out] -7/4*a^6*b^2/x^16-4/3*a*b^7/x^6-1/20*a^8/x^20-1/4*b^8/x^4-28/5*a^3*b^5/x^10
-4/9*a^7*b/x^18-35/6*a^4*b^4/x^12-7/2*a^2*b^6/x^8-4*a^5*b^3/x^14
```

maxima [B] time = 1.35, size = 92, normalized size = 2.30

$$-\frac{45\,b^8x^{16}+240\,ab^7x^{14}+630\,a^2b^6x^{12}+1008\,a^3b^5x^{10}+1050\,a^4b^4x^8+720\,a^5b^3x^6+315\,a^6b^2x^4+80\,a^7bx^2+9\,a^8}{180\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^21,x, algorithm="maxima")
```

```
[Out] -1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^10
 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 + 9*
a^8)/x^20
```

mupad [B] time = 0.08, size = 92, normalized size = 2.30

$$-\frac{\frac{a^8}{20}+\frac{4\,a^7\,b\,x^2}{9}+\frac{7\,a^6\,b^2\,x^4}{4}+4\,a^5\,b^3\,x^6+\frac{35\,a^4\,b^4\,x^8}{6}+\frac{28\,a^3\,b^5\,x^{10}}{5}+\frac{7\,a^2\,b^6\,x^{12}}{2}+\frac{4\,a\,b^7\,x^{14}}{3}+\frac{b^8\,x^{16}}{4}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^21,x)
```

```
[Out] -(a^8/20 + (b^8*x^16)/4 + (4*a^7*b*x^2)/9 + (4*a*b^7*x^14)/3 + (7*a^6*b^2*x
^4)/4 + 4*a^5*b^3*x^6 + (35*a^4*b^4*x^8)/6 + (28*a^3*b^5*x^10)/5 + (7*a^2*b
^6*x^12)/2)/x^20
```

sympy [B] time = 0.94, size = 99, normalized size = 2.48

$$\frac{-9a^8 - 80a^7bx^2 - 315a^6b^2x^4 - 720a^5b^3x^6 - 1050a^4b^4x^8 - 1008a^3b^5x^{10} - 630a^2b^6x^{12} - 240ab^7x^{14} - 45b^8x^{16}}{180x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**21,x)
```

```
[Out] (-9*a**8 - 80*a**7*b*x**2 - 315*a**6*b**2*x**4 - 720*a**5*b**3*x**6 - 1050*
a**4*b**4*x**8 - 1008*a**3*b**5*x**10 - 630*a**2*b**6*x**12 - 240*a*b**7*x*
*14 - 45*b**8*x**16)/(180*x**20)
```

3.103 $\int \frac{\left(a+bx^2\right)^8}{x^{23}} dx$

Optimal. Leaf size=62

$$-\frac{b^2\left(a+bx^2\right)^9}{990a^3x^{18}}+\frac{b\left(a+bx^2\right)^9}{110a^2x^{20}}-\frac{\left(a+bx^2\right)^9}{22ax^{22}}$$

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$-\frac{b^2\left(a+bx^2\right)^9}{990a^3x^{18}}+\frac{b\left(a+bx^2\right)^9}{110a^2x^{20}}-\frac{\left(a+bx^2\right)^9}{22ax^{22}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^23,x]
```

```
[Out] -(a + b*x^2)^9/(22*a*x^22) + (b*(a + b*x^2)^9)/(110*a^2*x^20) - (b^2*(a + b
*x^2)^9)/(990*a^3*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^8}{x^{23}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx,x,x^2\right)\\ &= -\frac{\left(a+bx^2\right)^9}{22ax^{22}}-\frac{b\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^2\right)}{11a}\\ &= -\frac{\left(a+bx^2\right)^9}{22ax^{22}}+\frac{b\left(a+bx^2\right)^9}{110a^2x^{20}}+\frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^2\right)}{110a^2}\\ &= -\frac{\left(a+bx^2\right)^9}{22ax^{22}}+\frac{b\left(a+bx^2\right)^9}{110a^2x^{20}}-\frac{b^2\left(a+bx^2\right)^9}{990a^3x^{18}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 104, normalized size = 1.68

$$-\frac{a^8}{22x^{22}}-\frac{2a^7b}{5x^{20}}-\frac{14a^6b^2}{9x^{18}}-\frac{7a^5b^3}{2x^{16}}-\frac{5a^4b^4}{x^{14}}-\frac{14a^3b^5}{3x^{12}}-\frac{14a^2b^6}{5x^{10}}-\frac{ab^7}{x^8}-\frac{b^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^23,x]
```

```
[Out] -1/22*a^8/x^22 - (2*a^7*b)/(5*x^20) - (14*a^6*b^2)/(9*x^18) - (7*a^5*b^3)/(
2*x^16) - (5*a^4*b^4)/x^14 - (14*a^3*b^5)/(3*x^12) - (14*a^2*b^6)/(5*x^10)
- (a*b^7)/x^8 - b^8/(6*x^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^{23}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^23,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^23, x]
```

fricas [A] time = 1.08, size = 92, normalized size = 1.48

$$-\frac{165\,b^8x^{16}+990\,ab^7x^{14}+2772\,a^2b^6x^{12}+4620\,a^3b^5x^{10}+4950\,a^4b^4x^8+3465\,a^5b^3x^6+1540\,a^6b^2x^4+396\,a^7bx^2+45\,a^8}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="fricas")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

giac [A] time = 1.06, size = 92, normalized size = 1.48

$$-\frac{165\,b^{8}x^{16} + 990\,ab^{7}x^{14} + 2772\,a^{2}b^{6}x^{12} + 4620\,a^{3}b^{5}x^{10} + 4950\,a^{4}b^{4}x^{8} + 3465\,a^{5}b^{3}x^{6} + 1540\,a^{6}b^{2}x^{4} + 396\,a^{7}bx^{2} + 45\,a^{8}}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="giac")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

maple [A] time = 0.01, size = 91, normalized size = 1.47

$$-\frac{b^{8}}{6x^{6}}-\frac{a\,b^{7}}{x^{8}}-\frac{14a^{2}b^{6}}{5x^{10}}-\frac{14a^{3}b^{5}}{3x^{12}}-\frac{5a^{4}b^{4}}{x^{14}}-\frac{7a^{5}b^{3}}{2x^{16}}-\frac{14a^{6}b^{2}}{9x^{18}}-\frac{2a^{7}b}{5x^{20}}-\frac{a^{8}}{22x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^23,x)
```

```
[Out] -1/6*b^8/x^6-7/2*a^5*b^3/x^16-2/5*a^7*b/x^20-14/9*a^6*b^2/x^18-14/5*a^2*b^6
/x^10-14/3*a^3*b^5/x^12-a*b^7/x^8-5*a^4*b^4/x^14-1/22*a^8/x^22
```

maxima [A] time = 1.36, size = 92, normalized size = 1.48

$$-\frac{165\,b^{8}x^{16} + 990\,ab^{7}x^{14} + 2772\,a^{2}b^{6}x^{12} + 4620\,a^{3}b^{5}x^{10} + 4950\,a^{4}b^{4}x^{8} + 3465\,a^{5}b^{3}x^{6} + 1540\,a^{6}b^{2}x^{4} + 396\,a^{7}bx^{2} + 45\,a^{8}}{990\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^23,x, algorithm="maxima")
```

```
[Out] -1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x^
10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2
 + 45*a^8)/x^22
```

mupad [B] time = 0.08, size = 91, normalized size = 1.47

$$-\frac{\frac{a^{8}}{22}+\frac{2\,a^{7}\,b\,x^{2}}{5}+\frac{14\,a^{6}\,b^{2}\,x^{4}}{9}+\frac{7\,a^{5}\,b^{3}\,x^{6}}{2}+5\,a^{4}\,b^{4}\,x^{8}+\frac{14\,a^{3}\,b^{5}\,x^{10}}{3}+\frac{14\,a^{2}\,b^{6}\,x^{12}}{5}+a\,b^{7}\,x^{14}+\frac{b^{8}\,x^{16}}{6}}{x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^23,x)
```

```
[Out] -(a^8/22 + (b^8*x^16)/6 + (2*a^7*b*x^2)/5 + a*b^7*x^14 + (14*a^6*b^2*x^4)/9
 + (7*a^5*b^3*x^6)/2 + 5*a^4*b^4*x^8 + (14*a^3*b^5*x^10)/3 + (14*a^2*b^6*x^
12)/5)/x^22
```

sympy [A] time = 0.99, size = 99, normalized size = 1.60

$$\frac{-45a^8 - 396a^7bx^2 - 1540a^6b^2x^4 - 3465a^5b^3x^6 - 4950a^4b^4x^8 - 4620a^3b^5x^{10} - 2772a^2b^6x^{12} - 990ab^7x^{14} - 165b^8x^{16}}{990x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**23,x)
```

```
[Out] (-45*a**8 - 396*a**7*b*x**2 - 1540*a**6*b**2*x**4 - 3465*a**5*b**3*x**6 - 4
950*a**4*b**4*x**8 - 4620*a**3*b**5*x**10 - 2772*a**2*b**6*x**12 - 990*a*b*
*7*x**14 - 165*b**8*x**16)/(990*x**22)
```

3.104 $\int \frac{(a+bx^2)^8}{x^{25}}\,dx$

Optimal. Leaf size=84

$$\frac{b^3(a+bx^2)^9}{3960a^4x^{18}}-\frac{b^2(a+bx^2)^9}{440a^3x^{20}}+\frac{b(a+bx^2)^9}{88a^2x^{22}}-\frac{(a+bx^2)^9}{24ax^{24}}$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{b^3(a+bx^2)^9}{3960a^4x^{18}}-\frac{b^2(a+bx^2)^9}{440a^3x^{20}}+\frac{b(a+bx^2)^9}{88a^2x^{22}}-\frac{(a+bx^2)^9}{24ax^{24}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^25,x]
```

```
[Out] -(a + b*x^2)^9/(24*a*x^24) + (b*(a + b*x^2)^9)/(88*a^2*x^22) - (b^2*(a + b*
x^2)^9)/(440*a^3*x^20) + (b^3*(a + b*x^2)^9)/(3960*a^4*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^8}{x^{25}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^9}{24ax^{24}}-\frac{b\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx,x,x^2\right)}{8a}\\
&= -\frac{\left(a+bx^2\right)^9}{24ax^{24}}+\frac{b\left(a+bx^2\right)^9}{88a^2x^{22}}+\frac{b^2\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^2\right)}{44a^2}\\
&= -\frac{\left(a+bx^2\right)^9}{24ax^{24}}+\frac{b\left(a+bx^2\right)^9}{88a^2x^{22}}-\frac{b^2\left(a+bx^2\right)^9}{440a^3x^{20}}-\frac{b^3\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^2\right)}{440a^3}\\
&= -\frac{\left(a+bx^2\right)^9}{24ax^{24}}+\frac{b\left(a+bx^2\right)^9}{88a^2x^{22}}-\frac{b^2\left(a+bx^2\right)^9}{440a^3x^{20}}+\frac{b^3\left(a+bx^2\right)^9}{3960a^4x^{18}}
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.26

$$
-\frac{a^8}{24x^{24}}-\frac{4a^7b}{11x^{22}}-\frac{7a^6b^2}{5x^{20}}-\frac{28a^5b^3}{9x^{18}}-\frac{35a^4b^4}{8x^{16}}-\frac{4a^3b^5}{x^{14}}-\frac{7a^2b^6}{3x^{12}}-\frac{4ab^7}{5x^{10}}-\frac{b^8}{8x^8}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^25,x]
```

```
[Out] -1/24*a^8/x^24 - (4*a^7*b)/(11*x^22) - (7*a^6*b^2)/(5*x^20) - (28*a^5*b^3)/
(9*x^18) - (35*a^4*b^4)/(8*x^16) - (4*a^3*b^5)/x^14 - (7*a^2*b^6)/(3*x^12)
- (4*a*b^7)/(5*x^10) - b^8/(8*x^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+bx^2\right)^8}{x^{25}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^25,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^25, x]
```

fricas [A] time = 1.13, size = 92, normalized size = 1.10

$$
-\frac{495\,b^8x^{16}+3168\,ab^7x^{14}+9240\,a^2b^6x^{12}+15840\,a^3b^5x^{10}+17325\,a^4b^4x^8+12320\,a^5b^3x^6+5544\,a^6b^2x^4+1440\,a^7bx^2+165\,a^8}{3960\,x^{24}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="fricas")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

giac [A] time = 1.20, size = 92, normalized size = 1.10

$$-\frac{495\, b^{8} x^{16} + 3168\, a b^{7} x^{14} + 9240\, a^{2} b^{6} x^{12} + 15840\, a^{3} b^{5} x^{10} + 17325\, a^{4} b^{4} x^{8} + 12320\, a^{5} b^{3} x^{6} + 5544\, a^{6} b^{2} x^{4} + 1440\, a^{7} b x^{2} + 165\, a^{8}}{3960\, x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="giac")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

maple [A] time = 0.01, size = 91, normalized size = 1.08

$$-\frac{b^{8}}{8x^{8}}-\frac{4a\,b^{7}}{5x^{10}}-\frac{7a^{2}b^{6}}{3x^{12}}-\frac{4a^{3}b^{5}}{x^{14}}-\frac{35a^{4}b^{4}}{8x^{16}}-\frac{28a^{5}b^{3}}{9x^{18}}-\frac{7a^{6}b^{2}}{5x^{20}}-\frac{4a^{7}b}{11x^{22}}-\frac{a^{8}}{24x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^25,x)
```

```
[Out] -28/9*a^5*b^3/x^18-35/8*a^4*b^4/x^16-7/5*a^6*b^2/x^20-1/24*a^8/x^24-4/5*a*b
^7/x^10-7/3*a^2*b^6/x^12-1/8*b^8/x^8-4/11*a^7*b/x^22-4*a^3*b^5/x^14
```

maxima [A] time = 1.39, size = 92, normalized size = 1.10

$$-\frac{495\, b^{8} x^{16} + 3168\, a b^{7} x^{14} + 9240\, a^{2} b^{6} x^{12} + 15840\, a^{3} b^{5} x^{10} + 17325\, a^{4} b^{4} x^{8} + 12320\, a^{5} b^{3} x^{6} + 5544\, a^{6} b^{2} x^{4} + 1440\, a^{7} b x^{2} + 165\, a^{8}}{3960\, x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^25,x, algorithm="maxima")
```

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5
*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7
*b*x^2 + 165*a^8)/x^24
```

mupad [B] time = 4.97, size = 92, normalized size = 1.10

$$-\frac{\frac{a^{8}}{24}+\frac{4\,a^{7}\,b\,x^{2}}{11}+\frac{7\,a^{6}\,b^{2}\,x^{4}}{5}+\frac{28\,a^{5}\,b^{3}\,x^{6}}{9}+\frac{35\,a^{4}\,b^{4}\,x^{8}}{8}+4\,a^{3}\,b^{5}\,x^{10}+\frac{7\,a^{2}\,b^{6}\,x^{12}}{3}+\frac{4\,a\,b^{7}\,x^{14}}{5}+\frac{b^{8}\,x^{16}}{8}}{x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^25,x)
```

```
[Out] -(a^8/24 + (b^8*x^16)/8 + (4*a^7*b*x^2)/11 + (4*a*b^7*x^14)/5 + (7*a^6*b^2*
x^4)/5 + (28*a^5*b^3*x^6)/9 + (35*a^4*b^4*x^8)/8 + 4*a^3*b^5*x^10 + (7*a^2*
b^6*x^12)/3)/x^24
```

sympy [A] time = 1.08, size = 99, normalized size = 1.18

$$\frac{-165a^8 - 1440a^7bx^2 - 5544a^6b^2x^4 - 12320a^5b^3x^6 - 17325a^4b^4x^8 - 15840a^3b^5x^{10} - 9240a^2b^6x^{12} - 3168ab^7x^{14} - 495b^8x^{16}}{3960x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**25,x)
```

```
[Out] (-165*a**8 - 1440*a**7*b*x**2 - 5544*a**6*b**2*x**4 - 12320*a**5*b**3*x**6
- 17325*a**4*b**4*x**8 - 15840*a**3*b**5*x**10 - 9240*a**2*b**6*x**12 - 316
8*a*b**7*x**14 - 495*b**8*x**16)/(3960*x**24)
```

3.105 $\int \frac{(a+bx^2)^8}{x^{27}} dx$

Optimal. Leaf size=106

$$-\frac{b^4 (a+bx^2)^9}{12870a^5x^{18}}+\frac{b^3 (a+bx^2)^9}{1430a^4x^{20}}-\frac{b^2 (a+bx^2)^9}{286a^3x^{22}}+\frac{b (a+bx^2)^9}{78a^2x^{24}}-\frac{(a+bx^2)^9}{26ax^{26}}$$

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$-\frac{b^4 (a+bx^2)^9}{12870a^5x^{18}}+\frac{b^3 (a+bx^2)^9}{1430a^4x^{20}}-\frac{b^2 (a+bx^2)^9}{286a^3x^{22}}+\frac{b (a+bx^2)^9}{78a^2x^{24}}-\frac{(a+bx^2)^9}{26ax^{26}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^27,x]
```

```
[Out] -(a + b*x^2)^9/(26*a*x^26) + (b*(a + b*x^2)^9)/(78*a^2*x^24) - (b^2*(a + b*
x^2)^9)/(286*a^3*x^22) + (b^3*(a + b*x^2)^9)/(1430*a^4*x^20) - (b^4*(a + b*
x^2)^9)/(12870*a^5*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^8}{x^{27}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{14}}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}} - \frac{(2b)\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx, x, x^2\right)}{13a}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}} + \frac{b\left(a+bx^2\right)^9}{78a^2x^{24}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx, x, x^2\right)}{26a^2}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}} + \frac{b\left(a+bx^2\right)^9}{78a^2x^{24}} - \frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}} - \frac{b^3\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^2\right)}{143a^3}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}} + \frac{b\left(a+bx^2\right)^9}{78a^2x^{24}} - \frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}} + \frac{b^3\left(a+bx^2\right)^9}{1430a^4x^{20}} + \frac{b^4\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^2\right)}{1430a^4}\\
&= -\frac{\left(a+bx^2\right)^9}{26ax^{26}} + \frac{b\left(a+bx^2\right)^9}{78a^2x^{24}} - \frac{b^2\left(a+bx^2\right)^9}{286a^3x^{22}} + \frac{b^3\left(a+bx^2\right)^9}{1430a^4x^{20}} - \frac{b^4\left(a+bx^2\right)^9}{12870a^5x^{18}}
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.00

$$
-\frac{a^8}{26x^{26}} - \frac{a^7b}{3x^{24}} - \frac{14a^6b^2}{11x^{22}} - \frac{14a^5b^3}{5x^{20}} - \frac{35a^4b^4}{9x^{18}} - \frac{7a^3b^5}{2x^{16}} - \frac{2a^2b^6}{x^{14}} - \frac{2ab^7}{3x^{12}} - \frac{b^8}{10x^{10}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^27,x]
```

```
[Out] -1/26*a^8/x^26 - (a^7*b)/(3*x^24) - (14*a^6*b^2)/(11*x^22) - (14*a^5*b^3)/(
5*x^20) - (35*a^4*b^4)/(9*x^18) - (7*a^3*b^5)/(2*x^16) - (2*a^2*b^6)/x^14 -
 (2*a*b^7)/(3*x^12) - b^8/(10*x^10)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+bx^2\right)^8}{x^{27}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^27,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^27, x]
```

fricas [A] time = 1.12, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b^2x^4+4290\,a^7bx^2+495\,a^8}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="fricas")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

giac [A] time = 1.06, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b^2x^4+4290\,a^7bx^2+495\,a^8}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="giac")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

maple [A] time = 0.01, size = 91, normalized size = 0.86

$$-\frac{b^8}{10x^{10}}-\frac{2a\,b^7}{3x^{12}}-\frac{2a^2b^6}{x^{14}}-\frac{7a^3b^5}{2x^{16}}-\frac{35a^4b^4}{9x^{18}}-\frac{14a^5b^3}{5x^{20}}-\frac{14a^6b^2}{11x^{22}}-\frac{a^7b}{3x^{24}}-\frac{a^8}{26x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^27,x)
```

```
[Out] -1/3*a^7*b/x^24-35/9*a^4*b^4/x^18-14/5*a^5*b^3/x^20-1/26*a^8/x^26-14/11*a^6
*b^2/x^22-1/10*b^8/x^10-2/3*a*b^7/x^12-2*a^2*b^6/x^14-7/2*a^3*b^5/x^16
```

maxima [A] time = 1.30, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b^2x^4+4290\,a^7bx^2+495\,a^8}{12870\,x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^27,x, algorithm="maxima")
```

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3*
b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290
*a^7*b*x^2 + 495*a^8)/x^26
```

mupad [B] time = 0.08, size = 92, normalized size = 0.87

$$-\frac{\frac{a^8}{26}+\frac{a^7\,b\,x^2}{3}+\frac{14\,a^6\,b^2\,x^4}{11}+\frac{14\,a^5\,b^3\,x^6}{5}+\frac{35\,a^4\,b^4\,x^8}{9}+\frac{7\,a^3\,b^5\,x^{10}}{2}+2\,a^2\,b^6\,x^{12}+\frac{2\,a\,b^7\,x^{14}}{3}+\frac{b^8\,x^{16}}{10}}{x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^27,x)
```

```
[Out] -(a^8/26 + (b^8*x^16)/10 + (a^7*b*x^2)/3 + (2*a*b^7*x^14)/3 + (14*a^6*b^2*x
^4)/11 + (14*a^5*b^3*x^6)/5 + (35*a^4*b^4*x^8)/9 + (7*a^3*b^5*x^10)/2 + 2*a
^2*b^6*x^12)/x^26
```

sympy [A] time = 1.14, size = 99, normalized size = 0.93

$$\frac{-495a^8-4290a^7bx^2-16380a^6b^2x^4-36036a^5b^3x^6-50050a^4b^4x^8-45045a^3b^5x^{10}-25740a^2b^6x^{12}-8580ab^7x^{14}-1287b^8x^{16}}{12870x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**27,x)
```

```
[Out] (-495*a**8 - 4290*a**7*b*x**2 - 16380*a**6*b**2*x**4 - 36036*a**5*b**3*x**6
 - 50050*a**4*b**4*x**8 - 45045*a**3*b**5*x**10 - 25740*a**2*b**6*x**12 - 8
580*a*b**7*x**14 - 1287*b**8*x**16)/(12870*x**26)
```

3.106 $\int \frac{(a+bx^2)^8}{x^{29}}\, dx$

Optimal. Leaf size=108

$$-\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4a^7b}{13x^{26}}-\frac{a^8}{28x^{28}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^29,x]
```

```
[Out] -a^8/(28*x^28) - (4*a^7*b)/(13*x^26) - (7*a^6*b^2)/(6*x^24) - (28*a^5*b^3)/
(11*x^22) - (7*a^4*b^4)/(2*x^20) - (28*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(4*x
^16) - (4*a*b^7)/(7*x^14) - b^8/(12*x^12)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^8}{x^{29}}\, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{15}}\, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{15}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{13}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{11}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^9}+\frac{8ab^7}{x^8}+\frac{b^8}{x^7}\right) d\\
&= -\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$-\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^29,x]
```

```
[Out] -1/28*a^8/x^28 - (4*a^7*b)/(13*x^26) - (7*a^6*b^2)/(6*x^24) - (28*a^5*b^3)/
(11*x^22) - (7*a^4*b^4)/(2*x^20) - (28*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(4*x
^16) - (4*a*b^7)/(7*x^14) - b^8/(12*x^12)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^8}{x^{29}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^29,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^29, x]
```

fricas [A] time = 0.60, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{16}+20592\,ab^7x^{14}+63063\,a^2b^6x^{12}+112112\,a^3b^5x^{10}+126126\,a^4b^4x^8+91728\,a^5b^3x^6+42042\,a^6b^2x^4+11088\,a^7bx^2+1287\,a^8}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^29,x, algorithm="fricas")
```

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^
3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 1
1088*a^7*b*x^2 + 1287*a^8)/x^28
```

giac [A] time = 1.05, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{16}+20592\,ab^7x^{14}+63063\,a^2b^6x^{12}+112112\,a^3b^5x^{10}+126126\,a^4b^4x^8+91728\,a^5b^3x^6+42042\,a^6b^2x^4+11088\,a^7bx^2+1287\,a^8}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^29,x, algorithm="giac")
```

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^
3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 1
1088*a^7*b*x^2 + 1287*a^8)/x^28
```

maple [A] time = 0.01, size = 91, normalized size = 0.84

$$-\frac{b^8}{12x^{12}}-\frac{4a\,b^7}{7x^{14}}-\frac{7a^2b^6}{4x^{16}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^6b^2}{6x^{24}}-\frac{4a^7b}{13x^{26}}-\frac{a^8}{28x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^29,x)
```

```
[Out] -1/28*a^8/x^28-4/13*a^7*b/x^26-7/6*a^6*b^2/x^24-28/11*a^5*b^3/x^22-7/2*a^4*
b^4/x^20-28/9*a^3*b^5/x^18-7/4*a^2*b^6/x^16-4/7*a*b^7/x^14-1/12*b^8/x^12
```

maxima [A] time = 1.41, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{16}+20592\,ab^7x^{14}+63063\,a^2b^6x^{12}+112112\,a^3b^5x^{10}+126126\,a^4b^4x^8+91728\,a^5b^3x^6+42042\,a^6b^2x^4+11088\,a^7bx^2+1287\,a^8}{36036\,x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^29,x, algorithm="maxima")
```

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a^
3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 1
1088*a^7*b*x^2 + 1287*a^8)/x^28
```

mupad [B] time = 4.89, size = 92, normalized size = 0.85

$$-\frac{\frac{a^8}{28}+\frac{4\,a^7\,b\,x^2}{13}+\frac{7\,a^6\,b^2\,x^4}{6}+\frac{28\,a^5\,b^3\,x^6}{11}+\frac{7\,a^4\,b^4\,x^8}{2}+\frac{28\,a^3\,b^5\,x^{10}}{9}+\frac{7\,a^2\,b^6\,x^{12}}{4}+\frac{4\,a\,b^7\,x^{14}}{7}+\frac{b^8\,x^{16}}{12}}{x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^29,x)
```

```
[Out] -(a^8/28 + (b^8*x^16)/12 + (4*a^7*b*x^2)/13 + (4*a*b^7*x^14)/7 + (7*a^6*b^2
*x^4)/6 + (28*a^5*b^3*x^6)/11 + (7*a^4*b^4*x^8)/2 + (28*a^3*b^5*x^10)/9 + (
7*a^2*b^6*x^12)/4)/x^28
```

sympy [A] time = 1.20, size = 99, normalized size = 0.92

$$\frac{-1287a^8-11088a^7bx^2-42042a^6b^2x^4-91728a^5b^3x^6-126126a^4b^4x^8-112112a^3b^5x^{10}-63063a^2b^6x^{12}-20592ab^7x^{14}-3003b^8x^{16}}{36036x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**29,x)
```

```
[Out] (-1287*a**8 - 11088*a**7*b*x**2 - 42042*a**6*b**2*x**4 - 91728*a**5*b**3*x*
*6 - 126126*a**4*b**4*x**8 - 112112*a**3*b**5*x**10 - 63063*a**2*b**6*x**12
 - 20592*a*b**7*x**14 - 3003*b**8*x**16)/(36036*x**28)
```

3.107 $\int \frac{(a+bx^2)^8}{x^{31}} dx$

Optimal. Leaf size=108

$$-\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{2a^7b}{7x^{28}}-\frac{a^8}{30x^{30}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^31,x]
```

```
[Out] -a^8/(30*x^30) - (2*a^7*b)/(7*x^28) - (14*a^6*b^2)/(13*x^26) - (7*a^5*b^3)/
(3*x^24) - (35*a^4*b^4)/(11*x^22) - (14*a^3*b^5)/(5*x^20) - (14*a^2*b^6)/(9
*x^18) - (a*b^7)/(2*x^16) - b^8/(14*x^14)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{31}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{16}} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{15}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{13}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{11}}+\frac{28a^2b^6}{x^{10}}+\frac{8ab^7}{x^9}+\frac{b^8}{x^8}\right) dx\right.\\ &= -\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$-\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{ab^7}{2x^{16}}-\frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^31,x]
```

```
[Out] -1/30*a^8/x^30 - (2*a^7*b)/(7*x^28) - (14*a^6*b^2)/(13*x^26) - (7*a^5*b^3)/
(3*x^24) - (35*a^4*b^4)/(11*x^22) - (14*a^3*b^5)/(5*x^20) - (14*a^2*b^6)/(9
*x^18) - (a*b^7)/(2*x^16) - b^8/(14*x^14)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^8}{x^{31}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^31,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^31, x]
```

fricas [A] time = 0.59, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{16}+45045\,ab^7x^{14}+140140\,a^2b^6x^{12}+252252\,a^3b^5x^{10}+286650\,a^4b^4x^8+210210\,a^5b^3x^6+97020\,a^6b^2x^4+25740\,a^7bx^2+3003\,a^8}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="fricas")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

giac [A] time = 1.22, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{16}+45045\,ab^7x^{14}+140140\,a^2b^6x^{12}+252252\,a^3b^5x^{10}+286650\,a^4b^4x^8+210210\,a^5b^3x^6+97020\,a^6b^2x^4+25740\,a^7bx^2+3003\,a^8}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="giac")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

maple [A] time = 0.01, size = 91, normalized size = 0.84

$$-\frac{b^{8}}{14x^{14}}-\frac{a\,b^{7}}{2x^{16}}-\frac{14a^{2}b^{6}}{9x^{18}}-\frac{14a^{3}b^{5}}{5x^{20}}-\frac{35a^{4}b^{4}}{11x^{22}}-\frac{7a^{5}b^{3}}{3x^{24}}-\frac{14a^{6}b^{2}}{13x^{26}}-\frac{2a^{7}b}{7x^{28}}-\frac{a^{8}}{30x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^31,x)
```

```
[Out] -1/30*a^8/x^30-2/7*a^7*b/x^28-14/13*a^6*b^2/x^26-7/3*a^5*b^3/x^24-35/11*a^4
*b^4/x^22-14/5*a^3*b^5/x^20-14/9*a^2*b^6/x^18-1/2*a*b^7/x^16-1/14*b^8/x^14
```

maxima [A] time = 1.31, size = 92, normalized size = 0.85

$$-\frac{6435\,b^{8}x^{16}+45045\,ab^{7}x^{14}+140140\,a^{2}b^{6}x^{12}+252252\,a^{3}b^{5}x^{10}+286650\,a^{4}b^{4}x^{8}+210210\,a^{5}b^{3}x^{6}+97020\,a^{6}b^{2}x^{4}+25740\,a^{7}bx^{2}+3003\,a^{8}}{90090\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^31,x, algorithm="maxima")
```

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*a
^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 +
 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

mupad [B] time = 4.86, size = 92, normalized size = 0.85

$$-\frac{\frac{a^{8}}{30}+\frac{2\,a^{7}\,b\,x^{2}}{7}+\frac{14\,a^{6}\,b^{2}\,x^{4}}{13}+\frac{7\,a^{5}\,b^{3}\,x^{6}}{3}+\frac{35\,a^{4}\,b^{4}\,x^{8}}{11}+\frac{14\,a^{3}\,b^{5}\,x^{10}}{5}+\frac{14\,a^{2}\,b^{6}\,x^{12}}{9}+\frac{a\,b^{7}\,x^{14}}{2}+\frac{b^{8}\,x^{16}}{14}}{x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^31,x)
```

```
[Out] -(a^8/30 + (b^8*x^16)/14 + (2*a^7*b*x^2)/7 + (a*b^7*x^14)/2 + (14*a^6*b^2*x
^4)/13 + (7*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/11 + (14*a^3*b^5*x^10)/5 + (1
4*a^2*b^6*x^12)/9)/x^30
```

sympy [A] time = 1.28, size = 99, normalized size = 0.92

$$\frac{-3003a^{8}-25740a^{7}bx^{2}-97020a^{6}b^{2}x^{4}-210210a^{5}b^{3}x^{6}-286650a^{4}b^{4}x^{8}-252252a^{3}b^{5}x^{10}-140140a^{2}b^{6}x^{12}-45045ab^{7}x^{14}-6435b^{8}x^{16}}{90090x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**31,x)
```

```
[Out] (-3003*a**8 - 25740*a**7*b*x**2 - 97020*a**6*b**2*x**4 - 210210*a**5*b**3*x
**6 - 286650*a**4*b**4*x**8 - 252252*a**3*b**5*x**10 - 140140*a**2*b**6*x**
12 - 45045*a*b**7*x**14 - 6435*b**8*x**16)/(90090*x**30)
```

3.108 $\int \frac{\left(a+bx^2\right)^8}{x^{33}}\,dx$

Optimal. Leaf size=106

$$-\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4a^7b}{15x^{30}}-\frac{a^8}{32x^{32}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^33,x]
```

```
[Out] -a^8/(32*x^32) - (4*a^7*b)/(15*x^30) - (a^6*b^2)/x^28 - (28*a^5*b^3)/(13*x^
26) - (35*a^4*b^4)/(12*x^24) - (28*a^3*b^5)/(11*x^22) - (7*a^2*b^6)/(5*x^20
) - (4*a*b^7)/(9*x^18) - b^8/(16*x^16)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^8}{x^{33}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^8}{x^{17}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^8}{x^{17}}+\frac{8a^7b}{x^{16}}+\frac{28a^6b^2}{x^{15}}+\frac{56a^5b^3}{x^{14}}+\frac{70a^4b^4}{x^{13}}+\frac{56a^3b^5}{x^{12}}+\frac{28a^2b^6}{x^{11}}+\frac{8ab^7}{x^{10}}+\frac{b^8}{x^9}\right) d\right.\\ &= -\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.00

$$-\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^33,x]
```

```
[Out] -1/32*a^8/x^32 - (4*a^7*b)/(15*x^30) - (a^6*b^2)/x^28 - (28*a^5*b^3)/(13*x^
26) - (35*a^4*b^4)/(12*x^24) - (28*a^3*b^5)/(11*x^22) - (7*a^2*b^6)/(5*x^20
) - (4*a*b^7)/(9*x^18) - b^8/(16*x^16)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^{33}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^33,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^33, x]
```

fricas [A] time = 0.77, size = 92, normalized size = 0.87

$$-\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+205920\,a^6b^2x^4+54912\,a^7bx^2+6435\,a^8}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="fricas")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

giac [A] time = 1.05, size = 92, normalized size = 0.87

$$-\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+205920\,a^6b^2x^4+54912\,a^7bx^2+6435\,a^8}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="giac")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

maple [A] time = 0.01, size = 91, normalized size = 0.86

$$-\frac{b^8}{16x^{16}}-\frac{4a\,b^7}{9x^{18}}-\frac{7a^2b^6}{5x^{20}}-\frac{28a^3b^5}{11x^{22}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^5b^3}{13x^{26}}-\frac{a^6b^2}{x^{28}}-\frac{4a^7b}{15x^{30}}-\frac{a^8}{32x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^33,x)
```

```
[Out] -1/32*a^8/x^32-4/15*a^7*b/x^30-a^6*b^2/x^28-28/13*a^5*b^3/x^26-35/12*a^4*b^
4/x^24-28/11*a^3*b^5/x^22-7/5*a^2*b^6/x^20-4/9*a*b^7/x^18-1/16*b^8/x^16
```

maxima [A] time = 1.34, size = 92, normalized size = 0.87

$$-\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+205920\,a^6b^2x^4+54912\,a^7bx^2+6435\,a^8}{205920\,x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^33,x, algorithm="maxima")
```

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160
*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^
4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

mupad [B] time = 0.08, size = 91, normalized size = 0.86

$$-\frac{\frac{a^8}{32}+\frac{4\,a^7\,b\,x^2}{15}+a^6\,b^2\,x^4+\frac{28\,a^5\,b^3\,x^6}{13}+\frac{35\,a^4\,b^4\,x^8}{12}+\frac{28\,a^3\,b^5\,x^{10}}{11}+\frac{7\,a^2\,b^6\,x^{12}}{5}+\frac{4\,a\,b^7\,x^{14}}{9}+\frac{b^8\,x^{16}}{16}}{x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^33,x)
```

```
[Out] -(a^8/32 + (b^8*x^16)/16 + (4*a^7*b*x^2)/15 + (4*a*b^7*x^14)/9 + a^6*b^2*x^
4 + (28*a^5*b^3*x^6)/13 + (35*a^4*b^4*x^8)/12 + (28*a^3*b^5*x^10)/11 + (7*a
^2*b^6*x^12)/5)/x^32
```

sympy [A] time = 1.34, size = 99, normalized size = 0.93

$$\frac{-6435a^8-54912a^7bx^2-205920a^6b^2x^4-443520a^5b^3x^6-600600a^4b^4x^8-524160a^3b^5x^{10}-288288a^2b^6x^{12}-91520ab^7x^{14}-12870b^8x^{16}}{205920x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**33,x)
```

```
[Out] (-6435*a**8 - 54912*a**7*b*x**2 - 205920*a**6*b**2*x**4 - 443520*a**5*b**3*
x**6 - 600600*a**4*b**4*x**8 - 524160*a**3*b**5*x**10 - 288288*a**2*b**6*x*
*12 - 91520*a*b**7*x**14 - 12870*b**8*x**16)/(205920*x**32)
```

3.109 $\int x^8 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=108

$$\frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{4}{3}a^2b^6x^{21}+\frac{56}{19}a^3b^5x^{19}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{15}a^5b^3x^{15}+\frac{28}{13}a^6b^2x^{13}+\frac{8}{11}a^7bx^{11}+\frac{a^8x^9}{9}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/
15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (8*
a*b^7*x^23)/23 + (b^8*x^25)/25
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^2\right)^8\,dx &= \int \left(a^8x^8+8a^7bx^{10}+28a^6b^2x^{12}+56a^5b^3x^{14}+70a^4b^4x^{16}+56a^3b^5x^{18}+28a^2b^6x^{20}+8ab^7\right.\\ &= \frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$\frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/
15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (8*
a*b^7*x^23)/23 + (b^8*x^25)/25
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \left(a+bx^2\right)^8 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x^8*(a + b*x^2)^8, x]
```

fricas [A] time = 0.55, size = 90, normalized size = 0.83

$$\frac{1}{25}x^{25}b^8 + \frac{8}{23}x^{23}b^7a + \frac{4}{3}x^{21}b^6a^2 + \frac{56}{19}x^{19}b^5a^3 + \frac{70}{17}x^{17}b^4a^4 + \frac{56}{15}x^{15}b^3a^5 + \frac{28}{13}x^{13}b^2a^6 + \frac{8}{11}x^{11}ba^7 + \frac{1}{9}x^9a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*b^8 + 8/23*x^23*b^7*a + 4/3*x^21*b^6*a^2 + 56/19*x^19*b^5*a^3 + 7
0/17*x^17*b^4*a^4 + 56/15*x^15*b^3*a^5 + 28/13*x^13*b^2*a^6 + 8/11*x^11*b*a
^7 + 1/9*x^9*a^8
```

giac [A] time = 1.10, size = 90, normalized size = 0.83

$$\frac{1}{25}\,b^8x^{25} + \frac{8}{23}\,ab^7x^{23} + \frac{4}{3}\,a^2b^6x^{21} + \frac{56}{19}\,a^3b^5x^{19} + \frac{70}{17}\,a^4b^4x^{17} + \frac{56}{15}\,a^5b^3x^{15} + \frac{28}{13}\,a^6b^2x^{13} + \frac{8}{11}\,a^7bx^{11} + \frac{1}{9}\,a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 + 7
0/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*x^
11 + 1/9*a^8*x^9
```

maple [A] time = 0.00, size = 91, normalized size = 0.84

$$\frac{1}{25}b^8x^{25} + \frac{8}{23}a\,b^7x^{23} + \frac{4}{3}a^2b^6x^{21} + \frac{56}{19}a^3b^5x^{19} + \frac{70}{17}a^4b^4x^{17} + \frac{56}{15}a^5b^3x^{15} + \frac{28}{13}a^6b^2x^{13} + \frac{8}{11}a^7b\,x^{11} + \frac{1}{9}a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^8,x)
```

```
[Out] 1/9*a^8*x^9+8/11*a^7*b*x^11+28/13*a^6*b^2*x^13+56/15*a^5*b^3*x^15+70/17*a^4
*b^4*x^17+56/19*a^3*b^5*x^19+4/3*a^2*b^6*x^21+8/23*a*b^7*x^23+1/25*b^8*x^25
```

maxima [A] time = 1.36, size = 90, normalized size = 0.83

$$\frac{1}{25}b^8x^{25}+\frac{8}{23}ab^7x^{23}+\frac{4}{3}a^2b^6x^{21}+\frac{56}{19}a^3b^5x^{19}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{15}a^5b^3x^{15}+\frac{28}{13}a^6b^2x^{13}+\frac{8}{11}a^7bx^{11}+\frac{1}{9}a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 + 7
0/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*x^
11 + 1/9*a^8*x^9
```

mupad [B] time = 4.94, size = 90, normalized size = 0.83

$$\frac{a^8\,x^9}{9}+\frac{8\,a^7\,b\,x^{11}}{11}+\frac{28\,a^6\,b^2\,x^{13}}{13}+\frac{56\,a^5\,b^3\,x^{15}}{15}+\frac{70\,a^4\,b^4\,x^{17}}{17}+\frac{56\,a^3\,b^5\,x^{19}}{19}+\frac{4\,a^2\,b^6\,x^{21}}{3}+\frac{8\,a\,b^7\,x^{23}}{23}+\frac{b^8\,x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^9)/9 + (b^8*x^25)/25 + (8*a^7*b*x^11)/11 + (8*a*b^7*x^23)/23 + (28*a
^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5
*x^19)/19 + (4*a^2*b^6*x^21)/3
```

sympy [A] time = 0.09, size = 107, normalized size = 0.99

$$\frac{a^8x^9}{9}+\frac{8a^7bx^{11}}{11}+\frac{28a^6b^2x^{13}}{13}+\frac{56a^5b^3x^{15}}{15}+\frac{70a^4b^4x^{17}}{17}+\frac{56a^3b^5x^{19}}{19}+\frac{4a^2b^6x^{21}}{3}+\frac{8ab^7x^{23}}{23}+\frac{b^8x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**9/9 + 8*a**7*b*x**11/11 + 28*a**6*b**2*x**13/13 + 56*a**5*b**3*x**1
5/15 + 70*a**4*b**4*x**17/17 + 56*a**3*b**5*x**19/19 + 4*a**2*b**6*x**21/3
+ 8*a*b**7*x**23/23 + b**8*x**25/25
```

3.110 $\int x^6 \left(a+bx^2\right)^8 dx$

Optimal. Leaf size=108

$$\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{19}a^2b^6x^{19}+\frac{56}{17}a^3b^5x^{17}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{13}a^5b^3x^{13}+\frac{28}{11}a^6b^2x^{11}+\frac{8}{9}a^7bx^9+\frac{a^8x^7}{7}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13
 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8*a
*b^7*x^21)/21 + (b^8*x^23)/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a+bx^2\right)^8 dx &= \int \left(a^8x^6+8a^7bx^8+28a^6b^2x^{10}+56a^5b^3x^{12}+70a^4b^4x^{14}+56a^3b^5x^{16}+28a^2b^6x^{18}+8ab^7\right. \\ &= \frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13
 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8*a
*b^7*x^21)/21 + (b^8*x^23)/23
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left(a + bx^2\right)^8 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x^6*(a + b*x^2)^8, x]
```

fricas [A] time = 0.53, size = 90, normalized size = 0.83

$$\frac{1}{23}x^{23}b^8 + \frac{8}{21}x^{21}b^7a + \frac{28}{19}x^{19}b^6a^2 + \frac{56}{17}x^{17}b^5a^3 + \frac{14}{3}x^{15}b^4a^4 + \frac{56}{13}x^{13}b^3a^5 + \frac{28}{11}x^{11}b^2a^6 + \frac{8}{9}x^9ba^7 + \frac{1}{7}x^7a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*b^8 + 8/21*x^21*b^7*a + 28/19*x^19*b^6*a^2 + 56/17*x^17*b^5*a^3 +
 14/3*x^15*b^4*a^4 + 56/13*x^13*b^3*a^5 + 28/11*x^11*b^2*a^6 + 8/9*x^9*b*a^
7 + 1/7*x^7*a^8
```

giac [A] time = 1.14, size = 90, normalized size = 0.83

$$\frac{1}{23}\,b^8x^{23} + \frac{8}{21}\,ab^7x^{21} + \frac{28}{19}\,a^2b^6x^{19} + \frac{56}{17}\,a^3b^5x^{17} + \frac{14}{3}\,a^4b^4x^{15} + \frac{56}{13}\,a^5b^3x^{13} + \frac{28}{11}\,a^6b^2x^{11} + \frac{8}{9}\,a^7bx^9 + \frac{1}{7}\,a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17 +
 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*x^
9 + 1/7*a^8*x^7
```

maple [A] time = 0.00, size = 91, normalized size = 0.84

$$\frac{1}{23}b^8x^{23} + \frac{8}{21}a\,b^7x^{21} + \frac{28}{19}a^2b^6x^{19} + \frac{56}{17}a^3b^5x^{17} + \frac{14}{3}a^4b^4x^{15} + \frac{56}{13}a^5b^3x^{13} + \frac{28}{11}a^6b^2x^{11} + \frac{8}{9}a^7b\,x^9 + \frac{1}{7}a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^8,x)
```

```
[Out] 1/7*a^8*x^7+8/9*a^7*b*x^9+28/11*a^6*b^2*x^11+56/13*a^5*b^3*x^13+14/3*a^4*b^
4*x^15+56/17*a^3*b^5*x^17+28/19*a^2*b^6*x^19+8/21*a*b^7*x^21+1/23*b^8*x^23
```

maxima [A] time = 1.32, size = 90, normalized size = 0.83

$$\frac{1}{23}\,b^8x^{23}+\frac{8}{21}\,ab^7x^{21}+\frac{28}{19}\,a^2b^6x^{19}+\frac{56}{17}\,a^3b^5x^{17}+\frac{14}{3}\,a^4b^4x^{15}+\frac{56}{13}\,a^5b^3x^{13}+\frac{28}{11}\,a^6b^2x^{11}+\frac{8}{9}\,a^7bx^9+\frac{1}{7}\,a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17 +
 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*x^
9 + 1/7*a^8*x^7
```

mupad [B] time = 0.10, size = 90, normalized size = 0.83

$$\frac{a^8\,x^7}{7}+\frac{8\,a^7\,b\,x^9}{9}+\frac{28\,a^6\,b^2\,x^{11}}{11}+\frac{56\,a^5\,b^3\,x^{13}}{13}+\frac{14\,a^4\,b^4\,x^{15}}{3}+\frac{56\,a^3\,b^5\,x^{17}}{17}+\frac{28\,a^2\,b^6\,x^{19}}{19}+\frac{8\,a\,b^7\,x^{21}}{21}+\frac{b^8\,x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^7)/7 + (b^8*x^23)/23 + (8*a^7*b*x^9)/9 + (8*a*b^7*x^21)/21 + (28*a^6
*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^
17)/17 + (28*a^2*b^6*x^19)/19
```

sympy [A] time = 0.09, size = 107, normalized size = 0.99

$$\frac{a^8x^7}{7}+\frac{8a^7bx^9}{9}+\frac{28a^6b^2x^{11}}{11}+\frac{56a^5b^3x^{13}}{13}+\frac{14a^4b^4x^{15}}{3}+\frac{56a^3b^5x^{17}}{17}+\frac{28a^2b^6x^{19}}{19}+\frac{8ab^7x^{21}}{21}+\frac{b^8x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**7/7 + 8*a**7*b*x**9/9 + 28*a**6*b**2*x**11/11 + 56*a**5*b**3*x**13/
13 + 14*a**4*b**4*x**15/3 + 56*a**3*b**5*x**17/17 + 28*a**2*b**6*x**19/19 +
 8*a*b**7*x**21/21 + b**8*x**23/23
```

3.111 $\int x^4 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=108

$$\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{17}a^2b^6x^{17}+\frac{56}{15}a^3b^5x^{15}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{11}a^5b^3x^{11}+\frac{28}{9}a^6b^2x^9+\frac{8}{7}a^7bx^7+\frac{a^8x^5}{5}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 +
 (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*a*
b^7*x^19)/19 + (b^8*x^21)/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^2\right)^8 \, dx &= \int \left(a^8x^4+8a^7bx^6+28a^6b^2x^8+56a^5b^3x^{10}+70a^4b^4x^{12}+56a^3b^5x^{14}+28a^2b^6x^{16}+8ab^7x^{1}\right.\\ &= \frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 +
 (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*a*
b^7*x^19)/19 + (b^8*x^21)/21
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a+bx^2\right)^8 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x^2)^8, x]
```

fricas [A] time = 0.53, size = 90, normalized size = 0.83

$$\frac{1}{21}x^{21}b^8+\frac{8}{19}x^{19}b^7a+\frac{28}{17}x^{17}b^6a^2+\frac{56}{15}x^{15}b^5a^3+\frac{70}{13}x^{13}b^4a^4+\frac{56}{11}x^{11}b^3a^5+\frac{28}{9}x^9b^2a^6+\frac{8}{7}x^7ba^7+\frac{1}{5}x^5a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^8 + 8/19*x^19*b^7*a + 28/17*x^17*b^6*a^2 + 56/15*x^15*b^5*a^3 +
 70/13*x^13*b^4*a^4 + 56/11*x^11*b^3*a^5 + 28/9*x^9*b^2*a^6 + 8/7*x^7*b*a^7
 + 1/5*x^5*a^8
```

giac [A] time = 1.10, size = 90, normalized size = 0.83

$$\frac{1}{21}\,b^8x^{21}+\frac{8}{19}\,ab^7x^{19}+\frac{28}{17}\,a^2b^6x^{17}+\frac{56}{15}\,a^3b^5x^{15}+\frac{70}{13}\,a^4b^4x^{13}+\frac{56}{11}\,a^5b^3x^{11}+\frac{28}{9}\,a^6b^2x^9+\frac{8}{7}\,a^7bx^7+\frac{1}{5}\,a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15 +
 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x^7
 + 1/5*a^8*x^5
```

maple [A] time = 0.00, size = 91, normalized size = 0.84

$$\frac{1}{21}b^8x^{21}+\frac{8}{19}a\,b^7x^{19}+\frac{28}{17}a^2b^6x^{17}+\frac{56}{15}a^3b^5x^{15}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{11}a^5b^3x^{11}+\frac{28}{9}a^6b^2x^9+\frac{8}{7}a^7b\,x^7+\frac{1}{5}a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^8,x)
```

```
[Out] 1/5*a^8*x^5+8/7*a^7*b*x^7+28/9*a^6*b^2*x^9+56/11*a^5*b^3*x^11+70/13*a^4*b^4
*x^13+56/15*a^3*b^5*x^15+28/17*a^2*b^6*x^17+8/19*a*b^7*x^19+1/21*b^8*x^21
```

maxima [A] time = 1.34, size = 90, normalized size = 0.83

$$\frac{1}{21}\,b^8x^{21}+\frac{8}{19}\,ab^7x^{19}+\frac{28}{17}\,a^2b^6x^{17}+\frac{56}{15}\,a^3b^5x^{15}+\frac{70}{13}\,a^4b^4x^{13}+\frac{56}{11}\,a^5b^3x^{11}+\frac{28}{9}\,a^6b^2x^9+\frac{8}{7}\,a^7bx^7+\frac{1}{5}\,a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15 +
 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x^7
 + 1/5*a^8*x^5
```

mupad [B] time = 0.10, size = 90, normalized size = 0.83

$$\frac{a^8\,x^5}{5}+\frac{8\,a^7\,b\,x^7}{7}+\frac{28\,a^6\,b^2\,x^9}{9}+\frac{56\,a^5\,b^3\,x^{11}}{11}+\frac{70\,a^4\,b^4\,x^{13}}{13}+\frac{56\,a^3\,b^5\,x^{15}}{15}+\frac{28\,a^2\,b^6\,x^{17}}{17}+\frac{8\,a\,b^7\,x^{19}}{19}+\frac{b^8\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^8,x)
```

```
[Out] (a^8*x^5)/5 + (b^8*x^21)/21 + (8*a^7*b*x^7)/7 + (8*a*b^7*x^19)/19 + (28*a^6
*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 + (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^1
5)/15 + (28*a^2*b^6*x^17)/17
```

sympy [A] time = 0.09, size = 107, normalized size = 0.99

$$\frac{a^8x^5}{5}+\frac{8a^7bx^7}{7}+\frac{28a^6b^2x^9}{9}+\frac{56a^5b^3x^{11}}{11}+\frac{70a^4b^4x^{13}}{13}+\frac{56a^3b^5x^{15}}{15}+\frac{28a^2b^6x^{17}}{17}+\frac{8ab^7x^{19}}{19}+\frac{b^8x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**8,x)
```

```
[Out] a**8*x**5/5 + 8*a**7*b*x**7/7 + 28*a**6*b**2*x**9/9 + 56*a**5*b**3*x**11/11
 + 70*a**4*b**4*x**13/13 + 56*a**3*b**5*x**15/15 + 28*a**2*b**6*x**17/17 +
8*a*b**7*x**19/19 + b**8*x**21/21
```

3.112 $\int x^2 \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=106

$$\frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{15}a^2b^6x^{15}+\frac{56}{13}a^3b^5x^{13}+\frac{70}{11}a^4b^4x^{11}+\frac{56}{9}a^5b^3x^9+4a^6b^2x^7+\frac{8}{5}a^7bx^5+\frac{a^8x^3}{3}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^
4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x^1
7)/17 + (b^8*x^19)/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right)^8 \, dx &= \int \left(a^8x^2+8a^7bx^4+28a^6b^2x^6+56a^5b^3x^8+70a^4b^4x^{10}+56a^3b^5x^{12}+28a^2b^6x^{14}+8ab^7x\right.\\ &= \frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^1\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.00

$$\frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^8,x]
```

```
[Out] (a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^
4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x^1
7)/17 + (b^8*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2\right)^8 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^8, x]
```

fricas [A] time = 0.41, size = 90, normalized size = 0.85

$$\frac{1}{19}x^{19}b^8 + \frac{8}{17}x^{17}b^7a + \frac{28}{15}x^{15}b^6a^2 + \frac{56}{13}x^{13}b^5a^3 + \frac{70}{11}x^{11}b^4a^4 + \frac{56}{9}x^9b^3a^5 + 4x^7b^2a^6 + \frac{8}{5}x^5ba^7 + \frac{1}{3}x^3a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^8 + 8/17*x^17*b^7*a + 28/15*x^15*b^6*a^2 + 56/13*x^13*b^5*a^3 +
 70/11*x^11*b^4*a^4 + 56/9*x^9*b^3*a^5 + 4*x^7*b^2*a^6 + 8/5*x^5*b*a^7 + 1/
3*x^3*a^8
```

giac [A] time = 0.91, size = 90, normalized size = 0.85

$$\frac{1}{19}\,b^8x^{19} + \frac{8}{17}\,ab^7x^{17} + \frac{28}{15}\,a^2b^6x^{15} + \frac{56}{13}\,a^3b^5x^{13} + \frac{70}{11}\,a^4b^4x^{11} + \frac{56}{9}\,a^5b^3x^9 + 4\,a^6b^2x^7 + \frac{8}{5}\,a^7bx^5 + \frac{1}{3}\,a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13 +
 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 + 1/
3*a^8*x^3
```

maple [A] time = 0.00, size = 91, normalized size = 0.86

$$\frac{1}{19}b^8x^{19} + \frac{8}{17}a\,b^7x^{17} + \frac{28}{15}a^2b^6x^{15} + \frac{56}{13}a^3b^5x^{13} + \frac{70}{11}a^4b^4x^{11} + \frac{56}{9}a^5b^3x^9 + 4a^6b^2x^7 + \frac{8}{5}a^7b\,x^5 + \frac{1}{3}a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^8,x)
```

[Out] 1/3*a^8*x^3+8/5*a^7*b*x^5+4*a^6*b^2*x^7+56/9*a^5*b^3*x^9+70/11*a^4*b^4*x^11 +56/13*a^3*b^5*x^13+28/15*a^2*b^6*x^15+8/17*a*b^7*x^17+1/19*b^8*x^19

maxima [A] time = 1.37, size = 90, normalized size = 0.85

$$\frac{1}{19}b^8x^{19} + \frac{8}{17}ab^7x^{17} + \frac{28}{15}a^2b^6x^{15} + \frac{56}{13}a^3b^5x^{13} + \frac{70}{11}a^4b^4x^{11} + \frac{56}{9}a^5b^3x^9 + 4a^6b^2x^7 + \frac{8}{5}a^7bx^5 + \frac{1}{3}a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+a)^8,x, algorithm="maxima")

[Out] 1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13 + 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 + 1/3*a^8*x^3

mupad [B] time = 4.97, size = 90, normalized size = 0.85

$$\frac{a^8\,x^3}{3} + \frac{8\,a^7\,b\,x^5}{5} + 4\,a^6\,b^2\,x^7 + \frac{56\,a^5\,b^3\,x^9}{9} + \frac{70\,a^4\,b^4\,x^{11}}{11} + \frac{56\,a^3\,b^5\,x^{13}}{13} + \frac{28\,a^2\,b^6\,x^{15}}{15} + \frac{8\,a\,b^7\,x^{17}}{17} + \frac{b^8\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*x^2)^8,x)

[Out] (a^8*x^3)/3 + (b^8*x^19)/19 + (8*a^7*b*x^5)/5 + (8*a*b^7*x^17)/17 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15

sympy [A] time = 0.09, size = 105, normalized size = 0.99

$$\frac{a^8x^3}{3} + \frac{8a^7bx^5}{5} + 4a^6b^2x^7 + \frac{56a^5b^3x^9}{9} + \frac{70a^4b^4x^{11}}{11} + \frac{56a^3b^5x^{13}}{13} + \frac{28a^2b^6x^{15}}{15} + \frac{8ab^7x^{17}}{17} + \frac{b^8x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**2+a)**8,x)

[Out] a**8*x**3/3 + 8*a**7*b*x**5/5 + 4*a**6*b**2*x**7 + 56*a**5*b**3*x**9/9 + 70*a**4*b**4*x**11/11 + 56*a**3*b**5*x**13/13 + 28*a**2*b**6*x**15/15 + 8*a*b**7*x**17/17 + b**8*x**19/19

3.113 $\int \left(a+bx^2\right)^8 \, dx$

Optimal. Leaf size=101

$$a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{28}{13}a^2b^6x^{13}+\frac{56}{11}a^3b^5x^{11}+\frac{70}{9}a^4b^4x^9+8a^5b^3x^7+\frac{28}{5}a^6b^2x^5+\frac{8}{3}a^7bx^3+a^8x+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8,x]
```

```
[Out] a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*
x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15 +
(b^8*x^17)/17
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^8 \, dx &= \int \left(a^8+8a^7bx^2+28a^6b^2x^4+56a^5b^3x^6+70a^4b^4x^8+56a^3b^5x^{10}+28a^2b^6x^{12}+8ab^7x^{14}+b^8x\right.\\ &= a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8}{1}\end{aligned}$$

Mathematica [A] time = 0.00, size = 101, normalized size = 1.00

$$a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8,x]
```

```
[Out] a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*
x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15 +
(b^8*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^8 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8, x]
```

fricas [A] time = 0.77, size = 87, normalized size = 0.86

$$\frac{1}{17}x^{17}b^8+\frac{8}{15}x^{15}b^7a+\frac{28}{13}x^{13}b^6a^2+\frac{56}{11}x^{11}b^5a^3+\frac{70}{9}x^9b^4a^4+8x^7b^3a^5+\frac{28}{5}x^5b^2a^6+\frac{8}{3}x^3ba^7+xa^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^8 + 8/15*x^15*b^7*a + 28/13*x^13*b^6*a^2 + 56/11*x^11*b^5*a^3 +
 70/9*x^9*b^4*a^4 + 8*x^7*b^3*a^5 + 28/5*x^5*b^2*a^6 + 8/3*x^3*b*a^7 + x*a^
8
```

giac [A] time = 1.18, size = 87, normalized size = 0.86

$$\frac{1}{17}b^8x^{17}+\frac{8}{15}ab^7x^{15}+\frac{28}{13}a^2b^6x^{13}+\frac{56}{11}a^3b^5x^{11}+\frac{70}{9}a^4b^4x^9+8a^5b^3x^7+\frac{28}{5}a^6b^2x^5+\frac{8}{3}a^7bx^3+a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] 1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11 +
 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^8*
x
```

maple [A] time = 0.00, size = 88, normalized size = 0.87

$$\frac{1}{17}b^8x^{17}+\frac{8}{15}a\,b^7x^{15}+\frac{28}{13}a^2b^6x^{13}+\frac{56}{11}a^3b^5x^{11}+\frac{70}{9}a^4b^4x^9+8a^5b^3x^7+\frac{28}{5}a^6b^2x^5+\frac{8}{3}a^7b\,x^3+a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8,x)
```

```
[Out] a^8*x+8/3*a^7*b*x^3+28/5*a^6*b^2*x^5+8*a^5*b^3*x^7+70/9*a^4*b^4*x^9+56/11*a
^3*b^5*x^11+28/13*a^2*b^6*x^13+8/15*a*b^7*x^15+1/17*b^8*x^17
```

maxima [A] time = 1.30, size = 87, normalized size = 0.86

$$\frac{1}{17}\,b^{8}x^{17}+\frac{8}{15}\,ab^{7}x^{15}+\frac{28}{13}\,a^{2}b^{6}x^{13}+\frac{56}{11}\,a^{3}b^{5}x^{11}+\frac{70}{9}\,a^{4}b^{4}x^{9}+8\,a^{5}b^{3}x^{7}+\frac{28}{5}\,a^{6}b^{2}x^{5}+\frac{8}{3}\,a^{7}bx^{3}+a^{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] 1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11 +
 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^8*
x
```

mupad [B] time = 0.05, size = 87, normalized size = 0.86

$$a^{8}\,x+\frac{8\,a^{7}\,b\,x^{3}}{3}+\frac{28\,a^{6}\,b^{2}\,x^{5}}{5}+8\,a^{5}\,b^{3}\,x^{7}+\frac{70\,a^{4}\,b^{4}\,x^{9}}{9}+\frac{56\,a^{3}\,b^{5}\,x^{11}}{11}+\frac{28\,a^{2}\,b^{6}\,x^{13}}{13}+\frac{8\,a\,b^{7}\,x^{15}}{15}+\frac{b^{8}\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8,x)
```

```
[Out] a^8*x + (b^8*x^17)/17 + (8*a^7*b*x^3)/3 + (8*a*b^7*x^15)/15 + (28*a^6*b^2*x
^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2
*b^6*x^13)/13
```

sympy [A] time = 0.08, size = 102, normalized size = 1.01

$$a^{8}x+\frac{8a^{7}bx^{3}}{3}+\frac{28a^{6}b^{2}x^{5}}{5}+8a^{5}b^{3}x^{7}+\frac{70a^{4}b^{4}x^{9}}{9}+\frac{56a^{3}b^{5}x^{11}}{11}+\frac{28a^{2}b^{6}x^{13}}{13}+\frac{8ab^{7}x^{15}}{15}+\frac{b^{8}x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8,x)
```

```
[Out] a**8*x + 8*a**7*b*x**3/3 + 28*a**6*b**2*x**5/5 + 8*a**5*b**3*x**7 + 70*a**4
*b**4*x**9/9 + 56*a**3*b**5*x**11/11 + 28*a**2*b**6*x**13/13 + 8*a*b**7*x**
15/15 + b**8*x**17/17
```

3.114 $\int \frac{(a+bx^2)^8}{x^2} dx$

Optimal. Leaf size=100

$$-\frac{a^8}{x} + 8a^7bx + \frac{28}{3}a^6b^2x^3 + \frac{56}{5}a^5b^3x^5 + 10a^4b^4x^7 + \frac{56}{9}a^3b^5x^9 + \frac{28}{11}a^2b^6x^{11} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{15}}{15}$$

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{11}a^2b^6x^{11} + \frac{56}{9}a^3b^5x^9 + 10a^4b^4x^7 + \frac{56}{5}a^5b^3x^5 + \frac{28}{3}a^6b^2x^3 + 8a^7bx - \frac{a^8}{x} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^2,x]
```

```
[Out] -(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4
*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b^8
*x^15)/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^2)^8}{x^2} dx &= \int \left(8a^7b + \frac{a^8}{x^2} + 28a^6b^2x^2 + 56a^5b^3x^4 + 70a^4b^4x^6 + 56a^3b^5x^8 + 28a^2b^6x^{10} + 8ab^7x^{12} + b^8x^{14}\right) dx \\ &= -\frac{a^8}{x} + 8a^7bx + \frac{28}{3}a^6b^2x^3 + \frac{56}{5}a^5b^3x^5 + 10a^4b^4x^7 + \frac{56}{9}a^3b^5x^9 + \frac{28}{11}a^2b^6x^{11} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{15}}{15} \end{aligned}$$

Mathematica [A] time = 0.02, size = 100, normalized size = 1.00

$$-\frac{a^8}{x} + 8a^7bx + \frac{28}{3}a^6b^2x^3 + \frac{56}{5}a^5b^3x^5 + 10a^4b^4x^7 + \frac{56}{9}a^3b^5x^9 + \frac{28}{11}a^2b^6x^{11} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^2,x]
```

```
[Out] -(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4
*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b^8
*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^2, x]
```

fricas [A] time = 1.00, size = 92, normalized size = 0.92

$$\frac{429\,b^8x^{16}+3960\,ab^7x^{14}+16380\,a^2b^6x^{12}+40040\,a^3b^5x^{10}+64350\,a^4b^4x^8+72072\,a^5b^3x^6+60060\,a^6b^2x^4+51480\,a^7bx^2-6435\,a^8}{6435\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="fricas")
```

```
[Out] 1/6435*(429*b^8*x^16 + 3960*a*b^7*x^14 + 16380*a^2*b^6*x^12 + 40040*a^3*b^5
*x^10 + 64350*a^4*b^4*x^8 + 72072*a^5*b^3*x^6 + 60060*a^6*b^2*x^4 + 51480*a
^7*b*x^2 - 6435*a^8)/x
```

giac [A] time = 1.07, size = 88, normalized size = 0.88

$$\frac{1}{15}\,b^8x^{15}+\frac{8}{13}\,ab^7x^{13}+\frac{28}{11}\,a^2b^6x^{11}+\frac{56}{9}\,a^3b^5x^9+10\,a^4b^4x^7+\frac{56}{5}\,a^5b^3x^5+\frac{28}{3}\,a^6b^2x^3+8\,a^7bx-\frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="giac")
```

```
[Out] 1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 + 1
0*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```

maple [A] time = 0.00, size = 89, normalized size = 0.89

$$\frac{b^8x^{15}}{15}+\frac{8a\,b^7x^{13}}{13}+\frac{28a^2b^6x^{11}}{11}+\frac{56a^3b^5x^9}{9}+10a^4b^4x^7+\frac{56a^5b^3x^5}{5}+\frac{28a^6b^2x^3}{3}+8a^7bx-\frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^2,x)
```

```
[Out] -a^8/x+8*a^7*b*x+28/3*a^6*b^2*x^3+56/5*a^5*b^3*x^5+10*a^4*b^4*x^7+56/9*a^3*
b^5*x^9+28/11*a^2*b^6*x^11+8/13*a*b^7*x^13+1/15*b^8*x^15
```

maxima [A] time = 1.30, size = 88, normalized size = 0.88

$$\frac{1}{15}\,b^{8}x^{15}+\frac{8}{13}\,ab^{7}x^{13}+\frac{28}{11}\,a^{2}b^{6}x^{11}+\frac{56}{9}\,a^{3}b^{5}x^{9}+10\,a^{4}b^{4}x^{7}+\frac{56}{5}\,a^{5}b^{3}x^{5}+\frac{28}{3}\,a^{6}b^{2}x^{3}+8\,a^{7}bx-\frac{a^{8}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 + 1
0*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```

mupad [B] time = 0.06, size = 88, normalized size = 0.88

$$\frac{b^{8}\,x^{15}}{15}-\frac{a^{8}}{x}+\frac{8\,a\,b^{7}\,x^{13}}{13}+\frac{28\,a^{6}\,b^{2}\,x^{3}}{3}+\frac{56\,a^{5}\,b^{3}\,x^{5}}{5}+10\,a^{4}\,b^{4}\,x^{7}+\frac{56\,a^{3}\,b^{5}\,x^{9}}{9}+\frac{28\,a^{2}\,b^{6}\,x^{11}}{11}+8\,a^{7}\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^2,x)
```

```
[Out] (b^8*x^15)/15 - a^8/x + (8*a*b^7*x^13)/13 + (28*a^6*b^2*x^3)/3 + (56*a^5*b^
3*x^5)/5 + 10*a^4*b^4*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + 8*a
^7*b*x
```

sympy [A] time = 0.18, size = 99, normalized size = 0.99

$$-\frac{a^{8}}{x}+8a^{7}bx+\frac{28a^{6}b^{2}x^{3}}{3}+\frac{56a^{5}b^{3}x^{5}}{5}+10a^{4}b^{4}x^{7}+\frac{56a^{3}b^{5}x^{9}}{9}+\frac{28a^{2}b^{6}x^{11}}{11}+\frac{8ab^{7}x^{13}}{13}+\frac{b^{8}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**2,x)
```

```
[Out] -a**8/x + 8*a**7*b*x + 28*a**6*b**2*x**3/3 + 56*a**5*b**3*x**5/5 + 10*a**4*
b**4*x**7 + 56*a**3*b**5*x**9/9 + 28*a**2*b**6*x**11/11 + 8*a*b**7*x**13/13
 + b**8*x**15/15
```

3.115 $\int \frac{(a+bx^2)^8}{x^4} dx$

Optimal. Leaf size=98

$$-\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{9}a^2b^6x^9+8a^3b^5x^7+14a^4b^4x^5+\frac{56}{3}a^5b^3x^3+28a^6b^2x-\frac{8a^7b}{x}-\frac{a^8}{3x^3}+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^4,x]
```

```
[Out] -a^8/(3*x^3) - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4
*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13)/
13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^4} dx &= \int \left(28a^6b^2+\frac{a^8}{x^4}+\frac{8a^7b}{x^2}+56a^5b^3x^2+70a^4b^4x^4+56a^3b^5x^6+28a^2b^6x^8+8ab^7x^{10}+b^8x^{12}\right) d\\ &= -\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 98, normalized size = 1.00

$$-\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^4,x]
```

```
[Out] -1/3*a^8/x^3 - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4
*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13)/
13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^4, x]
```

fricas [A] time = 1.03, size = 92, normalized size = 0.94

$$\frac{99\,b^8x^{16}+936\,ab^7x^{14}+4004\,a^2b^6x^{12}+10296\,a^3b^5x^{10}+18018\,a^4b^4x^8+24024\,a^5b^3x^6+36036\,a^6b^2x^4-10296\,a^7bx^2-429\,a^8}{1287\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="fricas")
```

```
[Out] 1/1287*(99*b^8*x^16 + 936*a*b^7*x^14 + 4004*a^2*b^6*x^12 + 10296*a^3*b^5*x^
10 + 18018*a^4*b^4*x^8 + 24024*a^5*b^3*x^6 + 36036*a^6*b^2*x^4 - 10296*a^7*
b*x^2 - 429*a^8)/x^3
```

giac [A] time = 1.10, size = 89, normalized size = 0.91

$$\frac{1}{13}\,b^8x^{13}+\frac{8}{11}\,ab^7x^{11}+\frac{28}{9}\,a^2b^6x^9+8\,a^3b^5x^7+14\,a^4b^4x^5+\frac{56}{3}\,a^5b^3x^3+28\,a^6b^2x-\frac{24\,a^7bx^2+a^8}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="giac")
```

```
[Out] 1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^4
*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3
```

maple [A] time = 0.01, size = 89, normalized size = 0.91

$$\frac{b^8x^{13}}{13}+\frac{8a\,b^7x^{11}}{11}+\frac{28a^2b^6x^9}{9}+8a^3b^5x^7+14a^4b^4x^5+\frac{56a^5b^3x^3}{3}+28a^6b^2x-\frac{8a^7b}{x}-\frac{a^8}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^4,x)
```

```
[Out] -1/3*a^8/x^3-8*a^7*b/x+28*a^6*b^2*x+56/3*a^5*b^3*x^3+14*a^4*b^4*x^5+8*a^3*b
^5*x^7+28/9*a^2*b^6*x^9+8/11*a*b^7*x^11+1/13*b^8*x^13
```

maxima [A] time = 1.38, size = 89, normalized size = 0.91

$$\frac{1}{13}\,b^8x^{13}+\frac{8}{11}\,ab^7x^{11}+\frac{28}{9}\,a^2b^6x^9+8\,a^3b^5x^7+14\,a^4b^4x^5+\frac{56}{3}\,a^5b^3x^3+28\,a^6b^2x-\frac{24\,a^7bx^2+a^8}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^4,x, algorithm="maxima")
```

```
[Out] 1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^4
*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3
```

mupad [B] time = 0.05, size = 91, normalized size = 0.93

$$\frac{b^8\,x^{13}}{13}-\frac{\frac{a^8}{3}+8\,b\,a^7\,x^2}{x^3}+28\,a^6\,b^2\,x+\frac{8\,a\,b^7\,x^{11}}{11}+\frac{56\,a^5\,b^3\,x^3}{3}+14\,a^4\,b^4\,x^5+8\,a^3\,b^5\,x^7+\frac{28\,a^2\,b^6\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^4,x)
```

```
[Out] (b^8*x^13)/13 - (a^8/3 + 8*a^7*b*x^2)/x^3 + 28*a^6*b^2*x + (8*a*b^7*x^11)/1
1 + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/
9
```

sympy [A] time = 0.22, size = 100, normalized size = 1.02

$$28a^6b^2x+\frac{56a^5b^3x^3}{3}+14a^4b^4x^5+8a^3b^5x^7+\frac{28a^2b^6x^9}{9}+\frac{8ab^7x^{11}}{11}+\frac{b^8x^{13}}{13}+\frac{-a^8-24a^7bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**4,x)
```

```
[Out] 28*a**6*b**2*x + 56*a**5*b**3*x**3/3 + 14*a**4*b**4*x**5 + 8*a**3*b**5*x**7
 + 28*a**2*b**6*x**9/9 + 8*a*b**7*x**11/11 + b**8*x**13/13 + (-a**8 - 24*a*
*7*b*x**2)/(3*x**3)
```

3.116 $\int \frac{(a+bx^2)^8}{x^6}\,dx$

Optimal. Leaf size=100

$$-\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$4a^2b^6x^7+\frac{56}{5}a^3b^5x^5+\frac{70}{3}a^4b^4x^3+56a^5b^3x-\frac{28a^6b^2}{x}-\frac{8a^7b}{3x^3}-\frac{a^8}{5x^5}+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^6,x]
```

```
[Out] -a^8/(5*x^5) - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*
b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^
11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^6}\,dx &= \int \left(56a^5b^3+\frac{a^8}{x^6}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^2}+70a^4b^4x^2+56a^3b^5x^4+28a^2b^6x^6+8ab^7x^8+b^8x^{10}\right)dx\\ &= -\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 100, normalized size = 1.00

$$-\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^6,x]
```

```
[Out] -1/5*a^8/x^5 - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*
b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^
11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^6, x]
```

fricas [A] time = 1.11, size = 92, normalized size = 0.92

$$\frac{45\,b^8x^{16}+440\,ab^7x^{14}+1980\,a^2b^6x^{12}+5544\,a^3b^5x^{10}+11550\,a^4b^4x^8+27720\,a^5b^3x^6-13860\,a^6b^2x^4-1320\,a^7bx^2-99\,a^8}{495\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="fricas")
```

```
[Out] 1/495*(45*b^8*x^16 + 440*a*b^7*x^14 + 1980*a^2*b^6*x^12 + 5544*a^3*b^5*x^10
 + 11550*a^4*b^4*x^8 + 27720*a^5*b^3*x^6 - 13860*a^6*b^2*x^4 - 1320*a^7*b*x
^2 - 99*a^8)/x^5
```

giac [A] time = 0.97, size = 91, normalized size = 0.91

$$\frac{1}{11}\,b^8x^{11}+\frac{8}{9}\,ab^7x^9+4\,a^2b^6x^7+\frac{56}{5}\,a^3b^5x^5+\frac{70}{3}\,a^4b^4x^3+56\,a^5b^3x-\frac{420\,a^6b^2x^4+40\,a^7bx^2+3\,a^8}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="giac")
```

```
[Out] 1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^4
*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x^5
```

maple [A] time = 0.01, size = 89, normalized size = 0.89

$$\frac{b^8x^{11}}{11}+\frac{8a\,b^7x^9}{9}+4a^2b^6x^7+\frac{56a^3b^5x^5}{5}+\frac{70a^4b^4x^3}{3}+56a^5b^3x-\frac{28a^6b^2}{x}-\frac{8a^7b}{3x^3}-\frac{a^8}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^6,x)
```

```
[Out] -1/5*a^8/x^5-8/3*a^7*b/x^3-28*a^6*b^2/x+56*a^5*b^3*x+70/3*a^4*b^4*x^3+56/5*
a^3*b^5*x^5+4*a^2*b^6*x^7+8/9*a*b^7*x^9+1/11*b^8*x^11
```

maxima [A] time = 1.21, size = 91, normalized size = 0.91

$$\frac{1}{11}\,b^8x^{11}+\frac{8}{9}\,ab^7x^9+4\,a^2b^6x^7+\frac{56}{5}\,a^3b^5x^5+\frac{70}{3}\,a^4b^4x^3+56\,a^5b^3x-\frac{420\,a^6b^2x^4+40\,a^7bx^2+3\,a^8}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^6,x, algorithm="maxima")
```

```
[Out] 1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^4
*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x^5
```

mupad [B] time = 4.96, size = 91, normalized size = 0.91

$$\frac{b^8\,x^{11}}{11}-\frac{\frac{a^8}{5}+\frac{8\,a^7\,b\,x^2}{3}+28\,a^6\,b^2\,x^4}{x^5}+56\,a^5\,b^3\,x+\frac{8\,a\,b^7\,x^9}{9}+\frac{70\,a^4\,b^4\,x^3}{3}+\frac{56\,a^3\,b^5\,x^5}{5}+4\,a^2\,b^6\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^6,x)
```

```
[Out] (b^8*x^11)/11 - (a^8/5 + (8*a^7*b*x^2)/3 + 28*a^6*b^2*x^4)/x^5 + 56*a^5*b^3
*x + (8*a*b^7*x^9)/9 + (70*a^4*b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*
x^7
```

sympy [A] time = 0.28, size = 102, normalized size = 1.02

$$56a^5b^3x+\frac{70a^4b^4x^3}{3}+\frac{56a^3b^5x^5}{5}+4a^2b^6x^7+\frac{8ab^7x^9}{9}+\frac{b^8x^{11}}{11}+\frac{-3a^8-40a^7bx^2-420a^6b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**6,x)
```

```
[Out] 56*a**5*b**3*x + 70*a**4*b**4*x**3/3 + 56*a**3*b**5*x**5/5 + 4*a**2*b**6*x*
*7 + 8*a*b**7*x**9/9 + b**8*x**11/11 + (-3*a**8 - 40*a**7*b*x**2 - 420*a**6
*b**2*x**4)/(15*x**5)
```

3.117 $\int \frac{(a+bx^2)^8}{x^8} dx$

Optimal. Leaf size=102

$$-\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{28}{5}a^2b^6x^5+\frac{56}{3}a^3b^5x^3-\frac{28a^6b^2}{3x^3}+70a^4b^4x-\frac{56a^5b^3}{x}-\frac{8a^7b}{5x^5}-\frac{a^8}{7x^7}+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^8,x]
```

```
[Out] -a^8/(7*x^7) - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 +
(b^8*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^8}\,dx &= \int \left(70a^4b^4+\frac{a^8}{x^8}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^2}+56a^3b^5x^2+28a^2b^6x^4+8ab^7x^6+b^8x^8\right)dx\\ &= -\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 102, normalized size = 1.00

$$-\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^8,x]
```

```
[Out] -1/7*a^8/x^7 - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 +
(b^8*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^8}{x^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^8, x]
```

fricas [A] time = 1.14, size = 92, normalized size = 0.90

$$\frac{35\,b^8x^{16} + 360\,ab^7x^{14} + 1764\,a^2b^6x^{12} + 5880\,a^3b^5x^{10} + 22050\,a^4b^4x^8 - 17640\,a^5b^3x^6 - 2940\,a^6b^2x^4 - 504\,a^7bx^2 - 45\,a^8}{315\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^8*x^16 + 360*a*b^7*x^14 + 1764*a^2*b^6*x^12 + 5880*a^3*b^5*x^10
 + 22050*a^4*b^4*x^8 - 17640*a^5*b^3*x^6 - 2940*a^6*b^2*x^4 - 504*a^7*b*x^2
 - 45*a^8)/x^7
```

giac [A] time = 0.93, size = 91, normalized size = 0.89

$$\frac{1}{9}\,b^8x^9 + \frac{8}{7}\,ab^7x^7 + \frac{28}{5}\,a^2b^6x^5 + \frac{56}{3}\,a^3b^5x^3 + 70\,a^4b^4x - \frac{5880\,a^5b^3x^6 + 980\,a^6b^2x^4 + 168\,a^7bx^2 + 15\,a^8}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="giac")
```

```
[Out] 1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4*
b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^8)
/x^7
```

maple [A] time = 0.00, size = 89, normalized size = 0.87

$$\frac{b^8x^9}{9} + \frac{8a\,b^7x^7}{7} + \frac{28a^2b^6x^5}{5} + \frac{56a^3b^5x^3}{3} + 70a^4b^4x - \frac{56a^5b^3}{x} - \frac{28a^6b^2}{3x^3} - \frac{8a^7b}{5x^5} - \frac{a^8}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^8,x)
```

```
[Out] -1/7*a^8/x^7-8/5*a^7*b/x^5-28/3*a^6*b^2/x^3-56*a^5*b^3/x+70*a^4*b^4*x+56/3*
a^3*b^5*x^3+28/5*a^2*b^6*x^5+8/7*a*b^7*x^7+1/9*b^8*x^9
```

maxima [A] time = 1.36, size = 91, normalized size = 0.89

$$\frac{1}{9}\,b^{8}x^{9}+\frac{8}{7}\,ab^{7}x^{7}+\frac{28}{5}\,a^{2}b^{6}x^{5}+\frac{56}{3}\,a^{3}b^{5}x^{3}+70\,a^{4}b^{4}x-\frac{5880\,a^{5}b^{3}x^{6}+980\,a^{6}b^{2}x^{4}+168\,a^{7}bx^{2}+15\,a^{8}}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^8,x, algorithm="maxima")
```

```
[Out] 1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4*
b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^8)
/x^7
```

mupad [B] time = 4.80, size = 91, normalized size = 0.89

$$\frac{b^{8}\,x^{9}}{9}-\frac{\frac{a^{8}}{7}+\frac{8\,a^{7}\,b\,x^{2}}{5}+\frac{28\,a^{6}\,b^{2}\,x^{4}}{3}+56\,a^{5}\,b^{3}\,x^{6}}{x^{7}}+70\,a^{4}\,b^{4}\,x+\frac{8\,a\,b^{7}\,x^{7}}{7}+\frac{56\,a^{3}\,b^{5}\,x^{3}}{3}+\frac{28\,a^{2}\,b^{6}\,x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^8,x)
```

```
[Out] (b^8*x^9)/9 - (a^8/7 + (8*a^7*b*x^2)/5 + (28*a^6*b^2*x^4)/3 + 56*a^5*b^3*x^
6)/x^7 + 70*a^4*b^4*x + (8*a*b^7*x^7)/7 + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*
x^5)/5
```

sympy [A] time = 0.33, size = 102, normalized size = 1.00

$$70a^{4}b^{4}x+\frac{56a^{3}b^{5}x^{3}}{3}+\frac{28a^{2}b^{6}x^{5}}{5}+\frac{8ab^{7}x^{7}}{7}+\frac{b^{8}x^{9}}{9}+\frac{-15a^{8}-168a^{7}bx^{2}-980a^{6}b^{2}x^{4}-5880a^{5}b^{3}x^{6}}{105x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**8,x)
```

```
[Out] 70*a**4*b**4*x + 56*a**3*b**5*x**3/3 + 28*a**2*b**6*x**5/5 + 8*a*b**7*x**7/
7 + b**8*x**9/9 + (-15*a**8 - 168*a**7*b*x**2 - 980*a**6*b**2*x**4 - 5880*a
**5*b**3*x**6)/(105*x**7)
```

3.118 $\int \frac{(a+bx^2)^8}{x^{10}}\,dx$

Optimal. Leaf size=102

$$-\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}+\frac{28}{3}a^2b^6x^3-\frac{70a^4b^4}{x}+56a^3b^5x-\frac{8a^7b}{7x^7}-\frac{a^8}{9x^9}+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^10,x]
```

```
[Out] -a^8/(9*x^9) - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x
^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5
+ (b^8*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^8}{x^{10}}\,dx &= \int \left(56a^3b^5+\frac{a^8}{x^{10}}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^2}+28a^2b^6x^2+8ab^7x^4+b^8x^6\right)dx\\ &= -\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 102, normalized size = 1.00

$$-\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^10,x]
```

```
[Out] -1/9*a^8/x^9 - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x
^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5
+ (b^8*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^10, x]
```

fricas [A] time = 0.96, size = 92, normalized size = 0.90

$$\frac{45\,b^8x^{16}+504\,ab^7x^{14}+2940\,a^2b^6x^{12}+17640\,a^3b^5x^{10}-22050\,a^4b^4x^8-5880\,a^5b^3x^6-1764\,a^6b^2x^4-360\,a^7bx^2-35\,a^8}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(45*b^8*x^16 + 504*a*b^7*x^14 + 2940*a^2*b^6*x^12 + 17640*a^3*b^5*x^1
0 - 22050*a^4*b^4*x^8 - 5880*a^5*b^3*x^6 - 1764*a^6*b^2*x^4 - 360*a^7*b*x^2
 - 35*a^8)/x^9
```

giac [A] time = 0.94, size = 91, normalized size = 0.89

$$\frac{1}{7}\,b^8x^7+\frac{8}{5}\,ab^7x^5+\frac{28}{3}\,a^2b^6x^3+56\,a^3b^5x-\frac{22050\,a^4b^4x^8+5880\,a^5b^3x^6+1764\,a^6b^2x^4+360\,a^7bx^2+35\,a^8}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="giac")
```

```
[Out] 1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(2205
0*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*a^
8)/x^9
```

maple [A] time = 0.01, size = 89, normalized size = 0.87

$$\frac{b^8x^7}{7}+\frac{8a\,b^7x^5}{5}+\frac{28a^2b^6x^3}{3}+56a^3b^5x-\frac{70a^4b^4}{x}-\frac{56a^5b^3}{3x^3}-\frac{28a^6b^2}{5x^5}-\frac{8a^7b}{7x^7}-\frac{a^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^10,x)
```

```
[Out] -1/9*a^8/x^9-8/7*a^7*b/x^7-28/5*a^6*b^2/x^5-56/3*a^5*b^3/x^3-70*a^4*b^4/x+5
6*a^3*b^5*x+28/3*a^2*b^6*x^3+8/5*a*b^7*x^5+1/7*b^8*x^7
```

maxima [A] time = 1.36, size = 91, normalized size = 0.89

$$\frac{1}{7}\,b^8x^7+\frac{8}{5}\,ab^7x^5+\frac{28}{3}\,a^2b^6x^3+56\,a^3b^5x-\frac{22050\,a^4b^4x^8+5880\,a^5b^3x^6+1764\,a^6b^2x^4+360\,a^7bx^2+35\,a^8}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^10,x, algorithm="maxima")
```

```
[Out] 1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(2205
0*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*a^
8)/x^9
```

mupad [B] time = 0.05, size = 91, normalized size = 0.89

$$\frac{b^8\,x^7}{7}-\frac{\frac{a^8}{9}+\frac{8\,a^7\,b\,x^2}{7}+\frac{28\,a^6\,b^2\,x^4}{5}+\frac{56\,a^5\,b^3\,x^6}{3}+70\,a^4\,b^4\,x^8}{x^9}+56\,a^3\,b^5\,x+\frac{8\,a\,b^7\,x^5}{5}+\frac{28\,a^2\,b^6\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^10,x)
```

```
[Out] (b^8*x^7)/7 - (a^8/9 + (8*a^7*b*x^2)/7 + (28*a^6*b^2*x^4)/5 + (56*a^5*b^3*x
^6)/3 + 70*a^4*b^4*x^8)/x^9 + 56*a^3*b^5*x + (8*a*b^7*x^5)/5 + (28*a^2*b^6*
x^3)/3
```

sympy [A] time = 0.42, size = 100, normalized size = 0.98

$$56a^3b^5x+\frac{28a^2b^6x^3}{3}+\frac{8ab^7x^5}{5}+\frac{b^8x^7}{7}+\frac{-35a^8-360a^7bx^2-1764a^6b^2x^4-5880a^5b^3x^6-22050a^4b^4x^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**10,x)
```

```
[Out] 56*a**3*b**5*x + 28*a**2*b**6*x**3/3 + 8*a*b**7*x**5/5 + b**8*x**7/7 + (-35
*a**8 - 360*a**7*b*x**2 - 1764*a**6*b**2*x**4 - 5880*a**5*b**3*x**6 - 22050
*a**4*b**4*x**8)/(315*x**9)
```

3.119 $\int \frac{(a+bx^2)^8}{x^{12}}\, dx$

Optimal. Leaf size=100

$$-\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x-\frac{8a^7b}{9x^9}-\frac{a^8}{11x^{11}}+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^12,x]
```

```
[Out] -a^8/(11*x^11) - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5)
 - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 +
 (b^8*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{12}}\, dx &= \int \left(28a^2b^6+\frac{a^8}{x^{12}}+\frac{8a^7b}{x^{10}}+\frac{28a^6b^2}{x^8}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^2}+8ab^7x^2+b^8x^4\right) dx\\ &= -\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 100, normalized size = 1.00

$$-\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^12,x]
```

```
[Out] -1/11*a^8/x^11 - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5)
 - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 +
 (b^8*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^12, x]
```

fricas [A] time = 1.16, size = 92, normalized size = 0.92

$$\frac{99\,b^8x^{16}+1320\,ab^7x^{14}+13860\,a^2b^6x^{12}-27720\,a^3b^5x^{10}-11550\,a^4b^4x^8-5544\,a^5b^3x^6-1980\,a^6b^2x^4-440\,a^7bx^2-45\,a^8}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="fricas")
```

```
[Out] 1/495*(99*b^8*x^16 + 1320*a*b^7*x^14 + 13860*a^2*b^6*x^12 - 27720*a^3*b^5*x
^10 - 11550*a^4*b^4*x^8 - 5544*a^5*b^3*x^6 - 1980*a^6*b^2*x^4 - 440*a^7*b*x
^2 - 45*a^8)/x^11
```

giac [A] time = 0.92, size = 91, normalized size = 0.91

$$\frac{1}{5}\,b^8x^5+\frac{8}{3}\,ab^7x^3+28\,a^2b^6x-\frac{27720\,a^3b^5x^{10}+11550\,a^4b^4x^8+5544\,a^5b^3x^6+1980\,a^6b^2x^4+440\,a^7bx^2+45\,a^8}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="giac")
```

```
[Out] 1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^10 + 11
550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 45*
a^8)/x^11
```

maple [A] time = 0.01, size = 89, normalized size = 0.89

$$\frac{b^8x^5}{5}+\frac{8a\,b^7x^3}{3}+28a^2b^6x-\frac{56a^3b^5}{x}-\frac{70a^4b^4}{3x^3}-\frac{56a^5b^3}{5x^5}-\frac{4a^6b^2}{x^7}-\frac{8a^7b}{9x^9}-\frac{a^8}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^12,x)
```

```
[Out] -1/11*a^8/x^11-8/9*a^7*b/x^9-4*a^6*b^2/x^7-56/5*a^5*b^3/x^5-70/3*a^4*b^4/x^
3-56*a^3*b^5/x+28*a^2*b^6*x+8/3*a*b^7*x^3+1/5*b^8*x^5
```

maxima [A] time = 1.40, size = 91, normalized size = 0.91

$$\frac{1}{5}\,b^8x^5+\frac{8}{3}\,ab^7x^3+28\,a^2b^6x-\frac{27720\,a^3b^5x^{10}+11550\,a^4b^4x^8+5544\,a^5b^3x^6+1980\,a^6b^2x^4+440\,a^7bx^2+45\,a^8}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^12,x, algorithm="maxima")
```

```
[Out] 1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^10 + 11
550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 45*
a^8)/x^11
```

mupad [B] time = 4.58, size = 91, normalized size = 0.91

$$\frac{b^8\,x^5}{5}-\frac{\frac{a^8}{11}+\frac{8\,a^7\,b\,x^2}{9}+4\,a^6\,b^2\,x^4+\frac{56\,a^5\,b^3\,x^6}{5}+\frac{70\,a^4\,b^4\,x^8}{3}+56\,a^3\,b^5\,x^{10}}{x^{11}}+28\,a^2\,b^6\,x+\frac{8\,a\,b^7\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^12,x)
```

```
[Out] (b^8*x^5)/5 - (a^8/11 + (8*a^7*b*x^2)/9 + 4*a^6*b^2*x^4 + (56*a^5*b^3*x^6)/
5 + (70*a^4*b^4*x^8)/3 + 56*a^3*b^5*x^10)/x^11 + 28*a^2*b^6*x + (8*a*b^7*x^
3)/3
```

sympy [A] time = 0.50, size = 99, normalized size = 0.99

$$28a^2b^6x+\frac{8ab^7x^3}{3}+\frac{b^8x^5}{5}+\frac{-45a^8-440a^7bx^2-1980a^6b^2x^4-5544a^5b^3x^6-11550a^4b^4x^8-27720a^3b^5x^{10}}{495x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**12,x)
```

```
[Out] 28*a**2*b**6*x + 8*a*b**7*x**3/3 + b**8*x**5/5 + (-45*a**8 - 440*a**7*b*x**
2 - 1980*a**6*b**2*x**4 - 5544*a**5*b**3*x**6 - 11550*a**4*b**4*x**8 - 2772
0*a**3*b**5*x**10)/(495*x**11)
```

3.120 $\int \frac{(a+bx^2)^8}{x^{14}}\,dx$

Optimal. Leaf size=98

$$-\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}$$

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}-\frac{8a^7b}{11x^{11}}-\frac{a^8}{13x^{13}}+8ab^7x+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^14,x]
```

```
[Out] -a^8/(13*x^13) - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x
^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x +
 (b^8*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{14}}\,dx &= \int \left(8ab^7+\frac{a^8}{x^{14}}+\frac{8a^7b}{x^{12}}+\frac{28a^6b^2}{x^{10}}+\frac{56a^5b^3}{x^8}+\frac{70a^4b^4}{x^6}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^2}+b^8x^2\right)dx\\ &= -\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 98, normalized size = 1.00

$$-\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^14,x]
```

```
[Out] -1/13*a^8/x^13 - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x
^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x +
 (b^8*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^14,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^14, x]
```

fricas [A] time = 1.29, size = 92, normalized size = 0.94

$$\frac{429\,b^8x^{16}+10296\,ab^7x^{14}-36036\,a^2b^6x^{12}-24024\,a^3b^5x^{10}-18018\,a^4b^4x^8-10296\,a^5b^3x^6-4004\,a^6b^2x^4-936\,a^7bx^2-99\,a^8}{1287\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="fricas")
```

```
[Out] 1/1287*(429*b^8*x^16 + 10296*a*b^7*x^14 - 36036*a^2*b^6*x^12 - 24024*a^3*b^
5*x^10 - 18018*a^4*b^4*x^8 - 10296*a^5*b^3*x^6 - 4004*a^6*b^2*x^4 - 936*a^7
*b*x^2 - 99*a^8)/x^13
```

giac [A] time = 1.10, size = 91, normalized size = 0.93

$$\frac{1}{3}\,b^8x^3+8\,ab^7x-\frac{36036\,a^2b^6x^{12}+24024\,a^3b^5x^{10}+18018\,a^4b^4x^8+10296\,a^5b^3x^6+4004\,a^6b^2x^4+936\,a^7bx^2+99\,a^8}{1287\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="giac")
```

```
[Out] 1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*b^5*x^10 +
 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2 +
 99*a^8)/x^13
```

maple [A] time = 0.01, size = 89, normalized size = 0.91

$$\frac{b^8x^3}{3}+8a\,b^7x-\frac{28a^2b^6}{x}-\frac{56a^3b^5}{3x^3}-\frac{14a^4b^4}{x^5}-\frac{8a^5b^3}{x^7}-\frac{28a^6b^2}{9x^9}-\frac{8a^7b}{11x^{11}}-\frac{a^8}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^14,x)
```

```
[Out] -1/13*a^8/x^13-8/11*a^7*b/x^11-28/9*a^6*b^2/x^9-8*a^5*b^3/x^7-14*a^4*b^4/x^
5-56/3*a^3*b^5/x^3-28*a^2*b^6/x+8*a*b^7*x+1/3*b^8*x^3
```

maxima [A] time = 1.33, size = 91, normalized size = 0.93

$$\frac{1}{3}\,b^8x^3+8\,ab^7x-\frac{36036\,a^2b^6x^{12}+24024\,a^3b^5x^{10}+18018\,a^4b^4x^8+10296\,a^5b^3x^6+4004\,a^6b^2x^4+936\,a^7bx^2+99\,a^8}{1287\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^14,x, algorithm="maxima")
```

```
[Out] 1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*b^5*x^10 +
 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2 +
 99*a^8)/x^13
```

mupad [B] time = 0.07, size = 92, normalized size = 0.94

$$-\frac{\frac{a^8}{13}+\frac{8\,a^7\,b\,x^2}{11}+\frac{28\,a^6\,b^2\,x^4}{9}+8\,a^5\,b^3\,x^6+14\,a^4\,b^4\,x^8+\frac{56\,a^3\,b^5\,x^{10}}{3}+28\,a^2\,b^6\,x^{12}-8\,a\,b^7\,x^{14}-\frac{b^8\,x^{16}}{3}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^14,x)
```

```
[Out] -(a^8/13 - (b^8*x^16)/3 + (8*a^7*b*x^2)/11 - 8*a*b^7*x^14 + (28*a^6*b^2*x^4
)/9 + 8*a^5*b^3*x^6 + 14*a^4*b^4*x^8 + (56*a^3*b^5*x^10)/3 + 28*a^2*b^6*x^1
2)/x^13
```

sympy [A] time = 0.57, size = 97, normalized size = 0.99

$$8ab^7x+\frac{b^8x^3}{3}+\frac{-99a^8-936a^7bx^2-4004a^6b^2x^4-10296a^5b^3x^6-18018a^4b^4x^8-24024a^3b^5x^{10}-36036a^2b^6x^{12}}{1287x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**14,x)
```

```
[Out] 8*a*b**7*x + b**8*x**3/3 + (-99*a**8 - 936*a**7*b*x**2 - 4004*a**6*b**2*x**
4 - 10296*a**5*b**3*x**6 - 18018*a**4*b**4*x**8 - 24024*a**3*b**5*x**10 - 3
6036*a**2*b**6*x**12)/(1287*x**13)
```

3.121 $\int \frac{(a+bx^2)^8}{x^{16}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x$$

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8a^7b}{13x^{13}}-\frac{a^8}{15x^{15}}-\frac{8ab^7}{x}+b^8x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^16,x]
```

```
[Out] -a^8/(15*x^15) - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^3
)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3)
- (8*a*b^7)/x + b^8*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{16}}\,dx &= \int \left(b^8+\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{12}}+\frac{56a^5b^3}{x^{10}}+\frac{70a^4b^4}{x^8}+\frac{56a^3b^5}{x^6}+\frac{28a^2b^6}{x^4}+\frac{8ab^7}{x^2}\right)dx\\ &= -\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x\end{aligned}$$

Mathematica [A] time = 0.01, size = 99, normalized size = 1.00

$$-\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^16,x]
```

```
[Out] -1/15*a^8/x^15 - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^3
)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3)
- (8*a*b^7)/x + b^8*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^16,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^16, x]
```

fricas [A] time = 1.02, size = 92, normalized size = 0.93

$$\frac{6435\,b^8x^{16}-51480\,ab^7x^{14}-60060\,a^2b^6x^{12}-72072\,a^3b^5x^{10}-64350\,a^4b^4x^8-40040\,a^5b^3x^6-16380\,a^6b^2x^4-3960\,a^7bx^2-429\,a^8}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="fricas")
```

```
[Out] 1/6435*(6435*b^8*x^16 - 51480*a*b^7*x^14 - 60060*a^2*b^6*x^12 - 72072*a^3*b
^5*x^10 - 64350*a^4*b^4*x^8 - 40040*a^5*b^3*x^6 - 16380*a^6*b^2*x^4 - 3960*
a^7*b*x^2 - 429*a^8)/x^15
```

giac [A] time = 0.87, size = 90, normalized size = 0.91

$$b^8x-\frac{51480\,ab^7x^{14}+60060\,a^2b^6x^{12}+72072\,a^3b^5x^{10}+64350\,a^4b^4x^8+40040\,a^5b^3x^6+16380\,a^6b^2x^4+3960\,a^7bx^2+429\,a^8}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="giac")
```

```
[Out] b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*b^5*x^10
+ 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 3960*a^7*b*x^
2 + 429*a^8)/x^15
```

maple [A] time = 0.01, size = 88, normalized size = 0.89

$$b^8x-\frac{8a\,b^7}{x}-\frac{28a^2b^6}{3x^3}-\frac{56a^3b^5}{5x^5}-\frac{10a^4b^4}{x^7}-\frac{56a^5b^3}{9x^9}-\frac{28a^6b^2}{11x^{11}}-\frac{8a^7b}{13x^{13}}-\frac{a^8}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^16,x)
```

```
[Out] -1/15*a^8/x^15-8/13*a^7*b/x^13-28/11*a^6*b^2/x^11-56/9*a^5*b^3/x^9-10*a^4*b
^4/x^7-56/5*a^3*b^5/x^5-28/3*a^2*b^6/x^3-8*a*b^7/x+b^8*x
```

maxima [A] time = 1.41, size = 90, normalized size = 0.91

$$b^8x - \frac{51480\,ab^7x^{14} + 60060\,a^2b^6x^{12} + 72072\,a^3b^5x^{10} + 64350\,a^4b^4x^8 + 40040\,a^5b^3x^6 + 16380\,a^6b^2x^4 + 3960\,a^7bx^2 + 429\,a^8}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^16,x, algorithm="maxima")
```

```
[Out] b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*b^5*x^10
+ 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 3960*a^7*b*x^
2 + 429*a^8)/x^15
```

mupad [B] time = 4.52, size = 90, normalized size = 0.91

$$b^8\,x - \frac{\frac{a^8}{15} + \frac{8\,a^7\,b\,x^2}{13} + \frac{28\,a^6\,b^2\,x^4}{11} + \frac{56\,a^5\,b^3\,x^6}{9} + 10\,a^4\,b^4\,x^8 + \frac{56\,a^3\,b^5\,x^{10}}{5} + \frac{28\,a^2\,b^6\,x^{12}}{3} + 8\,a\,b^7\,x^{14}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^16,x)
```

```
[Out] b^8*x - (a^8/15 + (8*a^7*b*x^2)/13 + 8*a*b^7*x^14 + (28*a^6*b^2*x^4)/11 + (
56*a^5*b^3*x^6)/9 + 10*a^4*b^4*x^8 + (56*a^3*b^5*x^10)/5 + (28*a^2*b^6*x^12
)/3)/x^15
```

sympy [A] time = 0.69, size = 95, normalized size = 0.96

$$b^8x + \frac{-429a^8 - 3960a^7bx^2 - 16380a^6b^2x^4 - 40040a^5b^3x^6 - 64350a^4b^4x^8 - 72072a^3b^5x^{10} - 60060a^2b^6x^{12} - 51480ab^7x^{14}}{6435x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**16,x)
```

```
[Out] b**8*x + (-429*a**8 - 3960*a**7*b*x**2 - 16380*a**6*b**2*x**4 - 40040*a**5*
b**3*x**6 - 64350*a**4*b**4*x**8 - 72072*a**3*b**5*x**10 - 60060*a**2*b**6*
x**12 - 51480*a*b**7*x**14)/(6435*x**15)
```

3.122 $\int \frac{(a+bx^2)^8}{x^{18}} \, dx$

Optimal. Leaf size=104

$$-\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

Rubi [A] time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8a^7b}{15x^{15}}-\frac{a^8}{17x^{17}}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^18,x]
```

```
[Out] -a^8/(17*x^17) - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^3
)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5)
 - (8*a*b^7)/(3*x^3) - b^8/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^8}{x^{18}} \, dx &= \int \left(\frac{a^8}{x^{18}}+\frac{8a^7b}{x^{16}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{10}}+\frac{56a^3b^5}{x^8}+\frac{28a^2b^6}{x^6}+\frac{8ab^7}{x^4}+\frac{b^8}{x^2}\right) dx\\ &= -\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 104, normalized size = 1.00

$$-\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^18,x]
```

```
[Out] -1/17*a^8/x^17 - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^3
)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5)
 - (8*a*b^7)/(3*x^3) - b^8/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^8}{x^{18}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^18,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^18, x]
```

fricas [A] time = 1.08, size = 92, normalized size = 0.88

$$-\frac{109395\, b^8 x^{16} + 291720\, a b^7 x^{14} + 612612\, a^2 b^6 x^{12} + 875160\, a^3 b^5 x^{10} + 850850\, a^4 b^4 x^8 + 556920\, a^5 b^3 x^6 + 235620\, a^6 b^2 x^4 + 58344\, a^7 b x^2 + 6435\, a^8}{109395\, x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="fricas")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

giac [A] time = 0.82, size = 92, normalized size = 0.88

$$-\frac{109395\, b^8 x^{16} + 291720\, a b^7 x^{14} + 612612\, a^2 b^6 x^{12} + 875160\, a^3 b^5 x^{10} + 850850\, a^4 b^4 x^8 + 556920\, a^5 b^3 x^6 + 235620\, a^6 b^2 x^4 + 58344\, a^7 b x^2 + 6435\, a^8}{109395\, x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="giac")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

maple [A] time = 0.01, size = 91, normalized size = 0.88

$$-\frac{b^8}{x} - \frac{8a\,b^7}{3x^3} - \frac{28a^2b^6}{5x^5} - \frac{8a^3b^5}{x^7} - \frac{70a^4b^4}{9x^9} - \frac{56a^5b^3}{11x^{11}} - \frac{28a^6b^2}{13x^{13}} - \frac{8a^7b}{15x^{15}} - \frac{a^8}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^18,x)
```

```
[Out] -1/17*a^8/x^17-8/15*a^7*b/x^15-28/13*a^6*b^2/x^13-56/11*a^5*b^3/x^11-70/9*a
^4*b^4/x^9-8*a^3*b^5/x^7-28/5*a^2*b^6/x^5-8/3*a*b^7/x^3-b^8/x
```

maxima [A] time = 1.36, size = 92, normalized size = 0.88

$$-\frac{109395\,b^{8}x^{16}+291720\,ab^{7}x^{14}+612612\,a^{2}b^{6}x^{12}+875160\,a^{3}b^{5}x^{10}+850850\,a^{4}b^{4}x^{8}+556920\,a^{5}b^{3}x^{6}+235620\,a^{6}b^{2}x^{4}+58344\,a^{7}bx^{2}+6435\,a^{8}}{109395\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^18,x, algorithm="maxima")
```

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 8751
60*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*
x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

mupad [B] time = 0.07, size = 91, normalized size = 0.88

$$-\frac{\frac{a^8}{17}+\frac{8\,a^7\,b\,x^2}{15}+\frac{28\,a^6\,b^2\,x^4}{13}+\frac{56\,a^5\,b^3\,x^6}{11}+\frac{70\,a^4\,b^4\,x^8}{9}+8\,a^3\,b^5\,x^{10}+\frac{28\,a^2\,b^6\,x^{12}}{5}+\frac{8\,a\,b^7\,x^{14}}{3}+b^8\,x^{16}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^18,x)
```

```
[Out] -(a^8/17 + b^8*x^16 + (8*a^7*b*x^2)/15 + (8*a*b^7*x^14)/3 + (28*a^6*b^2*x^4
)/13 + (56*a^5*b^3*x^6)/11 + (70*a^4*b^4*x^8)/9 + 8*a^3*b^5*x^10 + (28*a^2*
b^6*x^12)/5)/x^17
```

sympy [A] time = 0.79, size = 99, normalized size = 0.95

$$\frac{-6435a^{8}-58344a^{7}bx^{2}-235620a^{6}b^{2}x^{4}-556920a^{5}b^{3}x^{6}-850850a^{4}b^{4}x^{8}-875160a^{3}b^{5}x^{10}-612612a^{2}b^{6}x^{12}-291720ab^{7}x^{14}-109395b^{8}x^{16}}{109395x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**18,x)
```

```
[Out] (-6435*a**8 - 58344*a**7*b*x**2 - 235620*a**6*b**2*x**4 - 556920*a**5*b**3*
x**6 - 850850*a**4*b**4*x**8 - 875160*a**3*b**5*x**10 - 612612*a**2*b**6*x*
*12 - 291720*a*b**7*x**14 - 109395*b**8*x**16)/(109395*x**17)
```

3.123 $\int \frac{(a+bx^2)^8}{x^{20}} dx$

Optimal. Leaf size=106

$$-\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8a^7b}{17x^{17}}-\frac{a^8}{19x^{19}}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^8/x^20,x]
```

```
[Out] -a^8/(19*x^19) - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^3
)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x
^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^8}{x^{20}} dx &= \int \left(\frac{a^8}{x^{20}}+\frac{8a^7b}{x^{18}}+\frac{28a^6b^2}{x^{16}}+\frac{56a^5b^3}{x^{14}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^8}+\frac{8ab^7}{x^6}+\frac{b^8}{x^4}\right) dx\\ &= -\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 106, normalized size = 1.00

$$-\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^8/x^20,x]
```

```
[Out] -1/19*a^8/x^19 - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^3
)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x
^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^8}{x^{20}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^8/x^20,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^8/x^20, x]
```

fricas [A] time = 1.10, size = 92, normalized size = 0.87

$$-\frac{692835\,b^8x^{16}+3325608\,ab^7x^{14}+8314020\,a^2b^6x^{12}+12932920\,a^3b^5x^{10}+13226850\,a^4b^4x^8+8953560\,a^5b^3x^6+3879876\,a^6b^2x^4+978120\,a^7bx^2+109395\,a^8}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="fricas")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

giac [A] time = 1.14, size = 92, normalized size = 0.87

$$-\frac{692835\,b^8x^{16}+3325608\,ab^7x^{14}+8314020\,a^2b^6x^{12}+12932920\,a^3b^5x^{10}+13226850\,a^4b^4x^8+8953560\,a^5b^3x^6+3879876\,a^6b^2x^4+978120\,a^7bx^2+109395\,a^8}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="giac")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

maple [A] time = 0.01, size = 91, normalized size = 0.86

$$-\frac{b^8}{3x^3}-\frac{8a\,b^7}{5x^5}-\frac{4a^2b^6}{x^7}-\frac{56a^3b^5}{9x^9}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^5b^3}{13x^{13}}-\frac{28a^6b^2}{15x^{15}}-\frac{8a^7b}{17x^{17}}-\frac{a^8}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^8/x^20,x)
```

```
[Out] -1/19*a^8/x^19-8/17*a^7*b/x^17-28/15*a^6*b^2/x^15-56/13*a^5*b^3/x^13-70/11*
a^4*b^4/x^11-56/9*a^3*b^5/x^9-4*a^2*b^6/x^7-8/5*a*b^7/x^5-1/3*b^8/x^3
```

maxima [A] time = 1.36, size = 92, normalized size = 0.87

$$-\frac{692835\,b^8x^{16} + 3325608\,ab^7x^{14} + 8314020\,a^2b^6x^{12} + 12932920\,a^3b^5x^{10} + 13226850\,a^4b^4x^8 + 8953560\,a^5b^3x^6 + 3879876\,a^6b^2x^4 + 978120\,a^7bx^2 + 109395\,a^8}{2078505\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^8/x^20,x, algorithm="maxima")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 1
2932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876
*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

mupad [B] time = 0.08, size = 92, normalized size = 0.87

$$-\frac{\frac{a^8}{19} + \frac{8\,a^7\,b\,x^2}{17} + \frac{28\,a^6\,b^2\,x^4}{15} + \frac{56\,a^5\,b^3\,x^6}{13} + \frac{70\,a^4\,b^4\,x^8}{11} + \frac{56\,a^3\,b^5\,x^{10}}{9} + 4\,a^2\,b^6\,x^{12} + \frac{8\,a\,b^7\,x^{14}}{5} + \frac{b^8\,x^{16}}{3}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^8/x^20,x)
```

```
[Out] -(a^8/19 + (b^8*x^16)/3 + (8*a^7*b*x^2)/17 + (8*a*b^7*x^14)/5 + (28*a^6*b^2
*x^4)/15 + (56*a^5*b^3*x^6)/13 + (70*a^4*b^4*x^8)/11 + (56*a^3*b^5*x^10)/9
+ 4*a^2*b^6*x^12)/x^19
```

sympy [A] time = 0.84, size = 99, normalized size = 0.93

$$\frac{-109395a^8 - 978120a^7bx^2 - 3879876a^6b^2x^4 - 8953560a^5b^3x^6 - 13226850a^4b^4x^8 - 12932920a^3b^5x^{10} - 8314020a^2b^6x^{12} - 3325608ab^7x^{14} - 692835b^8x^{16}}{2078505x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**8/x**20,x)
```

```
[Out] (-109395*a**8 - 978120*a**7*b*x**2 - 3879876*a**6*b**2*x**4 - 8953560*a**5*
b**3*x**6 - 13226850*a**4*b**4*x**8 - 12932920*a**3*b**5*x**10 - 8314020*a*
*2*b**6*x**12 - 3325608*a*b**7*x**14 - 692835*b**8*x**16)/(2078505*x**19)
```

3.124 $\int \frac{x^{11}}{a+bx^2} \, dx$

Optimal. Leaf size=79

$$-\frac{a^5 \log \left(a+bx^2\right)}{2b^6}+\frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{6b^3}-\frac{a^3x^4}{4b^4}+\frac{a^4x^2}{2b^5}-\frac{a^5 \log \left(a+bx^2\right)}{2b^6}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2),x]
```

```
[Out] (a^4*x^2)/(2*b^5) - (a^3*x^4)/(4*b^4) + (a^2*x^6)/(6*b^3) - (a*x^8)/(8*b^2)
 + x^10/(10*b) - (a^5*Log[a + b*x^2])/(2*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{a+bx^2} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{x^5}{a+bx} \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right) \, dx, x, x^2\right)\\
&= \frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}-\frac{a^5 \log \left(a+bx^2\right)}{2b^6}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 79, normalized size = 1.00

$$-\frac{a^5\log\left(a+bx^2\right)}{2b^6}+\frac{a^4x^2}{2b^5}-\frac{a^3x^4}{4b^4}+\frac{a^2x^6}{6b^3}-\frac{ax^8}{8b^2}+\frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2),x]
```

```
[Out] (a^4*x^2)/(2*b^5) - (a^3*x^4)/(4*b^4) + (a^2*x^6)/(6*b^3) - (a*x^8)/(8*b^2)
 + x^10/(10*b) - (a^5*Log[a + b*x^2])/(2*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^11/(a + b*x^2), x]
```

fricas [A] time = 1.16, size = 67, normalized size = 0.85

$$\frac{12\,b^5x^{10}-15\,ab^4x^8+20\,a^2b^3x^6-30\,a^3b^2x^4+60\,a^4bx^2-60\,a^5\log\left(bx^2+a\right)}{120\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/120*(12*b^5*x^10 - 15*a*b^4*x^8 + 20*a^2*b^3*x^6 - 30*a^3*b^2*x^4 + 60*a^
4*b*x^2 - 60*a^5*log(b*x^2 + a))/b^6
```

giac [A] time = 0.99, size = 69, normalized size = 0.87

$$-\frac{a^5\log\left(\left|bx^2+a\right|\right)}{2\,b^6}+\frac{12\,b^4x^{10}-15\,ab^3x^8+20\,a^2b^2x^6-30\,a^3bx^4+60\,a^4x^2}{120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*a^5*log(abs(b*x^2 + a))/b^6 + 1/120*(12*b^4*x^10 - 15*a*b^3*x^8 + 20*a
^2*b^2*x^6 - 30*a^3*b*x^4 + 60*a^4*x^2)/b^5
```

maple [A] time = 0.00, size = 68, normalized size = 0.86

$$\frac{x^{10}}{10b}-\frac{a\,x^{8}}{8b^{2}}+\frac{a^{2}x^{6}}{6b^{3}}-\frac{a^{3}x^{4}}{4b^{4}}+\frac{a^{4}x^{2}}{2b^{5}}-\frac{a^{5}\ln\left(b\,x^{2}+a\right)}{2b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a),x)
```

```
[Out] 1/2*a^4*x^2/b^5-1/4*a^3*x^4/b^4+1/6*a^2*x^6/b^3-1/8*a*x^8/b^2+1/10*x^10/b-1
/2*a^5*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.36, size = 68, normalized size = 0.86

$$-\frac{a^{5}\log\left(bx^{2}+a\right)}{2\,b^{6}}+\frac{12\,b^{4}x^{10}-15\,ab^{3}x^{8}+20\,a^{2}b^{2}x^{6}-30\,a^{3}bx^{4}+60\,a^{4}x^{2}}{120\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*a^5*log(b*x^2 + a)/b^6 + 1/120*(12*b^4*x^10 - 15*a*b^3*x^8 + 20*a^2*b^
2*x^6 - 30*a^3*b*x^4 + 60*a^4*x^2)/b^5
```

mupad [B] time = 0.06, size = 67, normalized size = 0.85

$$\frac{x^{10}}{10\,b}-\frac{a\,x^{8}}{8\,b^{2}}-\frac{a^{5}\,\ln\left(b\,x^{2}+a\right)}{2\,b^{6}}+\frac{a^{2}\,x^{6}}{6\,b^{3}}-\frac{a^{3}\,x^{4}}{4\,b^{4}}+\frac{a^{4}\,x^{2}}{2\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2),x)
```

```
[Out] x^10/(10*b) - (a*x^8)/(8*b^2) - (a^5*log(a + b*x^2))/(2*b^6) + (a^2*x^6)/(6
*b^3) - (a^3*x^4)/(4*b^4) + (a^4*x^2)/(2*b^5)
```

sympy [A] time = 0.18, size = 68, normalized size = 0.86

$$-\frac{a^{5}\log\left(a+bx^{2}\right)}{2b^{6}}+\frac{a^{4}x^{2}}{2b^{5}}-\frac{a^{3}x^{4}}{4b^{4}}+\frac{a^{2}x^{6}}{6b^{3}}-\frac{ax^{8}}{8b^{2}}+\frac{x^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a),x)
```

```
[Out] -a**5*log(a + b*x**2)/(2*b**6) + a**4*x**2/(2*b**5) - a**3*x**4/(4*b**4) +
a**2*x**6/(6*b**3) - a*x**8/(8*b**2) + x**10/(10*b)
```

3.125 $\int \frac{x^{10}}{a+bx^2}\,dx$

Optimal. Leaf size=81

$$-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^2x^5}{5b^3}-\frac{a^3x^3}{3b^4}+\frac{a^4x}{b^5}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^9
/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{a+bx^2}\,dx &= \int \left(\frac{a^4}{b^5}-\frac{a^3x^2}{b^4}+\frac{a^2x^4}{b^3}-\frac{ax^6}{b^2}+\frac{x^8}{b}-\frac{a^5}{b^5\left(a+bx^2\right)}\right)dx\\ &= \frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^5\int \frac{1}{a+bx^2}\,dx}{b^5}\\ &= \frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 1.00

$$-\frac{a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^9
/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^10/(a + b*x^2), x]
```

fricas [A] time = 1.00, size = 170, normalized size = 2.10

$$\left[\frac{70\,b^4x^9-90\,ab^3x^7+126\,a^2b^2x^5-210\,a^3bx^3+315\,a^4\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+630\,a^4x}{630\,b^5},\frac{35\,b^4x^9-45\,ab^3x^7+63\,a^2b^2x^5-105\,a^3bx^3-315\,a^4\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+315\,a^4x}{315\,b^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*x^9 - 90*a*b^3*x^7 + 126*a^2*b^2*x^5 - 210*a^3*b*x^3 + 315*a
^4*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 630*a^4*x)/
b^5, 1/315*(35*b^4*x^9 - 45*a*b^3*x^7 + 63*a^2*b^2*x^5 - 105*a^3*b*x^3 - 31
5*a^4*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 315*a^4*x)/b^5]
```

giac [A] time = 1.08, size = 77, normalized size = 0.95

$$-\frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^5}+\frac{35\,b^8x^9-45\,ab^7x^7+63\,a^2b^6x^5-105\,a^3b^5x^3+315\,a^4b^4x}{315\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*x^9 - 45*a*b^7*x
^7 + 63*a^2*b^6*x^5 - 105*a^3*b^5*x^3 + 315*a^4*b^4*x)/b^9
```

maple [A] time = 0.00, size = 71, normalized size = 0.88

$$\frac{x^9}{9b} - \frac{a\,x^7}{7b^2} + \frac{a^2x^5}{5b^3} - \frac{a^3x^3}{3b^4} - \frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^5} + \frac{a^4x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a),x)
```

```
[Out] 1/9*x^9/b-1/7*a*x^7/b^2+1/5*a^2*x^5/b^3-1/3*a^3*x^3/b^4+a^4*x/b^5-a^5/b^5/(
a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

maxima [A] time = 2.92, size = 72, normalized size = 0.89

$$-\frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^5} + \frac{35\,b^4x^9 - 45\,ab^3x^7 + 63\,a^2b^2x^5 - 105\,a^3bx^3 + 315\,a^4x}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^4*x^9 - 45*a*b^3*x
^7 + 63*a^2*b^2*x^5 - 105*a^3*b*x^3 + 315*a^4*x)/b^5
```

mupad [B] time = 0.06, size = 65, normalized size = 0.80

$$\frac{x^9}{9\,b} - \frac{a\,x^7}{7\,b^2} + \frac{a^4\,x}{b^5} - \frac{a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}} + \frac{a^2\,x^5}{5\,b^3} - \frac{a^3\,x^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2),x)
```

```
[Out] x^9/(9*b) - (a*x^7)/(7*b^2) + (a^4*x)/b^5 - (a^(9/2)*atan((b^(1/2)*x)/a^(1/
2)))/b^(11/2) + (a^2*x^5)/(5*b^3) - (a^3*x^3)/(3*b^4)
```

sympy [A] time = 0.20, size = 119, normalized size = 1.47

$$\frac{a^4x}{b^5} - \frac{a^3x^3}{3b^4} + \frac{a^2x^5}{5b^3} - \frac{ax^7}{7b^2} + \frac{\sqrt{-\frac{a^9}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} - \frac{\sqrt{-\frac{a^9}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} + \frac{x^9}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a),x)
```

```
[Out] a**4*x/b**5 - a**3*x**3/(3*b**4) + a**2*x**5/(5*b**3) - a*x**7/(7*b**2) + s
qrt(-a**9/b**11)*log(x - b**5*sqrt(-a**9/b**11)/a**4)/2 - sqrt(-a**9/b**11)
*log(x + b**5*sqrt(-a**9/b**11)/a**4)/2 + x**9/(9*b)
```

3.126 $\int \frac{x^9}{a+bx^2}\,dx$

Optimal. Leaf size=66

$$\frac{a^4\log\left(a+bx^2\right)}{2b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}$$

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^4}{4b^3}-\frac{a^3x^2}{2b^4}+\frac{a^4\log\left(a+bx^2\right)}{2b^5}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2),x]
```

```
[Out] -(a^3*x^2)/(2*b^4) + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4
*Log[a + b*x^2])/(2*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}+\frac{a^4\log\left(a+bx^2\right)}{2b^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 66, normalized size = 1.00

$$\frac{a^4\log\left(a+bx^2\right)}{2b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2),x]
```

```
[Out] -1/2*(a^3*x^2)/b^4 + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4
*Log[a + b*x^2])/(2*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^9/(a + b*x^2), x]
```

fricas [A] time = 0.98, size = 56, normalized size = 0.85

$$\frac{3\,b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-12\,a^3bx^2+12\,a^4\log\left(bx^2+a\right)}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 12*a^3*b*x^2 + 12*a^4*log(b
*x^2 + a))/b^5
```

giac [A] time = 1.08, size = 58, normalized size = 0.88

$$\frac{a^4\log\left(\left|bx^2+a\right|\right)}{2\,b^5}+\frac{3\,b^3x^8-4\,ab^2x^6+6\,a^2bx^4-12\,a^3x^2}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*a^4*log(abs(b*x^2 + a))/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x
^4 - 12*a^3*x^2)/b^4
```

maple [A] time = 0.00, size = 57, normalized size = 0.86

$$\frac{x^8}{8b}-\frac{a\,x^6}{6b^2}+\frac{a^2x^4}{4b^3}-\frac{a^3x^2}{2b^4}+\frac{a^4\ln\left(b\,x^2+a\right)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a),x)
```

```
[Out] -1/2*a^3*x^2/b^4+1/4*a^2*x^4/b^3-1/6*a*x^6/b^2+1/8*x^8/b+1/2*a^4*ln(b*x^2+a
)/b^5
```

maxima [A] time = 1.35, size = 57, normalized size = 0.86

$$\frac{a^4\log\left(bx^2+a\right)}{2\,b^5}+\frac{3\,b^3x^8-4\,ab^2x^6+6\,a^2bx^4-12\,a^3x^2}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*a^4*log(b*x^2 + a)/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x^4 -
12*a^3*x^2)/b^4
```

mupad [B] time = 0.08, size = 56, normalized size = 0.85

$$\frac{x^8}{8\,b}-\frac{a\,x^6}{6\,b^2}+\frac{a^4\,\ln\left(b\,x^2+a\right)}{2\,b^5}+\frac{a^2\,x^4}{4\,b^3}-\frac{a^3\,x^2}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2),x)
```

```
[Out] x^8/(8*b) - (a*x^6)/(6*b^2) + (a^4*log(a + b*x^2))/(2*b^5) + (a^2*x^4)/(4*b
^3) - (a^3*x^2)/(2*b^4)
```

sympy [A] time = 0.17, size = 56, normalized size = 0.85

$$\frac{a^4\log\left(a+bx^2\right)}{2b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{ax^6}{6b^2}+\frac{x^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a),x)
```

```
[Out] a**4*log(a + b*x**2)/(2*b**5) - a**3*x**2/(2*b**4) + a**2*x**4/(4*b**3) - a
*x**6/(6*b**2) + x**8/(8*b)
```

3.127 $\int \frac{x^8}{a+bx^2} \, dx$

Optimal. Leaf size=68

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^2x^3}{3b^3}-\frac{a^3x}{b^4}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a+bx^2}\,dx &= \int \left(-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}+\frac{a^4}{b^4\left(a+bx^2\right)}\right)dx\\
&= -\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}+\frac{a^4\int \frac{1}{a+bx^2}\,dx}{b^4}\\
&= -\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.00

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a + b*x^2),x]

[Out] -((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{a+bx^2}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(a + b*x^2),x]

[Out] IntegrateAlgebraic[x^8/(a + b*x^2), x]

fricas [A] time = 0.97, size = 148, normalized size = 2.18

$$\left[\frac{30\,b^3x^7-42\,ab^2x^5+70\,a^2bx^3+105\,a^3\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-210\,a^3x}{210\,b^4},\frac{15\,b^3x^7-21\,ab^2x^5+35\,a^2bx^3+105\,a^3\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-105\,a^3x}{105\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^2+a),x, algorithm="fricas")

[Out] [1/210*(30*b^3*x^7 - 42*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*a^3*x)/b^4, 1/105*(15*b^3*x^7 - 21*a*b^2*x^5 + 35*a^2*b*x^3 + 105*a^3*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*a^3*x)/b^4]

giac [A] time = 1.10, size = 65, normalized size = 0.96

$$\frac{a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{15\,b^6x^7-21\,ab^5x^5+35\,a^2b^4x^3-105\,a^3b^3x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^2+a),x, algorithm="giac")

```
[Out] a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*x^7 - 21*a*b^5*x^
5 + 35*a^2*b^4*x^3 - 105*a^3*b^3*x)/b^7
```

maple [A] time = 0.00, size = 60, normalized size = 0.88

$$\frac{x^7}{7b} - \frac{a\,x^5}{5b^2} + \frac{a^2x^3}{3b^3} + \frac{a^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^4} - \frac{a^3x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a),x)
```

```
[Out] 1/7*x^7/b-1/5*a*x^5/b^2+1/3*a^2*x^3/b^3-a^3*x/b^4+a^4/b^4/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.94, size = 60, normalized size = 0.88

$$\frac{a^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,b^3x^7 - 21\,ab^2x^5 + 35\,a^2bx^3 - 105\,a^3x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*x^7 - 21*a*b^2*x^
5 + 35*a^2*b*x^3 - 105*a^3*x)/b^4
```

mupad [B] time = 0.05, size = 54, normalized size = 0.79

$$\frac{x^7}{7\,b} - \frac{a\,x^5}{5\,b^2} - \frac{a^3\,x}{b^4} + \frac{a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}} + \frac{a^2\,x^3}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2),x)
```

```
[Out] x^7/(7*b) - (a*x^5)/(5*b^2) - (a^3*x)/b^4 + (a^(7/2)*atan((b^(1/2)*x)/a^(1/
2)))/b^(9/2) + (a^2*x^3)/(3*b^3)
```

sympy [A] time = 0.19, size = 107, normalized size = 1.57

$$-\frac{a^3x}{b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^5}{5b^2} - \frac{\sqrt{-\frac{a^7}{b^9}}\log\left(x - \frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2} + \frac{\sqrt{-\frac{a^7}{b^9}}\log\left(x + \frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2} + \frac{x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a),x)
```

```
[Out] -a**3*x/b**4 + a**2*x**3/(3*b**3) - a*x**5/(5*b**2) - sqrt(-a**7/b**9)*log(
x - b**4*sqrt(-a**7/b**9)/a**3)/2 + sqrt(-a**7/b**9)*log(x + b**4*sqrt(-a**
7/b**9)/a**3)/2 + x**7/(7*b)
```

3.128 $\int \frac{x^7}{a+bx^2}\,dx$

Optimal. Leaf size=53

$$-\frac{a^3\log\left(a+bx^2\right)}{2b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}$$

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^2}{2b^3}-\frac{a^3\log\left(a+bx^2\right)}{2b^4}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2),x]
```

```
[Out] (a^2*x^2)/(2*b^3) - (a*x^4)/(4*b^2) + x^6/(6*b) - (a^3*Log[a + b*x^2])/(2*b
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{a+bx^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx, x, x^2\right)\\ &= \frac{a^2x^2}{2b^3}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}-\frac{a^3\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.00

$$-\frac{a^3 \log \left(a+b x^2\right)}{2 b^4}+\frac{a^2 x^2}{2 b^3}-\frac{a x^4}{4 b^2}+\frac{x^6}{6 b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2),x]
```

```
[Out] (a^2*x^2)/(2*b^3) - (a*x^4)/(4*b^2) + x^6/(6*b) - (a^3*Log[a + b*x^2])/(2*b
^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{a+b x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x^2), x]
```

fricas [A] time = 0.93, size = 45, normalized size = 0.85

$$\frac{2 b^3 x^6 - 3 a b^2 x^4 + 6 a^2 b x^2 - 6 a^3 \log \left(b x^2 + a\right)}{12 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^6 - 3*a*b^2*x^4 + 6*a^2*b*x^2 - 6*a^3*log(b*x^2 + a))/b^4
```

giac [A] time = 1.05, size = 47, normalized size = 0.89

$$-\frac{a^3 \log \left(\left|b x^2 + a\right|\right)}{2 b^4} + \frac{2 b^2 x^6 - 3 a b x^4 + 6 a^2 x^2}{12 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*a^3*log(abs(b*x^2 + a))/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)
/b^3
```

maple [A] time = 0.00, size = 46, normalized size = 0.87

$$\frac{x^6}{6b} - \frac{a\,x^4}{4b^2} + \frac{a^2x^2}{2b^3} - \frac{a^3\ln\left(b\,x^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a),x)
```

```
[Out] 1/2*a^2*x^2/b^3-1/4*a*x^4/b^2+1/6*x^6/b-1/2*a^3*ln(b*x^2+a)/b^4
```

maxima [A] time = 1.36, size = 46, normalized size = 0.87

$$-\frac{a^3\log\left(bx^2+a\right)}{2\,b^4} + \frac{2\,b^2x^6 - 3\,abx^4 + 6\,a^2x^2}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*a^3*log(b*x^2 + a)/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3
```

mupad [B] time = 4.73, size = 45, normalized size = 0.85

$$\frac{x^6}{6\,b} - \frac{a\,x^4}{4\,b^2} - \frac{a^3\,\ln\left(b\,x^2+a\right)}{2\,b^4} + \frac{a^2\,x^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2),x)
```

```
[Out] x^6/(6*b) - (a*x^4)/(4*b^2) - (a^3*log(a + b*x^2))/(2*b^4) + (a^2*x^2)/(2*b
^3)
```

sympy [A] time = 0.16, size = 44, normalized size = 0.83

$$-\frac{a^3\log\left(a+bx^2\right)}{2b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a),x)
```

```
[Out] -a**3*log(a + b*x**2)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4/(4*b**2) + x**
6/(6*b)
```

3.129 $\int \frac{x^6}{a+bx^2} \, dx$

Optimal. Leaf size=55

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}$$

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^2x}{b^3}-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{a+bx^2}\,dx &= \int \left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+bx^2\right)}\right)dx\\ &= \frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}-\frac{a^3\int \frac{1}{a+bx^2}\,dx}{b^3}\\ &= \frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.00

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(7/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x^2), x]
```

fricas [A] time = 1.26, size = 126, normalized size = 2.29

$$\left[\frac{6\,b^2x^5-10\,abx^3+15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\,a^2x}{30\,b^3},\frac{3\,b^2x^5-5\,abx^3-15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+15\,a^2x}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*x^5 - 10*a*b*x^3 + 15*a^2*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-
a/b) - a)/(b*x^2 + a)) + 30*a^2*x)/b^3, 1/15*(3*b^2*x^5 - 5*a*b*x^3 - 15*a^
2*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*a^2*x)/b^3]
```

giac [A] time = 0.99, size = 55, normalized size = 1.00

$$-\frac{a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{3\,b^4x^5-5\,ab^3x^3+15\,a^2b^2x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*x^5 - 5*a*b^3*x^3
+ 15*a^2*b^2*x)/b^5
```

maple [A] time = 0.00, size = 49, normalized size = 0.89

$$\frac{x^5}{5b}-\frac{a\,x^3}{3b^2}-\frac{a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3}+\frac{a^2x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a),x)
```

```
[Out] 1/5*x^5/b-1/3*a*x^3/b^2+a^2*x/b^3-a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x)
```

maxima [A] time = 2.84, size = 50, normalized size = 0.91

$$-\frac{a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{3\,b^2x^5-5\,abx^3+15\,a^2x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^2*x^5 - 5*a*b*x^3 +
15*a^2*x)/b^3
```

mupad [B] time = 0.07, size = 43, normalized size = 0.78

$$\frac{x^5}{5\,b}-\frac{a\,x^3}{3\,b^2}+\frac{a^2\,x}{b^3}-\frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2),x)
```

```
[Out] x^5/(5*b) - (a*x^3)/(3*b^2) + (a^2*x)/b^3 - (a^(5/2)*atan((b^(1/2)*x)/a^(1/
2)))/b^(7/2)
```

sympy [A] time = 0.18, size = 95, normalized size = 1.73

$$\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x-\frac{b^3\sqrt{-\frac{a^5}{b^7}}}{a^2}\right)}{2}-\frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x+\frac{b^3\sqrt{-\frac{a^5}{b^7}}}{a^2}\right)}{2}+\frac{x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a),x)
```

```
[Out] a**2*x/b**3 - a*x**3/(3*b**2) + sqrt(-a**5/b**7)*log(x - b**3*sqrt(-a**5/b*
*7)/a**2)/2 - sqrt(-a**5/b**7)*log(x + b**3*sqrt(-a**5/b**7)/a**2)/2 + x**5
/(5*b)
```

3.130 $\int \frac{x^5}{a+bx^2} \, dx$

Optimal. Leaf size=40

$$\frac{a^2 \log \left(a+bx^2\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}$$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2 \log \left(a+bx^2\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2),x]
```

```
[Out] -(a*x^2)/(2*b^2) + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a+bx^2} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{x^2}{a+bx} \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right) dx, x, x^2\right)\\
&= -\frac{ax^2}{2b^2}+\frac{x^4}{4b}+\frac{a^2 \log \left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{a^2 \log \left(a+bx^2\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2),x]
[Out] -1/2*(a*x^2)/b^2 + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2),x]
[Out] IntegrateAlgebraic[x^5/(a + b*x^2), x]
```

fricas [A] time = 0.92, size = 33, normalized size = 0.82

$$\frac{b^2x^4-2\,abx^2+2\,a^2\log \left(bx^2+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="fricas")
[Out] 1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

giac [A] time = 1.12, size = 35, normalized size = 0.88

$$\frac{a^2 \log \left(\left|bx^2+a\right|\right)}{2\,b^3}+\frac{bx^4-2\,ax^2}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="giac")
[Out] 1/2*a^2*log(abs(b*x^2 + a))/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

maple [A] time = 0.00, size = 35, normalized size = 0.88

$$\frac{x^4}{4b}-\frac{a\,x^2}{2b^2}+\frac{a^2 \ln \left(b\,x^2+a\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a),x)
```

```
[Out] -1/2*a*x^2/b^2+1/4*x^4/b+1/2*a^2*ln(b*x^2+a)/b^3
```

maxima [A] time = 1.31, size = 34, normalized size = 0.85

$$\frac{a^2\log\left(bx^2+a\right)}{2\,b^3}+\frac{bx^4-2\,ax^2}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(b*x^2 + a)/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

mupad [B] time = 4.65, size = 33, normalized size = 0.82

$$\frac{2\,a^2\,\ln\left(b\,x^2+a\right)+b^2\,x^4-2\,a\,b\,x^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2),x)
```

```
[Out] (2*a^2*log(a + b*x^2) + b^2*x^4 - 2*a*b*x^2)/(4*b^3)
```

sympy [A] time = 0.15, size = 32, normalized size = 0.80

$$\frac{a^2\log\left(a+bx^2\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a),x)
```

```
[Out] a**2*log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)
```

3.131 $\int \frac{x^4}{a+bx^2} \, dx$

Optimal. Leaf size=42

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}} - \frac{ax}{b^2} + \frac{x^3}{3b}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 205}

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}} - \frac{ax}{b^2} + \frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2),x]
```

```
[Out] -((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{a+bx^2} \, dx &= \int \left(-\frac{a}{b^2} + \frac{x^2}{b} + \frac{a^2}{b^2\left(a+bx^2\right)}\right) dx \\
&= -\frac{ax}{b^2} + \frac{x^3}{3b} + \frac{a^2 \int \frac{1}{a+bx^2} \, dx}{b^2} \\
&= -\frac{ax}{b^2} + \frac{x^3}{3b} + \frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2),x]
[Out] -((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^2),x]
[Out] IntegrateAlgebraic[x^4/(a + b*x^2), x]
```

fricas [A] time = 0.67, size = 99, normalized size = 2.36

$$\left[\frac{2\,bx^3+3\,a\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,ax}{6\,b^2},\ \frac{bx^3+3\,a\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,ax}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="fricas")
[Out] [1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*a*
x)/b^2]
```

giac [A] time = 1.01, size = 40, normalized size = 0.95

$$\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{b^2x^3-3\,abx}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3 - 3*a*b*x)/b^3
```

maple [A] time = 0.00, size = 38, normalized size = 0.90

$$\frac{x^3}{3b} + \frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ b^2} - \frac{ax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a),x)
[Out] 1/3*x^3/b-a*x/b^2+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.80, size = 37, normalized size = 0.88

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^2} + \frac{bx^3 - 3\, ax}{3\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a),x, algorithm="maxima")
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*x^3 - 3*a*x)/b^2
```

mupad [B] time = 0.07, size = 32, normalized size = 0.76

$$\frac{x^3}{3\, b} + \frac{a^{3/2} \operatorname{atan}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{b^{5/2}} - \frac{a\, x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2),x)
[Out] x^3/(3*b) + (a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(5/2) - (a*x)/b^2
```

sympy [B] time = 0.17, size = 80, normalized size = 1.90

$$-\frac{ax}{b^2} - \frac{\sqrt{-\frac{a^3}{b^5}} \log\left(x - \frac{b^2 \sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2} + \frac{\sqrt{-\frac{a^3}{b^5}} \log\left(x + \frac{b^2 \sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2} + \frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a),x)
[Out] -a*x/b**2 - sqrt(-a**3/b**5)*log(x - b**2*sqrt(-a**3/b**5)/a)/2 + sqrt(-a**
3/b**5)*log(x + b**2*sqrt(-a**3/b**5)/a)/2 + x**3/(3*b)
```

3.132 $\int \frac{x^3}{a+bx^2}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2b} - \frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2),x]
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2),x]
[Out] IntegrateAlgebraic[x^3/(a + b*x^2), x]
```

fricas [A] time = 0.96, size = 22, normalized size = 0.81

$$\frac{bx^2 - a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

giac [A] time = 1.09, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,b} - \frac{a\log\left(\left|bx^2+a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="giac")
[Out] 1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^2}{2b} - \frac{a\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a),x)
```

```
[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

maxima [A] time = 1.32, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,b} - \frac{a\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.04, size = 22, normalized size = 0.81

$$-\frac{a\,\ln\left(b\,x^2 + a\right) - b\,x^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2),x)
```

```
[Out] -(a*log(a + b*x^2) - b*x^2)/(2*b^2)
```

sympy [A] time = 0.13, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a + bx^2\right)}{2b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

3.133 $\int \frac{x^2}{a+bx^2} \, dx$

Optimal. Leaf size=31

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {321, 205}

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+bx^2}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x^2), x]
```

fricas [A] time = 0.83, size = 82, normalized size = 2.65

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},\,-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

giac [A] time = 1.12, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

maple [A] time = 0.00, size = 27, normalized size = 0.87

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a),x)
[Out] 1/b*x-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a),x, algorithm="maxima")
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

mupad [B] time = 0.03, size = 23, normalized size = 0.74

$$\frac{x}{b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2),x)
[Out] x/b - (a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

sympy [B] time = 0.15, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a),x)
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

3.134 $\int \frac{x}{a+bx^2} \, dx$

Optimal. Leaf size=15

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {260}

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{a+bx^2} \, dx = \frac{\log\left(a+bx^2\right)}{2b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+bx^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x^2),x]
[Out] IntegrateAlgebraic[x/(a + b*x^2), x]
```

fricas [A] time = 0.87, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*log(b*x^2 + a)/b
```

giac [A] time = 1.07, size = 14, normalized size = 0.93

$$\frac{\log\left(|bx^2 + a|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(b\,x^2 + a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a),x)
[Out] 1/2*ln(b*x^2+a)/b
```

maxima [A] time = 1.34, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

mupad [B] time = 4.63, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2),x)
[Out] log(a + b*x^2)/(2*b)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.67

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a),x)
[Out] log(a + b*x**2)/(2*b)
```

3.135 $\int \frac{1}{a+bx^2} dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2} dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^(-1), x]
```

fricas [A] time = 0.89, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

giac [A] time = 0.97, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.90, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 4.70, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2),x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.13, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

3.136 $\int \frac{1}{x(a+bx^2)} \, dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^2\right)}{2a}$$
$$= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)),x]
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)),x]
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2)), x]
```

fricas [A] time = 1.37, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^2+a\right) - 2\,\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

giac [A] time = 1.09, size = 24, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,a} - \frac{\log\left(\left|bx^2+a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a} - \frac{\ln\left(b\,x^2 + a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a),x)
```

```
[Out] ln(x)/a-1/2*ln(b*x^2+a)/a
```

maxima [A] time = 1.33, size = 23, normalized size = 1.05

$$-\frac{\log\left(bx^2 + a\right)}{2\,a} + \frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + a)/a + 1/2*log(x^2)/a
```

mupad [B] time = 0.08, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^2 + a\right) - 2\,\ln(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)),x)
```

```
[Out] -(log(a + b*x^2) - 2*log(x))/(2*a)
```

sympy [A] time = 0.20, size = 15, normalized size = 0.68

$$\frac{\log{(x)}}{a} - \frac{\log{\left(\frac{a}{b} + x^2\right)}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a),x)
```

```
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

3.137 $\int \frac{1}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)}\,dx &= -\frac{1}{ax}-\frac{b\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)),x]
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)),x]
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2)), x]
```

fricas [A] time = 1.26, size = 82, normalized size = 2.41

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax},\,-\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="fricas")
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

giac [A] time = 1.13, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

maple [A] time = 0.01, size = 30, normalized size = 0.88

$$-\frac{b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ a} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a),x)
```

```
[Out] -1/a/x-1/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.89, size = 29, normalized size = 0.85

$$-\frac{b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

mupad [B] time = 4.62, size = 26, normalized size = 0.76

$$-\frac{1}{a\,x} - \frac{\sqrt{b}\ \mathrm{atan}\left(\frac{\sqrt{b}\ x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)),x)
```

```
[Out] - 1/(a*x) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

sympy [B] time = 0.18, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{b}{a^3}}\ \log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b} + x\right)}{2} - \frac{\sqrt{-\frac{b}{a^3}}\ \log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b} + x\right)}{2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

3.138 $\int \frac{1}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=35

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)),x]
[Out] -1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)),x]
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2)), x]
```

fricas [A] time = 1.24, size = 33, normalized size = 0.94

$$\frac{bx^2\log\left(bx^2+a\right)-2\,bx^2\log(x)-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

giac [A] time = 1.15, size = 43, normalized size = 1.23

$$-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{b\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="giac")
[Out] -1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*
x^2)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(b\,x^2+a\right)}{2a^2}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a),x)
```

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.37, size = 33, normalized size = 0.94

$$\frac{b\log\left(bx^2+a\right)}{2\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

mupad [B] time = 0.07, size = 31, normalized size = 0.89

$$\frac{b\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{1}{2\,a\,x^2}-\frac{b\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)),x)
```

```
[Out] (b*log(a + b*x^2))/(2*a^2) - 1/(2*a*x^2) - (b*log(x))/a^2
```

sympy [A] time = 0.26, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

3.139 $\int \frac{1}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=43

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)}\,dx = -\frac{1}{3ax^3} - \frac{b\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a}$$
$$= -\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^2\int \frac{1}{a+bx^2}\,dx}{a^2}$$
$$= -\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b}{a^2x} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)),x]
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)),x]
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2)), x]
```

fricas [A] time = 0.98, size = 106, normalized size = 2.47

$$\left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3}, \frac{3\,bx^3\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

giac [A] time = 1.12, size = 40, normalized size = 0.93

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} + \frac{3\,bx^2 - a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

maple [A] time = 0.01, size = 39, normalized size = 0.91

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2} + \frac{b}{a^2x} - \frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a),x)
```

```
[Out] -1/3/a/x^3+b/a^2/x+b^2/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.88, size = 40, normalized size = 0.93

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} + \frac{3\,bx^2 - a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

mupad [B] time = 4.67, size = 37, normalized size = 0.86

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{1}{3\,a} - \frac{b\,x^2}{a^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)),x)
```

```
[Out] (b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (1/(3*a) - (b*x^2)/a^2)/x^3
```

sympy [B] time = 0.22, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

3.140 $\int \frac{1}{x^5(a+bx^2)}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a
^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx, x, x^2\right)\\
&= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{b^2 \log \left(a+b x^2\right)}{2 a^3}+\frac{b^2 \log (x)}{a^3}+\frac{b}{2 a^2 x^2}-\frac{1}{4 a x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2
*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a+b x^2\right)}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^2)), x]
```

fricas [A] time = 1.04, size = 45, normalized size = 0.92

$$-\frac{2 \, b^{2} x^{4} \log \left(b x^{2}+a\right)-4 \, b^{2} x^{4} \log (x)-2 \, a b x^{2}+a^{2}}{4 \, a^{3} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*x
^4)
```

giac [A] time = 1.19, size = 57, normalized size = 1.16

$$\frac{b^{2} \log \left(x^{2}\right)}{2 \, a^{3}}-\frac{b^{2} \log \left({\left| b x^{2}+a \right|}\right)}{2 \, a^{3}}-\frac{3 \, b^{2} x^{4}-2 \, a b x^{2}+a^{2}}{4 \, a^{3} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2
*a*b*x^2 + a^2)/(a^3*x^4)
```

maple [A] time = 0.01, size = 44, normalized size = 0.90

$$\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^3}+\frac{b}{2a^2x^2}-\frac{1}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a),x)
```

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.36, size = 47, normalized size = 0.96

$$-\frac{b^2\log\left(bx^2+a\right)}{2\,a^3}+\frac{b^2\log\left(x^2\right)}{2\,a^3}+\frac{2\,bx^2-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/a^3 + 1/2*b^2*log(x^2)/a^3 + 1/4*(2*b*x^2 - a)/(a^2
*x^4)
```

mupad [B] time = 0.08, size = 46, normalized size = 0.94

$$\frac{b^2\,\ln(x)}{a^3}-\frac{b^2\,\ln\left(b\,x^2+a\right)}{2\,a^3}-\frac{\frac{1}{4\,a}-\frac{b\,x^2}{2\,a^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)),x)
```

```
[Out] (b^2*log(x))/a^3 - (b^2*log(a + b*x^2))/(2*a^3) - (1/(4*a) - (b*x^2)/(2*a^2
))/x^4
```

sympy [A] time = 0.30, size = 42, normalized size = 0.86

$$\frac{-a+2bx^2}{4a^2x^4}+\frac{b^2\log\left(x\right)}{a^3}-\frac{b^2\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a),x)
```

```
[Out] (-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2*
a**3)
```

3.141 $\int \frac{1}{x^6(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}-\frac{b^2}{a^3x}+\frac{b}{3a^2x^3}-\frac{1}{5ax^5}$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$-\frac{b^2}{a^3x}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}+\frac{b}{3a^2x^3}-\frac{1}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/(5*a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/a^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6 \left(a+bx^2\right)}\,dx = -\frac{1}{5ax^5} - \frac{b\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{a}$$
$$= -\frac{1}{5ax^5} + \frac{b}{3a^2x^3} + \frac{b^2\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a^2}$$
$$= -\frac{1}{5ax^5} + \frac{b}{3a^2x^3} - \frac{b^2}{a^3x} - \frac{b^3\int \frac{1}{a+bx^2}\,dx}{a^3}$$
$$= -\frac{1}{5ax^5} + \frac{b}{3a^2x^3} - \frac{b^2}{a^3x} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.00

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}} - \frac{b^2}{a^3x} + \frac{b}{3a^2x^3} - \frac{1}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/5*1/(a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/a^(7/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a + b*x^2)), x]
```

fricas [A] time = 0.91, size = 132, normalized size = 2.28

$$\left[\frac{15\,b^2x^5\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-30\,b^2x^4+10\,abx^2-6\,a^2}{30\,a^3x^5},\ -\frac{15\,b^2x^5\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+15\,b^2x^4-5\,abx^2+3\,a^2}{15\,a^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^2*x^5*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)
) - 30*b^2*x^4 + 10*a*b*x^2 - 6*a^2)/(a^3*x^5), -1/15*(15*b^2*x^5*sqrt(b/a)
*arctan(x*sqrt(b/a)) + 15*b^2*x^4 - 5*a*b*x^2 + 3*a^2)/(a^3*x^5)]
```

giac [A] time = 0.62, size = 52, normalized size = 0.90

$$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^3} - \frac{15\, b^2 x^4 - 5\, abx^2 + 3\, a^2}{15\, a^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/15*(15*b^2*x^4 - 5*a*b*x^2 +
 3*a^2)/(a^3*x^5)
```

maple [A] time = 0.01, size = 52, normalized size = 0.90

$$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\; a^3} - \frac{b^2}{a^3 x} + \frac{b}{3a^2 x^3} - \frac{1}{5a\, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a),x)
```

```
[Out] -1/5/a/x^5-b^2/a^3/x+1/3*b/a^2/x^3-b^3/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)
*b*x)
```

maxima [A] time = 2.89, size = 52, normalized size = 0.90

$$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^3} - \frac{15\, b^2 x^4 - 5\, abx^2 + 3\, a^2}{15\, a^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/15*(15*b^2*x^4 - 5*a*b*x^2 +
 3*a^2)/(a^3*x^5)
```

mupad [B] time = 0.06, size = 48, normalized size = 0.83

$$-\frac{\frac{1}{5\,a}-\frac{b\,x^2}{3\,a^2}+\frac{b^2\,x^4}{a^3}}{x^5}-\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)),x)
```

```
[Out] - (1/(5*a) - (b*x^2)/(3*a^2) + (b^2*x^4)/a^3)/x^5 - (b^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/a^(7/2)
```

sympy [B] time = 0.26, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{b^5}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^5}{a^7}}}{b^3}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^5}{a^7}}}{b^3}+x\right)}{2}+\frac{-3a^2+5abx^2-15b^2x^4}{15a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**7)*log(-a**4*sqrt(-b**5/a**7)/b**3 + x)/2 - sqrt(-b**5/a**7)*
log(a**4*sqrt(-b**5/a**7)/b**3 + x)/2 + (-3*a**2 + 5*a*b*x**2 - 15*b**2*x**
4)/(15*a**3*x**5)
```

3.142 $\int \frac{1}{x^7\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=63

$$\frac{b^3\log\left(a+bx^2\right)}{2a^4}-\frac{b^3\log(x)}{a^4}-\frac{b^2}{2a^3x^2}+\frac{b}{4a^2x^4}-\frac{1}{6ax^6}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b^2}{2a^3x^2}+\frac{b^3\log\left(a+bx^2\right)}{2a^4}-\frac{b^3\log(x)}{a^4}+\frac{b}{4a^2x^4}-\frac{1}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/(6*a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*Log[x])/a^4 + (b^3*Lo
g[a + b*x^2])/(2*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(a+bx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{6ax^6}+\frac{b}{4a^2x^4}-\frac{b^2}{2a^3x^2}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(a+bx^2\right)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.00

$$\frac{b^3\log\left(a+bx^2\right)}{2a^4}-\frac{b^3\log(x)}{a^4}-\frac{b^2}{2a^3x^2}+\frac{b}{4a^2x^4}-\frac{1}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/6*1/(a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*Log[x])/a^4 + (b^3*
Log[a + b*x^2])/(2*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(a + b*x^2)), x]
```

fricas [A] time = 1.09, size = 58, normalized size = 0.92

$$\frac{6\,b^3x^6\log\left(bx^2+a\right)-12\,b^3x^6\log(x)-6\,ab^2x^4+3\,a^2bx^2-2\,a^3}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b^3*x^6*log(b*x^2 + a) - 12*b^3*x^6*log(x) - 6*a*b^2*x^4 + 3*a^2*b*
x^2 - 2*a^3)/(a^4*x^6)
```

giac [A] time = 0.63, size = 70, normalized size = 1.11

$$-\frac{b^3\log\left(x^2\right)}{2\,a^4}+\frac{b^3\log\left(\left|bx^2+a\right|\right)}{2\,a^4}+\frac{11\,b^3x^6-6\,ab^2x^4+3\,a^2bx^2-2\,a^3}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(x^2)/a^4 + 1/2*b^3*log(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6
- 6*a*b^2*x^4 + 3*a^2*b*x^2 - 2*a^3)/(a^4*x^6)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln\left(b\,x^2+a\right)}{2a^4}-\frac{b^2}{2a^3x^2}+\frac{b}{4a^2x^4}-\frac{1}{6a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a),x)
```

```
[Out] -1/6/a/x^6+1/4*b/a^2/x^4-1/2*b^2/a^3/x^2-b^3*ln(x)/a^4+1/2*b^3*ln(b*x^2+a)/
a^4
```

maxima [A] time = 1.36, size = 58, normalized size = 0.92

$$\frac{b^3\log\left(bx^2+a\right)}{2\,a^4}-\frac{b^3\log\left(x^2\right)}{2\,a^4}-\frac{6\,b^2x^4-3\,abx^2+2\,a^2}{12\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log(b*x^2 + a)/a^4 - 1/2*b^3*log(x^2)/a^4 - 1/12*(6*b^2*x^4 - 3*a*b
*x^2 + 2*a^2)/(a^3*x^6)
```

mupad [B] time = 4.64, size = 58, normalized size = 0.92

$$\frac{b^3\,\ln\left(b\,x^2+a\right)}{2\,a^4}-\frac{\frac{1}{6\,a}-\frac{b\,x^2}{4\,a^2}+\frac{b^2\,x^4}{2\,a^3}}{x^6}-\frac{b^3\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)),x)
```

```
[Out] (b^3*log(a + b*x^2))/(2*a^4) - (1/(6*a) - (b*x^2)/(4*a^2) + (b^2*x^4)/(2*a^
3))/x^6 - (b^3*log(x))/a^4
```

sympy [A] time = 0.35, size = 56, normalized size = 0.89

$$\frac{-2a^2+3abx^2-6b^2x^4}{12a^3x^6}-\frac{b^3\log\left(x\right)}{a^4}+\frac{b^3\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a),x)
```

```
[Out] (-2*a**2 + 3*a*b*x**2 - 6*b**2*x**4)/(12*a**3*x**6) - b**3*log(x)/a**4 + b*
*3*log(a/b + x**2)/(2*a**4)
```

3.143 $\int \frac{1}{x^8(a+bx^2)}\,dx$

Optimal. Leaf size=69

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b^3}{a^4x}-\frac{b^2}{3a^3x^3}+\frac{b}{5a^2x^5}-\frac{1}{7ax^7}$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 205}

$$-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b}{5a^2x^5}-\frac{1}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/(7*a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^(7/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^8\left(a+bx^2\right)}\,dx &= -\frac{1}{7ax^7}-\frac{b\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{a}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}+\frac{b^2\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{a^2}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}-\frac{b^3\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a^3}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^4\int \frac{1}{a+bx^2}\,dx}{a^4}\\
&= -\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 69, normalized size = 1.00

$$
\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b^3}{a^4x}-\frac{b^2}{3a^3x^3}+\frac{b}{5a^2x^5}-\frac{1}{7ax^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*1/(a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^(7/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^8\left(a+bx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^8*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^8*(a + b*x^2)), x]
```

fricas [A] time = 1.13, size = 154, normalized size = 2.23

$$
\left[\frac{105\,b^3x^7\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+210\,b^3x^6-70\,ab^2x^4+42\,a^2bx^2-30\,a^3}{210\,a^4x^7},\ \frac{105\,b^3x^7\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+105\,b^3x^6-35\,ab^2x^4+21\,a^2bx^2-15\,a^3}{105\,a^4x^7}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(105*b^3*x^7*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
a)) + 210*b^3*x^6 - 70*a*b^2*x^4 + 42*a^2*b*x^2 - 30*a^3)/(a^4*x^7), 1/105*
(105*b^3*x^7*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*b^3*x^6 - 35*a*b^2*x^4 + 2
1*a^2*b*x^2 - 15*a^3)/(a^4*x^7)]
```

giac [A] time = 0.64, size = 62, normalized size = 0.90

$$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^4} + \frac{105\, b^3 x^6 - 35\, ab^2 x^4 + 21\, a^2 b x^2 - 15\, a^3}{105\, a^4 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(105*b^3*x^6 - 35*a*b^2*x
^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)
```

maple [A] time = 0.01, size = 61, normalized size = 0.88

$$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\; a^4} + \frac{b^3}{a^4 x} - \frac{b^2}{3a^3 x^3} + \frac{b}{5a^2 x^5} - \frac{1}{7a\, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a),x)
```

```
[Out] -1/7/a/x^7-1/3*b^2/a^3/x^3+1/5*b/a^2/x^5+b^3/a^4/x+b^4/a^4/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.93, size = 62, normalized size = 0.90

$$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^4} + \frac{105\, b^3 x^6 - 35\, ab^2 x^4 + 21\, a^2 b x^2 - 15\, a^3}{105\, a^4 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(105*b^3*x^6 - 35*a*b^2*x
^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)
```

mupad [B] time = 0.06, size = 59, normalized size = 0.86

$$\frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}-\frac{\frac{1}{7\,a}-\frac{b\,x^2}{5\,a^2}+\frac{b^2\,x^4}{3\,a^3}-\frac{b^3\,x^6}{a^4}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^2)),x)
```

```
[Out] (b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(9/2) - (1/(7*a) - (b*x^2)/(5*a^2) +
(b^2*x^4)/(3*a^3) - (b^3*x^6)/a^4)/x^7
```

sympy [A] time = 0.30, size = 112, normalized size = 1.62

$$-\frac{\sqrt{-\frac{b^7}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^7}{a^9}}}{b^4}+x\right)}{2}+\frac{\sqrt{-\frac{b^7}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^7}{a^9}}}{b^4}+x\right)}{2}+\frac{-15a^3+21a^2bx^2-35ab^2x^4+105b^3x^6}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**7/a**9)*log(-a**5*sqrt(-b**7/a**9)/b**4 + x)/2 + sqrt(-b**7/a**9)
*log(a**5*sqrt(-b**7/a**9)/b**4 + x)/2 + (-15*a**3 + 21*a**2*b*x**2 - 35*a*
b**2*x**4 + 105*b**3*x**6)/(105*a**4*x**7)
```

3.144 $\int \frac{1}{x^9\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=75

$$-\frac{b^4\log\left(a+bx^2\right)}{2a^5}+\frac{b^4\log(x)}{a^5}+\frac{b^3}{2a^4x^2}-\frac{b^2}{4a^3x^4}+\frac{b}{6a^2x^6}-\frac{1}{8ax^8}$$

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b^3}{2a^4x^2}-\frac{b^2}{4a^3x^4}-\frac{b^4\log\left(a+bx^2\right)}{2a^5}+\frac{b^4\log(x)}{a^5}+\frac{b}{6a^2x^6}-\frac{1}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)),x]
```

```
[Out] -1/(8*a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*Log
[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^5(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^5}-\frac{b}{a^2x^4}+\frac{b^2}{a^3x^3}-\frac{b^3}{a^4x^2}+\frac{b^4}{a^5x}-\frac{b^5}{a^5(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{8ax^8}+\frac{b}{6a^2x^6}-\frac{b^2}{4a^3x^4}+\frac{b^3}{2a^4x^2}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log\left(a+bx^2\right)}{2a^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 1.00

$$-\frac{b^4\log\left(a+bx^2\right)}{2a^5}+\frac{b^4\log(x)}{a^5}+\frac{b^3}{2a^4x^2}-\frac{b^2}{4a^3x^4}+\frac{b}{6a^2x^6}-\frac{1}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)),x]
```

```
[Out] -1/8*1/(a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*L
og[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^9*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^9*(a + b*x^2)), x]
```

fricas [A] time = 0.74, size = 69, normalized size = 0.92

$$-\frac{12\,b^4x^8\log\left(bx^2+a\right)-24\,b^4x^8\log(x)-12\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+3\,a^4}{24\,a^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/24*(12*b^4*x^8*log(b*x^2 + a) - 24*b^4*x^8*log(x) - 12*a*b^3*x^6 + 6*a^2
*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)
```

giac [A] time = 0.63, size = 81, normalized size = 1.08

$$\frac{b^4\log\left(x^2\right)}{2\,a^5}-\frac{b^4\log\left(\left|bx^2+a\right|\right)}{2\,a^5}-\frac{25\,b^4x^8-12\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+3\,a^4}{24\,a^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(x^2)/a^5 - 1/2*b^4*log(abs(b*x^2 + a))/a^5 - 1/24*(25*b^4*x^8 -
 12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)
```

maple [A] time = 0.01, size = 66, normalized size = 0.88

$$\frac{b^4 \ln(x)}{a^5} - \frac{b^4 \ln\left(b\,x^2 + a\right)}{2a^5} + \frac{b^3}{2a^4x^2} - \frac{b^2}{4a^3x^4} + \frac{b}{6a^2x^6} - \frac{1}{8a\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a),x)
```

```
[Out] -1/8/a/x^8+1/6*b/a^2/x^6-1/4*b^2/a^3/x^4+1/2*b^3/a^4/x^2+b^4*ln(x)/a^5-1/2*
b^4*ln(b*x^2+a)/a^5
```

maxima [A] time = 1.33, size = 69, normalized size = 0.92

$$-\frac{b^4 \log\left(bx^2 + a\right)}{2\,a^5} + \frac{b^4 \log\left(x^2\right)}{2\,a^5} + \frac{12\,b^3x^6 - 6\,ab^2x^4 + 4\,a^2bx^2 - 3\,a^3}{24\,a^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b^4*log(b*x^2 + a)/a^5 + 1/2*b^4*log(x^2)/a^5 + 1/24*(12*b^3*x^6 - 6*a
*b^2*x^4 + 4*a^2*b*x^2 - 3*a^3)/(a^4*x^8)
```

mupad [B] time = 4.67, size = 68, normalized size = 0.91

$$\frac{b^4\,\ln(x)}{a^5} - \frac{b^4\,\ln\left(b\,x^2 + a\right)}{2\,a^5} - \frac{\frac{1}{8\,a} - \frac{b\,x^2}{6\,a^2} + \frac{b^2\,x^4}{4\,a^3} - \frac{b^3\,x^6}{2\,a^4}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^2)),x)
```

```
[Out] (b^4*log(x))/a^5 - (b^4*log(a + b*x^2))/(2*a^5) - (1/(8*a) - (b*x^2)/(6*a^2
) + (b^2*x^4)/(4*a^3) - (b^3*x^6)/(2*a^4))/x^8
```

sympy [A] time = 0.39, size = 68, normalized size = 0.91

$$\frac{-3a^3 + 4a^2bx^2 - 6ab^2x^4 + 12b^3x^6}{24a^4x^8} + \frac{b^4 \log(x)}{a^5} - \frac{b^4 \log\left(\frac{a}{b} + x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a),x)
```

```
[Out] (-3*a**3 + 4*a**2*b*x**2 - 6*a*b**2*x**4 + 12*b**3*x**6)/(24*a**4*x**8) + b
**4*log(x)/a**5 - b**4*log(a/b + x**2)/(2*a**5)
```

3.145 $\int \frac{x^{13}}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=94

$$-\frac{a^6}{2b^7\left(a+bx^2\right)}-\frac{3a^5\log\left(a+bx^2\right)}{b^7}+\frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}$$

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{2b^4}-\frac{a^3x^4}{b^5}+\frac{5a^4x^2}{2b^6}-\frac{a^6}{2b^7\left(a+bx^2\right)}-\frac{3a^5\log\left(a+bx^2\right)}{b^7}-\frac{ax^8}{4b^3}+\frac{x^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^2,x]
```

```
[Out] (5*a^4*x^2)/(2*b^6) - (a^3*x^4)/b^5 + (a^2*x^6)/(2*b^4) - (a*x^8)/(4*b^3) +
 x^10/(10*b^2) - a^6/(2*b^7*(a + b*x^2)) - (3*a^5*Log[a + b*x^2])/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{5a^4}{b^6} - \frac{4a^3x}{b^5} + \frac{3a^2x^2}{b^4} - \frac{2ax^3}{b^3} + \frac{x^4}{b^2} + \frac{a^6}{b^6(a+bx)^2} - \frac{6a^5}{b^6(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{5a^4x^2}{2b^6} - \frac{a^3x^4}{b^5} + \frac{a^2x^6}{2b^4} - \frac{ax^8}{4b^3} + \frac{x^{10}}{10b^2} - \frac{a^6}{2b^7\left(a+bx^2\right)} - \frac{3a^5\log\left(a+bx^2\right)}{b^7}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.88

$$\frac{-\frac{10a^6}{a+bx^2} - 60a^5\log\left(a+bx^2\right) + 50a^4bx^2 - 20a^3b^2x^4 + 10a^2b^3x^6 - 5ab^4x^8 + 2b^5x^{10}}{20b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^2,x]
```

```
[Out] (50*a^4*b*x^2 - 20*a^3*b^2*x^4 + 10*a^2*b^3*x^6 - 5*a*b^4*x^8 + 2*b^5*x^10
- (10*a^6)/(a + b*x^2) - 60*a^5*Log[a + b*x^2])/(20*b^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^13/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^13/(a + b*x^2)^2, x]
```

fricas [A] time = 0.85, size = 104, normalized size = 1.11

$$\frac{2\,b^6x^{12} - 3\,ab^5x^{10} + 5\,a^2b^4x^8 - 10\,a^3b^3x^6 + 30\,a^4b^2x^4 + 50\,a^5bx^2 - 10\,a^6 - 60\left(a^5bx^2 + a^6\right)\log\left(bx^2+a\right)}{20\left(b^8x^2 + ab^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^6*x^12 - 3*a*b^5*x^10 + 5*a^2*b^4*x^8 - 10*a^3*b^3*x^6 + 30*a^4*b
^2*x^4 + 50*a^5*b*x^2 - 10*a^6 - 60*(a^5*b*x^2 + a^6)*log(b*x^2 + a))/(b^8*
x^2 + a*b^7)
```

giac [A] time = 0.61, size = 103, normalized size = 1.10

$$-\frac{3\,a^5\log\left(\left|bx^2+a\right|\right)}{b^7}+\frac{6\,a^5bx^2+5\,a^6}{2\left(bx^2+a\right)b^7}+\frac{2\,b^8x^{10}-5\,ab^7x^8+10\,a^2b^6x^6-20\,a^3b^5x^4+50\,a^4b^4x^2}{20\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3*a^5*log(abs(b*x^2 + a))/b^7 + 1/2*(6*a^5*b*x^2 + 5*a^6)/((b*x^2 + a)*b^7
) + 1/20*(2*b^8*x^10 - 5*a*b^7*x^8 + 10*a^2*b^6*x^6 - 20*a^3*b^5*x^4 + 50*a
^4*b^4*x^2)/b^10
```

maple [A] time = 0.01, size = 85, normalized size = 0.90

$$\frac{x^{10}}{10b^2}-\frac{a\,x^8}{4b^3}+\frac{a^2x^6}{2b^4}-\frac{a^3x^4}{b^5}+\frac{5a^4x^2}{2b^6}-\frac{a^6}{2\left(b\,x^2+a\right)b^7}-\frac{3a^5\ln\left(b\,x^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^2,x)
```

```
[Out] 5/2*a^4*x^2/b^6-a^3*x^4/b^5+1/2*a^2*x^6/b^4-1/4*a*x^8/b^3+1/10*x^10/b^2-1/2
*a^6/b^7/(b*x^2+a)-3*a^5*ln(b*x^2+a)/b^7
```

maxima [A] time = 1.33, size = 88, normalized size = 0.94

$$-\frac{a^6}{2\left(b^8x^2+ab^7\right)}-\frac{3\,a^5\log\left(bx^2+a\right)}{b^7}+\frac{2\,b^4x^{10}-5\,ab^3x^8+10\,a^2b^2x^6-20\,a^3bx^4+50\,a^4x^2}{20\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^6/(b^8*x^2 + a*b^7) - 3*a^5*log(b*x^2 + a)/b^7 + 1/20*(2*b^4*x^10 -
5*a*b^3*x^8 + 10*a^2*b^2*x^6 - 20*a^3*b*x^4 + 50*a^4*x^2)/b^6
```

mupad [B] time = 0.09, size = 90, normalized size = 0.96

$$\frac{x^{10}}{10\,b^2}-\frac{a^6}{2\,b\,\left(b^7\,x^2+a\,b^6\right)}-\frac{a\,x^8}{4\,b^3}-\frac{3\,a^5\,\ln\left(b\,x^2+a\right)}{b^7}+\frac{a^2\,x^6}{2\,b^4}-\frac{a^3\,x^4}{b^5}+\frac{5\,a^4\,x^2}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^2)^2,x)
```

```
[Out] x^10/(10*b^2) - a^6/(2*b*(a*b^6 + b^7*x^2)) - (a*x^8)/(4*b^3) - (3*a^5*log(
a + b*x^2))/b^7 + (a^2*x^6)/(2*b^4) - (a^3*x^4)/b^5 + (5*a^4*x^2)/(2*b^6)
```

sympy [A] time = 0.31, size = 88, normalized size = 0.94

$$-\frac{a^{6}}{2ab^{7}+2b^{8}x^{2}}-\frac{3a^{5}\log\left(a+bx^{2}\right)}{b^{7}}+\frac{5a^{4}x^{2}}{2b^{6}}-\frac{a^{3}x^{4}}{b^{5}}+\frac{a^{2}x^{6}}{2b^{4}}-\frac{ax^{8}}{4b^{3}}+\frac{x^{10}}{10b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**2,x)
```

```
[Out] -a**6/(2*a*b**7 + 2*b**8*x**2) - 3*a**5*log(a + b*x**2)/b**7 + 5*a**4*x**2/
(2*b**6) - a**3*x**4/b**5 + a**2*x**6/(2*b**4) - a*x**8/(4*b**3) + x**10/(1
0*b**2)
```

3.146 $\int \frac{x^{12}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}+\frac{11a^4x}{2b^6}-\frac{11a^3x^3}{6b^5}+\frac{11a^2x^5}{10b^4}-\frac{11ax^7}{14b^3}-\frac{x^{11}}{2b\left(a+bx^2\right)}+\frac{11x^9}{18b^2}$$

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{11a^2x^5}{10b^4}-\frac{11a^3x^3}{6b^5}+\frac{11a^4x}{2b^6}-\frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}-\frac{11ax^7}{14b^3}-\frac{x^{11}}{2b\left(a+bx^2\right)}+\frac{11x^9}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^2,x]
```

```
[Out] (11*a^4*x)/(2*b^6) - (11*a^3*x^3)/(6*b^5) + (11*a^2*x^5)/(10*b^4) - (11*a*x
^7)/(14*b^3) + (11*x^9)/(18*b^2) - x^11/(2*b*(a + b*x^2)) - (11*a^(9/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{12}}{\left(a+bx^2\right)^2}\,dx &= -\frac{x^{11}}{2b\left(a+bx^2\right)}+\frac{11\int \frac{x^{10}}{a+bx^2}\,dx}{2b} \\
&= -\frac{x^{11}}{2b\left(a+bx^2\right)}+\frac{11\int \left(\frac{a^4}{b^5}-\frac{a^3x^2}{b^4}+\frac{a^2x^4}{b^3}-\frac{ax^6}{b^2}+\frac{x^8}{b}-\frac{a^5}{b^5\left(a+bx^2\right)}\right)dx}{2b} \\
&= \frac{11a^4x}{2b^6}-\frac{11a^3x^3}{6b^5}+\frac{11a^2x^5}{10b^4}-\frac{11ax^7}{14b^3}+\frac{11x^9}{18b^2}-\frac{x^{11}}{2b\left(a+bx^2\right)}-\frac{\left(11a^5\right)\int \frac{1}{a+bx^2}\,dx}{2b^6} \\
&= \frac{11a^4x}{2b^6}-\frac{11a^3x^3}{6b^5}+\frac{11a^2x^5}{10b^4}-\frac{11ax^7}{14b^3}+\frac{11x^9}{18b^2}-\frac{x^{11}}{2b\left(a+bx^2\right)}-\frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 93, normalized size = 0.89

$$
\frac{x\left(\frac{315a^5}{a+bx^2}+3150a^4-840a^3bx^2+378a^2b^2x^4-180ab^3x^6+70b^4x^8\right)}{630b^6}-\frac{11a^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^2,x]
```

```
[Out] (x*(3150*a^4 - 840*a^3*b*x^2 + 378*a^2*b^2*x^4 - 180*a*b^3*x^6 + 70*b^4*x^8
 + (315*a^5)/(a + b*x^2)))/(630*b^6) - (11*a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^{12}}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^12/(a + b*x^2)^2, x]
```

fricas [A] time = 0.97, size = 234, normalized size = 2.23

$$
\left[\frac{140\,b^5x^{11}-220\,ab^4x^9+396\,a^2b^3x^7-924\,a^3b^2x^5+4620\,a^4bx^3+6930\,a^5x+3465\,\left(a^4bx^2+a^5\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{1260\,\left(b^7x^2+ab^6\right)},\ \frac{70\,b^5x^{11}-110\,ab^4x^9+198\,a^2b^3x^7-462\,a^3b^2x^5+2310\,a^4bx^3+3465\,a^5x-3465\,\left(a^4bx^2+a^5\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{630\,\left(b^7x^2+ab^6\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(140*b^5*x^11 - 220*a*b^4*x^9 + 396*a^2*b^3*x^7 - 924*a^3*b^2*x^5 +
 4620*a^4*b*x^3 + 6930*a^5*x + 3465*(a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2
 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^2 + a*b^6), 1/630*(70*b^5*x^1
1 - 110*a*b^4*x^9 + 198*a^2*b^3*x^7 - 462*a^3*b^2*x^5 + 2310*a^4*b*x^3 + 34
65*a^5*x - 3465*(a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x
^2 + a*b^6)]
```

giac [A] time = 0.64, size = 95, normalized size = 0.90

$$-\frac{11\,a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^6}+\frac{a^5x}{2\left(bx^2+a\right)b^6}+\frac{35\,b^{16}x^9-90\,ab^{15}x^7+189\,a^2b^{14}x^5-420\,a^3b^{13}x^3+1575\,a^4b^{12}x}{315\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -11/2*a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/2*a^5*x/((b*x^2 + a)*b^
6) + 1/315*(35*b^16*x^9 - 90*a*b^15*x^7 + 189*a^2*b^14*x^5 - 420*a^3*b^13*x
^3 + 1575*a^4*b^12*x)/b^18
```

maple [A] time = 0.01, size = 90, normalized size = 0.86

$$\frac{x^9}{9b^2}-\frac{2a\,x^7}{7b^3}+\frac{3a^2x^5}{5b^4}-\frac{4a^3x^3}{3b^5}+\frac{a^5x}{2\left(b\,x^2+a\right)b^6}-\frac{11a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^6}+\frac{5a^4x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^2,x)
```

```
[Out] 1/9*x^9/b^2-2/7*a*x^7/b^3+3/5*a^2*x^5/b^4-4/3*a^3*x^3/b^5+5*a^4*x/b^6+1/2/b
^6*a^5*x/(b*x^2+a)-11/2/b^6*a^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.92, size = 93, normalized size = 0.89

$$\frac{a^5x}{2\left(b^7x^2+ab^6\right)}-\frac{11\,a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^6}+\frac{35\,b^4x^9-90\,ab^3x^7+189\,a^2b^2x^5-420\,a^3bx^3+1575\,a^4x}{315\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^5*x/(b^7*x^2 + a*b^6) - 11/2*a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6
) + 1/315*(35*b^4*x^9 - 90*a*b^3*x^7 + 189*a^2*b^2*x^5 - 420*a^3*b*x^3 + 15
75*a^4*x)/b^6
```

mupad [B] time = 0.07, size = 88, normalized size = 0.84

$$\frac{x^9}{9\,b^2}-\frac{2\,a\,x^7}{7\,b^3}+\frac{5\,a^4\,x}{b^6}-\frac{11\,a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{13/2}}+\frac{3\,a^2\,x^5}{5\,b^4}-\frac{4\,a^3\,x^3}{3\,b^5}+\frac{a^5\,x}{2\,\left(b^7\,x^2+a\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^2)^2,x)
```

```
[Out] x^9/(9*b^2) - (2*a*x^7)/(7*b^3) + (5*a^4*x)/b^6 - (11*a^(9/2)*atan((b^(1/2)
*x)/a^(1/2)))/(2*b^(13/2)) + (3*a^2*x^5)/(5*b^4) - (4*a^3*x^3)/(3*b^5) + (a
^5*x)/(2*(a*b^6 + b^7*x^2))
```

sympy [A] time = 0.34, size = 151, normalized size = 1.44

$$\frac{a^5x}{2ab^6+2b^7x^2}+\frac{5a^4x}{b^6}-\frac{4a^3x^3}{3b^5}+\frac{3a^2x^5}{5b^4}-\frac{2ax^7}{7b^3}+\frac{11\sqrt{-\frac{a^9}{b^{13}}}\,\log\left(x-\frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4}-\frac{11\sqrt{-\frac{a^9}{b^{13}}}\,\log\left(x+\frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4}+\frac{x^9}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**2,x)
```

```
[Out] a**5*x/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*x/b**6 - 4*a**3*x**3/(3*b**5) + 3*
a**2*x**5/(5*b**4) - 2*a*x**7/(7*b**3) + 11*sqrt(-a**9/b**13)*log(x - b**6*
sqrt(-a**9/b**13)/a**4)/4 - 11*sqrt(-a**9/b**13)*log(x + b**6*sqrt(-a**9/b*
*13)/a**4)/4 + x**9/(9*b**2)
```

3.147 $\int \frac{x^{11}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=83

$$\frac{a^5}{2b^6(a+bx^2)}+\frac{5a^4\log(a+bx^2)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^4}{4b^4}-\frac{2a^3x^2}{b^5}+\frac{a^5}{2b^6(a+bx^2)}+\frac{5a^4\log(a+bx^2)}{2b^6}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^2,x]
```

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^
5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{(a+bx^2)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^5}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6(a+bx^2)}+\frac{5a^4\log(a+bx^2)}{2b^6}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.87

$$\frac{\frac{12a^5}{a+bx^2}+60a^4\log(a+bx^2)-48a^3bx^2+18a^2b^2x^4-8ab^3x^6+3b^4x^8}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^2,x]
```

```
[Out] (-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a + b
*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a+bx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^11/(a + b*x^2)^2, x]
```

fricas [A] time = 0.99, size = 93, normalized size = 1.12

$$\frac{3\,b^5x^{10}-5\,ab^4x^8+10\,a^2b^3x^6-30\,a^3b^2x^4-48\,a^4bx^2+12\,a^5+60\,(a^4bx^2+a^5)\log(bx^2+a)}{24\,(b^7x^2+ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*b
*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

giac [A] time = 0.63, size = 92, normalized size = 1.11

$$\frac{5\,a^4\log\left(|bx^2+a|\right)}{2\,b^6}-\frac{5\,a^4bx^2+4\,a^5}{2\left(bx^2+a\right)b^6}+\frac{3\,b^6x^8-8\,ab^5x^6+18\,a^2b^4x^4-48\,a^3b^3x^2}{24\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*a^4*log(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^2 + a)*b^
6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48*a^3*b^3*x^2)/b^8
```

maple [A] time = 0.01, size = 74, normalized size = 0.89

$$\frac{x^8}{8b^2}-\frac{a\,x^6}{3b^3}+\frac{3a^2x^4}{4b^4}-\frac{2a^3x^2}{b^5}+\frac{a^5}{2\left(b\,x^2+a\right)b^6}+\frac{5a^4\ln\left(b\,x^2+a\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^2,x)
```

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2
+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.36, size = 77, normalized size = 0.93

$$\frac{a^5}{2\left(b^7x^2+ab^6\right)}+\frac{5\,a^4\log\left(bx^2+a\right)}{2\,b^6}+\frac{3\,b^3x^8-8\,ab^2x^6+18\,a^2bx^4-48\,a^3x^2}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 -
8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

mupad [B] time = 4.48, size = 79, normalized size = 0.95

$$\frac{x^8}{8\,b^2}+\frac{a^5}{2\,b\,\left(b^6\,x^2+a\,b^5\right)}-\frac{a\,x^6}{3\,b^3}+\frac{5\,a^4\,\ln\left(b\,x^2+a\right)}{2\,b^6}+\frac{3\,a^2\,x^4}{4\,b^4}-\frac{2\,a^3\,x^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2)^2,x)
```

```
[Out] x^8/(8*b^2) + a^5/(2*b*(a*b^5 + b^6*x^2)) - (a*x^6)/(3*b^3) + (5*a^4*log(a
+ b*x^2))/(2*b^6) + (3*a^2*x^4)/(4*b^4) - (2*a^3*x^2)/b^5
```

sympy [**A**] time = 0.29, size = 80, normalized size = 0.96

$$\frac{a^5}{2ab^6 + 2b^7x^2} + \frac{5a^4 \log\left(a + bx^2\right)}{2b^6} - \frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**2,x)
```

```
[Out] a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x*
*2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)
```

3.148 $\int \frac{x^{10}}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=92

$$\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{3a^2x^3}{2b^4}-\frac{9a^3x}{2b^5}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^2,x]
```

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14
*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^2}\,dx = -\frac{x^9}{2b\left(a+bx^2\right)} + \frac{9\int \frac{x^8}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^9}{2b\left(a+bx^2\right)} + \frac{9\int \left(-\frac{a^3}{b^4} + \frac{a^2x^2}{b^3} - \frac{ax^4}{b^2} + \frac{x^6}{b} + \frac{a^4}{b^4\left(a+bx^2\right)}\right) dx}{2b}$$

$$= -\frac{9a^3x}{2b^5} + \frac{3a^2x^3}{2b^4} - \frac{9ax^5}{10b^3} + \frac{9x^7}{14b^2} - \frac{x^9}{2b\left(a+bx^2\right)} + \frac{\left(9a^4\right)\int \frac{1}{a+bx^2}\,dx}{2b^5}$$

$$= -\frac{9a^3x}{2b^5} + \frac{3a^2x^3}{2b^4} - \frac{9ax^5}{10b^3} + \frac{9x^7}{14b^2} - \frac{x^9}{2b\left(a+bx^2\right)} + \frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.89

$$\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}} + \frac{x\left(-\frac{35a^4}{a+bx^2} - 280a^3 + 70a^2bx^2 - 28ab^2x^4 + 10b^3x^6\right)}{70b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^2,x]
```

```
[Out] (x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*x
^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^10/(a + b*x^2)^2, x]
```

fricas [A] time = 0.78, size = 212, normalized size = 2.30

$$\left[\frac{20\,b^4x^9 - 36\,ab^3x^7 + 84\,a^2b^2x^5 - 420\,a^3bx^3 - 630\,a^4x + 315\left(a^3bx^2 + a^4\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2 + 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right)}{140\left(b^6x^2 + ab^5\right)}, \frac{10\,b^4x^9 - 18\,ab^3x^7 + 42\,a^2b^2x^5 - 210\,a^3bx^3 - 315\,a^4x + 315\left(a^3bx^2 + a^4\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{70\left(b^6x^2 + ab^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a^
4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2*b^
2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a))/(b^6*x^2 + a*b^5)]
```

giac [A] time = 0.64, size = 84, normalized size = 0.91

$$\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}-\frac{a^4x}{2\left(bx^2+a\right)b^5}+\frac{5\,b^{12}x^7-14\,ab^{11}x^5+35\,a^2b^{10}x^3-140\,a^3b^9x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2 + a)*b^5)
 + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 - 140*a^3*b^9*x)/b^14
```

maple [A] time = 0.01, size = 78, normalized size = 0.85

$$\frac{x^7}{7b^2}-\frac{2a\,x^5}{5b^3}+\frac{a^2x^3}{b^4}-\frac{a^4x}{2\left(b\,x^2+a\right)b^5}+\frac{9a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^5}-\frac{4a^3x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^2,x)
```

```
[Out] 1/7*x^7/b^2-2/5*a*x^5/b^3+a^2*x^3/b^4-4*a^3*x/b^5-1/2/b^5*a^4*x/(b*x^2+a)+9
/2/b^5*a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.01, size = 82, normalized size = 0.89

$$-\frac{a^4x}{2\left(b^6x^2+ab^5\right)}+\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}+\frac{5\,b^3x^7-14\,ab^2x^5+35\,a^2bx^3-140\,a^3x}{35\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^4*x/(b^6*x^2 + a*b^5) + 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5
) + 1/35*(5*b^3*x^7 - 14*a*b^2*x^5 + 35*a^2*b*x^3 - 140*a^3*x)/b^5
```

mupad [B] time = 4.56, size = 77, normalized size = 0.84

$$\frac{x^7}{7\,b^2}-\frac{2\,a\,x^5}{5\,b^3}-\frac{4\,a^3\,x}{b^5}+\frac{9\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{11/2}}+\frac{a^2\,x^3}{b^4}-\frac{a^4\,x}{2\,\left(b^6\,x^2+a\,b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^2,x)
```

```
[Out] x^7/(7*b^2) - (2*a*x^5)/(5*b^3) - (4*a^3*x)/b^5 + (9*a^(7/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(11/2)) + (a^2*x^3)/b^4 - (a^4*x)/(2*(a*b^5 + b^6*x^2))
```

sympy [A] time = 0.33, size = 134, normalized size = 1.46

$$-\frac{a^4x}{2ab^5+2b^6x^2}-\frac{4a^3x}{b^5}+\frac{a^2x^3}{b^4}-\frac{2ax^5}{5b^3}-\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**2,x)
```

```
[Out] -a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x**
5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**7/b**11)/a**3)/4 + 9
*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b**11)/a**3)/4 + x**7/(7*b**2)
```

3.149 $\int \frac{x^9}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=70

$$-\frac{a^4}{2b^5(a+bx^2)}-\frac{2a^3\log(a+bx^2)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^2}{2b^4}-\frac{a^4}{2b^5(a+bx^2)}-\frac{2a^3\log(a+bx^2)}{b^5}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^2,x]
```

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2
)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a+bx)^2} - \frac{4a^3}{b^4(a+bx)}\right) dx, x, x^2\right)$$
$$= \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2} - \frac{a^4}{2b^5\left(a+bx^2\right)} - \frac{2a^3\log\left(a+bx^2\right)}{b^5}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.86

$$\frac{-\frac{3a^4}{a+bx^2} - 12a^3\log\left(a+bx^2\right) + 9a^2bx^2 - 3ab^2x^4 + b^3x^6}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^2,x]
```

```
[Out] (9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a +
 b*x^2])/(6*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^9/(a + b*x^2)^2, x]
```

fricas [A] time = 1.30, size = 81, normalized size = 1.16

$$\frac{b^4x^8 - 2\,ab^3x^6 + 6\,a^2b^2x^4 + 9\,a^3bx^2 - 3\,a^4 - 12\left(a^3bx^2 + a^4\right)\log\left(bx^2 + a\right)}{6\left(b^6x^2 + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4 - 12*(a^3*
b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

giac [A] time = 0.65, size = 80, normalized size = 1.14

$$-\frac{2\,a^{3}\log\left(\left|bx^{2}+a\right|\right)}{b^{5}}+\frac{b^{4}x^{6}-3\,ab^{3}x^{4}+9\,a^{2}b^{2}x^{2}}{6\,b^{6}}+\frac{4\,a^{3}bx^{2}+3\,a^{4}}{2\left(bx^{2}+a\right)b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^2*b^2*x^2
)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

maple [A] time = 0.01, size = 63, normalized size = 0.90

$$\frac{x^{6}}{6b^{2}}-\frac{a\,x^{4}}{2b^{3}}+\frac{3a^{2}x^{2}}{2b^{4}}-\frac{a^{4}}{2\left(b\,x^{2}+a\right)b^{5}}-\frac{2a^{3}\ln\left(b\,x^{2}+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^2,x)
```

```
[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*
x^2+a)/b^5
```

maxima [A] time = 1.35, size = 65, normalized size = 0.93

$$-\frac{a^{4}}{2\left(b^{6}x^{2}+ab^{5}\right)}-\frac{2\,a^{3}\log\left(bx^{2}+a\right)}{b^{5}}+\frac{b^{2}x^{6}-3\,abx^{4}+9\,a^{2}x^{2}}{6\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*x^6 - 3*a*
b*x^4 + 9*a^2*x^2)/b^4
```

mupad [B] time = 0.07, size = 68, normalized size = 0.97

$$\frac{x^{6}}{6\,b^{2}}-\frac{a^{4}}{2\,b\,\left(b^{5}\,x^{2}+a\,b^{4}\right)}-\frac{a\,x^{4}}{2\,b^{3}}-\frac{2\,a^{3}\,\ln\left(b\,x^{2}+a\right)}{b^{5}}+\frac{3\,a^{2}\,x^{2}}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^2,x)
```

```
[Out] x^6/(6*b^2) - a^4/(2*b*(a*b^4 + b^5*x^2)) - (a*x^4)/(2*b^3) - (2*a^3*log(a
+ b*x^2))/b^5 + (3*a^2*x^2)/(2*b^4)
```

sympy [A] time = 0.27, size = 66, normalized size = 0.94

$$-\frac{a^4}{2ab^5+2b^6x^2}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**2,x)
```

```
[Out] -a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2/
(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

3.150 $\int \frac{x^8}{(a+bx^2)^2} dx$

Optimal. Leaf size=79

$$-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}$$

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{7a^2x}{2b^4}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^2,x]
```

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2*b*(a + b*
x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(a+bx^2\right)^2}\,dx &= -\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7\int \frac{x^6}{a+bx^2}\,dx}{2b} \\
&= -\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7\int \left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+bx^2\right)}\right)dx}{2b} \\
&= \frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b\left(a+bx^2\right)}-\frac{\left(7a^3\right)\int \frac{1}{a+bx^2}\,dx}{2b^4} \\
&= \frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b\left(a+bx^2\right)}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.90

$$
\frac{x\left(\frac{15a^3}{a+bx^2}+90a^2-20abx^2+6b^2x^4\right)}{30b^4}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^2,x]
```

```
[Out] (x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*b^4) - (7*
a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^8}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x^2)^2, x]
```

fricas [A] time = 1.43, size = 190, normalized size = 2.41

$$
\left[\frac{12\,b^3x^7-28\,ab^2x^5+140\,a^2bx^3+210\,a^3x+105\left(a^2bx^2+a^3\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{60\left(b^5x^2+ab^4\right)},\ \frac{6\,b^3x^7-14\,ab^2x^5+70\,a^2bx^3+105\,a^3x-105\left(a^2bx^2+a^3\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{30\left(b^5x^2+ab^4\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 105*(a^2*b*x
^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*
x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*x - 1
05*(a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^2 + a*b^4)]
```

giac [A] time = 0.63, size = 73, normalized size = 0.92

$$-\frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{a^3x}{2\left(bx^2+a\right)b^4}+\frac{3\,b^8x^5-10\,ab^7x^3+45\,a^2b^6x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

maple [A] time = 0.01, size = 68, normalized size = 0.86

$$\frac{x^5}{5b^2}-\frac{2a\,x^3}{3b^3}+\frac{a^3x}{2\left(b\,x^2+a\right)b^4}-\frac{7a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^4}+\frac{3a^2x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^2,x)
```

```
[Out] 1/5*x^5/b^2-2/3*a*x^3/b^3+3*a^2*x/b^4+1/2/b^4*a^3*x/(b*x^2+a)-7/2/b^4*a^3/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 71, normalized size = 0.90

$$\frac{a^3x}{2\left(b^5x^2+ab^4\right)}-\frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{3\,b^2x^5-10\,abx^3+45\,a^2x}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x/(b^5*x^2 + a*b^4) - 7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/15*(3*b^2*x^5 - 10*a*b*x^3 + 45*a^2*x)/b^4
```

mupad [B] time = 4.59, size = 66, normalized size = 0.84

$$\frac{x^5}{5\,b^2}-\frac{2\,a\,x^3}{3\,b^3}+\frac{3\,a^2\,x}{b^4}-\frac{7\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{9/2}}+\frac{a^3\,x}{2\,\left(b^5\,x^2+a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^2,x)
```

```
[Out] x^5/(5*b^2) - (2*a*x^3)/(3*b^3) + (3*a^2*x)/b^4 - (7*a^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(9/2)) + (a^3*x)/(2*(a*b^4 + b^5*x^2))
```

sympy [A] time = 0.31, size = 124, normalized size = 1.57

$$\frac{a^3x}{2ab^4+2b^5x^2}+\frac{3a^2x}{b^4}-\frac{2ax^3}{3b^3}+\frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}-\frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}+\frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**2,x)
```

```
[Out] a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**3) + 7*sqr
t(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7*sqrt(-a**5/b**9)*lo
g(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5*b**2)
```

3.151 $\int \frac{x^7}{(a+bx^2)^2} dx$

Optimal. Leaf size=57

$$\frac{a^3}{2b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^3}{2b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^2,x]
```

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/(2*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.86

$$\frac{\frac{2a^3}{a+bx^2}+6a^2\log\left(a+bx^2\right)-4abx^2+b^2x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^2,x]
[Out] (-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2])/(4*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2)^2,x]
[Out] IntegrateAlgebraic[x^7/(a + b*x^2)^2, x]
```

fricas [A] time = 0.68, size = 70, normalized size = 1.23

$$\frac{b^3x^6-3\,ab^2x^4-4\,a^2bx^2+2\,a^3+6\left(a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 + a^3)*log(
b*x^2 + a))/(b^5*x^2 + a*b^4)
```

giac [A] time = 0.62, size = 67, normalized size = 1.18

$$\frac{3\,a^2\log\left(\left|bx^2+a\right|\right)}{2\,b^4}+\frac{b^2x^4-4\,abx^2}{4\,b^4}-\frac{3\,a^2bx^2+2\,a^3}{2\,\left(bx^2+a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/2*a^2*log(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 - 1/2*(3*a^
2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

maple [A] time = 0.01, size = 52, normalized size = 0.91

$$\frac{x^4}{4b^2}-\frac{a\,x^2}{b^3}+\frac{a^3}{2\left(b\,x^2+a\right)b^4}+\frac{3a^2\ln\left(b\,x^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^2,x)
```

```
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

maxima [A] time = 1.37, size = 54, normalized size = 0.95

$$\frac{a^3}{2\left(b^5x^2+ab^4\right)}+\frac{3\,a^2\log\left(bx^2+a\right)}{2\,b^4}+\frac{bx^4-4\,ax^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x^4 - 4*a*x
^2)/b^3
```

mupad [B] time = 0.08, size = 57, normalized size = 1.00

$$\frac{x^4}{4\,b^2}+\frac{a^3}{2\,b\,\left(b^4\,x^2+a\,b^3\right)}-\frac{a\,x^2}{b^3}+\frac{3\,a^2\,\ln\left(b\,x^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^2,x)
```

```
[Out] x^4/(4*b^2) + a^3/(2*b*(a*b^3 + b^4*x^2)) - (a*x^2)/b^3 + (3*a^2*log(a + b*
x^2))/(2*b^4)
```

sympy [A] time = 0.26, size = 53, normalized size = 0.93

$$\frac{a^3}{2ab^4 + 2b^5x^2} + \frac{3a^2 \log\left(a + bx^2\right)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**2,x)
```

```
[Out] a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*x**2/b*
*3 + x**4/(4*b**2)
```

3.152 $\int \frac{x^6}{(a+bx^2)^2} dx$

Optimal. Leaf size=66

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}} - \frac{5ax}{2b^3} - \frac{x^5}{2b\left(a+bx^2\right)} + \frac{5x^3}{6b^2}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}} - \frac{5ax}{2b^3} - \frac{x^5}{2b\left(a+bx^2\right)} + \frac{5x^3}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^2,x]
```

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^2}\,dx = -\frac{x^5}{2b\left(a+bx^2\right)} + \frac{5\int \frac{x^4}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^5}{2b\left(a+bx^2\right)} + \frac{5\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{2b}$$

$$= -\frac{5ax}{2b^3} + \frac{5x^3}{6b^2} - \frac{x^5}{2b\left(a+bx^2\right)} + \frac{\left(5a^2\right)\int \frac{1}{a+bx^2}\,dx}{2b^3}$$

$$= -\frac{5ax}{2b^3} + \frac{5x^3}{6b^2} - \frac{x^5}{2b\left(a+bx^2\right)} + \frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.91

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}} + \frac{x\left(-\frac{3a^2}{a+bx^2}-12a+2bx^2\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^2,x]
```

```
[Out] (x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x^2)^2, x]
```

fricas [A] time = 1.00, size = 164, normalized size = 2.48

$$\left[\frac{4\,b^2x^5-20\,abx^3-30\,a^2x+15\left(abx^2+a^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{12\left(b^4x^2+ab^3\right)},\ \frac{2\,b^2x^5-10\,abx^3-15\,a^2x+15\left(abx^2+a^2\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{6\left(b^4x^2+ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(-a/b)*lo
g((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2 + a*b^3), 1/6*(2*b^
2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 0.64, size = 61, normalized size = 0.92

$$\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}-\frac{a^2x}{2\left(bx^2+a\right)b^3}+\frac{b^4x^3-6\,ab^3x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3)
 + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{x^3}{3b^2}-\frac{a^2x}{2\left(b\,x^2+a\right)b^3}+\frac{5a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^3}-\frac{2ax}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^2,x)
```

```
[Out] 1/3*x^3/b^2-2*a*x/b^3-1/2/b^3*a^2*x/(b*x^2+a)+5/2/b^3*a^2/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 59, normalized size = 0.89

$$-\frac{a^2x}{2\left(b^4x^2+ab^3\right)}+\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{bx^3-6\,ax}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*x/(b^4*x^2 + a*b^3) + 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3
) + 1/3*(b*x^3 - 6*a*x)/b^3
```

mupad [B] time = 0.09, size = 56, normalized size = 0.85

$$\frac{x^3}{3\,b^2}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{7/2}}-\frac{a^2\,x}{2\,\left(b^4\,x^2+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^2,x)
```

```
[Out] x^3/(3*b^2) + (5*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(7/2)) - (a^2*x)/(
2*(a*b^3 + b^4*x^2)) - (2*a*x)/b^3
```

sympy [A] time = 0.29, size = 107, normalized size = 1.62

$$-\frac{a^2x}{2ab^3+2b^4x^2}-\frac{2ax}{b^3}-\frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4}+\frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4}+\frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**2,x)
```

```
[Out] -a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7)*log(x -
b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x + b**3*sqrt(-a**3/b**
7)/a)/4 + x**3/(3*b**2)
```

3.153 $\int \frac{x^5}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=44

$$-\frac{a^2}{2b^3(a+bx^2)} - \frac{a\log(a+bx^2)}{b^3} + \frac{x^2}{2b^2}$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{2b^3(a+bx^2)} - \frac{a\log(a+bx^2)}{b^3} + \frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^2,x]
```

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^2} + \frac{a^2}{b^2(a+bx)^2} - \frac{2a}{b^2(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{x^2}{2b^2} - \frac{a^2}{2b^3\left(a+bx^2\right)} - \frac{a\log\left(a+bx^2\right)}{b^3}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.86

$$\frac{-\frac{a^2}{a+bx^2} - 2a\log\left(a+bx^2\right) + bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^2,x]
[Out] (b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2)^2,x]
[Out] IntegrateAlgebraic[x^5/(a + b*x^2)^2, x]
```

fricas [A] time = 0.88, size = 56, normalized size = 1.27

$$\frac{b^2x^4 + abx^2 - a^2 - 2\left(abx^2 + a^2\right)\log\left(bx^2 + a\right)}{2\left(b^4x^2 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/(b^4*x^2 +
 a*b^3)
```

giac [A] time = 0.62, size = 49, normalized size = 1.11

$$\frac{x^2}{2\,b^2} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{b^3} + \frac{2\,abx^2 + a^2}{2\left(bx^2 + a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^2 - a*log(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 + a
)*b^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.93

$$\frac{x^2}{2b^2} - \frac{a^2}{2\left(b\,x^2 + a\right)b^3} - \frac{a\ln\left(b\,x^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

maxima [A] time = 1.39, size = 43, normalized size = 0.98

$$-\frac{a^2}{2\left(b^4x^2 + ab^3\right)} + \frac{x^2}{2\,b^2} - \frac{a\log\left(bx^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

mupad [B] time = 0.05, size = 45, normalized size = 1.02

$$\frac{x^2}{2\,b^2} - \frac{a^2}{2\,\left(b^4\,x^2 + a\,b^3\right)} - \frac{a\,\ln\left(b\,x^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^2,x)
```

```
[Out] x^2/(2*b^2) - a^2/(2*(a*b^3 + b^4*x^2)) - (a*log(a + b*x^2))/b^3
```

sympy [A] time = 0.23, size = 39, normalized size = 0.89

$$-\frac{a^2}{2ab^3+2b^4x^2}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**2,x)
```

```
[Out] -a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2*b**2)
```

3.154 $\int \frac{x^4}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 321, 205}

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^2,x]
```

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2}\,dx = -\frac{x^3}{2b\left(a+bx^2\right)} + \frac{3\int \frac{x^2}{a+bx^2}\,dx}{2b}$$
$$= \frac{3x}{2b^2} - \frac{x^3}{2b\left(a+bx^2\right)} - \frac{(3a)\int \frac{1}{a+bx^2}\,dx}{2b^2}$$
$$= \frac{3x}{2b^2} - \frac{x^3}{2b\left(a+bx^2\right)} - \frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.93

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}} + \frac{ax}{2b^2\left(a+bx^2\right)} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^2,x]
```

```
[Out] x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x^2)^2, x]
```

fricas [A] time = 1.01, size = 136, normalized size = 2.47

$$\left[\frac{4\,bx^3 + 3\left(bx^2+a\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 6\,ax}{4\left(b^3x^2+ab^2\right)},\ \frac{2\,bx^3 - 3\left(bx^2+a\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 3\,ax}{2\left(b^3x^2+ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3 - 3*(b*x^2 + a)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*a*x)/(b^3*x^2 + a*b^2)]
```

giac [A] time = 0.63, size = 42, normalized size = 0.76

$$-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{ax}{2\left(bx^2+a\right)b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 + a)*b^2) +
x/b^2
```

maple [A] time = 0.01, size = 43, normalized size = 0.78

$$\frac{ax}{2\left(b\,x^2+a\right)b^2}-\frac{3a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2,x)
```

```
[Out] x/b^2+1/2/b^2*a*x/(b*x^2+a)-3/2/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.94, size = 45, normalized size = 0.82

$$\frac{ax}{2\left(b^3x^2+ab^2\right)}-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*x/(b^3*x^2 + a*b^2) - 3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + x
/b^2
```

mupad [B] time = 4.59, size = 43, normalized size = 0.78

$$\frac{x}{b^2}+\frac{a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)}-\frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^2,x)
```

```
[Out] x/b^2 + (a*x)/(2*(a*b^2 + b^3*x^2)) - (3*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))
/(2*b^(5/2))
```

sympy [A] time = 0.26, size = 83, normalized size = 1.51

$$\frac{ax}{2ab^2+2b^3x^2}+\frac{3\sqrt{-\frac{a}{b^5}}\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{a}{b^5}}\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2,x)
```

```
[Out] a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/b**5) + x)
/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/b**2
```

3.155 $\int \frac{x^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=33

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^2,x]
```

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^2}+\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^2,x]
```

```
[Out] (a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x^2)^2, x]
```

fricas [A] time = 0.54, size = 35, normalized size = 1.06

$$\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)+a}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.64, size = 48, normalized size = 1.45

$$-\frac{\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b} - \frac{a}{(bx^2+a)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="giac")
[Out] -1/2*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{a}{2\left(b\,x^2+a\right)b^2} + \frac{\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2,x)
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

maxima [A] time = 1.32, size = 32, normalized size = 0.97

$$\frac{a}{2\left(b^3x^2+ab^2\right)} + \frac{\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.05, size = 29, normalized size = 0.88

$$\frac{\ln\left(b\,x^2+a\right)}{2\,b^2} + \frac{a}{2\,b^2\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^2,x)
[Out] log(a + b*x^2)/(2*b^2) + a/(2*b^2*(a + b*x^2))
```

sympy [A] time = 0.19, size = 29, normalized size = 0.88

$$\frac{a}{2ab^2 + 2b^3x^2} + \frac{\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2,x)
```

```
[Out] a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

3.156 $\int \frac{x^2}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^2,x]
```

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2\right)^2}\,dx &= -\frac{x}{2b\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2}\,dx}{2b}\\ &= -\frac{x}{2b\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^2,x]
```

```
[Out] -1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x^2)^2, x]
```

fricas [A] time = 1.45, size = 120, normalized size = 2.67

$$\left[-\frac{2\,abx+\left(bx^2+a\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(ab^3x^2+a^2b^2\right)},-\frac{abx-\left(bx^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(ab^3x^2+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x + (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*x - (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

giac [A] time = 0.64, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b}-\frac{x}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(b\,x^{2}+a\right)b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.96, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(b^{2}x^{2}+ab\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(b^2*x^2 + a*b) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 4.76, size = 33, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2\,b\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^2,x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2)) - x/(2*b*(a + b*x^2))
```

sympy [B] time = 0.21, size = 78, normalized size = 1.73

$$-\frac{x}{2ab+2b^{2}x^{2}}-\frac{\sqrt{-\frac{1}{ab^{3}}}\log\left(-ab\sqrt{-\frac{1}{ab^{3}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^{3}}}\log\left(ab\sqrt{-\frac{1}{ab^{3}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2,x)
```

```
[Out] -x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3)) + x
)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4
```

3.157 $\int \frac{x}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^2,x]
[Out] -1/(2*b*(a + b*x^2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^2}\,dx = -\frac{1}{2b\left(a+bx^2\right)}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^2,x]
[Out] -1/2*1/(b*(a + b*x^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x^2)^2, x]
```

fricas [A] time = 0.92, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

giac [A] time = 0.64, size = 14, normalized size = 0.88

$$-\frac{1}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(b\,x^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b/(b*x^2+a)
```

maxima [A] time = 1.30, size = 14, normalized size = 0.88

$$-\frac{1}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.88

$$-\frac{1}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^2,x)
```

```
[Out] -1/(2*b*(a + b*x^2))
```

sympy [A] time = 0.16, size = 15, normalized size = 0.94

$$-\frac{1}{2ab + 2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2,x)
```

```
[Out] -1/(2*a*b + 2*b**2*x**2)
```

3.158 $\int \frac{1}{(a+bx^2)^2} dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^2}\,dx &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2}\,dx}{2a}\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^(-2), x]
```

fricas [A] time = 0.76, size = 120, normalized size = 2.67

$$\left[\frac{2\,abx-\left(bx^2+a\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^2b^2x^2+a^3b\right)},\frac{abx+\left(bx^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

giac [A] time = 0.65, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}+\frac{x}{2\left(bx^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

maple [A] time = 0.00, size = 36, normalized size = 0.80

$$\frac{x}{2\left(b\,x^{2}+a\right)a}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.96, size = 35, normalized size = 0.78

$$\frac{x}{2\left(abx^{2}+a^{2}\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [B] time = 4.74, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(b\,x^{2}+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^2,x)
```

```
[Out] x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

sympy [B] time = 0.21, size = 78, normalized size = 1.73

$$\frac{x}{2a^{2}+2abx^{2}}-\frac{\sqrt{-\frac{1}{a^{3}b}}\log\left(-a^{2}\sqrt{-\frac{1}{a^{3}b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{3}b}}\log\left(a^{2}\sqrt{-\frac{1}{a^{3}b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

3.159 $\int \frac{1}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2),x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{1}{2a\left(a+bx^2\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2\right)}{2a^2}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2} - \log\left(a+bx^2\right) + 2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2),x]
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^2),x]
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2)^2), x]
```

fricas [A] time = 1.04, size = 47, normalized size = 1.24

$$-\frac{\left(bx^2+a\right)\log\left(bx^2+a\right) - 2\left(bx^2+a\right)\log(x) - a}{2\left(a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="fricas")
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

giac [A] time = 0.63, size = 47, normalized size = 1.24

$$\frac{\log\left(x^2\right)}{2\,a^2} - \frac{\log\left(\left|bx^2 + a\right|\right)}{2\,a^2} + \frac{bx^2 + 2\,a}{2\left(bx^2 + a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{2\left(b\,x^2 + a\right)a} + \frac{\ln(x)}{a^2} - \frac{\ln\left(b\,x^2 + a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.39, size = 37, normalized size = 0.97

$$\frac{1}{2\left(abx^2 + a^2\right)} - \frac{\log\left(bx^2 + a\right)}{2\,a^2} + \frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

mupad [B] time = 4.70, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2} + \frac{1}{2\,a\,\left(b\,x^2 + a\right)} - \frac{\ln\left(b\,x^2 + a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2),x)
```

```
[Out] log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

sympy [A] time = 0.30, size = 34, normalized size = 0.89

$$\frac{1}{2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2,x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

3.160 $\int \frac{1}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax\left(a+bx^2\right)} + \frac{3\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax\left(a+bx^2\right)} - \frac{(3b)\int \frac{1}{a+bx^2}\,dx}{2a^2}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax\left(a+bx^2\right)} - \frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{bx}{2a^2\left(a+bx^2\right)} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2), x]
```

fricas [A] time = 0.89, size = 136, normalized size = 2.39

$$\left[-\frac{6\,bx^2 - 3\left(bx^3+ax\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2 - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2+a}\right) + 4\,a}{4\left(a^2bx^3+a^3x\right)},\ -\frac{3\,bx^2 + 3\left(bx^3+ax\right)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right) + 2\,a}{2\left(a^2bx^3+a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

giac [A] time = 0.65, size = 47, normalized size = 0.82

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{3\,bx^2+2\,a}{2\,\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.81

$$-\frac{bx}{2\left(b\,x^2+a\right)a^2}-\frac{3b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^2}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/a^2/x-1/2/a^2*b*x/(b*x^2+a)-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)
```

maxima [A] time = 2.97, size = 49, normalized size = 0.86

$$-\frac{3\,bx^2+2\,a}{2\,\left(a^2bx^3+a^3x\right)}-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2)
```

mupad [B] time = 0.07, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{a}+\frac{3\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2),x)
```

```
[Out] - (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^(
1/2)))/(2*a^(5/2))
```

sympy [A] time = 0.29, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{-2a-3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2,x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

3.161 $\int \frac{1}{x^3(a+bx^2)^2}\,dx$

Optimal. Leaf size=49

$$\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}$$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^2)^2} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^2(a+bx)^2} \, dx, x, x^2\right)$$
$$= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right) dx, x, x^2\right)$$
$$= -\frac{1}{2a^2x^2} - \frac{b}{2a^2 (a+bx^2)} - \frac{2b \log(x)}{a^3} + \frac{b \log (a+bx^2)}{a^3}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{b}{a+bx^2} + \frac{1}{x^2}\right) - 2b \log (a+bx^2) + 4b \log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2),x]
[Out] -1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a+bx^2)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2),x]
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2), x]
```

fricas [A] time = 1.04, size = 73, normalized size = 1.49

$$-\frac{2\,abx^2 + a^2 - 2\left(b^2x^4 + abx^2\right) \log \left(bx^2 + a\right) + 4\left(b^2x^4 + abx^2\right) \log(x)}{2\left(a^3bx^4 + a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="fricas")
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

giac [A] time = 0.60, size = 51, normalized size = 1.04

$$-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{b}{2\left(b\,x^2+a\right)a^2}-\frac{2b\ln(x)}{a^3}+\frac{b\ln\left(b\,x^2+a\right)}{a^3}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.37, size = 52, normalized size = 1.06

$$-\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)}+\frac{b\log\left(bx^2+a\right)}{a^3}-\frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - b*log(x^2
)/a^3
```

mupad [B] time = 0.08, size = 51, normalized size = 1.04

$$\frac{b\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{\frac{1}{2\,a}+\frac{b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{2\,b\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*log
(x))/a^3
```

sympy [A] time = 0.36, size = 51, normalized size = 1.04

$$\frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

3.162 $\int \frac{1}{x^4(a+bx^2)^2}\,dx$

Optimal. Leaf size=68

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx &= \frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a}\\
&= -\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}-\frac{(5b)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^2}\\
&= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{\left(5b^2\right)\int \frac{1}{a+bx^2}\,dx}{2a^3}\\
&= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.99

$$
\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2), x]
```

fricas [A] time = 1.09, size = 172, normalized size = 2.53

$$
\left[\frac{30\,b^2x^4+20\,abx^2+15\left(b^2x^5+abx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-4\,a^2}{12\left(a^3bx^5+a^4x^3\right)},\ \frac{15\,b^2x^4+10\,abx^2+15\left(b^2x^5+abx^3\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-2\,a^2}{6\left(a^3bx^5+a^4x^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

giac [A] time = 0.61, size = 59, normalized size = 0.87

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}+\frac{b^2x}{2\left(bx^2+a\right)a^3}+\frac{6\,bx^2-a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

maple [A] time = 0.01, size = 59, normalized size = 0.87

$$\frac{b^2x}{2\left(b\,x^2+a\right)a^3}+\frac{5b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^3}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/3/a^2/x^3+2*b/a^3/x+1/2*b^2/a^3*x/(b*x^2+a)+5/2*b^2/a^3/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.01, size = 64, normalized size = 0.94

$$\frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\left(a^3bx^5+a^4x^3\right)}+\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 4.73, size = 58, normalized size = 0.85

$$\frac{\frac{5\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{5\,b^2\,x^4}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2),x)
```

```
[Out] ((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b^
(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

sympy [A] time = 0.35, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2,x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

3.163 $\int \frac{1}{x^5\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=66

$$-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b^2}{2a^3\left(a+bx^2\right)}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx, x, x^2\right)\\ &= -\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.86

$$\frac{-6b^2\log\left(a+bx^2\right)+a\left(\frac{2b^2}{a+bx^2}-\frac{a}{x^4}+\frac{4b}{x^2}\right)+12b^2\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^2),x]
```

```
[Out] (a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] - 6*b^2*Log
[a + b*x^2])/(4*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^2), x]
```

fricas [A] time = 1.08, size = 90, normalized size = 1.36

$$\frac{6\,ab^2x^4+3\,a^2bx^2-a^3-6\left(b^3x^6+ab^2x^4\right)\log\left(bx^2+a\right)+12\left(b^3x^6+ab^2x^4\right)\log(x)}{4\left(a^4bx^6+a^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*log(b*x^2 +
a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

giac [A] time = 0.63, size = 86, normalized size = 1.30

$$\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}-\frac{3\,b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^4}+\frac{3\,b^3x^2+4\,ab^2}{2\left(bx^2+a\right)a^4}-\frac{9\,b^2x^4-4\,abx^2+a^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(x^2)/a^4 - 3/2*b^2*log(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 + 4
*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

maple [A] time = 0.01, size = 61, normalized size = 0.92

$$\frac{b^2}{2\left(b\,x^2+a\right)a^3}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln\left(b\,x^2+a\right)}{2a^4}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^2,x)
```

```
[Out] -1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*x
^2+a)/a^4
```

maxima [A] time = 1.32, size = 70, normalized size = 1.06

$$\frac{6\,b^2x^4+3\,abx^2-a^2}{4\left(a^3bx^6+a^4x^4\right)}-\frac{3\,b^2\log\left(bx^2+a\right)}{2\,a^4}+\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*log(b*x^2
 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

mupad [B] time = 4.80, size = 67, normalized size = 1.02

$$\frac{\frac{3\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{3\,b^2\,x^4}{2\,a^3}}{b\,x^6+a\,x^4}-\frac{3\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^4}+\frac{3\,b^2\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^2),x)
```

```
[Out] ((3*b*x^2)/(4*a^2) - 1/(4*a) + (3*b^2*x^4)/(2*a^3))/(a*x^4 + b*x^6) - (3*b^
2*log(a + b*x^2))/(2*a^4) + (3*b^2*log(x))/a^4
```

sympy [A] time = 0.44, size = 68, normalized size = 1.03

$$\frac{-a^2 + 3abx^2 + 6b^2x^4}{4a^4x^4 + 4a^3bx^6} + \frac{3b^2 \log(x)}{a^4} - \frac{3b^2 \log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*l
og(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

3.164 $\int \frac{1}{x^6(a+bx^2)^2} dx$

Optimal. Leaf size=81

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{7b^2}{2a^4x}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{7b^2}{2a^4x}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b
*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(a+bx^2\right)^2}\,dx &= \frac{1}{2ax^5\left(a+bx^2\right)}+\frac{7\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{2a}\\
&= -\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{(7b)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a^2}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}+\frac{1}{2ax^5\left(a+bx^2\right)}+\frac{\left(7b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^3}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{\left(7b^3\right)\int \frac{1}{a+bx^2}\,dx}{2a^4}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.99

$$
-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{b^3x}{2a^4\left(a+bx^2\right)}-\frac{3b^2}{a^4x}+\frac{2b}{3a^3x^3}-\frac{1}{5a^2x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/5*1/(a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a
+ b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^6\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a + b*x^2)^2), x]
```

fricas [A] time = 0.89, size = 198, normalized size = 2.44

$$\left[-\frac{210\,b^{3}x^{6}+140\,ab^{2}x^{4}-28\,a^{2}bx^{2}+12\,a^{3}-105\,\left(b^{3}x^{7}+ab^{2}x^{5}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{60\,\left(a^{4}bx^{7}+a^{5}x^{5}\right)},\,-\frac{105\,b^{3}x^{6}+70\,ab^{2}x^{4}-14\,a^{2}bx^{2}+6\,a^{3}+105\,\left(b^{3}x^{7}+ab^{2}x^{5}\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{30\,\left(a^{4}bx^{7}+a^{5}x^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7
+ a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a
^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a
^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b*x^7 +
a^5*x^5)]
```

giac [A] time = 0.65, size = 70, normalized size = 0.86

$$-\frac{7\,b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{4}}-\frac{b^{3}x}{2\,\left(bx^{2}+a\right)a^{4}}-\frac{45\,b^{2}x^{4}-10\,abx^{2}+3\,a^{2}}{15\,a^{4}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^4
) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

maple [A] time = 0.01, size = 70, normalized size = 0.86

$$-\frac{b^{3}x}{2\left(b\,x^{2}+a\right)a^{4}}-\frac{7b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{4}}-\frac{3b^{2}}{a^{4}x}+\frac{2b}{3a^{3}x^{3}}-\frac{1}{5a^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^2,x)
```

```
[Out] -1/5/a^2/x^5-3*b^2/a^4/x+2/3*b/a^3/x^3-1/2*b^3/a^4*x/(b*x^2+a)-7/2*b^3/a^4/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.91, size = 75, normalized size = 0.93

$$-\frac{105\,b^{3}x^{6}+70\,ab^{2}x^{4}-14\,a^{2}bx^{2}+6\,a^{3}}{30\,\left(a^{4}bx^{7}+a^{5}x^{5}\right)}-\frac{7\,b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a^3)/(a^4*b*x^7 + a^5*
x^5) - 7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [B] time = 4.85, size = 70, normalized size = 0.86

$$-\frac{\frac{1}{5\,a}-\frac{7\,b\,x^2}{15\,a^2}+\frac{7\,b^2\,x^4}{3\,a^3}+\frac{7\,b^3\,x^6}{2\,a^4}}{b\,x^7+a\,x^5}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^2),x)
```

```
[Out] - (1/(5*a) - (7*b*x^2)/(15*a^2) + (7*b^2*x^4)/(3*a^3) + (7*b^3*x^6)/(2*a^4)
)/(a*x^5 + b*x^7) - (7*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(9/2))
```

sympy [A] time = 0.40, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{b^5}{a^9}}\,\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}-\frac{7\sqrt{-\frac{b^5}{a^9}}\,\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}+\frac{-6a^3+14a^2bx^2-70ab^2x^4-105b^3x^6}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**2,x)
```

```
[Out] 7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a*
*9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 + (-6*a**3 + 14*a**2*b*x**2 - 70*
a*b**2*x**4 - 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

3.165 $\int \frac{1}{x^7(a+bx^2)^2}\,dx$

Optimal. Leaf size=80

$$\frac{2b^3\log(a+bx^2)}{a^5}-\frac{4b^3\log(x)}{a^5}-\frac{b^3}{2a^4(a+bx^2)}-\frac{3b^2}{2a^4x^2}+\frac{b}{2a^3x^4}-\frac{1}{6a^2x^6}$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b^3}{2a^4(a+bx^2)}-\frac{3b^2}{2a^4x^2}+\frac{2b^3\log(a+bx^2)}{a^5}-\frac{4b^3\log(x)}{a^5}+\frac{b}{2a^3x^4}-\frac{1}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^2),x]
```

```
[Out] -1/(6*a^2*x^6) + b/(2*a^3*x^4) - (3*b^2)/(2*a^4*x^2) - b^3/(2*a^4*(a + b*x^
2)) - (4*b^3*Log[x])/a^5 + (2*b^3*Log[a + b*x^2])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^2x^4} - \frac{2b}{a^3x^3} + \frac{3b^2}{a^4x^2} - \frac{4b^3}{a^5x} + \frac{b^4}{a^4(a+bx)^2} + \frac{4b^4}{a^5(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{6a^2x^6} + \frac{b}{2a^3x^4} - \frac{3b^2}{2a^4x^2} - \frac{b^3}{2a^4\left(a+bx^2\right)} - \frac{4b^3\log(x)}{a^5} + \frac{2b^3\log\left(a+bx^2\right)}{a^5}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 0.85

$$\frac{a\left(-\frac{a^2}{x^6} - \frac{3b^3}{a+bx^2} + \frac{3ab}{x^4} - \frac{9b^2}{x^2}\right) + 12b^3\log\left(a+bx^2\right) - 24b^3\log(x)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^2),x]
```

```
[Out] (a*(-(a^2/x^6) + (3*a*b)/x^4 - (9*b^2)/x^2 - (3*b^3)/(a + b*x^2)) - 24*b^3*
Log[x] + 12*b^3*Log[a + b*x^2])/(6*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(a + b*x^2)^2), x]
```

fricas [A] time = 0.59, size = 99, normalized size = 1.24

$$-\frac{12\,ab^3x^6 + 6\,a^2b^2x^4 - 2\,a^3bx^2 + a^4 - 12\left(b^4x^8 + ab^3x^6\right)\log\left(bx^2+a\right) + 24\left(b^4x^8 + ab^3x^6\right)\log(x)}{6\left(a^5bx^8 + a^6x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4 - 12*(b^4*x^8 + a*b^
3*x^6)*log(b*x^2 + a) + 24*(b^4*x^8 + a*b^3*x^6)*log(x))/(a^5*b*x^8 + a^6*x
^6)
```

giac [A] time = 0.64, size = 99, normalized size = 1.24

$$-\frac{2\,b^3\log\left(x^2\right)}{a^5}+\frac{2\,b^3\log\left(\left|bx^2+a\right|\right)}{a^5}-\frac{4\,b^4x^2+5\,ab^3}{2\left(bx^2+a\right)a^5}+\frac{22\,b^3x^6-9\,ab^2x^4+3\,a^2bx^2-a^3}{6\,a^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*b^3*log(x^2)/a^5 + 2*b^3*log(abs(b*x^2 + a))/a^5 - 1/2*(4*b^4*x^2 + 5*a*
b^3)/((b*x^2 + a)*a^5) + 1/6*(22*b^3*x^6 - 9*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)
/(a^5*x^6)
```

maple [A] time = 0.01, size = 73, normalized size = 0.91

$$-\frac{b^3}{2\left(b\,x^2+a\right)a^4}-\frac{4b^3\ln(x)}{a^5}+\frac{2b^3\ln\left(b\,x^2+a\right)}{a^5}-\frac{3b^2}{2a^4x^2}+\frac{b}{2a^3x^4}-\frac{1}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^2,x)
```

```
[Out] -1/6/a^2/x^6+1/2*b/a^3/x^4-3/2*b^2/a^4/x^2-1/2*b^3/a^4/(b*x^2+a)-4*b^3*ln(x
)/a^5+2*b^3*ln(b*x^2+a)/a^5
```

maxima [A] time = 1.32, size = 79, normalized size = 0.99

$$-\frac{12\,b^3x^6+6\,ab^2x^4-2\,a^2bx^2+a^3}{6\left(a^4bx^8+a^5x^6\right)}+\frac{2\,b^3\log\left(bx^2+a\right)}{a^5}-\frac{2\,b^3\log\left(x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(12*b^3*x^6 + 6*a*b^2*x^4 - 2*a^2*b*x^2 + a^3)/(a^4*b*x^8 + a^5*x^6) +
 2*b^3*log(b*x^2 + a)/a^5 - 2*b^3*log(x^2)/a^5
```

mupad [B] time = 0.12, size = 78, normalized size = 0.98

$$\frac{2\,b^3\,\ln\left(b\,x^2+a\right)}{a^5}-\frac{\frac{1}{6\,a}-\frac{b\,x^2}{3\,a^2}+\frac{b^2\,x^4}{a^3}+\frac{2\,b^3\,x^6}{a^4}}{b\,x^8+a\,x^6}-\frac{4\,b^3\,\ln(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^2),x)
```

```
[Out] (2*b^3*log(a + b*x^2))/a^5 - (1/(6*a) - (b*x^2)/(3*a^2) + (b^2*x^4)/a^3 + (
2*b^3*x^6)/a^4)/(a*x^6 + b*x^8) - (4*b^3*log(x))/a^5
```

sympy [A] time = 0.47, size = 78, normalized size = 0.98

$$\frac{-a^3+2a^2bx^2-6ab^2x^4-12b^3x^6}{6a^5x^6+6a^4bx^8}-\frac{4b^3\log(x)}{a^5}+\frac{2b^3\log\left(\frac{a}{b}+x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x**2 - 6*a*b**2*x**4 - 12*b**3*x**6)/(6*a**5*x**6 + 6*a**
4*b*x**8) - 4*b**3*log(x)/a**5 + 2*b**3*log(a/b + x**2)/a**5
```

3.166 $\int \frac{1}{x^8(a+bx^2)^2} dx$

Optimal. Leaf size=94

$$\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{9b^3}{2a^5x}-\frac{3b^2}{2a^4x^3}+\frac{9b}{10a^3x^5}-\frac{9}{14a^2x^7}+\frac{1}{2ax^7\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{3b^2}{2a^4x^3}+\frac{9b^3}{2a^5x}+\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{9b}{10a^3x^5}-\frac{9}{14a^2x^7}+\frac{1}{2ax^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -9/(14*a^2*x^7) + (9*b)/(10*a^3*x^5) - (3*b^2)/(2*a^4*x^3) + (9*b^3)/(2*a^5
*x) + 1/(2*a*x^7*(a + b*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
a^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^8 \left(a+bx^2\right)^2}\,dx &= \frac{1}{2ax^7\left(a+bx^2\right)}+\frac{9\int \frac{1}{x^8\left(a+bx^2\right)}\,dx}{2a}\\
&= -\frac{9}{14a^2x^7}+\frac{1}{2ax^7\left(a+bx^2\right)}-\frac{(9b)\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{2a^2}\\
&= -\frac{9}{14a^2x^7}+\frac{9b}{10a^3x^5}+\frac{1}{2ax^7\left(a+bx^2\right)}+\frac{\left(9b^2\right)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a^3}\\
&= -\frac{9}{14a^2x^7}+\frac{9b}{10a^3x^5}-\frac{3b^2}{2a^4x^3}+\frac{1}{2ax^7\left(a+bx^2\right)}-\frac{\left(9b^3\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^4}\\
&= -\frac{9}{14a^2x^7}+\frac{9b}{10a^3x^5}-\frac{3b^2}{2a^4x^3}+\frac{9b^3}{2a^5x}+\frac{1}{2ax^7\left(a+bx^2\right)}+\frac{\left(9b^4\right)\int \frac{1}{a+bx^2}\,dx}{2a^5}\\
&= -\frac{9}{14a^2x^7}+\frac{9b}{10a^3x^5}-\frac{3b^2}{2a^4x^3}+\frac{9b^3}{2a^5x}+\frac{1}{2ax^7\left(a+bx^2\right)}+\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.97

$$
\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{b^4x}{2a^5\left(a+bx^2\right)}+\frac{4b^3}{a^5x}-\frac{b^2}{a^4x^3}+\frac{2b}{5a^3x^5}-\frac{1}{7a^2x^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -1/7*1/(a^2*x^7) + (2*b)/(5*a^3*x^5) - b^2/(a^4*x^3) + (4*b^3)/(a^5*x) + (b
^4*x)/(2*a^5*(a + b*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(1
1/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^8\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^8*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^8*(a + b*x^2)^2), x]
```

fricas [A] time = 0.64, size = 220, normalized size = 2.34

$$\left[\frac{630\,b^4x^8+420\,ab^3x^6-84\,a^2b^2x^4+36\,a^3bx^2-20\,a^4+315\left(b^4x^9+ab^3x^7\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{140\left(a^5bx^9+a^6x^7\right)},\ \frac{315\,b^4x^8+210\,ab^3x^6-42\,a^2b^2x^4+18\,a^3bx^2-10\,a^4+315\left(b^4x^9+ab^3x^7\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{70\left(a^5bx^9+a^6x^7\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(630*b^4*x^8 + 420*a*b^3*x^6 - 84*a^2*b^2*x^4 + 36*a^3*b*x^2 - 20*a^
4 + 315*(b^4*x^9 + a*b^3*x^7)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)
/(b*x^2 + a)))/(a^5*b*x^9 + a^6*x^7), 1/70*(315*b^4*x^8 + 210*a*b^3*x^6 - 4
2*a^2*b^2*x^4 + 18*a^3*b*x^2 - 10*a^4 + 315*(b^4*x^9 + a*b^3*x^7)*sqrt(b/a)
*arctan(x*sqrt(b/a)))/(a^5*b*x^9 + a^6*x^7)]
```

giac [A] time = 0.62, size = 81, normalized size = 0.86

$$\frac{9\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^5}+\frac{b^4x}{2\left(bx^2+a\right)a^5}+\frac{140\,b^3x^6-35\,ab^2x^4+14\,a^2bx^2-5\,a^3}{35\,a^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 9/2*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/2*b^4*x/((b*x^2 + a)*a^5)
 + 1/35*(140*b^3*x^6 - 35*a*b^2*x^4 + 14*a^2*b*x^2 - 5*a^3)/(a^5*x^7)
```

maple [A] time = 0.01, size = 81, normalized size = 0.86

$$\frac{b^4x}{2\left(b\,x^2+a\right)a^5}+\frac{9b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^5}+\frac{4b^3}{a^5x}-\frac{b^2}{a^4x^3}+\frac{2b}{5a^3x^5}-\frac{1}{7a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a)^2,x)
```

```
[Out] -1/7/a^2/x^7+4*b^3/a^5/x-b^2/a^4/x^3+2/5*b/a^3/x^5+1/2*b^4/a^5*x/(b*x^2+a)+
9/2*b^4/a^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 86, normalized size = 0.91

$$\frac{315\, b^4x^8 + 210\, ab^3x^6 - 42\, a^2b^2x^4 + 18\, a^3bx^2 - 10\, a^4}{70\left(a^5bx^9 + a^6x^7\right)} + \frac{9\, b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\, \sqrt{ab}\, a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/70*(315*b^4*x^8 + 210*a*b^3*x^6 - 42*a^2*b^2*x^4 + 18*a^3*b*x^2 - 10*a^4)
/(a^5*b*x^9 + a^6*x^7) + 9/2*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 4.56, size = 80, normalized size = 0.85

$$\frac{\frac{9\,b\,x^2}{35\,a^2} - \frac{1}{7\,a} - \frac{3\,b^2\,x^4}{5\,a^3} + \frac{3\,b^3\,x^6}{a^4} + \frac{9\,b^4\,x^8}{2\,a^5}}{b\,x^9 + a\,x^7} + \frac{9\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^2)^2),x)
```

```
[Out] ((9*b*x^2)/(35*a^2) - 1/(7*a) - (3*b^2*x^4)/(5*a^3) + (3*b^3*x^6)/a^4 + (9*
b^4*x^8)/(2*a^5))/(a*x^7 + b*x^9) + (9*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/(
2*a^(11/2))
```

sympy [A] time = 0.45, size = 138, normalized size = 1.47

$$-\frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^7}{a^{11}}}}{b^4} + x\right)}{4} + \frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^7}{a^{11}}}}{b^4} + x\right)}{4} + \frac{-10a^4 + 18a^3bx^2 - 42a^2b^2x^4 + 210ab^3x^6 + 315b^4x^8}{70a^6x^7 + 70a^5bx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a)**2,x)
```

```
[Out] -9*sqrt(-b**7/a**11)*log(-a**6*sqrt(-b**7/a**11)/b**4 + x)/4 + 9*sqrt(-b**7
/a**11)*log(a**6*sqrt(-b**7/a**11)/b**4 + x)/4 + (-10*a**4 + 18*a**3*b*x**2
 - 42*a**2*b**2*x**4 + 210*a*b**3*x**6 + 315*b**4*x**8)/(70*a**6*x**7 + 70*
a**5*b*x**9)
```

3.167 $\int \frac{1}{x^9(a+bx^2)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{5b^4\log\left(a+bx^2\right)}{2a^6}+\frac{5b^4\log(x)}{a^6}+\frac{b^4}{2a^5\left(a+bx^2\right)}+\frac{2b^3}{a^5x^2}-\frac{3b^2}{4a^4x^4}+\frac{b}{3a^3x^6}-\frac{1}{8a^2x^8}$$

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{b^4}{2a^5\left(a+bx^2\right)}+\frac{2b^3}{a^5x^2}-\frac{3b^2}{4a^4x^4}-\frac{5b^4\log\left(a+bx^2\right)}{2a^6}+\frac{5b^4\log(x)}{a^6}+\frac{b}{3a^3x^6}-\frac{1}{8a^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)^2),x]
```

```
[Out] -1/(8*a^2*x^8) + b/(3*a^3*x^6) - (3*b^2)/(4*a^4*x^4) + (2*b^3)/(a^5*x^2) +
b^4/(2*a^5*(a + b*x^2)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x^2])/(2*a^
6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^5(a+bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^2x^5} - \frac{2b}{a^3x^4} + \frac{3b^2}{a^4x^3} - \frac{4b^3}{a^5x^2} + \frac{5b^4}{a^6x} - \frac{b^5}{a^5(a+bx)^2} - \frac{5b^5}{a^6(a+bx)}\right) dx, x, x^2\right)$$
$$= -\frac{1}{8a^2x^8} + \frac{b}{3a^3x^6} - \frac{3b^2}{4a^4x^4} + \frac{2b^3}{a^5x^2} + \frac{b^4}{2a^5\left(a+bx^2\right)} + \frac{5b^4\log(x)}{a^6} - \frac{5b^4\log\left(a+bx^2\right)}{2a^6}$$

Mathematica [**A**] time = 0.08, size = 79, normalized size = 0.85

$$\frac{a\left(-\frac{3a^3}{x^8} + \frac{8a^2b}{x^6} + 12b^3\left(\frac{b}{a+bx^2} + \frac{4}{x^2}\right) - \frac{18ab^2}{x^4}\right) - 60b^4\log\left(a+bx^2\right) + 120b^4\log(x)}{24a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)^2),x]
```

```
[Out] (a*((-3*a^3)/x^8 + (8*a^2*b)/x^6 - (18*a*b^2)/x^4 + 12*b^3*(4/x^2 + b/(a +
b*x^2))) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2])/(24*a^6)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^9*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^9*(a + b*x^2)^2), x]
```

fricas [**A**] time = 0.75, size = 112, normalized size = 1.20

$$\frac{60\,ab^4x^8 + 30\,a^2b^3x^6 - 10\,a^3b^2x^4 + 5\,a^4bx^2 - 3\,a^5 - 60\left(b^5x^{10} + ab^4x^8\right)\log\left(bx^2+a\right) + 120\left(b^5x^{10} + ab^4x^8\right)\log(x)}{24\left(a^6bx^{10} + a^7x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(60*a*b^4*x^8 + 30*a^2*b^3*x^6 - 10*a^3*b^2*x^4 + 5*a^4*b*x^2 - 3*a^5
- 60*(b^5*x^10 + a*b^4*x^8)*log(b*x^2 + a) + 120*(b^5*x^10 + a*b^4*x^8)*log
(x))/(a^6*b*x^10 + a^7*x^8)
```

giac [A] time = 0.63, size = 110, normalized size = 1.18

$$\frac{5\,b^4\log\left(x^2\right)}{2\,a^6}-\frac{5\,b^4\log\left(\left|bx^2+a\right|\right)}{2\,a^6}+\frac{5\,b^5x^2+6\,ab^4}{2\left(bx^2+a\right)a^6}-\frac{125\,b^4x^8-48\,ab^3x^6+18\,a^2b^2x^4-8\,a^3bx^2+3\,a^4}{24\,a^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^4*log(x^2)/a^6 - 5/2*b^4*log(abs(b*x^2 + a))/a^6 + 1/2*(5*b^5*x^2 + 6
*a*b^4)/((b*x^2 + a)*a^6) - 1/24*(125*b^4*x^8 - 48*a*b^3*x^6 + 18*a^2*b^2*x
^4 - 8*a^3*b*x^2 + 3*a^4)/(a^6*x^8)
```

maple [A] time = 0.01, size = 84, normalized size = 0.90

$$\frac{b^4}{2\left(b\,x^2+a\right)a^5}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln\left(b\,x^2+a\right)}{2a^6}+\frac{2b^3}{a^5x^2}-\frac{3b^2}{4a^4x^4}+\frac{b}{3a^3x^6}-\frac{1}{8a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a)^2,x)
```

```
[Out] -1/8/a^2/x^8+1/3*b/a^3/x^6-3/4*b^2/a^4/x^4+2*b^3/a^5/x^2+1/2*b^4/a^5/(b*x^2
+a)+5*b^4*ln(x)/a^6-5/2*b^4*ln(b*x^2+a)/a^6
```

maxima [A] time = 1.30, size = 92, normalized size = 0.99

$$\frac{60\,b^4x^8+30\,ab^3x^6-10\,a^2b^2x^4+5\,a^3bx^2-3\,a^4}{24\left(a^5bx^{10}+a^6x^8\right)}-\frac{5\,b^4\log\left(bx^2+a\right)}{2\,a^6}+\frac{5\,b^4\log\left(x^2\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(60*b^4*x^8 + 30*a*b^3*x^6 - 10*a^2*b^2*x^4 + 5*a^3*b*x^2 - 3*a^4)/(a^
5*b*x^10 + a^6*x^8) - 5/2*b^4*log(b*x^2 + a)/a^6 + 5/2*b^4*log(x^2)/a^6
```

mupad [B] time = 4.73, size = 89, normalized size = 0.96

$$\frac{\frac{5\,b\,x^2}{24\,a^2}-\frac{1}{8\,a}-\frac{5\,b^2\,x^4}{12\,a^3}+\frac{5\,b^3\,x^6}{4\,a^4}+\frac{5\,b^4\,x^8}{2\,a^5}}{b\,x^{10}+a\,x^8}-\frac{5\,b^4\,\ln\left(b\,x^2+a\right)}{2\,a^6}+\frac{5\,b^4\,\ln(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^2)^2),x)
```

```
[Out] ((5*b*x^2)/(24*a^2) - 1/(8*a) - (5*b^2*x^4)/(12*a^3) + (5*b^3*x^6)/(4*a^4)
+ (5*b^4*x^8)/(2*a^5))/(a*x^8 + b*x^10) - (5*b^4*log(a + b*x^2))/(2*a^6) +
(5*b^4*log(x))/a^6
```

sympy [A] time = 0.52, size = 94, normalized size = 1.01

$$\frac{-3a^4+5a^3bx^2-10a^2b^2x^4+30ab^3x^6+60b^4x^8}{24a^6x^8+24a^5bx^{10}}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log\left(\frac{a}{b}+x^2\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x**2 - 10*a**2*b**2*x**4 + 30*a*b**3*x**6 + 60*b**4*x**
8)/(24*a**6*x**8 + 24*a**5*b*x**10) + 5*b**4*log(x)/a**6 - 5*b**4*log(a/b +
 x**2)/(2*a**6)
```

3.168 $\int \frac{x^{15}}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=114

$$\frac{a^7}{4b^8(a+bx^2)^2} - \frac{7a^6}{2b^8(a+bx^2)} - \frac{21a^5\log(a+bx^2)}{2b^8} + \frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3}$$

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2x^6}{b^5} - \frac{5a^3x^4}{2b^6} + \frac{15a^4x^2}{2b^7} - \frac{7a^6}{2b^8(a+bx^2)} + \frac{a^7}{4b^8(a+bx^2)^2} - \frac{21a^5\log(a+bx^2)}{2b^8} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^2)^3,x]
```

```
[Out] (15*a^4*x^2)/(2*b^7) - (5*a^3*x^4)/(2*b^6) + (a^2*x^6)/b^5 - (3*a*x^8)/(8*b
^4) + x^10/(10*b^3) + a^7/(4*b^8*(a + b*x^2)^2) - (7*a^6)/(2*b^8*(a + b*x^2
)) - (21*a^5*Log[a + b*x^2])/(2*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^7}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{15a^4}{b^7} - \frac{10a^3x}{b^6} + \frac{6a^2x^2}{b^5} - \frac{3ax^3}{b^4} + \frac{x^4}{b^3} - \frac{a^7}{b^7(a+bx)^3} + \frac{7a^6}{b^7(a+bx)^2} - \frac{21a^5}{b^7(a+bx)}\right)\right.$$
$$= \frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3} + \frac{a^7}{4b^8\left(a+bx^2\right)^2} - \frac{7a^6}{2b^8\left(a+bx^2\right)} - \frac{21a^5\log\left(a+bx^2\right.}{2b^8}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 0.85

$$\frac{\frac{10a^7}{\left(a+bx^2\right)^2} - \frac{140a^6}{a+bx^2} - 420a^5\log\left(a+bx^2\right) + 300a^4bx^2 - 100a^3b^2x^4 + 40a^2b^3x^6 - 15ab^4x^8 + 4b^5x^{10}}{40b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^2)^3,x]
```

```
[Out] (300*a^4*b*x^2 - 100*a^3*b^2*x^4 + 40*a^2*b^3*x^6 - 15*a*b^4*x^8 + 4*b^5*x^
10 + (10*a^7)/(a + b*x^2)^2 - (140*a^6)/(a + b*x^2) - 420*a^5*Log[a + b*x^2
])/(40*b^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^15/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^15/(a + b*x^2)^3, x]
```

fricas [A] time = 0.45, size = 137, normalized size = 1.20

$$\frac{4\,b^7x^{14} - 7\,ab^6x^{12} + 14\,a^2b^5x^{10} - 35\,a^3b^4x^8 + 140\,a^4b^3x^6 + 500\,a^5b^2x^4 + 160\,a^6bx^2 - 130\,a^7 - 420\left(a^5b^2x^4 + 2\,a^6bx^2 + a^7\right)\log\left(bx^2+a\right)}{40\left(b^{10}x^4 + 2\,ab^9x^2 + a^2b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(4*b^7*x^14 - 7*a*b^6*x^12 + 14*a^2*b^5*x^10 - 35*a^3*b^4*x^8 + 140*a^
4*b^3*x^6 + 500*a^5*b^2*x^4 + 160*a^6*b*x^2 - 130*a^7 - 420*(a^5*b^2*x^4 +
2*a^6*b*x^2 + a^7)*log(b*x^2 + a))/(b^10*x^4 + 2*a*b^9*x^2 + a^2*b^8)
```

giac [A] time = 0.63, size = 114, normalized size = 1.00

$$-\frac{21\,a^5\log\left(\left|bx^2+a\right|\right)}{2\,b^8}+\frac{63\,a^5b^2x^4+112\,a^6bx^2+50\,a^7}{4\left(bx^2+a\right)^2b^8}+\frac{4\,b^{12}x^{10}-15\,ab^{11}x^8+40\,a^2b^{10}x^6-100\,a^3b^9x^4+300\,a^4b^8x^2}{40\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -21/2*a^5*log(abs(b*x^2 + a))/b^8 + 1/4*(63*a^5*b^2*x^4 + 112*a^6*b*x^2 + 5
0*a^7)/((b*x^2 + a)^2*b^8) + 1/40*(4*b^12*x^10 - 15*a*b^11*x^8 + 40*a^2*b^1
0*x^6 - 100*a^3*b^9*x^4 + 300*a^4*b^8*x^2)/b^15
```

maple [A] time = 0.01, size = 101, normalized size = 0.89

$$\frac{x^{10}}{10b^3}-\frac{3a\,x^8}{8b^4}+\frac{a^2x^6}{b^5}-\frac{5a^3x^4}{2b^6}+\frac{a^7}{4\left(b\,x^2+a\right)^2b^8}+\frac{15a^4x^2}{2b^7}-\frac{7a^6}{2\left(b\,x^2+a\right)b^8}-\frac{21a^5\ln\left(b\,x^2+a\right)}{2b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^2+a)^3,x)
```

```
[Out] 15/2*a^4*x^2/b^7-5/2*a^3*x^4/b^6+a^2*x^6/b^5-3/8*a*x^8/b^4+1/10*x^10/b^3+1/
4*a^7/b^8/(b*x^2+a)^2-7/2*a^6/b^8/(b*x^2+a)-21/2*a^5*ln(b*x^2+a)/b^8
```

maxima [A] time = 1.36, size = 111, normalized size = 0.97

$$-\frac{14\,a^6bx^2+13\,a^7}{4\left(b^{10}x^4+2\,ab^9x^2+a^2b^8\right)}-\frac{21\,a^5\log\left(bx^2+a\right)}{2\,b^8}+\frac{4\,b^4x^{10}-15\,ab^3x^8+40\,a^2b^2x^6-100\,a^3bx^4+300\,a^4x^2}{40\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(14*a^6*b*x^2 + 13*a^7)/(b^10*x^4 + 2*a*b^9*x^2 + a^2*b^8) - 21/2*a^5*
log(b*x^2 + a)/b^8 + 1/40*(4*b^4*x^10 - 15*a*b^3*x^8 + 40*a^2*b^2*x^6 - 100
*a^3*b*x^4 + 300*a^4*x^2)/b^7
```

mupad [B] time = 4.72, size = 111, normalized size = 0.97

$$\frac{x^{10}}{10\,b^3}-\frac{\frac{13\,a^7}{4\,b}+\frac{7\,a^6\,x^2}{2}}{a^2\,b^7+2\,a\,b^8\,x^2+b^9\,x^4}-\frac{3\,a\,x^8}{8\,b^4}-\frac{21\,a^5\,\ln\left(b\,x^2+a\right)}{2\,b^8}+\frac{a^2\,x^6}{b^5}-\frac{5\,a^3\,x^4}{2\,b^6}+\frac{15\,a^4\,x^2}{2\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a + b*x^2)^3,x)
```

```
[Out] x^10/(10*b^3) - ((13*a^7)/(4*b) + (7*a^6*x^2)/2)/(a^2*b^7 + b^9*x^4 + 2*a*b
^8*x^2) - (3*a*x^8)/(8*b^4) - (21*a^5*log(a + b*x^2))/(2*b^8) + (a^2*x^6)/b
^5 - (5*a^3*x^4)/(2*b^6) + (15*a^4*x^2)/(2*b^7)
```

sympy [A] time = 0.48, size = 119, normalized size = 1.04

$$-\frac{21a^5\log\left(a+bx^2\right)}{2b^8}+\frac{15a^4x^2}{2b^7}-\frac{5a^3x^4}{2b^6}+\frac{a^2x^6}{b^5}-\frac{3ax^8}{8b^4}+\frac{-13a^7-14a^6bx^2}{4a^2b^8+8ab^9x^2+4b^{10}x^4}+\frac{x^{10}}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**2+a)**3,x)
```

```
[Out] -21*a**5*log(a + b*x**2)/(2*b**8) + 15*a**4*x**2/(2*b**7) - 5*a**3*x**4/(2*
b**6) + a**2*x**6/b**5 - 3*a*x**8/(8*b**4) + (-13*a**7 - 14*a**6*b*x**2)/(4
*a**2*b**8 + 8*a*b**9*x**2 + 4*b**10*x**4) + x**10/(10*b**3)
```

3.169 $\int \frac{x^{13}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{a^6}{4b^7\left(a+bx^2\right)^2}+\frac{3a^5}{b^7\left(a+bx^2\right)}+\frac{15a^4\log\left(a+bx^2\right)}{2b^7}-\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}$$

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^4}{2b^5}-\frac{5a^3x^2}{b^6}+\frac{3a^5}{b^7\left(a+bx^2\right)}-\frac{a^6}{4b^7\left(a+bx^2\right)^2}+\frac{15a^4\log\left(a+bx^2\right)}{2b^7}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^3,x]
```

```
[Out] (-5*a^3*x^2)/b^6 + (3*a^2*x^4)/(2*b^5) - (a*x^6)/(2*b^4) + x^8/(8*b^3) - a^
6/(4*b^7*(a + b*x^2)^2) + (3*a^5)/(b^7*(a + b*x^2)) + (15*a^4*Log[a + b*x^2
])/(2*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{10a^3}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{b^4}+\frac{x^3}{b^3}+\frac{a^6}{b^6(a+bx)^3}-\frac{6a^5}{b^6(a+bx)^2}+\frac{15a^4}{b^6(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{x^8}{8b^3}-\frac{a^6}{4b^7\left(a+bx^2\right)^2}+\frac{3a^5}{b^7\left(a+bx^2\right)}+\frac{15a^4\log\left(a+bx^2\right)}{2b^7}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.85

$$\frac{-\frac{2a^6}{\left(a+bx^2\right)^2}+\frac{24a^5}{a+bx^2}+60a^4\log\left(a+bx^2\right)-40a^3bx^2+12a^2b^2x^4-4ab^3x^6+b^4x^8}{8b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^3,x]
```

```
[Out] (-40*a^3*b*x^2 + 12*a^2*b^2*x^4 - 4*a*b^3*x^6 + b^4*x^8 - (2*a^6)/(a + b*x^
2)^2 + (24*a^5)/(a + b*x^2) + 60*a^4*Log[a + b*x^2])/(8*b^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^13/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^13/(a + b*x^2)^3, x]
```

fricas [A] time = 0.95, size = 125, normalized size = 1.25

$$\frac{b^6x^{12}-2\,ab^5x^{10}+5\,a^2b^4x^8-20\,a^3b^3x^6-68\,a^4b^2x^4-16\,a^5bx^2+22\,a^6+60\left(a^4b^2x^4+2\,a^5bx^2+a^6\right)\log\left(bx^2+a\right)}{8\left(b^9x^4+2\,ab^8x^2+a^2b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(b^6*x^12 - 2*a*b^5*x^10 + 5*a^2*b^4*x^8 - 20*a^3*b^3*x^6 - 68*a^4*b^2*
x^4 - 16*a^5*b*x^2 + 22*a^6 + 60*(a^4*b^2*x^4 + 2*a^5*b*x^2 + a^6)*log(b*x^
2 + a))/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)
```

giac [A] time = 0.61, size = 102, normalized size = 1.02

$$\frac{15\,a^4\log\left(\left|bx^2+a\right|\right)}{2\,b^7}-\frac{45\,a^4b^2x^4+78\,a^5bx^2+34\,a^6}{4\left(bx^2+a\right)^2b^7}+\frac{b^9x^8-4\,ab^8x^6+12\,a^2b^7x^4-40\,a^3b^6x^2}{8\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 15/2*a^4*log(abs(b*x^2 + a))/b^7 - 1/4*(45*a^4*b^2*x^4 + 78*a^5*b*x^2 + 34*
a^6)/((b*x^2 + a)^2*b^7) + 1/8*(b^9*x^8 - 4*a*b^8*x^6 + 12*a^2*b^7*x^4 - 40
*a^3*b^6*x^2)/b^12
```

maple [A] time = 0.01, size = 91, normalized size = 0.91

$$\frac{x^8}{8b^3}-\frac{a\,x^6}{2b^4}+\frac{3a^2x^4}{2b^5}-\frac{a^6}{4\left(b\,x^2+a\right)^2b^7}-\frac{5a^3x^2}{b^6}+\frac{3a^5}{\left(b\,x^2+a\right)b^7}+\frac{15a^4\ln\left(b\,x^2+a\right)}{2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^3,x)
```

```
[Out] -5*a^3*x^2/b^6+3/2*a^2*x^4/b^5-1/2*a*x^6/b^4+1/8*x^8/b^3-1/4*a^6/b^7/(b*x^2
+a)^2+3*a^5/b^7/(b*x^2+a)+15/2*a^4*ln(b*x^2+a)/b^7
```

maxima [A] time = 1.29, size = 99, normalized size = 0.99

$$\frac{12\,a^5bx^2+11\,a^6}{4\left(b^9x^4+2\,ab^8x^2+a^2b^7\right)}+\frac{15\,a^4\log\left(bx^2+a\right)}{2\,b^7}+\frac{b^3x^8-4\,ab^2x^6+12\,a^2bx^4-40\,a^3x^2}{8\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*a^5*b*x^2 + 11*a^6)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7) + 15/2*a^4*lo
g(b*x^2 + a)/b^7 + 1/8*(b^3*x^8 - 4*a*b^2*x^6 + 12*a^2*b*x^4 - 40*a^3*x^2)/
b^6
```

mupad [B] time = 0.08, size = 100, normalized size = 1.00

$$\frac{\frac{11\,a^6}{4\,b}+3\,a^5\,x^2}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}+\frac{x^8}{8\,b^3}-\frac{a\,x^6}{2\,b^4}+\frac{15\,a^4\,\ln\left(b\,x^2+a\right)}{2\,b^7}+\frac{3\,a^2\,x^4}{2\,b^5}-\frac{5\,a^3\,x^2}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^2)^3,x)
```

```
[Out] ((11*a^6)/(4*b) + 3*a^5*x^2)/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) + x^8/(8*b^3
) - (a*x^6)/(2*b^4) + (15*a^4*log(a + b*x^2))/(2*b^7) + (3*a^2*x^4)/(2*b^5)
 - (5*a^3*x^2)/b^6
```

sympy [A] time = 0.47, size = 104, normalized size = 1.04

$$\frac{15a^4\log\left(a+bx^2\right)}{2b^7}-\frac{5a^3x^2}{b^6}+\frac{3a^2x^4}{2b^5}-\frac{ax^6}{2b^4}+\frac{11a^6+12a^5bx^2}{4a^2b^7+8ab^8x^2+4b^9x^4}+\frac{x^8}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**3,x)
```

```
[Out] 15*a**4*log(a + b*x**2)/(2*b**7) - 5*a**3*x**2/b**6 + 3*a**2*x**4/(2*b**5)
- a*x**6/(2*b**4) + (11*a**6 + 12*a**5*b*x**2)/(4*a**2*b**7 + 8*a*b**8*x**2
 + 4*b**9*x**4) + x**8/(8*b**3)
```

3.170 $\int \frac{x^{11}}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=87

$$\frac{a^5}{4b^6\left(a+bx^2\right)^2}-\frac{5a^4}{2b^6\left(a+bx^2\right)}-\frac{5a^3\log\left(a+bx^2\right)}{b^6}+\frac{3a^2x^2}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^6}{6b^3}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3a^2x^2}{b^5}-\frac{5a^4}{2b^6\left(a+bx^2\right)}+\frac{a^5}{4b^6\left(a+bx^2\right)^2}-\frac{5a^3\log\left(a+bx^2\right)}{b^6}-\frac{3ax^4}{4b^4}+\frac{x^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^3,x]
```

```
[Out] (3*a^2*x^2)/b^5 - (3*a*x^4)/(4*b^4) + x^6/(6*b^3) + a^5/(4*b^6*(a + b*x^2)^
2) - (5*a^4)/(2*b^6*(a + b*x^2)) - (5*a^3*Log[a + b*x^2])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^5}{(a+bx)^3}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{6a^2}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{b^3}-\frac{a^5}{b^5(a+bx)^3}+\frac{5a^4}{b^5(a+bx)^2}-\frac{10a^3}{b^5(a+bx)}\right)dx,x,x^2\right)$$
$$= \frac{3a^2x^2}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^6}{6b^3}+\frac{a^5}{4b^6\left(a+bx^2\right)^2}-\frac{5a^4}{2b^6\left(a+bx^2\right)}-\frac{5a^3\log\left(a+bx^2\right)}{b^6}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.86

$$\frac{\frac{3a^5}{\left(a+bx^2\right)^2}-\frac{30a^4}{a+bx^2}-60a^3\log\left(a+bx^2\right)+36a^2bx^2-9ab^2x^4+2b^3x^6}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^3,x]
```

```
[Out] (36*a^2*b*x^2 - 9*a*b^2*x^4 + 2*b^3*x^6 + (3*a^5)/(a + b*x^2)^2 - (30*a^4)/
(a + b*x^2) - 60*a^3*Log[a + b*x^2])/(12*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^11/(a + b*x^2)^3, x]
```

fricas [A] time = 0.80, size = 115, normalized size = 1.32

$$\frac{2\,b^5x^{10}-5\,ab^4x^8+20\,a^2b^3x^6+63\,a^3b^2x^4+6\,a^4bx^2-27\,a^5-60\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)\log\left(bx^2+a\right)}{12\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 - 5*a*b^4*x^8 + 20*a^2*b^3*x^6 + 63*a^3*b^2*x^4 + 6*a^4*b*
x^2 - 27*a^5 - 60*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^8*x^
4 + 2*a*b^7*x^2 + a^2*b^6)
```

giac [A] time = 0.63, size = 92, normalized size = 1.06

$$-\frac{5\,a^3\log\left(\left|bx^2+a\right|\right)}{b^6}+\frac{30\,a^3b^2x^4+50\,a^4bx^2+21\,a^5}{4\left(bx^2+a\right)^2b^6}+\frac{2\,b^6x^6-9\,ab^5x^4+36\,a^2b^4x^2}{12\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5*a^3*log(abs(b*x^2 + a))/b^6 + 1/4*(30*a^3*b^2*x^4 + 50*a^4*b*x^2 + 21*a^
5)/((b*x^2 + a)^2*b^6) + 1/12*(2*b^6*x^6 - 9*a*b^5*x^4 + 36*a^2*b^4*x^2)/b^
9
```

maple [A] time = 0.01, size = 80, normalized size = 0.92

$$\frac{x^6}{6b^3}-\frac{3a\,x^4}{4b^4}+\frac{a^5}{4\left(b\,x^2+a\right)^2b^6}+\frac{3a^2x^2}{b^5}-\frac{5a^4}{2\left(b\,x^2+a\right)b^6}-\frac{5a^3\ln\left(b\,x^2+a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^3,x)
```

```
[Out] 3*a^2*x^2/b^5-3/4*a*x^4/b^4+1/6*x^6/b^3+1/4*a^5/b^6/(b*x^2+a)^2-5/2*a^4/b^6
/(b*x^2+a)-5*a^3*ln(b*x^2+a)/b^6
```

maxima [A] time = 1.34, size = 89, normalized size = 1.02

$$-\frac{10\,a^4bx^2+9\,a^5}{4\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)}-\frac{5\,a^3\log\left(bx^2+a\right)}{b^6}+\frac{2\,b^2x^6-9\,abx^4+36\,a^2x^2}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(10*a^4*b*x^2 + 9*a^5)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6) - 5*a^3*log(b
*x^2 + a)/b^6 + 1/12*(2*b^2*x^6 - 9*a*b*x^4 + 36*a^2*x^2)/b^5
```

mupad [B] time = 4.49, size = 90, normalized size = 1.03

$$\frac{x^6}{6\,b^3}-\frac{\frac{9\,a^5}{4\,b}+\frac{5\,a^4\,x^2}{2}}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}-\frac{3\,a\,x^4}{4\,b^4}-\frac{5\,a^3\,\ln\left(b\,x^2+a\right)}{b^6}+\frac{3\,a^2\,x^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2)^3,x)
```

```
[Out] x^6/(6*b^3) - ((9*a^5)/(4*b) + (5*a^4*x^2)/2)/(a^2*b^5 + b^7*x^4 + 2*a*b^6*
x^2) - (3*a*x^4)/(4*b^4) - (5*a^3*log(a + b*x^2))/b^6 + (3*a^2*x^2)/b^5
```

sympy [A] time = 0.44, size = 92, normalized size = 1.06

$$-\frac{5a^3\log\left(a+bx^2\right)}{b^6}+\frac{3a^2x^2}{b^5}-\frac{3ax^4}{4b^4}+\frac{-9a^5-10a^4bx^2}{4a^2b^6+8ab^7x^2+4b^8x^4}+\frac{x^6}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**3,x)
```

```
[Out] -5*a**3*log(a + b*x**2)/b**6 + 3*a**2*x**2/b**5 - 3*a*x**4/(4*b**4) + (-9*a
**5 - 10*a**4*b*x**2)/(4*a**2*b**6 + 8*a*b**7*x**2 + 4*b**8*x**4) + x**6/(6
*b**3)
```

3.171 $\int \frac{x^9}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=74

$$-\frac{a^4}{4b^5\left(a+bx^2\right)^2}+\frac{2a^3}{b^5\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^4}{4b^5\left(a+bx^2\right)^2}+\frac{2a^3}{b^5\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*x^2)/(2*b^4) + x^4/(4*b^3) - a^4/(4*b^5*(a + b*x^2)^2) + (2*a^3)/(b^5
*(a + b*x^2)) + (3*a^2*Log[a + b*x^2])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{(a+bx^2)^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^4}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(-\frac{3a}{b^4} + \frac{x}{b^3} + \frac{a^4}{b^4(a+bx)^3} - \frac{4a^3}{b^4(a+bx)^2} + \frac{6a^2}{b^4(a+bx)}\right) dx, x, x^2\right)$$
$$= -\frac{3ax^2}{2b^4} + \frac{x^4}{4b^3} - \frac{a^4}{4b^5(a+bx^2)^2} + \frac{2a^3}{b^5(a+bx^2)} + \frac{3a^2\log(a+bx^2)}{b^5}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.85

$$\frac{-\frac{a^4}{(a+bx^2)^2} + \frac{8a^3}{a+bx^2} + 12a^2\log(a+bx^2) - 6abx^2 + b^2x^4}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^3,x]
```

```
[Out] (-6*a*b*x^2 + b^2*x^4 - a^4/(a + b*x^2)^2 + (8*a^3)/(a + b*x^2) + 12*a^2*Lo
g[a + b*x^2])/(4*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a+bx^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^9/(a + b*x^2)^3, x]
```

fricas [A] time = 1.04, size = 103, normalized size = 1.39

$$\frac{b^4x^8 - 4\,ab^3x^6 - 11\,a^2b^2x^4 + 2\,a^3bx^2 + 7\,a^4 + 12\left(a^2b^2x^4 + 2\,a^3bx^2 + a^4\right)\log\left(bx^2 + a\right)}{4\left(b^7x^4 + 2\,ab^6x^2 + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^8 - 4*a*b^3*x^6 - 11*a^2*b^2*x^4 + 2*a^3*b*x^2 + 7*a^4 + 12*(a^2
*b^2*x^4 + 2*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*
b^5)
```

giac [A] time = 0.61, size = 80, normalized size = 1.08

$$\frac{3\,a^2\log\left(\left|bx^2+a\right|\right)}{b^5}+\frac{b^3x^4-6\,ab^2x^2}{4\,b^6}-\frac{18\,a^2b^2x^4+28\,a^3bx^2+11\,a^4}{4\,\left(bx^2+a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x^2 + a))/b^5 + 1/4*(b^3*x^4 - 6*a*b^2*x^2)/b^6 - 1/4*(18*a
^2*b^2*x^4 + 28*a^3*b*x^2 + 11*a^4)/((b*x^2 + a)^2*b^5)
```

maple [A] time = 0.01, size = 69, normalized size = 0.93

$$\frac{x^4}{4b^3}-\frac{a^4}{4\left(b\,x^2+a\right)^2b^5}-\frac{3a\,x^2}{2b^4}+\frac{2a^3}{\left(b\,x^2+a\right)b^5}+\frac{3a^2\ln\left(b\,x^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^3,x)
```

```
[Out] -3/2*a*x^2/b^4+1/4*x^4/b^3-1/4*a^4/b^5/(b*x^2+a)^2+2*a^3/b^5/(b*x^2+a)+3*a^
2*ln(b*x^2+a)/b^5
```

maxima [A] time = 1.34, size = 77, normalized size = 1.04

$$\frac{8\,a^3bx^2+7\,a^4}{4\,\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}+\frac{3\,a^2\log\left(bx^2+a\right)}{b^5}+\frac{bx^4-6\,ax^2}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^3*b*x^2 + 7*a^4)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) + 3*a^2*log(b*x
^2 + a)/b^5 + 1/4*(b*x^4 - 6*a*x^2)/b^4
```

mupad [B] time = 0.08, size = 78, normalized size = 1.05

$$\frac{\frac{7\,a^4}{4\,b}+2\,a^3\,x^2}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+\frac{x^4}{4\,b^3}-\frac{3\,a\,x^2}{2\,b^4}+\frac{3\,a^2\,\ln\left(b\,x^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^3,x)
```

```
[Out] ((7*a^4)/(4*b) + 2*a^3*x^2)/(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + x^4/(4*b^3)
 - (3*a*x^2)/(2*b^4) + (3*a^2*log(a + b*x^2))/b^5
```

sympy [A] time = 0.42, size = 78, normalized size = 1.05

$$\frac{3a^2\log\left(a+bx^2\right)}{b^5}-\frac{3ax^2}{2b^4}+\frac{7a^4+8a^3bx^2}{4a^2b^5+8ab^6x^2+4b^7x^4}+\frac{x^4}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**3,x)
```

```
[Out] 3*a**2*log(a + b*x**2)/b**5 - 3*a*x**2/(2*b**4) + (7*a**4 + 8*a**3*b*x**2)/
(4*a**2*b**5 + 8*a*b**6*x**2 + 4*b**7*x**4) + x**4/(4*b**3)
```

3.172 $\int \frac{x^7}{(a+bx^2)^3} dx$

Optimal. Leaf size=65

$$\frac{a^3}{4b^4(a+bx^2)^2} - \frac{3a^2}{2b^4(a+bx^2)} - \frac{3a\log(a+bx^2)}{2b^4} + \frac{x^2}{2b^3}$$

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^3}{4b^4(a+bx^2)^2} - \frac{3a^2}{2b^4(a+bx^2)} - \frac{3a\log(a+bx^2)}{2b^4} + \frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^3,x]
```

```
[Out] x^2/(2*b^3) + a^3/(4*b^4*(a + b*x^2)^2) - (3*a^2)/(2*b^4*(a + b*x^2)) - (3*
a*Log[a + b*x^2])/(2*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^3}-\frac{a^3}{b^3(a+bx)^3}+\frac{3a^2}{b^3(a+bx)^2}-\frac{3a}{b^3(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{x^2}{2b^3}+\frac{a^3}{4b^4\left(a+bx^2\right)^2}-\frac{3a^2}{2b^4\left(a+bx^2\right)}-\frac{3a\log\left(a+bx^2\right)}{2b^4}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 0.74

$$-\frac{\frac{a^2\left(5a+6bx^2\right)}{\left(a+bx^2\right)^2}+6a\log\left(a+bx^2\right)-2bx^2}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^3,x]
[Out] -1/4*(-2*b*x^2 + (a^2*(5*a + 6*b*x^2))/(a + b*x^2)^2 + 6*a*Log[a + b*x^2])/
b^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[x^7/(a + b*x^2)^3, x]
```

fricas [A] time = 0.84, size = 91, normalized size = 1.40

$$\frac{2\,b^3x^6+4\,ab^2x^4-4\,a^2bx^2-5\,a^3-6\left(ab^2x^4+2\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*b
*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

giac [A] time = 0.66, size = 62, normalized size = 0.95

$$\frac{x^2}{2\,b^3} - \frac{3\,a\log\left(\left|bx^2+a\right|\right)}{2\,b^4} + \frac{9\,ab^2x^4 + 12\,a^2bx^2 + 4\,a^3}{4\left(bx^2+a\right)^2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^3 - 3/2*a*log(abs(b*x^2 + a))/b^4 + 1/4*(9*a*b^2*x^4 + 12*a^2*b*x
^2 + 4*a^3)/((b*x^2 + a)^2*b^4)
```

maple [A] time = 0.01, size = 58, normalized size = 0.89

$$\frac{a^3}{4\left(b\,x^2+a\right)^2 b^4} + \frac{x^2}{2b^3} - \frac{3a^2}{2\left(b\,x^2+a\right)b^4} - \frac{3a\ln\left(b\,x^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^3,x)
```

```
[Out] 1/2*x^2/b^3+1/4*a^3/b^4/(b*x^2+a)^2-3/2*a^2/b^4/(b*x^2+a)-3/2*a*ln(b*x^2+a)
/b^4
```

maxima [A] time = 1.28, size = 66, normalized size = 1.02

$$-\frac{6\,a^2bx^2 + 5\,a^3}{4\left(b^6x^4 + 2\,ab^5x^2 + a^2b^4\right)} + \frac{x^2}{2\,b^3} - \frac{3\,a\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*a^2*b*x^2 + 5*a^3)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*x^2/b^3
- 3/2*a*log(b*x^2 + a)/b^4
```

mupad [B] time = 4.75, size = 68, normalized size = 1.05

$$\frac{x^2}{2\,b^3} - \frac{\frac{5\,a^3}{4\,b} + \frac{3\,a^2\,x^2}{2}}{a^2\,b^3 + 2\,a\,b^4\,x^2 + b^5\,x^4} - \frac{3\,a\,\ln\left(b\,x^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^3,x)
```

```
[Out] x^2/(2*b^3) - ((5*a^3)/(4*b) + (3*a^2*x^2)/2)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*
x^2) - (3*a*log(a + b*x^2))/(2*b^4)
```

sympy [A] time = 0.38, size = 68, normalized size = 1.05

$$-\frac{3a\log\left(a+bx^2\right)}{2b^4}+\frac{-5a^3-6a^2bx^2}{4a^2b^4+8ab^5x^2+4b^6x^4}+\frac{x^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**3,x)
```

```
[Out] -3*a*log(a + b*x**2)/(2*b**4) + (-5*a**3 - 6*a**2*b*x**2)/(4*a**2*b**4 + 8*
a*b**5*x**2 + 4*b**6*x**4) + x**2/(2*b**3)
```

3.173 $\int \frac{x^5}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=49

$$-\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}$$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{4b^3\left(a+bx^2\right)^2}+\frac{a}{b^3\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^3,x]
```

```
[Out] -a^2/(4*b^3*(a + b*x^2)^2) + a/(b^3*(a + b*x^2)) + Log[a + b*x^2]/(2*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^3} - \frac{2a}{b^2(a+bx)^2} + \frac{1}{b^2(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{4b^3\left(a+bx^2\right)^2} + \frac{a}{b^3\left(a+bx^2\right)} + \frac{\log\left(a+bx^2\right)}{2b^3}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.80

$$\frac{\frac{a\left(3a+4bx^2\right)}{\left(a+bx^2\right)^2} + 2\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^3,x]
[Out] ((a*(3*a + 4*b*x^2))/(a + b*x^2)^2 + 2*Log[a + b*x^2])/(4*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[x^5/(a + b*x^2)^3, x]
```

fricas [A] time = 0.73, size = 69, normalized size = 1.41

$$\frac{4\,abx^2 + 3\,a^2 + 2\left(b^2x^4 + 2\,abx^2 + a^2\right)\log\left(bx^2 + a\right)}{4\left(b^5x^4 + 2\,ab^4x^2 + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^5
*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [A] time = 0.62, size = 42, normalized size = 0.86

$$\frac{\log\left(\left|bx^2 + a\right|\right)}{2\,b^3} - \frac{3\,bx^4 + 2\,ax^2}{4\left(bx^2 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))/b^3 - 1/4*(3*b*x^4 + 2*a*x^2)/((b*x^2 + a)^2*b^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{a^2}{4\left(b\,x^2 + a\right)^2 b^3} + \frac{a}{\left(b\,x^2 + a\right) b^3} + \frac{\ln\left(b\,x^2 + a\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^3,x)
[Out] -1/4*a^2/b^3/(b*x^2+a)^2+a/b^3/(b*x^2+a)+1/2*ln(b*x^2+a)/b^3
```

maxima [A] time = 1.30, size = 55, normalized size = 1.12

$$\frac{4\,abx^2 + 3\,a^2}{4\left(b^5x^4 + 2\,ab^4x^2 + a^2b^3\right)} + \frac{\log\left(bx^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/4*(4*a*b*x^2 + 3*a^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + 1/2*log(b*x^2 +
 a)/b^3
```

mupad [B] time = 0.06, size = 52, normalized size = 1.06

$$\frac{\frac{3\,a^2}{4\,b^3} + \frac{a\,x^2}{b^2}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4} + \frac{\ln\left(b\,x^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^3,x)
[Out] ((3*a^2)/(4*b^3) + (a*x^2)/b^2)/(a^2 + b^2*x^4 + 2*a*b*x^2) + log(a + b*x^2
)/(2*b^3)
```

sympy [A] time = 0.32, size = 53, normalized size = 1.08

$$\frac{3a^2+4abx^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{\log\left(a+bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*b*x**2)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + log(a +
 b*x**2)/(2*b**3)
```

3.174 $\int \frac{x^3}{(a+bx^2)^3} dx$

Optimal. Leaf size=19

$$\frac{x^4}{4a(a+bx^2)^2}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{x^4}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^3,x]
```

```
[Out] x^4/(4*a*(a + b*x^2)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2)^3} dx = \frac{x^4}{4a(a+bx^2)^2}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{a+2bx^2}{4b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(a + 2*b*x^2)/(b^2*(a + b*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[x^3/(a + b*x^2)^3, x]
```

fricas [B] time = 0.85, size = 36, normalized size = 1.89

$$-\frac{2\,bx^2+a}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="fricas")
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.62, size = 22, normalized size = 1.16

$$-\frac{2\,bx^2+a}{4\left(bx^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="giac")
[Out] -1/4*(2*b*x^2 + a)/((b*x^2 + a)^2*b^2)
```

maple [A] time = 0.01, size = 31, normalized size = 1.63

$$\frac{a}{4\left(b\,x^2+a\right)^2b^2}-\frac{1}{2\left(b\,x^2+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^3,x)
[Out] -1/2/b^2/(b*x^2+a)+1/4*a/b^2/(b*x^2+a)^2
```

maxima [**B**] time = 1.35, size = 36, normalized size = 1.89

$$-\frac{2\,bx^2+a}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

mupad [**B**] time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{a}{4\,b^2}+\frac{x^2}{2\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^3,x)
```

```
[Out] -(a/(4*b^2) + x^2/(2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

sympy [**B**] time = 0.26, size = 36, normalized size = 1.89

$$\frac{-a-2bx^2}{4a^2b^2+8ab^3x^2+4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**3,x)
```

```
[Out] (-a - 2*b*x**2)/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x**4)
```

3.175 $\int \frac{x}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{4b\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^3,x]
```

```
[Out] -1/(4*b*(a + b*x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^3} \, dx = -\frac{1}{4b\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^3,x]
```

```
[Out] -1/4*1/(b*(a + b*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x^2)^3, x]
```

fricas [A] time = 0.69, size = 26, normalized size = 1.62

$$-\frac{1}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

giac [A] time = 0.63, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(bx^2+a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/4/((b*x^2 + a)^2*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{4\left(b\,x^2+a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^3,x)
```

```
[Out] -1/4/b/(b*x^2+a)^2
```

maxima [A] time = 1.33, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(bx^2+a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4/((b*x^2 + a)^2*b)
```

mupad [B] time = 4.62, size = 28, normalized size = 1.75

$$-\frac{1}{4\,a^2\,b+8\,a\,b^2\,x^2+4\,b^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^3,x)
```

```
[Out] -1/(4*a^2*b + 4*b^3*x^4 + 8*a*b^2*x^2)
```

sympy [A] time = 0.24, size = 27, normalized size = 1.69

$$-\frac{1}{4a^2b+8ab^2x^2+4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**3,x)
```

```
[Out] -1/(4*a**2*b + 8*a*b**2*x**2 + 4*b**3*x**4)
```

3.176 $\int \frac{1}{x(a+bx^2)^3} \, dx$

Optimal. Leaf size=54

$$-\frac{\log\left(a+bx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{1}{2a^2\left(a+bx^2\right)}+\frac{1}{4a\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2a^2\left(a+bx^2\right)}-\frac{\log\left(a+bx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{1}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^3),x]
```

```
[Out] 1/(4*a*(a + b*x^2)^2) + 1/(2*a^2*(a + b*x^2)) + Log[x]/a^3 - Log[a + b*x^2]
/(2*a^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right) dx, x, x^2\right)$$
$$= \frac{1}{4a\left(a+bx^2\right)^2} + \frac{1}{2a^2\left(a+bx^2\right)} + \frac{\log(x)}{a^3} - \frac{\log\left(a+bx^2\right)}{2a^3}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.80

$$\frac{\frac{a\left(3a+2bx^2\right)}{\left(a+bx^2\right)^2} - 2\log\left(a+bx^2\right) + 4\log(x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^3),x]
[Out] ((a*(3*a + 2*b*x^2))/(a + b*x^2)^2 + 4*Log[x] - 2*Log[a + b*x^2])/(4*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^3),x]
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2)^3), x]
```

fricas [A] time = 0.88, size = 90, normalized size = 1.67

$$\frac{2\,abx^2 + 3\,a^2 - 2\left(b^2x^4 + 2\,abx^2 + a^2\right)\log\left(bx^2+a\right) + 4\left(b^2x^4 + 2\,abx^2 + a^2\right)\log(x)}{4\left(a^3b^2x^4 + 2\,a^4bx^2 + a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a) + 4*(
b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)
```

giac [A] time = 0.62, size = 59, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,a^3}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^3}+\frac{3\,b^2x^4+8\,abx^2+6\,a^2}{4\left(bx^2+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^3 - 1/2*log(abs(b*x^2 + a))/a^3 + 1/4*(3*b^2*x^4 + 8*a*b*x^2
 + 6*a^2)/((b*x^2 + a)^2*a^3)
```

maple [A] time = 0.01, size = 49, normalized size = 0.91

$$\frac{1}{4\left(b\,x^2+a\right)^2a}+\frac{1}{2\left(b\,x^2+a\right)a^2}+\frac{\ln(x)}{a^3}-\frac{\ln\left(b\,x^2+a\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^3,x)
```

```
[Out] 1/4/a/(b*x^2+a)^2+1/2/a^2/(b*x^2+a)+ln(x)/a^3-1/2*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.32, size = 60, normalized size = 1.11

$$\frac{2\,bx^2+3\,a}{4\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^3}+\frac{\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 + a)/
a^3 + 1/2*log(x^2)/a^3
```

mupad [B] time = 4.68, size = 56, normalized size = 1.04

$$\frac{\ln(x)}{a^3}+\frac{\frac{3}{4\,a}+\frac{b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^3),x)
```

```
[Out] log(x)/a^3 + (3/(4*a) + (b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - log(
a + b*x^2)/(2*a^3)
```

sympy [**A**] time = 0.41, size = 56, normalized size = 1.04

$$\frac{3a + 2bx^2}{4a^4 + 8a^3bx^2 + 4a^2b^2x^4} + \frac{\log(x)}{a^3} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**3,x)
```

```
[Out] (3*a + 2*b*x**2)/(4*a**4 + 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + log(x)/a**3
- log(a/b + x**2)/(2*a**3)
```

3.177 $\int \frac{1}{x^3(a+bx^2)^3} dx$

Optimal. Leaf size=67

$$\frac{3b\log(a+bx^2)}{2a^4} - \frac{3b\log(x)}{a^4} - \frac{b}{a^3(a+bx^2)} - \frac{1}{2a^3x^2} - \frac{b}{4a^2(a+bx^2)^2}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{b}{a^3(a+bx^2)} - \frac{b}{4a^2(a+bx^2)^2} + \frac{3b\log(a+bx^2)}{2a^4} - \frac{3b\log(x)}{a^4} - \frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/(2*a^3*x^2) - b/(4*a^2*(a + b*x^2)^2) - b/(a^3*(a + b*x^2)) - (3*b*Log[x
])/a^4 + (3*b*Log[a + b*x^2])/(2*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^3x^2} - \frac{3b}{a^4x} + \frac{b^2}{a^2(a+bx)^3} + \frac{2b^2}{a^3(a+bx)^2} + \frac{3b^2}{a^4(a+bx)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{2a^3x^2} - \frac{b}{4a^2\left(a+bx^2\right)^2} - \frac{b}{a^3\left(a+bx^2\right)} - \frac{3b\log(x)}{a^4} + \frac{3b\log\left(a+bx^2\right)}{2a^4}$$

Mathematica [**A**] time = 0.06, size = 59, normalized size = 0.88

$$-\frac{\frac{a\left(2a^2+9abx^2+6b^2x^4\right)}{x^2\left(a+bx^2\right)^2} - 6b\log\left(a+bx^2\right) + 12b\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/4*((a*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4))/(x^2*(a + b*x^2)^2) + 12*b*Log[x]
 - 6*b*Log[a + b*x^2])/a^4
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^3), x]
```

fricas [**A**] time = 0.79, size = 119, normalized size = 1.78

$$-\frac{6\,ab^2x^4 + 9\,a^2bx^2 + 2\,a^3 - 6\left(b^3x^6 + 2\,ab^2x^4 + a^2bx^2\right)\log\left(bx^2+a\right) + 12\left(b^3x^6 + 2\,ab^2x^4 + a^2bx^2\right)\log(x)}{4\left(a^4b^2x^6 + 2\,a^5bx^4 + a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

[Out] -1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)

giac [A] time = 0.62, size = 82, normalized size = 1.22

$$-\frac{3\,b\log\left(x^2\right)}{2\,a^4}+\frac{3\,b\log\left(\left|bx^2+a\right|\right)}{2\,a^4}-\frac{9\,b^3x^4+22\,ab^2x^2+14\,a^2b}{4\left(bx^2+a\right)^2a^4}+\frac{3\,bx^2-a}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="giac")

[Out] -3/2*b*log(x^2)/a^4 + 3/2*b*log(abs(b*x^2 + a))/a^4 - 1/4*(9*b^3*x^4 + 22*a*b^2*x^2 + 14*a^2*b)/((b*x^2 + a)^2*a^4) + 1/2*(3*b*x^2 - a)/(a^4*x^2)

maple [A] time = 0.01, size = 62, normalized size = 0.93

$$-\frac{b}{4\left(b\,x^2+a\right)^2a^2}-\frac{b}{\left(b\,x^2+a\right)a^3}-\frac{3b\ln(x)}{a^4}+\frac{3b\ln\left(b\,x^2+a\right)}{2a^4}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x^2+a)^3,x)

[Out] -1/2/a^3/x^2-1/4*b/a^2/(b*x^2+a)^2-b/a^3/(b*x^2+a)-3*b*ln(x)/a^4+3/2*b*ln(b*x^2+a)/a^4

maxima [A] time = 1.34, size = 77, normalized size = 1.15

$$-\frac{6\,b^2x^4+9\,abx^2+2\,a^2}{4\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)}+\frac{3\,b\log\left(bx^2+a\right)}{2\,a^4}-\frac{3\,b\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x^2+a)^3,x, algorithm="maxima")

[Out] -1/4*(6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2) + 3/2*b*log(b*x^2 + a)/a^4 - 3/2*b*log(x^2)/a^4

mupad [B] time = 0.08, size = 75, normalized size = 1.12

$$\frac{3\,b\,\ln\left(b\,x^2+a\right)}{2\,a^4}-\frac{\frac{1}{2\,a}+\frac{9\,b\,x^2}{4\,a^2}+\frac{3\,b^2\,x^4}{2\,a^3}}{a^2\,x^2+2\,a\,b\,x^4+b^2\,x^6}-\frac{3\,b\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^3),x)
```

```
[Out] (3*b*log(a + b*x^2))/(2*a^4) - (1/(2*a) + (9*b*x^2)/(4*a^2) + (3*b^2*x^4)/(
2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4) - (3*b*log(x))/a^4
```

sympy [A] time = 0.51, size = 80, normalized size = 1.19

$$\frac{-2a^2 - 9abx^2 - 6b^2x^4}{4a^5x^2 + 8a^4bx^4 + 4a^3b^2x^6} - \frac{3b\log(x)}{a^4} + \frac{3b\log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**3,x)
```

```
[Out] (-2*a**2 - 9*a*b*x**2 - 6*b**2*x**4)/(4*a**5*x**2 + 8*a**4*b*x**4 + 4*a**3*
b**2*x**6) - 3*b*log(x)/a**4 + 3*b*log(a/b + x**2)/(2*a**4)
```

3.178 $\int \frac{1}{x^5(a+bx^2)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{3b^2\log\left(a+bx^2\right)}{a^5}+\frac{6b^2\log(x)}{a^5}+\frac{3b^2}{2a^4\left(a+bx^2\right)}+\frac{3b}{2a^4x^2}+\frac{b^2}{4a^3\left(a+bx^2\right)^2}-\frac{1}{4a^3x^4}$$

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{3b^2}{2a^4\left(a+bx^2\right)}+\frac{b^2}{4a^3\left(a+bx^2\right)^2}-\frac{3b^2\log\left(a+bx^2\right)}{a^5}+\frac{6b^2\log(x)}{a^5}+\frac{3b}{2a^4x^2}-\frac{1}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^3),x]
```

```
[Out] -1/(4*a^3*x^4) + (3*b)/(2*a^4*x^2) + b^2/(4*a^3*(a + b*x^2)^2) + (3*b^2)/(2
*a^4*(a + b*x^2)) + (6*b^2*Log[x])/a^5 - (3*b^2*Log[a + b*x^2])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^3x^3} - \frac{3b}{a^4x^2} + \frac{6b^2}{a^5x} - \frac{b^3}{a^3(a+bx)^3} - \frac{3b^3}{a^4(a+bx)^2} - \frac{6b^3}{a^5(a+bx)}\right) dx, x, x^2\right)$$
$$= -\frac{1}{4a^3x^4} + \frac{3b}{2a^4x^2} + \frac{b^2}{4a^3\left(a+bx^2\right)^2} + \frac{3b^2}{2a^4\left(a+bx^2\right)} + \frac{6b^2\log(x)}{a^5} - \frac{3b^2\log\left(a+bx^2\right)}{a^5}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.86

$$\frac{\frac{a\left(-a^3+4a^2bx^2+18ab^2x^4+12b^3x^6\right)}{x^4\left(a+bx^2\right)^2} - 12b^2\log\left(a+bx^2\right) + 24b^2\log(x)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^3),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*x^2 + 18*a*b^2*x^4 + 12*b^3*x^6))/(x^4*(a + b*x^2)^2) +
 24*b^2*Log[x] - 12*b^2*Log[a + b*x^2])/(4*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^3), x]
```

fricas [A] time = 0.55, size = 134, normalized size = 1.56

$$\frac{12\,ab^3x^6 + 18\,a^2b^2x^4 + 4\,a^3bx^2 - a^4 - 12\left(b^4x^8 + 2\,ab^3x^6 + a^2b^2x^4\right)\log\left(bx^2+a\right) + 24\left(b^4x^8 + 2\,ab^3x^6 + a^2b^2x^4\right)\log(x)}{4\left(a^5b^2x^8 + 2\,a^6bx^6 + a^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

[Out] 1/4*(12*a*b^3*x^6 + 18*a^2*b^2*x^4 + 4*a^3*b*x^2 - a^4 - 12*(b^4*x^8 + 2*a*b^3*x^6 + a^2*b^2*x^4)*log(b*x^2 + a) + 24*(b^4*x^8 + 2*a*b^3*x^6 + a^2*b^2*x^4)*log(x))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)

giac [A] time = 0.65, size = 80, normalized size = 0.93

$$\frac{3\,b^2\log\left(x^2\right)}{a^5}-\frac{3\,b^2\log\left(\left|bx^2+a\right|\right)}{a^5}+\frac{12\,b^3x^6+18\,ab^2x^4+4\,a^2bx^2-a^3}{4\left(bx^4+ax^2\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="giac")

[Out] 3*b^2*log(x^2)/a^5 - 3*b^2*log(abs(b*x^2 + a))/a^5 + 1/4*(12*b^3*x^6 + 18*a*b^2*x^4 + 4*a^2*b*x^2 - a^3)/((b*x^4 + a*x^2)^2*a^4)

maple [A] time = 0.02, size = 79, normalized size = 0.92

$$\frac{b^2}{4\left(b\,x^2+a\right)^2a^3}+\frac{3b^2}{2\left(b\,x^2+a\right)a^4}+\frac{6b^2\ln(x)}{a^5}-\frac{3b^2\ln\left(b\,x^2+a\right)}{a^5}+\frac{3b}{2a^4x^2}-\frac{1}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b*x^2+a)^3,x)

[Out] -1/4/a^3/x^4+3/2*b/a^4/x^2+1/4*b^2/a^3/(b*x^2+a)^2+3/2*b^2/a^4/(b*x^2+a)+6*b^2*ln(x)/a^5-3*b^2*ln(b*x^2+a)/a^5

maxima [A] time = 1.42, size = 92, normalized size = 1.07

$$\frac{12\,b^3x^6+18\,ab^2x^4+4\,a^2bx^2-a^3}{4\left(a^4b^2x^8+2\,a^5bx^6+a^6x^4\right)}-\frac{3\,b^2\log\left(bx^2+a\right)}{a^5}+\frac{3\,b^2\log\left(x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/4*(12*b^3*x^6 + 18*a*b^2*x^4 + 4*a^2*b*x^2 - a^3)/(a^4*b^2*x^8 + 2*a^5*b*x^6 + a^6*x^4) - 3*b^2*log(b*x^2 + a)/a^5 + 3*b^2*log(x^2)/a^5

mupad [B] time = 4.67, size = 88, normalized size = 1.02

$$\frac{\frac{b\,x^2}{a^2}-\frac{1}{4\,a}+\frac{9\,b^2\,x^4}{2\,a^3}+\frac{3\,b^3\,x^6}{a^4}}{a^2\,x^4+2\,a\,b\,x^6+b^2\,x^8}-\frac{3\,b^2\,\ln\left(b\,x^2+a\right)}{a^5}+\frac{6\,b^2\,\ln(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^3),x)
```

```
[Out] ((b*x^2)/a^2 - 1/(4*a) + (9*b^2*x^4)/(2*a^3) + (3*b^3*x^6)/a^4)/(a^2*x^4 +
b^2*x^8 + 2*a*b*x^6) - (3*b^2*log(a + b*x^2))/a^5 + (6*b^2*log(x))/a^5
```

sympy [A] time = 0.55, size = 90, normalized size = 1.05

$$\frac{-a^3 + 4a^2bx^2 + 18ab^2x^4 + 12b^3x^6}{4a^6x^4 + 8a^5bx^6 + 4a^4b^2x^8} + \frac{6b^2\log(x)}{a^5} - \frac{3b^2\log\left(\frac{a}{b} + x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**3,x)
```

```
[Out] (-a**3 + 4*a**2*b*x**2 + 18*a*b**2*x**4 + 12*b**3*x**6)/(4*a**6*x**4 + 8*a*
*5*b*x**6 + 4*a**4*b**2*x**8) + 6*b**2*log(x)/a**5 - 3*b**2*log(a/b + x**2)
/a**5
```

3.179 $\int \frac{1}{x^7 \left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=95

$$\frac{5b^3 \log \left(a+bx^2\right)}{a^6}-\frac{10b^3 \log (x)}{a^6}-\frac{2b^3}{a^5 \left(a+bx^2\right)}-\frac{3b^2}{a^5x^2}-\frac{b^3}{4a^4 \left(a+bx^2\right)^2}+\frac{3b}{4a^4x^4}-\frac{1}{6a^3x^6}$$

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{2b^3}{a^5 \left(a+bx^2\right)}-\frac{b^3}{4a^4 \left(a+bx^2\right)^2}-\frac{3b^2}{a^5x^2}+\frac{5b^3 \log \left(a+bx^2\right)}{a^6}-\frac{10b^3 \log (x)}{a^6}+\frac{3b}{4a^4x^4}-\frac{1}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -1/(6*a^3*x^6) + (3*b)/(4*a^4*x^4) - (3*b^2)/(a^5*x^2) - b^3/(4*a^4*(a + b*
x^2)^2) - (2*b^3)/(a^5*(a + b*x^2)) - (10*b^3*Log[x])/a^6 + (5*b^3*Log[a +
b*x^2])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^3x^4} - \frac{3b}{a^4x^3} + \frac{6b^2}{a^5x^2} - \frac{10b^3}{a^6x} + \frac{b^4}{a^4(a+bx)^3} + \frac{4b^4}{a^5(a+bx)^2} + \frac{10b^4}{a^6(a+bx)}\right) dx, x$$
$$= -\frac{1}{6a^3x^6} + \frac{3b}{4a^4x^4} - \frac{3b^2}{a^5x^2} - \frac{b^3}{4a^4\left(a+bx^2\right)^2} - \frac{2b^3}{a^5\left(a+bx^2\right)} - \frac{10b^3\log(x)}{a^6} + \frac{5b^3\log\left(a+bx^2\right.}{a^6}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.89

$$-\frac{\frac{a\left(2a^4-5a^3bx^2+20a^2b^2x^4+90ab^3x^6+60b^4x^8\right)}{x^6\left(a+bx^2\right)^2} - 60b^3\log\left(a+bx^2\right) + 120b^3\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -1/12*((a*(2*a^4 - 5*a^3*b*x^2 + 20*a^2*b^2*x^4 + 90*a*b^3*x^6 + 60*b^4*x^8
))/(x^6*(a + b*x^2)^2) + 120*b^3*Log[x] - 60*b^3*Log[a + b*x^2])/a^6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(a + b*x^2)^3), x]
```

fricas [A] time = 0.73, size = 145, normalized size = 1.53

$$-\frac{60\,ab^4x^8 + 90\,a^2b^3x^6 + 20\,a^3b^2x^4 - 5\,a^4bx^2 + 2\,a^5 - 60\left(b^5x^{10} + 2\,ab^4x^8 + a^2b^3x^6\right)\log\left(bx^2+a\right) + 120\left(b^5x^{10} + 2\,ab^4x^8 + a^2b^3x^6\right)\log(x)}{12\left(a^6b^2x^{10} + 2\,a^7bx^8 + a^8x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(60*a*b^4*x^8 + 90*a^2*b^3*x^6 + 20*a^3*b^2*x^4 - 5*a^4*b*x^2 + 2*a^5
 - 60*(b^5*x^10 + 2*a*b^4*x^8 + a^2*b^3*x^6)*log(b*x^2 + a) + 120*(b^5*x^10
```

```
 + 2*a*b^4*x^8 + a^2*b^3*x^6)*log(x))/(a^6*b^2*x^10 + 2*a^7*b*x^8 + a^8*x^6
)
```

giac [A] time = 0.65, size = 110, normalized size = 1.16

$$-\frac{5\,b^3\log\left(x^2\right)}{a^6}+\frac{5\,b^3\log\left(\left|bx^2+a\right|\right)}{a^6}-\frac{30\,b^5x^4+68\,ab^4x^2+39\,a^2b^3}{4\left(bx^2+a\right)^2a^6}+\frac{110\,b^3x^6-36\,ab^2x^4+9\,a^2bx^2-2\,a^3}{12\,a^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5*b^3*log(x^2)/a^6 + 5*b^3*log(abs(b*x^2 + a))/a^6 - 1/4*(30*b^5*x^4 + 68*
a*b^4*x^2 + 39*a^2*b^3)/((b*x^2 + a)^2*a^6) + 1/12*(110*b^3*x^6 - 36*a*b^2*
x^4 + 9*a^2*b*x^2 - 2*a^3)/(a^6*x^6)
```

maple [A] time = 0.01, size = 90, normalized size = 0.95

$$-\frac{b^3}{4\left(b\,x^2+a\right)^2a^4}-\frac{2b^3}{\left(b\,x^2+a\right)a^5}-\frac{10b^3\ln(x)}{a^6}+\frac{5b^3\ln\left(b\,x^2+a\right)}{a^6}-\frac{3b^2}{a^5x^2}+\frac{3b}{4a^4x^4}-\frac{1}{6a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^3,x)
```

```
[Out] -1/6/a^3/x^6+3/4*b/a^4/x^4-3*b^2/a^5/x^2-1/4*b^3/a^4/(b*x^2+a)^2-2*b^3/a^5/
(b*x^2+a)-10*b^3*ln(x)/a^6+5*b^3*ln(b*x^2+a)/a^6
```

maxima [A] time = 1.36, size = 103, normalized size = 1.08

$$-\frac{60\,b^4x^8+90\,ab^3x^6+20\,a^2b^2x^4-5\,a^3bx^2+2\,a^4}{12\left(a^5b^2x^{10}+2\,a^6bx^8+a^7x^6\right)}+\frac{5\,b^3\log\left(bx^2+a\right)}{a^6}-\frac{5\,b^3\log\left(x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(60*b^4*x^8 + 90*a*b^3*x^6 + 20*a^2*b^2*x^4 - 5*a^3*b*x^2 + 2*a^4)/(a
^5*b^2*x^10 + 2*a^6*b*x^8 + a^7*x^6) + 5*b^3*log(b*x^2 + a)/a^6 - 5*b^3*log
(x^2)/a^6
```

mupad [B] time = 4.69, size = 101, normalized size = 1.06

$$\frac{5\,b^3\,\ln\left(b\,x^2+a\right)}{a^6}-\frac{\frac{1}{6\,a}-\frac{5\,b\,x^2}{12\,a^2}+\frac{5\,b^2\,x^4}{3\,a^3}+\frac{15\,b^3\,x^6}{2\,a^4}+\frac{5\,b^4\,x^8}{a^5}}{a^2\,x^6+2\,a\,b\,x^8+b^2\,x^{10}}-\frac{10\,b^3\,\ln(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^3),x)
```

```
[Out] (5*b^3*log(a + b*x^2))/a^6 - (1/(6*a) - (5*b*x^2)/(12*a^2) + (5*b^2*x^4)/(3
*a^3) + (15*b^3*x^6)/(2*a^4) + (5*b^4*x^8)/a^5)/(a^2*x^6 + b^2*x^10 + 2*a*b
*x^8) - (10*b^3*log(x))/a^6
```

sympy [A] time = 0.59, size = 104, normalized size = 1.09

$$\frac{-2a^4 + 5a^3bx^2 - 20a^2b^2x^4 - 90ab^3x^6 - 60b^4x^8}{12a^7x^6 + 24a^6bx^8 + 12a^5b^2x^{10}} - \frac{10b^3 \log(x)}{a^6} + \frac{5b^3 \log\left(\frac{a}{b} + x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**3,x)
```

```
[Out] (-2*a**4 + 5*a**3*b*x**2 - 20*a**2*b**2*x**4 - 90*a*b**3*x**6 - 60*b**4*x**
8)/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5*b**2*x**10) - 10*b**3*log(x)/a*
*6 + 5*b**3*log(a/b + x**2)/a**6
```

3.180 $\int \frac{1}{x^9\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{15b^4\log\left(a+bx^2\right)}{2a^7}+\frac{15b^4\log(x)}{a^7}+\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{5b^3}{a^6x^2}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}-\frac{3b^2}{2a^5x^4}+\frac{b}{2a^4x^6}-\frac{1}{8a^3x^8}$$

Rubi [A] time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{5b^4}{2a^6\left(a+bx^2\right)}+\frac{b^4}{4a^5\left(a+bx^2\right)^2}+\frac{5b^3}{a^6x^2}-\frac{3b^2}{2a^5x^4}-\frac{15b^4\log\left(a+bx^2\right)}{2a^7}+\frac{15b^4\log(x)}{a^7}+\frac{b}{2a^4x^6}-\frac{1}{8a^3x^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^2)^3),x]
```

```
[Out] -1/(8*a^3*x^8) + b/(2*a^4*x^6) - (3*b^2)/(2*a^5*x^4) + (5*b^3)/(a^6*x^2) +
b^4/(4*a^5*(a + b*x^2)^2) + (5*b^4)/(2*a^6*(a + b*x^2)) + (15*b^4*Log[x])/a
^7 - (15*b^4*Log[a + b*x^2])/(2*a^7)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^5(a+bx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^3x^5} - \frac{3b}{a^4x^4} + \frac{6b^2}{a^5x^3} - \frac{10b^3}{a^6x^2} + \frac{15b^4}{a^7x} - \frac{b^5}{a^5(a+bx)^3} - \frac{5b^5}{a^6(a+bx)^2} - \frac{15b^5}{a^7(a+bx}\right.\right.$$

$$= -\frac{1}{8a^3x^8} + \frac{b}{2a^4x^6} - \frac{3b^2}{2a^5x^4} + \frac{5b^3}{a^6x^2} + \frac{b^4}{4a^5\left(a+bx^2\right)^2} + \frac{5b^4}{2a^6\left(a+bx^2\right)} + \frac{15b^4\log(x)}{a^7} - \frac{15b^4\,\mathrm{l}}{}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 0.86

$$\frac{\frac{a\left(-a^5+2a^4bx^2-5a^3b^2x^4+20a^2b^3x^6+90ab^4x^8+60b^5x^{10}\right)}{x^8\left(a+bx^2\right)^2} - 60b^4\log\left(a+bx^2\right) + 120b^4\log(x)}{8a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^2)^3),x]
```

```
[Out] ((a*(-a^5 + 2*a^4*b*x^2 - 5*a^3*b^2*x^4 + 20*a^2*b^3*x^6 + 90*a*b^4*x^8 + 6
0*b^5*x^10))/(x^8*(a + b*x^2)^2) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2])/
(8*a^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^9*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^9*(a + b*x^2)^3), x]
```

fricas [A] time = 1.19, size = 156, normalized size = 1.39

$$\frac{60\,ab^5x^{10} + 90\,a^2b^4x^8 + 20\,a^3b^3x^6 - 5\,a^4b^2x^4 + 2\,a^5bx^2 - a^6 - 60\left(b^6x^{12} + 2\,ab^5x^{10} + a^2b^4x^8\right)\log\left(bx^2+a\right) + 120\left(b^6x^{12} + 2\,ab^5x^{10} + a^2b^4x^8\right)\log(x)}{8\left(a^7b^2x^{12} + 2\,a^8bx^{10} + a^9x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(60*a*b^5*x^10 + 90*a^2*b^4*x^8 + 20*a^3*b^3*x^6 - 5*a^4*b^2*x^4 + 2*a^
5*b*x^2 - a^6 - 60*(b^6*x^12 + 2*a*b^5*x^10 + a^2*b^4*x^8)*log(b*x^2 + a) +
 120*(b^6*x^12 + 2*a*b^5*x^10 + a^2*b^4*x^8)*log(x))/(a^7*b^2*x^12 + 2*a^8*
b*x^10 + a^9*x^8)
```

giac [A] time = 0.63, size = 119, normalized size = 1.06

$$\frac{15\,b^4\log\left(x^2\right)}{2\,a^7}-\frac{15\,b^4\log\left(\left|bx^2+a\right|\right)}{2\,a^7}+\frac{45\,b^6x^4+100\,ab^5x^2+56\,a^2b^4}{4\left(bx^2+a\right)^2a^7}-\frac{125\,b^4x^8-40\,ab^3x^6+12\,a^2b^2x^4-4\,a^3bx^2+a^4}{8\,a^7x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 15/2*b^4*log(x^2)/a^7 - 15/2*b^4*log(abs(b*x^2 + a))/a^7 + 1/4*(45*b^6*x^4
+ 100*a*b^5*x^2 + 56*a^2*b^4)/((b*x^2 + a)^2*a^7) - 1/8*(125*b^4*x^8 - 40*a
*b^3*x^6 + 12*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)/(a^7*x^8)
```

maple [A] time = 0.01, size = 101, normalized size = 0.90

$$\frac{b^4}{4\left(b\,x^2+a\right)^2a^5}+\frac{5b^4}{2\left(b\,x^2+a\right)a^6}+\frac{15b^4\ln(x)}{a^7}-\frac{15b^4\ln\left(b\,x^2+a\right)}{2a^7}+\frac{5b^3}{a^6x^2}-\frac{3b^2}{2a^5x^4}+\frac{b}{2a^4x^6}-\frac{1}{8a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^2+a)^3,x)
```

```
[Out] -1/8/a^3/x^8+1/2*b/a^4/x^6-3/2*b^2/a^5/x^4+5*b^3/a^6/x^2+1/4*b^4/a^5/(b*x^2
+a)^2+5/2*b^4/a^6/(b*x^2+a)+15*b^4*ln(x)/a^7-15/2*b^4*ln(b*x^2+a)/a^7
```

maxima [A] time = 1.34, size = 114, normalized size = 1.02

$$\frac{60\,b^5x^{10}+90\,ab^4x^8+20\,a^2b^3x^6-5\,a^3b^2x^4+2\,a^4bx^2-a^5}{8\left(a^6b^2x^{12}+2\,a^7bx^{10}+a^8x^8\right)}-\frac{15\,b^4\log\left(bx^2+a\right)}{2\,a^7}+\frac{15\,b^4\log\left(x^2\right)}{2\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(60*b^5*x^10 + 90*a*b^4*x^8 + 20*a^2*b^3*x^6 - 5*a^3*b^2*x^4 + 2*a^4*b*
x^2 - a^5)/(a^6*b^2*x^12 + 2*a^7*b*x^10 + a^8*x^8) - 15/2*b^4*log(b*x^2 + a
)/a^7 + 15/2*b^4*log(x^2)/a^7
```

mupad [B] time = 4.86, size = 111, normalized size = 0.99

$$\frac{\frac{b\,x^2}{4\,a^2}-\frac{1}{8\,a}-\frac{5\,b^2\,x^4}{8\,a^3}+\frac{5\,b^3\,x^6}{2\,a^4}+\frac{45\,b^4\,x^8}{4\,a^5}+\frac{15\,b^5\,x^{10}}{2\,a^6}}{a^2\,x^8+2\,a\,b\,x^{10}+b^2\,x^{12}}-\frac{15\,b^4\,\ln\left(b\,x^2+a\right)}{2\,a^7}+\frac{15\,b^4\,\ln(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^2)^3),x)
```

```
[Out] ((b*x^2)/(4*a^2) - 1/(8*a) - (5*b^2*x^4)/(8*a^3) + (5*b^3*x^6)/(2*a^4) + (4
5*b^4*x^8)/(4*a^5) + (15*b^5*x^10)/(2*a^6))/(a^2*x^8 + b^2*x^12 + 2*a*b*x^1
0) - (15*b^4*log(a + b*x^2))/(2*a^7) + (15*b^4*log(x))/a^7
```

sympy [A] time = 0.65, size = 116, normalized size = 1.04

$$\frac{-a^5 + 2a^4bx^2 - 5a^3b^2x^4 + 20a^2b^3x^6 + 90ab^4x^8 + 60b^5x^{10}}{8a^8x^8 + 16a^7bx^{10} + 8a^6b^2x^{12}} + \frac{15b^4 \log(x)}{a^7} - \frac{15b^4 \log\left(\frac{a}{b} + x^2\right)}{2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**2+a)**3,x)
```

```
[Out] (-a**5 + 2*a**4*b*x**2 - 5*a**3*b**2*x**4 + 20*a**2*b**3*x**6 + 90*a*b**4*x
**8 + 60*b**5*x**10)/(8*a**8*x**8 + 16*a**7*b*x**10 + 8*a**6*b**2*x**12) +
15*b**4*log(x)/a**7 - 15*b**4*log(a/b + x**2)/(2*a**7)
```

3.181 $\int \frac{x^{12}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=111

$$\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{99a^3x}{8b^6}+\frac{33a^2x^3}{8b^5}-\frac{99ax^5}{40b^4}-\frac{11x^9}{8b^2\left(a+bx^2\right)}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}+\frac{99x^7}{56b^3}$$

Rubi [A] time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{33a^2x^3}{8b^5}-\frac{99a^3x}{8b^6}+\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{11x^9}{8b^2\left(a+bx^2\right)}-\frac{99ax^5}{40b^4}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}+\frac{99x^7}{56b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^3,x]
```

```
[Out] (-99*a^3*x)/(8*b^6) + (33*a^2*x^3)/(8*b^5) - (99*a*x^5)/(40*b^4) + (99*x^7)
/(56*b^3) - x^11/(4*b*(a + b*x^2)^2) - (11*x^9)/(8*b^2*(a + b*x^2)) + (99*a
^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{12}}{\left(a+bx^2\right)^3}\,dx &= -\frac{x^{11}}{4b\left(a+bx^2\right)^2}+\frac{11\int \frac{x^{10}}{\left(a+bx^2\right)^2}\,dx}{4b}\\
&= -\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{99\int \frac{x^8}{a+bx^2}\,dx}{8b^2}\\
&= -\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{99\int \left(-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}+\frac{a^4}{b^4\left(a+bx^2\right)}\right)dx}{8b^2}\\
&= -\frac{99a^3x}{8b^6}+\frac{33a^2x^3}{8b^5}-\frac{99ax^5}{40b^4}+\frac{99x^7}{56b^3}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{\left(99a^4\right)\int \frac{1}{a+bx^2}\,dx}{8b^6}\\
&= -\frac{99a^3x}{8b^6}+\frac{33a^2x^3}{8b^5}-\frac{99ax^5}{40b^4}+\frac{99x^7}{56b^3}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}-\frac{11x^9}{8b^2\left(a+bx^2\right)}+\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.89

$$
\frac{99a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{3465a^5x+5775a^4bx^3+1848a^3b^2x^5-264a^2b^3x^7+88ab^4x^9-40b^5x^{11}}{280b^6\left(a+bx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^3,x]
```

```
[Out] -1/280*(3465*a^5*x + 5775*a^4*b*x^3 + 1848*a^3*b^2*x^5 - 264*a^2*b^3*x^7 +
88*a*b^4*x^9 - 40*b^5*x^11)/(b^6*(a + b*x^2)^2) + (99*a^(7/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(8*b^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^{12}}{\left(a+bx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^12/(a + b*x^2)^3, x]
```

fricas [A] time = 0.72, size = 278, normalized size = 2.50

$$\left[\frac{80\,b^{5}x^{11}-176\,ab^{4}x^{9}+528\,a^{2}b^{3}x^{7}-3696\,a^{3}b^{2}x^{5}-11550\,a^{4}bx^{3}-6930\,a^{5}x+3465\left(a^{3}b^{2}x^{4}+2\,a^{4}bx^{2}+a^{5}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)}{560\left(b^{8}x^{4}+2\,ab^{7}x^{2}+a^{2}b^{6}\right)},\ \frac{40\,b^{5}x^{11}-88\,ab^{4}x^{9}+264\,a^{2}b^{3}x^{7}-1848\,a^{3}b^{2}x^{5}-5775\,a^{4}bx^{3}-3465\,a^{5}x+3465\left(a^{3}b^{2}x^{4}+2\,a^{4}bx^{2}+a^{5}\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{280\left(b^{8}x^{4}+2\,ab^{7}x^{2}+a^{2}b^{6}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(80*b^5*x^11 - 176*a*b^4*x^9 + 528*a^2*b^3*x^7 - 3696*a^3*b^2*x^5 -
11550*a^4*b*x^3 - 6930*a^5*x + 3465*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*sqrt(
-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^8*x^4 + 2*a*b^7*x
^2 + a^2*b^6), 1/280*(40*b^5*x^11 - 88*a*b^4*x^9 + 264*a^2*b^3*x^7 - 1848*a
^3*b^2*x^5 - 5775*a^4*b*x^3 - 3465*a^5*x + 3465*(a^3*b^2*x^4 + 2*a^4*b*x^2
+ a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)
]
```

giac [A] time = 0.61, size = 96, normalized size = 0.86

$$\frac{99\,a^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{6}}-\frac{21\,a^{4}bx^{3}+19\,a^{5}x}{8\left(bx^{2}+a\right)^{2}b^{6}}+\frac{5\,b^{18}x^{7}-21\,ab^{17}x^{5}+70\,a^{2}b^{16}x^{3}-350\,a^{3}b^{15}x}{35\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 99/8*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/8*(21*a^4*b*x^3 + 19*a^5
*x)/((b*x^2 + a)^2*b^6) + 1/35*(5*b^18*x^7 - 21*a*b^17*x^5 + 70*a^2*b^16*x^
3 - 350*a^3*b^15*x)/b^21
```

maple [A] time = 0.01, size = 99, normalized size = 0.89

$$\frac{x^{7}}{7b^{3}}-\frac{21a^{4}x^{3}}{8\left(b\,x^{2}+a\right)^{2}b^{5}}-\frac{3a\,x^{5}}{5b^{4}}-\frac{19a^{5}x}{8\left(b\,x^{2}+a\right)^{2}b^{6}}+\frac{2a^{2}x^{3}}{b^{5}}+\frac{99a^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^{6}}-\frac{10a^{3}x}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^3,x)
```

```
[Out] 1/7*x^7/b^3-3/5*a*x^5/b^4+2*a^2*x^3/b^5-10*a^3*x/b^6-21/8/b^5*a^4/(b*x^2+a)
^2*x^3-19/8/b^6*a^5/(b*x^2+a)^2*x+99/8/b^6*a^4/(a*b)^(1/2)*arctan(1/(a*b)^(
1/2)*b*x)
```

maxima [A] time = 2.95, size = 105, normalized size = 0.95

$$-\frac{21\,a^{4}bx^{3}+19\,a^{5}x}{8\left(b^{8}x^{4}+2\,ab^{7}x^{2}+a^{2}b^{6}\right)}+\frac{99\,a^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{6}}+\frac{5\,b^{3}x^{7}-21\,ab^{2}x^{5}+70\,a^{2}bx^{3}-350\,a^{3}x}{35\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(21*a^4*b*x^3 + 19*a^5*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6) + 99/8*a^4
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/35*(5*b^3*x^7 - 21*a*b^2*x^5 + 7
0*a^2*b*x^3 - 350*a^3*x)/b^6
```

mupad [B] time = 0.08, size = 99, normalized size = 0.89

$$\frac{x^{7}}{7\,b^{3}}-\frac{\frac{19\,a^{5}\,x}{8}+\frac{21\,b\,a^{4}\,x^{3}}{8}}{a^{2}\,b^{6}+2\,a\,b^{7}\,x^{2}+b^{8}\,x^{4}}-\frac{3\,a\,x^{5}}{5\,b^{4}}-\frac{10\,a^{3}\,x}{b^{6}}+\frac{99\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{13/2}}+\frac{2\,a^{2}\,x^{3}}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^2)^3,x)
```

```
[Out] x^7/(7*b^3) - ((19*a^5*x)/8 + (21*a^4*b*x^3)/8)/(a^2*b^6 + b^8*x^4 + 2*a*b^
7*x^2) - (3*a*x^5)/(5*b^4) - (10*a^3*x)/b^6 + (99*a^(7/2)*atan((b^(1/2)*x)/
a^(1/2)))/(8*b^(13/2)) + (2*a^2*x^3)/b^5
```

sympy [A] time = 0.49, size = 162, normalized size = 1.46

$$-\frac{10a^{3}x}{b^{6}}+\frac{2a^{2}x^{3}}{b^{5}}-\frac{3ax^{5}}{5b^{4}}-\frac{99\sqrt{-\frac{a^{7}}{b^{13}}}\log\left(x-\frac{b^{6}\sqrt{-\frac{a^{7}}{b^{13}}}}{a^{3}}\right)}{16}+\frac{99\sqrt{-\frac{a^{7}}{b^{13}}}\log\left(x+\frac{b^{6}\sqrt{-\frac{a^{7}}{b^{13}}}}{a^{3}}\right)}{16}+\frac{-19a^{5}x-21a^{4}bx^{3}}{8a^{2}b^{6}+16ab^{7}x^{2}+8b^{8}x^{4}}+\frac{x^{7}}{7b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**3,x)
```

```
[Out] -10*a**3*x/b**6 + 2*a**2*x**3/b**5 - 3*a*x**5/(5*b**4) - 99*sqrt(-a**7/b**1
3)*log(x - b**6*sqrt(-a**7/b**13)/a**3)/16 + 99*sqrt(-a**7/b**13)*log(x + b
**6*sqrt(-a**7/b**13)/a**3)/16 + (-19*a**5*x - 21*a**4*b*x**3)/(8*a**2*b**6
 + 16*a*b**7*x**2 + 8*b**8*x**4) + x**7/(7*b**3)
```

3.182 $\int \frac{x^{10}}{\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=98

$$-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}+\frac{63a^2x}{8b^5}-\frac{21ax^3}{8b^4}-\frac{9x^7}{8b^2\left(a+bx^2\right)}-\frac{x^9}{4b\left(a+bx^2\right)^2}+\frac{63x^5}{40b^3}$$

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{63a^2x}{8b^5}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}-\frac{9x^7}{8b^2\left(a+bx^2\right)}-\frac{21ax^3}{8b^4}-\frac{x^9}{4b\left(a+bx^2\right)^2}+\frac{63x^5}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^3,x]
```

```
[Out] (63*a^2*x)/(8*b^5) - (21*a*x^3)/(8*b^4) + (63*x^5)/(40*b^3) - x^9/(4*b*(a +
 b*x^2)^2) - (9*x^7)/(8*b^2*(a + b*x^2)) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^9}{4b\left(a+bx^2\right)^2}+\frac{9\int \frac{x^8}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{x^9}{4b\left(a+bx^2\right)^2}-\frac{9x^7}{8b^2\left(a+bx^2\right)}+\frac{63\int \frac{x^6}{a+bx^2}\,dx}{8b^2}$$

$$= -\frac{x^9}{4b\left(a+bx^2\right)^2}-\frac{9x^7}{8b^2\left(a+bx^2\right)}+\frac{63\int \left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+bx^2\right)}\right)dx}{8b^2}$$

$$= \frac{63a^2x}{8b^5}-\frac{21ax^3}{8b^4}+\frac{63x^5}{40b^3}-\frac{x^9}{4b\left(a+bx^2\right)^2}-\frac{9x^7}{8b^2\left(a+bx^2\right)}-\frac{\left(63a^3\right)\int \frac{1}{a+bx^2}\,dx}{8b^5}$$

$$= \frac{63a^2x}{8b^5}-\frac{21ax^3}{8b^4}+\frac{63x^5}{40b^3}-\frac{x^9}{4b\left(a+bx^2\right)^2}-\frac{9x^7}{8b^2\left(a+bx^2\right)}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.90

$$\frac{315a^4x+525a^3bx^3+168a^2b^2x^5-24ab^3x^7+8b^4x^9}{40b^5\left(a+bx^2\right)^2}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^3,x]
```

```
[Out] (315*a^4*x + 525*a^3*b*x^3 + 168*a^2*b^2*x^5 - 24*a*b^3*x^7 + 8*b^4*x^9)/(4
0*b^5*(a + b*x^2)^2) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/2)
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^10/(a + b*x^2)^3, x]
```

fricas [A] time = 0.93, size = 256, normalized size = 2.61

$$\left[\frac{16\,b^4x^9-48\,ab^3x^7+336\,a^2b^2x^5+1050\,a^3bx^3+630\,a^4x+315\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{80\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)},\frac{8\,b^4x^9-24\,ab^3x^7+168\,a^2b^2x^5+525\,a^3bx^3+315\,a^4x-315\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{40\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/80*(16*b^4*x^9 - 48*a*b^3*x^7 + 336*a^2*b^2*x^5 + 1050*a^3*b*x^3 + 630*a
^4*x + 315*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 - 2*b*x*
sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5), 1/40*(8*b^
4*x^9 - 24*a*b^3*x^7 + 168*a^2*b^2*x^5 + 525*a^3*b*x^3 + 315*a^4*x - 315*(a
^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x^4
 + 2*a*b^6*x^2 + a^2*b^5)]
```

giac [A] time = 0.64, size = 84, normalized size = 0.86

$$-\frac{63\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^5}+\frac{17\,a^3bx^3+15\,a^4x}{8\,{\left(bx^2+a\right)}^2b^5}+\frac{b^{12}x^5-5\,ab^{11}x^3+30\,a^2b^{10}x}{5\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -63/8*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/8*(17*a^3*b*x^3 + 15*a^
4*x)/((b*x^2 + a)^2*b^5) + 1/5*(b^12*x^5 - 5*a*b^11*x^3 + 30*a^2*b^10*x)/b^
15
```

maple [A] time = 0.01, size = 88, normalized size = 0.90

$$\frac{17a^3x^3}{8\left(b\,x^2+a\right)^2b^4}+\frac{x^5}{5b^3}+\frac{15a^4x}{8\left(b\,x^2+a\right)^2b^5}-\frac{a\,x^3}{b^4}-\frac{63a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^5}+\frac{6a^2x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^3,x)
```

```
[Out] 1/5*x^5/b^3-a*x^3/b^4+6*a^2*x/b^5+17/8/b^4*a^3/(b*x^2+a)^2*x^3+15/8/b^5*a^4
/(b*x^2+a)^2*x-63/8/b^5*a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.86, size = 93, normalized size = 0.95

$$\frac{17\,a^3bx^3+15\,a^4x}{8\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}-\frac{63\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^5}+\frac{b^2x^5-5\,abx^3+30\,a^2x}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(17*a^3*b*x^3 + 15*a^4*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) - 63/8*a^3*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/5*(b^2*x^5 - 5*a*b*x^3 + 30*a^2*x
)/b^5
```

mupad [B] time = 0.07, size = 87, normalized size = 0.89

$$\frac{\frac{15\,a^4\,x}{8}+\frac{17\,b\,a^3\,x^3}{8}}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}+\frac{x^5}{5\,b^3}-\frac{a\,x^3}{b^4}+\frac{6\,a^2\,x}{b^5}-\frac{63\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^3,x)
```

```
[Out] ((15*a^4*x)/8 + (17*a^3*b*x^3)/8)/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) + x^5/(
5*b^3) - (a*x^3)/b^4 + (6*a^2*x)/b^5 - (63*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)
))/(8*b^(11/2))
```

sympy [A] time = 0.47, size = 144, normalized size = 1.47

$$\frac{6a^2x}{b^5}-\frac{ax^3}{b^4}+\frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16}-\frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16}+\frac{15a^4x+17a^3bx^3}{8a^2b^5+16ab^6x^2+8b^7x^4}+\frac{x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**3,x)
```

```
[Out] 6*a**2*x/b**5 - a*x**3/b**4 + 63*sqrt(-a**5/b**11)*log(x - b**5*sqrt(-a**5/
b**11)/a**2)/16 - 63*sqrt(-a**5/b**11)*log(x + b**5*sqrt(-a**5/b**11)/a**2)
/16 + (15*a**4*x + 17*a**3*b*x**3)/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*b**7*x
**4) + x**5/(5*b**3)
```

3.183 $\int \frac{x^8}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=85

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}-\frac{35ax}{8b^4}-\frac{7x^5}{8b^2\left(a+bx^2\right)}-\frac{x^7}{4b\left(a+bx^2\right)^2}+\frac{35x^3}{24b^3}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}-\frac{7x^5}{8b^2\left(a+bx^2\right)}-\frac{35ax}{8b^4}-\frac{x^7}{4b\left(a+bx^2\right)^2}+\frac{35x^3}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^3,x]
```

```
[Out] (-35*a*x)/(8*b^4) + (35*x^3)/(24*b^3) - x^7/(4*b*(a + b*x^2)^2) - (7*x^5)/(
8*b^2*(a + b*x^2)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2\right)^3}\,dx = -\frac{x^7}{4b\left(a+bx^2\right)^2}+\frac{7\int \frac{x^6}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{x^7}{4b\left(a+bx^2\right)^2}-\frac{7x^5}{8b^2\left(a+bx^2\right)}+\frac{35\int \frac{x^4}{a+bx^2}\,dx}{8b^2}$$

$$= -\frac{x^7}{4b\left(a+bx^2\right)^2}-\frac{7x^5}{8b^2\left(a+bx^2\right)}+\frac{35\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{8b^2}$$

$$= -\frac{35ax}{8b^4}+\frac{35x^3}{24b^3}-\frac{x^7}{4b\left(a+bx^2\right)^2}-\frac{7x^5}{8b^2\left(a+bx^2\right)}+\frac{\left(35a^2\right)\int \frac{1}{a+bx^2}\,dx}{8b^4}$$

$$= -\frac{35ax}{8b^4}+\frac{35x^3}{24b^3}-\frac{x^7}{4b\left(a+bx^2\right)^2}-\frac{7x^5}{8b^2\left(a+bx^2\right)}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.91

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}-\frac{105a^3x+175a^2bx^3+56ab^2x^5-8b^3x^7}{24b^4\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^3,x]
```

```
[Out] -1/24*(105*a^3*x + 175*a^2*b*x^3 + 56*a*b^2*x^5 - 8*b^3*x^7)/(b^4*(a + b*x^
2)^2) + (35*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x^2)^3, x]
```

fricas [A] time = 0.88, size = 230, normalized size = 2.71

$$\left[\frac{16\,b^{3}x^{7}-112\,ab^{2}x^{5}-350\,a^{2}bx^{3}-210\,a^{3}x+105\,\left(ab^{2}x^{4}+2\,a^{2}bx^{2}+a^{3}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)}{48\,\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)},\ \frac{8\,b^{3}x^{7}-56\,ab^{2}x^{5}-175\,a^{2}bx^{3}-105\,a^{3}x+105\,\left(ab^{2}x^{4}+2\,a^{2}bx^{2}+a^{3}\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{24\,\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*b^3*x^7 - 112*a*b^2*x^5 - 350*a^2*b*x^3 - 210*a^3*x + 105*(a*b^2*
x^4 + 2*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/24*(8*b^3*x^7 - 56*a*b^2*x^5
 - 175*a^2*b*x^3 - 105*a^3*x + 105*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

giac [A] time = 0.63, size = 73, normalized size = 0.86

$$\frac{35\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{4}}-\frac{13\,a^{2}bx^{3}+11\,a^{3}x}{8\,\left(bx^{2}+a\right)^{2}b^{4}}+\frac{b^{6}x^{3}-9\,ab^{5}x}{3\,b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 35/8*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/8*(13*a^2*b*x^3 + 11*a^3
*x)/((b*x^2 + a)^2*b^4) + 1/3*(b^6*x^3 - 9*a*b^5*x)/b^9
```

maple [A] time = 0.01, size = 77, normalized size = 0.91

$$-\frac{13a^{2}x^{3}}{8\left(b\,x^{2}+a\right)^{2}b^{3}}-\frac{11a^{3}x}{8\left(b\,x^{2}+a\right)^{2}b^{4}}+\frac{x^{3}}{3b^{3}}+\frac{35a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^{4}}-\frac{3ax}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^3,x)
```

```
[Out] 1/3*x^3/b^3-3*a*x/b^4-13/8/b^3*a^2/(b*x^2+a)^2*x^3-11/8/b^4*a^3/(b*x^2+a)^2
*x+35/8/b^4*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 82, normalized size = 0.96

$$-\frac{13\,a^{2}bx^{3}+11\,a^{3}x}{8\,\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)}+\frac{35\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{4}}+\frac{bx^{3}-9\,ax}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(13*a^2*b*x^3 + 11*a^3*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 35/8*a^2
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/3*(b*x^3 - 9*a*x)/b^4
```

mupad [B] time = 4.72, size = 77, normalized size = 0.91

$$\frac{x^3}{3\,b^3}-\frac{\frac{11\,a^3\,x}{8}+\frac{13\,b\,a^2\,x^3}{8}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+\frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{9/2}}-\frac{3\,a\,x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^3,x)
```

```
[Out] x^3/(3*b^3) - ((11*a^3*x)/8 + (13*a^2*b*x^3)/8)/(a^2*b^4 + b^6*x^4 + 2*a*b^
5*x^2) + (35*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(9/2)) - (3*a*x)/b^4
```

sympy [A] time = 0.44, size = 133, normalized size = 1.56

$$-\frac{3ax}{b^4}-\frac{35\sqrt{-\frac{a^3}{b^9}}\,\log\left(x-\frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16}+\frac{35\sqrt{-\frac{a^3}{b^9}}\,\log\left(x+\frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16}+\frac{-11a^3x-13a^2bx^3}{8a^2b^4+16ab^5x^2+8b^6x^4}+\frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**3,x)
```

```
[Out] -3*a*x/b**4 - 35*sqrt(-a**3/b**9)*log(x - b**4*sqrt(-a**3/b**9)/a)/16 + 35*
sqrt(-a**3/b**9)*log(x + b**4*sqrt(-a**3/b**9)/a)/16 + (-11*a**3*x - 13*a**
2*b*x**3)/(8*a**2*b**4 + 16*a*b**5*x**2 + 8*b**6*x**4) + x**3/(3*b**3)
```

3.184 $\int \frac{x^6}{\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=74

$$-\frac{15\sqrt{a}\ \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{7/2}}-\frac{5x^3}{8b^2\left(a+bx^2\right)}-\frac{x^5}{4b\left(a+bx^2\right)^2}+\frac{15x}{8b^3}$$

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 321, 205}

$$-\frac{5x^3}{8b^2\left(a+bx^2\right)}-\frac{15\sqrt{a}\ \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{7/2}}-\frac{x^5}{4b\left(a+bx^2\right)^2}+\frac{15x}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^3,x]
```

```
[Out] (15*x)/(8*b^3) - x^5/(4*b*(a + b*x^2)^2) - (5*x^3)/(8*b^2*(a + b*x^2)) - (1
5*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^3}\,dx = -\frac{x^5}{4b\left(a+bx^2\right)^2} + \frac{5\int \frac{x^4}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{x^5}{4b\left(a+bx^2\right)^2} - \frac{5x^3}{8b^2\left(a+bx^2\right)} + \frac{15\int \frac{x^2}{a+bx^2}\,dx}{8b^2}$$

$$= \frac{15x}{8b^3} - \frac{x^5}{4b\left(a+bx^2\right)^2} - \frac{5x^3}{8b^2\left(a+bx^2\right)} - \frac{(15a)\int \frac{1}{a+bx^2}\,dx}{8b^3}$$

$$= \frac{15x}{8b^3} - \frac{x^5}{4b\left(a+bx^2\right)^2} - \frac{5x^3}{8b^2\left(a+bx^2\right)} - \frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.89

$$\frac{15a^2x+25abx^3+8b^2x^5}{8b^3\left(a+bx^2\right)^2} - \frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^3,x]
```

```
[Out] (15*a^2*x + 25*a*b*x^3 + 8*b^2*x^5)/(8*b^3*(a + b*x^2)^2) - (15*Sqrt[a]*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x^2)^3, x]
```

fricas [A] time = 1.02, size = 202, normalized size = 2.73

$$\left[\frac{16\,b^{2}x^{5}+50\,abx^{3}+30\,a^{2}x+15\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)}{16\left(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3}\right)},\frac{8\,b^{2}x^{5}+25\,abx^{3}+15\,a^{2}x-15\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{8\left(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^2*x^5 + 50*a*b*x^3 + 30*a^2*x + 15*(b^2*x^4 + 2*a*b*x^2 + a^2)*
sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*x^4 + 2*a*
b^4*x^2 + a^2*b^3), 1/8*(8*b^2*x^5 + 25*a*b*x^3 + 15*a^2*x - 15*(b^2*x^4 +
2*a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^4 + 2*a*b^4*x^2
+ a^2*b^3)]
```

giac [A] time = 0.64, size = 54, normalized size = 0.73

$$-\frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{3}}+\frac{x}{b^{3}}+\frac{9\,abx^{3}+7\,a^{2}x}{8\left(bx^{2}+a\right)^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3 + 1/8*(9*a*b*x^3 + 7*
a^2*x)/((b*x^2 + a)^2*b^3)
```

maple [A] time = 0.01, size = 63, normalized size = 0.85

$$\frac{9a\,x^{3}}{8\left(b\,x^{2}+a\right)^{2}b^{2}}+\frac{7a^{2}x}{8\left(b\,x^{2}+a\right)^{2}b^{3}}-\frac{15a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^{3}}+\frac{x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^3,x)
```

```
[Out] x/b^3+9/8/b^2*a/(b*x^2+a)^2*x^3+7/8/b^3*a^2/(b*x^2+a)^2*x-15/8/b^3*a/(a*b)^
(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.90, size = 68, normalized size = 0.92

$$\frac{9\,abx^{3}+7\,a^{2}x}{8\left(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3}\right)}-\frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{3}}+\frac{x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(9*a*b*x^3 + 7*a^2*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) - 15/8*a*arctan
(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3
```

mupad [B] time = 4.75, size = 64, normalized size = 0.86

$$\frac{\frac{7\,a^2\,x}{8}+\frac{9\,b\,a\,x^3}{8}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{x}{b^3}-\frac{15\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^3,x)
```

```
[Out] ((7*a^2*x)/8 + (9*a*b*x^3)/8)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + x/b^3 - (
15*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(7/2))
```

sympy [A] time = 0.40, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{a}{b^7}}\log\left(-b^3\sqrt{-\frac{a}{b^7}}+x\right)}{16}-\frac{15\sqrt{-\frac{a}{b^7}}\log\left(b^3\sqrt{-\frac{a}{b^7}}+x\right)}{16}+\frac{7a^2x+9abx^3}{8a^2b^3+16ab^4x^2+8b^5x^4}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**3,x)
```

```
[Out] 15*sqrt(-a/b**7)*log(-b**3*sqrt(-a/b**7) + x)/16 - 15*sqrt(-a/b**7)*log(b**
3*sqrt(-a/b**7) + x)/16 + (7*a**2*x + 9*a*b*x**3)/(8*a**2*b**3 + 16*a*b**4*
x**2 + 8*b**5*x**4) + x/b**3
```

3.185 $\int \frac{x^4}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=64

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}-\frac{3x}{8b^2\left(a+bx^2\right)}-\frac{x^3}{4b\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {288, 205}

$$-\frac{3x}{8b^2\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}-\frac{x^3}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^3,x]
```

```
[Out] -x^3/(4*b*(a + b*x^2)^2) - (3*x)/(8*b^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^3}\,dx = -\frac{x^3}{4b\left(a+bx^2\right)^2}+\frac{3\int \frac{x^2}{\left(a+bx^2\right)^2}\,dx}{4b}$$
$$= -\frac{x^3}{4b\left(a+bx^2\right)^2}-\frac{3x}{8b^2\left(a+bx^2\right)}+\frac{3\int \frac{1}{a+bx^2}\,dx}{8b^2}$$
$$= -\frac{x^3}{4b\left(a+bx^2\right)^2}-\frac{3x}{8b^2\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.86

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}}-\frac{3ax+5bx^3}{8b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^3,x]
```

```
[Out] -1/8*(3*a*x + 5*b*x^3)/(b^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*Sqrt[a]*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x^2)^3, x]
```

fricas [A] time = 0.86, size = 188, normalized size = 2.94

$$\left[-\frac{10\,ab^2x^3+6\,a^2bx+3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)},-\frac{5\,ab^2x^3+3\,a^2bx-3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*a*b^2*x^3 + 6*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^5*x^4 + 2*a^2*b^4*x^2
+ a^3*b^3), -1/8*(5*a*b^2*x^3 + 3*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)]
```

giac [A] time = 0.60, size = 45, normalized size = 0.70

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^2} - \frac{5\,bx^3 + 3\,ax}{8\left(bx^2 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/8*(5*b*x^3 + 3*a*x)/((b*x^2 +
 a)^2*b^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^2} + \frac{-\frac{5x^3}{8b} - \frac{3ax}{8b^2}}{\left(b\,x^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^3,x)
```

```
[Out] (-5/8/b*x^3-3/8*a/b^2*x)/(b*x^2+a)^2+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/
2)*b*x)
```

maxima [A] time = 3.00, size = 59, normalized size = 0.92

$$-\frac{5\,bx^3 + 3\,ax}{8\left(b^4x^4 + 2\,ab^3x^2 + a^2b^2\right)} + \frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 3/8*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [B] time = 4.77, size = 56, normalized size = 0.88

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}}-\frac{\frac{5\,x^3}{8\,b}+\frac{3\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*b^(5/2)) - ((5*x^3)/(8*b) + (3*a*x
)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

sympy [A] time = 0.33, size = 110, normalized size = 1.72

$$-\frac{3\sqrt{-\frac{1}{ab^5}}\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{ab^5}}\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{16}+\frac{-3ax-5bx^3}{8a^2b^2+16ab^3x^2+8b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a*b**5))*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/16 + 3*sqrt(-1/(a*b
**5))*log(a*b**2*sqrt(-1/(a*b**5)) + x)/16 + (-3*a*x - 5*b*x**3)/(8*a**2*b*
*2 + 16*a*b**3*x**2 + 8*b**4*x**4)
```

3.186 $\int \frac{x^2}{(a+bx^2)^3} dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}+\frac{x}{8ab\left(a+bx^2\right)}-\frac{x}{4b\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}+\frac{x}{8ab\left(a+bx^2\right)}-\frac{x}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^3,x]
```

```
[Out] -x/(4*b*(a + b*x^2)^2) + x/(8*a*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]
]/(8*a^(3/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2\right)^3}\,dx &= -\frac{x}{4b\left(a+bx^2\right)^2}+\frac{\int \frac{1}{\left(a+bx^2\right)^2}\,dx}{4b}\\ &= -\frac{x}{4b\left(a+bx^2\right)^2}+\frac{x}{8ab\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2}\,dx}{8ab}\\ &= -\frac{x}{4b\left(a+bx^2\right)^2}+\frac{x}{8ab\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{a}\sqrt{b}\,x\left(bx^2-a\right)}{\left(a+bx^2\right)^2}+\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-a + b*x^2))/(a + b*x^2)^2 + ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(8*a^(3/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x^2)^3, x]
```

fricas [A] time = 0.75, size = 190, normalized size = 2.92

$$\left[\frac{2\,ab^2x^3-2\,a^2bx-\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)},\ \frac{ab^2x^3-a^2bx+\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 - 2*a^2*b*x - (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*log
((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 +
a^4*b^2), 1/8*(a*b^2*x^3 - a^2*b*x + (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)]
```

giac [A] time = 0.62, size = 50, normalized size = 0.77

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab}+\frac{bx^3-ax}{8\left(bx^2+a\right)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/8*(b*x^3 - a*x)/((b*x^2 + a)^
2*a*b)
```

maple [A] time = 0.01, size = 49, normalized size = 0.75

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab}+\frac{\frac{x^3}{8a}-\frac{x}{8b}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^3,x)
```

```
[Out] (1/8/a*x^3-1/8/b*x)/(b*x^2+a)^2+1/8/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x)
```

maxima [A] time = 3.04, size = 62, normalized size = 0.95

$$\frac{bx^3-ax}{8\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 1/8*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a*b)
```

mupad [B] time = 4.74, size = 55, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}-\frac{\frac{x}{8\,b}-\frac{x^3}{8\,a}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^3,x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(8*a^(3/2)*b^(3/2)) - (x/(8*b) - x^3/(8*a))/(a^2
+ b^2*x^4 + 2*a*b*x^2)
```

sympy [B] time = 0.31, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{a^3b^3}}\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^3}}\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{16}+\frac{-ax+bx^3}{8a^3b+16a^2b^2x^2+8ab^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**3))*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/16 + sqrt(-1/(a
**3*b**3))*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/16 + (-a*x + b*x**3)/(8*a**
3*b + 16*a**2*b**2*x**2 + 8*a*b**3*x**4)
```

3.187 $\int \frac{1}{(a+bx^2)^3} dx$

Optimal. Leaf size=62

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{3x}{8a^2\left(a+bx^2\right)}+\frac{x}{4a\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{3x}{8a^2\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{x}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a + b*x^2)^2) + (3*x)/(8*a^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^3}\,dx = \frac{x}{4a\left(a+bx^2\right)^2} + \frac{3\int \frac{1}{\left(a+bx^2\right)^2}\,dx}{4a}$$
$$= \frac{x}{4a\left(a+bx^2\right)^2} + \frac{3x}{8a^2\left(a+bx^2\right)} + \frac{3\int \frac{1}{a+bx^2}\,dx}{8a^2}$$
$$= \frac{x}{4a\left(a+bx^2\right)^2} + \frac{3x}{8a^2\left(a+bx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.89

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{5ax+3bx^3}{8a^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3),x]
```

```
[Out] (5*a*x + 3*b*x^3)/(8*a^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(5/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^(-3), x]
```

fricas [A] time = 0.81, size = 188, normalized size = 3.03

$$\left[\frac{6\,ab^2x^3 + 10\,a^2bx - 3\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2+a}\right)}{16\left(a^3b^3x^4 + 2\,a^4b^2x^2 + a^5b\right)},\ \frac{3\,ab^2x^3 + 5\,a^2bx + 3\left(b^2x^4 + 2\,abx^2 + a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\left(a^3b^3x^4 + 2\,a^4b^2x^2 + a^5b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2
 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

giac [A] time = 0.63, size = 45, normalized size = 0.73

$$\frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab}\, a^2} + \frac{3\, bx^3 + 5\, ax}{8 \left(bx^2 + a\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/8*(3*b*x^3 + 5*a*x)/((b*x^2 +
 a)^2*a^2)
```

maple [A] time = 0.00, size = 51, normalized size = 0.82

$$\frac{x}{4 \left(b\, x^2 + a\right)^2 a} + \frac{3x}{8 \left(b\, x^2 + a\right) a^2} + \frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\ a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3,x)
```

```
[Out] 1/4*x/a/(b*x^2+a)^2+3/8*x/a^2/(b*x^2+a)+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)
```

maxima [A] time = 3.03, size = 58, normalized size = 0.94

$$\frac{3\, bx^3 + 5\, ax}{8 \left(a^2 b^2 x^4 + 2\, a^3 b x^2 + a^4\right)} + \frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab}\, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3/8*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 4.66, size = 55, normalized size = 0.89

$$\frac{\frac{5\,x}{8\,a}+\frac{3\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^3,x)
```

```
[Out] ((5*x)/(8*a) + (3*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (3*atan((b^
(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

sympy [A] time = 0.32, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16}+\frac{5ax+3bx^3}{8a^4+16a^3bx^2+8a^2b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*b))*log(-a**3*sqrt(-1/(a**5*b)) + x)/16 + 3*sqrt(-1/(a**5*
b))*log(a**3*sqrt(-1/(a**5*b)) + x)/16 + (5*a*x + 3*b*x**3)/(8*a**4 + 16*a*
*3*b*x**2 + 8*a**2*b**2*x**4)
```

3.188 $\int \frac{1}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=76

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{5}{8a^2x\left(a+bx^2\right)}+\frac{1}{4ax\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{5}{8a^2x\left(a+bx^2\right)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{1}{4ax\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -15/(8*a^3*x) + 1/(4*a*x*(a + b*x^2)^2) + 5/(8*a^2*x*(a + b*x^2)) - (15*Sqr
t[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax\left(a+bx^2\right)^2}+\frac{5\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{1}{4ax\left(a+bx^2\right)^2}+\frac{5}{8a^2x\left(a+bx^2\right)}+\frac{15\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= -\frac{15}{8a^3x}+\frac{1}{4ax\left(a+bx^2\right)^2}+\frac{5}{8a^2x\left(a+bx^2\right)}-\frac{(15b)\int \frac{1}{a+bx^2}\,dx}{8a^3}$$

$$= -\frac{15}{8a^3x}+\frac{1}{4ax\left(a+bx^2\right)^2}+\frac{5}{8a^2x\left(a+bx^2\right)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.89

$$-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{8a^2+25abx^2+15b^2x^4}{8a^3x\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -1/8*(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)/(a^3*x*(a + b*x^2)^2) - (15*Sqrt[b]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^3), x]
```

fricas [A] time = 0.81, size = 202, normalized size = 2.66

$$\left[-\frac{30\,b^{2}x^{4}+50\,abx^{2}-15\,\left(b^{2}x^{5}+2\,abx^{3}+a^{2}x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+16\,a^{2}}{16\,\left(a^{3}b^{2}x^{5}+2\,a^{4}bx^{3}+a^{5}x\right)},-\frac{15\,b^{2}x^{4}+25\,abx^{2}+15\,\left(b^{2}x^{5}+2\,abx^{3}+a^{2}x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+8\,a^{2}}{8\,\left(a^{3}b^{2}x^{5}+2\,a^{4}bx^{3}+a^{5}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5 +
 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a*b*
x^3 + a^2*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*
x^3 + a^5*x)]
```

giac [A] time = 0.64, size = 57, normalized size = 0.75

$$-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}-\frac{7\,b^{2}x^{3}+9\,abx}{8\,\left(bx^{2}+a\right)^{2}a^{3}}-\frac{1}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/8*(7*b^2*x^3 + 9*a*b*x)/(
(b*x^2 + a)^2*a^3) - 1/(a^3*x)
```

maple [A] time = 0.01, size = 66, normalized size = 0.87

$$-\frac{7b^{2}x^{3}}{8\left(b\,x^{2}+a\right)^{2}a^{3}}-\frac{9bx}{8\left(b\,x^{2}+a\right)^{2}a^{2}}-\frac{15b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^{3}}-\frac{1}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^3,x)
```

```
[Out] -1/a^3/x-7/8/a^3*b^2/(b*x^2+a)^2*x^3-9/8/a^2*b/(b*x^2+a)^2*x-15/8/a^3*b/(a*
b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 71, normalized size = 0.93

$$-\frac{15\,b^{2}x^{4}+25\,abx^{2}+8\,a^{2}}{8\,\left(a^{3}b^{2}x^{5}+2\,a^{4}bx^{3}+a^{5}x\right)}-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
- 15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 4.67, size = 66, normalized size = 0.87

$$-\frac{\frac{1}{a}+\frac{25\,b\,x^2}{8\,a^2}+\frac{15\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^3),x)
```

```
[Out] - (1/a + (25*b*x^2)/(8*a^2) + (15*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a*
b*x^3) - (15*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

sympy [A] time = 0.42, size = 116, normalized size = 1.53

$$\frac{15\sqrt{-\frac{b}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}}{b}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}}{b}+x\right)}{16}+\frac{-8a^2-25abx^2-15b^2x^4}{8a^5x+16a^4bx^3+8a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**3,x)
```

```
[Out] 15*sqrt(-b/a**7)*log(-a**4*sqrt(-b/a**7)/b + x)/16 - 15*sqrt(-b/a**7)*log(a
**4*sqrt(-b/a**7)/b + x)/16 + (-8*a**2 - 25*a*b*x**2 - 15*b**2*x**4)/(8*a**
5*x + 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

3.189 $\int \frac{1}{x^4(a+bx^2)^3}\,dx$

Optimal. Leaf size=87

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{35b}{8a^4x}-\frac{35}{24a^3x^3}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{1}{4ax^3\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{7}{8a^2x^3\left(a+bx^2\right)}+\frac{35b}{8a^4x}-\frac{35}{24a^3x^3}+\frac{1}{4ax^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -35/(24*a^3*x^3) + (35*b)/(8*a^4*x) + 1/(4*a*x^3*(a + b*x^2)^2) + 7/(8*a^2*
x^3*(a + b*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^3\left(a+bx^2\right)^2} + \frac{7\int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{1}{4ax^3\left(a+bx^2\right)^2} + \frac{7}{8a^2x^3\left(a+bx^2\right)} + \frac{35\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= -\frac{35}{24a^3x^3} + \frac{1}{4ax^3\left(a+bx^2\right)^2} + \frac{7}{8a^2x^3\left(a+bx^2\right)} - \frac{(35b)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^3}$$

$$= -\frac{35}{24a^3x^3} + \frac{35b}{8a^4x} + \frac{1}{4ax^3\left(a+bx^2\right)^2} + \frac{7}{8a^2x^3\left(a+bx^2\right)} + \frac{\left(35b^2\right)\int \frac{1}{a+bx^2}\,dx}{8a^4}$$

$$= -\frac{35}{24a^3x^3} + \frac{35b}{8a^4x} + \frac{1}{4ax^3\left(a+bx^2\right)^2} + \frac{7}{8a^2x^3\left(a+bx^2\right)} + \frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.91

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}} + \frac{-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6}{24a^4x^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6)/(24*a^4*x^3*(a + b*x^
2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^3), x]
```

fricas [A] time = 0.81, size = 238, normalized size = 2.74

$$\left[\frac{210\, b^{3} x^{6} + 350\, a b^{2} x^{4} + 112\, a^{2} b x^{2} - 16\, a^{3} + 105\, \left(b^{3} x^{7} + 2\, a b^{2} x^{5} + a^{2} b x^{3}\right) \sqrt{-\frac{b}{a}} \log\left(\frac{b x^{2} + 2\, a x \sqrt{-\frac{b}{a}} - a}{b x^{2} + a}\right)}{48\, \left(a^{4} b^{2} x^{7} + 2\, a^{5} b x^{5} + a^{6} x^{3}\right)}, \frac{105\, b^{3} x^{6} + 175\, a b^{2} x^{4} + 56\, a^{2} b x^{2} - 8\, a^{3} + 105\, \left(b^{3} x^{7} + 2\, a b^{2} x^{5} + a^{2} b x^{3}\right) \sqrt{\frac{b}{a}} \arctan\left(x \sqrt{\frac{b}{a}}\right)}{24\, \left(a^{4} b^{2} x^{7} + 2\, a^{5} b x^{5} + a^{6} x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105*(b^3*x^7
+ 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), 1/24*(105*b^3*x^6 + 175*
a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 + 105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

giac [A] time = 0.64, size = 71, normalized size = 0.82

$$\frac{35\, b^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{8\, \sqrt{a b} a^{4}} + \frac{11\, b^{3} x^{3} + 13\, a b^{2} x}{8\, {\left(b x^{2} + a\right)}^{2} a^{4}} + \frac{9\, b x^{2} - a}{3\, a^{4} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/8*(11*b^3*x^3 + 13*a*b^2
*x)/((b*x^2 + a)^2*a^4) + 1/3*(9*b*x^2 - a)/(a^4*x^3)
```

maple [A] time = 0.02, size = 79, normalized size = 0.91

$$\frac{11 b^{3} x^{3}}{8 \left(b \,x^{2}+a \right)^{2} a^{4}}+\frac{13 b^{2} x}{8 \left(b \,x^{2}+a \right)^{2} a^{3}}+\frac{35 b^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{8 \sqrt{a b}\, a^{4}}+\frac{3 b}{a^{4} x}-\frac{1}{3 a^{3} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^3,x)
```

```
[Out] -1/3/a^3/x^3+3*b/a^4/x+11/8/a^4*b^3/(b*x^2+a)^2*x^3+13/8/a^3*b^2/(b*x^2+a)^
2*x+35/8/a^4*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.96, size = 86, normalized size = 0.99

$$\frac{105\, b^{3} x^{6} + 175\, a b^{2} x^{4} + 56\, a^{2} b x^{2} - 8\, a^{3}}{24\, {\left(a^{4} b^{2} x^{7} + 2\, a^{5} b x^{5} + a^{6} x^{3}\right)}} + \frac{35\, b^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{8\, \sqrt{a b} a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(105*b^3*x^6 + 175*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3)/(a^4*b^2*x^7 + 2*
a^5*b*x^5 + a^6*x^3) + 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [B] time = 4.67, size = 80, normalized size = 0.92

$$\frac{\frac{7\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{175\,b^2\,x^4}{24\,a^3}+\frac{35\,b^3\,x^6}{8\,a^4}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^3),x)
```

```
[Out] ((7*b*x^2)/(3*a^2) - 1/(3*a) + (175*b^2*x^4)/(24*a^3) + (35*b^3*x^6)/(8*a^4
))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5) + (35*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))
/(8*a^(9/2))
```

sympy [A] time = 0.48, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{b^3}{a^9}}\,\log\left(-\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2}+x\right)}{16}+\frac{35\sqrt{-\frac{b^3}{a^9}}\,\log\left(\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2}+x\right)}{16}+\frac{-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6}{24a^6x^3+48a^5bx^5+24a^4b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**3,x)
```

```
[Out] -35*sqrt(-b**3/a**9)*log(-a**5*sqrt(-b**3/a**9)/b**2 + x)/16 + 35*sqrt(-b**
3/a**9)*log(a**5*sqrt(-b**3/a**9)/b**2 + x)/16 + (-8*a**3 + 56*a**2*b*x**2
+ 175*a*b**2*x**4 + 105*b**3*x**6)/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*a**4
*b**2*x**7)
```

3.190 $\int \frac{1}{x^6\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}-\frac{63b^2}{8a^5x}+\frac{21b}{8a^4x^3}-\frac{63}{40a^3x^5}+\frac{9}{8a^2x^5\left(a+bx^2\right)}+\frac{1}{4ax^5\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{63b^2}{8a^5x}-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}+\frac{21b}{8a^4x^3}+\frac{9}{8a^2x^5\left(a+bx^2\right)}-\frac{63}{40a^3x^5}+\frac{1}{4ax^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -63/(40*a^3*x^5) + (21*b)/(8*a^4*x^3) - (63*b^2)/(8*a^5*x) + 1/(4*a*x^5*(a
+ b*x^2)^2) + 9/(8*a^2*x^5*(a + b*x^2)) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(8*a^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rubi steps

$$\int \frac{1}{x^6 \left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^5\left(a+bx^2\right)^2} + \frac{9\int \frac{1}{x^6\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{1}{4ax^5\left(a+bx^2\right)^2} + \frac{9}{8a^2x^5\left(a+bx^2\right)} + \frac{63\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= -\frac{63}{40a^3x^5} + \frac{1}{4ax^5\left(a+bx^2\right)^2} + \frac{9}{8a^2x^5\left(a+bx^2\right)} - \frac{(63b)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{8a^3}$$

$$= -\frac{63}{40a^3x^5} + \frac{21b}{8a^4x^3} + \frac{1}{4ax^5\left(a+bx^2\right)^2} + \frac{9}{8a^2x^5\left(a+bx^2\right)} + \frac{\left(63b^2\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^4}$$

$$= -\frac{63}{40a^3x^5} + \frac{21b}{8a^4x^3} - \frac{63b^2}{8a^5x} + \frac{1}{4ax^5\left(a+bx^2\right)^2} + \frac{9}{8a^2x^5\left(a+bx^2\right)} - \frac{\left(63b^3\right)\int \frac{1}{a+bx^2}\,dx}{8a^5}$$

$$= -\frac{63}{40a^3x^5} + \frac{21b}{8a^4x^3} - \frac{63b^2}{8a^5x} + \frac{1}{4ax^5\left(a+bx^2\right)^2} + \frac{9}{8a^2x^5\left(a+bx^2\right)} - \frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.90

$$-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}} - \frac{8a^4-24a^3bx^2+168a^2b^2x^4+525ab^3x^6+315b^4x^8}{40a^5x^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -1/40*(8*a^4 - 24*a^3*b*x^2 + 168*a^2*b^2*x^4 + 525*a*b^3*x^6 + 315*b^4*x^8
)/(a^5*x^5*(a + b*x^2)^2) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a + b*x^2)^3), x]
```

fricas [A] time = 0.81, size = 264, normalized size = 2.64

$$\left[-\frac{630\,b^4x^8+1050\,ab^3x^6+336\,a^2b^2x^4-48\,a^3bx^2+16\,a^4-315\,\left(b^4x^9+2\,ab^3x^7+a^2b^2x^5\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{80\,\left(a^5b^2x^9+2\,a^6bx^7+a^7x^5\right)},\ -\frac{315\,b^4x^8+525\,ab^3x^6+168\,a^2b^2x^4-24\,a^3bx^2+8\,a^4+315\,\left(b^4x^9+2\,ab^3x^7+a^2b^2x^5\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{40\,\left(a^5b^2x^9+2\,a^6bx^7+a^7x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(630*b^4*x^8 + 1050*a*b^3*x^6 + 336*a^2*b^2*x^4 - 48*a^3*b*x^2 + 16*
a^4 - 315*(b^4*x^9 + 2*a*b^3*x^7 + a^2*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a
*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5), -1/
40*(315*b^4*x^8 + 525*a*b^3*x^6 + 168*a^2*b^2*x^4 - 24*a^3*b*x^2 + 8*a^4 +
315*(b^4*x^9 + 2*a*b^3*x^7 + a^2*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a
^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)]
```

giac [A] time = 0.64, size = 80, normalized size = 0.80

$$-\frac{63\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}-\frac{15\,b^4x^3+17\,ab^3x}{8\,\left(bx^2+a\right)^2a^5}-\frac{30\,b^2x^4-5\,abx^2+a^2}{5\,a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -63/8*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/8*(15*b^4*x^3 + 17*a*b^
3*x)/((b*x^2 + a)^2*a^5) - 1/5*(30*b^2*x^4 - 5*a*b*x^2 + a^2)/(a^5*x^5)
```

maple [A] time = 0.01, size = 89, normalized size = 0.89

$$-\frac{15b^4x^3}{8\left(b\,x^2+a\right)^2a^5}-\frac{17b^3x}{8\left(b\,x^2+a\right)^2a^4}-\frac{63b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^5}-\frac{6b^2}{a^5x}+\frac{b}{a^4x^3}-\frac{1}{5a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^3,x)
```

```
[Out] -1/5/a^3/x^5-6*b^2/a^5/x+b/a^4/x^3-15/8/a^5*b^4/(b*x^2+a)^2*x^3-17/8/a^4*b^
3/(b*x^2+a)^2*x-63/8/a^5*b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.91, size = 97, normalized size = 0.97

$$-\frac{315\,b^4x^8 + 525\,ab^3x^6 + 168\,a^2b^2x^4 - 24\,a^3bx^2 + 8\,a^4}{40\left(a^5b^2x^9 + 2\,a^6bx^7 + a^7x^5\right)} - \frac{63\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(315*b^4*x^8 + 525*a*b^3*x^6 + 168*a^2*b^2*x^4 - 24*a^3*b*x^2 + 8*a^4
)/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5) - 63/8*b^3*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*a^5)
```

mupad [B] time = 5.02, size = 92, normalized size = 0.92

$$-\frac{\frac{1}{5\,a} - \frac{3\,b\,x^2}{5\,a^2} + \frac{21\,b^2\,x^4}{5\,a^3} + \frac{105\,b^3\,x^6}{8\,a^4} + \frac{63\,b^4\,x^8}{8\,a^5}}{a^2\,x^5 + 2\,a\,b\,x^7 + b^2\,x^9} - \frac{63\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^3),x)
```

```
[Out] - (1/(5*a) - (3*b*x^2)/(5*a^2) + (21*b^2*x^4)/(5*a^3) + (105*b^3*x^6)/(8*a^
4) + (63*b^4*x^8)/(8*a^5))/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (63*b^(5/2)*at
an((b^(1/2)*x)/a^(1/2)))/(8*a^(11/2))
```

sympy [A] time = 0.53, size = 150, normalized size = 1.50

$$\frac{63\sqrt{-\frac{b^5}{a^{11}}}\,\log\left(-\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3} + x\right)}{16} - \frac{63\sqrt{-\frac{b^5}{a^{11}}}\,\log\left(\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3} + x\right)}{16} + \frac{-8a^4 + 24a^3bx^2 - 168a^2b^2x^4 - 525ab^3x^6 - 315b^4x^8}{40a^7x^5 + 80a^6bx^7 + 40a^5b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**3,x)
```

```
[Out] 63*sqrt(-b**5/a**11)*log(-a**6*sqrt(-b**5/a**11)/b**3 + x)/16 - 63*sqrt(-b*
*5/a**11)*log(a**6*sqrt(-b**5/a**11)/b**3 + x)/16 + (-8*a**4 + 24*a**3*b*x*
*2 - 168*a**2*b**2*x**4 - 525*a*b**3*x**6 - 315*b**4*x**8)/(40*a**7*x**5 +
80*a**6*b*x**7 + 40*a**5*b**2*x**9)
```

3.191 $\int \frac{1}{x^8(a+bx^2)^3}\,dx$

Optimal. Leaf size=113

$$\frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{13/2}}+\frac{99b^3}{8a^6x}-\frac{33b^2}{8a^5x^3}+\frac{99b}{40a^4x^5}-\frac{99}{56a^3x^7}+\frac{11}{8a^2x^7\left(a+bx^2\right)}+\frac{1}{4ax^7\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{33b^2}{8a^5x^3}+\frac{99b^3}{8a^6x}+\frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{13/2}}+\frac{99b}{40a^4x^5}+\frac{11}{8a^2x^7\left(a+bx^2\right)}-\frac{99}{56a^3x^7}+\frac{1}{4ax^7\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -99/(56*a^3*x^7) + (99*b)/(40*a^4*x^5) - (33*b^2)/(8*a^5*x^3) + (99*b^3)/(8
*a^6*x) + 1/(4*a*x^7*(a + b*x^2)^2) + 11/(8*a^2*x^7*(a + b*x^2)) + (99*b^(7
/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11\int \frac{1}{x^8\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}+\frac{99\int \frac{1}{x^8\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= -\frac{99}{56a^3x^7}+\frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}-\frac{(99b)\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{8a^3}$$

$$= -\frac{99}{56a^3x^7}+\frac{99b}{40a^4x^5}+\frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}+\frac{\left(99b^2\right)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{8a^4}$$

$$= -\frac{99}{56a^3x^7}+\frac{99b}{40a^4x^5}-\frac{33b^2}{8a^5x^3}+\frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}-\frac{\left(99b^3\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{8a^5}$$

$$= -\frac{99}{56a^3x^7}+\frac{99b}{40a^4x^5}-\frac{33b^2}{8a^5x^3}+\frac{99b^3}{8a^6x}+\frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}+\frac{\left(99b^4\right)\int \frac{1}{a+bx^2}}{8a^6}$$

$$= -\frac{99}{56a^3x^7}+\frac{99b}{40a^4x^5}-\frac{33b^2}{8a^5x^3}+\frac{99b^3}{8a^6x}+\frac{1}{4ax^7\left(a+bx^2\right)^2}+\frac{11}{8a^2x^7\left(a+bx^2\right)}+\frac{99b^{7/2}\tan^{-1}\left(\right.}{8a^{13/2}}$$

Mathematica [A] time = 0.06, size = 101, normalized size = 0.89

$$\frac{99b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{13/2}}+\frac{-40a^5+88a^4bx^2-264a^3b^2x^4+1848a^2b^3x^6+5775ab^4x^8+3465b^5x^{10}}{280a^6x^7\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^2)^3),x]

[Out] (-40*a^5 + 88*a^4*b*x^2 - 264*a^3*b^2*x^4 + 1848*a^2*b^3*x^6 + 5775*a*b^4*x
^8 + 3465*b^5*x^10)/(280*a^6*x^7*(a + b*x^2)^2) + (99*b^(7/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(8*a^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 \left(a + bx^2\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^8*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^8*(a + b*x^2)^3), x]
```

fricas [A] time = 0.89, size = 286, normalized size = 2.53

$$\left[\frac{6930\,b^5x^{10} + 11550\,ab^4x^8 + 3696\,a^2b^3x^6 - 528\,a^3b^2x^4 + 176\,a^4bx^2 - 80\,a^5 + 3465\left(b^5x^{11} + 2\,ab^4x^9 + a^2b^3x^7\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 + 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right)}{560\left(a^6b^2x^{11} + 2\,a^7bx^9 + a^8x^7\right)}, \frac{3465\,b^5x^{10} + 5775\,ab^4x^8 + 1848\,a^2b^3x^6 - 264\,a^3b^2x^4 + 88\,a^4bx^2 - 40\,a^5 + 3465\left(b^5x^{11} + 2\,ab^4x^9 + a^2b^3x^7\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{280\left(a^6b^2x^{11} + 2\,a^7bx^9 + a^8x^7\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(6930*b^5*x^10 + 11550*a*b^4*x^8 + 3696*a^2*b^3*x^6 - 528*a^3*b^2*x^
4 + 176*a^4*b*x^2 - 80*a^5 + 3465*(b^5*x^11 + 2*a*b^4*x^9 + a^2*b^3*x^7)*sq
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2
*a^7*b*x^9 + a^8*x^7), 1/280*(3465*b^5*x^10 + 5775*a*b^4*x^8 + 1848*a^2*b^3
*x^6 - 264*a^3*b^2*x^4 + 88*a^4*b*x^2 - 40*a^5 + 3465*(b^5*x^11 + 2*a*b^4*x
^9 + a^2*b^3*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^11 + 2*a^7*b*x^
9 + a^8*x^7)]
```

giac [A] time = 0.61, size = 93, normalized size = 0.82

$$\frac{99\,b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6} + \frac{19\,b^5x^3 + 21\,ab^4x}{8\left(bx^2 + a\right)^2 a^6} + \frac{350\,b^3x^6 - 70\,ab^2x^4 + 21\,a^2bx^2 - 5\,a^3}{35\,a^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 99/8*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*x^3 + 21*a*b^4
*x)/((b*x^2 + a)^2*a^6) + 1/35*(350*b^3*x^6 - 70*a*b^2*x^4 + 21*a^2*b*x^2 -
 5*a^3)/(a^6*x^7)
```

maple [A] time = 0.01, size = 101, normalized size = 0.89

$$\frac{19b^5x^3}{8\left(b\,x^2 + a\right)^2 a^6} + \frac{21b^4x}{8\left(b\,x^2 + a\right)^2 a^5} + \frac{99b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^6} + \frac{10b^3}{a^6x} - \frac{2b^2}{a^5x^3} + \frac{3b}{5a^4x^5} - \frac{1}{7a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^2+a)^3,x)
```

```
[Out] -1/7/a^3/x^7+10*b^3/a^6/x-2*b^2/a^5/x^3+3/5*b/a^4/x^5+19/8/a^6*b^5/(b*x^2+a
)^2*x^3+21/8/a^5*b^4/(b*x^2+a)^2*x+99/8/a^6*b^4/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)
```

maxima [A] time = 3.02, size = 108, normalized size = 0.96

$$\frac{3465\,b^5x^{10}+5775\,ab^4x^8+1848\,a^2b^3x^6-264\,a^3b^2x^4+88\,a^4bx^2-40\,a^5}{280\left(a^6b^2x^{11}+2\,a^7bx^9+a^8x^7\right)}+\frac{99\,b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/280*(3465*b^5*x^10 + 5775*a*b^4*x^8 + 1848*a^2*b^3*x^6 - 264*a^3*b^2*x^4
+ 88*a^4*b*x^2 - 40*a^5)/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7) + 99/8*b^4*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

mupad [B] time = 4.98, size = 102, normalized size = 0.90

$$\frac{\frac{11\,b\,x^2}{35\,a^2}-\frac{1}{7\,a}-\frac{33\,b^2\,x^4}{35\,a^3}+\frac{33\,b^3\,x^6}{5\,a^4}+\frac{165\,b^4\,x^8}{8\,a^5}+\frac{99\,b^5\,x^{10}}{8\,a^6}}{a^2\,x^7+2\,a\,b\,x^9+b^2\,x^{11}}+\frac{99\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^2)^3),x)
```

```
[Out] ((11*b*x^2)/(35*a^2) - 1/(7*a) - (33*b^2*x^4)/(35*a^3) + (33*b^3*x^6)/(5*a^
4) + (165*b^4*x^8)/(8*a^5) + (99*b^5*x^10)/(8*a^6))/(a^2*x^7 + b^2*x^11 + 2
*a*b*x^9) + (99*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(13/2))
```

sympy [A] time = 0.58, size = 162, normalized size = 1.43

$$-\frac{99\sqrt{-\frac{b^7}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4}+x\right)}{16}+\frac{99\sqrt{-\frac{b^7}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4}+x\right)}{16}+\frac{-40a^5+88a^4bx^2-264a^3b^2x^4+1848a^2b^3x^6+5775ab^4x^8+3465b^5x^{10}}{280a^8x^7+560a^7bx^9+280a^6b^2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**2+a)**3,x)
```

```
[Out] -99*sqrt(-b**7/a**13)*log(-a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + 99*sqrt(-b
**7/a**13)*log(a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + (-40*a**5 + 88*a**4*b*
x**2 - 264*a**3*b**2*x**4 + 1848*a**2*b**3*x**6 + 5775*a*b**4*x**8 + 3465*b
**5*x**10)/(280*a**8*x**7 + 560*a**7*b*x**9 + 280*a**6*b**2*x**11)
```

3.192 $\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=216

$$-\frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9}+\frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8}-\frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7}+\frac{55a^9}{3b^{13}\left(a+bx^2\right)^6}-\frac{99a^8}{2b^{13}\left(a+bx^2\right)^5}+\frac{99a^7}{b^{13}\left(a+bx^2\right)^4}-\frac{15}{b^{13}\left(a\right.}$$

Rubi [A] time = 0.26, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9}+\frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8}-\frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7}+\frac{55a^9}{3b^{13}\left(a+bx^2\right)^6}-\frac{99a^8}{2b^{13}\left(a+bx^2\right)^5}+\frac{99a^7}{b^{13}\left(a+bx^2\right)^4}-\frac{154a^6}{b^{13}\left(a+bx^2\right)^3}+\frac{198a^5}{b^{13}\left(a+bx^2\right)^2}-\frac{495a^4}{2b^{13}\left(a+bx^2\right)}+\frac{55a^2x^2}{2b^{12}}-\frac{110a^3\log\left(a+bx^2\right)}{b^{13}}-\frac{5ax^4}{2b^{11}}+\frac{x^6}{6b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^25/(a + b*x^2)^10,x]
```

```
[Out] (55*a^2*x^2)/(2*b^12) - (5*a*x^4)/(2*b^11) + x^6/(6*b^10) - a^12/(18*b^13*(
a + b*x^2)^9) + (3*a^11)/(4*b^13*(a + b*x^2)^8) - (33*a^10)/(7*b^13*(a + b*
x^2)^7) + (55*a^9)/(3*b^13*(a + b*x^2)^6) - (99*a^8)/(2*b^13*(a + b*x^2)^5)
 + (99*a^7)/(b^13*(a + b*x^2)^4) - (154*a^6)/(b^13*(a + b*x^2)^3) + (198*a^
5)/(b^13*(a + b*x^2)^2) - (495*a^4)/(2*b^13*(a + b*x^2)) - (110*a^3*Log[a +
 b*x^2])/b^13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{25}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^{12}}{(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{55a^2}{b^{12}} - \frac{10ax}{b^{11}} + \frac{x^2}{b^{10}} + \frac{a^{12}}{b^{12}(a+bx)^{10}} - \frac{12a^{11}}{b^{12}(a+bx)^9} + \frac{66a^{10}}{b^{12}(a+bx)^8} - \frac{220a^9}{b^{12}(a+b\cdots}\right.\right.$$

$$= \frac{55a^2x^2}{2b^{12}} - \frac{5ax^4}{2b^{11}} + \frac{x^6}{6b^{10}} - \frac{a^{12}}{18b^{13}\left(a+bx^2\right)^9} + \frac{3a^{11}}{4b^{13}\left(a+bx^2\right)^8} - \frac{33a^{10}}{7b^{13}\left(a+bx^2\right)^7} + \frac{55a^9}{3b^{13}\left(a+b\cdots\right.}$$

Mathematica [A] time = 0.05, size = 169, normalized size = 0.78

$$-\frac{35201a^{12} + 289089a^{11}bx^2 + 1031616a^{10}b^2x^4 + 2074464a^9b^3x^6 + 2529576a^8b^4x^8 + 1831032a^7b^5x^{10} + 638568a^6b^6x^{12} - 58968a^5b^7x^{14} - 139482a^4b^8x^{16} - 43218a^3b^9x^{18} + 27720a^3\left(a+bx^2\right)^9\log\left(a+bx^2\right) - 2772a^2b^{10}x^{20} + 252ab^{11}x^{22} - 42b^{12}x^{24}}{252b^{13}\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^25/(a + b*x^2)^10,x]
```

```
[Out] -1/252*(35201*a^12 + 289089*a^11*b*x^2 + 1031616*a^10*b^2*x^4 + 2074464*a^9
*b^3*x^6 + 2529576*a^8*b^4*x^8 + 1831032*a^7*b^5*x^10 + 638568*a^6*b^6*x^12
 - 58968*a^5*b^7*x^14 - 139482*a^4*b^8*x^16 - 43218*a^3*b^9*x^18 - 2772*a^2
*b^10*x^20 + 252*a*b^11*x^22 - 42*b^12*x^24 + 27720*a^3*(a + b*x^2)^9*Log[a
 + b*x^2])/(b^13*(a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{25}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^25/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^25/(a + b*x^2)^10, x]
```

fricas [A] time = 0.66, size = 346, normalized size = 1.60

$$\frac{42\,b^{12}x^{24} - 252\,ab^{11}x^{22} + 2772\,a^2b^{10}x^{20} + 43218\,a^3b^9x^{18} + 139482\,a^4b^8x^{16} + 58968\,a^5b^7x^{14} - 638568\,a^6b^6x^{12} - 1831032\,a^7b^5x^{10} - 2529576\,a^8b^4x^8 - 2074464\,a^9b^3x^6 - 1031616\,a^{10}b^2x^4 - 289089\,a^{11}bx^2 - 35201\,a^{12} - 27720\left(a^3b^9x^{18} + 9\,a^4b^8x^{16} + 36\,a^5b^7x^{14} + 84\,a^6b^6x^{12} + 126\,a^7b^5x^{10} + 126\,a^8b^4x^8 + 84\,a^9b^3x^6 + 36\,a^{10}b^2x^4 + 9\,a^{11}bx^2 + a^{12}\right)\log\left(bx^2+a\right)}{252\left(b^{22}x^{18} + 9\,ab^{21}x^{16} + 36\,a^2b^{20}x^{14} + 84\,a^3b^{19}x^{12} + 126\,a^4b^{18}x^{10} + 126\,a^5b^{17}x^8 + 84\,a^6b^{16}x^6 + 36\,a^7b^{15}x^4 + 9\,a^8b^{14}x^2 + a^9b^{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(42*b^12*x^24 - 252*a*b^11*x^22 + 2772*a^2*b^10*x^20 + 43218*a^3*b^9*
x^18 + 139482*a^4*b^8*x^16 + 58968*a^5*b^7*x^14 - 638568*a^6*b^6*x^12 - 183
1032*a^7*b^5*x^10 - 2529576*a^8*b^4*x^8 - 2074464*a^9*b^3*x^6 - 1031616*a^1
0*b^2*x^4 - 289089*a^11*b*x^2 - 35201*a^12 - 27720*(a^3*b^9*x^18 + 9*a^4*b^
8*x^16 + 36*a^5*b^7*x^14 + 84*a^6*b^6*x^12 + 126*a^7*b^5*x^10 + 126*a^8*b^4
*x^8 + 84*a^9*b^3*x^6 + 36*a^10*b^2*x^4 + 9*a^11*b*x^2 + a^12)*log(b*x^2 +
a))/(b^22*x^18 + 9*a*b^21*x^16 + 36*a^2*b^20*x^14 + 84*a^3*b^19*x^12 + 126*
a^4*b^18*x^10 + 126*a^5*b^17*x^8 + 84*a^6*b^16*x^6 + 36*a^7*b^15*x^4 + 9*a^
8*b^14*x^2 + a^9*b^13)
```

giac [A] time = 0.64, size = 168, normalized size = 0.78

$$-\frac{110\,a^3\log\left(\left|bx^2+a\right|\right)}{b^{13}}+\frac{78419\,a^3b^9x^{18}+643401\,a^4b^8x^{16}+2374020\,a^5b^7x^{14}+5151300\,a^6b^6x^{12}+7227990\,a^7b^5x^{10}+6791400\,a^8b^4x^8+4268880\,a^9b^3x^6+1729728\,a^{10}b^2x^4+409752\,a^{11}bx^2+43218\,a^{12}}{252\left(bx^2+a\right)^9b^{13}}+\frac{b^{20}x^6-15\,ab^{19}x^4+165\,a^2b^{18}x^2}{6\,b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -110*a^3*log(abs(b*x^2 + a))/b^13 + 1/252*(78419*a^3*b^9*x^18 + 643401*a^4*
b^8*x^16 + 2374020*a^5*b^7*x^14 + 5151300*a^6*b^6*x^12 + 7227990*a^7*b^5*x^
10 + 6791400*a^8*b^4*x^8 + 4268880*a^9*b^3*x^6 + 1729728*a^10*b^2*x^4 + 409
752*a^11*b*x^2 + 43218*a^12)/((b*x^2 + a)^9*b^13) + 1/6*(b^20*x^6 - 15*a*b^
19*x^4 + 165*a^2*b^18*x^2)/b^30
```

maple [A] time = 0.02, size = 199, normalized size = 0.92

$$-\frac{a^{12}}{18\left(bx^2+a\right)^9b^{13}}+\frac{3a^{11}}{4\left(bx^2+a\right)^8b^{13}}-\frac{33a^{10}}{7\left(bx^2+a\right)^7b^{13}}+\frac{55a^9}{3\left(bx^2+a\right)^6b^{13}}+\frac{x^6}{6b^{10}}-\frac{99a^8}{2\left(bx^2+a\right)^5b^{13}}+\frac{99a^7}{\left(bx^2+a\right)^4b^{13}}-\frac{5a\,x^4}{2b^{11}}-\frac{154a^6}{\left(bx^2+a\right)^3b^{13}}+\frac{198a^5}{\left(bx^2+a\right)^2b^{13}}+\frac{55a^2x^2}{2b^{12}}-\frac{495a^4}{2\left(bx^2+a\right)b^{13}}-\frac{110a^3\ln\left(bx^2+a\right)}{b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^25/(b*x^2+a)^10,x)
```

```
[Out] 55/2*a^2*x^2/b^12-5/2*a*x^4/b^11+1/6*x^6/b^10-1/18*a^12/b^13/(b*x^2+a)^9+3/
4*a^11/b^13/(b*x^2+a)^8-33/7*a^10/b^13/(b*x^2+a)^7+55/3*a^9/b^13/(b*x^2+a)^
6-99/2*a^8/b^13/(b*x^2+a)^5+99*a^7/b^13/(b*x^2+a)^4-154*a^6/b^13/(b*x^2+a)^
3+198*a^5/b^13/(b*x^2+a)^2-495/2*a^4/b^13/(b*x^2+a)-110*a^3*ln(b*x^2+a)/b^1
3
```

maxima [A] time = 1.62, size = 242, normalized size = 1.12

$$-\frac{62370\,a^4b^8x^{16}+449064\,a^5b^7x^{14}+1435896\,a^6b^6x^{12}+2652804\,a^7b^5x^{10}+3089394\,a^8b^4x^8+2318316\,a^9b^3x^6+1093356\,a^{10}b^2x^4+296019\,a^{11}bx^2+35201\,a^{12}}{252\left(b^{22}x^{18}+9\,ab^{21}x^{16}+36\,a^2b^{20}x^{14}+84\,a^3b^{19}x^{12}+126\,a^4b^{18}x^{10}+126\,a^5b^{17}x^8+84\,a^6b^{16}x^6+36\,a^7b^{15}x^4+9\,a^8b^{14}x^2+a^9b^{13}\right)}-\frac{110\,a^3\log\left(bx^2+a\right)}{b^{13}}+\frac{b^2x^6-15\,abx^4+165\,a^2x^2}{6\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^25/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(62370*a^4*b^8*x^16 + 449064*a^5*b^7*x^14 + 1435896*a^6*b^6*x^12 + 2
652804*a^7*b^5*x^10 + 3089394*a^8*b^4*x^8 + 2318316*a^9*b^3*x^6 + 1093356*a
^10*b^2*x^4 + 296019*a^11*b*x^2 + 35201*a^12)/(b^22*x^18 + 9*a*b^21*x^16 +
36*a^2*b^20*x^14 + 84*a^3*b^19*x^12 + 126*a^4*b^18*x^10 + 126*a^5*b^17*x^8
+ 84*a^6*b^16*x^6 + 36*a^7*b^15*x^4 + 9*a^8*b^14*x^2 + a^9*b^13) - 110*a^3*
log(b*x^2 + a)/b^13 + 1/6*(b^2*x^6 - 15*a*b*x^4 + 165*a^2*x^2)/b^12
```

mupad [B] time = 5.29, size = 242, normalized size = 1.12

$$\frac{x^6}{6\,b^{10}}-\frac{\frac{35201\,a^{12}}{252\,b}+\frac{32891\,a^{11}\,x^2}{28}+\frac{30371\,a^{10}\,b\,x^4}{7}+\frac{27599\,a^9\,b^2\,x^6}{3}+\frac{24519\,a^8\,b^3\,x^8}{2}+10527\,a^7\,b^4\,x^{10}+5698\,a^6\,b^5\,x^{12}+1782\,a^5\,b^6\,x^{14}+\frac{495\,a^4\,b^7\,x^{16}}{2}}{a^9\,b^{12}+9\,a^8\,b^{13}\,x^2+36\,a^7\,b^{14}\,x^4+84\,a^6\,b^{15}\,x^6+126\,a^5\,b^{16}\,x^8+126\,a^4\,b^{17}\,x^{10}+84\,a^3\,b^{18}\,x^{12}+36\,a^2\,b^{19}\,x^{14}+9\,a\,b^{20}\,x^{16}+b^{21}\,x^{18}}-\frac{5\,a\,x^4}{2\,b^{11}}-\frac{110\,a^3\,\ln\left(b\,x^2+a\right)}{b^{13}}+\frac{55\,a^2\,x^2}{2\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^25/(a + b*x^2)^10,x)
```

```
[Out] x^6/(6*b^10) - ((35201*a^12)/(252*b) + (32891*a^11*x^2)/28 + (30371*a^10*b*
x^4)/7 + (27599*a^9*b^2*x^6)/3 + (24519*a^8*b^3*x^8)/2 + 10527*a^7*b^4*x^10
 + 5698*a^6*b^5*x^12 + 1782*a^5*b^6*x^14 + (495*a^4*b^7*x^16)/2)/(a^9*b^12
+ b^21*x^18 + 9*a*b^20*x^16 + 9*a^8*b^13*x^2 + 36*a^7*b^14*x^4 + 84*a^6*b^1
5*x^6 + 126*a^5*b^16*x^8 + 126*a^4*b^17*x^10 + 84*a^3*b^18*x^12 + 36*a^2*b^
19*x^14) - (5*a*x^4)/(2*b^11) - (110*a^3*log(a + b*x^2))/b^13 + (55*a^2*x^2
)/(2*b^12)
```

sympy [A] time = 2.15, size = 260, normalized size = 1.20

$$-\frac{110a^3\log\left(a+bx^2\right)}{b^{13}}+\frac{55a^2x^2}{2b^{12}}-\frac{5ax^4}{2b^{11}}+\frac{-35201a^{12}-296019a^{11}bx^2-1093356a^{10}b^2x^4-2318316a^9b^3x^6-3089394a^8b^4x^8-2652804a^7b^5x^{10}-1435896a^6b^6x^{12}-449064a^5b^7x^{14}-62370a^4b^8x^{16}}{252a^9b^{13}+2268a^8b^{14}x^2+9072a^7b^{15}x^4+21168a^6b^{16}x^6+31752a^5b^{17}x^8+31752a^4b^{18}x^{10}+21168a^3b^{19}x^{12}+9072a^2b^{20}x^{14}+2268ab^{21}x^{16}+252b^{22}x^{18}}+\frac{x^6}{6b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**25/(b*x**2+a)**10,x)
```

```
[Out] -110*a**3*log(a + b*x**2)/b**13 + 55*a**2*x**2/(2*b**12) - 5*a*x**4/(2*b**1
1) + (-35201*a**12 - 296019*a**11*b*x**2 - 1093356*a**10*b**2*x**4 - 231831
6*a**9*b**3*x**6 - 3089394*a**8*b**4*x**8 - 2652804*a**7*b**5*x**10 - 14358
96*a**6*b**6*x**12 - 449064*a**5*b**7*x**14 - 62370*a**4*b**8*x**16)/(252*a
**9*b**13 + 2268*a**8*b**14*x**2 + 9072*a**7*b**15*x**4 + 21168*a**6*b**16*
x**6 + 31752*a**5*b**17*x**8 + 31752*a**4*b**18*x**10 + 21168*a**3*b**19*x*
*12 + 9072*a**2*b**20*x**14 + 2268*a*b**21*x**16 + 252*b**22*x**18) + x**6/
(6*b**10)
```

3.193 $\int \frac{x^{23}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=205

$$\frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9}-\frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8}+\frac{55a^9}{14b^{12}\left(a+bx^2\right)^7}-\frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}+\frac{33a^7}{b^{12}\left(a+bx^2\right)^5}-\frac{231a^6}{4b^{12}\left(a+bx^2\right)^4}+\frac{7\cdots}{b^{12}\left(a\cdots\right.}$$

Rubi [A] time = 0.21, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9}-\frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8}+\frac{55a^9}{14b^{12}\left(a+bx^2\right)^7}-\frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}+\frac{33a^7}{b^{12}\left(a+bx^2\right)^5}-\frac{231a^6}{4b^{12}\left(a+bx^2\right)^4}+\frac{77a^5}{b^{12}\left(a+bx^2\right)^3}-\frac{165a^4}{2b^{12}\left(a+bx^2\right)^2}+\frac{165a^3}{2b^{12}\left(a+bx^2\right)}+\frac{55a^2\log\left(a+bx^2\right)}{2b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^4}{4b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/(a + b*x^2)^10,x]
```

```
[Out] (-5*a*x^2)/b^11 + x^4/(4*b^10) + a^11/(18*b^12*(a + b*x^2)^9) - (11*a^10)/(
16*b^12*(a + b*x^2)^8) + (55*a^9)/(14*b^12*(a + b*x^2)^7) - (55*a^8)/(4*b^1
2*(a + b*x^2)^6) + (33*a^7)/(b^12*(a + b*x^2)^5) - (231*a^6)/(4*b^12*(a + b
*x^2)^4) + (77*a^5)/(b^12*(a + b*x^2)^3) - (165*a^4)/(2*b^12*(a + b*x^2)^2)
 + (165*a^3)/(2*b^12*(a + b*x^2)) + (55*a^2*Log[a + b*x^2])/(2*b^12)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{23}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^{11}}{(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{10a}{b^{11}} + \frac{x}{b^{10}} - \frac{a^{11}}{b^{11}(a+bx)^{10}} + \frac{11a^{10}}{b^{11}(a+bx)^9} - \frac{55a^9}{b^{11}(a+bx)^8} + \frac{165a^8}{b^{11}(a+bx)^7} - \frac{}{b^{}}\right.\right.$$

$$= -\frac{5ax^2}{b^{11}} + \frac{x^4}{4b^{10}} + \frac{a^{11}}{18b^{12}\left(a+bx^2\right)^9} - \frac{11a^{10}}{16b^{12}\left(a+bx^2\right)^8} + \frac{55a^9}{14b^{12}\left(a+bx^2\right)^7} - \frac{55a^8}{4b^{12}\left(a+bx^2\right)^6}$$

Mathematica [A] time = 0.03, size = 158, normalized size = 0.77

$$\frac{42131a^{11} + 351459a^{10}bx^2 + 1281096a^9b^2x^4 + 2656584a^8b^3x^6 + 3402756a^7b^4x^8 + 2704212a^6b^5x^{10} + 1220688a^5b^6x^{12} + 190512a^4b^7x^{14} - 77112a^3b^8x^{16} - 36288a^2b^9x^{18} + 27720a^2\left(a+bx^2\right)^9\log\left(a+bx^2\right) - 2772ab^{10}x^{20} + 252b^{11}x^{22}}{1008b^{12}\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/(a + b*x^2)^10,x]
```

```
[Out] (42131*a^11 + 351459*a^10*b*x^2 + 1281096*a^9*b^2*x^4 + 2656584*a^8*b^3*x^6
 + 3402756*a^7*b^4*x^8 + 2704212*a^6*b^5*x^10 + 1220688*a^5*b^6*x^12 + 1905
12*a^4*b^7*x^14 - 77112*a^3*b^8*x^16 - 36288*a^2*b^9*x^18 - 2772*a*b^10*x^2
0 + 252*b^11*x^22 + 27720*a^2*(a + b*x^2)^9*Log[a + b*x^2])/(1008*b^12*(a +
 b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{23}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^23/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^23/(a + b*x^2)^10, x]
```

fricas [A] time = 0.89, size = 335, normalized size = 1.63

$$\frac{252\,b^{11}x^{22} - 2772\,ab^{10}x^{20} - 36288\,a^2b^9x^{18} - 77112\,a^3b^8x^{16} + 190512\,a^4b^7x^{14} + 1220688\,a^5b^6x^{12} + 2704212\,a^6b^5x^{10} + 3402756\,a^7b^4x^8 + 2656584\,a^8b^3x^6 + 1281096\,a^9b^2x^4 + 351459\,a^{10}bx^2 + 42131\,a^{11} + 27720\left(a^2b^9x^{18} + 9\,a^3b^8x^{16} + 36\,a^4b^7x^{14} + 84\,a^5b^6x^{12} + 126\,a^6b^5x^{10} + 126\,a^7b^4x^8 + 84\,a^8b^3x^6 + 36\,a^9b^2x^4 + 9\,a^{10}bx^2 + a^{11}\right)\log\left(bx^2+a\right)}{1008\left(b^{21}x^{18} + 9\,ab^{20}x^{16} + 36\,a^2b^{19}x^{14} + 84\,a^3b^{18}x^{12} + 126\,a^4b^{17}x^{10} + 126\,a^5b^{16}x^8 + 84\,a^6b^{15}x^6 + 36\,a^7b^{14}x^4 + 9\,a^8b^{13}x^2 + a^9b^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/1008*(252*b^11*x^22 - 2772*a*b^10*x^20 - 36288*a^2*b^9*x^18 - 77112*a^3*b
^8*x^16 + 190512*a^4*b^7*x^14 + 1220688*a^5*b^6*x^12 + 2704212*a^6*b^5*x^10
 + 3402756*a^7*b^4*x^8 + 2656584*a^8*b^3*x^6 + 1281096*a^9*b^2*x^4 + 351459
*a^10*b*x^2 + 42131*a^11 + 27720*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^4*b^
7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8*b^3*
x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*log(b*x^2 + a))/(b^21*x^18 + 9*
a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126
*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^
12)
```

giac [A] time = 0.63, size = 157, normalized size = 0.77

$$\frac{55\,a^2\log\left(\left|bx^2+a\right|\right)}{2\,b^{12}}+\frac{b^{10}x^4-20\,ab^9x^2}{4\,b^{20}}-\frac{78419\,a^2b^9x^{18}+622611\,a^3b^8x^{16}+2240964\,a^4b^7x^{14}+4763220\,a^5b^6x^{12}+6562710\,a^6b^5x^{10}+6063750\,a^7b^4x^8+3751440\,a^8b^3x^6+1496880\,a^9b^2x^4+349272\,a^{10}bx^2+36288\,a^{11}}{1008\left(bx^2+a\right)^9b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/2*a^2*log(abs(b*x^2 + a))/b^12 + 1/4*(b^10*x^4 - 20*a*b^9*x^2)/b^20 - 1/
1008*(78419*a^2*b^9*x^18 + 622611*a^3*b^8*x^16 + 2240964*a^4*b^7*x^14 + 476
3220*a^5*b^6*x^12 + 6562710*a^6*b^5*x^10 + 6063750*a^7*b^4*x^8 + 3751440*a^
8*b^3*x^6 + 1496880*a^9*b^2*x^4 + 349272*a^10*b*x^2 + 36288*a^11)/((b*x^2 +
 a)^9*b^12)
```

maple [A] time = 0.02, size = 188, normalized size = 0.92

$$\frac{a^{11}}{18\left(bx^2+a\right)^9b^{12}}-\frac{11a^{10}}{16\left(bx^2+a\right)^8b^{12}}+\frac{55a^9}{14\left(bx^2+a\right)^7b^{12}}-\frac{55a^8}{4\left(bx^2+a\right)^6b^{12}}+\frac{33a^7}{\left(bx^2+a\right)^5b^{12}}-\frac{231a^6}{4\left(bx^2+a\right)^4b^{12}}+\frac{x^4}{4b^{10}}+\frac{77a^5}{\left(bx^2+a\right)^3b^{12}}-\frac{165a^4}{2\left(bx^2+a\right)^2b^{12}}-\frac{5a\,x^2}{b^{11}}+\frac{165a^3}{2\left(bx^2+a\right)b^{12}}+\frac{55a^2\ln\left(bx^2+a\right)}{2b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^2+a)^10,x)
```

```
[Out] -5*a*x^2/b^11+1/4*x^4/b^10+1/18*a^11/b^12/(b*x^2+a)^9-11/16*a^10/b^12/(b*x^
2+a)^8+55/14*a^9/b^12/(b*x^2+a)^7-55/4*a^8/b^12/(b*x^2+a)^6+33*a^7/b^12/(b*
x^2+a)^5-231/4*a^6/b^12/(b*x^2+a)^4+77*a^5/b^12/(b*x^2+a)^3-165/2*a^4/b^12/
(b*x^2+a)^2+165/2*a^3/b^12/(b*x^2+a)+55/2*a^2*ln(b*x^2+a)/b^12
```

maxima [A] time = 1.58, size = 231, normalized size = 1.13

$$\frac{83160\,a^3b^8x^{16}+582120\,a^4b^7x^{14}+1823976\,a^5b^6x^{12}+3318084\,a^6b^5x^{10}+3817044\,a^7b^4x^8+2835756\,a^8b^3x^6+1326204\,a^9b^2x^4+356499\,a^{10}bx^2+42131\,a^{11}}{1008\left(b^{21}x^{18}+9\,ab^{20}x^{16}+36\,a^2b^{19}x^{14}+84\,a^3b^{18}x^{12}+126\,a^4b^{17}x^{10}+126\,a^5b^{16}x^8+84\,a^6b^{15}x^6+36\,a^7b^{14}x^4+9\,a^8b^{13}x^2+a^9b^{12}\right)}+\frac{55\,a^2\log\left(bx^2+a\right)}{2\,b^{12}}+\frac{bx^4-20\,ax^2}{4\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/1008*(83160*a^3*b^8*x^16 + 582120*a^4*b^7*x^14 + 1823976*a^5*b^6*x^12 + 3
318084*a^6*b^5*x^10 + 3817044*a^7*b^4*x^8 + 2835756*a^8*b^3*x^6 + 1326204*a
```

```
^9*b^2*x^4 + 356499*a^10*b*x^2 + 42131*a^11)/(b^21*x^18 + 9*a*b^20*x^16 + 3
6*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 +
 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12) + 55/2*a^2*
log(b*x^2 + a)/b^12 + 1/4*(b*x^4 - 20*a*x^2)/b^11
```

mupad [B] time = 0.40, size = 230, normalized size = 1.12

$$\frac{\frac{42131\,a^{11}}{1008\,b}+\frac{39611\,a^{10}\,x^2}{112}+\frac{36839\,a^9\,b\,x^4}{28}+\frac{11253\,a^8\,b^2\,x^6}{4}+\frac{15147\,a^7\,b^3\,x^8}{4}+\frac{13167\,a^6\,b^4\,x^{10}}{4}+\frac{3619\,a^5\,b^5\,x^{12}}{2}+\frac{1155\,a^4\,b^6\,x^{14}}{2}+\frac{165\,a^3\,b^7\,x^{16}}{2}}{a^9\,b^{11}+9\,a^8\,b^{12}\,x^2+36\,a^7\,b^{13}\,x^4+84\,a^6\,b^{14}\,x^6+126\,a^5\,b^{15}\,x^8+126\,a^4\,b^{16}\,x^{10}+84\,a^3\,b^{17}\,x^{12}+36\,a^2\,b^{18}\,x^{14}+9\,a\,b^{19}\,x^{16}+b^{20}\,x^{18}}+\frac{x^4}{4\,b^{10}}-\frac{5\,a\,x^2}{b^{11}}+\frac{55\,a^2\,\ln\left(b\,x^2+a\right)}{2\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(a + b*x^2)^10,x)
```

```
[Out] ((42131*a^11)/(1008*b) + (39611*a^10*x^2)/112 + (36839*a^9*b*x^4)/28 + (112
53*a^8*b^2*x^6)/4 + (15147*a^7*b^3*x^8)/4 + (13167*a^6*b^4*x^10)/4 + (3619*
a^5*b^5*x^12)/2 + (1155*a^4*b^6*x^14)/2 + (165*a^3*b^7*x^16)/2)/(a^9*b^11 +
 b^20*x^18 + 9*a*b^19*x^16 + 9*a^8*b^12*x^2 + 36*a^7*b^13*x^4 + 84*a^6*b^14
*x^6 + 126*a^5*b^15*x^8 + 126*a^4*b^16*x^10 + 84*a^3*b^17*x^12 + 36*a^2*b^1
8*x^14) + x^4/(4*b^10) - (5*a*x^2)/b^11 + (55*a^2*log(a + b*x^2))/(2*b^12)
```

sympy [A] time = 2.06, size = 245, normalized size = 1.20

$$\frac{55a^2\log\left(a+bx^2\right)}{2b^{12}}-\frac{5ax^2}{b^{11}}+\frac{42131a^{11}+356499a^{10}bx^2+1326204a^9b^2x^4+2835756a^8b^3x^6+3817044a^7b^4x^8+3318084a^6b^5x^{10}+1823976a^5b^6x^{12}+582120a^4b^7x^{14}+83160a^3b^8x^{16}}{1008a^9b^{12}+9072a^8b^{13}x^2+36288a^7b^{14}x^4+84672a^6b^{15}x^6+127008a^5b^{16}x^8+127008a^4b^{17}x^{10}+84672a^3b^{18}x^{12}+36288a^2b^{19}x^{14}+9072ab^{20}x^{16}+1008b^{21}x^{18}}+\frac{x^4}{4b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**2+a)**10,x)
```

```
[Out] 55*a**2*log(a + b*x**2)/(2*b**12) - 5*a*x**2/b**11 + (42131*a**11 + 356499*
a**10*b*x**2 + 1326204*a**9*b**2*x**4 + 2835756*a**8*b**3*x**6 + 3817044*a*
*7*b**4*x**8 + 3318084*a**6*b**5*x**10 + 1823976*a**5*b**6*x**12 + 582120*a
**4*b**7*x**14 + 83160*a**3*b**8*x**16)/(1008*a**9*b**12 + 9072*a**8*b**13*
x**2 + 36288*a**7*b**14*x**4 + 84672*a**6*b**15*x**6 + 127008*a**5*b**16*x*
*8 + 127008*a**4*b**17*x**10 + 84672*a**3*b**18*x**12 + 36288*a**2*b**19*x*
*14 + 9072*a*b**20*x**16 + 1008*b**21*x**18) + x**4/(4*b**10)
```

3.194 $\int \frac{x^{21}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=188

$$-\frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9}+\frac{5a^9}{8b^{11}\left(a+bx^2\right)^8}-\frac{45a^8}{14b^{11}\left(a+bx^2\right)^7}+\frac{10a^7}{b^{11}\left(a+bx^2\right)^6}-\frac{21a^6}{b^{11}\left(a+bx^2\right)^5}+\frac{63a^5}{2b^{11}\left(a+bx^2\right)^4}-\frac{35}{b^{11}\left(a\right.}$$

Rubi [A] time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9}+\frac{5a^9}{8b^{11}\left(a+bx^2\right)^8}-\frac{45a^8}{14b^{11}\left(a+bx^2\right)^7}+\frac{10a^7}{b^{11}\left(a+bx^2\right)^6}-\frac{21a^6}{b^{11}\left(a+bx^2\right)^5}+\frac{63a^5}{2b^{11}\left(a+bx^2\right)^4}-\frac{35a^4}{b^{11}\left(a+bx^2\right)^3}+\frac{30a^3}{b^{11}\left(a+bx^2\right)^2}-\frac{45a^2}{2b^{11}\left(a+bx^2\right)}-\frac{5a\log\left(a+bx^2\right)}{b^{11}}+\frac{x^2}{2b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^21/(a + b*x^2)^10,x]
```

```
[Out] x^2/(2*b^10) - a^10/(18*b^11*(a + b*x^2)^9) + (5*a^9)/(8*b^11*(a + b*x^2)^8
) - (45*a^8)/(14*b^11*(a + b*x^2)^7) + (10*a^7)/(b^11*(a + b*x^2)^6) - (21*
a^6)/(b^11*(a + b*x^2)^5) + (63*a^5)/(2*b^11*(a + b*x^2)^4) - (35*a^4)/(b^1
1*(a + b*x^2)^3) + (30*a^3)/(b^11*(a + b*x^2)^2) - (45*a^2)/(2*b^11*(a + b*
x^2)) - (5*a*Log[a + b*x^2])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{21}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^{10}}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^{10}} + \frac{a^{10}}{b^{10}(a+bx)^{10}} - \frac{10a^9}{b^{10}(a+bx)^9} + \frac{45a^8}{b^{10}(a+bx)^8} - \frac{120a^7}{b^{10}(a+bx)^7} + \frac{210a^6}{b^{10}(a+bx)}\right.\right.$$
$$= \frac{x^2}{2b^{10}} - \frac{a^{10}}{18b^{11}\left(a+bx^2\right)^9} + \frac{5a^9}{8b^{11}\left(a+bx^2\right)^8} - \frac{45a^8}{14b^{11}\left(a+bx^2\right)^7} + \frac{10a^7}{b^{11}\left(a+bx^2\right)^6} - \frac{21a^6}{b^{11}\left(a+bx\right.}$$

Mathematica [A] time = 0.04, size = 145, normalized size = 0.77

$$-\frac{4861a^{10} + 41229a^9bx^2 + 153576a^8b^2x^4 + 328104a^7b^3x^6 + 439236a^6b^4x^8 + 375732a^5b^5x^{10} + 197568a^4b^6x^{12} + 54432a^3b^7x^{14} + 2268a^2b^8x^{16} - 2268ab^9x^{18} + 2520a\left(a+bx^2\right)^9\log\left(a+bx^2\right) - 252b^{10}x^{20}}{504b^{11}\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^21/(a + b*x^2)^10,x]
```

```
[Out] -1/504*(4861*a^10 + 41229*a^9*b*x^2 + 153576*a^8*b^2*x^4 + 328104*a^7*b^3*x
^6 + 439236*a^6*b^4*x^8 + 375732*a^5*b^5*x^10 + 197568*a^4*b^6*x^12 + 54432
*a^3*b^7*x^14 + 2268*a^2*b^8*x^16 - 2268*a*b^9*x^18 - 252*b^10*x^20 + 2520*
a*(a + b*x^2)^9*Log[a + b*x^2])/(b^11*(a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{21}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^21/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^21/(a + b*x^2)^10, x]
```

fricas [A] time = 0.71, size = 322, normalized size = 1.71

$$\frac{252\,b^{10}x^{20} + 2268\,ab^9x^{18} - 2268\,a^2b^8x^{16} - 54432\,a^3b^7x^{14} - 197568\,a^4b^6x^{12} - 375732\,a^5b^5x^{10} - 439236\,a^6b^4x^8 - 328104\,a^7b^3x^6 - 153576\,a^8b^2x^4 - 41229\,a^9bx^2 - 4861\,a^{10} - 2520\left(ab^9x^{18} + 9\,a^2b^8x^{16} + 36\,a^3b^7x^{14} + 84\,a^4b^6x^{12} + 126\,a^5b^5x^{10} + 126\,a^6b^4x^8 + 84\,a^7b^3x^6 + 36\,a^8b^2x^4 + 9\,a^9bx^2 + a^{10}\right)\log\left(bx^2+a\right)}{504\left(b^{20}x^{18} + 9\,ab^{19}x^{16} + 36\,a^2b^{18}x^{14} + 84\,a^3b^{17}x^{12} + 126\,a^4b^{16}x^{10} + 126\,a^5b^{15}x^8 + 84\,a^6b^{14}x^6 + 36\,a^7b^{13}x^4 + 9\,a^8b^{12}x^2 + a^9b^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^10*x^20 + 2268*a*b^9*x^18 - 2268*a^2*b^8*x^16 - 54432*a^3*b^7*
x^14 - 197568*a^4*b^6*x^12 - 375732*a^5*b^5*x^10 - 439236*a^6*b^4*x^8 - 328
```

```
104*a^7*b^3*x^6 - 153576*a^8*b^2*x^4 - 41229*a^9*b*x^2 - 4861*a^10 - 2520*(
a*b^9*x^18 + 9*a^2*b^8*x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b
^5*x^10 + 126*a^6*b^4*x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 +
 a^10)*log(b*x^2 + a))/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a
^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*
a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11)
```

giac [A] time = 0.63, size = 139, normalized size = 0.74

$$\frac{x^2}{2\,b^{10}} - \frac{5\,a\log\left(\left|bx^2 + a\right|\right)}{b^{11}} + \frac{7129\,ab^9x^{18} + 52821\,a^2b^8x^{16} + 181044\,a^3b^7x^{14} + 369516\,a^4b^6x^{12} + 490770\,a^5b^5x^{10} + 437850\,a^6b^4x^8 + 261660\,a^7b^3x^6 + 100800\,a^8b^2x^4 + 22680\,a^9bx^2 + 2268\,a^{10}}{504\left(bx^2 + a\right)^9 b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^10 - 5*a*log(abs(b*x^2 + a))/b^11 + 1/504*(7129*a*b^9*x^18 + 5282
1*a^2*b^8*x^16 + 181044*a^3*b^7*x^14 + 369516*a^4*b^6*x^12 + 490770*a^5*b^5
*x^10 + 437850*a^6*b^4*x^8 + 261660*a^7*b^3*x^6 + 100800*a^8*b^2*x^4 + 2268
0*a^9*b*x^2 + 2268*a^10)/((b*x^2 + a)^9*b^11)
```

maple [A] time = 0.02, size = 177, normalized size = 0.94

$$-\frac{a^{10}}{18\left(b\,x^2 + a\right)^9 b^{11}} + \frac{5a^9}{8\left(b\,x^2 + a\right)^8 b^{11}} - \frac{45a^8}{14\left(b\,x^2 + a\right)^7 b^{11}} + \frac{10a^7}{\left(b\,x^2 + a\right)^6 b^{11}} - \frac{21a^6}{\left(b\,x^2 + a\right)^5 b^{11}} + \frac{63a^5}{2\left(b\,x^2 + a\right)^4 b^{11}} - \frac{35a^4}{\left(b\,x^2 + a\right)^3 b^{11}} + \frac{30a^3}{\left(b\,x^2 + a\right)^2 b^{11}} + \frac{x^2}{2b^{10}} - \frac{45a^2}{2\left(b\,x^2 + a\right)b^{11}} - \frac{5a\ln\left(b\,x^2 + a\right)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^21/(b*x^2+a)^10,x)
```

```
[Out] 1/2*x^2/b^10-1/18*a^10/b^11/(b*x^2+a)^9+5/8*a^9/b^11/(b*x^2+a)^8-45/14*a^8/
b^11/(b*x^2+a)^7+10*a^7/b^11/(b*x^2+a)^6-21*a^6/b^11/(b*x^2+a)^5+63/2*a^5/b
^11/(b*x^2+a)^4-35*a^4/b^11/(b*x^2+a)^3+30*a^3/b^11/(b*x^2+a)^2-45/2*a^2/b^
11/(b*x^2+a)-5*a*ln(b*x^2+a)/b^11
```

maxima [A] time = 1.60, size = 220, normalized size = 1.17

$$-\frac{11340\,a^2b^8x^{16} + 75600\,a^3b^7x^{14} + 229320\,a^4b^6x^{12} + 407484\,a^5b^5x^{10} + 460404\,a^6b^4x^8 + 337176\,a^7b^3x^6 + 155844\,a^8b^2x^4 + 41481\,a^9bx^2 + 4861\,a^{10}}{504\left(b^{20}x^{18} + 9\,ab^{19}x^{16} + 36\,a^2b^{18}x^{14} + 84\,a^3b^{17}x^{12} + 126\,a^4b^{16}x^{10} + 126\,a^5b^{15}x^8 + 84\,a^6b^{14}x^6 + 36\,a^7b^{13}x^4 + 9\,a^8b^{12}x^2 + a^9b^{11}\right)} + \frac{x^2}{2\,b^{10}} - \frac{5\,a\log\left(bx^2 + a\right)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^21/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(11340*a^2*b^8*x^16 + 75600*a^3*b^7*x^14 + 229320*a^4*b^6*x^12 + 407
484*a^5*b^5*x^10 + 460404*a^6*b^4*x^8 + 337176*a^7*b^3*x^6 + 155844*a^8*b^2
*x^4 + 41481*a^9*b*x^2 + 4861*a^10)/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^1
8*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b
```

```
^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11) + 1/2*x^2/b^10 - 5*a
*log(b*x^2 + a)/b^11
```

mupad [B] time = 0.43, size = 220, normalized size = 1.17

$$\frac{x^2}{2\,b^{10}}-\frac{\frac{4861\,a^{10}}{504\,b}+\frac{4609\,a^9\,x^2}{56}+\frac{4329\,a^8\,b\,x^4}{14}+669\,a^7\,b^2\,x^6+\frac{1827\,a^6\,b^3\,x^8}{2}+\frac{1617\,a^5\,b^4\,x^{10}}{2}+455\,a^4\,b^5\,x^{12}+150\,a^3\,b^6\,x^{14}+\frac{45\,a^2\,b^7\,x^{16}}{2}}{a^9\,b^{10}+9\,a^8\,b^{11}\,x^2+36\,a^7\,b^{12}\,x^4+84\,a^6\,b^{13}\,x^6+126\,a^5\,b^{14}\,x^8+126\,a^4\,b^{15}\,x^{10}+84\,a^3\,b^{16}\,x^{12}+36\,a^2\,b^{17}\,x^{14}+9\,a\,b^{18}\,x^{16}+b^{19}\,x^{18}}-\frac{5\,a\,\ln\left(b\,x^2+a\right)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^21/(a + b*x^2)^10,x)
```

```
[Out] x^2/(2*b^10) - ((4861*a^10)/(504*b) + (4609*a^9*x^2)/56 + (4329*a^8*b*x^4)/
14 + 669*a^7*b^2*x^6 + (1827*a^6*b^3*x^8)/2 + (1617*a^5*b^4*x^10)/2 + 455*a
^4*b^5*x^12 + 150*a^3*b^6*x^14 + (45*a^2*b^7*x^16)/2)/(a^9*b^10 + b^19*x^18
 + 9*a*b^18*x^16 + 9*a^8*b^11*x^2 + 36*a^7*b^12*x^4 + 84*a^6*b^13*x^6 + 126
*a^5*b^14*x^8 + 126*a^4*b^15*x^10 + 84*a^3*b^16*x^12 + 36*a^2*b^17*x^14) -
(5*a*log(a + b*x^2))/b^11
```

sympy [A] time = 2.03, size = 233, normalized size = 1.24

$$-\frac{5a\log\left(a+bx^2\right)}{b^{11}}+\frac{-4861a^{10}-41481a^9bx^2-155844a^8b^2x^4-337176a^7b^3x^6-460404a^6b^4x^8-407484a^5b^5x^{10}-229320a^4b^6x^{12}-75600a^3b^7x^{14}-11340a^2b^8x^{16}}{504a^9b^{11}+4536a^8b^{12}x^2+18144a^7b^{13}x^4+42336a^6b^{14}x^6+63504a^5b^{15}x^8+63504a^4b^{16}x^{10}+42336a^3b^{17}x^{12}+18144a^2b^{18}x^{14}+4536ab^{19}x^{16}+504b^{20}x^{18}}+\frac{x^2}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**21/(b*x**2+a)**10,x)
```

```
[Out] -5*a*log(a + b*x**2)/b**11 + (-4861*a**10 - 41481*a**9*b*x**2 - 155844*a**8
*b**2*x**4 - 337176*a**7*b**3*x**6 - 460404*a**6*b**4*x**8 - 407484*a**5*b*
*5*x**10 - 229320*a**4*b**6*x**12 - 75600*a**3*b**7*x**14 - 11340*a**2*b**8
*x**16)/(504*a**9*b**11 + 4536*a**8*b**12*x**2 + 18144*a**7*b**13*x**4 + 42
336*a**6*b**14*x**6 + 63504*a**5*b**15*x**8 + 63504*a**4*b**16*x**10 + 4233
6*a**3*b**17*x**12 + 18144*a**2*b**18*x**14 + 4536*a*b**19*x**16 + 504*b**2
0*x**18) + x**2/(2*b**10)
```

3.195 $\int \frac{x^{19}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=179

$$\frac{a^9}{18b^{10}\left(a+bx^2\right)^9}-\frac{9a^8}{16b^{10}\left(a+bx^2\right)^8}+\frac{18a^7}{7b^{10}\left(a+bx^2\right)^7}-\frac{7a^6}{b^{10}\left(a+bx^2\right)^6}+\frac{63a^5}{5b^{10}\left(a+bx^2\right)^5}-\frac{63a^4}{4b^{10}\left(a+bx^2\right)^4}+\frac{14}{b^{10}\left(a-\right.}$$

Rubi [A] time = 0.17, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^9}{18b^{10}\left(a+bx^2\right)^9}-\frac{9a^8}{16b^{10}\left(a+bx^2\right)^8}+\frac{18a^7}{7b^{10}\left(a+bx^2\right)^7}-\frac{7a^6}{b^{10}\left(a+bx^2\right)^6}+\frac{63a^5}{5b^{10}\left(a+bx^2\right)^5}-\frac{63a^4}{4b^{10}\left(a+bx^2\right)^4}+\frac{14a^3}{b^{10}\left(a+bx^2\right)^3}-\frac{9a^2}{b^{10}\left(a+bx^2\right)^2}+\frac{9a}{2b^{10}\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a + b*x^2)^10,x]
```

```
[Out] a^9/(18*b^10*(a + b*x^2)^9) - (9*a^8)/(16*b^10*(a + b*x^2)^8) + (18*a^7)/(7
*b^10*(a + b*x^2)^7) - (7*a^6)/(b^10*(a + b*x^2)^6) + (63*a^5)/(5*b^10*(a +
 b*x^2)^5) - (63*a^4)/(4*b^10*(a + b*x^2)^4) + (14*a^3)/(b^10*(a + b*x^2)^3
) - (9*a^2)/(b^10*(a + b*x^2)^2) + (9*a)/(2*b^10*(a + b*x^2)) + Log[a + b*x
^2]/(2*b^10)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{19}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^9}{(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^9}{b^9(a+bx)^{10}} + \frac{9a^8}{b^9(a+bx)^9} - \frac{36a^7}{b^9(a+bx)^8} + \frac{84a^6}{b^9(a+bx)^7} - \frac{126a^5}{b^9(a+bx)^6} + \frac{1}{b^9(a\right.\right.$$

$$= \frac{a^9}{18b^{10}\left(a+bx^2\right)^9} - \frac{9a^8}{16b^{10}\left(a+bx^2\right)^8} + \frac{18a^7}{7b^{10}\left(a+bx^2\right)^7} - \frac{7a^6}{b^{10}\left(a+bx^2\right)^6} + \frac{63a^5}{5b^{10}\left(a+bx^2\right)^5} -$$

Mathematica [A] time = 0.03, size = 116, normalized size = 0.65

$$\frac{\frac{a\left(7129a^8+61641a^7bx^2+235224a^6b^2x^4+518616a^5b^3x^6+725004a^4b^4x^8+661500a^3b^5x^{10}+388080a^2b^6x^{12}+136080ab^7x^{14}+22680b^8x^{16}\right)}{\left(a+bx^2\right)^9}+2520\log\left(a+bx^2\right)}{5040b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a + b*x^2)^10,x]
```

```
[Out] ((a*(7129*a^8 + 61641*a^7*b*x^2 + 235224*a^6*b^2*x^4 + 518616*a^5*b^3*x^6 +
 725004*a^4*b^4*x^8 + 661500*a^3*b^5*x^10 + 388080*a^2*b^6*x^12 + 136080*a*
b^7*x^14 + 22680*b^8*x^16))/(a + b*x^2)^9 + 2520*Log[a + b*x^2])/(5040*b^10
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^19/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^19/(a + b*x^2)^10, x]
```

fricas [A] time = 1.30, size = 300, normalized size = 1.68

$$\frac{22680\,ab^8x^{16}+136080\,a^2b^7x^{14}+388080\,a^3b^6x^{12}+661500\,a^4b^5x^{10}+725004\,a^5b^4x^8+518616\,a^6b^3x^6+235224\,a^7b^2x^4+61641\,a^8bx^2+7129\,a^9+2520\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\log\left(bx^2+a\right)}{5040\left(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}x^{14}+84\,a^3b^{16}x^{12}+126\,a^4b^{15}x^{10}+126\,a^5b^{14}x^8+84\,a^6b^{13}x^6+36\,a^7b^{12}x^4+9\,a^8b^{11}x^2+a^9b^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/5040*(22680*a*b^8*x^16 + 136080*a^2*b^7*x^14 + 388080*a^3*b^6*x^12 + 6615
00*a^4*b^5*x^10 + 725004*a^5*b^4*x^8 + 518616*a^6*b^3*x^6 + 235224*a^7*b^2*
x^4 + 61641*a^8*b*x^2 + 7129*a^9 + 2520*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b
^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3
*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(b*x^2 + a))/(b^19*x^18 + 9*a
*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*
a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^1
0)
```

giac [A] time = 0.61, size = 119, normalized size = 0.66

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b^{10}}-\frac{7129\,b^8x^{18}+41481\,ab^7x^{16}+120564\,a^2b^6x^{14}+210756\,a^3b^5x^{12}+236754\,a^4b^4x^{10}+173250\,a^5b^3x^8+80220\,a^6b^2x^6+21420\,a^7bx^4+2520\,a^8x^2}{5040\left(bx^2+a\right)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^10 - 1/5040*(7129*b^8*x^18 + 41481*a*b^7*x^16 + 1
20564*a^2*b^6*x^14 + 210756*a^3*b^5*x^12 + 236754*a^4*b^4*x^10 + 173250*a^5
*b^3*x^8 + 80220*a^6*b^2*x^6 + 21420*a^7*b*x^4 + 2520*a^8*x^2)/((b*x^2 + a)
^9*b^9)
```

maple [A] time = 0.01, size = 166, normalized size = 0.93

$$\frac{a^9}{18\left(bx^2+a\right)^9b^{10}}-\frac{9a^8}{16\left(bx^2+a\right)^8b^{10}}+\frac{18a^7}{7\left(bx^2+a\right)^7b^{10}}-\frac{7a^6}{\left(bx^2+a\right)^6b^{10}}+\frac{63a^5}{5\left(bx^2+a\right)^5b^{10}}-\frac{63a^4}{4\left(bx^2+a\right)^4b^{10}}+\frac{14a^3}{\left(bx^2+a\right)^3b^{10}}-\frac{9a^2}{\left(bx^2+a\right)^2b^{10}}+\frac{9a}{2\left(bx^2+a\right)b^{10}}+\frac{\ln\left(bx^2+a\right)}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^2+a)^10,x)
```

```
[Out] 1/18*a^9/b^10/(b*x^2+a)^9-9/16*a^8/b^10/(b*x^2+a)^8+18/7*a^7/b^10/(b*x^2+a)
^7-7*a^6/b^10/(b*x^2+a)^6+63/5*a^5/b^10/(b*x^2+a)^5-63/4*a^4/b^10/(b*x^2+a)
^4+14*a^3/b^10/(b*x^2+a)^3-9*a^2/b^10/(b*x^2+a)^2+9/2*a/b^10/(b*x^2+a)+1/2*
ln(b*x^2+a)/b^10
```

maxima [A] time = 1.52, size = 209, normalized size = 1.17

$$\frac{22680\,ab^8x^{16}+136080\,a^2b^7x^{14}+388080\,a^3b^6x^{12}+661500\,a^4b^5x^{10}+725004\,a^5b^4x^8+518616\,a^6b^3x^6+235224\,a^7b^2x^4+61641\,a^8bx^2+7129\,a^9}{5040\left(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}x^{14}+84\,a^3b^{16}x^{12}+126\,a^4b^{15}x^{10}+126\,a^5b^{14}x^8+84\,a^6b^{13}x^6+36\,a^7b^{12}x^4+9\,a^8b^{11}x^2+a^9b^{10}\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/5040*(22680*a*b^8*x^16 + 136080*a^2*b^7*x^14 + 388080*a^3*b^6*x^12 + 6615
00*a^4*b^5*x^10 + 725004*a^5*b^4*x^8 + 518616*a^6*b^3*x^6 + 235224*a^7*b^2*
```

```
x^4 + 61641*a^8*b*x^2 + 7129*a^9)/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*
x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^1
3*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10) + 1/2*log(b*x^2 + a)/b
^10
```

mupad [B] time = 5.36, size = 207, normalized size = 1.16

$$\frac{\frac{7129\,a^9}{5040\,b^{10}}+\frac{9\,a\,x^{16}}{2\,b^2}+\frac{27\,a^2\,x^{14}}{b^3}+\frac{77\,a^3\,x^{12}}{b^4}+\frac{525\,a^4\,x^{10}}{4\,b^5}+\frac{2877\,a^5\,x^8}{20\,b^6}+\frac{1029\,a^6\,x^6}{10\,b^7}+\frac{3267\,a^7\,x^4}{70\,b^8}+\frac{6849\,a^8\,x^2}{560\,b^9}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{\ln\left(b\,x^2+a\right)}{2\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(a + b*x^2)^10,x)
```

```
[Out] ((7129*a^9)/(5040*b^10) + (9*a*x^16)/(2*b^2) + (27*a^2*x^14)/b^3 + (77*a^3*
x^12)/b^4 + (525*a^4*x^10)/(4*b^5) + (2877*a^5*x^8)/(20*b^6) + (1029*a^6*x^
6)/(10*b^7) + (3267*a^7*x^4)/(70*b^8) + (6849*a^8*x^2)/(560*b^9))/(a^9 + b^
9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126
*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) + log(
a + b*x^2)/(2*b^10)
```

sympy [A] time = 1.84, size = 219, normalized size = 1.22

$$\frac{7129a^9+61641a^8bx^2+235224a^7b^2x^4+518616a^6b^3x^6+725004a^5b^4x^8+661500a^4b^5x^{10}+388080a^3b^6x^{12}+136080a^2b^7x^{14}+22680ab^8x^{16}}{5040a^9b^{10}+45360a^8b^{11}x^2+181440a^7b^{12}x^4+423360a^6b^{13}x^6+635040a^5b^{14}x^8+635040a^4b^{15}x^{10}+423360a^3b^{16}x^{12}+181440a^2b^{17}x^{14}+45360ab^{18}x^{16}+5040b^{19}x^{18}}+\frac{\log\left(a+bx^2\right)}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**2+a)**10,x)
```

```
[Out] (7129*a**9 + 61641*a**8*b*x**2 + 235224*a**7*b**2*x**4 + 518616*a**6*b**3*x
**6 + 725004*a**5*b**4*x**8 + 661500*a**4*b**5*x**10 + 388080*a**3*b**6*x**
12 + 136080*a**2*b**7*x**14 + 22680*a*b**8*x**16)/(5040*a**9*b**10 + 45360*
a**8*b**11*x**2 + 181440*a**7*b**12*x**4 + 423360*a**6*b**13*x**6 + 635040*
a**5*b**14*x**8 + 635040*a**4*b**15*x**10 + 423360*a**3*b**16*x**12 + 18144
0*a**2*b**17*x**14 + 45360*a*b**18*x**16 + 5040*b**19*x**18) + log(a + b*x*
*2)/(2*b**10)
```

3.196 $\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=19

$$\frac{x^{18}}{18a(a+bx^2)^9}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{x^{18}}{18a(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/(a + b*x^2)^10,x]
```

```
[Out] x^18/(18*a*(a + b*x^2)^9)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx = \frac{x^{18}}{18a(a+bx^2)^9}$$

Mathematica [B] time = 0.02, size = 101, normalized size = 5.32

$$-\frac{a^8+9a^7bx^2+36a^6b^2x^4+84a^5b^3x^6+126a^4b^4x^8+126a^3b^5x^{10}+84a^2b^6x^{12}+36ab^7x^{14}+9b^8x^{16}}{18b^9(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/(a + b*x^2)^10,x]
```

```
[Out] -1/18*(a^8 + 9*a^7*b*x^2 + 36*a^6*b^2*x^4 + 84*a^5*b^3*x^6 + 126*a^4*b^4*x^
8 + 126*a^3*b^5*x^10 + 84*a^2*b^6*x^12 + 36*a*b^7*x^14 + 9*b^8*x^16)/(b^9*(
a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^17/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^17/(a + b*x^2)^10, x]
```

fricas [B] time = 1.35, size = 190, normalized size = 10.00

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\left(b^{18}x^{18}+9\,ab^{17}x^{16}+36\,a^2b^{16}x^{14}+84\,a^3b^{15}x^{12}+126\,a^4b^{14}x^{10}+126\,a^5b^{13}x^8+84\,a^6b^{12}x^6+36\,a^7b^{11}x^4+9\,a^8b^{10}x^2+a^9b^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^18*
x^18 + 9*a*b^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x
^10 + 126*a^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2
 + a^9*b^9)
```

giac [B] time = 0.64, size = 99, normalized size = 5.21

$$-\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\left(bx^2+a\right)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/((b*x^
2 + a)^9*b^9)
```

maple [B] time = 0.01, size = 150, normalized size = 7.89

$$-\frac{a^8}{18\left(bx^2+a\right)^9b^9}+\frac{a^7}{2\left(bx^2+a\right)^8b^9}-\frac{2a^6}{\left(bx^2+a\right)^7b^9}+\frac{14a^5}{3\left(bx^2+a\right)^6b^9}-\frac{7a^4}{\left(bx^2+a\right)^5b^9}+\frac{7a^3}{\left(bx^2+a\right)^4b^9}-\frac{14a^2}{3\left(bx^2+a\right)^3b^9}+\frac{2a}{\left(bx^2+a\right)^2b^9}-\frac{1}{2\left(bx^2+a\right)b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^2+a)^10,x)
```

```
[Out] 1/2*a^7/b^9/(b*x^2+a)^8-1/2/b^9/(b*x^2+a)-2*a^6/b^9/(b*x^2+a)^7-7*a^4/b^9/(
b*x^2+a)^5+7*a^3/b^9/(b*x^2+a)^4-14/3*a^2/b^9/(b*x^2+a)^3+14/3*a^5/b^9/(b*x
^2+a)^6-1/18*a^8/b^9/(b*x^2+a)^9+2*a/b^9/(b*x^2+a)^2
```

maxima [B] time = 1.50, size = 190, normalized size = 10.00

$$-\frac{9 b^{8} x^{16} + 36 a b^{7} x^{14} + 84 a^{2} b^{6} x^{12} + 126 a^{3} b^{5} x^{10} + 126 a^{4} b^{4} x^{8} + 84 a^{5} b^{3} x^{6} + 36 a^{6} b^{2} x^{4} + 9 a^{7} b x^{2} + a^{8}}{18 \left(b^{18} x^{18} + 9 a b^{17} x^{16} + 36 a^{2} b^{16} x^{14} + 84 a^{3} b^{15} x^{12} + 126 a^{4} b^{14} x^{10} + 126 a^{5} b^{13} x^{8} + 84 a^{6} b^{12} x^{6} + 36 a^{7} b^{11} x^{4} + 9 a^{8} b^{10} x^{2} + a^{9} b^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 12
6*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^18*
x^18 + 9*a*b^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x
^10 + 126*a^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2
 + a^9*b^9)
```

mupad [B] time = 0.12, size = 192, normalized size = 10.11

$$-\frac{a^{8} + 9 a^{7} b x^{2} + 36 a^{6} b^{2} x^{4} + 84 a^{5} b^{3} x^{6} + 126 a^{4} b^{4} x^{8} + 126 a^{3} b^{5} x^{10} + 84 a^{2} b^{6} x^{12} + 36 a b^{7} x^{14} + 9 b^{8} x^{16}}{18 a^{9} b^{9} + 162 a^{8} b^{10} x^{2} + 648 a^{7} b^{11} x^{4} + 1512 a^{6} b^{12} x^{6} + 2268 a^{5} b^{13} x^{8} + 2268 a^{4} b^{14} x^{10} + 1512 a^{3} b^{15} x^{12} + 648 a^{2} b^{16} x^{14} + 162 a b^{17} x^{16} + 18 b^{18} x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(a + b*x^2)^10,x)
```

```
[Out] -(a^8 + 9*b^8*x^16 + 9*a^7*b*x^2 + 36*a*b^7*x^14 + 36*a^6*b^2*x^4 + 84*a^5*
b^3*x^6 + 126*a^4*b^4*x^8 + 126*a^3*b^5*x^10 + 84*a^2*b^6*x^12)/(18*a^9*b^9
 + 18*b^18*x^18 + 162*a*b^17*x^16 + 162*a^8*b^10*x^2 + 648*a^7*b^11*x^4 + 1
512*a^6*b^12*x^6 + 2268*a^5*b^13*x^8 + 2268*a^4*b^14*x^10 + 1512*a^3*b^15*x
^12 + 648*a^2*b^16*x^14)
```

sympy [B] time = 1.70, size = 202, normalized size = 10.63

$$\frac{-a^{8} - 9a^{7}bx^{2} - 36a^{6}b^{2}x^{4} - 84a^{5}b^{3}x^{6} - 126a^{4}b^{4}x^{8} - 126a^{3}b^{5}x^{10} - 84a^{2}b^{6}x^{12} - 36ab^{7}x^{14} - 9b^{8}x^{16}}{18a^{9}b^{9} + 162a^{8}b^{10}x^{2} + 648a^{7}b^{11}x^{4} + 1512a^{6}b^{12}x^{6} + 2268a^{5}b^{13}x^{8} + 2268a^{4}b^{14}x^{10} + 1512a^{3}b^{15}x^{12} + 648a^{2}b^{16}x^{14} + 162ab^{17}x^{16} + 18b^{18}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**2+a)**10,x)
```

```
[Out] (-a**8 - 9*a**7*b*x**2 - 36*a**6*b**2*x**4 - 84*a**5*b**3*x**6 - 126*a**4*b
**4*x**8 - 126*a**3*b**5*x**10 - 84*a**2*b**6*x**12 - 36*a*b**7*x**14 - 9*b
**8*x**16)/(18*a**9*b**9 + 162*a**8*b**10*x**2 + 648*a**7*b**11*x**4 + 1512
*a**6*b**12*x**6 + 2268*a**5*b**13*x**8 + 2268*a**4*b**14*x**10 + 1512*a**3
*b**15*x**12 + 648*a**2*b**16*x**14 + 162*a*b**17*x**16 + 18*b**18*x**18)
```

3.197 $\int \frac{x^{15}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=39

$$\frac{x^{16}}{144a^2\left(a+bx^2\right)^8}+\frac{x^{16}}{18a\left(a+bx^2\right)^9}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{x^{16}}{144a^2\left(a+bx^2\right)^8}+\frac{x^{16}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^2)^10,x]
```

```
[Out] x^16/(18*a*(a + b*x^2)^9) + x^16/(144*a^2*(a + b*x^2)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^7}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{x^{16}}{18a\left(a+bx^2\right)^9} + \frac{\text{Subst}\left(\int \frac{x^7}{(a+bx)^9}\,dx, x, x^2\right)}{18a}$$
$$= \frac{x^{16}}{18a\left(a+bx^2\right)^9} + \frac{x^{16}}{144a^2\left(a+bx^2\right)^8}$$

Mathematica [B] time = 0.02, size = 90, normalized size = 2.31

$$-\frac{a^7+9a^6bx^2+36a^5b^2x^4+84a^4b^3x^6+126a^3b^4x^8+126a^2b^5x^{10}+84ab^6x^{12}+36b^7x^{14}}{144b^8\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^2)^10,x]
```

```
[Out] -1/144*(a^7 + 9*a^6*b*x^2 + 36*a^5*b^2*x^4 + 84*a^4*b^3*x^6 + 126*a^3*b^4*x
^8 + 126*a^2*b^5*x^10 + 84*a*b^6*x^12 + 36*b^7*x^14)/(b^8*(a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^15/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^15/(a + b*x^2)^10, x]
```

fricas [B] time = 0.86, size = 179, normalized size = 4.59

$$-\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2+a^7}{144\left(b^{17}x^{18}+9\,ab^{16}x^{16}+36\,a^2b^{15}x^{14}+84\,a^3b^{14}x^{12}+126\,a^4b^{13}x^{10}+126\,a^5b^{12}x^8+84\,a^6b^{11}x^6+36\,a^7b^{10}x^4+9\,a^8b^9x^2+a^9b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="fricas")
```

[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 + 84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/(b^17*x^18 + 9*a*b^16*x^16 + 36*a^2*b^15*x^14 + 84*a^3*b^14*x^12 + 126*a^4*b^13*x^10 + 126*a^5*b^12*x^8 + 84*a^6*b^11*x^6 + 36*a^7*b^10*x^4 + 9*a^8*b^9*x^2 + a^9*b^8)

giac [B] time = 0.63, size = 88, normalized size = 2.26

$$-\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2+a^7}{144\left(bx^2+a\right)^9b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="giac")

[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 + 84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/((b*x^2 + a)^9*b^8)

maple [B] time = 0.01, size = 133, normalized size = 3.41

$$\frac{a^7}{18\left(bx^2+a\right)^9b^8}-\frac{7a^6}{16\left(bx^2+a\right)^8b^8}+\frac{3a^5}{2\left(bx^2+a\right)^7b^8}-\frac{35a^4}{12\left(bx^2+a\right)^6b^8}+\frac{7a^3}{2\left(bx^2+a\right)^5b^8}-\frac{21a^2}{8\left(bx^2+a\right)^4b^8}+\frac{7a}{6\left(bx^2+a\right)^3b^8}-\frac{1}{4\left(bx^2+a\right)^2b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^15/(b*x^2+a)^10,x)

[Out] -7/16*a^6/b^8/(b*x^2+a)^8+3/2*a^5/b^8/(b*x^2+a)^7+7/2*a^3/b^8/(b*x^2+a)^5-21/8*a^2/b^8/(b*x^2+a)^4+7/6*a/b^8/(b*x^2+a)^3+1/18*a^7/b^8/(b*x^2+a)^9-35/12*a^4/b^8/(b*x^2+a)^6-1/4/b^8/(b*x^2+a)^2

maxima [B] time = 1.47, size = 179, normalized size = 4.59

$$-\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2+a^7}{144\left(b^{17}x^{18}+9\,ab^{16}x^{16}+36\,a^2b^{15}x^{14}+84\,a^3b^{14}x^{12}+126\,a^4b^{13}x^{10}+126\,a^5b^{12}x^8+84\,a^6b^{11}x^6+36\,a^7b^{10}x^4+9\,a^8b^9x^2+a^9b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b*x^2+a)^10,x, algorithm="maxima")

[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 + 84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/(b^17*x^18 + 9*a*b^16*x^16 + 36*a^2*b^15*x^14 + 84*a^3*b^14*x^12 + 126*a^4*b^13*x^10 + 126*a^5*b^12*x^8 + 84*a^6*b^11*x^6 + 36*a^7*b^10*x^4 + 9*a^8*b^9*x^2 + a^9*b^8)

mupad [B] time = 4.89, size = 181, normalized size = 4.64

$$-\frac{a^7+9\,a^6\,b\,x^2+36\,a^5\,b^2\,x^4+84\,a^4\,b^3\,x^6+126\,a^3\,b^4\,x^8+126\,a^2\,b^5\,x^{10}+84\,a\,b^6\,x^{12}+36\,b^7\,x^{14}}{144\,a^9\,b^8+1296\,a^8\,b^9\,x^2+5184\,a^7\,b^{10}\,x^4+12096\,a^6\,b^{11}\,x^6+18144\,a^5\,b^{12}\,x^8+18144\,a^4\,b^{13}\,x^{10}+12096\,a^3\,b^{14}\,x^{12}+5184\,a^2\,b^{15}\,x^{14}+1296\,a\,b^{16}\,x^{16}+144\,b^{17}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a + b*x^2)^10,x)
```

```
[Out] -(a^7 + 36*b^7*x^14 + 9*a^6*b*x^2 + 84*a*b^6*x^12 + 36*a^5*b^2*x^4 + 84*a^4
*b^3*x^6 + 126*a^3*b^4*x^8 + 126*a^2*b^5*x^10)/(144*a^9*b^8 + 144*b^17*x^18
 + 1296*a*b^16*x^16 + 1296*a^8*b^9*x^2 + 5184*a^7*b^10*x^4 + 12096*a^6*b^11
*x^6 + 18144*a^5*b^12*x^8 + 18144*a^4*b^13*x^10 + 12096*a^3*b^14*x^12 + 518
4*a^2*b^15*x^14)
```

sympy [B] time = 1.42, size = 190, normalized size = 4.87

$$\frac{-a^7 - 9a^6bx^2 - 36a^5b^2x^4 - 84a^4b^3x^6 - 126a^3b^4x^8 - 126a^2b^5x^{10} - 84ab^6x^{12} - 36b^7x^{14}}{144a^9b^8 + 1296a^8b^9x^2 + 5184a^7b^{10}x^4 + 12096a^6b^{11}x^6 + 18144a^5b^{12}x^8 + 18144a^4b^{13}x^{10} + 12096a^3b^{14}x^{12} + 5184a^2b^{15}x^{14} + 1296ab^{16}x^{16} + 144b^{17}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**2+a)**10,x)
```

```
[Out] (-a**7 - 9*a**6*b*x**2 - 36*a**5*b**2*x**4 - 84*a**4*b**3*x**6 - 126*a**3*b
**4*x**8 - 126*a**2*b**5*x**10 - 84*a*b**6*x**12 - 36*b**7*x**14)/(144*a**9
*b**8 + 1296*a**8*b**9*x**2 + 5184*a**7*b**10*x**4 + 12096*a**6*b**11*x**6
+ 18144*a**5*b**12*x**8 + 18144*a**4*b**13*x**10 + 12096*a**3*b**14*x**12 +
 5184*a**2*b**15*x**14 + 1296*a*b**16*x**16 + 144*b**17*x**18)
```

3.198 $\int \frac{x^{13}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=58

$$\frac{x^{14}}{504a^3\left(a+bx^2\right)^7}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{x^{14}}{18a\left(a+bx^2\right)^9}$$

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{x^{14}}{504a^3\left(a+bx^2\right)^7}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{x^{14}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^2)^10,x]
```

```
[Out] x^14/(18*a*(a + b*x^2)^9) + x^14/(72*a^2*(a + b*x^2)^8) + x^14/(504*a^3*(a
+ b*x^2)^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{(a+bx)^{10}}\,dx,x,x^2\right)$$
$$= \frac{x^{14}}{18a\left(a+bx^2\right)^9}+\frac{\text{Subst}\left(\int \frac{x^6}{(a+bx)^9}\,dx,x,x^2\right)}{9a}$$
$$= \frac{x^{14}}{18a\left(a+bx^2\right)^9}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{\text{Subst}\left(\int \frac{x^6}{(a+bx)^8}\,dx,x,x^2\right)}{72a^2}$$
$$= \frac{x^{14}}{18a\left(a+bx^2\right)^9}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{x^{14}}{504a^3\left(a+bx^2\right)^7}$$

Mathematica [A] time = 0.02, size = 79, normalized size = 1.36

$$-\frac{a^6+9a^5bx^2+36a^4b^2x^4+84a^3b^3x^6+126a^2b^4x^8+126ab^5x^{10}+84b^6x^{12}}{504b^7\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^2)^10,x]
```

```
[Out] -1/504*(a^6 + 9*a^5*b*x^2 + 36*a^4*b^2*x^4 + 84*a^3*b^3*x^6 + 126*a^2*b^4*x
^8 + 126*a*b^5*x^10 + 84*b^6*x^12)/(b^7*(a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^13/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^13/(a + b*x^2)^10, x]
```

fricas [B] time = 0.87, size = 168, normalized size = 2.90

$$-\frac{84\,b^6x^{12}+126\,ab^5x^{10}+126\,a^2b^4x^8+84\,a^3b^3x^6+36\,a^4b^2x^4+9\,a^5bx^2+a^6}{504\left(b^{16}x^{18}+9\,ab^{15}x^{16}+36\,a^2b^{14}x^{14}+84\,a^3b^{13}x^{12}+126\,a^4b^{12}x^{10}+126\,a^5b^{11}x^8+84\,a^6b^{10}x^6+36\,a^7b^9x^4+9\,a^8b^8x^2+a^9b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^14
*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6*b^
10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)
```

giac [A] time = 0.63, size = 77, normalized size = 1.33

$$-\frac{84\,b^6x^{12}+126\,ab^5x^{10}+126\,a^2b^4x^8+84\,a^3b^3x^6+36\,a^4b^2x^4+9\,a^5bx^2+a^6}{504\left(bx^2+a\right)^9b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/((b*x^2 + a)^9*b^7)
```

maple [B] time = 0.01, size = 116, normalized size = 2.00

$$-\frac{a^6}{18\left(b\,x^2+a\right)^9b^7}+\frac{3a^5}{8\left(b\,x^2+a\right)^8b^7}-\frac{15a^4}{14\left(b\,x^2+a\right)^7b^7}+\frac{5a^3}{3\left(b\,x^2+a\right)^6b^7}-\frac{3a^2}{2\left(b\,x^2+a\right)^5b^7}+\frac{3a}{4\left(b\,x^2+a\right)^4b^7}-\frac{1}{6\left(b\,x^2+a\right)^3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2+a)^10,x)
```

```
[Out] 3/8*a^5/b^7/(b*x^2+a)^8-15/14*a^4/b^7/(b*x^2+a)^7-1/6/b^7/(b*x^2+a)^3-3/2*a
^2/b^7/(b*x^2+a)^5+3/4*a/b^7/(b*x^2+a)^4-1/18*a^6/b^7/(b*x^2+a)^9+5/3*a^3/b
^7/(b*x^2+a)^6
```

maxima [B] time = 1.46, size = 168, normalized size = 2.90

$$-\frac{84\,b^6x^{12}+126\,ab^5x^{10}+126\,a^2b^4x^8+84\,a^3b^3x^6+36\,a^4b^2x^4+9\,a^5bx^2+a^6}{504\left(b^{16}x^{18}+9\,ab^{15}x^{16}+36\,a^2b^{14}x^{14}+84\,a^3b^{13}x^{12}+126\,a^4b^{12}x^{10}+126\,a^5b^{11}x^8+84\,a^6b^{10}x^6+36\,a^7b^9x^4+9\,a^8b^8x^2+a^9b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 3
6*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^14
*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6*b^
10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)
```

mupad [**B**] time = 0.10, size = 170, normalized size = 2.93

$$-\frac{a^6+9\,a^5\,b\,x^2+36\,a^4\,b^2\,x^4+84\,a^3\,b^3\,x^6+126\,a^2\,b^4\,x^8+126\,a\,b^5\,x^{10}+84\,b^6\,x^{12}}{504\,a^9\,b^7+4536\,a^8\,b^8\,x^2+18144\,a^7\,b^9\,x^4+42336\,a^6\,b^{10}\,x^6+63504\,a^5\,b^{11}\,x^8+63504\,a^4\,b^{12}\,x^{10}+42336\,a^3\,b^{13}\,x^{12}+18144\,a^2\,b^{14}\,x^{14}+4536\,a\,b^{15}\,x^{16}+504\,b^{16}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^2)^10,x)
```

```
[Out] -(a^6 + 84*b^6*x^12 + 9*a^5*b*x^2 + 126*a*b^5*x^10 + 36*a^4*b^2*x^4 + 84*a^
3*b^3*x^6 + 126*a^2*b^4*x^8)/(504*a^9*b^7 + 504*b^16*x^18 + 4536*a*b^15*x^1
6 + 4536*a^8*b^8*x^2 + 18144*a^7*b^9*x^4 + 42336*a^6*b^10*x^6 + 63504*a^5*b
^11*x^8 + 63504*a^4*b^12*x^10 + 42336*a^3*b^13*x^12 + 18144*a^2*b^14*x^14)
```

sympy [**B**] time = 1.33, size = 178, normalized size = 3.07

$$\frac{-a^6-9a^5bx^2-36a^4b^2x^4-84a^3b^3x^6-126a^2b^4x^8-126ab^5x^{10}-84b^6x^{12}}{504a^9b^7+4536a^8b^8x^2+18144a^7b^9x^4+42336a^6b^{10}x^6+63504a^5b^{11}x^8+63504a^4b^{12}x^{10}+42336a^3b^{13}x^{12}+18144a^2b^{14}x^{14}+4536ab^{15}x^{16}+504b^{16}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**2+a)**10,x)
```

```
[Out] (-a**6 - 9*a**5*b*x**2 - 36*a**4*b**2*x**4 - 84*a**3*b**3*x**6 - 126*a**2*b
**4*x**8 - 126*a*b**5*x**10 - 84*b**6*x**12)/(504*a**9*b**7 + 4536*a**8*b**
8*x**2 + 18144*a**7*b**9*x**4 + 42336*a**6*b**10*x**6 + 63504*a**5*b**11*x*
*8 + 63504*a**4*b**12*x**10 + 42336*a**3*b**13*x**12 + 18144*a**2*b**14*x**
14 + 4536*a*b**15*x**16 + 504*b**16*x**18)
```

3.199 $\int \frac{x^{11}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=77

$$\frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}+\frac{x^{12}}{168a^3\left(a+bx^2\right)^7}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{x^{12}}{18a\left(a+bx^2\right)^9}$$

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 45, 37}

$$\frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}+\frac{x^{12}}{168a^3\left(a+bx^2\right)^7}+\frac{x^{12}}{48a^2\left(a+bx^2\right)^8}+\frac{x^{12}}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2)^10,x]
```

```
[Out] x^12/(18*a*(a + b*x^2)^9) + x^12/(48*a^2*(a + b*x^2)^8) + x^12/(168*a^3*(a
+ b*x^2)^7) + x^12/(1008*a^4*(a + b*x^2)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{x^{12}}{18a\left(a+bx^2\right)^9} + \frac{\text{Subst}\left(\int \frac{x^5}{(a+bx)^9}\,dx, x, x^2\right)}{6a}$$
$$= \frac{x^{12}}{18a\left(a+bx^2\right)^9} + \frac{x^{12}}{48a^2\left(a+bx^2\right)^8} + \frac{\text{Subst}\left(\int \frac{x^5}{(a+bx)^8}\,dx, x, x^2\right)}{24a^2}$$
$$= \frac{x^{12}}{18a\left(a+bx^2\right)^9} + \frac{x^{12}}{48a^2\left(a+bx^2\right)^8} + \frac{x^{12}}{168a^3\left(a+bx^2\right)^7} + \frac{\text{Subst}\left(\int \frac{x^5}{(a+bx)^7}\,dx, x, x^2\right)}{168a^3}$$
$$= \frac{x^{12}}{18a\left(a+bx^2\right)^9} + \frac{x^{12}}{48a^2\left(a+bx^2\right)^8} + \frac{x^{12}}{168a^3\left(a+bx^2\right)^7} + \frac{x^{12}}{1008a^4\left(a+bx^2\right)^6}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.88

$$-\frac{a^5+9a^4bx^2+36a^3b^2x^4+84a^2b^3x^6+126ab^4x^8+126b^5x^{10}}{1008b^6\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2)^10,x]
```

```
[Out] -1/1008*(a^5 + 9*a^4*b*x^2 + 36*a^3*b^2*x^4 + 84*a^2*b^3*x^6 + 126*a*b^4*x^
8 + 126*b^5*x^10)/(b^6*(a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^11/(a + b*x^2)^10, x]
```

fricas [B] time = 1.84, size = 157, normalized size = 2.04

$$-\frac{126\,b^{5}x^{10}+126\,ab^{4}x^{8}+84\,a^{2}b^{3}x^{6}+36\,a^{3}b^{2}x^{4}+9\,a^{4}bx^{2}+a^{5}}{1008\,\left(b^{15}x^{18}+9\,ab^{14}x^{16}+36\,a^{2}b^{13}x^{14}+84\,a^{3}b^{12}x^{12}+126\,a^{4}b^{11}x^{10}+126\,a^{5}b^{10}x^{8}+84\,a^{6}b^{9}x^{6}+36\,a^{7}b^{8}x^{4}+9\,a^{8}b^{7}x^{2}+a^{9}b^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/(b^15*x^18 + 9*a*b^14*x^16 + 36*a^2*b^13*x^14 + 84*a^3*b^
12*x^12 + 126*a^4*b^11*x^10 + 126*a^5*b^10*x^8 + 84*a^6*b^9*x^6 + 36*a^7*b^
8*x^4 + 9*a^8*b^7*x^2 + a^9*b^6)
```

giac [A] time = 0.63, size = 66, normalized size = 0.86

$$-\frac{126\,b^{5}x^{10}+126\,ab^{4}x^{8}+84\,a^{2}b^{3}x^{6}+36\,a^{3}b^{2}x^{4}+9\,a^{4}bx^{2}+a^{5}}{1008\,\left(bx^{2}+a\right)^{9}b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/((b*x^2 + a)^9*b^6)
```

maple [A] time = 0.01, size = 99, normalized size = 1.29

$$\frac{a^{5}}{18\left(b\,x^{2}+a\right)^{9}b^{6}}-\frac{5a^{4}}{16\left(b\,x^{2}+a\right)^{8}b^{6}}+\frac{5a^{3}}{7\left(b\,x^{2}+a\right)^{7}b^{6}}-\frac{5a^{2}}{6\left(b\,x^{2}+a\right)^{6}b^{6}}+\frac{a}{2\left(b\,x^{2}+a\right)^{5}b^{6}}-\frac{1}{8\left(b\,x^{2}+a\right)^{4}b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2+a)^10,x)
```

```
[Out] -5/16*a^4/b^6/(b*x^2+a)^8+5/7*a^3/b^6/(b*x^2+a)^7+1/2*a/b^6/(b*x^2+a)^5-1/8
/b^6/(b*x^2+a)^4-5/6*a^2/b^6/(b*x^2+a)^6+1/18*a^5/b^6/(b*x^2+a)^9
```

maxima [B] time = 1.43, size = 157, normalized size = 2.04

$$-\frac{126\,b^{5}x^{10}+126\,ab^{4}x^{8}+84\,a^{2}b^{3}x^{6}+36\,a^{3}b^{2}x^{4}+9\,a^{4}bx^{2}+a^{5}}{1008\,\left(b^{15}x^{18}+9\,ab^{14}x^{16}+36\,a^{2}b^{13}x^{14}+84\,a^{3}b^{12}x^{12}+126\,a^{4}b^{11}x^{10}+126\,a^{5}b^{10}x^{8}+84\,a^{6}b^{9}x^{6}+36\,a^{7}b^{8}x^{4}+9\,a^{8}b^{7}x^{2}+a^{9}b^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9
*a^4*b*x^2 + a^5)/(b^15*x^18 + 9*a*b^14*x^16 + 36*a^2*b^13*x^14 + 84*a^3*b^
12*x^12 + 126*a^4*b^11*x^10 + 126*a^5*b^10*x^8 + 84*a^6*b^9*x^6 + 36*a^7*b^
8*x^4 + 9*a^8*b^7*x^2 + a^9*b^6)
```

mupad [B] time = 0.10, size = 159, normalized size = 2.06

$$-\frac{a^5+9\,a^4\,b\,x^2+36\,a^3\,b^2\,x^4+84\,a^2\,b^3\,x^6+126\,a\,b^4\,x^8+126\,b^5\,x^{10}}{1008\,a^9\,b^6+9072\,a^8\,b^7\,x^2+36288\,a^7\,b^8\,x^4+84672\,a^6\,b^9\,x^6+127008\,a^5\,b^{10}\,x^8+127008\,a^4\,b^{11}\,x^{10}+84672\,a^3\,b^{12}\,x^{12}+36288\,a^2\,b^{13}\,x^{14}+9072\,a\,b^{14}\,x^{16}+1008\,b^{15}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2)^10,x)
```

```
[Out] -(a^5 + 126*b^5*x^10 + 9*a^4*b*x^2 + 126*a*b^4*x^8 + 36*a^3*b^2*x^4 + 84*a^
2*b^3*x^6)/(1008*a^9*b^6 + 1008*b^15*x^18 + 9072*a*b^14*x^16 + 9072*a^8*b^7
*x^2 + 36288*a^7*b^8*x^4 + 84672*a^6*b^9*x^6 + 127008*a^5*b^10*x^8 + 127008
*a^4*b^11*x^10 + 84672*a^3*b^12*x^12 + 36288*a^2*b^13*x^14)
```

sympy [B] time = 1.18, size = 167, normalized size = 2.17

$$\frac{-a^5-9a^4bx^2-36a^3b^2x^4-84a^2b^3x^6-126ab^4x^8-126b^5x^{10}}{1008a^9b^6+9072a^8b^7x^2+36288a^7b^8x^4+84672a^6b^9x^6+127008a^5b^{10}x^8+127008a^4b^{11}x^{10}+84672a^3b^{12}x^{12}+36288a^2b^{13}x^{14}+9072ab^{14}x^{16}+1008b^{15}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**2+a)**10,x)
```

```
[Out] (-a**5 - 9*a**4*b*x**2 - 36*a**3*b**2*x**4 - 84*a**2*b**3*x**6 - 126*a*b**4
*x**8 - 126*b**5*x**10)/(1008*a**9*b**6 + 9072*a**8*b**7*x**2 + 36288*a**7*
b**8*x**4 + 84672*a**6*b**9*x**6 + 127008*a**5*b**10*x**8 + 127008*a**4*b**
11*x**10 + 84672*a**3*b**12*x**12 + 36288*a**2*b**13*x**14 + 9072*a*b**14*x
**16 + 1008*b**15*x**18)
```

3.200 $\int \frac{x^9}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=91

$$-\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}$$

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^4}{18b^5\left(a+bx^2\right)^9}+\frac{a^3}{4b^5\left(a+bx^2\right)^8}-\frac{3a^2}{7b^5\left(a+bx^2\right)^7}+\frac{a}{3b^5\left(a+bx^2\right)^6}-\frac{1}{10b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^10,x]
```

```
[Out] -a^4/(18*b^5*(a + b*x^2)^9) + a^3/(4*b^5*(a + b*x^2)^8) - (3*a^2)/(7*b^5*(a
 + b*x^2)^7) + a/(3*b^5*(a + b*x^2)^6) - 1/(10*b^5*(a + b*x^2)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^4}{b^4(a+bx)^{10}} - \frac{4a^3}{b^4(a+bx)^9} + \frac{6a^2}{b^4(a+bx)^8} - \frac{4a}{b^4(a+bx)^7} + \frac{1}{b^4(a+bx)^6}\right) dx, x$$
$$= -\frac{a^4}{18b^5\left(a+bx^2\right)^9} + \frac{a^3}{4b^5\left(a+bx^2\right)^8} - \frac{3a^2}{7b^5\left(a+bx^2\right)^7} + \frac{a}{3b^5\left(a+bx^2\right)^6} - \frac{1}{10b^5\left(a+bx^2\right)^5}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.63

$$-\frac{a^4+9a^3bx^2+36a^2b^2x^4+84ab^3x^6+126b^4x^8}{1260b^5\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^10,x]
```

```
[Out] -1/1260*(a^4 + 9*a^3*b*x^2 + 36*a^2*b^2*x^4 + 84*a*b^3*x^6 + 126*b^4*x^8)/(
b^5*(a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^9/(a + b*x^2)^10, x]
```

fricas [A] time = 1.11, size = 146, normalized size = 1.60

$$-\frac{126\,b^4x^8+84\,ab^3x^6+36\,a^2b^2x^4+9\,a^3bx^2+a^4}{1260\left(b^{14}x^{18}+9\,ab^{13}x^{16}+36\,a^2b^{12}x^{14}+84\,a^3b^{11}x^{12}+126\,a^4b^{10}x^{10}+126\,a^5b^9x^8+84\,a^6b^8x^6+36\,a^7b^7x^4+9\,a^8b^6x^2+a^9b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
b^14*x^18 + 9*a*b^13*x^16 + 36*a^2*b^12*x^14 + 84*a^3*b^11*x^12 + 126*a^4*b
```

```
^10*x^10 + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6*x^
2 + a^9*b^5)
```

giac [A] time = 0.64, size = 55, normalized size = 0.60

$$-\frac{126\,b^4x^8 + 84\,ab^3x^6 + 36\,a^2b^2x^4 + 9\,a^3bx^2 + a^4}{1260\left(bx^2 + a\right)^9 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
(b*x^2 + a)^9*b^5)
```

maple [A] time = 0.01, size = 82, normalized size = 0.90

$$-\frac{a^4}{18\left(b\,x^2+a\right)^9 b^5}+\frac{a^3}{4\left(b\,x^2+a\right)^8 b^5}-\frac{3a^2}{7\left(b\,x^2+a\right)^7 b^5}+\frac{a}{3\left(b\,x^2+a\right)^6 b^5}-\frac{1}{10\left(b\,x^2+a\right)^5 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^10,x)
```

```
[Out] -1/18*a^4/b^5/(b*x^2+a)^9+1/4*a^3/b^5/(b*x^2+a)^8-3/7*a^2/b^5/(b*x^2+a)^7+1
/3*a/b^5/(b*x^2+a)^6-1/10/b^5/(b*x^2+a)^5
```

maxima [A] time = 1.44, size = 146, normalized size = 1.60

$$-\frac{126\,b^4x^8 + 84\,ab^3x^6 + 36\,a^2b^2x^4 + 9\,a^3bx^2 + a^4}{1260\left(b^{14}x^{18} + 9\,ab^{13}x^{16} + 36\,a^2b^{12}x^{14} + 84\,a^3b^{11}x^{12} + 126\,a^4b^{10}x^{10} + 126\,a^5b^9x^8 + 84\,a^6b^8x^6 + 36\,a^7b^7x^4 + 9\,a^8b^6x^2 + a^9b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(
b^14*x^18 + 9*a*b^13*x^16 + 36*a^2*b^12*x^14 + 84*a^3*b^11*x^12 + 126*a^4*b
^10*x^10 + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6*x^
2 + a^9*b^5)
```

mupad [B] time = 4.82, size = 148, normalized size = 1.63

$$-\frac{a^4 + 9\,a^3\,b\,x^2 + 36\,a^2\,b^2\,x^4 + 84\,a\,b^3\,x^6 + 126\,b^4\,x^8}{1260\,a^9\,b^5 + 11340\,a^8\,b^6\,x^2 + 45360\,a^7\,b^7\,x^4 + 105840\,a^6\,b^8\,x^6 + 158760\,a^5\,b^9\,x^8 + 158760\,a^4\,b^{10}\,x^{10} + 105840\,a^3\,b^{11}\,x^{12} + 45360\,a^2\,b^{12}\,x^{14} + 11340\,a\,b^{13}\,x^{16} + 1260\,b^{14}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^10,x)
```

```
[Out] -(a^4 + 126*b^4*x^8 + 9*a^3*b*x^2 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4)/(1260*a^
9*b^5 + 1260*b^14*x^18 + 11340*a*b^13*x^16 + 11340*a^8*b^6*x^2 + 45360*a^7*
b^7*x^4 + 105840*a^6*b^8*x^6 + 158760*a^5*b^9*x^8 + 158760*a^4*b^10*x^10 +
105840*a^3*b^11*x^12 + 45360*a^2*b^12*x^14)
```

sympy [A] time = 1.08, size = 155, normalized size = 1.70

$$\frac{-a^4 - 9a^3bx^2 - 36a^2b^2x^4 - 84ab^3x^6 - 126b^4x^8}{1260a^9b^5 + 11340a^8b^6x^2 + 45360a^7b^7x^4 + 105840a^6b^8x^6 + 158760a^5b^9x^8 + 158760a^4b^{10}x^{10} + 105840a^3b^{11}x^{12} + 45360a^2b^{12}x^{14} + 11340ab^{13}x^{16} + 1260b^{14}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**10,x)
```

```
[Out] (-a**4 - 9*a**3*b*x**2 - 36*a**2*b**2*x**4 - 84*a*b**3*x**6 - 126*b**4*x**8
)/(1260*a**9*b**5 + 11340*a**8*b**6*x**2 + 45360*a**7*b**7*x**4 + 105840*a*
*6*b**8*x**6 + 158760*a**5*b**9*x**8 + 158760*a**4*b**10*x**10 + 105840*a**
3*b**11*x**12 + 45360*a**2*b**12*x**14 + 11340*a*b**13*x**16 + 1260*b**14*x
**18)
```

3.201 $\int \frac{x^7}{\left(a+bx^2\right)^{10}} \, dx$

Optimal. Leaf size=72

$$\frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}$$

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^10,x]
```

```
[Out] a^3/(18*b^4*(a + b*x^2)^9) - (3*a^2)/(16*b^4*(a + b*x^2)^8) + (3*a)/(14*b^4
*(a + b*x^2)^7) - 1/(12*b^4*(a + b*x^2)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{10}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{10}}+\frac{3a^2}{b^3(a+bx)^9}-\frac{3a}{b^3(a+bx)^8}+\frac{1}{b^3(a+bx)^7}\right)dx, x, x^2\right)$$
$$= \frac{a^3}{18b^4\left(a+bx^2\right)^9}-\frac{3a^2}{16b^4\left(a+bx^2\right)^8}+\frac{3a}{14b^4\left(a+bx^2\right)^7}-\frac{1}{12b^4\left(a+bx^2\right)^6}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.64

$$-\frac{a^3+9a^2bx^2+36ab^2x^4+84b^3x^6}{1008b^4\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^10,x]
[Out] -1/1008*(a^3 + 9*a^2*b*x^2 + 36*a*b^2*x^4 + 84*b^3*x^6)/(b^4*(a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2)^10,x]
[Out] IntegrateAlgebraic[x^7/(a + b*x^2)^10, x]
```

fricas [B] time = 0.86, size = 135, normalized size = 1.88

$$-\frac{84\,b^3x^6+36\,ab^2x^4+9\,a^2bx^2+a^3}{1008\left(b^{13}x^{18}+9\,ab^{12}x^{16}+36\,a^2b^{11}x^{14}+84\,a^3b^{10}x^{12}+126\,a^4b^9x^{10}+126\,a^5b^8x^8+84\,a^6b^7x^6+36\,a^7b^6x^4+9\,a^8b^5x^2+a^9b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="fricas")
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^13*x^18 + 9*a*b^
12*x^16 + 36*a^2*b^11*x^14 + 84*a^3*b^10*x^12 + 126*a^4*b^9*x^10 + 126*a^5*
b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)
```

giac [A] time = 0.64, size = 44, normalized size = 0.61

$$-\frac{84\,b^{3}x^{6}+36\,ab^{2}x^{4}+9\,a^{2}bx^{2}+a^{3}}{1008\,\left(bx^{2}+a\right)^{9}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/((b*x^2 + a)^9*b^4)
```

maple [A] time = 0.01, size = 65, normalized size = 0.90

$$\frac{a^{3}}{18\left(b\,x^{2}+a\right)^{9}b^{4}}-\frac{3a^{2}}{16\left(b\,x^{2}+a\right)^{8}b^{4}}+\frac{3a}{14\left(b\,x^{2}+a\right)^{7}b^{4}}-\frac{1}{12\left(b\,x^{2}+a\right)^{6}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^10,x)
```

```
[Out] 1/18*a^3/b^4/(b*x^2+a)^9-3/16*a^2/b^4/(b*x^2+a)^8+3/14*a/b^4/(b*x^2+a)^7-1/
12/b^4/(b*x^2+a)^6
```

maxima [B] time = 1.45, size = 135, normalized size = 1.88

$$-\frac{84\,b^{3}x^{6}+36\,ab^{2}x^{4}+9\,a^{2}bx^{2}+a^{3}}{1008\,\left(b^{13}x^{18}+9\,ab^{12}x^{16}+36\,a^{2}b^{11}x^{14}+84\,a^{3}b^{10}x^{12}+126\,a^{4}b^{9}x^{10}+126\,a^{5}b^{8}x^{8}+84\,a^{6}b^{7}x^{6}+36\,a^{7}b^{6}x^{4}+9\,a^{8}b^{5}x^{2}+a^{9}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^13*x^18 + 9*a*b^
12*x^16 + 36*a^2*b^11*x^14 + 84*a^3*b^10*x^12 + 126*a^4*b^9*x^10 + 126*a^5*
b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)
```

mupad [B] time = 0.10, size = 136, normalized size = 1.89

$$-\frac{\frac{a^{3}}{1008\,b^{4}}+\frac{x^{6}}{12\,b}+\frac{a\,x^{4}}{28\,b^{2}}+\frac{a^{2}\,x^{2}}{112\,b^{3}}}{a^{9}+9\,a^{8}\,b\,x^{2}+36\,a^{7}\,b^{2}\,x^{4}+84\,a^{6}\,b^{3}\,x^{6}+126\,a^{5}\,b^{4}\,x^{8}+126\,a^{4}\,b^{5}\,x^{10}+84\,a^{3}\,b^{6}\,x^{12}+36\,a^{2}\,b^{7}\,x^{14}+9\,a\,b^{8}\,x^{16}+b^{9}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^10,x)
```

```
[Out] -(a^3/(1008*b^4) + x^6/(12*b) + (a*x^4)/(28*b^2) + (a^2*x^2)/(112*b^3))/(a^
9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6
 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [B] time = 1.01, size = 143, normalized size = 1.99

$$\frac{-a^3-9a^2bx^2-36ab^2x^4-84b^3x^6}{1008a^9b^4+9072a^8b^5x^2+36288a^7b^6x^4+84672a^6b^7x^6+127008a^5b^8x^8+127008a^4b^9x^{10}+84672a^3b^{10}x^{12}+36288a^2b^{11}x^{14}+9072ab^{12}x^{16}+1008b^{13}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**10,x)
```

```
[Out] (-a**3 - 9*a**2*b*x**2 - 36*a*b**2*x**4 - 84*b**3*x**6)/(1008*a**9*b**4 + 9
072*a**8*b**5*x**2 + 36288*a**7*b**6*x**4 + 84672*a**6*b**7*x**6 + 127008*a
**5*b**8*x**8 + 127008*a**4*b**9*x**10 + 84672*a**3*b**10*x**12 + 36288*a**
2*b**11*x**14 + 9072*a*b**12*x**16 + 1008*b**13*x**18)
```

3.202 $\int \frac{x^5}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2}{18b^3\left(a+bx^2\right)^9}+\frac{a}{8b^3\left(a+bx^2\right)^8}-\frac{1}{14b^3\left(a+bx^2\right)^7}$$

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^2}{18b^3\left(a+bx^2\right)^9}+\frac{a}{8b^3\left(a+bx^2\right)^8}-\frac{1}{14b^3\left(a+bx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^10,x]
```

```
[Out] -a^2/(18*b^3*(a + b*x^2)^9) + a/(8*b^3*(a + b*x^2)^8) - 1/(14*b^3*(a + b*x^
2)^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{10}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{10}}-\frac{2a}{b^2(a+bx)^9}+\frac{1}{b^2(a+bx)^8}\right)dx, x, x^2\right)\\
&= -\frac{a^2}{18b^3\left(a+bx^2\right)^9}+\frac{a}{8b^3\left(a+bx^2\right)^8}-\frac{1}{14b^3\left(a+bx^2\right)^7}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.66

$$
-\frac{a^2+9abx^2+36b^2x^4}{504b^3\left(a+bx^2\right)^9}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^10,x]
[Out] -1/504*(a^2 + 9*a*b*x^2 + 36*b^2*x^4)/(b^3*(a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^5}{\left(a+bx^2\right)^{10}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2)^10,x]
[Out] IntegrateAlgebraic[x^5/(a + b*x^2)^10, x]
```

fricas [B] time = 0.89, size = 124, normalized size = 2.34

$$
-\frac{36\,b^2x^4+9\,abx^2+a^2}{504\left(b^{12}x^{18}+9\,ab^{11}x^{16}+36\,a^2b^{10}x^{14}+84\,a^3b^9x^{12}+126\,a^4b^8x^{10}+126\,a^5b^7x^8+84\,a^6b^6x^6+36\,a^7b^5x^4+9\,a^8b^4x^2+a^9b^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="fricas")
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^12*x^18 + 9*a*b^11*x^16 + 36*a^2*b
^10*x^14 + 84*a^3*b^9*x^12 + 126*a^4*b^8*x^10 + 126*a^5*b^7*x^8 + 84*a^6*b^
6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)
```

giac [A] time = 0.64, size = 33, normalized size = 0.62

$$-\frac{36\,b^2x^4+9\,abx^2+a^2}{504\,\left(bx^2+a\right)^9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/((b*x^2 + a)^9*b^3)
```

maple [A] time = 0.01, size = 48, normalized size = 0.91

$$-\frac{a^2}{18\left(b\,x^2+a\right)^9b^3}+\frac{a}{8\left(b\,x^2+a\right)^8b^3}-\frac{1}{14\left(b\,x^2+a\right)^7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^10,x)
```

```
[Out] -1/18*a^2/b^3/(b*x^2+a)^9+1/8*a/b^3/(b*x^2+a)^8-1/14/b^3/(b*x^2+a)^7
```

maxima [B] time = 1.51, size = 124, normalized size = 2.34

$$-\frac{36\,b^2x^4+9\,abx^2+a^2}{504\,\left(b^{12}x^{18}+9\,ab^{11}x^{16}+36\,a^2b^{10}x^{14}+84\,a^3b^9x^{12}+126\,a^4b^8x^{10}+126\,a^5b^7x^8+84\,a^6b^6x^6+36\,a^7b^5x^4+9\,a^8b^4x^2+a^9b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^12*x^18 + 9*a*b^11*x^16 + 36*a^2*b
^10*x^14 + 84*a^3*b^9*x^12 + 126*a^4*b^8*x^10 + 126*a^5*b^7*x^8 + 84*a^6*b^
6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)
```

mupad [B] time = 4.83, size = 125, normalized size = 2.36

$$-\frac{\frac{a^2}{504\,b^3}+\frac{x^4}{14\,b}+\frac{a\,x^2}{56\,b^2}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^10,x)
```

```
[Out] -(a^2/(504*b^3) + x^4/(14*b) + (a*x^2)/(56*b^2))/(a^9 + b^9*x^18 + 9*a^8*b*
x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 12
6*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [B] time = 0.95, size = 131, normalized size = 2.47

$$\frac{-a^2-9abx^2-36b^2x^4}{504a^9b^3+4536a^8b^4x^2+18144a^7b^5x^4+42336a^6b^6x^6+63504a^5b^7x^8+63504a^4b^8x^{10}+42336a^3b^9x^{12}+18144a^2b^{10}x^{14}+4536ab^{11}x^{16}+504b^{12}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**10,x)
```

```
[Out] (-a**2 - 9*a*b*x**2 - 36*b**2*x**4)/(504*a**9*b**3 + 4536*a**8*b**4*x**2 +
18144*a**7*b**5*x**4 + 42336*a**6*b**6*x**6 + 63504*a**5*b**7*x**8 + 63504*
a**4*b**8*x**10 + 42336*a**3*b**9*x**12 + 18144*a**2*b**10*x**14 + 4536*a*b
**11*x**16 + 504*b**12*x**18)
```

3.203 $\int \frac{x^3}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=34

$$\frac{a}{18b^2\left(a+bx^2\right)^9}-\frac{1}{16b^2\left(a+bx^2\right)^8}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a}{18b^2\left(a+bx^2\right)^9}-\frac{1}{16b^2\left(a+bx^2\right)^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^10,x]
```

```
[Out] a/(18*b^2*(a + b*x^2)^9) - 1/(16*b^2*(a + b*x^2)^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^{10}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{10}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{10}}+\frac{1}{b(a+bx)^9}\right)dx, x, x^2\right)\\ &= \frac{a}{18b^2\left(a+bx^2\right)^9}-\frac{1}{16b^2\left(a+bx^2\right)^8}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$-\frac{a+9bx^2}{144b^2\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^10,x]
```

```
[Out] -1/144*(a + 9*b*x^2)/(b^2*(a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x^2)^10, x]
```

fricas [B] time = 0.57, size = 113, normalized size = 3.32

$$-\frac{9\,bx^2+a}{144\left(b^{11}x^{18}+9\,ab^{10}x^{16}+36\,a^2b^9x^{14}+84\,a^3b^8x^{12}+126\,a^4b^7x^{10}+126\,a^5b^6x^8+84\,a^6b^5x^6+36\,a^7b^4x^4+9\,a^8b^3x^2+a^9b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/144*(9*b*x^2 + a)/(b^11*x^18 + 9*a*b^10*x^16 + 36*a^2*b^9*x^14 + 84*a^3*
b^8*x^12 + 126*a^4*b^7*x^10 + 126*a^5*b^6*x^8 + 84*a^6*b^5*x^6 + 36*a^7*b^4
*x^4 + 9*a^8*b^3*x^2 + a^9*b^2)
```

giac [A] time = 0.62, size = 22, normalized size = 0.65

$$-\frac{9\,bx^2+a}{144\left(bx^2+a\right)^9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/144*(9*b*x^2 + a)/((b*x^2 + a)^9*b^2)
```

maple [A] time = 0.01, size = 31, normalized size = 0.91

$$\frac{a}{18\left(b\,x^{2}+a\right)^{9} b^{2}}-\frac{1}{16\left(b\,x^{2}+a\right)^{8} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^10,x)
```

```
[Out] 1/18*a/b^2/(b*x^2+a)^9-1/16/b^2/(b*x^2+a)^8
```

maxima [B] time = 1.39, size = 113, normalized size = 3.32

$$-\frac{9\,bx^{2}+a}{144\left(b^{11}x^{18}+9\,ab^{10}x^{16}+36\,a^{2}b^{9}x^{14}+84\,a^{3}b^{8}x^{12}+126\,a^{4}b^{7}x^{10}+126\,a^{5}b^{6}x^{8}+84\,a^{6}b^{5}x^{6}+36\,a^{7}b^{4}x^{4}+9\,a^{8}b^{3}x^{2}+a^{9}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/144*(9*b*x^2 + a)/(b^11*x^18 + 9*a*b^10*x^16 + 36*a^2*b^9*x^14 + 84*a^3*
b^8*x^12 + 126*a^4*b^7*x^10 + 126*a^5*b^6*x^8 + 84*a^6*b^5*x^6 + 36*a^7*b^4
*x^4 + 9*a^8*b^3*x^2 + a^9*b^2)
```

mupad [B] time = 0.11, size = 114, normalized size = 3.35

$$-\frac{\frac{a}{144\,b^{2}}+\frac{x^{2}}{16\,b}}{a^{9}+9\,a^{8}\,b\,x^{2}+36\,a^{7}\,b^{2}\,x^{4}+84\,a^{6}\,b^{3}\,x^{6}+126\,a^{5}\,b^{4}\,x^{8}+126\,a^{4}\,b^{5}\,x^{10}+84\,a^{3}\,b^{6}\,x^{12}+36\,a^{2}\,b^{7}\,x^{14}+9\,a\,b^{8}\,x^{16}+b^{9}\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^10,x)
```

```
[Out] -(a/(144*b^2) + x^2/(16*b))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 +
36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a
^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [B] time = 0.93, size = 119, normalized size = 3.50

$$\frac{-a-9bx^{2}}{144a^{9}b^{2}+1296a^{8}b^{3}x^{2}+5184a^{7}b^{4}x^{4}+12096a^{6}b^{5}x^{6}+18144a^{5}b^{6}x^{8}+18144a^{4}b^{7}x^{10}+12096a^{3}b^{8}x^{12}+5184a^{2}b^{9}x^{14}+1296ab^{10}x^{16}+144b^{11}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**10,x)
```

```
[Out] (-a - 9*b*x**2)/(144*a**9*b**2 + 1296*a**8*b**3*x**2 + 5184*a**7*b**4*x**4
+ 12096*a**6*b**5*x**6 + 18144*a**5*b**6*x**8 + 18144*a**4*b**7*x**10 + 120
96*a**3*b**8*x**12 + 5184*a**2*b**9*x**14 + 1296*a*b**10*x**16 + 144*b**11*
x**18)
```

3.204 $\int \frac{x}{(a+bx^2)^{10}} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{18b(a+bx^2)^9}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{18b(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^10,x]
```

```
[Out] -1/(18*b*(a + b*x^2)^9)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{10}} \, dx = -\frac{1}{18b(a+bx^2)^9}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{18b(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^10,x]
```

```
[Out] -1/18*1/(b*(a + b*x^2)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x^2)^10, x]
```

fricas [B] time = 0.73, size = 103, normalized size = 6.44

$$-\frac{1}{18\left(b^{10}x^{18}+9\,ab^{9}x^{16}+36\,a^{2}b^{8}x^{14}+84\,a^{3}b^{7}x^{12}+126\,a^{4}b^{6}x^{10}+126\,a^{5}b^{5}x^{8}+84\,a^{6}b^{4}x^{6}+36\,a^{7}b^{3}x^{4}+9\,a^{8}b^{2}x^{2}+a^{9}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/18/(b^10*x^18 + 9*a*b^9*x^16 + 36*a^2*b^8*x^14 + 84*a^3*b^7*x^12 + 126*a
^4*b^6*x^10 + 126*a^5*b^5*x^8 + 84*a^6*b^4*x^6 + 36*a^7*b^3*x^4 + 9*a^8*b^2
*x^2 + a^9*b)
```

giac [A] time = 0.59, size = 14, normalized size = 0.88

$$-\frac{1}{18\left(bx^2+a\right)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -1/18/((b*x^2 + a)^9*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{18\left(b\,x^2+a\right)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^10,x)
```

```
[Out] -1/18/b/(b*x^2+a)^9
```

maxima [A] time = 1.30, size = 14, normalized size = 0.88

$$-\frac{1}{18\left(bx^2+a\right)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/18/((b*x^2 + a)^9*b)
```

mupad [B] time = 0.13, size = 14, normalized size = 0.88

$$-\frac{1}{18\,b\left(b\,x^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^10,x)
```

```
[Out] -1/(18*b*(a + b*x^2)^9)
```

sympy [B] time = 0.89, size = 110, normalized size = 6.88

$$-\frac{1}{18a^9b+162a^8b^2x^2+648a^7b^3x^4+1512a^6b^4x^6+2268a^5b^5x^8+2268a^4b^6x^{10}+1512a^3b^7x^{12}+648a^2b^8x^{14}+162ab^9x^{16}+18b^{10}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**10,x)
```

```
[Out] -1/(18*a**9*b + 162*a**8*b**2*x**2 + 648*a**7*b**3*x**4 + 1512*a**6*b**4*x*
*6 + 2268*a**5*b**5*x**8 + 2268*a**4*b**6*x**10 + 1512*a**3*b**7*x**12 + 64
8*a**2*b**8*x**14 + 162*a*b**9*x**16 + 18*b**10*x**18)
```

3.205 $\int \frac{1}{x\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=166

$$-\frac{\log\left(a+bx^2\right)}{2a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{2a^9\left(a+bx^2\right)}+\frac{1}{4a^8\left(a+bx^2\right)^2}+\frac{1}{6a^7\left(a+bx^2\right)^3}+\frac{1}{8a^6\left(a+bx^2\right)^4}+\frac{1}{10a^5\left(a+bx^2\right)^5}+\frac{}{12a^4}$$

Rubi [A] time = 0.13, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2a^9\left(a+bx^2\right)}+\frac{1}{4a^8\left(a+bx^2\right)^2}+\frac{1}{6a^7\left(a+bx^2\right)^3}+\frac{1}{8a^6\left(a+bx^2\right)^4}+\frac{1}{10a^5\left(a+bx^2\right)^5}+\frac{1}{12a^4\left(a+bx^2\right)^6}+\frac{1}{14a^3\left(a+bx^2\right)^7}+\frac{1}{16a^2\left(a+bx^2\right)^8}-\frac{\log\left(a+bx^2\right)}{2a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^10),x]
```

```
[Out] 1/(18*a*(a + b*x^2)^9) + 1/(16*a^2*(a + b*x^2)^8) + 1/(14*a^3*(a + b*x^2)^7
) + 1/(12*a^4*(a + b*x^2)^6) + 1/(10*a^5*(a + b*x^2)^5) + 1/(8*a^6*(a + b*x
^2)^4) + 1/(6*a^7*(a + b*x^2)^3) + 1/(4*a^8*(a + b*x^2)^2) + 1/(2*a^9*(a +
b*x^2)) + Log[x]/a^10 - Log[a + b*x^2]/(2*a^10)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^{10}x} - \frac{b}{a(a+bx)^{10}} - \frac{b}{a^2(a+bx)^9} - \frac{b}{a^3(a+bx)^8} - \frac{b}{a^4(a+bx)^7} - \frac{b}{a^5(a+bx)}\right.\right.$$

$$= \frac{1}{18a\left(a+bx^2\right)^9} + \frac{1}{16a^2\left(a+bx^2\right)^8} + \frac{1}{14a^3\left(a+bx^2\right)^7} + \frac{1}{12a^4\left(a+bx^2\right)^6} + \frac{1}{10a^5\left(a+bx^2\right)^5}$$

Mathematica [A] time = 0.10, size = 120, normalized size = 0.72

$$\frac{\frac{a\left(7129a^8+41481a^7bx^2+120564a^6b^2x^4+210756a^5b^3x^6+236754a^4b^4x^8+173250a^3b^5x^{10}+80220a^2b^6x^{12}+21420ab^7x^{14}+2520b^8x^{16}\right)}{\left(a+bx^2\right)^9} - 2520\log\left(a+bx^2\right) + 5040\log(x)}{5040a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^10),x]
```

```
[Out] ((a*(7129*a^8 + 41481*a^7*b*x^2 + 120564*a^6*b^2*x^4 + 210756*a^5*b^3*x^6 +
 236754*a^4*b^4*x^8 + 173250*a^3*b^5*x^10 + 80220*a^2*b^6*x^12 + 21420*a*b^
7*x^14 + 2520*b^8*x^16))/(a + b*x^2)^9 + 5040*Log[x] - 2520*Log[a + b*x^2])
/(5040*a^10)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2)^10), x]
```

fricas [B] time = 1.01, size = 398, normalized size = 2.40

$$\frac{2520\,ab^8x^{16} + 21420\,a^2b^7x^{14} + 80220\,a^3b^6x^{12} + 173250\,a^4b^5x^{10} + 236754\,a^5b^4x^8 + 210756\,a^6b^3x^6 + 120564\,a^7b^2x^4 + 41481\,a^8bx^2 + 7129\,a^9 - 2520\left(b^9x^{18} + 9\,ab^8x^{16} + 36\,a^2b^7x^{14} + 84\,a^3b^6x^{12} + 126\,a^4b^5x^{10} + 126\,a^5b^4x^8 + 84\,a^6b^3x^6 + 36\,a^7b^2x^4 + 9\,a^8bx^2 + a^9\right)\log\left(bx^2+a\right) + 5040\left(b^9x^{18} + 9\,ab^8x^{16} + 36\,a^2b^7x^{14} + 84\,a^3b^6x^{12} + 126\,a^4b^5x^{10} + 126\,a^5b^4x^8 + 84\,a^6b^3x^6 + 36\,a^7b^2x^4 + 9\,a^8bx^2 + a^9\right)\log(x)}{5040\left(a^{10}b^9x^{18} + 9\,a^{11}b^8x^{16} + 36\,a^{12}b^7x^{14} + 84\,a^{13}b^6x^{12} + 126\,a^{14}b^5x^{10} + 126\,a^{15}b^4x^8 + 84\,a^{16}b^3x^6 + 36\,a^{17}b^2x^4 + 9\,a^{18}bx^2 + a^{19}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/5040*(2520*a*b^8*x^16 + 21420*a^2*b^7*x^14 + 80220*a^3*b^6*x^12 + 173250*
a^4*b^5*x^10 + 236754*a^5*b^4*x^8 + 210756*a^6*b^3*x^6 + 120564*a^7*b^2*x^4
```

```
 + 41481*a^8*b*x^2 + 7129*a^9 - 2520*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*
x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^
6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(b*x^2 + a) + 5040*(b^9*x^18 + 9
*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^
5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(x))/(a
^10*b^9*x^18 + 9*a^11*b^8*x^16 + 36*a^12*b^7*x^14 + 84*a^13*b^6*x^12 + 126*
a^14*b^5*x^10 + 126*a^15*b^4*x^8 + 84*a^16*b^3*x^6 + 36*a^17*b^2*x^4 + 9*a^
18*b*x^2 + a^19)
```

giac [A] time = 0.63, size = 136, normalized size = 0.82

$$\frac{\log\left(x^2\right)}{2\,a^{10}} - \frac{\log\left(\left|bx^2 + a\right|\right)}{2\,a^{10}} + \frac{7129\,b^9x^{18} + 66681\,ab^8x^{16} + 278064\,a^2b^7x^{14} + 679056\,a^3b^6x^{12} + 1071504\,a^4b^5x^{10} + 1135008\,a^5b^4x^8 + 809592\,a^6b^3x^6 + 377208\,a^7b^2x^4 + 105642\,a^8bx^2 + 14258\,a^9}{5040\,\left(bx^2 + a\right)^9 a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^10 - 1/2*log(abs(b*x^2 + a))/a^10 + 1/5040*(7129*b^9*x^18 +
66681*a*b^8*x^16 + 278064*a^2*b^7*x^14 + 679056*a^3*b^6*x^12 + 1071504*a^4*
b^5*x^10 + 1135008*a^5*b^4*x^8 + 809592*a^6*b^3*x^6 + 377208*a^7*b^2*x^4 +
105642*a^8*b*x^2 + 14258*a^9)/((b*x^2 + a)^9*a^10)
```

maple [A] time = 0.02, size = 147, normalized size = 0.89

$$\frac{1}{18\left(b\,x^2+a\right)^9 a}+\frac{1}{16\left(b\,x^2+a\right)^8 a^2}+\frac{1}{14\left(b\,x^2+a\right)^7 a^3}+\frac{1}{12\left(b\,x^2+a\right)^6 a^4}+\frac{1}{10\left(b\,x^2+a\right)^5 a^5}+\frac{1}{8\left(b\,x^2+a\right)^4 a^6}+\frac{1}{6\left(b\,x^2+a\right)^3 a^7}+\frac{1}{4\left(b\,x^2+a\right)^2 a^8}+\frac{1}{2\left(b\,x^2+a\right) a^9}+\frac{\ln(x)}{a^{10}}-\frac{\ln\left(b\,x^2+a\right)}{2a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^10,x)
```

```
[Out] 1/18/a/(b*x^2+a)^9+1/16/a^2/(b*x^2+a)^8+1/14/a^3/(b*x^2+a)^7+1/12/a^4/(b*x^
2+a)^6+1/10/a^5/(b*x^2+a)^5+1/8/a^6/(b*x^2+a)^4+1/6/a^7/(b*x^2+a)^3+1/4/a^8
/(b*x^2+a)^2+1/2/a^9/(b*x^2+a)+ln(x)/a^10-1/2*ln(b*x^2+a)/a^10
```

maxima [A] time = 1.65, size = 214, normalized size = 1.29

$$\frac{2520\,b^8x^{16} + 21420\,ab^7x^{14} + 80220\,a^2b^6x^{12} + 173250\,a^3b^5x^{10} + 236754\,a^4b^4x^8 + 210756\,a^5b^3x^6 + 120564\,a^6b^2x^4 + 41481\,a^7bx^2 + 7129\,a^8}{5040\left(a^9b^9x^{18} + 9\,a^{10}b^8x^{16} + 36\,a^{11}b^7x^{14} + 84\,a^{12}b^6x^{12} + 126\,a^{13}b^5x^{10} + 126\,a^{14}b^4x^8 + 84\,a^{15}b^3x^6 + 36\,a^{16}b^2x^4 + 9\,a^{17}bx^2 + a^{18}\right)} - \frac{\log\left(bx^2 + a\right)}{2\,a^{10}} + \frac{\log\left(x^2\right)}{2\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/5040*(2520*b^8*x^16 + 21420*a*b^7*x^14 + 80220*a^2*b^6*x^12 + 173250*a^3*
b^5*x^10 + 236754*a^4*b^4*x^8 + 210756*a^5*b^3*x^6 + 120564*a^6*b^2*x^4 + 4
1481*a^7*b*x^2 + 7129*a^8)/(a^9*b^9*x^18 + 9*a^10*b^8*x^16 + 36*a^11*b^7*x^
14 + 84*a^12*b^6*x^12 + 126*a^13*b^5*x^10 + 126*a^14*b^4*x^8 + 84*a^15*b^3*
```

```
x^6 + 36*a^16*b^2*x^4 + 9*a^17*b*x^2 + a^18) - 1/2*log(b*x^2 + a)/a^10 + 1/
2*log(x^2)/a^10
```

mupad [B] time = 5.46, size = 210, normalized size = 1.27

$$\frac{\ln(x)}{a^{10}}+\frac{\frac{7129}{5040\,a}+\frac{4609\,b\,x^2}{560\,a^2}+\frac{3349\,b^2\,x^4}{140\,a^3}+\frac{2509\,b^3\,x^6}{60\,a^4}+\frac{1879\,b^4\,x^8}{40\,a^5}+\frac{275\,b^5\,x^{10}}{8\,a^6}+\frac{191\,b^6\,x^{12}}{12\,a^7}+\frac{17\,b^7\,x^{14}}{4\,a^8}+\frac{b^8\,x^{16}}{2\,a^9}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^10),x)
```

```
[Out] log(x)/a^10 + (7129/(5040*a) + (4609*b*x^2)/(560*a^2) + (3349*b^2*x^4)/(140
*a^3) + (2509*b^3*x^6)/(60*a^4) + (1879*b^4*x^8)/(40*a^5) + (275*b^5*x^10)/
(8*a^6) + (191*b^6*x^12)/(12*a^7) + (17*b^7*x^14)/(4*a^8) + (b^8*x^16)/(2*a
^9))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6
*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^
7*x^14) - log(a + b*x^2)/(2*a^10)
```

sympy [A] time = 1.31, size = 223, normalized size = 1.34

$$\frac{7129a^8+41481a^7bx^2+120564a^6b^2x^4+210756a^5b^3x^6+236754a^4b^4x^8+173250a^3b^5x^{10}+80220a^2b^6x^{12}+21420ab^7x^{14}+2520b^8x^{16}}{5040a^{18}+45360a^{17}bx^2+181440a^{16}b^2x^4+423360a^{15}b^3x^6+635040a^{14}b^4x^8+635040a^{13}b^5x^{10}+423360a^{12}b^6x^{12}+181440a^{11}b^7x^{14}+45360a^{10}b^8x^{16}+5040a^9b^9x^{18}}+\frac{\log(x)}{a^{10}}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**10,x)
```

```
[Out] (7129*a**8 + 41481*a**7*b*x**2 + 120564*a**6*b**2*x**4 + 210756*a**5*b**3*x
**6 + 236754*a**4*b**4*x**8 + 173250*a**3*b**5*x**10 + 80220*a**2*b**6*x**1
2 + 21420*a*b**7*x**14 + 2520*b**8*x**16)/(5040*a**18 + 45360*a**17*b*x**2
+ 181440*a**16*b**2*x**4 + 423360*a**15*b**3*x**6 + 635040*a**14*b**4*x**8
+ 635040*a**13*b**5*x**10 + 423360*a**12*b**6*x**12 + 181440*a**11*b**7*x**
14 + 45360*a**10*b**8*x**16 + 5040*a**9*b**9*x**18) + log(x)/a**10 - log(a/
b + x**2)/(2*a**10)
```

3.206 $\int \frac{1}{x^3 \left(a+b x^2\right)^{10}} \, dx$

Optimal. Leaf size=184

$$\frac{5b\log\left(a+bx^2\right)}{a^{11}}-\frac{10b\log(x)}{a^{11}}-\frac{9b}{2a^{10}\left(a+bx^2\right)}-\frac{1}{2a^{10}x^2}-\frac{2b}{a^9\left(a+bx^2\right)^2}-\frac{7b}{6a^8\left(a+bx^2\right)^3}-\frac{3b}{4a^7\left(a+bx^2\right)^4}-\frac{b}{2a^6\left(a+b\right.}$$

Rubi [A] time = 0.19, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{9b}{2a^{10}\left(a+bx^2\right)}-\frac{2b}{a^9\left(a+bx^2\right)^2}-\frac{7b}{6a^8\left(a+bx^2\right)^3}-\frac{3b}{4a^7\left(a+bx^2\right)^4}-\frac{b}{2a^6\left(a+bx^2\right)^5}-\frac{b}{3a^5\left(a+bx^2\right)^6}-\frac{3b}{14a^4\left(a+bx^2\right)^7}-\frac{b}{8a^3\left(a+bx^2\right)^8}-\frac{b}{18a^2\left(a+bx^2\right)^9}+\frac{5b\log\left(a+bx^2\right)}{a^{11}}-\frac{10b\log(x)}{a^{11}}-\frac{1}{2a^{10}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^10),x]
```

```
[Out] -1/(2*a^10*x^2) - b/(18*a^2*(a + b*x^2)^9) - b/(8*a^3*(a + b*x^2)^8) - (3*b
)/(14*a^4*(a + b*x^2)^7) - b/(3*a^5*(a + b*x^2)^6) - b/(2*a^6*(a + b*x^2)^5
) - (3*b)/(4*a^7*(a + b*x^2)^4) - (7*b)/(6*a^8*(a + b*x^2)^3) - (2*b)/(a^9*
(a + b*x^2)^2) - (9*b)/(2*a^10*(a + b*x^2)) - (10*b*Log[x])/a^11 + (5*b*Log
[a + b*x^2])/a^11
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^{10}x^2} - \frac{10b}{a^{11}x} + \frac{b^2}{a^2(a+bx)^{10}} + \frac{2b^2}{a^3(a+bx)^9} + \frac{3b^2}{a^4(a+bx)^8} + \frac{4b^2}{a^5(a+bx)^7} + \right.\right.$$

$$= -\frac{1}{2a^{10}x^2} - \frac{b}{18a^2\left(a+bx^2\right)^9} - \frac{b}{8a^3\left(a+bx^2\right)^8} - \frac{3b}{14a^4\left(a+bx^2\right)^7} - \frac{b}{3a^5\left(a+bx^2\right)^6} - \frac{}{2a^6\left(\right.}$$

Mathematica [A] time = 0.13, size = 136, normalized size = 0.74

$$-\frac{\frac{a\left(252a^9+7129a^8bx^2+41481a^7b^2x^4+120564a^6b^3x^6+210756a^5b^4x^8+236754a^4b^5x^{10}+173250a^3b^6x^{12}+80220a^2b^7x^{14}+21420ab^8x^{16}+2520b^9x^{18}\right)}{x^2\left(a+bx^2\right)^9}-2520b\log\left(a+bx^2\right)+5040b\log(x)}{504a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^10),x]
```

```
[Out] -1/504*((a*(252*a^9 + 7129*a^8*b*x^2 + 41481*a^7*b^2*x^4 + 120564*a^6*b^3*x
^6 + 210756*a^5*b^4*x^8 + 236754*a^4*b^5*x^10 + 173250*a^3*b^6*x^12 + 80220
*a^2*b^7*x^14 + 21420*a*b^8*x^16 + 2520*b^9*x^18))/(x^2*(a + b*x^2)^9) + 50
40*b*Log[x] - 2520*b*Log[a + b*x^2])/a^11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^10), x]
```

fricas [B] time = 0.68, size = 427, normalized size = 2.32

$$-\frac{2520\,ab^9x^{18}+21420\,a^2b^8x^{16}+80220\,a^3b^7x^{14}+173250\,a^4b^6x^{12}+236754\,a^5b^5x^{10}+210756\,a^6b^4x^8+120564\,a^7b^3x^6+41481\,a^8b^2x^4+7129\,a^9bx^2+252\,a^{10}-2520\left(b^{10}x^{20}+9\,ab^9x^{18}+36\,a^2b^8x^{16}+84\,a^3b^7x^{14}+126\,a^4b^6x^{12}+126\,a^5b^5x^{10}+84\,a^6b^4x^8+36\,a^7b^3x^6+9\,a^8b^2x^4+a^9bx^2\right)\log\left(bx^2+a\right)+5040\left(b^{10}x^{20}+9\,ab^9x^{18}+36\,a^2b^8x^{16}+84\,a^3b^7x^{14}+126\,a^4b^6x^{12}+126\,a^5b^5x^{10}+84\,a^6b^4x^8+36\,a^7b^3x^6+9\,a^8b^2x^4+a^9bx^2\right)\log(x)}{504\left(a^{11}b^9x^{20}+9\,a^{12}b^8x^{18}+36\,a^{13}b^7x^{16}+84\,a^{14}b^6x^{14}+126\,a^{15}b^5x^{12}+126\,a^{16}b^4x^{10}+84\,a^{17}b^3x^8+36\,a^{18}b^2x^6+9\,a^{19}bx^4+a^{20}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(2520*a*b^9*x^18 + 21420*a^2*b^8*x^16 + 80220*a^3*b^7*x^14 + 173250*
a^4*b^6*x^12 + 236754*a^5*b^5*x^10 + 210756*a^6*b^4*x^8 + 120564*a^7*b^3*x^
```

```
6 + 41481*a^8*b^2*x^4 + 7129*a^9*b*x^2 + 252*a^10 - 2520*(b^10*x^20 + 9*a*b
^9*x^18 + 36*a^2*b^8*x^16 + 84*a^3*b^7*x^14 + 126*a^4*b^6*x^12 + 126*a^5*b^
5*x^10 + 84*a^6*b^4*x^8 + 36*a^7*b^3*x^6 + 9*a^8*b^2*x^4 + a^9*b*x^2)*log(b
*x^2 + a) + 5040*(b^10*x^20 + 9*a*b^9*x^18 + 36*a^2*b^8*x^16 + 84*a^3*b^7*x
^14 + 126*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 84*a^6*b^4*x^8 + 36*a^7*b^3*x^6
 + 9*a^8*b^2*x^4 + a^9*b*x^2)*log(x))/(a^11*b^9*x^20 + 9*a^12*b^8*x^18 + 36
*a^13*b^7*x^16 + 84*a^14*b^6*x^14 + 126*a^15*b^5*x^12 + 126*a^16*b^4*x^10 +
 84*a^17*b^3*x^8 + 36*a^18*b^2*x^6 + 9*a^19*b*x^4 + a^20*x^2)
```

giac [A] time = 0.59, size = 159, normalized size = 0.86

$$-\frac{5\,b\log\left(x^{2}\right)}{a^{11}}+\frac{5\,b\log\left(\left|bx^{2}+a\right|\right)}{a^{11}}+\frac{10\,bx^{2}-a}{2\,a^{11}x^{2}}-\frac{7129\,b^{10}x^{18}+66429\,ab^{9}x^{16}+275796\,a^{2}b^{8}x^{14}+669984\,a^{3}b^{7}x^{12}+1050336\,a^{4}b^{6}x^{10}+1103256\,a^{5}b^{5}x^{8}+777840\,a^{6}b^{4}x^{6}+356040\,a^{7}b^{3}x^{4}+96570\,a^{8}b^{2}x^{2}+11990\,a^{9}b}{504\left(bx^{2}+a\right)^{9}a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -5*b*log(x^2)/a^11 + 5*b*log(abs(b*x^2 + a))/a^11 + 1/2*(10*b*x^2 - a)/(a^1
1*x^2) - 1/504*(7129*b^10*x^18 + 66429*a*b^9*x^16 + 275796*a^2*b^8*x^14 + 6
69984*a^3*b^7*x^12 + 1050336*a^4*b^6*x^10 + 1103256*a^5*b^5*x^8 + 777840*a^
6*b^4*x^6 + 356040*a^7*b^3*x^4 + 96570*a^8*b^2*x^2 + 11990*a^9*b)/((b*x^2 +
 a)^9*a^11)
```

maple [A] time = 0.02, size = 167, normalized size = 0.91

$$-\frac{b}{18\left(bx^{2}+a\right)^{9}a^{2}}-\frac{b}{8\left(bx^{2}+a\right)^{8}a^{3}}-\frac{3b}{14\left(bx^{2}+a\right)^{7}a^{4}}-\frac{b}{3\left(bx^{2}+a\right)^{6}a^{5}}-\frac{b}{2\left(bx^{2}+a\right)^{5}a^{6}}-\frac{3b}{4\left(bx^{2}+a\right)^{4}a^{7}}-\frac{7b}{6\left(bx^{2}+a\right)^{3}a^{8}}-\frac{2b}{\left(bx^{2}+a\right)^{2}a^{9}}-\frac{9b}{2\left(bx^{2}+a\right)a^{10}}-\frac{10b\ln(x)}{a^{11}}+\frac{5b\ln\left(bx^{2}+a\right)}{a^{11}}-\frac{1}{2a^{10}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^10,x)
```

```
[Out] -1/2/a^10/x^2-1/18*b/a^2/(b*x^2+a)^9-1/8*b/a^3/(b*x^2+a)^8-3/14*b/a^4/(b*x^
2+a)^7-1/3*b/a^5/(b*x^2+a)^6-1/2*b/a^6/(b*x^2+a)^5-3/4*b/a^7/(b*x^2+a)^4-7/
6*b/a^8/(b*x^2+a)^3-2*b/a^9/(b*x^2+a)^2-9/2*b/a^10/(b*x^2+a)-10*b*ln(x)/a^1
1+5*b*ln(b*x^2+a)/a^11
```

maxima [A] time = 1.67, size = 231, normalized size = 1.26

$$-\frac{2520\,b^{9}x^{18}+21420\,ab^{8}x^{16}+80220\,a^{2}b^{7}x^{14}+173250\,a^{3}b^{6}x^{12}+236754\,a^{4}b^{5}x^{10}+210756\,a^{5}b^{4}x^{8}+120564\,a^{6}b^{3}x^{6}+41481\,a^{7}b^{2}x^{4}+7129\,a^{8}bx^{2}+252\,a^{9}}{504\left(a^{10}b^{9}x^{20}+9\,a^{11}b^{8}x^{18}+36\,a^{12}b^{7}x^{16}+84\,a^{13}b^{6}x^{14}+126\,a^{14}b^{5}x^{12}+126\,a^{15}b^{4}x^{10}+84\,a^{16}b^{3}x^{8}+36\,a^{17}b^{2}x^{6}+9\,a^{18}bx^{4}+a^{19}x^{2}\right)}+\frac{5\,b\log\left(bx^{2}+a\right)}{a^{11}}-\frac{5\,b\log\left(x^{2}\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(2520*b^9*x^18 + 21420*a*b^8*x^16 + 80220*a^2*b^7*x^14 + 173250*a^3*
b^6*x^12 + 236754*a^4*b^5*x^10 + 210756*a^5*b^4*x^8 + 120564*a^6*b^3*x^6 +
```

```
41481*a^7*b^2*x^4 + 7129*a^8*b*x^2 + 252*a^9)/(a^10*b^9*x^20 + 9*a^11*b^8*x
^18 + 36*a^12*b^7*x^16 + 84*a^13*b^6*x^14 + 126*a^14*b^5*x^12 + 126*a^15*b^
4*x^10 + 84*a^16*b^3*x^8 + 36*a^17*b^2*x^6 + 9*a^18*b*x^4 + a^19*x^2) + 5*b
*log(b*x^2 + a)/a^11 - 5*b*log(x^2)/a^11
```

mupad [B] time = 0.52, size = 229, normalized size = 1.24

$$\frac{5\,b\,\ln\left(b\,x^2+a\right)}{a^{11}}-\frac{\frac{1}{2\,a}+\frac{7129\,b\,x^2}{504\,a^2}+\frac{4609\,b^2\,x^4}{56\,a^3}+\frac{3349\,b^3\,x^6}{14\,a^4}+\frac{2509\,b^4\,x^8}{6\,a^5}+\frac{1879\,b^5\,x^{10}}{4\,a^6}+\frac{1375\,b^6\,x^{12}}{4\,a^7}+\frac{955\,b^7\,x^{14}}{6\,a^8}+\frac{85\,b^8\,x^{16}}{2\,a^9}+\frac{5\,b^9\,x^{18}}{a^{10}}}{a^9\,x^2+9\,a^8\,b\,x^4+36\,a^7\,b^2\,x^6+84\,a^6\,b^3\,x^8+126\,a^5\,b^4\,x^{10}+126\,a^4\,b^5\,x^{12}+84\,a^3\,b^6\,x^{14}+36\,a^2\,b^7\,x^{16}+9\,a\,b^8\,x^{18}+b^9\,x^{20}}-\frac{10\,b\,\ln(x)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^10),x)
```

```
[Out] (5*b*log(a + b*x^2))/a^11 - (1/(2*a) + (7129*b*x^2)/(504*a^2) + (4609*b^2*x
^4)/(56*a^3) + (3349*b^3*x^6)/(14*a^4) + (2509*b^4*x^8)/(6*a^5) + (1879*b^5
*x^10)/(4*a^6) + (1375*b^6*x^12)/(4*a^7) + (955*b^7*x^14)/(6*a^8) + (85*b^8
*x^16)/(2*a^9) + (5*b^9*x^18)/a^10)/(a^9*x^2 + b^9*x^20 + 9*a^8*b*x^4 + 9*a
*b^8*x^18 + 36*a^7*b^2*x^6 + 84*a^6*b^3*x^8 + 126*a^5*b^4*x^10 + 126*a^4*b^
5*x^12 + 84*a^3*b^6*x^14 + 36*a^2*b^7*x^16) - (10*b*log(x))/a^11
```

sympy [A] time = 1.46, size = 245, normalized size = 1.33

$$\frac{-252a^9-7129a^8bx^2-41481a^7b^2x^4-120564a^6b^3x^6-210756a^5b^4x^8-236754a^4b^5x^{10}-173250a^3b^6x^{12}-80220a^2b^7x^{14}-21420ab^8x^{16}-2520b^9x^{18}}{504a^{19}x^2+4536a^{18}bx^4+18144a^{17}b^2x^6+42336a^{16}b^3x^8+63504a^{15}b^4x^{10}+63504a^{14}b^5x^{12}+42336a^{13}b^6x^{14}+18144a^{12}b^7x^{16}+4536a^{11}b^8x^{18}+504a^{10}b^9x^{20}}-\frac{10b\log(x)}{a^{11}}+\frac{5b\log\left(\frac{a}{b}+x^2\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**10,x)
```

```
[Out] (-252*a**9 - 7129*a**8*b*x**2 - 41481*a**7*b**2*x**4 - 120564*a**6*b**3*x**
6 - 210756*a**5*b**4*x**8 - 236754*a**4*b**5*x**10 - 173250*a**3*b**6*x**12
 - 80220*a**2*b**7*x**14 - 21420*a*b**8*x**16 - 2520*b**9*x**18)/(504*a**19
*x**2 + 4536*a**18*b*x**4 + 18144*a**17*b**2*x**6 + 42336*a**16*b**3*x**8 +
 63504*a**15*b**4*x**10 + 63504*a**14*b**5*x**12 + 42336*a**13*b**6*x**14 +
 18144*a**12*b**7*x**16 + 4536*a**11*b**8*x**18 + 504*a**10*b**9*x**20) - 1
0*b*log(x)/a**11 + 5*b*log(a/b + x**2)/a**11
```

3.207 $\int \frac{1}{x^5\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=217

$$-\frac{55b^2\log\left(a+bx^2\right)}{2a^{12}}+\frac{55b^2\log(x)}{a^{12}}+\frac{45b^2}{2a^{11}\left(a+bx^2\right)}+\frac{5b}{a^{11}x^2}+\frac{9b^2}{a^{10}\left(a+bx^2\right)^2}-\frac{1}{4a^{10}x^4}+\frac{14b^2}{3a^9\left(a+bx^2\right)^3}+\frac{21b^2}{8a^8\left(a+bx^2\right.}$$

Rubi [A] time = 0.22, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{45b^2}{2a^{11}\left(a+bx^2\right)}+\frac{9b^2}{a^{10}\left(a+bx^2\right)^2}+\frac{14b^2}{3a^9\left(a+bx^2\right)^3}+\frac{21b^2}{8a^8\left(a+bx^2\right)^4}+\frac{3b^2}{2a^7\left(a+bx^2\right)^5}+\frac{5b^2}{6a^6\left(a+bx^2\right)^6}+\frac{3b^2}{7a^5\left(a+bx^2\right)^7}+\frac{3b^2}{16a^4\left(a+bx^2\right)^8}+\frac{b^2}{18a^3\left(a+bx^2\right)^9}-\frac{55b^2\log\left(a+bx^2\right)}{2a^{12}}+\frac{55b^2\log(x)}{a^{12}}+\frac{5b}{a^{11}x^2}-\frac{1}{4a^{10}x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^10),x]
```

```
[Out] -1/(4*a^10*x^4) + (5*b)/(a^11*x^2) + b^2/(18*a^3*(a + b*x^2)^9) + (3*b^2)/(
16*a^4*(a + b*x^2)^8) + (3*b^2)/(7*a^5*(a + b*x^2)^7) + (5*b^2)/(6*a^6*(a +
 b*x^2)^6) + (3*b^2)/(2*a^7*(a + b*x^2)^5) + (21*b^2)/(8*a^8*(a + b*x^2)^4)
 + (14*b^2)/(3*a^9*(a + b*x^2)^3) + (9*b^2)/(a^10*(a + b*x^2)^2) + (45*b^2)
/(2*a^11*(a + b*x^2)) + (55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x^2])/(2*a
^12)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5 \left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^{10}x^3} - \frac{10b}{a^{11}x^2} + \frac{55b^2}{a^{12}x} - \frac{b^3}{a^3(a+bx)^{10}} - \frac{3b^3}{a^4(a+bx)^9} - \frac{6b^3}{a^5(a+bx)^8} - \cdots\right.\right.$$

$$= -\frac{1}{4a^{10}x^4} + \frac{5b}{a^{11}x^2} + \frac{b^2}{18a^3\left(a+bx^2\right)^9} + \frac{3b^2}{16a^4\left(a+bx^2\right)^8} + \frac{3b^2}{7a^5\left(a+bx^2\right)^7} + \frac{5b^2}{6a^6\left(a+bx^2\right)\cdots}$$

Mathematica [A] time = 0.10, size = 151, normalized size = 0.70

$$\frac{\frac{a\left(-252a^{10}+2772a^9bx^2+78419a^8b^2x^4+456291a^7b^3x^6+1326204a^6b^4x^8+2318316a^5b^5x^{10}+2604294a^4b^6x^{12}+1905750a^3b^7x^{14}+882420a^2b^8x^{16}+235620ab^9x^{18}+27720b^{10}x^{20}\right)}{x^4\left(a+bx^2\right)^9} - 27720b^2\log\left(a+bx^2\right) + 55440b^2\log(x)}{1008a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^10),x]
```

```
[Out] ((a*(-252*a^10 + 2772*a^9*b*x^2 + 78419*a^8*b^2*x^4 + 456291*a^7*b^3*x^6 +
1326204*a^6*b^4*x^8 + 2318316*a^5*b^5*x^10 + 2604294*a^4*b^6*x^12 + 1905750
*a^3*b^7*x^14 + 882420*a^2*b^8*x^16 + 235620*a*b^9*x^18 + 27720*b^10*x^20))
/(x^4*(a + b*x^2)^9) + 55440*b^2*Log[x] - 27720*b^2*Log[a + b*x^2])/(1008*a
^12)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^10), x]
```

fricas [B] time = 0.61, size = 442, normalized size = 2.04

$$\frac{27720\,ab^{10}x^{20} + 235620\,a^2b^9x^{18} + 882420\,a^3b^8x^{16} + 1905750\,a^4b^7x^{14} + 2604294\,a^5b^6x^{12} + 2318316\,a^6b^5x^{10} + 1326204\,a^7b^4x^8 + 456291\,a^8b^3x^6 + 78419\,a^9b^2x^4 + 2772\,a^{10}bx^2 - 252\,a^{11} - 27720\left(b^{11}x^{22} + 9\,ab^{10}x^{20} + 36\,a^2b^9x^{18} + 84\,a^3b^8x^{16} + 126\,a^4b^7x^{14} + 126\,a^5b^6x^{12} + 84\,a^6b^5x^{10} + 36\,a^7b^4x^8 + 9\,a^8b^3x^6 + a^9b^2x^4\right)\log\left(bx^2+a\right) + 55440\left(b^{11}x^{22} + 9\,ab^{10}x^{20} + 36\,a^2b^9x^{18} + 84\,a^3b^8x^{16} + 126\,a^4b^7x^{14} + 126\,a^5b^6x^{12} + 84\,a^6b^5x^{10} + 36\,a^7b^4x^8 + 9\,a^8b^3x^6 + a^9b^2x^4\right)\log(x)}{1008\left(a^{12}b^9x^{22} + 9\,a^{13}b^8x^{20} + 36\,a^{14}b^7x^{18} + 84\,a^{15}b^6x^{16} + 126\,a^{16}b^5x^{14} + 126\,a^{17}b^4x^{12} + 84\,a^{18}b^3x^{10} + 36\,a^{19}b^2x^8 + 9\,a^{20}bx^6 + a^{21}x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] 1/1008*(27720*a*b^10*x^20 + 235620*a^2*b^9*x^18 + 882420*a^3*b^8*x^16 + 190
5750*a^4*b^7*x^14 + 2604294*a^5*b^6*x^12 + 2318316*a^6*b^5*x^10 + 1326204*a
^7*b^4*x^8 + 456291*a^8*b^3*x^6 + 78419*a^9*b^2*x^4 + 2772*a^10*b*x^2 - 252
*a^11 - 27720*(b^11*x^22 + 9*a*b^10*x^20 + 36*a^2*b^9*x^18 + 84*a^3*b^8*x^1
6 + 126*a^4*b^7*x^14 + 126*a^5*b^6*x^12 + 84*a^6*b^5*x^10 + 36*a^7*b^4*x^8
+ 9*a^8*b^3*x^6 + a^9*b^2*x^4)*log(b*x^2 + a) + 55440*(b^11*x^22 + 9*a*b^10
*x^20 + 36*a^2*b^9*x^18 + 84*a^3*b^8*x^16 + 126*a^4*b^7*x^14 + 126*a^5*b^6*
x^12 + 84*a^6*b^5*x^10 + 36*a^7*b^4*x^8 + 9*a^8*b^3*x^6 + a^9*b^2*x^4)*log(
x))/(a^12*b^9*x^22 + 9*a^13*b^8*x^20 + 36*a^14*b^7*x^18 + 84*a^15*b^6*x^16
+ 126*a^16*b^5*x^14 + 126*a^17*b^4*x^12 + 84*a^18*b^3*x^10 + 36*a^19*b^2*x^
8 + 9*a^20*b*x^6 + a^21*x^4)
```

giac [A] time = 0.61, size = 174, normalized size = 0.80

$$\frac{55\,b^2\log\left(x^2\right)}{2\,a^{12}}-\frac{55\,b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^{12}}-\frac{165\,b^2x^4-20\,abx^2+a^2}{4\,a^{12}x^4}+\frac{78419\,b^{11}x^{18}+728451\,ab^{10}x^{16}+3013596\,a^2b^9x^{14}+7290444\,a^3b^8x^{12}+11372256\,a^4b^7x^{10}+11871216\,a^5b^6x^8+8302224\,a^6b^5x^6+3757680\,a^7b^4x^4+1001790\,a^8b^3x^2+120550\,a^9b^2}{1008\,\left(bx^2+a\right)^9a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/2*b^2*log(x^2)/a^12 - 55/2*b^2*log(abs(b*x^2 + a))/a^12 - 1/4*(165*b^2*x
^4 - 20*a*b*x^2 + a^2)/(a^12*x^4) + 1/1008*(78419*b^11*x^18 + 728451*a*b^10
*x^16 + 3013596*a^2*b^9*x^14 + 7290444*a^3*b^8*x^12 + 11372256*a^4*b^7*x^10
 + 11871216*a^5*b^6*x^8 + 8302224*a^6*b^5*x^6 + 3757680*a^7*b^4*x^4 + 10017
90*a^8*b^3*x^2 + 120550*a^9*b^2)/((b*x^2 + a)^9*a^12)
```

maple [A] time = 0.02, size = 198, normalized size = 0.91

$$\frac{b^2}{18\left(bx^2+a\right)^9a^3}+\frac{3b^2}{16\left(bx^2+a\right)^8a^4}+\frac{3b^2}{7\left(bx^2+a\right)^7a^5}+\frac{5b^2}{6\left(bx^2+a\right)^6a^6}+\frac{3b^2}{2\left(bx^2+a\right)^5a^7}+\frac{21b^2}{8\left(bx^2+a\right)^4a^8}+\frac{14b^2}{3\left(bx^2+a\right)^3a^9}+\frac{9b^2}{\left(bx^2+a\right)^2a^{10}}+\frac{45b^2}{2\left(bx^2+a\right)a^{11}}+\frac{55b^2\ln(x)}{a^{12}}-\frac{55b^2\ln\left(bx^2+a\right)}{2a^{12}}+\frac{5b}{a^{11}x^2}-\frac{1}{4a^{10}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^10,x)
```

```
[Out] -1/4/a^10/x^4+5*b/a^11/x^2+1/18*b^2/a^3/(b*x^2+a)^9+3/16*b^2/a^4/(b*x^2+a)^
8+3/7*b^2/a^5/(b*x^2+a)^7+5/6*b^2/a^6/(b*x^2+a)^6+3/2*b^2/a^7/(b*x^2+a)^5+2
1/8*b^2/a^8/(b*x^2+a)^4+14/3*b^2/a^9/(b*x^2+a)^3+9*b^2/a^10/(b*x^2+a)^2+45/
2*b^2/a^11/(b*x^2+a)+55*b^2*ln(x)/a^12-55/2*b^2*ln(b*x^2+a)/a^12
```

maxima [A] time = 1.71, size = 246, normalized size = 1.13

$$\frac{27720\,b^{10}x^{20}+235620\,ab^9x^{18}+882420\,a^2b^8x^{16}+1905750\,a^3b^7x^{14}+2604294\,a^4b^6x^{12}+2318316\,a^5b^5x^{10}+1326204\,a^6b^4x^8+456291\,a^7b^3x^6+78419\,a^8b^2x^4+2772\,a^9bx^2-252\,a^{10}}{1008\left(a^{11}b^9x^{22}+9\,a^{12}b^8x^{20}+36\,a^{13}b^7x^{18}+84\,a^{14}b^6x^{16}+126\,a^{15}b^5x^{14}+126\,a^{16}b^4x^{12}+84\,a^{17}b^3x^{10}+36\,a^{18}b^2x^8+9\,a^{19}bx^6+a^{20}x^4\right)}-\frac{55\,b^2\log\left(bx^2+a\right)}{2\,a^{12}}+\frac{55\,b^2\log\left(x^2\right)}{2\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/1008*(27720*b^10*x^20 + 235620*a*b^9*x^18 + 882420*a^2*b^8*x^16 + 1905750
*a^3*b^7*x^14 + 2604294*a^4*b^6*x^12 + 2318316*a^5*b^5*x^10 + 1326204*a^6*b
^4*x^8 + 456291*a^7*b^3*x^6 + 78419*a^8*b^2*x^4 + 2772*a^9*b*x^2 - 252*a^10
)/(a^11*b^9*x^22 + 9*a^12*b^8*x^20 + 36*a^13*b^7*x^18 + 84*a^14*b^6*x^16 +
126*a^15*b^5*x^14 + 126*a^16*b^4*x^12 + 84*a^17*b^3*x^10 + 36*a^18*b^2*x^8
+ 9*a^19*b*x^6 + a^20*x^4) - 55/2*b^2*log(b*x^2 + a)/a^12 + 55/2*b^2*log(x^
2)/a^12
```

mupad [B] time = 5.78, size = 243, normalized size = 1.12

$$\frac{\frac{11\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{78419\,b^2\,x^4}{1008\,a^3}+\frac{50699\,b^3\,x^6}{112\,a^4}+\frac{36839\,b^4\,x^8}{28\,a^5}+\frac{27599\,b^5\,x^{10}}{12\,a^6}+\frac{20669\,b^6\,x^{12}}{8\,a^7}+\frac{15125\,b^7\,x^{14}}{8\,a^8}+\frac{10505\,b^8\,x^{16}}{12\,a^9}+\frac{935\,b^9\,x^{18}}{4\,a^{10}}+\frac{55\,b^{10}\,x^{20}}{2\,a^{11}}}{a^9\,x^4+9\,a^8\,b\,x^6+36\,a^7\,b^2\,x^8+84\,a^6\,b^3\,x^{10}+126\,a^5\,b^4\,x^{12}+126\,a^4\,b^5\,x^{14}+84\,a^3\,b^6\,x^{16}+36\,a^2\,b^7\,x^{18}+9\,a\,b^8\,x^{20}+b^9\,x^{22}}-\frac{55\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^{12}}+\frac{55\,b^2\,\ln(x)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^10),x)
```

```
[Out] ((11*b*x^2)/(4*a^2) - 1/(4*a) + (78419*b^2*x^4)/(1008*a^3) + (50699*b^3*x^6
)/(112*a^4) + (36839*b^4*x^8)/(28*a^5) + (27599*b^5*x^10)/(12*a^6) + (20669
*b^6*x^12)/(8*a^7) + (15125*b^7*x^14)/(8*a^8) + (10505*b^8*x^16)/(12*a^9) +
 (935*b^9*x^18)/(4*a^10) + (55*b^10*x^20)/(2*a^11))/(a^9*x^4 + b^9*x^22 + 9
*a^8*b*x^6 + 9*a*b^8*x^20 + 36*a^7*b^2*x^8 + 84*a^6*b^3*x^10 + 126*a^5*b^4*
x^12 + 126*a^4*b^5*x^14 + 84*a^3*b^6*x^16 + 36*a^2*b^7*x^18) - (55*b^2*log(
a + b*x^2))/(2*a^12) + (55*b^2*log(x))/a^12
```

sympy [A] time = 1.55, size = 260, normalized size = 1.20

$$\frac{-252a^{10}+2772a^9bx^2+78419a^8b^2x^4+456291a^7b^3x^6+1326204a^6b^4x^8+2318316a^5b^5x^{10}+2604294a^4b^6x^{12}+1905750a^3b^7x^{14}+882420a^2b^8x^{16}+235620ab^9x^{18}+27720b^{10}x^{20}}{1008a^{20}x^4+9072a^{19}bx^6+36288a^{18}b^2x^8+84672a^{17}b^3x^{10}+127008a^{16}b^4x^{12}+127008a^{15}b^5x^{14}+84672a^{14}b^6x^{16}+36288a^{13}b^7x^{18}+9072a^{12}b^8x^{20}+1008a^{11}b^9x^{22}}+\frac{55b^2\log{(x)}}{a^{12}}-\frac{55b^2\log{\left(\frac{a}{b}+x^2\right)}}{2a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**10,x)
```

```
[Out] (-252*a**10 + 2772*a**9*b*x**2 + 78419*a**8*b**2*x**4 + 456291*a**7*b**3*x*
*6 + 1326204*a**6*b**4*x**8 + 2318316*a**5*b**5*x**10 + 2604294*a**4*b**6*x
**12 + 1905750*a**3*b**7*x**14 + 882420*a**2*b**8*x**16 + 235620*a*b**9*x**
18 + 27720*b**10*x**20)/(1008*a**20*x**4 + 9072*a**19*b*x**6 + 36288*a**18*
b**2*x**8 + 84672*a**17*b**3*x**10 + 127008*a**16*b**4*x**12 + 127008*a**15
*b**5*x**14 + 84672*a**14*b**6*x**16 + 36288*a**13*b**7*x**18 + 9072*a**12*
b**8*x**20 + 1008*a**11*b**9*x**22) + 55*b**2*log(x)/a**12 - 55*b**2*log(a/
b + x**2)/(2*a**12)
```

3.208 $\int \frac{1}{x^7\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=226

$$\frac{110b^3\log\left(a+bx^2\right)}{a^{13}}-\frac{220b^3\log(x)}{a^{13}}-\frac{165b^3}{2a^{12}\left(a+bx^2\right)}-\frac{55b^2}{2a^{12}x^2}-\frac{30b^3}{a^{11}\left(a+bx^2\right)^2}+\frac{5b}{2a^{11}x^4}-\frac{14b^3}{a^{10}\left(a+bx^2\right)^3}-\frac{1}{6a^{10}x^6}-\frac{}{a^9}$$

Rubi [A] time = 0.23, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{165b^3}{2a^{12}\left(a+bx^2\right)}-\frac{30b^3}{a^{11}\left(a+bx^2\right)^2}-\frac{14b^3}{a^{10}\left(a+bx^2\right)^3}-\frac{7b^3}{a^9\left(a+bx^2\right)^4}-\frac{7b^3}{2a^8\left(a+bx^2\right)^5}-\frac{5b^3}{3a^7\left(a+bx^2\right)^6}-\frac{5b^3}{7a^6\left(a+bx^2\right)^7}-\frac{b^3}{4a^5\left(a+bx^2\right)^8}-\frac{b^3}{18a^4\left(a+bx^2\right)^9}-\frac{55b^2}{2a^{12}x^2}+\frac{110b^3\log\left(a+bx^2\right)}{a^{13}}-\frac{220b^3\log(x)}{a^{13}}+\frac{5b}{2a^{11}x^4}-\frac{1}{6a^{10}x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^10),x]
```

```
[Out] -1/(6*a^10*x^6) + (5*b)/(2*a^11*x^4) - (55*b^2)/(2*a^12*x^2) - b^3/(18*a^4*
(a + b*x^2)^9) - b^3/(4*a^5*(a + b*x^2)^8) - (5*b^3)/(7*a^6*(a + b*x^2)^7)
- (5*b^3)/(3*a^7*(a + b*x^2)^6) - (7*b^3)/(2*a^8*(a + b*x^2)^5) - (7*b^3)/(
a^9*(a + b*x^2)^4) - (14*b^3)/(a^10*(a + b*x^2)^3) - (30*b^3)/(a^11*(a + b*
x^2)^2) - (165*b^3)/(2*a^12*(a + b*x^2)) - (220*b^3*Log[x])/a^13 + (110*b^3
*Log[a + b*x^2])/a^13
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^2\right)^{10}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)^{10}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^{10}x^4} - \frac{10b}{a^{11}x^3} + \frac{55b^2}{a^{12}x^2} - \frac{220b^3}{a^{13}x} + \frac{b^4}{a^4(a+bx)^{10}} + \frac{4b^4}{a^5(a+bx)^9} + \frac{10b^4}{a^6(a+bx \ldots}\right.\right.$$

$$= -\frac{1}{6a^{10}x^6} + \frac{5b}{2a^{11}x^4} - \frac{55b^2}{2a^{12}x^2} - \frac{b^3}{18a^4\left(a+bx^2\right)^9} - \frac{b^3}{4a^5\left(a+bx^2\right)^8} - \frac{5b^3}{7a^6\left(a+bx^2\right)^7} - \frac{\ldots}{3a^7 \ldots}$$

Mathematica [A] time = 0.13, size = 162, normalized size = 0.72

$$-\frac{-27720b^3\log\left(a+bx^2\right) + \frac{a\left(42a^{11}-252a^{10}bx^2+2772a^9b^2x^4+78419a^8b^3x^6+456291a^7b^4x^8+1326204a^6b^5x^{10}+2318316a^5b^6x^{12}+2604294a^4b^7x^{14}+1905750a^3b^8x^{16}+882420a^2b^9x^{18}+235620ab^{10}x^{20}+27720b^{11}x^{22}\right)}{x^6\left(a+bx^2\right)^9} + 55440b^3\log(x)}{252a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^10),x]
```

```
[Out] -1/252*((a*(42*a^11 - 252*a^10*b*x^2 + 2772*a^9*b^2*x^4 + 78419*a^8*b^3*x^6
 + 456291*a^7*b^4*x^8 + 1326204*a^6*b^5*x^10 + 2318316*a^5*b^6*x^12 + 26042
94*a^4*b^7*x^14 + 1905750*a^3*b^8*x^16 + 882420*a^2*b^9*x^18 + 235620*a*b^1
0*x^20 + 27720*b^11*x^22))/(x^6*(a + b*x^2)^9) + 55440*b^3*Log[x] - 27720*b
^3*Log[a + b*x^2])/a^13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^2)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(a + b*x^2)^10), x]
```

fricas [B] time = 1.08, size = 453, normalized size = 2.00

$$-\frac{27720\,ab^{11}x^{22} + 235620\,a^2b^{10}x^{20} + 882420\,a^3b^9x^{18} + 1905750\,a^4b^8x^{16} + 2604294\,a^5b^7x^{14} + 2318316\,a^6b^6x^{12} + 1326204\,a^7b^5x^{10} + 456291\,a^8b^4x^8 + 78419\,a^9b^3x^6 + 2772\,a^{10}b^2x^4 - 252\,a^{11}bx^2 + 42\,a^{12} - 27720\left(b^{12}x^{24} + 9\,ab^{11}x^{22} + 36\,a^2b^{10}x^{20} + 84\,a^3b^9x^{18} + 126\,a^4b^8x^{16} + 126\,a^5b^7x^{14} + 84\,a^6b^6x^{12} + 36\,a^7b^5x^{10} + 9\,a^8b^4x^8 + a^9b^3x^6\right)\log\left(bx^2+a\right) + 55440\left(b^{12}x^{24} + 9\,ab^{11}x^{22} + 36\,a^2b^{10}x^{20} + 84\,a^3b^9x^{18} + 126\,a^4b^8x^{16} + 126\,a^5b^7x^{14} + 84\,a^6b^6x^{12} + 36\,a^7b^5x^{10} + 9\,a^8b^4x^8 + a^9b^3x^6\right)\log(x)}{252\left(a^{13}b^9x^{24} + 9\,a^{14}b^8x^{22} + 36\,a^{15}b^7x^{20} + 84\,a^{16}b^6x^{18} + 126\,a^{17}b^5x^{16} + 126\,a^{18}b^4x^{14} + 84\,a^{19}b^3x^{12} + 36\,a^{20}b^2x^{10} + 9\,a^{21}bx^8 + a^{22}x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(27720*a*b^11*x^22 + 235620*a^2*b^10*x^20 + 882420*a^3*b^9*x^18 + 19
05750*a^4*b^8*x^16 + 2604294*a^5*b^7*x^14 + 2318316*a^6*b^6*x^12 + 1326204*
a^7*b^5*x^10 + 456291*a^8*b^4*x^8 + 78419*a^9*b^3*x^6 + 2772*a^10*b^2*x^4 -
 252*a^11*b*x^2 + 42*a^12 - 27720*(b^12*x^24 + 9*a*b^11*x^22 + 36*a^2*b^10*
x^20 + 84*a^3*b^9*x^18 + 126*a^4*b^8*x^16 + 126*a^5*b^7*x^14 + 84*a^6*b^6*x
^12 + 36*a^7*b^5*x^10 + 9*a^8*b^4*x^8 + a^9*b^3*x^6)*log(b*x^2 + a) + 55440
*(b^12*x^24 + 9*a*b^11*x^22 + 36*a^2*b^10*x^20 + 84*a^3*b^9*x^18 + 126*a^4*
b^8*x^16 + 126*a^5*b^7*x^14 + 84*a^6*b^6*x^12 + 36*a^7*b^5*x^10 + 9*a^8*b^4
*x^8 + a^9*b^3*x^6)*log(x))/(a^13*b^9*x^24 + 9*a^14*b^8*x^22 + 36*a^15*b^7*
x^20 + 84*a^16*b^6*x^18 + 126*a^17*b^5*x^16 + 126*a^18*b^4*x^14 + 84*a^19*b
^3*x^12 + 36*a^20*b^2*x^10 + 9*a^21*b*x^8 + a^22*x^6)
```

giac [A] time = 0.64, size = 187, normalized size = 0.83

$$-\frac{110\,b^3\log\left(x^2\right)}{a^{13}}+\frac{110\,b^3\log\left(|bx^2+a|\right)}{a^{13}}+\frac{1210\,b^3x^6-165\,ab^2x^4+15\,a^2bx^2-a^3}{6\,a^{13}x^6}-\frac{78419\,b^{12}x^{18}+726561\,ab^{11}x^{16}+2996964\,a^2b^{10}x^{14}+7225764\,a^3b^9x^{12}+11226726\,a^4b^8x^{10}+11663316\,a^5b^7x^8+8108184\,a^6b^6x^6+3641256\,a^7b^5x^4+960210\,a^8b^4x^2+113620\,a^9b^3}{252\left(bx^2+a\right)^9a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -110*b^3*log(x^2)/a^13 + 110*b^3*log(abs(b*x^2 + a))/a^13 + 1/6*(1210*b^3*x
^6 - 165*a*b^2*x^4 + 15*a^2*b*x^2 - a^3)/(a^13*x^6) - 1/252*(78419*b^12*x^1
8 + 726561*a*b^11*x^16 + 2996964*a^2*b^10*x^14 + 7225764*a^3*b^9*x^12 + 112
26726*a^4*b^8*x^10 + 11663316*a^5*b^7*x^8 + 8108184*a^6*b^6*x^6 + 3641256*a
^7*b^5*x^4 + 960210*a^8*b^4*x^2 + 113620*a^9*b^3)/((b*x^2 + a)^9*a^13)
```

maple [A] time = 0.02, size = 209, normalized size = 0.92

$$-\frac{b^3}{18\left(b\,x^2+a\right)^9a^4}-\frac{b^3}{4\left(b\,x^2+a\right)^8a^5}-\frac{5b^3}{7\left(b\,x^2+a\right)^7a^6}-\frac{5b^3}{3\left(b\,x^2+a\right)^6a^7}-\frac{7b^3}{2\left(b\,x^2+a\right)^5a^8}-\frac{7b^3}{\left(b\,x^2+a\right)^4a^9}-\frac{14b^3}{\left(b\,x^2+a\right)^3a^{10}}-\frac{30b^3}{\left(b\,x^2+a\right)^2a^{11}}-\frac{165b^3}{2\left(b\,x^2+a\right)a^{12}}-\frac{220b^3\ln(x)}{a^{13}}+\frac{110b^3\ln\left(b\,x^2+a\right)}{a^{13}}-\frac{55b^2}{2a^{12}x^2}+\frac{5b}{2a^{11}x^4}-\frac{1}{6a^{10}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^10,x)
```

```
[Out] -1/6/a^10/x^6+5/2*b/a^11/x^4-55/2*b^2/a^12/x^2-1/18*b^3/a^4/(b*x^2+a)^9-1/4
*b^3/a^5/(b*x^2+a)^8-5/7*b^3/a^6/(b*x^2+a)^7-5/3*b^3/a^7/(b*x^2+a)^6-7/2*b^
3/a^8/(b*x^2+a)^5-7*b^3/a^9/(b*x^2+a)^4-14*b^3/a^10/(b*x^2+a)^3-30*b^3/a^11
/(b*x^2+a)^2-165/2*b^3/a^12/(b*x^2+a)-220*b^3*ln(x)/a^13+110*b^3*ln(b*x^2+a
)/a^13
```

maxima [A] time = 1.71, size = 257, normalized size = 1.14

$$-\frac{27720\,b^{11}x^{22}+235620\,ab^{10}x^{20}+882420\,a^2b^9x^{18}+1905750\,a^3b^8x^{16}+2604294\,a^4b^7x^{14}+2318316\,a^5b^6x^{12}+1326204\,a^6b^5x^{10}+456291\,a^7b^4x^8+78419\,a^8b^3x^6+2772\,a^9b^2x^4-252\,a^{10}bx^2+42\,a^{11}}{252\left(a^{12}b^9x^{24}+9\,a^{13}b^8x^{22}+36\,a^{14}b^7x^{20}+84\,a^{15}b^6x^{18}+126\,a^{16}b^5x^{16}+126\,a^{17}b^4x^{14}+84\,a^{18}b^3x^{12}+36\,a^{19}b^2x^{10}+9\,a^{20}bx^8+a^{21}x^6\right)}+\frac{110\,b^3\log\left(bx^2+a\right)}{a^{13}}-\frac{110\,b^3\log\left(x^2\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(27720*b^11*x^22 + 235620*a*b^10*x^20 + 882420*a^2*b^9*x^18 + 190575
0*a^3*b^8*x^16 + 2604294*a^4*b^7*x^14 + 2318316*a^5*b^6*x^12 + 1326204*a^6*
b^5*x^10 + 456291*a^7*b^4*x^8 + 78419*a^8*b^3*x^6 + 2772*a^9*b^2*x^4 - 252*
a^10*b*x^2 + 42*a^11)/(a^12*b^9*x^24 + 9*a^13*b^8*x^22 + 36*a^14*b^7*x^20 +
 84*a^15*b^6*x^18 + 126*a^16*b^5*x^16 + 126*a^17*b^4*x^14 + 84*a^18*b^3*x^1
2 + 36*a^19*b^2*x^10 + 9*a^20*b*x^8 + a^21*x^6) + 110*b^3*log(b*x^2 + a)/a^
13 - 110*b^3*log(x^2)/a^13
```

mupad [B] time = 1.09, size = 255, normalized size = 1.13

$$\frac{110\,b^3\,\ln\left(b\,x^2+a\right)}{a^{13}}-\frac{\frac{1}{6\,a}-\frac{b\,x^2}{a^2}+\frac{11\,b^2\,x^4}{a^3}+\frac{78419\,b^3\,x^6}{252\,a^4}+\frac{50699\,b^4\,x^8}{28\,a^5}+\frac{36839\,b^5\,x^{10}}{7\,a^6}+\frac{27599\,b^6\,x^{12}}{3\,a^7}+\frac{20669\,b^7\,x^{14}}{2\,a^8}+\frac{15125\,b^8\,x^{16}}{2\,a^9}+\frac{10505\,b^9\,x^{18}}{3\,a^{10}}+\frac{935\,b^{10}\,x^{20}}{a^{11}}+\frac{110\,b^{11}\,x^{22}}{a^{12}}}{a^9\,x^6+9\,a^8\,b\,x^8+36\,a^7\,b^2\,x^{10}+84\,a^6\,b^3\,x^{12}+126\,a^5\,b^4\,x^{14}+126\,a^4\,b^5\,x^{16}+84\,a^3\,b^6\,x^{18}+36\,a^2\,b^7\,x^{20}+9\,a\,b^8\,x^{22}+b^9\,x^{24}}-\frac{220\,b^3\,\ln(x)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^10),x)
```

```
[Out] (110*b^3*log(a + b*x^2))/a^13 - (1/(6*a) - (b*x^2)/a^2 + (11*b^2*x^4)/a^3 +
 (78419*b^3*x^6)/(252*a^4) + (50699*b^4*x^8)/(28*a^5) + (36839*b^5*x^10)/(7
*a^6) + (27599*b^6*x^12)/(3*a^7) + (20669*b^7*x^14)/(2*a^8) + (15125*b^8*x^
16)/(2*a^9) + (10505*b^9*x^18)/(3*a^10) + (935*b^10*x^20)/a^11 + (110*b^11*
x^22)/a^12)/(a^9*x^6 + b^9*x^24 + 9*a^8*b*x^8 + 9*a*b^8*x^22 + 36*a^7*b^2*x
^10 + 84*a^6*b^3*x^12 + 126*a^5*b^4*x^14 + 126*a^4*b^5*x^16 + 84*a^3*b^6*x^
18 + 36*a^2*b^7*x^20) - (220*b^3*log(x))/a^13
```

sympy [A] time = 1.60, size = 270, normalized size = 1.19

$$\frac{-42a^{11}+252a^{10}bx^2-2772a^9b^2x^4-78419a^8b^3x^6-456291a^7b^4x^8-1326204a^6b^5x^{10}-2318316a^5b^6x^{12}-2604294a^4b^7x^{14}-1905750a^3b^8x^{16}-882420a^2b^9x^{18}-235620ab^{10}x^{20}-27720b^{11}x^{22}}{252a^{21}x^6+2268a^{20}bx^8+9072a^{19}b^2x^{10}+21168a^{18}b^3x^{12}+31752a^{17}b^4x^{14}+31752a^{16}b^5x^{16}+21168a^{15}b^6x^{18}+9072a^{14}b^7x^{20}+2268a^{13}b^8x^{22}+252a^{12}b^9x^{24}}-\frac{220b^3\log(x)}{a^{13}}+\frac{110b^3\log\left(\frac{a}{b}+x^2\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**10,x)
```

```
[Out] (-42*a**11 + 252*a**10*b*x**2 - 2772*a**9*b**2*x**4 - 78419*a**8*b**3*x**6
- 456291*a**7*b**4*x**8 - 1326204*a**6*b**5*x**10 - 2318316*a**5*b**6*x**12
 - 2604294*a**4*b**7*x**14 - 1905750*a**3*b**8*x**16 - 882420*a**2*b**9*x**
18 - 235620*a*b**10*x**20 - 27720*b**11*x**22)/(252*a**21*x**6 + 2268*a**20
*b*x**8 + 9072*a**19*b**2*x**10 + 21168*a**18*b**3*x**12 + 31752*a**17*b**4
*x**14 + 31752*a**16*b**5*x**16 + 21168*a**15*b**6*x**18 + 9072*a**14*b**7*
x**20 + 2268*a**13*b**8*x**22 + 252*a**12*b**9*x**24) - 220*b**3*log(x)/a**
13 + 110*b**3*log(a/b + x**2)/a**13
```

3.209 $\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=231

$$-\frac{7436429a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{25/2}}+\frac{7436429a^2x}{65536b^{12}}-\frac{7436429ax^3}{196608b^{11}}-\frac{1062347x^7}{65536b^9\left(a+bx^2\right)}-\frac{1062347x^9}{294912b^8\left(a+bx^2\right)^2}-\frac{96577x^1}{73728b^7\left(a+\right.}$$

Rubi [A] time = 0.17, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{7436429a^2x}{65536b^{12}}-\frac{7436429a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{25/2}}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{19}}{192b^3\left(a+bx^2\right)^7}-\frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6}-\frac{7429x^{15}}{23040b^5\left(a+bx^2\right)^5}-\frac{7429x^{13}}{12288b^6\left(a+bx^2\right)^4}-\frac{96577x^{11}}{73728b^7\left(a+bx^2\right)^3}-\frac{1062347x^9}{294912b^8\left(a+bx^2\right)^2}-\frac{1062347x^7}{65536b^9\left(a+bx^2\right)}-\frac{7436429ax^3}{196608b^{11}}-\frac{x^{23}}{18b\left(a+bx^2\right)^9}+\frac{7436429x^5}{327680b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^24/(a + b*x^2)^10,x]
```

```
[Out] (7436429*a^2*x)/(65536*b^12) - (7436429*a*x^3)/(196608*b^11) + (7436429*x^5
)/(327680*b^10) - x^23/(18*b*(a + b*x^2)^9) - (23*x^21)/(288*b^2*(a + b*x^2
)^8) - (23*x^19)/(192*b^3*(a + b*x^2)^7) - (437*x^17)/(2304*b^4*(a + b*x^2)
^6) - (7429*x^15)/(23040*b^5*(a + b*x^2)^5) - (7429*x^13)/(12288*b^6*(a + b
*x^2)^4) - (96577*x^11)/(73728*b^7*(a + b*x^2)^3) - (1062347*x^9)/(294912*b
^8*(a + b*x^2)^2) - (1062347*x^7)/(65536*b^9*(a + b*x^2)) - (7436429*a^(5/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(25/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{24}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{23}}{18b\left(a+bx^2\right)^9}+\frac{23\int \frac{x^{22}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}+\frac{161\int \frac{x^{20}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{19}}{192b^3\left(a+bx^2\right)^7}+\frac{437\int \frac{x^{18}}{\left(a+bx^2\right)^7}\,dx}{192b^3}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{19}}{192b^3\left(a+bx^2\right)^7}-\frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6}+\frac{7429\int \frac{x^{16}}{\left(a+bx\cdots\right.}\cdots}{2304b^4\cdots}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{19}}{192b^3\left(a+bx^2\right)^7}-\frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6}-\frac{7429x^{1\cdots}}{23040b^5\left(a+\cdots\right.}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{19}}{192b^3\left(a+bx^2\right)^7}-\frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6}-\frac{7429x^{1\cdots}}{23040b^5\left(a+\cdots\right.}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{19}}{192b^3\left(a+bx^2\right)^7}-\frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6}-\frac{7429x^{1\cdots}}{23040b^5\left(a+\cdots\right.}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{19}}{192b^3\left(a+bx^2\right)^7}-\frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6}-\frac{7429x^{1\cdots}}{23040b^5\left(a+\cdots\right.}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{19}}{192b^3\left(a+bx^2\right)^7}-\frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6}-\frac{7429x^{1\cdots}}{23040b^5\left(a+\cdots\right.}$$

$$= -\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{19}}{192b^3\left(a+bx^2\right)^7}-\frac{437x^{17}}{2304b^4\left(a+bx^2\right)^6}-\frac{7429x^{1\cdots}}{23040b^5\left(a+\cdots\right.}$$

$$= \frac{7436429a^2x}{65536b^{12}}-\frac{7436429ax^3}{196608b^{11}}+\frac{7436429x^5}{327680b^{10}}-\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{1\cdots}}{192b^3\left(a+\cdots\right.}$$

$$= \frac{7436429a^2x}{65536b^{12}}-\frac{7436429ax^3}{196608b^{11}}+\frac{7436429x^5}{327680b^{10}}-\frac{x^{23}}{18b\left(a+bx^2\right)^9}-\frac{23x^{21}}{288b^2\left(a+bx^2\right)^8}-\frac{23x^{1\cdots}}{192b^3\left(a+\cdots\right.}$$

Mathematica [A] time = 0.09, size = 166, normalized size = 0.72

$$\frac{\frac{\sqrt{b}x(334639305a^{11}+2900207310a^{10}bx^2+11110024926a^9b^2x^4+24648575094a^8b^3x^6+34810986496a^7b^4x^8+32314857354a^6b^5x^{10}+19562592546a^5b^6x^{12}+7323998514a^4b^7x^{14}+1469632311a^3b^8x^{16}+94961664a^2b^9x^{18}-4521984ab^{10}x^{20}+589824b^{11}x^{22})}{(a+bx^2)^9}-334639305a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2949120b^{25/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[b]*x*(334639305*a^11 + 2900207310*a^10*b*x^2 + 11110024926*a^9*b^2*x
^4 + 24648575094*a^8*b^3*x^6 + 34810986496*a^7*b^4*x^8 + 32314857354*a^6*b^
5*x^10 + 19562592546*a^5*b^6*x^12 + 7323998514*a^4*b^7*x^14 + 1469632311*a^
3*b^8*x^16 + 94961664*a^2*b^9*x^18 - 4521984*a*b^10*x^20 + 589824*b^11*x^22
))/(a + b*x^2)^9 - 334639305*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2949120*
b^(25/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{24}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^24/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^24/(a + b*x^2)^10, x]
```

fricas [A] time = 1.09, size = 718, normalized size = 3.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/5898240*(1179648*b^11*x^23 - 9043968*a*b^10*x^21 + 189923328*a^2*b^9*x^1
9 + 2939264622*a^3*b^8*x^17 + 14647997028*a^4*b^7*x^15 + 39125185092*a^5*b^
6*x^13 + 64629714708*a^6*b^5*x^11 + 69621972992*a^7*b^4*x^9 + 49297150188*a
^8*b^3*x^7 + 22220049852*a^9*b^2*x^5 + 5800414620*a^10*b*x^3 + 669278610*a^
11*x + 334639305*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^4*b^7*x^14 + 84*a^5*
b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8*b^3*x^6 + 36*a^9*b^2
*x^4 + 9*a^10*b*x^2 + a^11)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^
12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x
^4 + 9*a^8*b^13*x^2 + a^9*b^12), 1/2949120*(589824*b^11*x^23 - 4521984*a*b^
10*x^21 + 94961664*a^2*b^9*x^19 + 1469632311*a^3*b^8*x^17 + 7323998514*a^4*
b^7*x^15 + 19562592546*a^5*b^6*x^13 + 32314857354*a^6*b^5*x^11 + 3481098649
```

```
6*a^7*b^4*x^9 + 24648575094*a^8*b^3*x^7 + 11110024926*a^9*b^2*x^5 + 2900207
310*a^10*b*x^3 + 334639305*a^11*x - 334639305*(a^2*b^9*x^18 + 9*a^3*b^8*x^1
6 + 36*a^4*b^7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8
+ 84*a^8*b^3*x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*sqrt(a/b)*arctan(b
*x*sqrt(a/b)/a))/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^1
8*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^
14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12)]
```

giac [A] time = 0.59, size = 162, normalized size = 0.70

$$-\frac{7436429\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,b^{12}}+\frac{314167095\,a^{3}b^{8}x^{17}+2236176690\,a^{4}b^{7}x^{15}+7101970722\,a^{5}b^{6}x^{13}+13066540938\,a^{6}b^{5}x^{11}+15178104832\,a^{7}b^{4}x^{9}+11372226678\,a^{8}b^{3}x^{7}+5358651102\,a^{9}b^{2}x^{5}+1450223310\,a^{10}bx^{3}+172437705\,a^{11}x}{2949120\,(bx^{2}+a)^{9}b^{12}}+\frac{3\,b^{40}x^{5}-50\,ab^{39}x^{3}+825\,a^{2}b^{38}x}{15\,b^{50}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -7436429/65536*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^12) + 1/2949120*(3141
67095*a^3*b^8*x^17 + 2236176690*a^4*b^7*x^15 + 7101970722*a^5*b^6*x^13 + 13
066540938*a^6*b^5*x^11 + 15178104832*a^7*b^4*x^9 + 11372226678*a^8*b^3*x^7
+ 5358651102*a^9*b^2*x^5 + 1450223310*a^10*b*x^3 + 172437705*a^11*x)/((b*x^
2 + a)^9*b^12) + 1/15*(3*b^40*x^5 - 50*a*b^39*x^3 + 825*a^2*b^38*x)/b^50
```

maple [A] time = 0.02, size = 228, normalized size = 0.99

$$\frac{6981491a^{3}x^{17}}{65536\,(bx^{2}+a)^{9}b^{4}}+\frac{74539223a^{4}x^{15}}{98304\,(bx^{2}+a)^{9}b^{5}}+\frac{394553929a^{5}x^{13}}{163840\,(bx^{2}+a)^{9}b^{6}}+\frac{725918941a^{6}x^{11}}{163840\,(bx^{2}+a)^{9}b^{7}}+\frac{463199a^{7}x^{9}}{90\,(bx^{2}+a)^{9}b^{8}}+\frac{631790371a^{8}x^{7}}{163840\,(bx^{2}+a)^{9}b^{9}}+\frac{297702839a^{9}x^{5}}{163840\,(bx^{2}+a)^{9}b^{10}}+\frac{48340777a^{10}x^{3}}{98304\,(bx^{2}+a)^{9}b^{11}}+\frac{3831949a^{11}x}{65536\,(bx^{2}+a)^{9}b^{12}}+\frac{x^{5}}{5b^{10}}-\frac{10a\,x^{3}}{3b^{11}}-\frac{7436429a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,b^{12}}+\frac{55a^{2}x}{b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(b*x^2+a)^10,x)
```

```
[Out] 1/5*x^5/b^10-10/3*a*x^3/b^11+55*a^2*x/b^12+3831949/65536/b^12*a^11/(b*x^2+a
)^9*x+48340777/98304/b^11*a^10/(b*x^2+a)^9*x^3+297702839/163840/b^10*a^9/(b
*x^2+a)^9*x^5+631790371/163840/b^9*a^8/(b*x^2+a)^9*x^7+463199/90/b^8*a^7/(b
*x^2+a)^9*x^9+725918941/163840/b^7*a^6/(b*x^2+a)^9*x^11+394553929/163840/b^
6*a^5/(b*x^2+a)^9*x^13+74539223/98304/b^5*a^4/(b*x^2+a)^9*x^15+6981491/6553
6/b^4*a^3/(b*x^2+a)^9*x^17-7436429/65536/b^12*a^3/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*b*x)
```

maxima [A] time = 3.39, size = 248, normalized size = 1.07

$$\frac{314167095\,a^{3}b^{8}x^{17}+2236176690\,a^{4}b^{7}x^{15}+7101970722\,a^{5}b^{6}x^{13}+13066540938\,a^{6}b^{5}x^{11}+15178104832\,a^{7}b^{4}x^{9}+11372226678\,a^{8}b^{3}x^{7}+5358651102\,a^{9}b^{2}x^{5}+1450223310\,a^{10}bx^{3}+172437705\,a^{11}x}{2949120\,(b^{21}x^{18}+9\,ab^{20}x^{16}+36\,a^{2}b^{19}x^{14}+84\,a^{3}b^{18}x^{12}+126\,a^{4}b^{17}x^{10}+126\,a^{5}b^{16}x^{8}+84\,a^{6}b^{15}x^{6}+36\,a^{7}b^{14}x^{4}+9\,a^{8}b^{13}x^{2}+a^{9}b^{12})}-\frac{7436429\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,b^{12}}+\frac{3\,b^{2}x^{5}-50\,abx^{3}+825\,a^{2}x}{15\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2949120*(314167095*a^3*b^8*x^17 + 2236176690*a^4*b^7*x^15 + 7101970722*a^
5*b^6*x^13 + 13066540938*a^6*b^5*x^11 + 15178104832*a^7*b^4*x^9 + 113722266
78*a^8*b^3*x^7 + 5358651102*a^9*b^2*x^5 + 1450223310*a^10*b*x^3 + 172437705
*a^11*x)/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 +
 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 +
 9*a^8*b^13*x^2 + a^9*b^12) - 7436429/65536*a^3*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^12) + 1/15*(3*b^2*x^5 - 50*a*b*x^3 + 825*a^2*x)/b^12
```

mupad [B] time = 4.90, size = 241, normalized size = 1.04

$$\frac{\frac{3831949\,a^{11}\,x}{65536}+\frac{48340777\,a^{10}\,b\,x^3}{98304}+\frac{297702839\,a^9\,b^2\,x^5}{163840}+\frac{631790371\,a^8\,b^3\,x^7}{163840}+\frac{463199\,a^7\,b^4\,x^9}{90}+\frac{725918941\,a^6\,b^5\,x^{11}}{163840}+\frac{394553929\,a^5\,b^6\,x^{13}}{163840}+\frac{74539223\,a^4\,b^7\,x^{15}}{98304}+\frac{6981491\,a^3\,b^8\,x^{17}}{65536}}{a^9\,b^{12}+9\,a^8\,b^{13}\,x^2+36\,a^7\,b^{14}\,x^4+84\,a^6\,b^{15}\,x^6+126\,a^5\,b^{16}\,x^8+126\,a^4\,b^{17}\,x^{10}+84\,a^3\,b^{18}\,x^{12}+36\,a^2\,b^{19}\,x^{14}+9\,a\,b^{20}\,x^{16}+b^{21}\,x^{18}}+\frac{x^5}{5\,b^{10}}-\frac{10\,a\,x^3}{3\,b^{11}}+\frac{55\,a^2\,x}{b^{12}}-\frac{7436429\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,b^{25/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(a + b*x^2)^10,x)
```

```
[Out] ((3831949*a^11*x)/65536 + (48340777*a^10*b*x^3)/98304 + (297702839*a^9*b^2*
x^5)/163840 + (631790371*a^8*b^3*x^7)/163840 + (463199*a^7*b^4*x^9)/90 + (7
25918941*a^6*b^5*x^11)/163840 + (394553929*a^5*b^6*x^13)/163840 + (74539223
*a^4*b^7*x^15)/98304 + (6981491*a^3*b^8*x^17)/65536)/(a^9*b^12 + b^21*x^18
+ 9*a*b^20*x^16 + 9*a^8*b^13*x^2 + 36*a^7*b^14*x^4 + 84*a^6*b^15*x^6 + 126*
a^5*b^16*x^8 + 126*a^4*b^17*x^10 + 84*a^3*b^18*x^12 + 36*a^2*b^19*x^14) + x
^5/(5*b^10) - (10*a*x^3)/(3*b^11) + (55*a^2*x)/b^12 - (7436429*a^(5/2)*atan
((b^(1/2)*x)/a^(1/2)))/(65536*b^(25/2))
```

sympy [A] time = 2.04, size = 314, normalized size = 1.36

$$\frac{55a^2x}{b^{12}}-\frac{10ax^3}{3b^{11}}+\frac{7436429\sqrt{-\frac{a^5}{b^{25}}}\log\left(x-\frac{b^{12}\sqrt{-\frac{a^5}{b^{25}}}}{a^2}\right)}{131072}-\frac{7436429\sqrt{-\frac{a^5}{b^{25}}}\log\left(x+\frac{b^{12}\sqrt{-\frac{a^5}{b^{25}}}}{a^2}\right)}{131072}+\frac{172437705a^{11}x+1450223310a^{10}bx^3+5358651102a^9b^2x^5+11372226678a^8b^3x^7+15178104832a^7b^4x^9+13066540938a^6b^5x^{11}+7101970722a^5b^6x^{13}+2236176690a^4b^7x^{15}+314167095a^3b^8x^{17}}{2949120a^9b^{12}+26542080a^8b^{13}x^2+106168320a^7b^{14}x^4+247726080a^6b^{15}x^6+371589120a^5b^{16}x^8+371589120a^4b^{17}x^{10}+247726080a^3b^{18}x^{12}+106168320a^2b^{19}x^{14}+26542080ab^{20}x^{16}+2949120b^{21}x^{18}}+\frac{x^5}{5b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24/(b*x**2+a)**10,x)
```

```
[Out] 55*a**2*x/b**12 - 10*a*x**3/(3*b**11) + 7436429*sqrt(-a**5/b**25)*log(x - b
**12*sqrt(-a**5/b**25)/a**2)/131072 - 7436429*sqrt(-a**5/b**25)*log(x + b**
12*sqrt(-a**5/b**25)/a**2)/131072 + (172437705*a**11*x + 1450223310*a**10*b
*x**3 + 5358651102*a**9*b**2*x**5 + 11372226678*a**8*b**3*x**7 + 1517810483
2*a**7*b**4*x**9 + 13066540938*a**6*b**5*x**11 + 7101970722*a**5*b**6*x**13
 + 2236176690*a**4*b**7*x**15 + 314167095*a**3*b**8*x**17)/(2949120*a**9*b*
*12 + 26542080*a**8*b**13*x**2 + 106168320*a**7*b**14*x**4 + 247726080*a**6
*b**15*x**6 + 371589120*a**5*b**16*x**8 + 371589120*a**4*b**17*x**10 + 2477
26080*a**3*b**18*x**12 + 106168320*a**2*b**19*x**14 + 26542080*a*b**20*x**1
6 + 2949120*b**21*x**18) + x**5/(5*b**10)
```

3.210 $\int \frac{x^{22}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=218

$$\frac{1616615a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{23/2}}-\frac{1616615ax}{65536b^{11}}-\frac{323323x^5}{65536b^9\left(a+bx^2\right)}-\frac{46189x^7}{32768b^8\left(a+bx^2\right)^2}-\frac{46189x^9}{73728b^7\left(a+bx^2\right)^3}-\frac{4199}{12288b^6\left(a\right.}$$

Rubi [A] time = 0.14, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 302, 205}

$$\frac{1616615a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{23/2}}-\frac{7x^{19}}{96b^2\left(a+bx^2\right)^8}-\frac{19x^{17}}{192b^3\left(a+bx^2\right)^7}-\frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6}-\frac{323x^{13}}{1536b^5\left(a+bx^2\right)^5}-\frac{4199x^{11}}{12288b^6\left(a+bx^2\right)^4}-\frac{46189x^9}{73728b^7\left(a+bx^2\right)^3}-\frac{46189x^7}{32768b^8\left(a+bx^2\right)^2}-\frac{323323x^5}{65536b^9\left(a+bx^2\right)}-\frac{1616615ax}{65536b^{11}}-\frac{x^{21}}{18b\left(a+bx^2\right)^9}+\frac{1616615x^3}{196608b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^22/(a + b*x^2)^10,x]
```

```
[Out] (-1616615*a*x)/(65536*b^11) + (1616615*x^3)/(196608*b^10) - x^21/(18*b*(a +
 b*x^2)^9) - (7*x^19)/(96*b^2*(a + b*x^2)^8) - (19*x^17)/(192*b^3*(a + b*x^
2)^7) - (323*x^15)/(2304*b^4*(a + b*x^2)^6) - (323*x^13)/(1536*b^5*(a + b*x
^2)^5) - (4199*x^11)/(12288*b^6*(a + b*x^2)^4) - (46189*x^9)/(73728*b^7*(a
+ b*x^2)^3) - (46189*x^7)/(32768*b^8*(a + b*x^2)^2) - (323323*x^5)/(65536*b
^9*(a + b*x^2)) + (1616615*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(2
3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{22}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{21}}{18b\left(a+bx^2\right)^9} + \frac{7\int \frac{x^{20}}{\left(a+bx^2\right)^9}\,dx}{6b}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} + \frac{133\int \frac{x^{18}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} + \frac{323\int \frac{x^{16}}{\left(a+bx^2\right)^7}\,dx}{192b^3}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} + \frac{1615\int \frac{x^{14}}{(a+bx^2)}}{768b^4}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{323x^{15}}{2304b^4\left(a+bx^2\right)^6} - \frac{323x^{13}}{1536b^5\left(a+b\right.}$$

$$= -\frac{1616615ax}{65536b^{11}} + \frac{1616615x^3}{196608b^{10}} - \frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{32}{2304b^4}$$

$$= -\frac{1616615ax}{65536b^{11}} + \frac{1616615x^3}{196608b^{10}} - \frac{x^{21}}{18b\left(a+bx^2\right)^9} - \frac{7x^{19}}{96b^2\left(a+bx^2\right)^8} - \frac{19x^{17}}{192b^3\left(a+bx^2\right)^7} - \frac{32}{2304b^4}$$

Mathematica [A] time = 0.08, size = 155, normalized size = 0.71

$$\frac{14549535a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\frac{\sqrt{b}x\left(-14549535a^{10}-126095970a^9bx^2-483044562a^8b^2x^4-1071677178a^7b^3x^6-1513521152a^6b^4x^8-1404993798a^5b^5x^{10}-850547502a^4b^6x^{12}-318434718a^3b^7x^{14}-63897057a^2b^8x^{16}-4128768ab^9x^{18}+196608b^{10}x^{20}\right)}{(a+bx^2)^9}}{589824b^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^22/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[b]*x*(-14549535*a^10 - 126095970*a^9*b*x^2 - 483044562*a^8*b^2*x^4 -
 1071677178*a^7*b^3*x^6 - 1513521152*a^6*b^4*x^8 - 1404993798*a^5*b^5*x^10
- 850547502*a^4*b^6*x^12 - 318434718*a^3*b^7*x^14 - 63897057*a^2*b^8*x^16 -
 4128768*a*b^9*x^18 + 196608*b^10*x^20))/(a + b*x^2)^9 + 14549535*a^(3/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(589824*b^(23/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{22}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^22/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^22/(a + b*x^2)^10, x]
```

fricas [A] time = 0.81, size = 692, normalized size = 3.17

$$\left[\frac{393216\,b^{10}x^{21}-8257536\,ab^9x^{19}-127794114\,a^2b^8x^{17}-636869436\,a^3b^7x^{15}-1701095004\,a^4b^6x^{13}-2809987596\,a^5b^5x^{11}-3027042304\,a^6b^4x^9-2143354356\,a^7b^3x^7-966089124\,a^8b^2x^5-252191940\,a^9bx^3-29099070\,a^{10}x+14549535\left(ab^9x^{18}+9\,a^2b^8x^{16}+36\,a^3b^7x^{14}+84\,a^4b^6x^{12}+126\,a^5b^5x^{10}+126\,a^6b^4x^8+84\,a^7b^3x^6+36\,a^8b^2x^4+9\,a^9bx^2+a^{10}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{1179648\left(b^{20}x^{18}+9\,ab^{19}x^{16}+36\,a^2b^{18}x^{14}+84\,a^3b^{17}x^{12}+126\,a^4b^{16}x^{10}+126\,a^5b^{15}x^8+84\,a^6b^{14}x^6+36\,a^7b^{13}x^4+9\,a^8b^{12}x^2+a^9b^{11}\right)},\ \frac{196608\,b^{10}x^{21}-4128768\,ab^9x^{19}-63897057\,a^2b^8x^{17}-318434718\,a^3b^7x^{15}-850547502\,a^4b^6x^{13}-1404993798\,a^5b^5x^{11}-1513521152\,a^6b^4x^9-1071677178\,a^7b^3x^7-483044562\,a^8b^2x^5-126095970\,a^9bx^3-14549535\,a^{10}x+14549535\left(ab^9x^{18}+9\,a^2b^8x^{16}+36\,a^3b^7x^{14}+84\,a^4b^6x^{12}+126\,a^5b^5x^{10}+126\,a^6b^4x^8+84\,a^7b^3x^6+36\,a^8b^2x^4+9\,a^9bx^2+a^{10}\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{589824\left(b^{20}x^{18}+9\,ab^{19}x^{16}+36\,a^2b^{18}x^{14}+84\,a^3b^{17}x^{12}+126\,a^4b^{16}x^{10}+126\,a^5b^{15}x^8+84\,a^6b^{14}x^6+36\,a^7b^{13}x^4+9\,a^8b^{12}x^2+a^9b^{11}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(393216*b^10*x^21 - 8257536*a*b^9*x^19 - 127794114*a^2*b^8*x^17
- 636869436*a^3*b^7*x^15 - 1701095004*a^4*b^6*x^13 - 2809987596*a^5*b^5*x^1
1 - 3027042304*a^6*b^4*x^9 - 2143354356*a^7*b^3*x^7 - 966089124*a^8*b^2*x^5
 - 252191940*a^9*b*x^3 - 29099070*a^10*x + 14549535*(a*b^9*x^18 + 9*a^2*b^8
*x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 126*a^6*b^4*
x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^10)*sqrt(-a/b)*log(
(b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^20*x^18 + 9*a*b^19*x^16 + 3
6*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 +
 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11), 1/589824*(
196608*b^10*x^21 - 4128768*a*b^9*x^19 - 63897057*a^2*b^8*x^17 - 318434718*a
^3*b^7*x^15 - 850547502*a^4*b^6*x^13 - 1404993798*a^5*b^5*x^11 - 1513521152
*a^6*b^4*x^9 - 1071677178*a^7*b^3*x^7 - 483044562*a^8*b^2*x^5 - 126095970*a
^9*b*x^3 - 14549535*a^10*x + 14549535*(a*b^9*x^18 + 9*a^2*b^8*x^16 + 36*a^3
```

```
*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 126*a^6*b^4*x^8 + 84*a^7*b
^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^10)*sqrt(a/b)*arctan(b*x*sqrt(a/b
)/a))/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 12
6*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*
a^8*b^12*x^2 + a^9*b^11)]
```

giac [A] time = 0.65, size = 150, normalized size = 0.69

$$\frac{1616615\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,b^{11}}-\frac{17890785\,a^2b^8x^{17}+122613150\,a^3b^7x^{15}+379867950\,a^4b^6x^{13}+686588166\,a^5b^5x^{11}+786857984\,a^6b^4x^9+583302906\,a^7b^3x^7+272477394\,a^8b^2x^5+73208418\,a^9bx^3+8651295\,a^{10}x}{589824\left(bx^2+a\right)^9b^{11}}+\frac{b^{20}x^3-30\,ab^{19}x}{3\,b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1616615/65536*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^11) - 1/589824*(178907
85*a^2*b^8*x^17 + 122613150*a^3*b^7*x^15 + 379867950*a^4*b^6*x^13 + 6865881
66*a^5*b^5*x^11 + 786857984*a^6*b^4*x^9 + 583302906*a^7*b^3*x^7 + 272477394
*a^8*b^2*x^5 + 73208418*a^9*b*x^3 + 8651295*a^10*x)/((b*x^2 + a)^9*b^11) +
1/3*(b^20*x^3 - 30*a*b^19*x)/b^30
```

maple [A] time = 0.02, size = 217, normalized size = 1.00

$$-\frac{1987865a^2x^{17}}{65536\left(bx^2+a\right)^9b^3}-\frac{20435525a^3x^{15}}{98304\left(bx^2+a\right)^9b^4}-\frac{21103775a^4x^{13}}{32768\left(bx^2+a\right)^9b^5}-\frac{38143787a^5x^{11}}{32768\left(bx^2+a\right)^9b^6}-\frac{24013a^6x^9}{18\left(bx^2+a\right)^9b^7}-\frac{32405717a^7x^7}{32768\left(bx^2+a\right)^9b^8}-\frac{15137633a^8x^5}{32768\left(bx^2+a\right)^9b^9}-\frac{12201403a^9x^3}{98304\left(bx^2+a\right)^9b^{10}}-\frac{961255a^{10}x}{65536\left(bx^2+a\right)^9b^{11}}+\frac{x^3}{3b^{10}}+\frac{1616615a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,b^{11}}-\frac{10ax}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^22/(b*x^2+a)^10,x)
```

```
[Out] 1/3*x^3/b^10-10*a*x/b^11-961255/65536/b^11*a^10/(b*x^2+a)^9*x-12201403/9830
4/b^10*a^9/(b*x^2+a)^9*x^3-15137633/32768/b^9*a^8/(b*x^2+a)^9*x^5-32405717/
32768/b^8*a^7/(b*x^2+a)^9*x^7-24013/18/b^7*a^6/(b*x^2+a)^9*x^9-38143787/327
68/b^6*a^5/(b*x^2+a)^9*x^11-21103775/32768/b^5*a^4/(b*x^2+a)^9*x^13-2043552
5/98304/b^4*a^3/(b*x^2+a)^9*x^15-1987865/65536/b^3*a^2/(b*x^2+a)^9*x^17+161
6615/65536/b^11*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.25, size = 236, normalized size = 1.08

$$-\frac{17890785\,a^2b^8x^{17}+122613150\,a^3b^7x^{15}+379867950\,a^4b^6x^{13}+686588166\,a^5b^5x^{11}+786857984\,a^6b^4x^9+583302906\,a^7b^3x^7+272477394\,a^8b^2x^5+73208418\,a^9bx^3+8651295\,a^{10}x}{589824\left(b^{20}x^{18}+9\,ab^{19}x^{16}+36\,a^2b^{18}x^{14}+84\,a^3b^{17}x^{12}+126\,a^4b^{16}x^{10}+126\,a^5b^{15}x^8+84\,a^6b^{14}x^6+36\,a^7b^{13}x^4+9\,a^8b^{12}x^2+a^9b^{11}\right)}+\frac{1616615\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,b^{11}}+\frac{bx^3-30\,ax}{3\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^22/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/589824*(17890785*a^2*b^8*x^17 + 122613150*a^3*b^7*x^15 + 379867950*a^4*b
^6*x^13 + 686588166*a^5*b^5*x^11 + 786857984*a^6*b^4*x^9 + 583302906*a^7*b^
3*x^7 + 272477394*a^8*b^2*x^5 + 73208418*a^9*b*x^3 + 8651295*a^10*x)/(b^20*
```

```
x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x
^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2
 + a^9*b^11) + 1616615/65536*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^11) + 1
/3*(b*x^3 - 30*a*x)/b^11
```

mupad [B] time = 0.40, size = 231, normalized size = 1.06

$$\frac{x^3}{3\,b^{10}}-\frac{\frac{961255\,a^{10}\,x}{65536}+\frac{12201403\,a^9\,b\,x^3}{98304}+\frac{15137633\,a^8\,b^2\,x^5}{32768}+\frac{32405717\,a^7\,b^3\,x^7}{32768}+\frac{24013\,a^6\,b^4\,x^9}{18}+\frac{38143787\,a^5\,b^5\,x^{11}}{32768}+\frac{21103775\,a^4\,b^6\,x^{13}}{32768}+\frac{20435525\,a^3\,b^7\,x^{15}}{98304}+\frac{1987865\,a^2\,b^8\,x^{17}}{65536}}{a^9\,b^{11}+9\,a^8\,b^{12}\,x^2+36\,a^7\,b^{13}\,x^4+84\,a^6\,b^{14}\,x^6+126\,a^5\,b^{15}\,x^8+126\,a^4\,b^{16}\,x^{10}+84\,a^3\,b^{17}\,x^{12}+36\,a^2\,b^{18}\,x^{14}+9\,a\,b^{19}\,x^{16}+b^{20}\,x^{18}}+\frac{1616615\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,b^{23/2}}-\frac{10\,a\,x}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^22/(a + b*x^2)^10,x)
```

```
[Out] x^3/(3*b^10) - ((961255*a^10*x)/65536 + (12201403*a^9*b*x^3)/98304 + (15137
633*a^8*b^2*x^5)/32768 + (32405717*a^7*b^3*x^7)/32768 + (24013*a^6*b^4*x^9)
/18 + (38143787*a^5*b^5*x^11)/32768 + (21103775*a^4*b^6*x^13)/32768 + (2043
5525*a^3*b^7*x^15)/98304 + (1987865*a^2*b^8*x^17)/65536)/(a^9*b^11 + b^20*x
^18 + 9*a*b^19*x^16 + 9*a^8*b^12*x^2 + 36*a^7*b^13*x^4 + 84*a^6*b^14*x^6 +
126*a^5*b^15*x^8 + 126*a^4*b^16*x^10 + 84*a^3*b^17*x^12 + 36*a^2*b^18*x^14)
 + (1616615*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*b^(23/2)) - (10*a*x)/
b^11
```

sympy [A] time = 1.92, size = 299, normalized size = 1.37

$$-\frac{10ax}{b^{11}}-\frac{1616615\sqrt{-\frac{a^3}{b^{23}}}\,\log\left(x-\frac{b^{11}\sqrt{-\frac{a^3}{b^{23}}}}{a}\right)}{131072}+\frac{1616615\sqrt{-\frac{a^3}{b^{23}}}\,\log\left(x+\frac{b^{11}\sqrt{-\frac{a^3}{b^{23}}}}{a}\right)}{131072}+\frac{-8651295a^{10}x-73208418a^9bx^3-272477394a^8b^2x^5-583302906a^7b^3x^7-786857984a^6b^4x^9-686588166a^5b^5x^{11}-379867950a^4b^6x^{13}-122613150a^3b^7x^{15}-17890785a^2b^8x^{17}}{589824a^9b^{11}+5308416a^8b^{12}x^2+21233664a^7b^{13}x^4+49545216a^6b^{14}x^6+74317824a^5b^{15}x^8+74317824a^4b^{16}x^{10}+49545216a^3b^{17}x^{12}+21233664a^2b^{18}x^{14}+5308416ab^{19}x^{16}+589824b^{20}x^{18}}+\frac{x^3}{3b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**22/(b*x**2+a)**10,x)
```

```
[Out] -10*a*x/b**11 - 1616615*sqrt(-a**3/b**23)*log(x - b**11*sqrt(-a**3/b**23)/a
)/131072 + 1616615*sqrt(-a**3/b**23)*log(x + b**11*sqrt(-a**3/b**23)/a)/131
072 + (-8651295*a**10*x - 73208418*a**9*b*x**3 - 272477394*a**8*b**2*x**5 -
 583302906*a**7*b**3*x**7 - 786857984*a**6*b**4*x**9 - 686588166*a**5*b**5*
x**11 - 379867950*a**4*b**6*x**13 - 122613150*a**3*b**7*x**15 - 17890785*a*
*2*b**8*x**17)/(589824*a**9*b**11 + 5308416*a**8*b**12*x**2 + 21233664*a**7
*b**13*x**4 + 49545216*a**6*b**14*x**6 + 74317824*a**5*b**15*x**8 + 7431782
4*a**4*b**16*x**10 + 49545216*a**3*b**17*x**12 + 21233664*a**2*b**18*x**14
+ 5308416*a*b**19*x**16 + 589824*b**20*x**18) + x**3/(3*b**10)
```

3.211 $\int \frac{x^{20}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=207

$$-\frac{230945\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{21/2}}-\frac{230945x^3}{196608b^9\left(a+bx^2\right)}-\frac{46189x^5}{98304b^8\left(a+bx^2\right)^2}-\frac{46189x^7}{172032b^7\left(a+bx^2\right)^3}-\frac{46189x^9}{258048b^6\left(a+bx^2\right)^4}$$

Rubi [A] time = 0.12, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 321, 205}

$$-\frac{19x^{17}}{288b^2\left(a+bx^2\right)^8}-\frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7}-\frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6}-\frac{4199x^{11}}{32256b^5\left(a+bx^2\right)^5}-\frac{46189x^9}{258048b^6\left(a+bx^2\right)^4}-\frac{46189x^7}{172032b^7\left(a+bx^2\right)^3}-\frac{46189x^5}{98304b^8\left(a+bx^2\right)^2}-\frac{230945x^3}{196608b^9\left(a+bx^2\right)}-\frac{230945\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536b^{21/2}}-\frac{x^{19}}{18b\left(a+bx^2\right)^9}+\frac{230945x}{65536b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^20/(a + b*x^2)^10,x]
```

```
[Out] (230945*x)/(65536*b^10) - x^19/(18*b*(a + b*x^2)^9) - (19*x^17)/(288*b^2*(a
 + b*x^2)^8) - (323*x^15)/(4032*b^3*(a + b*x^2)^7) - (1615*x^13)/(16128*b^4
*(a + b*x^2)^6) - (4199*x^11)/(32256*b^5*(a + b*x^2)^5) - (46189*x^9)/(2580
48*b^6*(a + b*x^2)^4) - (46189*x^7)/(172032*b^7*(a + b*x^2)^3) - (46189*x^5
)/(98304*b^8*(a + b*x^2)^2) - (230945*x^3)/(196608*b^9*(a + b*x^2)) - (2309
45*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^(21/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{20}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{19}}{18b\left(a+bx^2\right)^9} + \frac{19\int \frac{x^{18}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9} - \frac{19x^{17}}{288b^2\left(a+bx^2\right)^8} + \frac{323\int \frac{x^{16}}{\left(a+bx^2\right)^8}\,dx}{288b^2}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9} - \frac{19x^{17}}{288b^2\left(a+bx^2\right)^8} - \frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7} + \frac{1615\int \frac{x^{14}}{\left(a+bx^2\right)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9} - \frac{19x^{17}}{288b^2\left(a+bx^2\right)^8} - \frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7} - \frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6} + \frac{20995\int}{1612}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9} - \frac{19x^{17}}{288b^2\left(a+bx^2\right)^8} - \frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7} - \frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6} - \frac{4199}{32256b^5\,(}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9} - \frac{19x^{17}}{288b^2\left(a+bx^2\right)^8} - \frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7} - \frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6} - \frac{4199}{32256b^5\,(}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9} - \frac{19x^{17}}{288b^2\left(a+bx^2\right)^8} - \frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7} - \frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6} - \frac{4199}{32256b^5\,(}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9} - \frac{19x^{17}}{288b^2\left(a+bx^2\right)^8} - \frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7} - \frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6} - \frac{4199}{32256b^5\,(}$$

$$= -\frac{x^{19}}{18b\left(a+bx^2\right)^9} - \frac{19x^{17}}{288b^2\left(a+bx^2\right)^8} - \frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7} - \frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6} - \frac{4199}{32256b^5\,(}$$

$$= \frac{230945x}{65536b^{10}} - \frac{x^{19}}{18b\left(a+bx^2\right)^9} - \frac{19x^{17}}{288b^2\left(a+bx^2\right)^8} - \frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7} - \frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6} -$$

$$= \frac{230945x}{65536b^{10}} - \frac{x^{19}}{18b\left(a+bx^2\right)^9} - \frac{19x^{17}}{288b^2\left(a+bx^2\right)^8} - \frac{323x^{15}}{4032b^3\left(a+bx^2\right)^7} - \frac{1615x^{13}}{16128b^4\left(a+bx^2\right)^6} -$$

Mathematica [A] time = 0.08, size = 144, normalized size = 0.70

$$\frac{\frac{\sqrt{b}x\left(14549535a^9+126095970a^8bx^2+483044562a^7b^2x^4+1071677178a^6b^3x^6+1513521152a^5b^4x^8+1404993798a^4b^5x^{10}+850547502a^3b^6x^{12}+318434718a^2b^7x^{14}+63897057ab^8x^{16}+4128768b^9x^{18}\right)}{\left(a+bx^2\right)^9}-14549535\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{4128768b^{21/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^20/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[b]*x*(14549535*a^9 + 126095970*a^8*b*x^2 + 483044562*a^7*b^2*x^4 + 1
071677178*a^6*b^3*x^6 + 1513521152*a^5*b^4*x^8 + 1404993798*a^4*b^5*x^10 +
850547502*a^3*b^6*x^12 + 318434718*a^2*b^7*x^14 + 63897057*a*b^8*x^16 + 412
8768*b^9*x^18))/(a + b*x^2)^9 - 14549535*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(4128768*b^(21/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{20}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^20/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^20/(a + b*x^2)^10, x]
```

fricas [A] time = 0.76, size = 664, normalized size = 3.21

$$\left[\frac{8257536\,b^9x^{19}+127794114\,ab^8x^{17}+636869436\,a^2b^7x^{15}+1701095004\,a^3b^6x^{13}+2809987596\,a^4b^5x^{11}+3027042304\,a^5b^4x^9+2143354356\,a^6b^3x^7+966089124\,a^7b^2x^5+252191940\,a^8bx^3+29099070\,a^9x+14549535\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{8257536\left(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}x^{14}+84\,a^3b^{16}x^{12}+126\,a^4b^{15}x^{10}+126\,a^5b^{14}x^8+84\,a^6b^{13}x^6+36\,a^7b^{12}x^4+9\,a^8b^{11}x^2+a^9b^{10}\right)},\ \frac{4128768\,b^9x^{19}+63897057\,ab^8x^{17}+318434718\,a^2b^7x^{15}+850547502\,a^3b^6x^{13}+1404993798\,a^4b^5x^{11}+1513521152\,a^5b^4x^9+1071677178\,a^6b^3x^7+483044562\,a^7b^2x^5+126095970\,a^8bx^3+14549535\,a^9x-14549535\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{4128768\left(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}x^{14}+84\,a^3b^{16}x^{12}+126\,a^4b^{15}x^{10}+126\,a^5b^{14}x^8+84\,a^6b^{13}x^6+36\,a^7b^{12}x^4+9\,a^8b^{11}x^2+a^9b^{10}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(8257536*b^9*x^19 + 127794114*a*b^8*x^17 + 636869436*a^2*b^7*x^1
5 + 1701095004*a^3*b^6*x^13 + 2809987596*a^4*b^5*x^11 + 3027042304*a^5*b^4*
x^9 + 2143354356*a^6*b^3*x^7 + 966089124*a^7*b^2*x^5 + 252191940*a^8*b*x^3
+ 29099070*a^9*x + 14549535*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84
*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^
7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a
)/(b*x^2 + a)))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16
*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^1
2*x^4 + 9*a^8*b^11*x^2 + a^9*b^10), 1/4128768*(4128768*b^9*x^19 + 63897057*
a*b^8*x^17 + 318434718*a^2*b^7*x^15 + 850547502*a^3*b^6*x^13 + 1404993798*a
^4*b^5*x^11 + 1513521152*a^5*b^4*x^9 + 1071677178*a^6*b^3*x^7 + 483044562*a
^7*b^2*x^5 + 126095970*a^8*b*x^3 + 14549535*a^9*x - 14549535*(b^9*x^18 + 9*
```

```
a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5
*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a/b)*a
rctan(b*x*sqrt(a/b)/a))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*
a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36
*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10)]
```

giac [A] time = 0.64, size = 131, normalized size = 0.63

$$-\frac{230945\, a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\, \sqrt{ab}\, b^{10}} + \frac{x}{b^{10}} + \frac{26738145\, ab^8x^{17} + 169799070\, a^2b^7x^{15} + 503730990\, a^3b^6x^{13} + 884769030\, a^4b^5x^{11} + 993296384\, a^5b^4x^9 + 724860666\, a^6b^3x^7 + 334408914\, a^7b^2x^5 + 88937058\, a^8bx^3 + 10420767\, a^9x}{4128768\, \left(bx^2 + a\right)^9 b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -230945/65536*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^10) + x/b^10 + 1/4128768
*(26738145*a*b^8*x^17 + 169799070*a^2*b^7*x^15 + 503730990*a^3*b^6*x^13 + 8
84769030*a^4*b^5*x^11 + 993296384*a^5*b^4*x^9 + 724860666*a^6*b^3*x^7 + 334
408914*a^7*b^2*x^5 + 88937058*a^8*b*x^3 + 10420767*a^9*x)/((b*x^2 + a)^9*b^
10)
```

maple [A] time = 0.02, size = 203, normalized size = 0.98

$$\frac{424415 a\, x^{17}}{65536 \left(b\, x^2 + a\right)^9 b^2} + \frac{4042835 a^2 x^{15}}{98304 \left(b\, x^2 + a\right)^9 b^3} + \frac{3997865 a^3 x^{13}}{32768 \left(b\, x^2 + a\right)^9 b^4} + \frac{49153835 a^4 x^{11}}{229376 \left(b\, x^2 + a\right)^9 b^5} + \frac{30313 a^5 x^9}{126 \left(b\, x^2 + a\right)^9 b^6} + \frac{40270037 a^6 x^7}{229376 \left(b\, x^2 + a\right)^9 b^7} + \frac{2654039 a^7 x^5}{32768 \left(b\, x^2 + a\right)^9 b^8} + \frac{2117549 a^8 x^3}{98304 \left(b\, x^2 + a\right)^9 b^9} + \frac{165409 a^9 x}{65536 \left(b\, x^2 + a\right)^9 b^{10}} - \frac{230945 a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, b^{10}} + \frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20/(b*x^2+a)^10,x)
```

```
[Out] x/b^10+165409/65536/b^10*a^9/(b*x^2+a)^9*x+2117549/98304/b^9*a^8/(b*x^2+a)^
9*x^3+2654039/32768/b^8*a^7/(b*x^2+a)^9*x^5+40270037/229376/b^7*a^6/(b*x^2+
a)^9*x^7+30313/126/b^6*a^5/(b*x^2+a)^9*x^9+49153835/229376/b^5*a^4/(b*x^2+a
)^9*x^11+3997865/32768/b^4*a^3/(b*x^2+a)^9*x^13+4042835/98304/b^3*a^2/(b*x^
2+a)^9*x^15+424415/65536/b^2*a/(b*x^2+a)^9*x^17-230945/65536/b^10*a/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.17, size = 222, normalized size = 1.07

$$\frac{26738145\, ab^8x^{17} + 169799070\, a^2b^7x^{15} + 503730990\, a^3b^6x^{13} + 884769030\, a^4b^5x^{11} + 993296384\, a^5b^4x^9 + 724860666\, a^6b^3x^7 + 334408914\, a^7b^2x^5 + 88937058\, a^8bx^3 + 10420767\, a^9x}{4128768\, \left(b^{19}x^{18} + 9\, ab^{18}x^{16} + 36\, a^2b^{17}x^{14} + 84\, a^3b^{16}x^{12} + 126\, a^4b^{15}x^{10} + 126\, a^5b^{14}x^8 + 84\, a^6b^{13}x^6 + 36\, a^7b^{12}x^4 + 9\, a^8b^{11}x^2 + a^9b^{10}\right)} - \frac{230945\, a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\, \sqrt{ab}\, b^{10}} + \frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(26738145*a*b^8*x^17 + 169799070*a^2*b^7*x^15 + 503730990*a^3*b^6
*x^13 + 884769030*a^4*b^5*x^11 + 993296384*a^5*b^4*x^9 + 724860666*a^6*b^3*
x^7 + 334408914*a^7*b^2*x^5 + 88937058*a^8*b*x^3 + 10420767*a^9*x)/(b^19*x^
```

```
18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^1
0 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 +
 a^9*b^10) - 230945/65536*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^10) + x/b^10
```

mupad [B] time = 0.44, size = 218, normalized size = 1.05

$$\frac{\frac{165409\,a^9\,x}{65536}+\frac{2117549\,a^8\,b\,x^3}{98304}+\frac{2654039\,a^7\,b^2\,x^5}{32768}+\frac{40270037\,a^6\,b^3\,x^7}{229376}+\frac{30313\,a^5\,b^4\,x^9}{126}+\frac{49153835\,a^4\,b^5\,x^{11}}{229376}+\frac{3997865\,a^3\,b^6\,x^{13}}{32768}+\frac{4042835\,a^2\,b^7\,x^{15}}{98304}+\frac{424415\,a\,b^8\,x^{17}}{65536}}{a^9\,b^{10}+9\,a^8\,b^{11}\,x^2+36\,a^7\,b^{12}\,x^4+84\,a^6\,b^{13}\,x^6+126\,a^5\,b^{14}\,x^8+126\,a^4\,b^{15}\,x^{10}+84\,a^3\,b^{16}\,x^{12}+36\,a^2\,b^{17}\,x^{14}+9\,a\,b^{18}\,x^{16}+b^{19}\,x^{18}}+\frac{x}{b^{10}}-\frac{230945\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,b^{21/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20/(a + b*x^2)^10,x)
```

```
[Out] ((165409*a^9*x)/65536 + (2117549*a^8*b*x^3)/98304 + (424415*a*b^8*x^17)/655
36 + (2654039*a^7*b^2*x^5)/32768 + (40270037*a^6*b^3*x^7)/229376 + (30313*a
^5*b^4*x^9)/126 + (49153835*a^4*b^5*x^11)/229376 + (3997865*a^3*b^6*x^13)/3
2768 + (4042835*a^2*b^7*x^15)/98304)/(a^9*b^10 + b^19*x^18 + 9*a*b^18*x^16
+ 9*a^8*b^11*x^2 + 36*a^7*b^12*x^4 + 84*a^6*b^13*x^6 + 126*a^5*b^14*x^8 + 1
26*a^4*b^15*x^10 + 84*a^3*b^16*x^12 + 36*a^2*b^17*x^14) + x/b^10 - (230945*
a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*b^(21/2))
```

sympy [A] time = 1.82, size = 274, normalized size = 1.32

$$\frac{230945\sqrt{-\frac{a}{b^{21}}}\log\left(-b^{10}\sqrt{-\frac{a}{b^{21}}}+x\right)}{131072}-\frac{230945\sqrt{-\frac{a}{b^{21}}}\log\left(b^{10}\sqrt{-\frac{a}{b^{21}}}+x\right)}{131072}+\frac{10420767a^9x+88937058a^8bx^3+334408914a^7b^2x^5+724860666a^6b^3x^7+993296384a^5b^4x^9+884769030a^4b^5x^{11}+503730990a^3b^6x^{13}+169799070a^2b^7x^{15}+26738145ab^8x^{17}}{4128768a^9b^{10}+37158912a^8b^{11}x^2+148635648a^7b^{12}x^4+346816512a^6b^{13}x^6+520224768a^5b^{14}x^8+520224768a^4b^{15}x^{10}+346816512a^3b^{16}x^{12}+148635648a^2b^{17}x^{14}+37158912ab^{18}x^{16}+4128768b^{19}x^{18}}+\frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**20/(b*x**2+a)**10,x)
```

```
[Out] 230945*sqrt(-a/b**21)*log(-b**10*sqrt(-a/b**21) + x)/131072 - 230945*sqrt(-
a/b**21)*log(b**10*sqrt(-a/b**21) + x)/131072 + (10420767*a**9*x + 88937058
*a**8*b*x**3 + 334408914*a**7*b**2*x**5 + 724860666*a**6*b**3*x**7 + 993296
384*a**5*b**4*x**9 + 884769030*a**4*b**5*x**11 + 503730990*a**3*b**6*x**13
+ 169799070*a**2*b**7*x**15 + 26738145*a*b**8*x**17)/(4128768*a**9*b**10 +
37158912*a**8*b**11*x**2 + 148635648*a**7*b**12*x**4 + 346816512*a**6*b**13
*x**6 + 520224768*a**5*b**14*x**8 + 520224768*a**4*b**15*x**10 + 346816512*
a**3*b**16*x**12 + 148635648*a**2*b**17*x**14 + 37158912*a*b**18*x**16 + 41
28768*b**19*x**18) + x/b**10
```

3.212 $\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=197

$$\frac{12155\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\sqrt{a}\,b^{19/2}}-\frac{12155x}{65536b^9\left(a+bx^2\right)}-\frac{12155x^3}{98304b^8\left(a+bx^2\right)^2}-\frac{2431x^5}{24576b^7\left(a+bx^2\right)^3}-\frac{2431x^7}{28672b^6\left(a+bx^2\right)^4}-\frac{24\ldots}{32256b^5\ldots}$$

Rubi [A] time = 0.11, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {288, 205}

$$-\frac{17x^{15}}{288b^2\left(a+bx^2\right)^8}-\frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7}-\frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6}-\frac{2431x^9}{32256b^5\left(a+bx^2\right)^5}-\frac{2431x^7}{28672b^6\left(a+bx^2\right)^4}-\frac{2431x^5}{24576b^7\left(a+bx^2\right)^3}-\frac{12155x^3}{98304b^8\left(a+bx^2\right)^2}-\frac{12155x}{65536b^9\left(a+bx^2\right)}+\frac{12155\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\sqrt{a}\,b^{19/2}}-\frac{x^{17}}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^18/(a + b*x^2)^10,x]
```

```
[Out] -x^17/(18*b*(a + b*x^2)^9) - (17*x^15)/(288*b^2*(a + b*x^2)^8) - (85*x^13)/
(1344*b^3*(a + b*x^2)^7) - (1105*x^11)/(16128*b^4*(a + b*x^2)^6) - (2431*x^
9)/(32256*b^5*(a + b*x^2)^5) - (2431*x^7)/(28672*b^6*(a + b*x^2)^4) - (2431
*x^5)/(24576*b^7*(a + b*x^2)^3) - (12155*x^3)/(98304*b^8*(a + b*x^2)^2) - (
12155*x)/(65536*b^9*(a + b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(655
36*Sqrt[a]*b^(19/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{18}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{17}}{18b\left(a+bx^2\right)^9} + \frac{17\int \frac{x^{16}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} + \frac{85\int \frac{x^{14}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} + \frac{1105\int \frac{x^{12}}{\left(a+bx^2\right)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} + \frac{12155\int}{161}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{24}{32256b^5}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{24}{32256b^5}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{24}{32256b^5}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{24}{32256b^5}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{24}{32256b^5}$$

$$= -\frac{x^{17}}{18b\left(a+bx^2\right)^9} - \frac{17x^{15}}{288b^2\left(a+bx^2\right)^8} - \frac{85x^{13}}{1344b^3\left(a+bx^2\right)^7} - \frac{1105x^{11}}{16128b^4\left(a+bx^2\right)^6} - \frac{24}{32256b^5}$$

Mathematica [A] time = 0.08, size = 134, normalized size = 0.68

$$\frac{\frac{765765\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{b}x\left(765765a^8+6636630a^7bx^2+25423398a^6b^2x^4+56404062a^5b^3x^6+79659008a^4b^4x^8+73947042a^3b^5x^{10}+44765658a^2b^6x^{12}+16759722ab^7x^{14}+3363003b^8x^{16}\right)}{\left(a+bx^2\right)^9}}{4128768b^{19/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^18/(a + b*x^2)^10,x]
```

```
[Out] (-((Sqrt[b]*x*(765765*a^8 + 6636630*a^7*b*x^2 + 25423398*a^6*b^2*x^4 + 5640
4062*a^5*b^3*x^6 + 79659008*a^4*b^4*x^8 + 73947042*a^3*b^5*x^10 + 44765658*
a^2*b^6*x^12 + 16759722*a*b^7*x^14 + 3363003*b^8*x^16))/(a + b*x^2)^9) + (7
65765*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a])/(4128768*b^(19/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{18}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^18/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^18/(a + b*x^2)^10, x]
```

fricas [A] time = 0.59, size = 650, normalized size = 3.30

$$\left[-\frac{6726006\,ab^9x^{17}+33519444\,a^2b^8x^{15}+89531316\,a^3b^7x^{13}+147894084\,a^4b^6x^{11}+159318016\,a^5b^5x^9+112808124\,a^6b^4x^7+50846796\,a^7b^3x^5+13273260\,a^8b^2x^3+1531530\,a^9bx+765765\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{8257536\left(ab^{19}x^{18}+9\,a^2b^{18}x^{16}+36\,a^3b^{17}x^{14}+84\,a^4b^{16}x^{12}+126\,a^5b^{15}x^{10}+126\,a^6b^{14}x^8+84\,a^7b^{13}x^6+36\,a^8b^{12}x^4+9\,a^9b^{11}x^2+a^{10}b^{10}\right)},\ -\frac{3363003\,ab^9x^{17}+16759722\,a^2b^8x^{15}+44765658\,a^3b^7x^{13}+73947042\,a^4b^6x^{11}+79659008\,a^5b^5x^9+56404062\,a^6b^4x^7+25423398\,a^7b^3x^5+6636630\,a^8b^2x^3+765765\,a^9bx-765765\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{4128768\left(ab^{19}x^{18}+9\,a^2b^{18}x^{16}+36\,a^3b^{17}x^{14}+84\,a^4b^{16}x^{12}+126\,a^5b^{15}x^{10}+126\,a^6b^{14}x^8+84\,a^7b^{13}x^6+36\,a^8b^{12}x^4+9\,a^9b^{11}x^2+a^{10}b^{10}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/8257536*(6726006*a*b^9*x^17 + 33519444*a^2*b^8*x^15 + 89531316*a^3*b^7*
x^13 + 147894084*a^4*b^6*x^11 + 159318016*a^5*b^5*x^9 + 112808124*a^6*b^4*x
^7 + 50846796*a^7*b^3*x^5 + 13273260*a^8*b^2*x^3 + 1531530*a^9*b*x + 765765
*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5
*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a
^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^19*x^18
+ 9*a^2*b^18*x^16 + 36*a^3*b^17*x^14 + 84*a^4*b^16*x^12 + 126*a^5*b^15*x^10
 + 126*a^6*b^14*x^8 + 84*a^7*b^13*x^6 + 36*a^8*b^12*x^4 + 9*a^9*b^11*x^2 +
a^10*b^10), -1/4128768*(3363003*a*b^9*x^17 + 16759722*a^2*b^8*x^15 + 447656
58*a^3*b^7*x^13 + 73947042*a^4*b^6*x^11 + 79659008*a^5*b^5*x^9 + 56404062*a
^6*b^4*x^7 + 25423398*a^7*b^3*x^5 + 6636630*a^8*b^2*x^3 + 765765*a^9*b*x -
765765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a
^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x
```

```
^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^19*x^18 + 9*a^2*b^18*x^16 +
 36*a^3*b^17*x^14 + 84*a^4*b^16*x^12 + 126*a^5*b^15*x^10 + 126*a^6*b^14*x^8
 + 84*a^7*b^13*x^6 + 36*a^8*b^12*x^4 + 9*a^9*b^11*x^2 + a^10*b^10)]
```

giac [A] time = 0.62, size = 122, normalized size = 0.62

$$\frac{12155 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, b^9} - \frac{3363003\, b^8 x^{17} + 16759722\, a b^7 x^{15} + 44765658\, a^2 b^6 x^{13} + 73947042\, a^3 b^5 x^{11} + 79659008\, a^4 b^4 x^9 + 56404062\, a^5 b^3 x^7 + 25423398\, a^6 b^2 x^5 + 6636630\, a^7 b x^3 + 765765\, a^8 x}{4128768 \left(bx^2 + a\right)^9 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^9) - 1/4128768*(3363003*b^8*
x^17 + 16759722*a*b^7*x^15 + 44765658*a^2*b^6*x^13 + 73947042*a^3*b^5*x^11
+ 79659008*a^4*b^4*x^9 + 56404062*a^5*b^3*x^7 + 25423398*a^6*b^2*x^5 + 6636
630*a^7*b*x^3 + 765765*a^8*x)/((b*x^2 + a)^9*b^9)
```

maple [A] time = 0.02, size = 124, normalized size = 0.63

$$\frac{12155 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, b^9} + \frac{-\frac{53381 x^{17}}{65536 b} - \frac{399041 a x^{15}}{98304 b^2} - \frac{355283 a^2 x^{13}}{32768 b^3} - \frac{4108169 a^3 x^{11}}{229376 b^4} - \frac{2431 a^4 x^9}{126 b^5} - \frac{3133559 a^5 x^7}{229376 b^6} - \frac{201773 a^6 x^5}{32768 b^7} - \frac{158015 a^7 x^3}{98304 b^8} - \frac{12155 a^8 x}{65536 b^9}}{\left(b x^2 + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^18/(b*x^2+a)^10,x)
```

```
[Out] (-12155/65536*a^8/b^9*x-158015/98304*a^7/b^8*x^3-201773/32768*a^6/b^7*x^5-3
133559/229376*a^5/b^6*x^7-2431/126*a^4/b^5*x^9-4108169/229376*a^3/b^4*x^11-
355283/32768*a^2/b^3*x^13-399041/98304*a/b^2*x^15-53381/65536/b*x^17)/(b*x^
2+a)^9+12155/65536/b^9/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.17, size = 213, normalized size = 1.08

$$-\frac{3363003\, b^8 x^{17} + 16759722\, a b^7 x^{15} + 44765658\, a^2 b^6 x^{13} + 73947042\, a^3 b^5 x^{11} + 79659008\, a^4 b^4 x^9 + 56404062\, a^5 b^3 x^7 + 25423398\, a^6 b^2 x^5 + 6636630\, a^7 b x^3 + 765765\, a^8 x}{4128768 \left(b^{18} x^{18} + 9\, a b^{17} x^{16} + 36\, a^2 b^{16} x^{14} + 84\, a^3 b^{15} x^{12} + 126\, a^4 b^{14} x^{10} + 126\, a^5 b^{13} x^8 + 84\, a^6 b^{12} x^6 + 36\, a^7 b^{11} x^4 + 9\, a^8 b^{10} x^2 + a^9 b^9\right)} + \frac{12155 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^18/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/4128768*(3363003*b^8*x^17 + 16759722*a*b^7*x^15 + 44765658*a^2*b^6*x^13
+ 73947042*a^3*b^5*x^11 + 79659008*a^4*b^4*x^9 + 56404062*a^5*b^3*x^7 + 254
23398*a^6*b^2*x^5 + 6636630*a^7*b*x^3 + 765765*a^8*x)/(b^18*x^18 + 9*a*b^17
*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x^10 + 126*a^5*b
^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2 + a^9*b^9) + 1
2155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^9)
```

mupad [B] time = 4.96, size = 210, normalized size = 1.07

$$\frac{12155\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,\sqrt{a}\,b^{19/2}}-\frac{\frac{53381\,x^{17}}{65536\,b}+\frac{399041\,a\,x^{15}}{98304\,b^2}+\frac{12155\,a^8\,x}{65536\,b^9}+\frac{355283\,a^2\,x^{13}}{32768\,b^3}+\frac{4108169\,a^3\,x^{11}}{229376\,b^4}+\frac{2431\,a^4\,x^9}{126\,b^5}+\frac{3133559\,a^5\,x^7}{229376\,b^6}+\frac{201773\,a^6\,x^5}{32768\,b^7}+\frac{158015\,a^7\,x^3}{98304\,b^8}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^18/(a + b*x^2)^10,x)
```

```
[Out] (12155*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(1/2)*b^(19/2)) - ((53381*x^17)/
(65536*b) + (399041*a*x^15)/(98304*b^2) + (12155*a^8*x)/(65536*b^9) + (3552
83*a^2*x^13)/(32768*b^3) + (4108169*a^3*x^11)/(229376*b^4) + (2431*a^4*x^9)
/(126*b^5) + (3133559*a^5*x^7)/(229376*b^6) + (201773*a^6*x^5)/(32768*b^7)
+ (158015*a^7*x^3)/(98304*b^8))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^1
6 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 +
84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [A] time = 1.55, size = 277, normalized size = 1.41

$$-\frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(-ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072}+\frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072}+\frac{-765765a^8x-6636630a^7bx^3-25423398a^6b^2x^5-56404062a^5b^3x^7-79659008a^4b^4x^9-73947042a^3b^5x^{11}-44765658a^2b^6x^{13}-16759722ab^7x^{15}-3363003b^8x^{17}}{4128768a^9b^9+37158912a^8b^{10}x^2+148635648a^7b^{11}x^4+346816512a^6b^{12}x^6+520224768a^5b^{13}x^8+520224768a^4b^{14}x^{10}+346816512a^3b^{15}x^{12}+148635648a^2b^{16}x^{14}+37158912ab^{17}x^{16}+4128768b^{18}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**18/(b*x**2+a)**10,x)
```

```
[Out] -12155*sqrt(-1/(a*b**19))*log(-a*b**9*sqrt(-1/(a*b**19)) + x)/131072 + 1215
5*sqrt(-1/(a*b**19))*log(a*b**9*sqrt(-1/(a*b**19)) + x)/131072 + (-765765*a
**8*x - 6636630*a**7*b*x**3 - 25423398*a**6*b**2*x**5 - 56404062*a**5*b**3*
x**7 - 79659008*a**4*b**4*x**9 - 73947042*a**3*b**5*x**11 - 44765658*a**2*b
**6*x**13 - 16759722*a*b**7*x**15 - 3363003*b**8*x**17)/(4128768*a**9*b**9
+ 37158912*a**8*b**10*x**2 + 148635648*a**7*b**11*x**4 + 346816512*a**6*b**
12*x**6 + 520224768*a**5*b**13*x**8 + 520224768*a**4*b**14*x**10 + 34681651
2*a**3*b**15*x**12 + 148635648*a**2*b**16*x**14 + 37158912*a*b**17*x**16 +
4128768*b**18*x**18)
```

3.213 $\int \frac{x^{16}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=198

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}}+\frac{715x}{65536ab^8\left(a+bx^2\right)}-\frac{715x}{32768b^8\left(a+bx^2\right)^2}-\frac{715x^3}{24576b^7\left(a+bx^2\right)^3}-\frac{143x^5}{4096b^6\left(a+bx^2\right)^4}-\frac{143}{3584b^5\left(\right.}$$

Rubi [A] time = 0.12, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}}-\frac{5x^{13}}{96b^2\left(a+bx^2\right)^8}-\frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7}-\frac{715x^9}{16128b^4\left(a+bx^2\right)^6}-\frac{143x^7}{3584b^5\left(a+bx^2\right)^5}-\frac{143x^5}{4096b^6\left(a+bx^2\right)^4}-\frac{715x^3}{24576b^7\left(a+bx^2\right)^3}+\frac{715x}{65536ab^8\left(a+bx^2\right)}-\frac{715x}{32768b^8\left(a+bx^2\right)^2}-\frac{x^{15}}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^16/(a + b*x^2)^10,x]
```

```
[Out] -x^15/(18*b*(a + b*x^2)^9) - (5*x^13)/(96*b^2*(a + b*x^2)^8) - (65*x^11)/(1
344*b^3*(a + b*x^2)^7) - (715*x^9)/(16128*b^4*(a + b*x^2)^6) - (143*x^7)/(3
584*b^5*(a + b*x^2)^5) - (143*x^5)/(4096*b^6*(a + b*x^2)^4) - (715*x^3)/(24
576*b^7*(a + b*x^2)^3) - (715*x)/(32768*b^8*(a + b*x^2)^2) + (715*x)/(65536
*a*b^8*(a + b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(3/2)*b^(1
7/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{16}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{15}}{18b\left(a+bx^2\right)^9} + \frac{5\int \frac{x^{14}}{\left(a+bx^2\right)^9}\,dx}{6b}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} + \frac{65\int \frac{x^{12}}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} + \frac{715\int \frac{x^{10}}{\left(a+bx^2\right)^7}\,dx}{1344b^3}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} + \frac{715\int \frac{x}{(a+b}}{1792}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143}{3584b^5\left(a\right.}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143}{3584b^5\left(a\right.}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143}{3584b^5\left(a\right.}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143}{3584b^5\left(a\right.}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143}{3584b^5\left(a\right.}$$

$$= -\frac{x^{15}}{18b\left(a+bx^2\right)^9} - \frac{5x^{13}}{96b^2\left(a+bx^2\right)^8} - \frac{65x^{11}}{1344b^3\left(a+bx^2\right)^7} - \frac{715x^9}{16128b^4\left(a+bx^2\right)^6} - \frac{143}{3584b^5\left(a\right.}$$

Mathematica [A] time = 0.07, size = 138, normalized size = 0.70

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-45045a^8-390390a^7bx^2-1495494a^6b^2x^4-3317886a^5b^3x^6-4685824a^4b^4x^8-4349826a^3b^5x^{10}-2633274a^2b^6x^{12}-985866ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9}+45045\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{4128768a^{3/2}b^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^16/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 - 1495494*a^6*b^2*x^4 -
3317886*a^5*b^3*x^6 - 4685824*a^4*b^4*x^8 - 4349826*a^3*b^5*x^10 - 2633274*
a^2*b^6*x^12 - 985866*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(3/2)*b^(17/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{16}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^16/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^16/(a + b*x^2)^10, x]
```

fricas [A] time = 0.98, size = 654, normalized size = 3.30

$$\left[\frac{90090\,ab^9x^{17}-1971732\,a^2b^8x^{15}-5266548\,a^3b^7x^{13}-8699652\,a^4b^6x^{11}-9371648\,a^5b^5x^9-6635772\,a^6b^4x^7-2990988\,a^7b^3x^5-780780\,a^8b^2x^3-90090\,a^9bx-45045\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{8257536\left(a^2b^{18}x^{18}+9\,a^3b^{17}x^{16}+36\,a^4b^{16}x^{14}+84\,a^5b^{15}x^{12}+126\,a^6b^{14}x^{10}+126\,a^7b^{13}x^8+84\,a^8b^{12}x^6+36\,a^9b^{11}x^4+9\,a^{10}b^{10}x^2+a^{11}b^9\right)},\ \frac{45045\,ab^9x^{17}-985866\,a^2b^8x^{15}-2633274\,a^3b^7x^{13}-4349826\,a^4b^6x^{11}-4685824\,a^5b^5x^9-3317886\,a^6b^4x^7-1495494\,a^7b^3x^5-390390\,a^8b^2x^3-45045\,a^9bx+45045\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{4128768\left(a^2b^{18}x^{18}+9\,a^3b^{17}x^{16}+36\,a^4b^{16}x^{14}+84\,a^5b^{15}x^{12}+126\,a^6b^{14}x^{10}+126\,a^7b^{13}x^8+84\,a^8b^{12}x^6+36\,a^9b^{11}x^4+9\,a^{10}b^{10}x^2+a^{11}b^9\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(90090*a*b^9*x^17 - 1971732*a^2*b^8*x^15 - 5266548*a^3*b^7*x^13
- 8699652*a^4*b^6*x^11 - 9371648*a^5*b^5*x^9 - 6635772*a^6*b^4*x^7 - 299098
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^18*x^18 + 9*a^3*b^17*x^
16 + 36*a^4*b^16*x^14 + 84*a^5*b^15*x^12 + 126*a^6*b^14*x^10 + 126*a^7*b^13
*x^8 + 84*a^8*b^12*x^6 + 36*a^9*b^11*x^4 + 9*a^10*b^10*x^2 + a^11*b^9), 1/4
128768*(45045*a*b^9*x^17 - 985866*a^2*b^8*x^15 - 2633274*a^3*b^7*x^13 - 434
9826*a^4*b^6*x^11 - 4685824*a^5*b^5*x^9 - 3317886*a^6*b^4*x^7 - 1495494*a^7
*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*x
^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^
8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(s
```

```
qrt(a*b)*x/a))/(a^2*b^18*x^18 + 9*a^3*b^17*x^16 + 36*a^4*b^16*x^14 + 84*a^5
*b^15*x^12 + 126*a^6*b^14*x^10 + 126*a^7*b^13*x^8 + 84*a^8*b^12*x^6 + 36*a^
9*b^11*x^4 + 9*a^10*b^10*x^2 + a^11*b^9)]
```

giac [A] time = 0.63, size = 128, normalized size = 0.65

$$\frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}ab^8} + \frac{45045 b^8x^{17} - 985866 ab^7x^{15} - 2633274 a^2b^6x^{13} - 4349826 a^3b^5x^{11} - 4685824 a^4b^4x^9 - 3317886 a^5b^3x^7 - 1495494 a^6b^2x^5 - 390390 a^7bx^3 - 45045 a^8x}{4128768 \left(bx^2 + a\right)^9 ab^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^8) + 1/4128768*(45045*b^8*x^
17 - 985866*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349826*a^3*b^5*x^11 - 4685
824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*b^2*x^5 - 390390*a^7*b*
x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a*b^8)
```

maple [A] time = 0.02, size = 124, normalized size = 0.63

$$\frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\, a\, b^8} + \frac{\frac{715x^{17}}{65536a} - \frac{23473x^{15}}{98304b} - \frac{20899a\, x^{13}}{32768b^2} - \frac{241657a^2x^{11}}{229376b^3} - \frac{143a^3x^9}{126b^4} - \frac{184327a^4x^7}{229376b^5} - \frac{11869a^5x^5}{32768b^6} - \frac{9295a^6x^3}{98304b^7} - \frac{715a^7x}{65536b^8}}{\left(b\, x^2 + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(b*x^2+a)^10,x)
```

```
[Out] (-715/65536*a^7/b^8*x-9295/98304*a^6/b^7*x^3-11869/32768*a^5/b^6*x^5-184327
/229376*a^4/b^5*x^7-143/126*a^3/b^4*x^9-241657/229376*a^2/b^3*x^11-20899/32
768*a/b^2*x^13-23473/98304/b*x^15+715/65536/a*x^17)/(b*x^2+a)^9+715/65536/a
/b^8/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.16, size = 219, normalized size = 1.11

$$\frac{45045 b^8x^{17} - 985866 ab^7x^{15} - 2633274 a^2b^6x^{13} - 4349826 a^3b^5x^{11} - 4685824 a^4b^4x^9 - 3317886 a^5b^3x^7 - 1495494 a^6b^2x^5 - 390390 a^7bx^3 - 45045 a^8x}{4128768 \left(ab^{17}x^{18} + 9 a^2b^{16}x^{16} + 36 a^3b^{15}x^{14} + 84 a^4b^{14}x^{12} + 126 a^5b^{13}x^{10} + 126 a^6b^{12}x^8 + 84 a^7b^{11}x^6 + 36 a^8b^{10}x^4 + 9 a^9b^9x^2 + a^{10}b^8\right)} + \frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}ab^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(45045*b^8*x^17 - 985866*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349
826*a^3*b^5*x^11 - 4685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*
b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a*b^17*x^18 + 9*a^2*b^16*x^16 +
36*a^3*b^15*x^14 + 84*a^4*b^14*x^12 + 126*a^5*b^13*x^10 + 126*a^6*b^12*x^8
+ 84*a^7*b^11*x^6 + 36*a^8*b^10*x^4 + 9*a^9*b^9*x^2 + a^10*b^8) + 715/65536
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^8)
```

mupad [B] time = 4.76, size = 207, normalized size = 1.05

$$\frac{715\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{3/2}\,b^{17/2}}-\frac{\frac{23473\,x^{15}}{98304\,b}-\frac{715\,x^{17}}{65536\,a}+\frac{20899\,a\,x^{13}}{32768\,b^2}+\frac{715\,a^7\,x}{65536\,b^8}+\frac{241657\,a^2\,x^{11}}{229376\,b^3}+\frac{143\,a^3\,x^9}{126\,b^4}+\frac{184327\,a^4\,x^7}{229376\,b^5}+\frac{11869\,a^5\,x^5}{32768\,b^6}+\frac{9295\,a^6\,x^3}{98304\,b^7}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(a + b*x^2)^10,x)
```

```
[Out] (715*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(3/2)*b^(17/2)) - ((23473*x^15)/(9
8304*b) - (715*x^17)/(65536*a) + (20899*a*x^13)/(32768*b^2) + (715*a^7*x)/(
65536*b^8) + (241657*a^2*x^11)/(229376*b^3) + (143*a^3*x^9)/(126*b^4) + (18
4327*a^4*x^7)/(229376*b^5) + (11869*a^5*x^5)/(32768*b^6) + (9295*a^6*x^3)/(
98304*b^7))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 +
 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36
*a^2*b^7*x^14)
```

sympy [A] time = 1.46, size = 289, normalized size = 1.46

$$-\frac{715\sqrt{-\frac{1}{a^3b^{17}}}\log\left(-a^2b^8\sqrt{-\frac{1}{a^3b^{17}}}+x\right)}{131072}+\frac{715\sqrt{-\frac{1}{a^3b^{17}}}\log\left(a^2b^8\sqrt{-\frac{1}{a^3b^{17}}}+x\right)}{131072}+\frac{-45045a^8x-390390a^7bx^3-1495494a^6b^2x^5-3317886a^5b^3x^7-4685824a^4b^4x^9-4349826a^3b^5x^{11}-2633274a^2b^6x^{13}-985866ab^7x^{15}+45045b^8x^{17}}{4128768a^{10}b^8+37158912a^9b^9x^2+148635648a^8b^{10}x^4+346816512a^7b^{11}x^6+520224768a^6b^{12}x^8+520224768a^5b^{13}x^{10}+346816512a^4b^{14}x^{12}+148635648a^3b^{15}x^{14}+37158912a^2b^{16}x^{16}+4128768ab^{17}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**16/(b*x**2+a)**10,x)
```

```
[Out] -715*sqrt(-1/(a**3*b**17))*log(-a**2*b**8*sqrt(-1/(a**3*b**17)) + x)/131072
 + 715*sqrt(-1/(a**3*b**17))*log(a**2*b**8*sqrt(-1/(a**3*b**17)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 - 1495494*a**6*b**2*x**5 - 3317886*
a**5*b**3*x**7 - 4685824*a**4*b**4*x**9 - 4349826*a**3*b**5*x**11 - 2633274
*a**2*b**6*x**13 - 985866*a*b**7*x**15 + 45045*b**8*x**17)/(4128768*a**10*b
**8 + 37158912*a**9*b**9*x**2 + 148635648*a**8*b**10*x**4 + 346816512*a**7*
b**11*x**6 + 520224768*a**6*b**12*x**8 + 520224768*a**5*b**13*x**10 + 34681
6512*a**4*b**14*x**12 + 148635648*a**3*b**15*x**14 + 37158912*a**2*b**16*x*
*16 + 4128768*a*b**17*x**18)
```

3.214 $\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=199

$$\frac{143\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}}+\frac{143x}{65536a^2b^7\left(a+bx^2\right)}+\frac{143x}{98304ab^7\left(a+bx^2\right)^2}-\frac{143x}{24576b^7\left(a+bx^2\right)^3}-\frac{143x^3}{12288b^6\left(a+bx^2\right)^4}-\frac{\quad}{7680b}$$

Rubi [A] time = 0.12, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{143x}{65536a^2b^7\left(a+bx^2\right)}+\frac{143\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}}-\frac{13x^{11}}{288b^2\left(a+bx^2\right)^8}-\frac{143x^9}{4032b^3\left(a+bx^2\right)^7}-\frac{143x^7}{5376b^4\left(a+bx^2\right)^6}-\frac{143x^5}{7680b^5\left(a+bx^2\right)^5}-\frac{143x^3}{12288b^6\left(a+bx^2\right)^4}+\frac{143x}{98304ab^7\left(a+bx^2\right)^2}-\frac{143x}{24576b^7\left(a+bx^2\right)^3}-\frac{x^{13}}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^2)^10,x]
```

```
[Out] -x^13/(18*b*(a + b*x^2)^9) - (13*x^11)/(288*b^2*(a + b*x^2)^8) - (143*x^9)/
(4032*b^3*(a + b*x^2)^7) - (143*x^7)/(5376*b^4*(a + b*x^2)^6) - (143*x^5)/(
7680*b^5*(a + b*x^2)^5) - (143*x^3)/(12288*b^6*(a + b*x^2)^4) - (143*x)/(24
576*b^7*(a + b*x^2)^3) + (143*x)/(98304*a*b^7*(a + b*x^2)^2) + (143*x)/(655
36*a^2*b^7*(a + b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(5/2)*
b^(15/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{13}}{18b\left(a+bx^2\right)^9} + \frac{13\int \frac{x^{12}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} + \frac{143\int \frac{x^{10}}{\left(a+bx^2\right)^8}\,dx}{288b^2}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} + \frac{143\int \frac{x^8}{\left(a+bx^2\right)^7}\,dx}{448b^3}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} + \frac{143\int \frac{}{(a+}}{768}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143}{7680b^5(a}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143}{7680b^5(a}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143}{7680b^5(a}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143}{7680b^5(a}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143}{7680b^5(a}$$

$$= -\frac{x^{13}}{18b\left(a+bx^2\right)^9} - \frac{13x^{11}}{288b^2\left(a+bx^2\right)^8} - \frac{143x^9}{4032b^3\left(a+bx^2\right)^7} - \frac{143x^7}{5376b^4\left(a+bx^2\right)^6} - \frac{143}{7680b^5(a}$$

Mathematica [A] time = 0.06, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-45045a^8-390390a^7bx^2-1495494a^6b^2x^4-3317886a^5b^3x^6-4685824a^4b^4x^8-4349826a^3b^5x^{10}-2633274a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9}+45045\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{20643840a^{5/2}b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 - 1495494*a^6*b^2*x^4 -
3317886*a^5*b^3*x^6 - 4685824*a^4*b^4*x^8 - 4349826*a^3*b^5*x^10 - 2633274*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(20643840*a^(5/2)*b^(15/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^14/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^14/(a + b*x^2)^10, x]
```

fricas [A] time = 0.80, size = 654, normalized size = 3.29

$$\left[\frac{90090\,ab^9x^{17}+780780\,a^2b^8x^{15}-5266548\,a^3b^7x^{13}-8699652\,a^4b^6x^{11}-9371648\,a^5b^5x^9-6635772\,a^6b^4x^7-2990988\,a^7b^3x^5-780780\,a^8b^2x^3-90090\,a^9bx-45045\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{41287680\left(a^3b^{17}x^{18}+9\,a^4b^{16}x^{16}+36\,a^5b^{15}x^{14}+84\,a^6b^{14}x^{12}+126\,a^7b^{13}x^{10}+126\,a^8b^{12}x^8+84\,a^9b^{11}x^6+36\,a^{10}b^{10}x^4+9\,a^{11}b^9x^2+a^{12}b^8\right)},\ \frac{45045\,ab^9x^{17}+390390\,a^2b^8x^{15}-2633274\,a^3b^7x^{13}-4349826\,a^4b^6x^{11}-4685824\,a^5b^5x^9-3317886\,a^6b^4x^7-1495494\,a^7b^3x^5-390390\,a^8b^2x^3-45045\,a^9bx+45045\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{20643840\left(a^3b^{17}x^{18}+9\,a^4b^{16}x^{16}+36\,a^5b^{15}x^{14}+84\,a^6b^{14}x^{12}+126\,a^7b^{13}x^{10}+126\,a^8b^{12}x^8+84\,a^9b^{11}x^6+36\,a^{10}b^{10}x^4+9\,a^{11}b^9x^2+a^{12}b^8\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/41287680*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 - 5266548*a^3*b^7*x^13
- 8699652*a^4*b^6*x^11 - 9371648*a^5*b^5*x^9 - 6635772*a^6*b^4*x^7 - 299098
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^17*x^18 + 9*a^4*b^16*x^
16 + 36*a^5*b^15*x^14 + 84*a^6*b^14*x^12 + 126*a^7*b^13*x^10 + 126*a^8*b^12
*x^8 + 84*a^9*b^11*x^6 + 36*a^10*b^10*x^4 + 9*a^11*b^9*x^2 + a^12*b^8), 1/2
0643840*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 - 2633274*a^3*b^7*x^13 - 43
49826*a^4*b^6*x^11 - 4685824*a^5*b^5*x^9 - 3317886*a^6*b^4*x^7 - 1495494*a^
7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*
x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x
^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(
```

```
sqrt(a*b)*x/a))/(a^3*b^17*x^18 + 9*a^4*b^16*x^16 + 36*a^5*b^15*x^14 + 84*a^
6*b^14*x^12 + 126*a^7*b^13*x^10 + 126*a^8*b^12*x^8 + 84*a^9*b^11*x^6 + 36*a
^10*b^10*x^4 + 9*a^11*b^9*x^2 + a^12*b^8)]
```

giac [A] time = 0.64, size = 128, normalized size = 0.64

$$\frac{143 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^2 b^7} + \frac{45045 \, b^8 x^{17} + 390390 \, a b^7 x^{15} - 2633274 \, a^2 b^6 x^{13} - 4349826 \, a^3 b^5 x^{11} - 4685824 \, a^4 b^4 x^9 - 3317886 \, a^5 b^3 x^7 - 1495494 \, a^6 b^2 x^5 - 390390 \, a^7 b x^3 - 45045 \, a^8 x}{20643840 \, (b x^2 + a)^9 a^2 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 143/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^7) + 1/20643840*(45045*b^8
*x^17 + 390390*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349826*a^3*b^5*x^11 - 4
685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*b^2*x^5 - 390390*a^7
*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^2*b^7)
```

maple [A] time = 0.02, size = 122, normalized size = 0.61

$$\frac{143 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\; a^2 b^7} + \frac{\frac{143 b\, x^{17}}{65536 a^2} + \frac{1859 x^{15}}{98304 a} - \frac{20899 x^{13}}{163840 b} - \frac{241657 a\, x^{11}}{1146880 b^2} - \frac{143 a^2 x^9}{630 b^3} - \frac{184327 a^3 x^7}{1146880 b^4} - \frac{11869 a^4 x^5}{163840 b^5} - \frac{1859 a^5 x^3}{98304 b^6} - \frac{143 a^6 x}{65536 b^7}}{\left(b\, x^2 + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^2+a)^10,x)
```

```
[Out] (-143/65536*a^6/b^7*x-1859/98304*a^5/b^6*x^3-11869/163840*a^4/b^5*x^5-18432
7/1146880*a^3/b^4*x^7-143/630*a^2/b^3*x^9-241657/1146880*a/b^2*x^11-20899/1
63840/b*x^13+1859/98304/a*x^15+143/65536/a^2*b*x^17)/(b*x^2+a)^9+143/65536/
a^2/b^7/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.16, size = 221, normalized size = 1.11

$$\frac{45045 \, b^8 x^{17} + 390390 \, a b^7 x^{15} - 2633274 \, a^2 b^6 x^{13} - 4349826 \, a^3 b^5 x^{11} - 4685824 \, a^4 b^4 x^9 - 3317886 \, a^5 b^3 x^7 - 1495494 \, a^6 b^2 x^5 - 390390 \, a^7 b x^3 - 45045 \, a^8 x}{20643840 \, \left(a^2 b^{16} x^{18} + 9 \, a^3 b^{15} x^{16} + 36 \, a^4 b^{14} x^{14} + 84 \, a^5 b^{13} x^{12} + 126 \, a^6 b^{12} x^{10} + 126 \, a^7 b^{11} x^8 + 84 \, a^8 b^{10} x^6 + 36 \, a^9 b^9 x^4 + 9 \, a^{10} b^8 x^2 + a^{11} b^7\right)} + \frac{143 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^2 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20643840*(45045*b^8*x^17 + 390390*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 434
9826*a^3*b^5*x^11 - 4685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6
*b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a^2*b^16*x^18 + 9*a^3*b^15*x^16
 + 36*a^4*b^14*x^14 + 84*a^5*b^13*x^12 + 126*a^6*b^12*x^10 + 126*a^7*b^11*x
^8 + 84*a^8*b^10*x^6 + 36*a^9*b^9*x^4 + 9*a^10*b^8*x^2 + a^11*b^7) + 143/65
536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^7)
```

mupad [B] time = 4.75, size = 205, normalized size = 1.03

$$\frac{143\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{5/2}\,b^{15/2}}-\frac{\frac{20899\,x^{13}}{163840\,b}-\frac{1859\,x^{15}}{98304\,a}+\frac{241657\,a\,x^{11}}{1146880\,b^2}+\frac{143\,a^6\,x}{65536\,b^7}-\frac{143\,b\,x^{17}}{65536\,a^2}+\frac{143\,a^2\,x^9}{630\,b^3}+\frac{184327\,a^3\,x^7}{1146880\,b^4}+\frac{11869\,a^4\,x^5}{163840\,b^5}+\frac{1859\,a^5\,x^3}{98304\,b^6}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^2)^10,x)
```

```
[Out] (143*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(5/2)*b^(15/2)) - ((20899*x^13)/(1
63840*b) - (1859*x^15)/(98304*a) + (241657*a*x^11)/(1146880*b^2) + (143*a^6
*x)/(65536*b^7) - (143*b*x^17)/(65536*a^2) + (143*a^2*x^9)/(630*b^3) + (184
327*a^3*x^7)/(1146880*b^4) + (11869*a^4*x^5)/(163840*b^5) + (1859*a^5*x^3)/
(98304*b^6))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4
+ 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 3
6*a^2*b^7*x^14)
```

sympy [A] time = 1.35, size = 291, normalized size = 1.46

$$-\frac{143\sqrt{-\frac{1}{a^5b^{15}}}\log\left(-a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072}+\frac{143\sqrt{-\frac{1}{a^5b^{15}}}\log\left(a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072}+\frac{-45045a^8x-390390a^7bx^3-1495494a^6b^2x^5-3317886a^5b^3x^7-4685824a^4b^4x^9-4349826a^3b^5x^{11}-2633274a^2b^6x^{13}+390390ab^7x^{15}+45045b^8x^{17}}{20643840a^{11}b^7+185794560a^{10}b^8x^2+743178240a^9b^9x^4+1734082560a^8b^{10}x^6+2601123840a^7b^{11}x^8+2601123840a^6b^{12}x^{10}+1734082560a^5b^{13}x^{12}+743178240a^4b^{14}x^{14}+185794560a^3b^{15}x^{16}+20643840a^2b^{16}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**2+a)**10,x)
```

```
[Out] -143*sqrt(-1/(a**5*b**15))*log(-a**3*b**7*sqrt(-1/(a**5*b**15)) + x)/131072
 + 143*sqrt(-1/(a**5*b**15))*log(a**3*b**7*sqrt(-1/(a**5*b**15)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 - 1495494*a**6*b**2*x**5 - 3317886*
a**5*b**3*x**7 - 4685824*a**4*b**4*x**9 - 4349826*a**3*b**5*x**11 - 2633274
*a**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(20643840*a**11*
b**7 + 185794560*a**10*b**8*x**2 + 743178240*a**9*b**9*x**4 + 1734082560*a*
*8*b**10*x**6 + 2601123840*a**7*b**11*x**8 + 2601123840*a**6*b**12*x**10 +
1734082560*a**5*b**13*x**12 + 743178240*a**4*b**14*x**14 + 185794560*a**3*b
**15*x**16 + 20643840*a**2*b**16*x**18)
```

3.215 $\int \frac{x^{12}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=200

$$\frac{55\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}}+\frac{55x}{65536a^3b^6\left(a+bx^2\right)}+\frac{55x}{98304a^2b^6\left(a+bx^2\right)^2}+\frac{11x}{24576ab^6\left(a+bx^2\right)^3}-\frac{11x}{4096b^6\left(a+bx^2\right)^4}-\frac{}{1536\cdots}$$

Rubi [A] time = 0.12, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{55x}{65536a^3b^6\left(a+bx^2\right)}+\frac{55x}{98304a^2b^6\left(a+bx^2\right)^2}+\frac{55\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}}-\frac{11x^9}{288b^2\left(a+bx^2\right)^8}-\frac{11x^7}{448b^3\left(a+bx^2\right)^7}-\frac{11x^5}{768b^4\left(a+bx^2\right)^6}-\frac{11x^3}{1536b^5\left(a+bx^2\right)^5}+\frac{11x}{24576ab^6\left(a+bx^2\right)^3}-\frac{11x}{4096b^6\left(a+bx^2\right)^4}-\frac{x^{11}}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^2)^10,x]
```

```
[Out] -x^11/(18*b*(a + b*x^2)^9) - (11*x^9)/(288*b^2*(a + b*x^2)^8) - (11*x^7)/(4
48*b^3*(a + b*x^2)^7) - (11*x^5)/(768*b^4*(a + b*x^2)^6) - (11*x^3)/(1536*b
^5*(a + b*x^2)^5) - (11*x)/(4096*b^6*(a + b*x^2)^4) + (11*x)/(24576*a*b^6*(
a + b*x^2)^3) + (55*x)/(98304*a^2*b^6*(a + b*x^2)^2) + (55*x)/(65536*a^3*b^
6*(a + b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(7/2)*b^(13/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^{11}}{18b\left(a+bx^2\right)^9} + \frac{11\int \frac{x^{10}}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} + \frac{11\int \frac{x^8}{\left(a+bx^2\right)^8}\,dx}{32b^2}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} + \frac{11\int \frac{x^6}{\left(a+bx^2\right)^7}\,dx}{64b^3}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} + \frac{55\int \frac{x^4}{\left(a+bx^2\right)^6}\,d}{768b^4}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

$$= -\frac{x^{11}}{18b\left(a+bx^2\right)^9} - \frac{11x^9}{288b^2\left(a+bx^2\right)^8} - \frac{11x^7}{448b^3\left(a+bx^2\right)^7} - \frac{11x^5}{768b^4\left(a+bx^2\right)^6} - \frac{11x^3}{1536b^5\left(a+b\right.}$$

Mathematica [A] time = 0.07, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-3465a^8-30030a^7bx^2-115038a^6b^2x^4-255222a^5b^3x^6-360448a^4b^4x^8-334602a^3b^5x^{10}+115038a^2b^6x^{12}+30030ab^7x^{14}+3465b^8x^{16}\right)}{\left(a+bx^2\right)^9}+3465\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{4128768a^{7/2}b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-3465*a^8 - 30030*a^7*b*x^2 - 115038*a^6*b^2*x^4 - 255
222*a^5*b^3*x^6 - 360448*a^4*b^4*x^8 - 334602*a^3*b^5*x^10 + 115038*a^2*b^6
*x^12 + 30030*a*b^7*x^14 + 3465*b^8*x^16))/(a + b*x^2)^9 + 3465*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(4128768*a^(7/2)*b^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^12/(a + b*x^2)^10, x]
```

fricas [A] time = 0.86, size = 654, normalized size = 3.27

$$\left[\frac{6930\,ab^9x^{17}+60060\,a^2b^8x^{15}+230076\,a^3b^7x^{13}-669204\,a^4b^6x^{11}-720896\,a^5b^5x^9-510444\,a^6b^4x^7-230076\,a^7b^3x^5-60060\,a^8b^2x^3-6930\,a^9bx-3465\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{8257536\left(a^4b^{16}x^{18}+9\,a^5b^{15}x^{16}+36\,a^6b^{14}x^{14}+84\,a^7b^{13}x^{12}+126\,a^8b^{12}x^{10}+126\,a^9b^{11}x^8+84\,a^{10}b^{10}x^6+36\,a^{11}b^9x^4+9\,a^{12}b^8x^2+a^{13}b^7\right)},\ \frac{3465\,ab^9x^{17}+30030\,a^2b^8x^{15}+115038\,a^3b^7x^{13}-334602\,a^4b^6x^{11}-360448\,a^5b^5x^9-255222\,a^6b^4x^7-115038\,a^7b^3x^5-30030\,a^8b^2x^3-3465\,a^9bx+3465\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{4128768\left(a^4b^{16}x^{18}+9\,a^5b^{15}x^{16}+36\,a^6b^{14}x^{14}+84\,a^7b^{13}x^{12}+126\,a^8b^{12}x^{10}+126\,a^9b^{11}x^8+84\,a^{10}b^{10}x^6+36\,a^{11}b^9x^4+9\,a^{12}b^8x^2+a^{13}b^7\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(6930*a*b^9*x^17 + 60060*a^2*b^8*x^15 + 230076*a^3*b^7*x^13 - 66
9204*a^4*b^6*x^11 - 720896*a^5*b^5*x^9 - 510444*a^6*b^4*x^7 - 230076*a^7*b^
3*x^5 - 60060*a^8*b^2*x^3 - 6930*a^9*b*x - 3465*(b^9*x^18 + 9*a*b^8*x^16 +
36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84
*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^16*x^18 + 9*a^5*b^15*x^16 + 36*a^6
*b^14*x^14 + 84*a^7*b^13*x^12 + 126*a^8*b^12*x^10 + 126*a^9*b^11*x^8 + 84*a
^10*b^10*x^6 + 36*a^11*b^9*x^4 + 9*a^12*b^8*x^2 + a^13*b^7), 1/4128768*(346
5*a*b^9*x^17 + 30030*a^2*b^8*x^15 + 115038*a^3*b^7*x^13 - 334602*a^4*b^6*x^
11 - 360448*a^5*b^5*x^9 - 255222*a^6*b^4*x^7 - 115038*a^7*b^3*x^5 - 30030*a
^8*b^2*x^3 - 3465*a^9*b*x + 3465*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14
 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 +
36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b
```

```
^16*x^18 + 9*a^5*b^15*x^16 + 36*a^6*b^14*x^14 + 84*a^7*b^13*x^12 + 126*a^8*
b^12*x^10 + 126*a^9*b^11*x^8 + 84*a^10*b^10*x^6 + 36*a^11*b^9*x^4 + 9*a^12*
b^8*x^2 + a^13*b^7)]
```

giac [A] time = 0.63, size = 128, normalized size = 0.64

$$\frac{55 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} a^3 b^6} + \frac{3465 b^8 x^{17} + 30030 a b^7 x^{15} + 115038 a^2 b^6 x^{13} - 334602 a^3 b^5 x^{11} - 360448 a^4 b^4 x^9 - 255222 a^5 b^3 x^7 - 115038 a^6 b^2 x^5 - 30030 a^7 b x^3 - 3465 a^8 x}{4128768 \left(bx^2 + a\right)^9 a^3 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^6) + 1/4128768*(3465*b^8*x^
17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 - 334602*a^3*b^5*x^11 - 360448*
a^4*b^4*x^9 - 255222*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3 - 3
465*a^8*x)/((b*x^2 + a)^9*a^3*b^6)
```

maple [A] time = 0.02, size = 122, normalized size = 0.61

$$\frac{55 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\, a^3 b^6} + \frac{\frac{55 b^2 x^{17}}{65536 a^3} + \frac{715 b\, x^{15}}{98304 a^2} + \frac{913 x^{13}}{32768 a} - \frac{18589 x^{11}}{229376 b} - \frac{11 a\, x^9}{126 b^2} - \frac{14179 a^2 x^7}{229376 b^3} - \frac{913 a^3 x^5}{32768 b^4} - \frac{715 a^4 x^3}{98304 b^5} - \frac{55 a^5 x}{65536 b^6}}{\left(b\, x^2 + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2+a)^10,x)
```

```
[Out] (-55/65536*a^5/b^6*x-715/98304*a^4/b^5*x^3-913/32768*a^3/b^4*x^5-14179/2293
76*a^2/b^3*x^7-11/126*a/b^2*x^9-18589/229376/b*x^11+913/32768/a*x^13+715/98
304/a^2*b*x^15+55/65536*b^2/a^3*x^17)/(b*x^2+a)^9+55/65536/a^3/b^6/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.09, size = 221, normalized size = 1.10

$$\frac{3465 b^8 x^{17} + 30030 a b^7 x^{15} + 115038 a^2 b^6 x^{13} - 334602 a^3 b^5 x^{11} - 360448 a^4 b^4 x^9 - 255222 a^5 b^3 x^7 - 115038 a^6 b^2 x^5 - 30030 a^7 b x^3 - 3465 a^8 x}{4128768 \left(a^3 b^{15} x^{18} + 9 a^4 b^{14} x^{16} + 36 a^5 b^{13} x^{14} + 84 a^6 b^{12} x^{12} + 126 a^7 b^{11} x^{10} + 126 a^8 b^{10} x^8 + 84 a^9 b^9 x^6 + 36 a^{10} b^8 x^4 + 9 a^{11} b^7 x^2 + a^{12} b^6\right)} + \frac{55 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} a^3 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(3465*b^8*x^17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 - 334602*
a^3*b^5*x^11 - 360448*a^4*b^4*x^9 - 255222*a^5*b^3*x^7 - 115038*a^6*b^2*x^5
 - 30030*a^7*b*x^3 - 3465*a^8*x)/(a^3*b^15*x^18 + 9*a^4*b^14*x^16 + 36*a^5*
b^13*x^14 + 84*a^6*b^12*x^12 + 126*a^7*b^11*x^10 + 126*a^8*b^10*x^8 + 84*a^
9*b^9*x^6 + 36*a^10*b^8*x^4 + 9*a^11*b^7*x^2 + a^12*b^6) + 55/65536*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^6)
```

mupad [B] time = 0.15, size = 205, normalized size = 1.02

$$\frac{55\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{7/2}\,b^{13/2}}-\frac{\frac{18589\,x^{11}}{229376\,b}-\frac{913\,x^{13}}{32768\,a}+\frac{11\,a\,x^9}{126\,b^2}+\frac{55\,a^5\,x}{65536\,b^6}-\frac{715\,b\,x^{15}}{98304\,a^2}+\frac{14179\,a^2\,x^7}{229376\,b^3}+\frac{913\,a^3\,x^5}{32768\,b^4}+\frac{715\,a^4\,x^3}{98304\,b^5}-\frac{55\,b^2\,x^{17}}{65536\,a^3}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^2)^10,x)
```

```
[Out] (55*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(7/2)*b^(13/2)) - ((18589*x^11)/(22
9376*b) - (913*x^13)/(32768*a) + (11*a*x^9)/(126*b^2) + (55*a^5*x)/(65536*b
^6) - (715*b*x^15)/(98304*a^2) + (14179*a^2*x^7)/(229376*b^3) + (913*a^3*x^
5)/(32768*b^4) + (715*a^4*x^3)/(98304*b^5) - (55*b^2*x^17)/(65536*a^3))/(a^
9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6
 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [A] time = 1.27, size = 291, normalized size = 1.46

$$-\frac{55\sqrt{-\frac{1}{a^7b^{13}}}\log\left(-a^4b^6\sqrt{-\frac{1}{a^7b^{13}}}+x\right)}{131072}+\frac{55\sqrt{-\frac{1}{a^7b^{13}}}\log\left(a^4b^6\sqrt{-\frac{1}{a^7b^{13}}}+x\right)}{131072}+\frac{-3465a^8x-30030a^7bx^3-115038a^6b^2x^5-255222a^5b^3x^7-360448a^4b^4x^9-334602a^3b^5x^{11}+115038a^2b^6x^{13}+30030ab^7x^{15}+3465b^8x^{17}}{4128768a^{12}b^6+37158912a^{11}b^7x^2+148635648a^{10}b^8x^4+346816512a^9b^9x^6+520224768a^8b^{10}x^8+520224768a^7b^{11}x^{10}+346816512a^6b^{12}x^{12}+148635648a^5b^{13}x^{14}+37158912a^4b^{14}x^{16}+4128768a^3b^{15}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**2+a)**10,x)
```

```
[Out] -55*sqrt(-1/(a**7*b**13))*log(-a**4*b**6*sqrt(-1/(a**7*b**13)) + x)/131072
+ 55*sqrt(-1/(a**7*b**13))*log(a**4*b**6*sqrt(-1/(a**7*b**13)) + x)/131072
+ (-3465*a**8*x - 30030*a**7*b*x**3 - 115038*a**6*b**2*x**5 - 255222*a**5*b
**3*x**7 - 360448*a**4*b**4*x**9 - 334602*a**3*b**5*x**11 + 115038*a**2*b**
6*x**13 + 30030*a*b**7*x**15 + 3465*b**8*x**17)/(4128768*a**12*b**6 + 37158
912*a**11*b**7*x**2 + 148635648*a**10*b**8*x**4 + 346816512*a**9*b**9*x**6
+ 520224768*a**8*b**10*x**8 + 520224768*a**7*b**11*x**10 + 346816512*a**6*b
**12*x**12 + 148635648*a**5*b**13*x**14 + 37158912*a**4*b**14*x**16 + 41287
68*a**3*b**15*x**18)
```

3.216 $\int \frac{x^{10}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=201

$$\frac{35\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{9/2}b^{11/2}}+\frac{35x}{65536a^4b^5\left(a+bx^2\right)}+\frac{35x}{98304a^3b^5\left(a+bx^2\right)^2}+\frac{7x}{24576a^2b^5\left(a+bx^2\right)^3}+\frac{x}{4096ab^5\left(a+bx^2\right)^4}-\frac{}{512b}$$

Rubi [A] time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{35x}{65536a^4b^5\left(a+bx^2\right)}+\frac{35x}{98304a^3b^5\left(a+bx^2\right)^2}+\frac{7x}{24576a^2b^5\left(a+bx^2\right)^3}+\frac{35\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{9/2}b^{11/2}}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}-\frac{5x^3}{768b^4\left(a+bx^2\right)^6}+\frac{x}{4096ab^5\left(a+bx^2\right)^4}-\frac{x}{512b^5\left(a+bx^2\right)^5}-\frac{x^9}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^10,x]
```

```
[Out] -x^9/(18*b*(a + b*x^2)^9) - x^7/(32*b^2*(a + b*x^2)^8) - x^5/(64*b^3*(a + b
*x^2)^7) - (5*x^3)/(768*b^4*(a + b*x^2)^6) - x/(512*b^5*(a + b*x^2)^5) + x/
(4096*a*b^5*(a + b*x^2)^4) + (7*x)/(24576*a^2*b^5*(a + b*x^2)^3) + (35*x)/(
98304*a^3*b^5*(a + b*x^2)^2) + (35*x)/(65536*a^4*b^5*(a + b*x^2)) + (35*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(9/2)*b^(11/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^9}{18b\left(a+bx^2\right)^9}+\frac{\int \frac{x^8}{\left(a+bx^2\right)^9}\,dx}{2b}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}+\frac{7\int \frac{x^6}{\left(a+bx^2\right)^8}\,dx}{32b^2}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}+\frac{5\int \frac{x^4}{\left(a+bx^2\right)^7}\,dx}{64b^3}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}-\frac{5x^3}{768b^4\left(a+bx^2\right)^6}+\frac{5\int \frac{x^2}{\left(a+bx^2\right)^6}\,dx}{256b^4}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}-\frac{5x^3}{768b^4\left(a+bx^2\right)^6}-\frac{x}{512b^5\left(a+bx^2\right.}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}-\frac{5x^3}{768b^4\left(a+bx^2\right)^6}-\frac{x}{512b^5\left(a+bx^2\right.}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}-\frac{5x^3}{768b^4\left(a+bx^2\right)^6}-\frac{x}{512b^5\left(a+bx^2\right.}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}-\frac{5x^3}{768b^4\left(a+bx^2\right)^6}-\frac{x}{512b^5\left(a+bx^2\right.}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}-\frac{5x^3}{768b^4\left(a+bx^2\right)^6}-\frac{x}{512b^5\left(a+bx^2\right.}$$

$$= -\frac{x^9}{18b\left(a+bx^2\right)^9}-\frac{x^7}{32b^2\left(a+bx^2\right)^8}-\frac{x^5}{64b^3\left(a+bx^2\right)^7}-\frac{5x^3}{768b^4\left(a+bx^2\right)^6}-\frac{x}{512b^5\left(a+bx^2\right.}$$

Mathematica [A] time = 0.07, size = 138, normalized size = 0.69

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-315a^8-2730a^7bx^2-10458a^6b^2x^4-23202a^5b^3x^6-32768a^4b^4x^8+23202a^3b^5x^{10}+10458a^2b^6x^{12}+2730ab^7x^{14}+315b^8x^{16}\right)}{\left(a+bx^2\right)^9}+315\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{589824a^{9/2}b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-315*a^8 - 2730*a^7*b*x^2 - 10458*a^6*b^2*x^4 - 23202*
a^5*b^3*x^6 - 32768*a^4*b^4*x^8 + 23202*a^3*b^5*x^10 + 10458*a^2*b^6*x^12 +
 2730*a*b^7*x^14 + 315*b^8*x^16))/(a + b*x^2)^9 + 315*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(589824*a^(9/2)*b^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^10/(a + b*x^2)^10, x]
```

fricas [A] time = 0.90, size = 654, normalized size = 3.25

$$\left[\frac{630\,ab^9x^{17}+5460\,a^2b^8x^{15}+20916\,a^3b^7x^{13}+46404\,a^4b^6x^{11}-65536\,a^5b^5x^9-46404\,a^6b^4x^7-20916\,a^7b^3x^5-5460\,a^8b^2x^3-630\,a^9bx-315\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{1179648\left(a^5b^{15}x^{18}+9\,a^6b^{14}x^{16}+36\,a^7b^{13}x^{14}+84\,a^8b^{12}x^{12}+126\,a^9b^{11}x^{10}+126\,a^{10}b^{10}x^8+84\,a^{11}b^9x^6+36\,a^{12}b^8x^4+9\,a^{13}b^7x^2+a^{14}b^6\right)},\ \frac{315\,ab^9x^{17}+2730\,a^2b^8x^{15}+10458\,a^3b^7x^{13}+23202\,a^4b^6x^{11}-32768\,a^5b^5x^9-23202\,a^6b^4x^7-10458\,a^7b^3x^5-2730\,a^8b^2x^3-315\,a^9bx+315\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{589824\left(a^5b^{15}x^{18}+9\,a^6b^{14}x^{16}+36\,a^7b^{13}x^{14}+84\,a^8b^{12}x^{12}+126\,a^9b^{11}x^{10}+126\,a^{10}b^{10}x^8+84\,a^{11}b^9x^6+36\,a^{12}b^8x^4+9\,a^{13}b^7x^2+a^{14}b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(630*a*b^9*x^17 + 5460*a^2*b^8*x^15 + 20916*a^3*b^7*x^13 + 46404
*a^4*b^6*x^11 - 65536*a^5*b^5*x^9 - 46404*a^6*b^4*x^7 - 20916*a^7*b^3*x^5 -
 5460*a^8*b^2*x^3 - 630*a^9*b*x - 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7
*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x
^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^5*b^15*x^18 + 9*a^6*b^14*x^16 + 36*a^7*b^13*x^14
 + 84*a^8*b^12*x^12 + 126*a^9*b^11*x^10 + 126*a^10*b^10*x^8 + 84*a^11*b^9*x
^6 + 36*a^12*b^8*x^4 + 9*a^13*b^7*x^2 + a^14*b^6), 1/589824*(315*a*b^9*x^17
 + 2730*a^2*b^8*x^15 + 10458*a^3*b^7*x^13 + 23202*a^4*b^6*x^11 - 32768*a^5*
b^5*x^9 - 23202*a^6*b^4*x^7 - 10458*a^7*b^3*x^5 - 2730*a^8*b^2*x^3 - 315*a^
9*b*x + 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 +
126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^
8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^15*x^18 + 9*a^6*b^14
```

```
*x^16 + 36*a^7*b^13*x^14 + 84*a^8*b^12*x^12 + 126*a^9*b^11*x^10 + 126*a^10*
b^10*x^8 + 84*a^11*b^9*x^6 + 36*a^12*b^8*x^4 + 9*a^13*b^7*x^2 + a^14*b^6)]
```

giac [A] time = 0.63, size = 128, normalized size = 0.64

$$\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^4b^5}+\frac{315\,b^8x^{17}+2730\,ab^7x^{15}+10458\,a^2b^6x^{13}+23202\,a^3b^5x^{11}-32768\,a^4b^4x^9-23202\,a^5b^3x^7-10458\,a^6b^2x^5-2730\,a^7bx^3-315\,a^8x}{589824\left(bx^2+a\right)^9a^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b^5) + 1/589824*(315*b^8*x^17
 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 - 32768*a^4*b^
4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)
/((b*x^2 + a)^9*a^4*b^5)
```

maple [A] time = 0.02, size = 122, normalized size = 0.61

$$\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^4b^5}+\frac{\frac{35b^3x^{17}}{65536a^4}+\frac{455b^2x^{15}}{98304a^3}+\frac{581b\,x^{13}}{32768a^2}+\frac{1289x^{11}}{32768a}-\frac{x^9}{18b}-\frac{1289a\,x^7}{32768b^2}-\frac{581a^2x^5}{32768b^3}-\frac{455a^3x^3}{98304b^4}-\frac{35a^4x}{65536b^5}}{\left(b\,x^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^10,x)
```

```
[Out] (-35/65536*a^4/b^5*x-455/98304*a^3/b^4*x^3-581/32768*a^2/b^3*x^5-1289/32768
*a/b^2*x^7-1/18/b*x^9+1289/32768/a*x^11+581/32768/a^2*b*x^13+455/98304*b^2/
a^3*x^15+35/65536*b^3/a^4*x^17)/(b*x^2+a)^9+35/65536/a^4/b^5/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.08, size = 221, normalized size = 1.10

$$\frac{315\,b^8x^{17}+2730\,ab^7x^{15}+10458\,a^2b^6x^{13}+23202\,a^3b^5x^{11}-32768\,a^4b^4x^9-23202\,a^5b^3x^7-10458\,a^6b^2x^5-2730\,a^7bx^3-315\,a^8x}{589824\left(a^4b^{14}x^{18}+9\,a^5b^{13}x^{16}+36\,a^6b^{12}x^{14}+84\,a^7b^{11}x^{12}+126\,a^8b^{10}x^{10}+126\,a^9b^9x^8+84\,a^{10}b^8x^6+36\,a^{11}b^7x^4+9\,a^{12}b^6x^2+a^{13}b^5\right)}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/589824*(315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b
^5*x^11 - 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*
a^7*b*x^3 - 315*a^8*x)/(a^4*b^14*x^18 + 9*a^5*b^13*x^16 + 36*a^6*b^12*x^14
+ 84*a^7*b^11*x^12 + 126*a^8*b^10*x^10 + 126*a^9*b^9*x^8 + 84*a^10*b^8*x^6
+ 36*a^11*b^7*x^4 + 9*a^12*b^6*x^2 + a^13*b^5) + 35/65536*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^4*b^5)
```

mupad [B] time = 4.77, size = 205, normalized size = 1.02

$$\frac{35\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{9/2}\,b^{11/2}}-\frac{\frac{x^9}{18\,b}-\frac{1289\,x^{11}}{32768\,a}+\frac{1289\,a\,x^7}{32768\,b^2}+\frac{35\,a^4\,x}{65536\,b^5}-\frac{581\,b\,x^{13}}{32768\,a^2}+\frac{581\,a^2\,x^5}{32768\,b^3}+\frac{455\,a^3\,x^3}{98304\,b^4}-\frac{455\,b^2\,x^{15}}{98304\,a^3}-\frac{35\,b^3\,x^{17}}{65536\,a^4}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^10,x)
```

```
[Out] (35*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(9/2)*b^(11/2)) - (x^9/(18*b) - (12
89*x^11)/(32768*a) + (1289*a*x^7)/(32768*b^2) + (35*a^4*x)/(65536*b^5) - (5
81*b*x^13)/(32768*a^2) + (581*a^2*x^5)/(32768*b^3) + (455*a^3*x^3)/(98304*b
^4) - (455*b^2*x^15)/(98304*a^3) - (35*b^3*x^17)/(65536*a^4))/(a^9 + b^9*x^
18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5
*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

sympy [A] time = 1.19, size = 291, normalized size = 1.45

$$-\frac{35\sqrt{-\frac{1}{a^9b^{11}}}\log\left(-a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072}+\frac{35\sqrt{-\frac{1}{a^9b^{11}}}\log\left(a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072}+\frac{-315a^8x-2730a^7bx^3-10458a^6b^2x^5-23202a^5b^3x^7-32768a^4b^4x^9+23202a^3b^5x^{11}+10458a^2b^6x^{13}+2730ab^7x^{15}+315b^8x^{17}}{589824a^{13}b^5+5308416a^{12}b^6x^2+21233664a^{11}b^7x^4+49545216a^{10}b^8x^6+74317824a^9b^9x^8+74317824a^8b^{10}x^{10}+49545216a^7b^{11}x^{12}+21233664a^6b^{12}x^{14}+5308416a^5b^{13}x^{16}+589824a^4b^{14}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**10,x)
```

```
[Out] -35*sqrt(-1/(a**9*b**11))*log(-a**5*b**5*sqrt(-1/(a**9*b**11)) + x)/131072
+ 35*sqrt(-1/(a**9*b**11))*log(a**5*b**5*sqrt(-1/(a**9*b**11)) + x)/131072
+ (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b**3*
x**7 - 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x**13
 + 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**13*b**5 + 5308416*a**12*b
**6*x**2 + 21233664*a**11*b**7*x**4 + 49545216*a**10*b**8*x**6 + 74317824*a
**9*b**9*x**8 + 74317824*a**8*b**10*x**10 + 49545216*a**7*b**11*x**12 + 212
33664*a**6*b**12*x**14 + 5308416*a**5*b**13*x**16 + 589824*a**4*b**14*x**18
)
```

3.217 $\int \frac{x^8}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=202

$$\frac{35\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{11/2}b^{9/2}}+\frac{35x}{65536a^5b^4\left(a+bx^2\right)}+\frac{35x}{98304a^4b^4\left(a+bx^2\right)^2}+\frac{7x}{24576a^3b^4\left(a+bx^2\right)^3}+\frac{x}{4096a^2b^4\left(a+bx^2\right)^4}+$$ 46

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{35x}{65536a^5b^4\left(a+bx^2\right)}+\frac{35x}{98304a^4b^4\left(a+bx^2\right)^2}+\frac{7x}{24576a^3b^4\left(a+bx^2\right)^3}+\frac{x}{4096a^2b^4\left(a+bx^2\right)^4}+\frac{35\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{11/2}b^{9/2}}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}-\frac{5x^3}{576b^3\left(a+bx^2\right)^7}+\frac{x}{4608ab^4\left(a+bx^2\right)^5}-\frac{5x}{2304b^4\left(a+bx^2\right)^6}-\frac{x^7}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^10,x]
```

```
[Out] -x^7/(18*b*(a + b*x^2)^9) - (7*x^5)/(288*b^2*(a + b*x^2)^8) - (5*x^3)/(576*
b^3*(a + b*x^2)^7) - (5*x)/(2304*b^4*(a + b*x^2)^6) + x/(4608*a*b^4*(a + b*
x^2)^5) + x/(4096*a^2*b^4*(a + b*x^2)^4) + (7*x)/(24576*a^3*b^4*(a + b*x^2)
^3) + (35*x)/(98304*a^4*b^4*(a + b*x^2)^2) + (35*x)/(65536*a^5*b^4*(a + b*x
^2)) + (35*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(11/2)*b^(9/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^7}{18b\left(a+bx^2\right)^9}+\frac{7\int \frac{x^6}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}+\frac{35\int \frac{x^4}{\left(a+bx^2\right)^8}\,dx}{288b^2}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}-\frac{5x^3}{576b^3\left(a+bx^2\right)^7}+\frac{5\int \frac{x^2}{\left(a+bx^2\right)^7}\,dx}{192b^3}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}-\frac{5x^3}{576b^3\left(a+bx^2\right)^7}-\frac{5x}{2304b^4\left(a+bx^2\right)^6}+\frac{5\int \frac{1}{\left(a+bx^2\right)^6}\,d}{2304b^4}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}-\frac{5x^3}{576b^3\left(a+bx^2\right)^7}-\frac{5x}{2304b^4\left(a+bx^2\right)^6}+\frac{x}{4608ab^4\left(a\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}-\frac{5x^3}{576b^3\left(a+bx^2\right)^7}-\frac{5x}{2304b^4\left(a+bx^2\right)^6}+\frac{x}{4608ab^4\left(a\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}-\frac{5x^3}{576b^3\left(a+bx^2\right)^7}-\frac{5x}{2304b^4\left(a+bx^2\right)^6}+\frac{x}{4608ab^4\left(a\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}-\frac{5x^3}{576b^3\left(a+bx^2\right)^7}-\frac{5x}{2304b^4\left(a+bx^2\right)^6}+\frac{x}{4608ab^4\left(a\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}-\frac{5x^3}{576b^3\left(a+bx^2\right)^7}-\frac{5x}{2304b^4\left(a+bx^2\right)^6}+\frac{x}{4608ab^4\left(a\right.}$$

$$= -\frac{x^7}{18b\left(a+bx^2\right)^9}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}-\frac{5x^3}{576b^3\left(a+bx^2\right)^7}-\frac{5x}{2304b^4\left(a+bx^2\right)^6}+\frac{x}{4608ab^4\left(a\right.}$$

Mathematica [A]   time = 0.06, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-315a^8-2730a^7bx^2-10458a^6b^2x^4-23202a^5b^3x^6+32768a^4b^4x^8+23202a^3b^5x^{10}+10458a^2b^6x^{12}+2730ab^7x^{14}+315b^8x^{16}\right)}{\left(a+bx^2\right)^9}+315\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{589824a^{11/2}b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-315*a^8 - 2730*a^7*b*x^2 - 10458*a^6*b^2*x^4 - 23202*
a^5*b^3*x^6 + 32768*a^4*b^4*x^8 + 23202*a^3*b^5*x^10 + 10458*a^2*b^6*x^12 +
 2730*a*b^7*x^14 + 315*b^8*x^16))/(a + b*x^2)^9 + 315*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(589824*a^(11/2)*b^(9/2))
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x^2)^10, x]
```

fricas [A]   time = 0.85, size = 654, normalized size = 3.24

$$\left[\frac{630\,ab^9x^{17}+5460\,a^2b^8x^{15}+20916\,a^3b^7x^{13}+46404\,a^4b^6x^{11}+65536\,a^5b^5x^9-46404\,a^6b^4x^7-20916\,a^7b^3x^5-5460\,a^8b^2x^3-630\,a^9bx-315\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{1179648\left(a^6b^{14}x^{18}+9\,a^7b^{13}x^{16}+36\,a^8b^{12}x^{14}+84\,a^9b^{11}x^{12}+126\,a^{10}b^{10}x^{10}+126\,a^{11}b^9x^8+84\,a^{12}b^8x^6+36\,a^{13}b^7x^4+9\,a^{14}b^6x^2+a^{15}b^5\right)},\frac{315\,ab^9x^{17}+2730\,a^2b^8x^{15}+10458\,a^3b^7x^{13}+23202\,a^4b^6x^{11}+32768\,a^5b^5x^9-23202\,a^6b^4x^7-10458\,a^7b^3x^5-2730\,a^8b^2x^3-315\,a^9bx+315\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{589824\left(a^6b^{14}x^{18}+9\,a^7b^{13}x^{16}+36\,a^8b^{12}x^{14}+84\,a^9b^{11}x^{12}+126\,a^{10}b^{10}x^{10}+126\,a^{11}b^9x^8+84\,a^{12}b^8x^6+36\,a^{13}b^7x^4+9\,a^{14}b^6x^2+a^{15}b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(630*a*b^9*x^17 + 5460*a^2*b^8*x^15 + 20916*a^3*b^7*x^13 + 46404
*a^4*b^6*x^11 + 65536*a^5*b^5*x^9 - 46404*a^6*b^4*x^7 - 20916*a^7*b^3*x^5 -
 5460*a^8*b^2*x^3 - 630*a^9*b*x - 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7
*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x
^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^6*b^14*x^18 + 9*a^7*b^13*x^16 + 36*a^8*b^12*x^14
 + 84*a^9*b^11*x^12 + 126*a^10*b^10*x^10 + 126*a^11*b^9*x^8 + 84*a^12*b^8*x
^6 + 36*a^13*b^7*x^4 + 9*a^14*b^6*x^2 + a^15*b^5), 1/589824*(315*a*b^9*x^17
 + 2730*a^2*b^8*x^15 + 10458*a^3*b^7*x^13 + 23202*a^4*b^6*x^11 + 32768*a^5*
b^5*x^9 - 23202*a^6*b^4*x^7 - 10458*a^7*b^3*x^5 - 2730*a^8*b^2*x^3 - 315*a^
9*b*x + 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 +
126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^
8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^6*b^14*x^18 + 9*a^7*b^13
```

```
*x^16 + 36*a^8*b^12*x^14 + 84*a^9*b^11*x^12 + 126*a^10*b^10*x^10 + 126*a^11
*b^9*x^8 + 84*a^12*b^8*x^6 + 36*a^13*b^7*x^4 + 9*a^14*b^6*x^2 + a^15*b^5)]
```

giac [A] time = 0.60, size = 128, normalized size = 0.63

$$\frac{35 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^5 b^4} + \frac{315 \, b^8 x^{17} + 2730 \, a b^7 x^{15} + 10458 \, a^2 b^6 x^{13} + 23202 \, a^3 b^5 x^{11} + 32768 \, a^4 b^4 x^9 - 23202 \, a^5 b^3 x^7 - 10458 \, a^6 b^2 x^5 - 2730 \, a^7 b x^3 - 315 \, a^8 x}{589824 \, \left(bx^2 + a\right)^9 a^5 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5*b^4) + 1/589824*(315*b^8*x^17
 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 + 32768*a^4*b^
4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)
/((b*x^2 + a)^9*a^5*b^4)
```

maple [A] time = 0.02, size = 122, normalized size = 0.60

$$\frac{35 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\; a^5 b^4} + \frac{\frac{35 b^4 x^{17}}{65536 a^5} + \frac{455 b^3 x^{15}}{98304 a^4} + \frac{581 b^2 x^{13}}{32768 a^3} + \frac{1289 b\, x^{11}}{32768 a^2} + \frac{x^9}{18 a} - \frac{1289 x^7}{32768 b} - \frac{581 a\, x^5}{32768 b^2} - \frac{455 a^2 x^3}{98304 b^3} - \frac{35 a^3 x}{65536 b^4}}{\left(b\, x^2 + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^10,x)
```

```
[Out] (-35/65536*a^3/b^4*x-455/98304*a^2/b^3*x^3-581/32768*a/b^2*x^5-1289/32768/b
*x^7+1/18/a*x^9+1289/32768/a^2*b*x^11+581/32768*b^2/a^3*x^13+455/98304*b^3/
a^4*x^15+35/65536*b^4/a^5*x^17)/(b*x^2+a)^9+35/65536/a^5/b^4/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.08, size = 221, normalized size = 1.09

$$\frac{315 \, b^8 x^{17} + 2730 \, a b^7 x^{15} + 10458 \, a^2 b^6 x^{13} + 23202 \, a^3 b^5 x^{11} + 32768 \, a^4 b^4 x^9 - 23202 \, a^5 b^3 x^7 - 10458 \, a^6 b^2 x^5 - 2730 \, a^7 b x^3 - 315 \, a^8 x}{589824 \, \left(a^5 b^{13} x^{18} + 9 \, a^6 b^{12} x^{16} + 36 \, a^7 b^{11} x^{14} + 84 \, a^8 b^{10} x^{12} + 126 \, a^9 b^9 x^{10} + 126 \, a^{10} b^8 x^8 + 84 \, a^{11} b^7 x^6 + 36 \, a^{12} b^6 x^4 + 9 \, a^{13} b^5 x^2 + a^{14} b^4\right)} + \frac{35 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^5 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/589824*(315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b
^5*x^11 + 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*
a^7*b*x^3 - 315*a^8*x)/(a^5*b^13*x^18 + 9*a^6*b^12*x^16 + 36*a^7*b^11*x^14
+ 84*a^8*b^10*x^12 + 126*a^9*b^9*x^10 + 126*a^10*b^8*x^8 + 84*a^11*b^7*x^6
+ 36*a^12*b^6*x^4 + 9*a^13*b^5*x^2 + a^14*b^4) + 35/65536*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^5*b^4)
```

mupad [B] time = 4.74, size = 204, normalized size = 1.01

$$\frac{\frac{x^9}{18\,a}-\frac{1289\,x^7}{32768\,b}-\frac{581\,a\,x^5}{32768\,b^2}-\frac{35\,a^3\,x}{65536\,b^4}+\frac{1289\,b\,x^{11}}{32768\,a^2}-\frac{455\,a^2\,x^3}{98304\,b^3}+\frac{581\,b^2\,x^{13}}{32768\,a^3}+\frac{455\,b^3\,x^{15}}{98304\,a^4}+\frac{35\,b^4\,x^{17}}{65536\,a^5}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{35\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{11/2}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^10,x)
```

```
[Out] (x^9/(18*a) - (1289*x^7)/(32768*b) - (581*a*x^5)/(32768*b^2) - (35*a^3*x)/(
65536*b^4) + (1289*b*x^11)/(32768*a^2) - (455*a^2*x^3)/(98304*b^3) + (581*b
^2*x^13)/(32768*a^3) + (455*b^3*x^15)/(98304*a^4) + (35*b^4*x^17)/(65536*a^
5))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*
b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7
*x^14) + (35*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(11/2)*b^(9/2))
```

sympy [A] time = 1.15, size = 291, normalized size = 1.44

$$-\frac{35\sqrt{-\frac{1}{a^{11}b^9}}\log\left(-a^6b^4\sqrt{-\frac{1}{a^{11}b^9}}+x\right)}{131072}+\frac{35\sqrt{-\frac{1}{a^{11}b^9}}\log\left(a^6b^4\sqrt{-\frac{1}{a^{11}b^9}}+x\right)}{131072}+\frac{-315a^8x-2730a^7bx^3-10458a^6b^2x^5-23202a^5b^3x^7+32768a^4b^4x^9+23202a^3b^5x^{11}+10458a^2b^6x^{13}+2730ab^7x^{15}+315b^8x^{17}}{589824a^{14}b^4+5308416a^{13}b^5x^2+21233664a^{12}b^6x^4+49545216a^{11}b^7x^6+74317824a^{10}b^8x^8+74317824a^9b^9x^{10}+49545216a^8b^{10}x^{12}+21233664a^7b^{11}x^{14}+5308416a^6b^{12}x^{16}+589824a^5b^{13}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**10,x)
```

```
[Out] -35*sqrt(-1/(a**11*b**9))*log(-a**6*b**4*sqrt(-1/(a**11*b**9)) + x)/131072
+ 35*sqrt(-1/(a**11*b**9))*log(a**6*b**4*sqrt(-1/(a**11*b**9)) + x)/131072
+ (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b**3*
x**7 + 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x**13
 + 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**14*b**4 + 5308416*a**13*b
**5*x**2 + 21233664*a**12*b**6*x**4 + 49545216*a**11*b**7*x**6 + 74317824*a
**10*b**8*x**8 + 74317824*a**9*b**9*x**10 + 49545216*a**8*b**10*x**12 + 212
33664*a**7*b**11*x**14 + 5308416*a**6*b**12*x**16 + 589824*a**5*b**13*x**18
)
```

3.218 $\int \frac{x^6}{\left(a+bx^2\right)^{10}} \, dx$

Optimal. Leaf size=203

$$\frac{55\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{13/2}b^{7/2}}+\frac{55x}{65536a^6b^3\left(a+bx^2\right)}+\frac{55x}{98304a^5b^3\left(a+bx^2\right)^2}+\frac{11x}{24576a^4b^3\left(a+bx^2\right)^3}+\frac{11x}{28672a^3b^3\left(a+bx^2\right)^4}+\frac{}{32}$$

Rubi [A] time = 0.11, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{55x}{65536a^6b^3\left(a+bx^2\right)}+\frac{55x}{98304a^5b^3\left(a+bx^2\right)^2}+\frac{11x}{24576a^4b^3\left(a+bx^2\right)^3}+\frac{11x}{28672a^3b^3\left(a+bx^2\right)^4}+\frac{11x}{32256a^2b^3\left(a+bx^2\right)^5}+\frac{55\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{13/2}b^{7/2}}-\frac{5x^3}{288b^2\left(a+bx^2\right)^8}+\frac{5x}{16128ab^3\left(a+bx^2\right)^6}-\frac{5x}{1344b^3\left(a+bx^2\right)^7}-\frac{x^5}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^10,x]
```

```
[Out] -x^5/(18*b*(a + b*x^2)^9) - (5*x^3)/(288*b^2*(a + b*x^2)^8) - (5*x)/(1344*b
^3*(a + b*x^2)^7) + (5*x)/(16128*a*b^3*(a + b*x^2)^6) + (11*x)/(32256*a^2*b
^3*(a + b*x^2)^5) + (11*x)/(28672*a^3*b^3*(a + b*x^2)^4) + (11*x)/(24576*a^
4*b^3*(a + b*x^2)^3) + (55*x)/(98304*a^5*b^3*(a + b*x^2)^2) + (55*x)/(65536
*a^6*b^3*(a + b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(13/2)*b^
(7/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^5}{18b\left(a+bx^2\right)^9} + \frac{5\int \frac{x^4}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9} - \frac{5x^3}{288b^2\left(a+bx^2\right)^8} + \frac{5\int \frac{x^2}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9} - \frac{5x^3}{288b^2\left(a+bx^2\right)^8} - \frac{5x}{1344b^3\left(a+bx^2\right)^7} + \frac{5\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344b^3}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9} - \frac{5x^3}{288b^2\left(a+bx^2\right)^8} - \frac{5x}{1344b^3\left(a+bx^2\right)^7} + \frac{5x}{16128ab^3\left(a+bx^2\right)^6} + \frac{55\int \frac{1}{\left(a+b\right.}}{16128}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9} - \frac{5x^3}{288b^2\left(a+bx^2\right)^8} - \frac{5x}{1344b^3\left(a+bx^2\right)^7} + \frac{5x}{16128ab^3\left(a+bx^2\right)^6} + \frac{}{32256a^2}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9} - \frac{5x^3}{288b^2\left(a+bx^2\right)^8} - \frac{5x}{1344b^3\left(a+bx^2\right)^7} + \frac{5x}{16128ab^3\left(a+bx^2\right)^6} + \frac{}{32256a^2}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9} - \frac{5x^3}{288b^2\left(a+bx^2\right)^8} - \frac{5x}{1344b^3\left(a+bx^2\right)^7} + \frac{5x}{16128ab^3\left(a+bx^2\right)^6} + \frac{}{32256a^2}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9} - \frac{5x^3}{288b^2\left(a+bx^2\right)^8} - \frac{5x}{1344b^3\left(a+bx^2\right)^7} + \frac{5x}{16128ab^3\left(a+bx^2\right)^6} + \frac{}{32256a^2}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9} - \frac{5x^3}{288b^2\left(a+bx^2\right)^8} - \frac{5x}{1344b^3\left(a+bx^2\right)^7} + \frac{5x}{16128ab^3\left(a+bx^2\right)^6} + \frac{}{32256a^2}$$

$$= -\frac{x^5}{18b\left(a+bx^2\right)^9} - \frac{5x^3}{288b^2\left(a+bx^2\right)^8} - \frac{5x}{1344b^3\left(a+bx^2\right)^7} + \frac{5x}{16128ab^3\left(a+bx^2\right)^6} + \frac{}{32256a^2}$$

Mathematica [A] time = 0.07, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-3465a^8-30030a^7bx^2-115038a^6b^2x^4+334602a^5b^3x^6+360448a^4b^4x^8+255222a^3b^5x^{10}+115038a^2b^6x^{12}+30030ab^7x^{14}+3465b^8x^{16}\right)}{\left(a+bx^2\right)^9}+3465\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{4128768a^{13/2}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-3465*a^8 - 30030*a^7*b*x^2 - 115038*a^6*b^2*x^4 + 334
602*a^5*b^3*x^6 + 360448*a^4*b^4*x^8 + 255222*a^3*b^5*x^10 + 115038*a^2*b^6
*x^12 + 30030*a*b^7*x^14 + 3465*b^8*x^16))/(a + b*x^2)^9 + 3465*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(4128768*a^(13/2)*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x^2)^10, x]
```

fricas [A] time = 0.56, size = 654, normalized size = 3.22

$$\left[\frac{6930\,ab^9x^{17}+60060\,a^2b^8x^{15}+230076\,a^3b^7x^{13}+510444\,a^4b^6x^{11}+720896\,a^5b^5x^9+669204\,a^6b^4x^7-230076\,a^7b^3x^5-60060\,a^8b^2x^3-6930\,a^9bx-3465\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{8257536\left(a^7b^{13}x^{18}+9\,a^8b^{12}x^{16}+36\,a^9b^{11}x^{14}+84\,a^{10}b^{10}x^{12}+126\,a^{11}b^9x^{10}+126\,a^{12}b^8x^8+84\,a^{13}b^7x^6+36\,a^{14}b^6x^4+9\,a^{15}b^5x^2+a^{16}b^4\right)},\ \frac{3465\,ab^9x^{17}+30030\,a^2b^8x^{15}+115038\,a^3b^7x^{13}+255222\,a^4b^6x^{11}+360448\,a^5b^5x^9+334602\,a^6b^4x^7-115038\,a^7b^3x^5-30030\,a^8b^2x^3-3465\,a^9bx+3465\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{4128768\left(a^7b^{13}x^{18}+9\,a^8b^{12}x^{16}+36\,a^9b^{11}x^{14}+84\,a^{10}b^{10}x^{12}+126\,a^{11}b^9x^{10}+126\,a^{12}b^8x^8+84\,a^{13}b^7x^6+36\,a^{14}b^6x^4+9\,a^{15}b^5x^2+a^{16}b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(6930*a*b^9*x^17 + 60060*a^2*b^8*x^15 + 230076*a^3*b^7*x^13 + 51
0444*a^4*b^6*x^11 + 720896*a^5*b^5*x^9 + 669204*a^6*b^4*x^7 - 230076*a^7*b^
3*x^5 - 60060*a^8*b^2*x^3 - 6930*a^9*b*x - 3465*(b^9*x^18 + 9*a*b^8*x^16 +
36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84
*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^7*b^13*x^18 + 9*a^8*b^12*x^16 + 36*a^9
*b^11*x^14 + 84*a^10*b^10*x^12 + 126*a^11*b^9*x^10 + 126*a^12*b^8*x^8 + 84*
a^13*b^7*x^6 + 36*a^14*b^6*x^4 + 9*a^15*b^5*x^2 + a^16*b^4), 1/4128768*(346
5*a*b^9*x^17 + 30030*a^2*b^8*x^15 + 115038*a^3*b^7*x^13 + 255222*a^4*b^6*x^
11 + 360448*a^5*b^5*x^9 + 334602*a^6*b^4*x^7 - 115038*a^7*b^3*x^5 - 30030*a
^8*b^2*x^3 - 3465*a^9*b*x + 3465*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14
 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 +
36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^7*b
```

```
^13*x^18 + 9*a^8*b^12*x^16 + 36*a^9*b^11*x^14 + 84*a^10*b^10*x^12 + 126*a^1
1*b^9*x^10 + 126*a^12*b^8*x^8 + 84*a^13*b^7*x^6 + 36*a^14*b^6*x^4 + 9*a^15*
b^5*x^2 + a^16*b^4)]
```

giac [A] time = 0.61, size = 128, normalized size = 0.63

$$\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^6b^3}+\frac{3465\,b^8x^{17}+30030\,ab^7x^{15}+115038\,a^2b^6x^{13}+255222\,a^3b^5x^{11}+360448\,a^4b^4x^9+334602\,a^5b^3x^7-115038\,a^6b^2x^5-30030\,a^7bx^3-3465\,a^8x}{4128768\left(bx^2+a\right)^9a^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6*b^3) + 1/4128768*(3465*b^8*x^
17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 + 255222*a^3*b^5*x^11 + 360448*
a^4*b^4*x^9 + 334602*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3 - 3
465*a^8*x)/((b*x^2 + a)^9*a^6*b^3)
```

maple [A] time = 0.02, size = 122, normalized size = 0.60

$$\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\;a^6b^3}+\frac{\frac{55b^5x^{17}}{65536a^6}+\frac{715b^4x^{15}}{98304a^5}+\frac{913b^3x^{13}}{32768a^4}+\frac{14179b^2x^{11}}{229376a^3}+\frac{11b\,x^9}{126a^2}+\frac{18589x^7}{229376a}-\frac{913x^5}{32768b}-\frac{715a\,x^3}{98304b^2}-\frac{55a^2x}{65536b^3}}{\left(b\,x^2+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^10,x)
```

```
[Out] (-55/65536*a^2/b^3*x-715/98304*a/b^2*x^3-913/32768/b*x^5+18589/229376/a*x^7
+11/126/a^2*b*x^9+14179/229376*b^2/a^3*x^11+913/32768*b^3/a^4*x^13+715/9830
4*b^4/a^5*x^15+55/65536/a^6*b^5*x^17)/(b*x^2+a)^9+55/65536/a^6/b^3/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.14, size = 221, normalized size = 1.09

$$\frac{3465\,b^8x^{17}+30030\,ab^7x^{15}+115038\,a^2b^6x^{13}+255222\,a^3b^5x^{11}+360448\,a^4b^4x^9+334602\,a^5b^3x^7-115038\,a^6b^2x^5-30030\,a^7bx^3-3465\,a^8x}{4128768\left(a^6b^{12}x^{18}+9\,a^7b^{11}x^{16}+36\,a^8b^{10}x^{14}+84\,a^9b^9x^{12}+126\,a^{10}b^8x^{10}+126\,a^{11}b^7x^8+84\,a^{12}b^6x^6+36\,a^{13}b^5x^4+9\,a^{14}b^4x^2+a^{15}b^3\right)}+\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(3465*b^8*x^17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 + 255222*
a^3*b^5*x^11 + 360448*a^4*b^4*x^9 + 334602*a^5*b^3*x^7 - 115038*a^6*b^2*x^5
 - 30030*a^7*b*x^3 - 3465*a^8*x)/(a^6*b^12*x^18 + 9*a^7*b^11*x^16 + 36*a^8*
b^10*x^14 + 84*a^9*b^9*x^12 + 126*a^10*b^8*x^10 + 126*a^11*b^7*x^8 + 84*a^1
2*b^6*x^6 + 36*a^13*b^5*x^4 + 9*a^14*b^4*x^2 + a^15*b^3) + 55/65536*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^6*b^3)
```

mupad [B] time = 4.79, size = 204, normalized size = 1.00

$$\frac{\frac{18589\,x^7}{229376\,a}-\frac{913\,x^5}{32768\,b}-\frac{715\,a\,x^3}{98304\,b^2}-\frac{55\,a^2\,x}{65536\,b^3}+\frac{11\,b\,x^9}{126\,a^2}+\frac{14179\,b^2\,x^{11}}{229376\,a^3}+\frac{913\,b^3\,x^{13}}{32768\,a^4}+\frac{715\,b^4\,x^{15}}{98304\,a^5}+\frac{55\,b^5\,x^{17}}{65536\,a^6}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{55\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{13/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^10,x)
```

```
[Out] ((18589*x^7)/(229376*a) - (913*x^5)/(32768*b) - (715*a*x^3)/(98304*b^2) - (
55*a^2*x)/(65536*b^3) + (11*b*x^9)/(126*a^2) + (14179*b^2*x^11)/(229376*a^3
) + (913*b^3*x^13)/(32768*a^4) + (715*b^4*x^15)/(98304*a^5) + (55*b^5*x^17)
/(65536*a^6))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4
 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 +
36*a^2*b^7*x^14) + (55*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(13/2)*b^(7/2))
```

sympy [A] time = 1.08, size = 291, normalized size = 1.43

$$-\frac{55\sqrt{-\frac{1}{a^{13}b^7}}\log\left(-a^7b^3\sqrt{-\frac{1}{a^{13}b^7}}+x\right)}{131072}+\frac{55\sqrt{-\frac{1}{a^{13}b^7}}\log\left(a^7b^3\sqrt{-\frac{1}{a^{13}b^7}}+x\right)}{131072}+\frac{-3465a^8x-30030a^7bx^3-115038a^6b^2x^5+334602a^5b^3x^7+360448a^4b^4x^9+255222a^3b^5x^{11}+115038a^2b^6x^{13}+30030ab^7x^{15}+3465b^8x^{17}}{4128768a^{15}b^3+37158912a^{14}b^4x^2+148635648a^{13}b^5x^4+346816512a^{12}b^6x^6+520224768a^{11}b^7x^8+520224768a^{10}b^8x^{10}+346816512a^9b^9x^{12}+148635648a^8b^{10}x^{14}+37158912a^7b^{11}x^{16}+4128768a^6b^{12}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**10,x)
```

```
[Out] -55*sqrt(-1/(a**13*b**7))*log(-a**7*b**3*sqrt(-1/(a**13*b**7)) + x)/131072
+ 55*sqrt(-1/(a**13*b**7))*log(a**7*b**3*sqrt(-1/(a**13*b**7)) + x)/131072
+ (-3465*a**8*x - 30030*a**7*b*x**3 - 115038*a**6*b**2*x**5 + 334602*a**5*b
**3*x**7 + 360448*a**4*b**4*x**9 + 255222*a**3*b**5*x**11 + 115038*a**2*b**
6*x**13 + 30030*a*b**7*x**15 + 3465*b**8*x**17)/(4128768*a**15*b**3 + 37158
912*a**14*b**4*x**2 + 148635648*a**13*b**5*x**4 + 346816512*a**12*b**6*x**6
 + 520224768*a**11*b**7*x**8 + 520224768*a**10*b**8*x**10 + 346816512*a**9*
b**9*x**12 + 148635648*a**8*b**10*x**14 + 37158912*a**7*b**11*x**16 + 41287
68*a**6*b**12*x**18)
```

3.219 $\int \frac{x^4}{(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=204

$$\frac{143\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{15/2}b^{5/2}}+\frac{143x}{65536a^7b^2\left(a+bx^2\right)}+\frac{143x}{98304a^6b^2\left(a+bx^2\right)^2}+\frac{143x}{122880a^5b^2\left(a+bx^2\right)^3}+\frac{143x}{143360a^4b^2\left(a+bx^2\right)^4}+$$

Rubi [A] time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {288, 199, 205}

$$\frac{143x}{65536a^7b^2\left(a+bx^2\right)}+\frac{143x}{98304a^6b^2\left(a+bx^2\right)^2}+\frac{143x}{122880a^5b^2\left(a+bx^2\right)^3}+\frac{143x}{143360a^4b^2\left(a+bx^2\right)^4}+\frac{143x}{161280a^3b^2\left(a+bx^2\right)^5}+\frac{13x}{16128a^2b^2\left(a+bx^2\right)^6}+\frac{143\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{15/2}b^{5/2}}+\frac{x}{1344ab^2\left(a+bx^2\right)^7}-\frac{x}{96b^2\left(a+bx^2\right)^8}-\frac{x^3}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^10,x]
```

```
[Out] -x^3/(18*b*(a + b*x^2)^9) - x/(96*b^2*(a + b*x^2)^8) + x/(1344*a*b^2*(a + b
*x^2)^7) + (13*x)/(16128*a^2*b^2*(a + b*x^2)^6) + (143*x)/(161280*a^3*b^2*(
a + b*x^2)^5) + (143*x)/(143360*a^4*b^2*(a + b*x^2)^4) + (143*x)/(122880*a^
5*b^2*(a + b*x^2)^3) + (143*x)/(98304*a^6*b^2*(a + b*x^2)^2) + (143*x)/(655
36*a^7*b^2*(a + b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(15/2)
*b^(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x^3}{18b\left(a+bx^2\right)^9} + \frac{\int \frac{x^2}{\left(a+bx^2\right)^9}\,dx}{6b}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{\int \frac{1}{\left(a+bx^2\right)^8}\,dx}{96b^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344ab^2}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{143\int \ldots}{1612\ldots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\ldots}{161280\ldots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\ldots}{161280\ldots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\ldots}{161280\ldots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\ldots}{161280\ldots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\ldots}{161280\ldots}$$

$$= -\frac{x^3}{18b\left(a+bx^2\right)^9} - \frac{x}{96b^2\left(a+bx^2\right)^8} + \frac{x}{1344ab^2\left(a+bx^2\right)^7} + \frac{13x}{16128a^2b^2\left(a+bx^2\right)^6} + \frac{\ldots}{161280\ldots}$$

Mathematica [A] time = 0.06, size = 138, normalized size = 0.68

$$\frac{\frac{\sqrt{a}\sqrt{b}x(-45045a^8-390390a^7bx^2+2633274a^6b^2x^4+4349826a^5b^3x^6+4685824a^4b^4x^8+3317886a^3b^5x^{10}+1495494a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16})}{(a+bx^2)^9}+45045\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{20643840a^{15/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 + 2633274*a^6*b^2*x^4 +
4349826*a^5*b^3*x^6 + 4685824*a^4*b^4*x^8 + 3317886*a^3*b^5*x^10 + 1495494*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(20643840*a^(15/2)*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x^2)^10, x]
```

fricas [A] time = 0.81, size = 654, normalized size = 3.21

$$\left[\frac{90090\,ab^9x^{17}+780780\,a^2b^8x^{15}+2990988\,a^3b^7x^{13}+6635772\,a^4b^6x^{11}+9371648\,a^5b^5x^9+8699652\,a^6b^4x^7+5266548\,a^7b^3x^5-780780\,a^8b^2x^3-90090\,a^9bx-45045\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{41287680\left(a^8b^{12}x^{18}+9\,a^9b^{11}x^{16}+36\,a^{10}b^{10}x^{14}+84\,a^{11}b^9x^{12}+126\,a^{12}b^8x^{10}+126\,a^{13}b^7x^8+84\,a^{14}b^6x^6+36\,a^{15}b^5x^4+9\,a^{16}b^4x^2+a^{17}b^3\right)},\ \frac{45045\,ab^9x^{17}+390390\,a^2b^8x^{15}+1495494\,a^3b^7x^{13}+3317886\,a^4b^6x^{11}+4685824\,a^5b^5x^9+4349826\,a^6b^4x^7+2633274\,a^7b^3x^5-390390\,a^8b^2x^3-45045\,a^9bx+45045\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{20643840\left(a^8b^{12}x^{18}+9\,a^9b^{11}x^{16}+36\,a^{10}b^{10}x^{14}+84\,a^{11}b^9x^{12}+126\,a^{12}b^8x^{10}+126\,a^{13}b^7x^8+84\,a^{14}b^6x^6+36\,a^{15}b^5x^4+9\,a^{16}b^4x^2+a^{17}b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/41287680*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 + 2990988*a^3*b^7*x^13
+ 6635772*a^4*b^6*x^11 + 9371648*a^5*b^5*x^9 + 8699652*a^6*b^4*x^7 + 526654
8*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^8*b^12*x^18 + 9*a^9*b^11*x^
16 + 36*a^10*b^10*x^14 + 84*a^11*b^9*x^12 + 126*a^12*b^8*x^10 + 126*a^13*b^
7*x^8 + 84*a^14*b^6*x^6 + 36*a^15*b^5*x^4 + 9*a^16*b^4*x^2 + a^17*b^3), 1/2
0643840*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 + 1495494*a^3*b^7*x^13 + 33
17886*a^4*b^6*x^11 + 4685824*a^5*b^5*x^9 + 4349826*a^6*b^4*x^7 + 2633274*a^
7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*
x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x
^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(
```

```
sqrt(a*b)*x/a))/(a^8*b^12*x^18 + 9*a^9*b^11*x^16 + 36*a^10*b^10*x^14 + 84*a
^11*b^9*x^12 + 126*a^12*b^8*x^10 + 126*a^13*b^7*x^8 + 84*a^14*b^6*x^6 + 36*
a^15*b^5*x^4 + 9*a^16*b^4*x^2 + a^17*b^3)]
```

giac [A] time = 0.63, size = 128, normalized size = 0.63

$$\frac{143 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^7 b^2} + \frac{45045\, b^8 x^{17} + 390390\, a b^7 x^{15} + 1495494\, a^2 b^6 x^{13} + 3317886\, a^3 b^5 x^{11} + 4685824\, a^4 b^4 x^9 + 4349826\, a^5 b^3 x^7 + 2633274\, a^6 b^2 x^5 - 390390\, a^7 b x^3 - 45045\, a^8 x}{20643840 \left(b x^2 + a\right)^9 a^7 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 143/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7*b^2) + 1/20643840*(45045*b^8
*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 + 4
685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 - 390390*a^7
*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^7*b^2)
```

maple [A] time = 0.02, size = 122, normalized size = 0.60

$$\frac{143 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\; a^7 b^2} + \frac{\frac{143 b^6 x^{17}}{65536 a^7} + \frac{1859 b^5 x^{15}}{98304 a^6} + \frac{11869 b^4 x^{13}}{163840 a^5} + \frac{184327 b^3 x^{11}}{1146880 a^4} + \frac{143 b^2 x^9}{630 a^3} + \frac{241657 b\, x^7}{1146880 a^2} + \frac{20899 x^5}{163840 a} - \frac{1859 x^3}{98304 b} - \frac{143 a x}{65536 b^2}}{\left(b\, x^2 + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^10,x)
```

```
[Out] (-143/65536*a/b^2*x-1859/98304/b*x^3+20899/163840/a*x^5+241657/1146880/a^2*
b*x^7+143/630*b^2/a^3*x^9+184327/1146880*b^3/a^4*x^11+11869/163840*b^4/a^5*
x^13+1859/98304/a^6*b^5*x^15+143/65536/a^7*b^6*x^17)/(b*x^2+a)^9+143/65536/
a^7/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.19, size = 221, normalized size = 1.08

$$\frac{45045\, b^8 x^{17} + 390390\, a b^7 x^{15} + 1495494\, a^2 b^6 x^{13} + 3317886\, a^3 b^5 x^{11} + 4685824\, a^4 b^4 x^9 + 4349826\, a^5 b^3 x^7 + 2633274\, a^6 b^2 x^5 - 390390\, a^7 b x^3 - 45045\, a^8 x}{20643840 \left(a^7 b^{11} x^{18} + 9\, a^8 b^{10} x^{16} + 36\, a^9 b^9 x^{14} + 84\, a^{10} b^8 x^{12} + 126\, a^{11} b^7 x^{10} + 126\, a^{12} b^6 x^8 + 84\, a^{13} b^5 x^6 + 36\, a^{14} b^4 x^4 + 9\, a^{15} b^3 x^2 + a^{16} b^2\right)} + \frac{143 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^7 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20643840*(45045*b^8*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 331
7886*a^3*b^5*x^11 + 4685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6
*b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a^7*b^11*x^18 + 9*a^8*b^10*x^16
 + 36*a^9*b^9*x^14 + 84*a^10*b^8*x^12 + 126*a^11*b^7*x^10 + 126*a^12*b^6*x^
8 + 84*a^13*b^5*x^6 + 36*a^14*b^4*x^4 + 9*a^15*b^3*x^2 + a^16*b^2) + 143/65
536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7*b^2)
```

mupad [B] time = 4.74, size = 204, normalized size = 1.00

$$\frac{\frac{20899\,x^5}{163840\,a}-\frac{1859\,x^3}{98304\,b}+\frac{241657\,b\,x^7}{1146880\,a^2}+\frac{143\,b^2\,x^9}{630\,a^3}+\frac{184327\,b^3\,x^{11}}{1146880\,a^4}+\frac{11869\,b^4\,x^{13}}{163840\,a^5}+\frac{1859\,b^5\,x^{15}}{98304\,a^6}+\frac{143\,b^6\,x^{17}}{65536\,a^7}-\frac{143\,a\,x}{65536\,b^2}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{143\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{15/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^10,x)
```

```
[Out] ((20899*x^5)/(163840*a) - (1859*x^3)/(98304*b) + (241657*b*x^7)/(1146880*a^
2) + (143*b^2*x^9)/(630*a^3) + (184327*b^3*x^11)/(1146880*a^4) + (11869*b^4
*x^13)/(163840*a^5) + (1859*b^5*x^15)/(98304*a^6) + (143*b^6*x^17)/(65536*a
^7) - (143*a*x)/(65536*b^2))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 +
 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*
a^3*b^6*x^12 + 36*a^2*b^7*x^14) + (143*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^
(15/2)*b^(5/2))
```

sympy [A] time = 1.04, size = 291, normalized size = 1.43

$$-\frac{143\sqrt{-\frac{1}{a^{15}b^5}}\log\left(-a^8b^2\sqrt{-\frac{1}{a^{15}b^5}}+x\right)}{131072}+\frac{143\sqrt{-\frac{1}{a^{15}b^5}}\log\left(a^8b^2\sqrt{-\frac{1}{a^{15}b^5}}+x\right)}{131072}+\frac{-45045a^8x-390390a^7bx^3+2633274a^6b^2x^5+4349826a^5b^3x^7+4685824a^4b^4x^9+3317886a^3b^5x^{11}+1495494a^2b^6x^{13}+390390ab^7x^{15}+45045b^8x^{17}}{20643840a^{16}b^2+185794560a^{15}b^3x^2+743178240a^{14}b^4x^4+1734082560a^{13}b^5x^6+2601123840a^{12}b^6x^8+2601123840a^{11}b^7x^{10}+1734082560a^{10}b^8x^{12}+743178240a^9b^9x^{14}+185794560a^8b^{10}x^{16}+20643840a^7b^{11}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**10,x)
```

```
[Out] -143*sqrt(-1/(a**15*b**5))*log(-a**8*b**2*sqrt(-1/(a**15*b**5)) + x)/131072
 + 143*sqrt(-1/(a**15*b**5))*log(a**8*b**2*sqrt(-1/(a**15*b**5)) + x)/13107
2 + (-45045*a**8*x - 390390*a**7*b*x**3 + 2633274*a**6*b**2*x**5 + 4349826*
a**5*b**3*x**7 + 4685824*a**4*b**4*x**9 + 3317886*a**3*b**5*x**11 + 1495494
*a**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(20643840*a**16*
b**2 + 185794560*a**15*b**3*x**2 + 743178240*a**14*b**4*x**4 + 1734082560*a
**13*b**5*x**6 + 2601123840*a**12*b**6*x**8 + 2601123840*a**11*b**7*x**10 +
 1734082560*a**10*b**8*x**12 + 743178240*a**9*b**9*x**14 + 185794560*a**8*b
**10*x**16 + 20643840*a**7*b**11*x**18)
```

3.220 $\int \frac{x^2}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=205

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}}+\frac{715x}{65536a^8b\left(a+bx^2\right)}+\frac{715x}{98304a^7b\left(a+bx^2\right)^2}+\frac{143x}{24576a^6b\left(a+bx^2\right)^3}+\frac{143x}{28672a^5b\left(a+bx^2\right)^4}+\frac{}{3225}$$

Rubi [A] time = 0.11, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {288, 199, 205}

$$\frac{715\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}}+\frac{715x}{65536a^8b\left(a+bx^2\right)}+\frac{715x}{98304a^7b\left(a+bx^2\right)^2}+\frac{143x}{24576a^6b\left(a+bx^2\right)^3}+\frac{143x}{28672a^5b\left(a+bx^2\right)^4}+\frac{143x}{32256a^4b\left(a+bx^2\right)^5}+\frac{65x}{16128a^3b\left(a+bx^2\right)^6}+\frac{5x}{1344a^2b\left(a+bx^2\right)^7}+\frac{x}{288ab\left(a+bx^2\right)^8}-\frac{x}{18b\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^10,x]
```

```
[Out] -x/(18*b*(a + b*x^2)^9) + x/(288*a*b*(a + b*x^2)^8) + (5*x)/(1344*a^2*b*(a
+ b*x^2)^7) + (65*x)/(16128*a^3*b*(a + b*x^2)^6) + (143*x)/(32256*a^4*b*(a
+ b*x^2)^5) + (143*x)/(28672*a^5*b*(a + b*x^2)^4) + (143*x)/(24576*a^6*b*(a
 + b*x^2)^3) + (715*x)/(98304*a^7*b*(a + b*x^2)^2) + (715*x)/(65536*a^8*b*(
a + b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(17/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{10}}\,dx = -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{\int \frac{1}{\left(a+bx^2\right)^9}\,dx}{18b}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5\int \frac{1}{\left(a+bx^2\right)^8}\,dx}{96ab}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344a^2b}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{715\int}{16}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{}{3225}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{}{3225}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{}{3225}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{}{3225}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{}{3225}$$

$$= -\frac{x}{18b\left(a+bx^2\right)^9} + \frac{x}{288ab\left(a+bx^2\right)^8} + \frac{5x}{1344a^2b\left(a+bx^2\right)^7} + \frac{65x}{16128a^3b\left(a+bx^2\right)^6} + \frac{}{3225}$$

Mathematica [A] time = 0.06, size = 138, normalized size = 0.67

$$\frac{\frac{\sqrt{a}\sqrt{b}x\left(-45045a^8+985866a^7bx^2+2633274a^6b^2x^4+4349826a^5b^3x^6+4685824a^4b^4x^8+3317886a^3b^5x^{10}+1495494a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16}\right)}{\left(a+bx^2\right)^9}+45045\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{4128768a^{17/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^10,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 + 985866*a^7*b*x^2 + 2633274*a^6*b^2*x^4 +
4349826*a^5*b^3*x^6 + 4685824*a^4*b^4*x^8 + 3317886*a^3*b^5*x^10 + 1495494*
a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 45045*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(17/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^2)^10,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x^2)^10, x]
```

fricas [A] time = 0.59, size = 654, normalized size = 3.19

$$\left[\frac{90090\,ab^9x^{17}+780780\,a^2b^8x^{15}+2990988\,a^3b^7x^{13}+6635772\,a^4b^6x^{11}+9371648\,a^5b^5x^9+8699652\,a^6b^4x^7+5266548\,a^7b^3x^5+1971732\,a^8b^2x^3-90090\,a^9bx-45045\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{8257536\left(a^9b^{11}x^{18}+9\,a^{10}b^{10}x^{16}+36\,a^{11}b^9x^{14}+84\,a^{12}b^8x^{12}+126\,a^{13}b^7x^{10}+126\,a^{14}b^6x^8+84\,a^{15}b^5x^6+36\,a^{16}b^4x^4+9\,a^{17}b^3x^2+a^{18}b^2\right)},\ \frac{45045\,ab^9x^{17}+390390\,a^2b^8x^{15}+1495494\,a^3b^7x^{13}+3317886\,a^4b^6x^{11}+4685824\,a^5b^5x^9+4349826\,a^6b^4x^7+2633274\,a^7b^3x^5+985866\,a^8b^2x^3-45045\,a^9bx+45045\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{4128768\left(a^9b^{11}x^{18}+9\,a^{10}b^{10}x^{16}+36\,a^{11}b^9x^{14}+84\,a^{12}b^8x^{12}+126\,a^{13}b^7x^{10}+126\,a^{14}b^6x^8+84\,a^{15}b^5x^6+36\,a^{16}b^4x^4+9\,a^{17}b^3x^2+a^{18}b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 + 2990988*a^3*b^7*x^13 +
 6635772*a^4*b^6*x^11 + 9371648*a^5*b^5*x^9 + 8699652*a^6*b^4*x^7 + 5266548
*a^7*b^3*x^5 + 1971732*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9*a*
b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b
^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^9*b^11*x^18 + 9*a^10*b^10*x
^16 + 36*a^11*b^9*x^14 + 84*a^12*b^8*x^12 + 126*a^13*b^7*x^10 + 126*a^14*b^
6*x^8 + 84*a^15*b^5*x^6 + 36*a^16*b^4*x^4 + 9*a^17*b^3*x^2 + a^18*b^2), 1/4
128768*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 + 1495494*a^3*b^7*x^13 + 331
7886*a^4*b^6*x^11 + 4685824*a^5*b^5*x^9 + 4349826*a^6*b^4*x^7 + 2633274*a^7
*b^3*x^5 + 985866*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18 + 9*a*b^8*x
^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^
8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(s
```

```
qrt(a*b)*x/a))/(a^9*b^11*x^18 + 9*a^10*b^10*x^16 + 36*a^11*b^9*x^14 + 84*a^
12*b^8*x^12 + 126*a^13*b^7*x^10 + 126*a^14*b^6*x^8 + 84*a^15*b^5*x^6 + 36*a
^16*b^4*x^4 + 9*a^17*b^3*x^2 + a^18*b^2)]
```

giac [A] time = 0.62, size = 128, normalized size = 0.62

$$\frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^8 b} + \frac{45045\, b^8 x^{17} + 390390\, a b^7 x^{15} + 1495494\, a^2 b^6 x^{13} + 3317886\, a^3 b^5 x^{11} + 4685824\, a^4 b^4 x^9 + 4349826\, a^5 b^3 x^7 + 2633274\, a^6 b^2 x^5 + 985866\, a^7 b x^3 - 45045\, a^8 x}{4128768 \left(bx^2 + a\right)^9 a^8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8*b) + 1/4128768*(45045*b^8*x^
17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 + 4685
824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 + 985866*a^7*b*
x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^8*b)
```

maple [A] time = 0.02, size = 124, normalized size = 0.60

$$\frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\; a^8 b} + \frac{\frac{715 b^7 x^{17}}{65536 a^8} + \frac{9295 b^6 x^{15}}{98304 a^7} + \frac{11869 b^5 x^{13}}{32768 a^6} + \frac{184327 b^4 x^{11}}{229376 a^5} + \frac{143 b^3 x^9}{126 a^4} + \frac{241657 b^2 x^7}{229376 a^3} + \frac{20899 b\, x^5}{32768 a^2} + \frac{23473 x^3}{98304 a} - \frac{715 x}{65536 b}}{\left(b\, x^2 + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^10,x)
```

```
[Out] (-715/65536/b*x+23473/98304/a*x^3+20899/32768/a^2*b*x^5+241657/229376*b^2/a
^3*x^7+143/126*b^3/a^4*x^9+184327/229376*b^4/a^5*x^11+11869/32768/a^6*b^5*x
^13+9295/98304/a^7*b^6*x^15+715/65536/a^8*b^7*x^17)/(b*x^2+a)^9+715/65536/a
^8/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.09, size = 219, normalized size = 1.07

$$\frac{45045\, b^8 x^{17} + 390390\, a b^7 x^{15} + 1495494\, a^2 b^6 x^{13} + 3317886\, a^3 b^5 x^{11} + 4685824\, a^4 b^4 x^9 + 4349826\, a^5 b^3 x^7 + 2633274\, a^6 b^2 x^5 + 985866\, a^7 b x^3 - 45045\, a^8 x}{4128768 \left(a^8 b^{10} x^{18} + 9\, a^9 b^9 x^{16} + 36\, a^{10} b^8 x^{14} + 84\, a^{11} b^7 x^{12} + 126\, a^{12} b^6 x^{10} + 126\, a^{13} b^5 x^8 + 84\, a^{14} b^4 x^6 + 36\, a^{15} b^3 x^4 + 9\, a^{16} b^2 x^2 + a^{17} b\right)} + \frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(45045*b^8*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317
886*a^3*b^5*x^11 + 4685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*
b^2*x^5 + 985866*a^7*b*x^3 - 45045*a^8*x)/(a^8*b^10*x^18 + 9*a^9*b^9*x^16 +
 36*a^10*b^8*x^14 + 84*a^11*b^7*x^12 + 126*a^12*b^6*x^10 + 126*a^13*b^5*x^8
 + 84*a^14*b^4*x^6 + 36*a^15*b^3*x^4 + 9*a^16*b^2*x^2 + a^17*b) + 715/65536
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8*b)
```

mupad [B] time = 0.17, size = 206, normalized size = 1.00

$$\frac{\frac{23473\,x^3}{98304\,a}-\frac{715\,x}{65536\,b}+\frac{20899\,b\,x^5}{32768\,a^2}+\frac{241657\,b^2\,x^7}{229376\,a^3}+\frac{143\,b^3\,x^9}{126\,a^4}+\frac{184327\,b^4\,x^{11}}{229376\,a^5}+\frac{11869\,b^5\,x^{13}}{32768\,a^6}+\frac{9295\,b^6\,x^{15}}{98304\,a^7}+\frac{715\,b^7\,x^{17}}{65536\,a^8}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{715\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{17/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^10,x)
```

```
[Out] ((23473*x^3)/(98304*a) - (715*x)/(65536*b) + (20899*b*x^5)/(32768*a^2) + (2
41657*b^2*x^7)/(229376*a^3) + (143*b^3*x^9)/(126*a^4) + (184327*b^4*x^11)/(
229376*a^5) + (11869*b^5*x^13)/(32768*a^6) + (9295*b^6*x^15)/(98304*a^7) +
(715*b^7*x^17)/(65536*a^8))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 +
36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a
^3*b^6*x^12 + 36*a^2*b^7*x^14) + (715*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(
17/2)*b^(3/2))
```

sympy [A] time = 1.03, size = 286, normalized size = 1.40

$$-\frac{715\sqrt{-\frac{1}{a^{17}b^{3}}}\log\left(-a^{9}b\sqrt{-\frac{1}{a^{17}b^{3}}}+x\right)}{131072}+\frac{715\sqrt{-\frac{1}{a^{17}b^{3}}}\log\left(a^{9}b\sqrt{-\frac{1}{a^{17}b^{3}}}+x\right)}{131072}+\frac{-45045a^{8}x+985866a^{7}bx^{3}+2633274a^{6}b^{2}x^{5}+4349826a^{5}b^{3}x^{7}+4685824a^{4}b^{4}x^{9}+3317886a^{3}b^{5}x^{11}+1495494a^{2}b^{6}x^{13}+390390ab^{7}x^{15}+45045b^{8}x^{17}}{4128768a^{17}b+37158912a^{16}b^{2}x^{2}+148635648a^{15}b^{3}x^{4}+346816512a^{14}b^{4}x^{6}+520224768a^{13}b^{5}x^{8}+520224768a^{12}b^{6}x^{10}+346816512a^{11}b^{7}x^{12}+148635648a^{10}b^{8}x^{14}+37158912a^{9}b^{9}x^{16}+4128768a^{8}b^{10}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**10,x)
```

```
[Out] -715*sqrt(-1/(a**17*b**3))*log(-a**9*b*sqrt(-1/(a**17*b**3)) + x)/131072 +
715*sqrt(-1/(a**17*b**3))*log(a**9*b*sqrt(-1/(a**17*b**3)) + x)/131072 + (-
45045*a**8*x + 985866*a**7*b*x**3 + 2633274*a**6*b**2*x**5 + 4349826*a**5*b
**3*x**7 + 4685824*a**4*b**4*x**9 + 3317886*a**3*b**5*x**11 + 1495494*a**2*
b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(4128768*a**17*b + 371
58912*a**16*b**2*x**2 + 148635648*a**15*b**3*x**4 + 346816512*a**14*b**4*x*
*6 + 520224768*a**13*b**5*x**8 + 520224768*a**12*b**6*x**10 + 346816512*a**
11*b**7*x**12 + 148635648*a**10*b**8*x**14 + 37158912*a**9*b**9*x**16 + 412
8768*a**8*b**10*x**18)
```

3.221 $\int \frac{1}{\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=181

$$\frac{12155\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{19/2}\sqrt{b}}+\frac{12155x}{65536a^9\left(a+bx^2\right)}+\frac{12155x}{98304a^8\left(a+bx^2\right)^2}+\frac{2431x}{24576a^7\left(a+bx^2\right)^3}+\frac{2431x}{28672a^6\left(a+bx^2\right)^4}+\text{32256}$$

Rubi [A] time = 0.10, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 205}

$$\frac{12155x}{65536a^9\left(a+bx^2\right)}+\frac{12155x}{98304a^8\left(a+bx^2\right)^2}+\frac{2431x}{24576a^7\left(a+bx^2\right)^3}+\frac{2431x}{28672a^6\left(a+bx^2\right)^4}+\frac{2431x}{32256a^5\left(a+bx^2\right)^5}+\frac{1105x}{16128a^4\left(a+bx^2\right)^6}+\frac{85x}{1344a^3\left(a+bx^2\right)^7}+\frac{17x}{288a^2\left(a+bx^2\right)^8}+\frac{12155\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{19/2}\sqrt{b}}+\frac{x}{18a\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-10),x]
```

```
[Out] x/(18*a*(a + b*x^2)^9) + (17*x)/(288*a^2*(a + b*x^2)^8) + (85*x)/(1344*a^3*
(a + b*x^2)^7) + (1105*x)/(16128*a^4*(a + b*x^2)^6) + (2431*x)/(32256*a^5*(
a + b*x^2)^5) + (2431*x)/(28672*a^6*(a + b*x^2)^4) + (2431*x)/(24576*a^7*(a
 + b*x^2)^3) + (12155*x)/(98304*a^8*(a + b*x^2)^2) + (12155*x)/(65536*a^9*(
a + b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(19/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{10}}\,dx = \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17\int \frac{1}{\left(a+bx^2\right)^9}\,dx}{18a}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85\int \frac{1}{\left(a+bx^2\right)^8}\,dx}{96a^2}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105\int \frac{1}{\left(a+bx^2\right)^7}\,dx}{1344a^3}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{12155\int \frac{\cdots}{(a\cdots}}{16128\cdots}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5\left(a\cdots\right.}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5\left(a\cdots\right.}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5\left(a\cdots\right.}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5\left(a\cdots\right.}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5\left(a\cdots\right.}$$

$$= \frac{x}{18a\left(a+bx^2\right)^9} + \frac{17x}{288a^2\left(a+bx^2\right)^8} + \frac{85x}{1344a^3\left(a+bx^2\right)^7} + \frac{1105x}{16128a^4\left(a+bx^2\right)^6} + \frac{2431\cdots}{32256a^5\left(a\cdots\right.}$$

Mathematica [A]   time = 0.10, size = 131, normalized size = 0.72

$$\frac{\frac{765765\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{19/2}\sqrt{b}}+\frac{3363003a^8x+16759722a^7bx^3+44765658a^6b^2x^5+73947042a^5b^3x^7+79659008a^4b^4x^9+56404062a^3b^5x^{11}+25423398a^2b^6x^{13}+6636630ab^7x^{15}+765765b^8x^{17}}{a^9\left(a+bx^2\right)^9}}{4128768}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-10),x]
```

```
[Out] ((3363003*a^8*x + 16759722*a^7*b*x^3 + 44765658*a^6*b^2*x^5 + 73947042*a^5*
b^3*x^7 + 79659008*a^4*b^4*x^9 + 56404062*a^3*b^5*x^11 + 25423398*a^2*b^6*x
^13 + 6636630*a*b^7*x^15 + 765765*b^8*x^17)/(a^9*(a + b*x^2)^9) + (765765*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(19/2)*Sqrt[b]))/4128768
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-10),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^(-10), x]
```

fricas [B]   time = 0.82, size = 650, normalized size = 3.59

$$\left[\frac{1531530\,ab^9x^{17}+13273260\,a^2b^8x^{15}+50846796\,a^3b^7x^{13}+112808124\,a^4b^6x^{11}+159318016\,a^5b^5x^9+147894084\,a^6b^4x^7+89531316\,a^7b^3x^5+33519444\,a^8b^2x^3+6726006\,a^9bx-765765\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{8257536\left(a^{10}b^{10}x^{18}+9\,a^{11}b^9x^{16}+36\,a^{12}b^8x^{14}+84\,a^{13}b^7x^{12}+126\,a^{14}b^6x^{10}+126\,a^{15}b^5x^8+84\,a^{16}b^4x^6+36\,a^{17}b^3x^4+9\,a^{18}b^2x^2+a^{19}b\right)},\ \frac{765765\,ab^9x^{17}+6636630\,a^2b^8x^{15}+25423398\,a^3b^7x^{13}+56404062\,a^4b^6x^{11}+79659008\,a^5b^5x^9+73947042\,a^6b^4x^7+44765658\,a^7b^3x^5+16759722\,a^8b^2x^3+3363003\,a^9bx+765765\left(b^9x^{18}+9\,ab^8x^{16}+36\,a^2b^7x^{14}+84\,a^3b^6x^{12}+126\,a^4b^5x^{10}+126\,a^5b^4x^8+84\,a^6b^3x^6+36\,a^7b^2x^4+9\,a^8bx^2+a^9\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{4128768\left(a^{10}b^{10}x^{18}+9\,a^{11}b^9x^{16}+36\,a^{12}b^8x^{14}+84\,a^{13}b^7x^{12}+126\,a^{14}b^6x^{10}+126\,a^{15}b^5x^8+84\,a^{16}b^4x^6+36\,a^{17}b^3x^4+9\,a^{18}b^2x^2+a^{19}b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/8257536*(1531530*a*b^9*x^17 + 13273260*a^2*b^8*x^15 + 50846796*a^3*b^7*x
^13 + 112808124*a^4*b^6*x^11 + 159318016*a^5*b^5*x^9 + 147894084*a^6*b^4*x^
7 + 89531316*a^7*b^3*x^5 + 33519444*a^8*b^2*x^3 + 6726006*a^9*b*x - 765765*
(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*
x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^
9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^10*b^10*x^1
8 + 9*a^11*b^9*x^16 + 36*a^12*b^8*x^14 + 84*a^13*b^7*x^12 + 126*a^14*b^6*x^
10 + 126*a^15*b^5*x^8 + 84*a^16*b^4*x^6 + 36*a^17*b^3*x^4 + 9*a^18*b^2*x^2
+ a^19*b), 1/4128768*(765765*a*b^9*x^17 + 6636630*a^2*b^8*x^15 + 25423398*a
^3*b^7*x^13 + 56404062*a^4*b^6*x^11 + 79659008*a^5*b^5*x^9 + 73947042*a^6*b
^4*x^7 + 44765658*a^7*b^3*x^5 + 16759722*a^8*b^2*x^3 + 3363003*a^9*b*x + 76
5765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4
*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2
```

```
 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^10*b^10*x^18 + 9*a^11*b^9*x^16
+ 36*a^12*b^8*x^14 + 84*a^13*b^7*x^12 + 126*a^14*b^6*x^10 + 126*a^15*b^5*x^
8 + 84*a^16*b^4*x^6 + 36*a^17*b^3*x^4 + 9*a^18*b^2*x^2 + a^19*b)]
```

giac [A] time = 0.60, size = 122, normalized size = 0.67

$$\frac{12155 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^9} + \frac{765765 b^8x^{17} + 6636630 ab^7x^{15} + 25423398 a^2b^6x^{13} + 56404062 a^3b^5x^{11} + 79659008 a^4b^4x^9 + 73947042 a^5b^3x^7 + 44765658 a^6b^2x^5 + 16759722 a^7bx^3 + 3363003 a^8x}{4128768 \left(bx^2+a\right)^9 a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^9) + 1/4128768*(765765*b^8*x
^17 + 6636630*a*b^7*x^15 + 25423398*a^2*b^6*x^13 + 56404062*a^3*b^5*x^11 +
79659008*a^4*b^4*x^9 + 73947042*a^5*b^3*x^7 + 44765658*a^6*b^2*x^5 + 167597
22*a^7*b*x^3 + 3363003*a^8*x)/((b*x^2 + a)^9*a^9)
```

maple [A] time = 0.01, size = 156, normalized size = 0.86

$$\frac{x}{18 \left(b x^2+a\right)^9 a} + \frac{17x}{288 \left(b x^2+a\right)^8 a^2} + \frac{85x}{1344 \left(b x^2+a\right)^7 a^3} + \frac{1105x}{16128 \left(b x^2+a\right)^6 a^4} + \frac{2431x}{32256 \left(b x^2+a\right)^5 a^5} + \frac{2431x}{28672 \left(b x^2+a\right)^4 a^6} + \frac{2431x}{24576 \left(b x^2+a\right)^3 a^7} + \frac{12155x}{98304 \left(b x^2+a\right)^2 a^8} + \frac{12155x}{65536 \left(b x^2+a\right) a^9} + \frac{12155 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^10,x)
```

```
[Out] 1/18*x/a/(b*x^2+a)^9+17/288*x/a^2/(b*x^2+a)^8+85/1344*x/a^3/(b*x^2+a)^7+110
5/16128*x/a^4/(b*x^2+a)^6+2431/32256*x/a^5/(b*x^2+a)^5+2431/28672*x/a^6/(b*
x^2+a)^4+2431/24576*x/a^7/(b*x^2+a)^3+12155/98304*x/a^8/(b*x^2+a)^2+12155/6
5536*x/a^9/(b*x^2+a)+12155/65536/a^9/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.11, size = 212, normalized size = 1.17

$$\frac{765765 b^8x^{17} + 6636630 ab^7x^{15} + 25423398 a^2b^6x^{13} + 56404062 a^3b^5x^{11} + 79659008 a^4b^4x^9 + 73947042 a^5b^3x^7 + 44765658 a^6b^2x^5 + 16759722 a^7bx^3 + 3363003 a^8x}{4128768 \left(a^9b^9x^{18} + 9 a^{10}b^8x^{16} + 36 a^{11}b^7x^{14} + 84 a^{12}b^6x^{12} + 126 a^{13}b^5x^{10} + 126 a^{14}b^4x^8 + 84 a^{15}b^3x^6 + 36 a^{16}b^2x^4 + 9 a^{17}bx^2 + a^{18}\right)} + \frac{12155 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/4128768*(765765*b^8*x^17 + 6636630*a*b^7*x^15 + 25423398*a^2*b^6*x^13 + 5
6404062*a^3*b^5*x^11 + 79659008*a^4*b^4*x^9 + 73947042*a^5*b^3*x^7 + 447656
58*a^6*b^2*x^5 + 16759722*a^7*b*x^3 + 3363003*a^8*x)/(a^9*b^9*x^18 + 9*a^10
*b^8*x^16 + 36*a^11*b^7*x^14 + 84*a^12*b^6*x^12 + 126*a^13*b^5*x^10 + 126*a
^14*b^4*x^8 + 84*a^15*b^3*x^6 + 36*a^16*b^2*x^4 + 9*a^17*b*x^2 + a^18) + 12
155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^9)
```

mupad [B] time = 4.74, size = 209, normalized size = 1.15

$$\frac{\frac{53381\,x}{65536\,a}+\frac{399041\,b\,x^3}{98304\,a^2}+\frac{355283\,b^2\,x^5}{32768\,a^3}+\frac{4108169\,b^3\,x^7}{229376\,a^4}+\frac{2431\,b^4\,x^9}{126\,a^5}+\frac{3133559\,b^5\,x^{11}}{229376\,a^6}+\frac{201773\,b^6\,x^{13}}{32768\,a^7}+\frac{158015\,b^7\,x^{15}}{98304\,a^8}+\frac{12155\,b^8\,x^{17}}{65536\,a^9}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8\,x^{16}+b^9\,x^{18}}+\frac{12155\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{19/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^10,x)
```

```
[Out] ((53381*x)/(65536*a) + (399041*b*x^3)/(98304*a^2) + (355283*b^2*x^5)/(32768
*a^3) + (4108169*b^3*x^7)/(229376*a^4) + (2431*b^4*x^9)/(126*a^5) + (313355
9*b^5*x^11)/(229376*a^6) + (201773*b^6*x^13)/(32768*a^7) + (158015*b^7*x^15
)/(98304*a^8) + (12155*b^8*x^17)/(65536*a^9))/(a^9 + b^9*x^18 + 9*a^8*b*x^2
 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a
^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) + (12155*atan((b^(1/2)*x)/
a^(1/2)))/(65536*a^(19/2)*b^(1/2))
```

sympy [A] time = 1.04, size = 272, normalized size = 1.50

$$-\frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(-a^{10}\sqrt{-\frac{1}{a^{19}b}}+x\right)}{131072}+\frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(a^{10}\sqrt{-\frac{1}{a^{19}b}}+x\right)}{131072}+\frac{3363003a^8x+16759722a^7bx^3+44765658a^6b^2x^5+73947042a^5b^3x^7+79659008a^4b^4x^9+56404062a^3b^5x^{11}+25423398a^2b^6x^{13}+6636630ab^7x^{15}+765765b^8x^{17}}{4128768a^{18}+37158912a^{17}bx^2+148635648a^{16}b^2x^4+346816512a^{15}b^3x^6+520224768a^{14}b^4x^8+520224768a^{13}b^5x^{10}+346816512a^{12}b^6x^{12}+148635648a^{11}b^7x^{14}+37158912a^{10}b^8x^{16}+4128768a^9b^9x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**10,x)
```

```
[Out] -12155*sqrt(-1/(a**19*b))*log(-a**10*sqrt(-1/(a**19*b)) + x)/131072 + 12155
*sqrt(-1/(a**19*b))*log(a**10*sqrt(-1/(a**19*b)) + x)/131072 + (3363003*a**
8*x + 16759722*a**7*b*x**3 + 44765658*a**6*b**2*x**5 + 73947042*a**5*b**3*x
**7 + 79659008*a**4*b**4*x**9 + 56404062*a**3*b**5*x**11 + 25423398*a**2*b*
*6*x**13 + 6636630*a*b**7*x**15 + 765765*b**8*x**17)/(4128768*a**18 + 37158
912*a**17*b*x**2 + 148635648*a**16*b**2*x**4 + 346816512*a**15*b**3*x**6 +
520224768*a**14*b**4*x**8 + 520224768*a**13*b**5*x**10 + 346816512*a**12*b*
*6*x**12 + 148635648*a**11*b**7*x**14 + 37158912*a**10*b**8*x**16 + 4128768
*a**9*b**9*x**18)
```

3.222 $\int \frac{1}{x^2(a+bx^2)^{10}}\,dx$

Optimal. Leaf size=209

$$-\frac{230945\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{21/2}}-\frac{230945}{65536a^{10}x}+\frac{230945}{196608a^9x\,(a+bx^2)}+\frac{46189}{98304a^8x\,(a+bx^2)^2}+\frac{46189}{172032a^7x\,(a+bx^2)^3}+\frac{\quad}{258}$$

Rubi [A] time = 0.13, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$\frac{230945}{196608a^9x\,(a+bx^2)}+\frac{46189}{98304a^8x\,(a+bx^2)^2}+\frac{46189}{172032a^7x\,(a+bx^2)^3}+\frac{46189}{258048a^6x\,(a+bx^2)^4}+\frac{4199}{32256a^5x\,(a+bx^2)^5}+\frac{1615}{16128a^4x\,(a+bx^2)^6}+\frac{323}{4032a^3x\,(a+bx^2)^7}+\frac{19}{288a^2x\,(a+bx^2)^8}-\frac{230945\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{21/2}}-\frac{230945}{65536a^{10}x}+\frac{1}{18ax\,(a+bx^2)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^10),x]
```

```
[Out] -230945/(65536*a^10*x) + 1/(18*a*x*(a + b*x^2)^9) + 19/(288*a^2*x*(a + b*x^
2)^8) + 323/(4032*a^3*x*(a + b*x^2)^7) + 1615/(16128*a^4*x*(a + b*x^2)^6) +
 4199/(32256*a^5*x*(a + b*x^2)^5) + 46189/(258048*a^6*x*(a + b*x^2)^4) + 46
189/(172032*a^7*x*(a + b*x^2)^3) + 46189/(98304*a^8*x*(a + b*x^2)^2) + 2309
45/(196608*a^9*x*(a + b*x^2)) - (230945*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(65536*a^(21/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{10}}\,dx = \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19\int \frac{1}{x^2\left(a+bx^2\right)^9}\,dx}{18a}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323\int \frac{1}{x^2\left(a+bx^2\right)^8}\,dx}{288a^2}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615\int \frac{1}{x^2\left(a+bx^2\right)^7}\,dx}{1344a^3}$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} + \frac{1615}{16128a^4x\left(a+bx^2\right)^6} +$$

$$= -\frac{230945}{65536a^{10}x} + \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} +$$

$$= -\frac{230945}{65536a^{10}x} + \frac{1}{18ax\left(a+bx^2\right)^9} + \frac{19}{288a^2x\left(a+bx^2\right)^8} + \frac{323}{4032a^3x\left(a+bx^2\right)^7} +$$

Mathematica [A] time = 0.10, size = 147, normalized size = 0.70

$$\frac{-\frac{\sqrt{a}\left(4128768a^9+63897057a^8bx^2+318434718a^7b^2x^4+850547502a^6b^3x^6+1404993798a^5b^4x^8+1513521152a^4b^5x^{10}+1071677178a^3b^6x^{12}+483044562a^2b^7x^{14}+126095970ab^8x^{16}+14549535b^9x^{18}\right)}{x\left(a+bx^2\right)^9}-14549535\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{4128768a^{21/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^10),x]
```

```
[Out] (-((Sqrt[a]*(4128768*a^9 + 63897057*a^8*b*x^2 + 318434718*a^7*b^2*x^4 + 850
547502*a^6*b^3*x^6 + 1404993798*a^5*b^4*x^8 + 1513521152*a^4*b^5*x^10 + 107
1677178*a^3*b^6*x^12 + 483044562*a^2*b^7*x^14 + 126095970*a*b^8*x^16 + 1454
9535*b^9*x^18))/(x*(a + b*x^2)^9)) - 14549535*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(4128768*a^(21/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^10), x]
```

fricas [A] time = 0.97, size = 664, normalized size = 3.18

$$\left[-\frac{29099070\,b^9x^{18}+252191940\,ab^8x^{16}+966089124\,a^2b^7x^{14}+2143354356\,a^3b^6x^{12}+3027042304\,a^4b^5x^{10}+2809987596\,a^5b^4x^8+1701095004\,a^6b^3x^6+636869436\,a^7b^2x^4+127794114\,a^8bx^2+8257536\,a^9-14549535\left(b^9x^{19}+9\,ab^8x^{17}+36\,a^2b^7x^{15}+84\,a^3b^6x^{13}+126\,a^4b^5x^{11}+126\,a^5b^4x^9+84\,a^6b^3x^7+36\,a^7b^2x^5+9\,a^8bx^3+a^9x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{8257536\left(a^{10}b^9x^{19}+9\,a^{11}b^8x^{17}+36\,a^{12}b^7x^{15}+84\,a^{13}b^6x^{13}+126\,a^{14}b^5x^{11}+126\,a^{15}b^4x^9+84\,a^{16}b^3x^7+36\,a^{17}b^2x^5+9\,a^{18}bx^3+a^{19}x\right)},\ -\frac{14549535\,b^9x^{18}+126095970\,ab^8x^{16}+483044562\,a^2b^7x^{14}+1071677178\,a^3b^6x^{12}+1513521152\,a^4b^5x^{10}+1404993798\,a^5b^4x^8+850547502\,a^6b^3x^6+318434718\,a^7b^2x^4+63897057\,a^8bx^2+4128768\,a^9+14549535\left(b^9x^{19}+9\,ab^8x^{17}+36\,a^2b^7x^{15}+84\,a^3b^6x^{13}+126\,a^4b^5x^{11}+126\,a^5b^4x^9+84\,a^6b^3x^7+36\,a^7b^2x^5+9\,a^8bx^3+a^9x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{4128768\left(a^{10}b^9x^{19}+9\,a^{11}b^8x^{17}+36\,a^{12}b^7x^{15}+84\,a^{13}b^6x^{13}+126\,a^{14}b^5x^{11}+126\,a^{15}b^4x^9+84\,a^{16}b^3x^7+36\,a^{17}b^2x^5+9\,a^{18}bx^3+a^{19}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/8257536*(29099070*b^9*x^18 + 252191940*a*b^8*x^16 + 966089124*a^2*b^7*x
^14 + 2143354356*a^3*b^6*x^12 + 3027042304*a^4*b^5*x^10 + 2809987596*a^5*b^
4*x^8 + 1701095004*a^6*b^3*x^6 + 636869436*a^7*b^2*x^4 + 127794114*a^8*b*x^
2 + 8257536*a^9 - 14549535*(b^9*x^19 + 9*a*b^8*x^17 + 36*a^2*b^7*x^15 + 84*
a^3*b^6*x^13 + 126*a^4*b^5*x^11 + 126*a^5*b^4*x^9 + 84*a^6*b^3*x^7 + 36*a^7
*b^2*x^5 + 9*a^8*b*x^3 + a^9*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)))/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15 + 84*a
^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^7 + 36*
a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x), -1/4128768*(14549535*b^9*x^18 + 1260
95970*a*b^8*x^16 + 483044562*a^2*b^7*x^14 + 1071677178*a^3*b^6*x^12 + 15135
21152*a^4*b^5*x^10 + 1404993798*a^5*b^4*x^8 + 850547502*a^6*b^3*x^6 + 31843
4718*a^7*b^2*x^4 + 63897057*a^8*b*x^2 + 4128768*a^9 + 14549535*(b^9*x^19 +
9*a*b^8*x^17 + 36*a^2*b^7*x^15 + 84*a^3*b^6*x^13 + 126*a^4*b^5*x^11 + 126*a
```

```
^5*b^4*x^9 + 84*a^6*b^3*x^7 + 36*a^7*b^2*x^5 + 9*a^8*b*x^3 + a^9*x)*sqrt(b/
a)*arctan(x*sqrt(b/a)))/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15
 + 84*a^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^
7 + 36*a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x)]
```

giac [A] time = 0.63, size = 134, normalized size = 0.64

$$-\frac{230945\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{10}}-\frac{1}{a^{10}x}-\frac{10420767\,b^9x^{17}+88937058\,ab^8x^{15}+334408914\,a^2b^7x^{13}+724860666\,a^3b^6x^{11}+993296384\,a^4b^5x^9+884769030\,a^5b^4x^7+503730990\,a^6b^3x^5+169799070\,a^7b^2x^3+26738145\,a^8bx}{4128768\left(bx^2+a\right)^9a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -230945/65536*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^10) - 1/(a^10*x) - 1/412
8768*(10420767*b^9*x^17 + 88937058*a*b^8*x^15 + 334408914*a^2*b^7*x^13 + 72
4860666*a^3*b^6*x^11 + 993296384*a^4*b^5*x^9 + 884769030*a^5*b^4*x^7 + 5037
30990*a^6*b^3*x^5 + 169799070*a^7*b^2*x^3 + 26738145*a^8*b*x)/((b*x^2 + a)^
9*a^10)
```

maple [A] time = 0.02, size = 206, normalized size = 0.99

$$-\frac{165409b^9x^{17}}{65536\left(bx^2+a\right)^9a^{10}}-\frac{2117549b^8x^{15}}{98304\left(bx^2+a\right)^9a^9}-\frac{2654039b^7x^{13}}{32768\left(bx^2+a\right)^9a^8}-\frac{40270037b^6x^{11}}{229376\left(bx^2+a\right)^9a^7}-\frac{30313b^5x^9}{126\left(bx^2+a\right)^9a^6}-\frac{49153835b^4x^7}{229376\left(bx^2+a\right)^9a^5}-\frac{3997865b^3x^5}{32768\left(bx^2+a\right)^9a^4}-\frac{4042835b^2x^3}{98304\left(bx^2+a\right)^9a^3}-\frac{424415bx}{65536\left(bx^2+a\right)^9a^2}-\frac{230945b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^{10}}-\frac{1}{a^{10}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^10,x)
```

```
[Out] -1/a^10/x-424415/65536/a^2*b/(b*x^2+a)^9*x-4042835/98304/a^3*b^2/(b*x^2+a)^
9*x^3-3997865/32768/a^4*b^3/(b*x^2+a)^9*x^5-49153835/229376/a^5*b^4/(b*x^2+
a)^9*x^7-30313/126/a^6*b^5/(b*x^2+a)^9*x^9-40270037/229376/a^7*b^6/(b*x^2+a
)^9*x^11-2654039/32768/a^8*b^7/(b*x^2+a)^9*x^13-2117549/98304/a^9*b^8/(b*x^
2+a)^9*x^15-165409/65536/a^10*b^9/(b*x^2+a)^9*x^17-230945/65536/a^10*b/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.13, size = 225, normalized size = 1.08

$$-\frac{14549535\,b^9x^{18}+126095970\,ab^8x^{16}+483044562\,a^2b^7x^{14}+1071677178\,a^3b^6x^{12}+1513521152\,a^4b^5x^{10}+1404993798\,a^5b^4x^8+850547502\,a^6b^3x^6+318434718\,a^7b^2x^4+63897057\,a^8bx^2+4128768\,a^9}{4128768\left(a^{10}b^9x^{19}+9\,a^{11}b^8x^{17}+36\,a^{12}b^7x^{15}+84\,a^{13}b^6x^{13}+126\,a^{14}b^5x^{11}+126\,a^{15}b^4x^9+84\,a^{16}b^3x^7+36\,a^{17}b^2x^5+9\,a^{18}bx^3+a^{19}x\right)}-\frac{230945\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/4128768*(14549535*b^9*x^18 + 126095970*a*b^8*x^16 + 483044562*a^2*b^7*x^
14 + 1071677178*a^3*b^6*x^12 + 1513521152*a^4*b^5*x^10 + 1404993798*a^5*b^4
*x^8 + 850547502*a^6*b^3*x^6 + 318434718*a^7*b^2*x^4 + 63897057*a^8*b*x^2 +
 4128768*a^9)/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15 + 84*a^13
```

```
*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^7 + 36*a^1
7*b^2*x^5 + 9*a^18*b*x^3 + a^19*x) - 230945/65536*b*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^10)
```

mupad [B] time = 5.09, size = 220, normalized size = 1.05

$$-\frac{\frac{1}{a}+\frac{1014239\,b\,x^2}{65536\,a^2}+\frac{7581779\,b^2\,x^4}{98304\,a^3}+\frac{6750377\,b^3\,x^6}{32768\,a^4}+\frac{78055211\,b^4\,x^8}{229376\,a^5}+\frac{46189\,b^5\,x^{10}}{126\,a^6}+\frac{59537621\,b^6\,x^{12}}{229376\,a^7}+\frac{3833687\,b^7\,x^{14}}{32768\,a^8}+\frac{3002285\,b^8\,x^{16}}{98304\,a^9}+\frac{230945\,b^9\,x^{18}}{65536\,a^{10}}}{a^9\,x+9\,a^8\,b\,x^3+36\,a^7\,b^2\,x^5+84\,a^6\,b^3\,x^7+126\,a^5\,b^4\,x^9+126\,a^4\,b^5\,x^{11}+84\,a^3\,b^6\,x^{13}+36\,a^2\,b^7\,x^{15}+9\,a\,b^8\,x^{17}+b^9\,x^{19}}-\frac{230945\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{21/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^10),x)
```

```
[Out] - (1/a + (1014239*b*x^2)/(65536*a^2) + (7581779*b^2*x^4)/(98304*a^3) + (675
0377*b^3*x^6)/(32768*a^4) + (78055211*b^4*x^8)/(229376*a^5) + (46189*b^5*x^
10)/(126*a^6) + (59537621*b^6*x^12)/(229376*a^7) + (3833687*b^7*x^14)/(3276
8*a^8) + (3002285*b^8*x^16)/(98304*a^9) + (230945*b^9*x^18)/(65536*a^10))/(
a^9*x + b^9*x^19 + 9*a^8*b*x^3 + 9*a*b^8*x^17 + 36*a^7*b^2*x^5 + 84*a^6*b^3
*x^7 + 126*a^5*b^4*x^9 + 126*a^4*b^5*x^11 + 84*a^3*b^6*x^13 + 36*a^2*b^7*x^
15) - (230945*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(21/2))
```

sympy [A] time = 1.35, size = 282, normalized size = 1.35

$$\frac{230945\sqrt{-\frac{b}{a^{21}}}\log\left(-\frac{a^{11}\sqrt{-\frac{b}{a^{21}}}}{b}+x\right)}{131072}-\frac{230945\sqrt{-\frac{b}{a^{21}}}\log\left(\frac{a^{11}\sqrt{-\frac{b}{a^{21}}}}{b}+x\right)}{131072}+\frac{-4128768a^9-63897057a^8bx^2-318434718a^7b^2x^4-850547502a^6b^3x^6-1404993798a^5b^4x^8-1513521152a^4b^5x^{10}-1071677178a^3b^6x^{12}-483044562a^2b^7x^{14}-126095970ab^8x^{16}-14549535b^9x^{18}}{4128768a^{19}x+37158912a^{18}bx^3+148635648a^{17}b^2x^5+346816512a^{16}b^3x^7+520224768a^{15}b^4x^9+520224768a^{14}b^5x^{11}+346816512a^{13}b^6x^{13}+148635648a^{12}b^7x^{15}+37158912a^{11}b^8x^{17}+4128768a^{10}b^9x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**10,x)
```

```
[Out] 230945*sqrt(-b/a**21)*log(-a**11*sqrt(-b/a**21)/b + x)/131072 - 230945*sqrt
(-b/a**21)*log(a**11*sqrt(-b/a**21)/b + x)/131072 + (-4128768*a**9 - 638970
57*a**8*b*x**2 - 318434718*a**7*b**2*x**4 - 850547502*a**6*b**3*x**6 - 1404
993798*a**5*b**4*x**8 - 1513521152*a**4*b**5*x**10 - 1071677178*a**3*b**6*x
**12 - 483044562*a**2*b**7*x**14 - 126095970*a*b**8*x**16 - 14549535*b**9*x
**18)/(4128768*a**19*x + 37158912*a**18*b*x**3 + 148635648*a**17*b**2*x**5
+ 346816512*a**16*b**3*x**7 + 520224768*a**15*b**4*x**9 + 520224768*a**14*b
**5*x**11 + 346816512*a**13*b**6*x**13 + 148635648*a**12*b**7*x**15 + 37158
912*a**11*b**8*x**17 + 4128768*a**10*b**9*x**19)
```

3.223 $\int \frac{1}{x^4(a+bx^2)^{10}}\,dx$

<u>**Optimal**</u>. Leaf size=220

$$\frac{1616615b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{23/2}}+\frac{1616615b}{65536a^{11}x}-\frac{1616615}{196608a^{10}x^3}+\frac{323323}{65536a^9x^3\left(a+bx^2\right)}+\frac{46189}{32768a^8x^3\left(a+bx^2\right)^2}+\frac{4618}{73728a^7x^3(}$$

<u>**Rubi**</u> [A] time = 0.14, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {290, 325, 205}

$$\frac{1616615b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{23/2}}+\frac{323323}{65536a^9x^3\left(a+bx^2\right)}+\frac{46189}{32768a^8x^3\left(a+bx^2\right)^2}+\frac{46189}{73728a^7x^3\left(a+bx^2\right)^3}+\frac{4199}{12288a^6x^3\left(a+bx^2\right)^4}+\frac{323}{1536a^5x^3\left(a+bx^2\right)^5}+\frac{323}{2304a^4x^3\left(a+bx^2\right)^6}+\frac{19}{192a^3x^3\left(a+bx^2\right)^7}+\frac{7}{96a^2x^3\left(a+bx^2\right)^8}+\frac{1616615b}{65536a^{11}x}-\frac{1616615}{196608a^{10}x^3}+\frac{1}{18ax^3\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^10),x]
```

```
[Out] -1616615/(196608*a^10*x^3) + (1616615*b)/(65536*a^11*x) + 1/(18*a*x^3*(a +
b*x^2)^9) + 7/(96*a^2*x^3*(a + b*x^2)^8) + 19/(192*a^3*x^3*(a + b*x^2)^7) +
 323/(2304*a^4*x^3*(a + b*x^2)^6) + 323/(1536*a^5*x^3*(a + b*x^2)^5) + 4199
/(12288*a^6*x^3*(a + b*x^2)^4) + 46189/(73728*a^7*x^3*(a + b*x^2)^3) + 4618
9/(32768*a^8*x^3*(a + b*x^2)^2) + 323323/(65536*a^9*x^3*(a + b*x^2)) + (161
6615*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(23/2))
```

<u>Rule 205</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 290</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

<u>Rule 325</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{10}}\,dx = \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7\int \frac{1}{x^4\left(a+bx^2\right)^9}\,dx}{6a}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{133\int \frac{1}{x^4\left(a+bx^2\right)^8}\,dx}{96a^2}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{19}{192a^3x^3\left(a+bx^2\right)^7} + \frac{323\int \frac{1}{x^4\left(a+bx^2\right)^7}\,dx}{192a^3}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{19}{192a^3x^3\left(a+bx^2\right)^7} + \frac{323}{2304a^4x^3\left(a+bx^2\right)^6} + \frac{161\cdots}{\cdots}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{19}{192a^3x^3\left(a+bx^2\right)^7} + \frac{323}{2304a^4x^3\left(a+bx^2\right)^6} + \frac{\cdots}{153\cdots}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{19}{192a^3x^3\left(a+bx^2\right)^7} + \frac{323}{2304a^4x^3\left(a+bx^2\right)^6} + \frac{\cdots}{153\cdots}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{19}{192a^3x^3\left(a+bx^2\right)^7} + \frac{323}{2304a^4x^3\left(a+bx^2\right)^6} + \frac{\cdots}{153\cdots}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{19}{192a^3x^3\left(a+bx^2\right)^7} + \frac{323}{2304a^4x^3\left(a+bx^2\right)^6} + \frac{\cdots}{153\cdots}$$

$$= \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{19}{192a^3x^3\left(a+bx^2\right)^7} + \frac{323}{2304a^4x^3\left(a+bx^2\right)^6} + \frac{\cdots}{153\cdots}$$

$$= -\frac{1616615}{196608a^{10}x^3} + \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{19}{192a^3x^3\left(a+bx^2\right)^7} + \frac{3\cdots}{2304a^4x^3\cdots}$$

$$= -\frac{1616615}{196608a^{10}x^3} + \frac{1616615b}{65536a^{11}x} + \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{19}{192a^3x^3\left(a+bx^2\cdots\right.}$$

$$= -\frac{1616615}{196608a^{10}x^3} + \frac{1616615b}{65536a^{11}x} + \frac{1}{18ax^3\left(a+bx^2\right)^9} + \frac{7}{96a^2x^3\left(a+bx^2\right)^8} + \frac{19}{192a^3x^3\left(a+bx^2\cdots\right.}$$

Mathematica [A] time = 0.09, size = 157, normalized size = 0.71

$$\frac{\frac{\sqrt{a}\left(-196608a^{10}+4128768a^9bx^2+63897057a^8b^2x^4+318434718a^7b^3x^6+850547502a^6b^4x^8+1404993798a^5b^5x^{10}+1513521152a^4b^6x^{12}+1071677178a^3b^7x^{14}+483044562a^2b^8x^{16}+126095970ab^9x^{18}+14549535b^{10}x^{20}\right)}{x^3\left(a+bx^2\right)^9}+14549535b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{589824a^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^10),x]
```

```
[Out] ((Sqrt[a]*(-196608*a^10 + 4128768*a^9*b*x^2 + 63897057*a^8*b^2*x^4 + 318434
718*a^7*b^3*x^6 + 850547502*a^6*b^4*x^8 + 1404993798*a^5*b^5*x^10 + 1513521
152*a^4*b^6*x^12 + 1071677178*a^3*b^7*x^14 + 483044562*a^2*b^8*x^16 + 12609
5970*a*b^9*x^18 + 14549535*b^10*x^20))/(x^3*(a + b*x^2)^9) + 14549535*b^(3/
2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(589824*a^(23/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^10), x]
```

fricas [A] time = 0.87, size = 700, normalized size = 3.18

$$\left[\frac{29099070b^{10}x^{20}+252191940ab^9x^{18}+966089124a^2b^8x^{16}+2143354356a^3b^7x^{14}+3027042304a^4b^6x^{12}+2809987596a^5b^5x^{10}+1701095004a^6b^4x^8+636869436a^7b^3x^6+127794114a^8b^2x^4+8257536a^9bx^2-393216a^{10}+14549535\left(b^{10}x^{21}+9ab^9x^{19}+36a^2b^8x^{17}+84a^3b^7x^{15}+126a^4b^6x^{13}+126a^5b^5x^{11}+84a^6b^4x^9+36a^7b^3x^7+9a^8b^2x^5+a^9bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{1179648\left(a^{11}b^9x^{21}+9a^{12}b^8x^{19}+36a^{13}b^7x^{17}+84a^{14}b^6x^{15}+126a^{15}b^5x^{13}+126a^{16}b^4x^{11}+84a^{17}b^3x^9+36a^{18}b^2x^7+9a^{19}bx^5+a^{20}x^3\right)}, \frac{14549535b^{10}x^{20}+126095970ab^9x^{18}+483044562a^2b^8x^{16}+1071677178a^3b^7x^{14}+1513521152a^4b^6x^{12}+1404993798a^5b^5x^{10}+850547502a^6b^4x^8+318434718a^7b^3x^6+63897057a^8b^2x^4+4128768a^9bx^2-196608a^{10}+14549535\left(b^{10}x^{21}+9ab^9x^{19}+36a^2b^8x^{17}+84a^3b^7x^{15}+126a^4b^6x^{13}+126a^5b^5x^{11}+84a^6b^4x^9+36a^7b^3x^7+9a^8b^2x^5+a^9bx^3\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{589824\left(a^{11}b^9x^{21}+9a^{12}b^8x^{19}+36a^{13}b^7x^{17}+84a^{14}b^6x^{15}+126a^{15}b^5x^{13}+126a^{16}b^4x^{11}+84a^{17}b^3x^9+36a^{18}b^2x^7+9a^{19}bx^5+a^{20}x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [1/1179648*(29099070*b^10*x^20 + 252191940*a*b^9*x^18 + 966089124*a^2*b^8*x
^16 + 2143354356*a^3*b^7*x^14 + 3027042304*a^4*b^6*x^12 + 2809987596*a^5*b^
5*x^10 + 1701095004*a^6*b^4*x^8 + 636869436*a^7*b^3*x^6 + 127794114*a^8*b^2
*x^4 + 8257536*a^9*b*x^2 - 393216*a^10 + 14549535*(b^10*x^21 + 9*a*b^9*x^19
 + 36*a^2*b^8*x^17 + 84*a^3*b^7*x^15 + 126*a^4*b^6*x^13 + 126*a^5*b^5*x^11
+ 84*a^6*b^4*x^9 + 36*a^7*b^3*x^7 + 9*a^8*b^2*x^5 + a^9*b*x^3)*sqrt(-b/a)*l
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^11*b^9*x^21 + 9*a^12*b^8
*x^19 + 36*a^13*b^7*x^17 + 84*a^14*b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*
b^4*x^11 + 84*a^17*b^3*x^9 + 36*a^18*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3), 1/
589824*(14549535*b^10*x^20 + 126095970*a*b^9*x^18 + 483044562*a^2*b^8*x^16
+ 1071677178*a^3*b^7*x^14 + 1513521152*a^4*b^6*x^12 + 1404993798*a^5*b^5*x^
10 + 850547502*a^6*b^4*x^8 + 318434718*a^7*b^3*x^6 + 63897057*a^8*b^2*x^4 +
 4128768*a^9*b*x^2 - 196608*a^10 + 14549535*(b^10*x^21 + 9*a*b^9*x^19 + 36*
```

```
a^2*b^8*x^17 + 84*a^3*b^7*x^15 + 126*a^4*b^6*x^13 + 126*a^5*b^5*x^11 + 84*a
^6*b^4*x^9 + 36*a^7*b^3*x^7 + 9*a^8*b^2*x^5 + a^9*b*x^3)*sqrt(b/a)*arctan(x
*sqrt(b/a)))/(a^11*b^9*x^21 + 9*a^12*b^8*x^19 + 36*a^13*b^7*x^17 + 84*a^14*
b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*b^4*x^11 + 84*a^17*b^3*x^9 + 36*a^1
8*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3)]
```

giac [A] time = 0.63, size = 148, normalized size = 0.67

$$\frac{1616615\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{11}}+\frac{30\,bx^{2}-a}{3\,a^{11}x^{3}}+\frac{8651295\,b^{10}x^{17}+73208418\,ab^{9}x^{15}+272477394\,a^{2}b^{8}x^{13}+583302906\,a^{3}b^{7}x^{11}+786857984\,a^{4}b^{6}x^{9}+686588166\,a^{5}b^{5}x^{7}+379867950\,a^{6}b^{4}x^{5}+122613150\,a^{7}b^{3}x^{3}+17890785\,a^{8}b^{2}x}{589824\,\left(bx^{2}+a\right)^{9}a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] 1616615/65536*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^11) + 1/3*(30*b*x^2 -
a)/(a^11*x^3) + 1/589824*(8651295*b^10*x^17 + 73208418*a*b^9*x^15 + 2724773
94*a^2*b^8*x^13 + 583302906*a^3*b^7*x^11 + 786857984*a^4*b^6*x^9 + 68658816
6*a^5*b^5*x^7 + 379867950*a^6*b^4*x^5 + 122613150*a^7*b^3*x^3 + 17890785*a^
8*b^2*x)/((b*x^2 + a)^9*a^11)
```

maple [A] time = 0.02, size = 219, normalized size = 1.00

$$\frac{961255 b^{10} x^{17}}{65536 \left(b\,x^{2}+a\right)^{9} a^{11}}+\frac{12201403 b^{9} x^{15}}{98304 \left(b\,x^{2}+a\right)^{9} a^{10}}+\frac{15137633 b^{8} x^{13}}{32768 \left(b\,x^{2}+a\right)^{9} a^{9}}+\frac{32405717 b^{7} x^{11}}{32768 \left(b\,x^{2}+a\right)^{9} a^{8}}+\frac{24013 b^{6} x^{9}}{18 \left(b\,x^{2}+a\right)^{9} a^{7}}+\frac{38143787 b^{5} x^{7}}{32768 \left(b\,x^{2}+a\right)^{9} a^{6}}+\frac{21103775 b^{4} x^{5}}{32768 \left(b\,x^{2}+a\right)^{9} a^{5}}+\frac{20435525 b^{3} x^{3}}{98304 \left(b\,x^{2}+a\right)^{9} a^{4}}+\frac{1987865 b^{2} x}{65536 \left(b\,x^{2}+a\right)^{9} a^{3}}+\frac{1616615 b^{2} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab}\, a^{11}}+\frac{10 b}{a^{11} x}-\frac{1}{3 a^{10} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^10,x)
```

```
[Out] -1/3/a^10/x^3+10*b/a^11/x+1987865/65536/a^3*b^2/(b*x^2+a)^9*x+20435525/9830
4/a^4*b^3/(b*x^2+a)^9*x^3+21103775/32768/a^5*b^4/(b*x^2+a)^9*x^5+38143787/3
2768/a^6*b^5/(b*x^2+a)^9*x^7+24013/18/a^7*b^6/(b*x^2+a)^9*x^9+32405717/3276
8/a^8*b^7/(b*x^2+a)^9*x^11+15137633/32768/a^9*b^8/(b*x^2+a)^9*x^13+12201403
/98304/a^10*b^9/(b*x^2+a)^9*x^15+961255/65536/a^11*b^10/(b*x^2+a)^9*x^17+16
16615/65536/a^11*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.30, size = 240, normalized size = 1.09

$$\frac{14549535\,b^{10}x^{20}+126095970\,ab^{9}x^{18}+483044562\,a^{2}b^{8}x^{16}+1071677178\,a^{3}b^{7}x^{14}+1513521152\,a^{4}b^{6}x^{12}+1404993798\,a^{5}b^{5}x^{10}+850547502\,a^{6}b^{4}x^{8}+318434718\,a^{7}b^{3}x^{6}+63897057\,a^{8}b^{2}x^{4}+4128768\,a^{9}bx^{2}-196608\,a^{10}}{589824\,\left(a^{11}b^{9}x^{21}+9\,a^{12}b^{8}x^{19}+36\,a^{13}b^{7}x^{17}+84\,a^{14}b^{6}x^{15}+126\,a^{15}b^{5}x^{13}+126\,a^{16}b^{4}x^{11}+84\,a^{17}b^{3}x^{9}+36\,a^{18}b^{2}x^{7}+9\,a^{19}bx^{5}+a^{20}x^{3}\right)}+\frac{1616615\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] 1/589824*(14549535*b^10*x^20 + 126095970*a*b^9*x^18 + 483044562*a^2*b^8*x^1
6 + 1071677178*a^3*b^7*x^14 + 1513521152*a^4*b^6*x^12 + 1404993798*a^5*b^5*
x^10 + 850547502*a^6*b^4*x^8 + 318434718*a^7*b^3*x^6 + 63897057*a^8*b^2*x^4
```

```
 + 4128768*a^9*b*x^2 - 196608*a^10)/(a^11*b^9*x^21 + 9*a^12*b^8*x^19 + 36*a
^13*b^7*x^17 + 84*a^14*b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*b^4*x^11 + 8
4*a^17*b^3*x^9 + 36*a^18*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3) + 1616615/65536
*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^11)
```

mupad [B] time = 5.11, size = 234, normalized size = 1.06

$$\frac{\frac{7\,b\,x^2}{a^2}-\frac{1}{3\,a}+\frac{7099673\,b^2\,x^4}{65536\,a^3}+\frac{53072453\,b^3\,x^6}{98304\,a^4}+\frac{47252639\,b^4\,x^8}{32768\,a^5}+\frac{78055211\,b^5\,x^{10}}{32768\,a^6}+\frac{46189\,b^6\,x^{12}}{18\,a^7}+\frac{59537621\,b^7\,x^{14}}{32768\,a^8}+\frac{26835809\,b^8\,x^{16}}{32768\,a^9}+\frac{21015995\,b^9\,x^{18}}{98304\,a^{10}}+\frac{1616615\,b^{10}\,x^{20}}{65536\,a^{11}}}{a^9\,x^3+9\,a^8\,b\,x^5+36\,a^7\,b^2\,x^7+84\,a^6\,b^3\,x^9+126\,a^5\,b^4\,x^{11}+126\,a^4\,b^5\,x^{13}+84\,a^3\,b^6\,x^{15}+36\,a^2\,b^7\,x^{17}+9\,a\,b^8\,x^{19}+b^9\,x^{21}}+\frac{1616615\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{23/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^10),x)
```

```
[Out] ((7*b*x^2)/a^2 - 1/(3*a) + (7099673*b^2*x^4)/(65536*a^3) + (53072453*b^3*x^
6)/(98304*a^4) + (47252639*b^4*x^8)/(32768*a^5) + (78055211*b^5*x^10)/(3276
8*a^6) + (46189*b^6*x^12)/(18*a^7) + (59537621*b^7*x^14)/(32768*a^8) + (268
35809*b^8*x^16)/(32768*a^9) + (21015995*b^9*x^18)/(98304*a^10) + (1616615*b
^10*x^20)/(65536*a^11))/(a^9*x^3 + b^9*x^21 + 9*a^8*b*x^5 + 9*a*b^8*x^19 +
36*a^7*b^2*x^7 + 84*a^6*b^3*x^9 + 126*a^5*b^4*x^11 + 126*a^4*b^5*x^13 + 84*
a^3*b^6*x^15 + 36*a^2*b^7*x^17) + (1616615*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)
))/(65536*a^(23/2))
```

sympy [A] time = 1.45, size = 304, normalized size = 1.38

$$-\frac{1616615\sqrt{-\frac{b^3}{a^{23}}}\log\left(-\frac{a^{12}\sqrt{-\frac{b^3}{a^{23}}}}{b^2}+x\right)}{131072}+\frac{1616615\sqrt{-\frac{b^3}{a^{23}}}\log\left(\frac{a^{12}\sqrt{-\frac{b^3}{a^{23}}}}{b^2}+x\right)}{131072}+\frac{-196608a^{10}+4128768a^9bx^2+63897057a^8b^2x^4+318434718a^7b^3x^6+850547502a^6b^4x^8+1404993798a^5b^5x^{10}+1513521152a^4b^6x^{12}+1071677178a^3b^7x^{14}+483044562a^2b^8x^{16}+126095970ab^9x^{18}+14549535b^{10}x^{20}}{589824a^{20}x^3+5308416a^{19}bx^5+21233664a^{18}b^2x^7+49545216a^{17}b^3x^9+74317824a^{16}b^4x^{11}+74317824a^{15}b^5x^{13}+49545216a^{14}b^6x^{15}+21233664a^{13}b^7x^{17}+5308416a^{12}b^8x^{19}+589824a^{11}b^9x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**10,x)
```

```
[Out] -1616615*sqrt(-b**3/a**23)*log(-a**12*sqrt(-b**3/a**23)/b**2 + x)/131072 +
1616615*sqrt(-b**3/a**23)*log(a**12*sqrt(-b**3/a**23)/b**2 + x)/131072 + (-
196608*a**10 + 4128768*a**9*b*x**2 + 63897057*a**8*b**2*x**4 + 318434718*a*
*7*b**3*x**6 + 850547502*a**6*b**4*x**8 + 1404993798*a**5*b**5*x**10 + 1513
521152*a**4*b**6*x**12 + 1071677178*a**3*b**7*x**14 + 483044562*a**2*b**8*x
**16 + 126095970*a*b**9*x**18 + 14549535*b**10*x**20)/(589824*a**20*x**3 +
5308416*a**19*b*x**5 + 21233664*a**18*b**2*x**7 + 49545216*a**17*b**3*x**9
+ 74317824*a**16*b**4*x**11 + 74317824*a**15*b**5*x**13 + 49545216*a**14*b*
*6*x**15 + 21233664*a**13*b**7*x**17 + 5308416*a**12*b**8*x**19 + 589824*a*
*11*b**9*x**21)
```

3.224 $\int \frac{1}{x^6\left(a+bx^2\right)^{10}}\,dx$

Optimal. Leaf size=233

$$-\frac{7436429b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{25/2}}-\frac{7436429b^2}{65536a^{12}x}+\frac{7436429b}{196608a^{11}x^3}-\frac{7436429}{327680a^{10}x^5}+\frac{1062347}{65536a^9x^5\left(a+bx^2\right)}+\frac{1062347}{294912a^8x^5\left(a+b\right.}$$

Rubi **[A]** time = 0.16, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {290, 325, 205}

$$-\frac{7436429b^2}{65536a^{12}x}-\frac{7436429b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536a^{25/2}}+\frac{7436429b}{196608a^{11}x^3}+\frac{1062347}{65536a^9x^5\left(a+bx^2\right)}+\frac{1062347}{294912a^8x^5\left(a+bx^2\right)^2}+\frac{96577}{73728a^7x^5\left(a+bx^2\right)^3}+\frac{7429}{12288a^6x^5\left(a+bx^2\right)^4}+\frac{7429}{23040a^5x^5\left(a+bx^2\right)^5}+\frac{437}{2304a^4x^5\left(a+bx^2\right)^6}+\frac{23}{192a^3x^5\left(a+bx^2\right)^7}+\frac{23}{288a^2x^5\left(a+bx^2\right)^8}-\frac{7436429}{327680a^{10}x^5}+\frac{1}{18ax^5\left(a+bx^2\right)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^10),x]
```

```
[Out] -7436429/(327680*a^10*x^5) + (7436429*b)/(196608*a^11*x^3) - (7436429*b^2)/
(65536*a^12*x) + 1/(18*a*x^5*(a + b*x^2)^9) + 23/(288*a^2*x^5*(a + b*x^2)^8
) + 23/(192*a^3*x^5*(a + b*x^2)^7) + 437/(2304*a^4*x^5*(a + b*x^2)^6) + 742
9/(23040*a^5*x^5*(a + b*x^2)^5) + 7429/(12288*a^6*x^5*(a + b*x^2)^4) + 9657
7/(73728*a^7*x^5*(a + b*x^2)^3) + 1062347/(294912*a^8*x^5*(a + b*x^2)^2) +
1062347/(65536*a^9*x^5*(a + b*x^2)) - (7436429*b^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(65536*a^(25/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^{10}}\,dx = \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23\int \frac{1}{x^6\left(a+bx^2\right)^9}\,dx}{18a}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{161\int \frac{1}{x^6\left(a+bx^2\right)^8}\,dx}{96a^2}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437\int \frac{1}{x^6\left(a+bx^2\right)^7}\,dx}{192a^3}$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} + \frac{437}{2304a^4x^5\left(a+bx^2\right)^6} +$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} + \frac{23}{192a^3x^5\left(a+bx^2\right)^7} +$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} +$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429b^2}{65536a^{12}x} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} +$$

$$= -\frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429b^2}{65536a^{12}x} + \frac{1}{18ax^5\left(a+bx^2\right)^9} + \frac{23}{288a^2x^5\left(a+bx^2\right)^8} +$$

Mathematica [A] time = 0.09, size = 169, normalized size = 0.73

$$\frac{-334639305b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{\sqrt{a}\left(589824a^{11}-4521984a^{10}bx^2+94961664a^9b^2x^4+1469632311a^8b^3x^6+7323998514a^7b^4x^8+19562592546a^6b^5x^{10}+32314857354a^5b^6x^{12}+34810986496a^4b^7x^{14}+24648575094a^3b^8x^{16}+11110024926a^2b^9x^{18}+2900207310ab^{10}x^{20}+334639305b^{11}x^{22}\right)}{x^5(a+bx^2)^9}}{2949120a^{25/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^10),x]
```

```
[Out] (-((Sqrt[a]*(589824*a^11 - 4521984*a^10*b*x^2 + 94961664*a^9*b^2*x^4 + 1469
632311*a^8*b^3*x^6 + 7323998514*a^7*b^4*x^8 + 19562592546*a^6*b^5*x^10 + 32
314857354*a^5*b^6*x^12 + 34810986496*a^4*b^7*x^14 + 24648575094*a^3*b^8*x^1
6 + 11110024926*a^2*b^9*x^18 + 2900207310*a*b^10*x^20 + 334639305*b^11*x^22
))/(x^5*(a + b*x^2)^9)) - 334639305*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
949120*a^(25/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(a+bx^2\right)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a + b*x^2)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a + b*x^2)^10), x]
```

fricas [A] time = 0.85, size = 726, normalized size = 3.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="fricas")
```

```
[Out] [-1/5898240*(669278610*b^11*x^22 + 5800414620*a*b^10*x^20 + 22220049852*a^2
*b^9*x^18 + 49297150188*a^3*b^8*x^16 + 69621972992*a^4*b^7*x^14 + 646297147
08*a^5*b^6*x^12 + 39125185092*a^6*b^5*x^10 + 14647997028*a^7*b^4*x^8 + 2939
264622*a^8*b^3*x^6 + 189923328*a^9*b^2*x^4 - 9043968*a^10*b*x^2 + 1179648*a
^11 - 334639305*(b^11*x^23 + 9*a*b^10*x^21 + 36*a^2*b^9*x^19 + 84*a^3*b^8*x
^17 + 126*a^4*b^7*x^15 + 126*a^5*b^6*x^13 + 84*a^6*b^5*x^11 + 36*a^7*b^4*x^
9 + 9*a^8*b^3*x^7 + a^9*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)))/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7*x^19 + 84*
a^15*b^6*x^17 + 126*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*b^3*x^11 +
36*a^19*b^2*x^9 + 9*a^20*b*x^7 + a^21*x^5), -1/2949120*(334639305*b^11*x^22
 + 2900207310*a*b^10*x^20 + 11110024926*a^2*b^9*x^18 + 24648575094*a^3*b^8*
x^16 + 34810986496*a^4*b^7*x^14 + 32314857354*a^5*b^6*x^12 + 19562592546*a^
```

```
6*b^5*x^10 + 7323998514*a^7*b^4*x^8 + 1469632311*a^8*b^3*x^6 + 94961664*a^9
*b^2*x^4 - 4521984*a^10*b*x^2 + 589824*a^11 + 334639305*(b^11*x^23 + 9*a*b^
10*x^21 + 36*a^2*b^9*x^19 + 84*a^3*b^8*x^17 + 126*a^4*b^7*x^15 + 126*a^5*b^
6*x^13 + 84*a^6*b^5*x^11 + 36*a^7*b^4*x^9 + 9*a^8*b^3*x^7 + a^9*b^2*x^5)*sq
rt(b/a)*arctan(x*sqrt(b/a)))/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7
*x^19 + 84*a^15*b^6*x^17 + 126*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*
b^3*x^11 + 36*a^19*b^2*x^9 + 9*a^20*b*x^7 + a^21*x^5)]
```

giac [A] time = 0.60, size = 159, normalized size = 0.68

$$-\frac{7436429\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{12}}-\frac{825\,b^2x^4-50\,abx^2+3\,a^2}{15\,a^{12}x^5}-\frac{172437705\,b^{11}x^{17}+1450223310\,ab^{10}x^{15}+5358651102\,a^2b^9x^{13}+11372226678\,a^3b^8x^{11}+15178104832\,a^4b^7x^9+13066540938\,a^5b^6x^7+7101970722\,a^6b^5x^5+2236176690\,a^7b^4x^3+314167095\,a^8b^3x}{2949120\,(bx^2+a)^9\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="giac")
```

```
[Out] -7436429/65536*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^12) - 1/15*(825*b^2*x
^4 - 50*a*b*x^2 + 3*a^2)/(a^12*x^5) - 1/2949120*(172437705*b^11*x^17 + 1450
223310*a*b^10*x^15 + 5358651102*a^2*b^9*x^13 + 11372226678*a^3*b^8*x^11 + 1
5178104832*a^4*b^7*x^9 + 13066540938*a^5*b^6*x^7 + 7101970722*a^6*b^5*x^5 +
 2236176690*a^7*b^4*x^3 + 314167095*a^8*b^3*x)/((b*x^2 + a)^9*a^12)
```

maple [A] time = 0.03, size = 230, normalized size = 0.99

$$-\frac{3831949b^{11}x^{17}}{65536\,(b\,x^2+a)^9\,a^{12}}-\frac{48340777b^{10}x^{15}}{98304\,(b\,x^2+a)^9\,a^{11}}-\frac{297702839b^9x^{13}}{163840\,(b\,x^2+a)^9\,a^{10}}-\frac{631790371b^8x^{11}}{163840\,(b\,x^2+a)^9\,a^9}-\frac{463199b^7x^9}{90\,(b\,x^2+a)^9\,a^8}-\frac{725918941b^6x^7}{163840\,(b\,x^2+a)^9\,a^7}-\frac{394553929b^5x^5}{163840\,(b\,x^2+a)^9\,a^6}-\frac{74539223b^4x^3}{98304\,(b\,x^2+a)^9\,a^5}-\frac{6981491b^3x}{65536\,(b\,x^2+a)^9\,a^4}-\frac{7436429b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}\,a^{12}}-\frac{55b^2}{a^{12}x}+\frac{10b}{3a^{11}x^3}-\frac{1}{5a^{10}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^10,x)
```

```
[Out] -1/5/a^10/x^5-55*b^2/a^12/x+10/3*b/a^11/x^3-6981491/65536/a^4*b^3/(b*x^2+a)
^9*x-74539223/98304/a^5*b^4/(b*x^2+a)^9*x^3-394553929/163840/a^6*b^5/(b*x^2
+a)^9*x^5-725918941/163840/a^7*b^6/(b*x^2+a)^9*x^7-463199/90/a^8*b^7/(b*x^2
+a)^9*x^9-631790371/163840/a^9*b^8/(b*x^2+a)^9*x^11-297702839/163840/a^10*b
^9/(b*x^2+a)^9*x^13-48340777/98304/a^11*b^10/(b*x^2+a)^9*x^15-3831949/65536
/a^12*b^11/(b*x^2+a)^9*x^17-7436429/65536/a^12*b^3/(a*b)^(1/2)*arctan(1/(a*
b)^(1/2)*b*x)
```

maxima [A] time = 3.35, size = 251, normalized size = 1.08

$$-\frac{334639305\,b^{11}x^{22}+2900207310\,ab^{10}x^{20}+11110024926\,a^2b^9x^{18}+24648575094\,a^3b^8x^{16}+34810986496\,a^4b^7x^{14}+32314857354\,a^5b^6x^{12}+19562592546\,a^6b^5x^{10}+7323998514\,a^7b^4x^8+1469632311\,a^8b^3x^6+94961664\,a^9b^2x^4-4521984\,a^{10}bx^2+589824\,a^{11}}{2949120\,\left(a^{12}b^9x^{23}+9\,a^{13}b^8x^{21}+36\,a^{14}b^7x^{19}+84\,a^{15}b^6x^{17}+126\,a^{16}b^5x^{15}+126\,a^{17}b^4x^{13}+84\,a^{18}b^3x^{11}+36\,a^{19}b^2x^9+9\,a^{20}bx^7+a^{21}x^5\right)}-\frac{7436429\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^10,x, algorithm="maxima")
```

```
[Out] -1/2949120*(334639305*b^11*x^22 + 2900207310*a*b^10*x^20 + 11110024926*a^2*
b^9*x^18 + 24648575094*a^3*b^8*x^16 + 34810986496*a^4*b^7*x^14 + 3231485735
4*a^5*b^6*x^12 + 19562592546*a^6*b^5*x^10 + 7323998514*a^7*b^4*x^8 + 146963
2311*a^8*b^3*x^6 + 94961664*a^9*b^2*x^4 - 4521984*a^10*b*x^2 + 589824*a^11)
/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7*x^19 + 84*a^15*b^6*x^17 + 1
26*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*b^3*x^11 + 36*a^19*b^2*x^9 +
 9*a^20*b*x^7 + a^21*x^5) - 7436429/65536*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a^12)
```

mupad [B] time = 5.89, size = 246, normalized size = 1.06

$$-\frac{\frac{1}{5\,a}-\frac{23\,b\,x^2}{15\,a^2}+\frac{161\,b^2\,x^4}{5\,a^3}+\frac{163292479\,b^3\,x^6}{327680\,a^4}+\frac{1220666419\,b^4\,x^8}{491520\,a^5}+\frac{1086810697\,b^5\,x^{10}}{163840\,a^6}+\frac{1795269853\,b^6\,x^{12}}{163840\,a^7}+\frac{1062347\,b^7\,x^{14}}{90\,a^8}+\frac{1369365283\,b^8\,x^{16}}{163840\,a^9}+\frac{617223607\,b^9\,x^{18}}{163840\,a^{10}}+\frac{96673577\,b^{10}\,x^{20}}{98304\,a^{11}}+\frac{7436429\,b^{11}\,x^{22}}{65536\,a^{12}}}{a^9\,x^5+9\,a^8\,b\,x^7+36\,a^7\,b^2\,x^9+84\,a^6\,b^3\,x^{11}+126\,a^5\,b^4\,x^{13}+126\,a^4\,b^5\,x^{15}+84\,a^3\,b^6\,x^{17}+36\,a^2\,b^7\,x^{19}+9\,a\,b^8\,x^{21}+b^9\,x^{23}}-\frac{7436429\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{25/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^10),x)
```

```
[Out] - (1/(5*a) - (23*b*x^2)/(15*a^2) + (161*b^2*x^4)/(5*a^3) + (163292479*b^3*x
^6)/(327680*a^4) + (1220666419*b^4*x^8)/(491520*a^5) + (1086810697*b^5*x^10
)/(163840*a^6) + (1795269853*b^6*x^12)/(163840*a^7) + (1062347*b^7*x^14)/(9
0*a^8) + (1369365283*b^8*x^16)/(163840*a^9) + (617223607*b^9*x^18)/(163840*
a^10) + (96673577*b^10*x^20)/(98304*a^11) + (7436429*b^11*x^22)/(65536*a^12
))/(a^9*x^5 + b^9*x^23 + 9*a^8*b*x^7 + 9*a*b^8*x^21 + 36*a^7*b^2*x^9 + 84*a
^6*b^3*x^11 + 126*a^5*b^4*x^13 + 126*a^4*b^5*x^15 + 84*a^3*b^6*x^17 + 36*a^
2*b^7*x^19) - (7436429*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(25/2))
```

sympy [A] time = 1.50, size = 316, normalized size = 1.36

$$\frac{7436429\sqrt{-\frac{b^5}{a^{25}}}\log\left(-\frac{a^{13}\sqrt{-\frac{b^5}{a^{25}}}}{b^3}+x\right)}{131072}-\frac{7436429\sqrt{-\frac{b^5}{a^{25}}}\log\left(\frac{a^{13}\sqrt{-\frac{b^5}{a^{25}}}}{b^3}+x\right)}{131072}+\frac{-589824a^{11}+4521984a^{10}bx^2-94961664a^9b^2x^4-1469632311a^8b^3x^6-7323998514a^7b^4x^8-19562592546a^6b^5x^{10}-32314857354a^5b^6x^{12}-34810986496a^4b^7x^{14}-24648575094a^3b^8x^{16}-11110024926a^2b^9x^{18}-2900207310ab^{10}x^{20}-334639305b^{11}x^{22}}{2949120a^{21}x^5+26542080a^{20}bx^7+106168320a^{19}b^2x^9+247726080a^{18}b^3x^{11}+371589120a^{17}b^4x^{13}+371589120a^{16}b^5x^{15}+247726080a^{15}b^6x^{17}+106168320a^{14}b^7x^{19}+26542080a^{13}b^8x^{21}+2949120a^{12}b^9x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**10,x)
```

```
[Out] 7436429*sqrt(-b**5/a**25)*log(-a**13*sqrt(-b**5/a**25)/b**3 + x)/131072 - 7
436429*sqrt(-b**5/a**25)*log(a**13*sqrt(-b**5/a**25)/b**3 + x)/131072 + (-5
89824*a**11 + 4521984*a**10*b*x**2 - 94961664*a**9*b**2*x**4 - 1469632311*a
**8*b**3*x**6 - 7323998514*a**7*b**4*x**8 - 19562592546*a**6*b**5*x**10 - 3
2314857354*a**5*b**6*x**12 - 34810986496*a**4*b**7*x**14 - 24648575094*a**3
*b**8*x**16 - 11110024926*a**2*b**9*x**18 - 2900207310*a*b**10*x**20 - 3346
39305*b**11*x**22)/(2949120*a**21*x**5 + 26542080*a**20*b*x**7 + 106168320*
a**19*b**2*x**9 + 247726080*a**18*b**3*x**11 + 371589120*a**17*b**4*x**13 +
 371589120*a**16*b**5*x**15 + 247726080*a**15*b**6*x**17 + 106168320*a**14*
b**7*x**19 + 26542080*a**13*b**8*x**21 + 2949120*a**12*b**9*x**23)
```

3.225 $\int \frac{x^3}{a-bx^2} dx$

Optimal. Leaf size=28

$$-\frac{a\log\left(a-bx^2\right)}{2b^2}-\frac{x^2}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 43}

$$-\frac{a\log\left(a-bx^2\right)}{2b^2}-\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2),x]
```

```
[Out] -x^2/(2*b) - (a*Log[a - b*x^2])/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a-bx^2} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a-bx} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{b}-\frac{a}{b(-a+bx)}\right) dx, x, x^2\right)\\
&= -\frac{x^2}{2b}-\frac{a\log\left(a-bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{a\log\left(a-bx^2\right)}{2b^2}-\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2),x]
[Out] -1/2*x^2/b - (a*Log[a - b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{a-bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a - b*x^2),x]
[Out] IntegrateAlgebraic[x^3/(a - b*x^2), x]
```

fricas [A] time = 0.73, size = 23, normalized size = 0.82

$$-\frac{bx^2+a\log\left(bx^2-a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(b*x^2 + a*log(b*x^2 - a))/b^2
```

giac [A] time = 0.62, size = 26, normalized size = 0.93

$$-\frac{x^2}{2\,b}-\frac{a\log\left(\left|bx^2-a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="giac")
[Out] -1/2*x^2/b - 1/2*a*log(abs(b*x^2 - a))/b^2
```

maple [A] time = 0.00, size = 26, normalized size = 0.93

$$-\frac{x^2}{2b}-\frac{a\ln\left(b\,x^2-a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a),x)
```

```
[Out] -1/2/b*x^2-1/2*a/b^2*ln(b*x^2-a)
```

maxima [A] time = 1.36, size = 25, normalized size = 0.89

$$-\frac{x^2}{2\,b}-\frac{a\log\left(bx^2-a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^2/b - 1/2*a*log(b*x^2 - a)/b^2
```

mupad [B] time = 0.05, size = 23, normalized size = 0.82

$$-\frac{b\,x^2+a\,\ln\left(b\,x^2-a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2),x)
```

```
[Out] -(b*x^2 + a*log(b*x^2 - a))/(2*b^2)
```

sympy [A] time = 0.14, size = 22, normalized size = 0.79

$$-\frac{a\log\left(-a+bx^2\right)}{2b^2}-\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a),x)
```

```
[Out] -a*log(-a + b*x**2)/(2*b**2) - x**2/(2*b)
```

3.226 $\int \frac{x^2}{a-bx^2}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {321, 208}

$$\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2),x]
```

```
[Out] -(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a-bx^2}\,dx &= -\frac{x}{b}+\frac{a\int \frac{1}{a-bx^2}\,dx}{b}\\ &= -\frac{x}{b}+\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2),x]
```

```
[Out] -(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a-bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a - b*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a - b*x^2), x]
```

fricas [A] time = 0.66, size = 80, normalized size = 2.58

$$\left[\frac{\sqrt{\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{\frac{a}{b}}+a}{bx^2-a}\right)-2\,x}{2\,b},\,-\frac{\sqrt{-\frac{a}{b}}\arctan\left(\frac{bx\sqrt{-\frac{a}{b}}}{a}\right)+x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a/b)*log((b*x^2 + 2*b*x*sqrt(a/b) + a)/(b*x^2 - a)) - 2*x)/b, -(
sqrt(-a/b)*arctan(b*x*sqrt(-a/b)/a) + x)/b]
```

giac [A] time = 0.63, size = 29, normalized size = 0.94

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b) - x/b
```

maple [A] time = 0.00, size = 27, normalized size = 0.87

$$\frac{a \operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ b} - \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a),x)
```

```
[Out] -1/b*x+a/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.97, size = 42, normalized size = 1.35

$$-\frac{a \log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}\,b} - \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*a*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*b) - x/b
```

mupad [B] time = 4.57, size = 23, normalized size = 0.74

$$\frac{\sqrt{a}\ \operatorname{atanh}\left(\frac{\sqrt{b}\ x}{\sqrt{a}}\right)}{b^{3/2}} - \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2),x)
```

```
[Out] (a^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/b^(3/2) - x/b
```

sympy [A] time = 0.15, size = 49, normalized size = 1.58

$$-\frac{\sqrt{\frac{a}{b^3}} \log\left(-b\sqrt{\frac{a}{b^3}} + x\right)}{2} + \frac{\sqrt{\frac{a}{b^3}} \log\left(b\sqrt{\frac{a}{b^3}} + x\right)}{2} - \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a),x)
```

```
[Out] -sqrt(a/b**3)*log(-b*sqrt(a/b**3) + x)/2 + sqrt(a/b**3)*log(b*sqrt(a/b**3)
+ x)/2 - x/b
```

3.227 $\int \frac{x}{a-bx^2} \, dx$

Optimal. Leaf size=16

$$-\frac{\log\left(a-bx^2\right)}{2b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {260}

$$-\frac{\log\left(a-bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2),x]
```

```
[Out] -Log[a - b*x^2]/(2*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{a-bx^2} \, dx = -\frac{\log\left(a-bx^2\right)}{2b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\log\left(a-bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2),x]
```

```
[Out] -1/2*Log[a - b*x^2]/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a-bx^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a - b*x^2),x]
```

```
[Out] IntegrateAlgebraic[x/(a - b*x^2), x]
```

fricas [A] time = 0.50, size = 15, normalized size = 0.94

$$-\frac{\log\left(bx^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*log(b*x^2 - a)/b
```

giac [A] time = 0.61, size = 16, normalized size = 1.00

$$-\frac{\log\left(\left|bx^2-a\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(b*x^2 - a))/b
```

maple [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\ln\left(b\,x^2-a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a),x)
```

```
[Out] -1/2/b*ln(b*x^2-a)
```

maxima [A] time = 1.29, size = 15, normalized size = 0.94

$$-\frac{\log\left(bx^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 - a)/b
```

mupad [B] time = 0.03, size = 15, normalized size = 0.94

$$-\frac{\ln\left(b\,x^2-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2),x)
```

```
[Out] -log(b*x^2 - a)/(2*b)
```

sympy [A] time = 0.12, size = 12, normalized size = 0.75

$$-\frac{\log\left(-a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a),x)
```

```
[Out] -log(-a + b*x**2)/(2*b)
```

3.228 $\int \frac{1}{a-bx^2} \, dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-bx^2} \, dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - bx^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a - b*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a - b*x^2)^(-1), x]
```

fricas [A] time = 0.76, size = 68, normalized size = 2.83

$$\left[\frac{\sqrt{ab}\log\left(\frac{bx^2+2\sqrt{ab}\,x+a}{bx^2-a}\right)}{2\,ab}, -\frac{\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a))/(a*b), -sqrt(-a
*b)*arctan(sqrt(-a*b)*x/a)/(a*b)]
```

giac [A] time = 0.59, size = 18, normalized size = 0.75

$$-\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -arctan(b*x/sqrt(-a*b))/sqrt(-a*b)
```

maple [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.00, size = 31, normalized size = 1.29

$$-\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/sqrt(a*b)
```

mupad [B] time = 0.18, size = 16, normalized size = 0.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2),x)
```

```
[Out] atanh((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.14, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{ab}}\log\left(-a\sqrt{\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{\frac{1}{ab}}\log\left(a\sqrt{\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a),x)
```

```
[Out] -sqrt(1/(a*b))*log(-a*sqrt(1/(a*b)) + x)/2 + sqrt(1/(a*b))*log(a*sqrt(1/(a*
b)) + x)/2
```

3.229 $\int \frac{1}{x(a-bx^2)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a}-\frac{\log\left(a-bx^2\right)}{2a}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a-bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)),x]
```

```
[Out] Log[x]/a - Log[a - b*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x(a-bx^2)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a-bx)}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} + \frac{b\,\text{Subst}\left(\int \frac{1}{a-bx}\,dx, x, x^2\right)}{2a}$$
$$= \frac{\log(x)}{a} - \frac{\log(a-bx^2)}{2a}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log(a-bx^2)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)),x]
[Out] Log[x]/a - Log[a - b*x^2]/(2*a)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a-bx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a - b*x^2)),x]
[Out] IntegrateAlgebraic[1/(x*(a - b*x^2)), x]
```

fricas [A] time = 0.86, size = 20, normalized size = 0.87

$$-\frac{\log(bx^2-a) - 2\,\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(log(b*x^2 - a) - 2*log(x))/a
```

giac [A] time = 0.58, size = 26, normalized size = 1.13

$$\frac{\log(x^2)}{2\,a} - \frac{\log(|bx^2-a|)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="giac")
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 - a))/a
```

maple [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^{2}-a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a),x)
[Out] 1/a*ln(x)-1/2/a*ln(b*x^2-a)
```

maxima [A] time = 1.32, size = 25, normalized size = 1.09

$$-\frac{\log\left(bx^{2}-a\right)}{2\,a}+\frac{\log\left(x^{2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 - a)/a + 1/2*log(x^2)/a
```

mupad [B] time = 4.54, size = 21, normalized size = 0.91

$$\frac{\ln(x)}{a}-\frac{\ln\left(a-b\,x^{2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)),x)
[Out] log(x)/a - log(a - b*x^2)/(2*a)
```

sympy [A] time = 0.21, size = 15, normalized size = 0.65

$$\frac{\log\left(x\right)}{a}-\frac{\log\left(-\frac{a}{b}+x^{2}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a),x)
[Out] log(x)/a - log(-a/b + x**2)/(2*a)
```

3.230 $\int \frac{1}{x^2(a-bx^2)}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {325, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a-bx^2\right)}\,dx &= -\frac{1}{ax}+\frac{b\int \frac{1}{a-bx^2}\,dx}{a}\\ &= -\frac{1}{ax}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2(a-bx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a - b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a - b*x^2)), x]
```

fricas [A] time = 0.55, size = 82, normalized size = 2.48

$$\left[\frac{x\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)-2}{2\,ax},-\frac{x\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)) - 2)/(a*x)
, -(x*sqrt(-b/a)*arctan(x*sqrt(-b/a)) + 1)/(a*x)]
```

giac [A] time = 0.62, size = 31, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/(a*x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.88

$$\frac{b\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a),x)
```

```
[Out] -1/a/x+1/a*b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.89, size = 44, normalized size = 1.33

$$-\frac{b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a) - 1/(a*x)
```

mupad [B] time = 4.61, size = 25, normalized size = 0.76

$$\frac{\sqrt{b}\;\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)),x)
```

```
[Out] (b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/a^(3/2) - 1/(a*x)
```

sympy [B] time = 0.19, size = 58, normalized size = 1.76

$$-\frac{\sqrt{\frac{b}{a^3}}\;\log\left(-\frac{a^2\sqrt{\frac{b}{a^3}}}{b}+x\right)}{2}+\frac{\sqrt{\frac{b}{a^3}}\;\log\left(\frac{a^2\sqrt{\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a),x)
```

```
[Out] -sqrt(b/a**3)*log(-a**2*sqrt(b/a**3)/b + x)/2 + sqrt(b/a**3)*log(a**2*sqrt(
b/a**3)/b + x)/2 - 1/(a*x)
```

3.231 $\int \frac{1}{x^3(a-bx^2)}\,dx$

Optimal. Leaf size=35

$$-\frac{b\log\left(a-bx^2\right)}{2a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$-\frac{b\log\left(a-bx^2\right)}{2a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)),x]
```

```
[Out] -1/(2*a*x^2) + (b*Log[x])/a^2 - (b*Log[a - b*x^2])/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a-bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a-bx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{b}{a^2x}+\frac{b^2}{a^2(a-bx)}\right)dx, x, x^2\right)\\ &= -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$-\frac{b\log\left(a-bx^2\right)}{2a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)),x]
[Out] -1/2*1/(a*x^2) + (b*Log[x])/a^2 - (b*Log[a - b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\left(a-bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a - b*x^2)),x]
[Out] IntegrateAlgebraic[1/(x^3*(a - b*x^2)), x]
```

fricas [A] time = 1.07, size = 33, normalized size = 0.94

$$-\frac{bx^2\log\left(bx^2-a\right)-2\,bx^2\log(x)+a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="fricas")
[Out] -1/2*(b*x^2*log(b*x^2 - a) - 2*b*x^2*log(x) + a)/(a^2*x^2)
```

giac [A] time = 0.58, size = 43, normalized size = 1.23

$$\frac{b\log\left(x^2\right)}{2\,a^2}-\frac{b\log\left(\left|bx^2-a\right|\right)}{2\,a^2}-\frac{bx^2+a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="giac")
[Out] 1/2*b*log(x^2)/a^2 - 1/2*b*log(abs(b*x^2 - a))/a^2 - 1/2*(b*x^2 + a)/(a^2*x
^2)
```

maple [A] time = 0.01, size = 33, normalized size = 0.94

$$\frac{b\ln(x)}{a^2}-\frac{b\ln\left(b\,x^2-a\right)}{2a^2}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a),x)
```

```
[Out] -1/2/a/x^2+1/a^2*b*ln(x)-1/2*b/a^2*ln(b*x^2-a)
```

maxima [A] time = 1.39, size = 35, normalized size = 1.00

$$-\frac{b\log\left(bx^2-a\right)}{2\,a^2}+\frac{b\log\left(x^2\right)}{2\,a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*b*log(b*x^2 - a)/a^2 + 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

mupad [B] time = 0.07, size = 31, normalized size = 0.89

$$\frac{b\,\ln(x)}{a^2}-\frac{b\,\ln\left(a-b\,x^2\right)}{2\,a^2}-\frac{1}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)),x)
```

```
[Out] (b*log(x))/a^2 - (b*log(a - b*x^2))/(2*a^2) - 1/(2*a*x^2)
```

sympy [A] time = 0.27, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2}+\frac{b\log\left(x\right)}{a^2}-\frac{b\log\left(-\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a),x)
```

```
[Out] -1/(2*a*x**2) + b*log(x)/a**2 - b*log(-a/b + x**2)/(2*a**2)
```

3.232 $\int \frac{x^3}{(a-bx^2)^2}\,dx$

Optimal. Leaf size=35

$$\frac{a}{2b^2\left(a-bx^2\right)}+\frac{\log\left(a-bx^2\right)}{2b^2}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 43}

$$\frac{a}{2b^2\left(a-bx^2\right)}+\frac{\log\left(a-bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2)^2,x]
```

```
[Out] a/(2*b^2*(a - b*x^2)) + Log[a - b*x^2]/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a-bx^2\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a-bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a}{b(-a+bx)^2}+\frac{1}{b(-a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{a}{2b^2\left(a-bx^2\right)}+\frac{\log\left(a-bx^2\right)}{2b^2}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.83

$$\frac{\frac{a}{a-bx^2}+\log\left(a-bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^2,x]
```

```
[Out] (a/(a - b*x^2) + Log[a - b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a-bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a - b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a - b*x^2)^2, x]
```

fricas [A] time = 0.40, size = 42, normalized size = 1.20

$$\frac{\left(bx^2-a\right)\log\left(bx^2-a\right)-a}{2\left(b^3x^2-ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x^2 - a)*log(b*x^2 - a) - a)/(b^3*x^2 - a*b^2)
```

giac [A] time = 0.63, size = 53, normalized size = 1.51

$$-\frac{\frac{\log\left(\frac{|bx^2-a|}{\left(bx^2-a\right)^2|b|}\right)}{b}+\frac{a}{\left(bx^2-a\right)b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(b*x^2 - a)/((b*x^2 - a)^2*abs(b)))/b + a/((b*x^2 - a)*b))/b
```

maple [A] time = 0.01, size = 34, normalized size = 0.97

$$-\frac{a}{2\left(b\,x^2-a\right)b^2}+\frac{\ln\left(b\,x^2-a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^2,x)
```

```
[Out] 1/2/b^2*ln(b*x^2-a)-1/2*a/b^2/(b*x^2-a)
```

maxima [A] time = 1.35, size = 35, normalized size = 1.00

$$-\frac{a}{2\left(b^3x^2-ab^2\right)}+\frac{\log\left(bx^2-a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a/(b^3*x^2 - a*b^2) + 1/2*log(b*x^2 - a)/b^2
```

mupad [B] time = 0.04, size = 32, normalized size = 0.91

$$\frac{\ln\left(b\,x^2-a\right)}{2\,b^2}+\frac{a}{2\,b^2\,\left(a-b\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2)^2,x)
```

```
[Out] log(b*x^2 - a)/(2*b^2) + a/(2*b^2*(a - b*x^2))
```

sympy [A] time = 0.20, size = 29, normalized size = 0.83

$$-\frac{a}{-2ab^2+2b^3x^2}+\frac{\log\left(-a+bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**2,x)
```

```
[Out] -a/(-2*a*b**2 + 2*b**3*x**2) + log(-a + b*x**2)/(2*b**2)
```

3.233 $\int \frac{x^2}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=46

$$\frac{x}{2b\left(a-bx^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {288, 208}

$$\frac{x}{2b\left(a-bx^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^2,x]
```

```
[Out] x/(2*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a-bx^2\right)^2}\,dx &= \frac{x}{2b\left(a-bx^2\right)}-\frac{\int \frac{1}{a-bx^2}\,dx}{2b}\\ &= \frac{x}{2b\left(a-bx^2\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.02

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b(bx^2-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^2,x]
[Out] -1/2*x/(b*(-a + b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{(a-bx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a - b*x^2)^2,x]
[Out] IntegrateAlgebraic[x^2/(a - b*x^2)^2, x]
```

fricas [A] time = 0.53, size = 127, normalized size = 2.76

$$\left[-\frac{2\,abx-(bx^2-a)\sqrt{ab}\,\log\left(\frac{bx^2-2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{4\,(ab^3x^2-a^2b^2)},-\frac{abx-(bx^2-a)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{2\,(ab^3x^2-a^2b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] [-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 - 2*sqrt(a*b)*x + a)/(b*x
^2 - a)))/(a*b^3*x^2 - a^2*b^2), -1/2*(a*b*x - (b*x^2 - a)*sqrt(-a*b)*arcta
n(sqrt(-a*b)*x/a))/(a*b^3*x^2 - a^2*b^2)]
```

giac [A] time = 0.61, size = 39, normalized size = 0.85

$$\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}\,b}-\frac{x}{2\,(bx^2-a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b) - 1/2*x/((b*x^2 - a)*b)
```

maple [A] time = 0.01, size = 38, normalized size = 0.83

$$-\frac{x}{2\left(b\,x^{2}-a\right)b}-\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^2,x)
```

```
[Out] -1/2/b*x/(b*x^2-a)-1/2/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 3.00, size = 52, normalized size = 1.13

$$-\frac{x}{2\left(b^{2}x^{2}-ab\right)}+\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(b^2*x^2 - a*b) + 1/4*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt
(a*b)*b)
```

mupad [B] time = 4.68, size = 34, normalized size = 0.74

$$\frac{x}{2\,b\,\left(a-b\,x^{2}\right)}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^2,x)
```

```
[Out] x/(2*b*(a - b*x^2)) - atanh((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2))
```

sympy [A] time = 0.21, size = 71, normalized size = 1.54

$$-\frac{x}{-2ab+2b^{2}x^{2}}+\frac{\sqrt{\frac{1}{ab^{3}}}\log\left(-ab\sqrt{\frac{1}{ab^{3}}}+x\right)}{4}-\frac{\sqrt{\frac{1}{ab^{3}}}\log\left(ab\sqrt{\frac{1}{ab^{3}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**2,x)
```

```
[Out] -x/(-2*a*b + 2*b**2*x**2) + sqrt(1/(a*b**3))*log(-a*b*sqrt(1/(a*b**3)) + x)
/4 - sqrt(1/(a*b**3))*log(a*b*sqrt(1/(a*b**3)) + x)/4
```

3.234 $\int \frac{x}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2b\left(a-bx^2\right)}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {261}

$$\frac{1}{2b\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2)^2,x]
```

```
[Out] 1/(2*b*(a - b*x^2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a-bx^2\right)^2}\,dx=\frac{1}{2b\left(a-bx^2\right)}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2b\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2)^2,x]
```

```
[Out] 1/(2*b*(a - b*x^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a-bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a - b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x/(a - b*x^2)^2, x]
```

fricas [A] time = 0.45, size = 16, normalized size = 0.94

$$-\frac{1}{2\left(b^2x^2-ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 - a*b)
```

giac [A] time = 0.61, size = 16, normalized size = 0.94

$$-\frac{1}{2\left(bx^2-a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 - a)*b)
```

maple [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2\left(b\,x^2-a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^2,x)
```

```
[Out] -1/2/b/(b*x^2-a)
```

maxima [A] time = 1.38, size = 16, normalized size = 0.94

$$-\frac{1}{2\left(bx^2-a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x^2 - a)*b)
```

mupad [B] time = 0.02, size = 15, normalized size = 0.88

$$\frac{1}{2\,b\,\left(a-b\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2)^2,x)
```

```
[Out] 1/(2*b*(a - b*x^2))
```

sympy [A] time = 0.16, size = 15, normalized size = 0.88

$$-\frac{1}{-2ab+2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**2,x)
```

```
[Out] -1/(-2*a*b + 2*b**2*x**2)
```

3.235 $\int \frac{1}{\left(a-bx^2\right)^2}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a-bx^2\right)}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-bx^2\right)^2}\,dx &= \frac{x}{2a\left(a-bx^2\right)}+\frac{\int \frac{1}{a-bx^2}\,dx}{2a}\\ &= \frac{x}{2a\left(a-bx^2\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.02

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}-\frac{x}{2a\left(bx^2-a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-2),x]
```

```
[Out] -1/2*x/(a*(-a + b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a-bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a - b*x^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a - b*x^2)^(-2), x]
```

fricas [A] time = 0.81, size = 126, normalized size = 2.74

$$\left[-\frac{2\,abx-\left(bx^2-a\right)\sqrt{ab}\,\log\left(\frac{bx^2+2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{4\left(a^2b^2x^2-a^3b\right)},-\frac{abx+\left(bx^2-a\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{2\left(a^2b^2x^2-a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x
^2 - a)))/(a^2*b^2*x^2 - a^3*b), -1/2*(a*b*x + (b*x^2 - a)*sqrt(-a*b)*arcta
n(sqrt(-a*b)*x/a))/(a^2*b^2*x^2 - a^3*b)]
```

giac [A] time = 0.58, size = 39, normalized size = 0.85

$$-\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}\,a}-\frac{x}{2\left(bx^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2*x/((b*x^2 - a)*a)
```

maple [A] time = 0.00, size = 38, normalized size = 0.83

$$-\frac{x}{2\left(b\,x^{2}-a\right)a}+\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^2,x)
```

```
[Out] -1/2*x/a/(b*x^2-a)+1/2/a/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.96, size = 52, normalized size = 1.13

$$-\frac{x}{2\left(abx^{2}-a^{2}\right)}-\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(a*b*x^2 - a^2) - 1/4*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt
(a*b)*a)
```

mupad [B] time = 4.51, size = 34, normalized size = 0.74

$$\frac{x}{2\,a\,\left(a-b\,x^{2}\right)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^2,x)
```

```
[Out] x/(2*a*(a - b*x^2)) + atanh((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

sympy [A] time = 0.22, size = 71, normalized size = 1.54

$$-\frac{x}{-2a^{2}+2abx^{2}}-\frac{\sqrt{\frac{1}{a^{3}b}}\log\left(-a^{2}\sqrt{\frac{1}{a^{3}b}}+x\right)}{4}+\frac{\sqrt{\frac{1}{a^{3}b}}\log\left(a^{2}\sqrt{\frac{1}{a^{3}b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**2,x)
```

```
[Out] -x/(-2*a**2 + 2*a*b*x**2) - sqrt(1/(a**3*b))*log(-a**2*sqrt(1/(a**3*b)) + x
)/4 + sqrt(1/(a**3*b))*log(a**2*sqrt(1/(a**3*b)) + x)/4
```

3.236 $\int \frac{1}{x\left(a-bx^2\right)^2} \, dx$

Optimal. Leaf size=40

$$-\frac{\log\left(a-bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a-bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$-\frac{\log\left(a-bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)^2),x]
```

```
[Out] 1/(2*a*(a - b*x^2)) + Log[x]/a^2 - Log[a - b*x^2]/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a-bx^2\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x(a-bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{1}{a^2x}+\frac{b}{a(a-bx)^2}+\frac{b}{a^2(a-bx)}\right)dx, x, x^2\right)$$
$$= \frac{1}{2a\left(a-bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a-bx^2\right)}{2a^2}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.88

$$\frac{\frac{a}{a-bx^2}-\log\left(a-bx^2\right)+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^2),x]
```

```
[Out] (a/(a - b*x^2) + 2*Log[x] - Log[a - b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a-bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a - b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a - b*x^2)^2), x]
```

fricas [A] time = 0.50, size = 53, normalized size = 1.32

$$-\frac{\left(bx^2-a\right)\log\left(bx^2-a\right)-2\left(bx^2-a\right)\log(x)+a}{2\left(a^2bx^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 - a)*log(b*x^2 - a) - 2*(b*x^2 - a)*log(x) + a)/(a^2*b*x^2 - a
^3)
```

giac [A] time = 0.63, size = 51, normalized size = 1.28

$$\frac{\log\left(x^2\right)}{2\,a^2} - \frac{\log\left(\left|bx^2 - a\right|\right)}{2\,a^2} + \frac{bx^2 - 2\,a}{2\left(bx^2 - a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 - a))/a^2 + 1/2*(b*x^2 - 2*a)/((b*x^2
- a)*a^2)
```

maple [A] time = 0.01, size = 39, normalized size = 0.98

$$-\frac{1}{2\left(b\,x^2 - a\right)a} + \frac{\ln(x)}{a^2} - \frac{\ln\left(b\,x^2 - a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^2,x)
```

```
[Out] 1/a^2*ln(x)-1/2/a^2*ln(b*x^2-a)-1/2/a/(b*x^2-a)
```

maxima [A] time = 1.36, size = 41, normalized size = 1.02

$$-\frac{1}{2\left(abx^2 - a^2\right)} - \frac{\log\left(bx^2 - a\right)}{2\,a^2} + \frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(a*b*x^2 - a^2) - 1/2*log(b*x^2 - a)/a^2 + 1/2*log(x^2)/a^2
```

mupad [B] time = 0.06, size = 36, normalized size = 0.90

$$\frac{\ln(x)}{a^2} + \frac{1}{2\,a\,\left(a - b\,x^2\right)} - \frac{\ln\left(a - b\,x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)^2),x)
```

```
[Out] log(x)/a^2 + 1/(2*a*(a - b*x^2)) - log(a - b*x^2)/(2*a^2)
```

sympy [A] time = 0.30, size = 34, normalized size = 0.85

$$-\frac{1}{-2a^2+2abx^2}+\frac{\log(x)}{a^2}-\frac{\log\left(-\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**2,x)
```

```
[Out] -1/(-2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(-a/b + x**2)/(2*a**2)
```

3.237 $\int \frac{1}{x^2(a-bx^2)^2}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a-bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {290, 325, 208}

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a-bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a - b*x^2)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(5/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^2\right)^2}\,dx = \frac{1}{2ax\left(a-bx^2\right)} + \frac{3\int \frac{1}{x^2\left(a-bx^2\right)}\,dx}{2a}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax\left(a-bx^2\right)} + \frac{(3b)\int \frac{1}{a-bx^2}\,dx}{2a^2}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax\left(a-bx^2\right)} + \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.97

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{bx}{2a^2\left(bx^2-a\right)} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^2),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(-a + b*x^2)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a-bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a - b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a - b*x^2)^2), x]
```

fricas [A] time = 0.65, size = 140, normalized size = 2.41

$$\left[-\frac{6\,bx^2 - 3\left(bx^3-ax\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right) - 4\,a}{4\left(a^2bx^3-a^3x\right)}, -\frac{3\,bx^2 + 3\left(bx^3-ax\right)\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right) - 2\,a}{2\left(a^2bx^3-a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 - a*x)*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a
)/(b*x^2 - a)) - 4*a)/(a^2*b*x^3 - a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 - a*x)*
sqrt(-b/a)*arctan(x*sqrt(-b/a)) - 2*a)/(a^2*b*x^3 - a^3*x)]
```

giac [A] time = 0.58, size = 50, normalized size = 0.86

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}\,a^2}-\frac{3\,bx^2-2\,a}{2\,\left(bx^3-ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/2*(3*b*x^2 - 2*a)/((b*x^
3 - a*x)*a^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.81

$$-\frac{\left(\frac{x}{2b\,x^2-2a}-\frac{3\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)b}{a^2}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^2,x)
```

```
[Out] -1/a^2/x-1/a^2*b*(1/2*x/(b*x^2-a)-3/2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x
))
```

maxima [A] time = 2.98, size = 65, normalized size = 1.12

$$-\frac{3\,bx^2-2\,a}{2\,\left(a^2bx^3-a^3x\right)}-\frac{3\,b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 - 2*a)/(a^2*b*x^3 - a^3*x) - 3/4*b*log((b*x - sqrt(a*b))/(b*x
 + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

mupad [B] time = 4.63, size = 45, normalized size = 0.78

$$\frac{3\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{\frac{1}{a}-\frac{3\,b\,x^2}{2\,a^2}}{a\,x-b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^2),x)
```

```
[Out] (3*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(2*a^(5/2)) - (1/a - (3*b*x^2)/(2*a^
2))/(a*x - b*x^3)
```

sympy [A] time = 0.30, size = 83, normalized size = 1.43

$$-\frac{3\sqrt{\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{3\sqrt{\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{2a-3bx^2}{-2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**2,x)
```

```
[Out] -3*sqrt(b/a**5)*log(-a**3*sqrt(b/a**5)/b + x)/4 + 3*sqrt(b/a**5)*log(a**3*s
qrt(b/a**5)/b + x)/4 + (2*a - 3*b*x**2)/(-2*a**3*x + 2*a**2*b*x**3)
```

3.238 $\int \frac{1}{x^3(a-bx^2)^2} dx$

Optimal. Leaf size=52

$$-\frac{b \log \left(a-bx^2\right)}{a^3}+\frac{2b \log(x)}{a^3}+\frac{b}{2a^2 \left(a-bx^2\right)}-\frac{1}{2a^2x^2}$$

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{b}{2a^2 \left(a-bx^2\right)}-\frac{b \log \left(a-bx^2\right)}{a^3}+\frac{2b \log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^2),x]
```

```
[Out] -1/(2*a^2*x^2) + b/(2*a^2*(a - b*x^2)) + (2*b*Log[x])/a^3 - (b*Log[a - b*x^
2])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3(a-bx^2)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a-bx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^2x^2} + \frac{2b}{a^3x} + \frac{b^2}{a^2(a-bx)^2} + \frac{2b^2}{a^3(a-bx)}\right) dx, x, x^2\right)$$
$$= -\frac{1}{2a^2x^2} + \frac{b}{2a^2(a-bx^2)} + \frac{2b\log(x)}{a^3} - \frac{b\log(a-bx^2)}{a^3}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.85

$$\frac{\frac{ab}{a-bx^2} - 2b\log(a-bx^2) - \frac{a}{x^2} + 4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^2),x]
[Out] (-(a/x^2) + (a*b)/(a - b*x^2) + 4*b*Log[x] - 2*b*Log[a - b*x^2])/(2*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a-bx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a - b*x^2)^2),x]
[Out] IntegrateAlgebraic[1/(x^3*(a - b*x^2)^2), x]
```

fricas [A] time = 0.55, size = 80, normalized size = 1.54

$$-\frac{2\,abx^2 - a^2 + 2\,(b^2x^4 - abx^2)\log(bx^2 - a) - 4\,(b^2x^4 - abx^2)\log(x)}{2\,(a^3bx^4 - a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] -1/2*(2*a*b*x^2 - a^2 + 2*(b^2*x^4 - a*b*x^2)*log(b*x^2 - a) - 4*(b^2*x^4 -
 a*b*x^2)*log(x))/(a^3*b*x^4 - a^4*x^2)
```

giac [A] time = 0.60, size = 56, normalized size = 1.08

$$\frac{b\log\left(x^2\right)}{a^3}-\frac{b\log\left(\left|bx^2-a\right|\right)}{a^3}-\frac{2\,bx^2-a}{2\left(bx^4-ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] b*log(x^2)/a^3 - b*log(abs(b*x^2 - a))/a^3 - 1/2*(2*b*x^2 - a)/((b*x^4 - a*
x^2)*a^2)
```

maple [A] time = 0.01, size = 51, normalized size = 0.98

$$-\frac{b}{2\left(b\,x^2-a\right)a^2}+\frac{2b\ln(x)}{a^3}-\frac{b\ln\left(b\,x^2-a\right)}{a^3}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2/x^2+2/a^3*b*ln(x)-b/a^3*ln(b*x^2-a)-1/2*b/a^2/(b*x^2-a)
```

maxima [A] time = 1.31, size = 57, normalized size = 1.10

$$-\frac{2\,bx^2-a}{2\left(a^2bx^4-a^3x^2\right)}-\frac{b\log\left(bx^2-a\right)}{a^3}+\frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 - a)/(a^2*b*x^4 - a^3*x^2) - b*log(b*x^2 - a)/a^3 + b*log(x^2
)/a^3
```

mupad [B] time = 4.59, size = 55, normalized size = 1.06

$$\frac{2\,b\,\ln(x)}{a^3}-\frac{b\,\ln\left(a-b\,x^2\right)}{a^3}-\frac{\frac{1}{2\,a}-\frac{b\,x^2}{a^2}}{a\,x^2-b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)^2),x)
```

```
[Out] (2*b*log(x))/a^3 - (b*log(a - b*x^2))/a^3 - (1/(2*a) - (b*x^2)/a^2)/(a*x^2
- b*x^4)
```

sympy [A] time = 0.37, size = 49, normalized size = 0.94

$$\frac{a-2bx^2}{-2a^3x^2+2a^2bx^4}+\frac{2b\log(x)}{a^3}-\frac{b\log\left(-\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**2,x)
```

```
[Out] (a - 2*b*x**2)/(-2*a**3*x**2 + 2*a**2*b*x**4) + 2*b*log(x)/a**3 - b*log(-a/
b + x**2)/a**3
```

3.239 $\int \frac{x^3}{(a-bx^2)^3} dx$

Optimal. Leaf size=20

$$\frac{x^4}{4a(a-bx^2)^2}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {264}

$$\frac{x^4}{4a(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2)^3,x]
```

```
[Out] x^4/(4*a*(a - b*x^2)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(a-bx^2)^3} dx = \frac{x^4}{4a(a-bx^2)^2}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.25

$$-\frac{a-2bx^2}{4b^2(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^3,x]
```

```
[Out] -1/4*(a - 2*b*x^2)/(b^2*(a - b*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a-bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a - b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a - b*x^2)^3, x]
```

fricas [A] time = 0.49, size = 38, normalized size = 1.90

$$\frac{2\,bx^2-a}{4\left(b^4x^4-2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.62, size = 26, normalized size = 1.30

$$\frac{2\,bx^2-a}{4\left(bx^2-a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b*x^2 - a)/((b*x^2 - a)^2*b^2)
```

maple [A] time = 0.01, size = 35, normalized size = 1.75

$$\frac{a}{4\left(b\,x^2-a\right)^2b^2}+\frac{1}{2\left(b\,x^2-a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^3,x)
```

```
[Out] 1/4*a/b^2/(b*x^2-a)^2+1/2/b^2/(b*x^2-a)
```

maxima [A] time = 1.31, size = 38, normalized size = 1.90

$$\frac{2\,bx^2-a}{4\left(b^4x^4-2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)
```

mupad [B] time = 0.04, size = 37, normalized size = 1.85

$$-\frac{\frac{a}{4\,b^2}-\frac{x^2}{2\,b}}{a^2-2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2)^3,x)
```

```
[Out] -(a/(4*b^2) - x^2/(2*b))/(a^2 + b^2*x^4 - 2*a*b*x^2)
```

sympy [B] time = 0.27, size = 36, normalized size = 1.80

$$-\frac{a-2bx^2}{4a^2b^2-8ab^3x^2+4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**3,x)
```

```
[Out] -(a - 2*b*x**2)/(4*a**2*b**2 - 8*a*b**3*x**2 + 4*b**4*x**4)
```

3.240 $\int \frac{x^2}{\left(a-bx^2\right)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}-\frac{x}{8ab\left(a-bx^2\right)}+\frac{x}{4b\left(a-bx^2\right)^2}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {288, 199, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}-\frac{x}{8ab\left(a-bx^2\right)}+\frac{x}{4b\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^3,x]
```

```
[Out] x/(4*b*(a - b*x^2)^2) - x/(8*a*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]
]/(8*a^(3/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^2\right)^3}\,dx = \frac{x}{4b\left(a-bx^2\right)^2} - \frac{\int \frac{1}{\left(a-bx^2\right)^2}\,dx}{4b}$$
$$= \frac{x}{4b\left(a-bx^2\right)^2} - \frac{x}{8ab\left(a-bx^2\right)} - \frac{\int \frac{1}{a-bx^2}\,dx}{8ab}$$
$$= \frac{x}{4b\left(a-bx^2\right)^2} - \frac{x}{8ab\left(a-bx^2\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.84

$$\frac{x\left(a+bx^2\right)}{8ab\left(a-bx^2\right)^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2))/(8*a*b*(a - b*x^2)^2) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(8*a^(
3/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a-bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a - b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a - b*x^2)^3, x]
```

fricas [A] time = 0.58, size = 188, normalized size = 2.81

$$\left[\frac{2\,ab^2x^3 + 2\,a^2bx + \left(b^2x^4 - 2\,abx^2 + a^2\right)\sqrt{ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{ab}\,x + a}{bx^2 - a}\right)}{16\left(a^2b^4x^4 - 2\,a^3b^3x^2 + a^4b^2\right)}, \frac{ab^2x^3 + a^2bx + \left(b^2x^4 - 2\,abx^2 + a^2\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{8\left(a^2b^4x^4 - 2\,a^3b^3x^2 + a^4b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 + 2*a^2*b*x + (b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)*log(
(b*x^2 - 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^2*b^4*x^4 - 2*a^3*b^3*x^2 + a^
4*b^2), 1/8*(a*b^2*x^3 + a^2*b*x + (b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-a*b)*a
rctan(sqrt(-a*b)*x/a))/(a^2*b^4*x^4 - 2*a^3*b^3*x^2 + a^4*b^2)]
```

giac [A] time = 0.63, size = 53, normalized size = 0.79

$$\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\sqrt{-ab}\,ab} + \frac{bx^3 + ax}{8\left(bx^2 - a\right)^2 ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a*b) + 1/8*(b*x^3 + a*x)/((b*x^2 - a
)^2*a*b)
```

maple [A] time = 0.01, size = 52, normalized size = 0.78

$$-\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;ab} - \frac{-\frac{x^3}{8a} - \frac{x}{8b}}{\left(b\,x^2 - a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^3,x)
```

```
[Out] -(-1/8/a*x^3-1/8/b*x)/(b*x^2-a)^2-1/8/b/a/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)
*b*x)
```

maxima [A] time = 2.89, size = 76, normalized size = 1.13

$$\frac{bx^3 + ax}{8\left(ab^3x^4 - 2\,a^2b^2x^2 + a^3b\right)} + \frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^3 + a*x)/(a*b^3*x^4 - 2*a^2*b^2*x^2 + a^3*b) + 1/16*log((b*x - sqr
t(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a*b)
```

mupad [B] time = 4.62, size = 54, normalized size = 0.81

$$\frac{\frac{x}{8\,b}+\frac{x^3}{8\,a}}{a^2-2\,a\,b\,x^2+b^2\,x^4}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^3,x)
```

```
[Out] (x/(8*b) + x^3/(8*a))/(a^2 + b^2*x^4 - 2*a*b*x^2) - atanh((b^(1/2)*x)/a^(1/
2))/(8*a^(3/2)*b^(3/2))
```

sympy [B] time = 0.32, size = 105, normalized size = 1.57

$$\frac{\sqrt{\frac{1}{a^3b^3}}\log\left(-a^2b\sqrt{\frac{1}{a^3b^3}}+x\right)}{16}-\frac{\sqrt{\frac{1}{a^3b^3}}\log\left(a^2b\sqrt{\frac{1}{a^3b^3}}+x\right)}{16}-\frac{-ax-bx^3}{8a^3b-16a^2b^2x^2+8ab^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**3,x)
```

```
[Out] sqrt(1/(a**3*b**3))*log(-a**2*b*sqrt(1/(a**3*b**3)) + x)/16 - sqrt(1/(a**3*
b**3))*log(a**2*b*sqrt(1/(a**3*b**3)) + x)/16 - (-a*x - b*x**3)/(8*a**3*b -
 16*a**2*b**2*x**2 + 8*a*b**3*x**4)
```

3.241 $\int \frac{x}{(a-bx^2)^3} \, dx$

Optimal. Leaf size=17

$$\frac{1}{4b(a-bx^2)^2}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {261}

$$\frac{1}{4b(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2)^3,x]
```

```
[Out] 1/(4*b*(a - b*x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a-bx^2)^3} \, dx = \frac{1}{4b(a-bx^2)^2}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{4b(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2)^3,x]
```

```
[Out] 1/(4*b*(a - b*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a-bx^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a - b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x/(a - b*x^2)^3, x]
```

fricas [A] time = 0.65, size = 26, normalized size = 1.53

$$\frac{1}{4\left(b^3x^4-2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4/(b^3*x^4 - 2*a*b^2*x^2 + a^2*b)
```

giac [A] time = 0.61, size = 16, normalized size = 0.94

$$\frac{1}{4\left(bx^2-a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/4/((b*x^2 - a)^2*b)
```

maple [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{4\left(b\,x^2-a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^3,x)
```

```
[Out] 1/4/b/(b*x^2-a)^2
```

maxima [A] time = 1.34, size = 16, normalized size = 0.94

$$\frac{1}{4\left(bx^2-a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4/((b*x^2 - a)^2*b)
```

mupad [B] time = 0.03, size = 26, normalized size = 1.53

$$\frac{1}{4\,a^2\,b-8\,a\,b^2\,x^2+4\,b^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2)^3,x)
```

```
[Out] 1/(4*a^2*b + 4*b^3*x^4 - 8*a*b^2*x^2)
```

sympy [B] time = 0.24, size = 26, normalized size = 1.53

$$\frac{1}{4a^2b-8ab^2x^2+4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**3,x)
```

```
[Out] 1/(4*a**2*b - 8*a*b**2*x**2 + 4*b**3*x**4)
```

3.242 $\int \frac{1}{(a-bx^2)^3} dx$

Optimal. Leaf size=64

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{x}{4a\left(a-bx^2\right)^2}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {199, 208}

$$\frac{3x}{8a^2\left(a-bx^2\right)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{x}{4a\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a - b*x^2)^2) + (3*x)/(8*a^2*(a - b*x^2)) + (3*ArcTanh[(Sqrt[b]*x)/
Sqrt[a]])/(8*a^(5/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^3}\,dx = \frac{x}{4a(a-bx^2)^2} + \frac{3\int \frac{1}{(a-bx^2)^2}\,dx}{4a}$$
$$= \frac{x}{4a(a-bx^2)^2} + \frac{3x}{8a^2(a-bx^2)} + \frac{3\int \frac{1}{a-bx^2}\,dx}{8a^2}$$
$$= \frac{x}{4a(a-bx^2)^2} + \frac{3x}{8a^2(a-bx^2)} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.88

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{5ax-3bx^3}{8a^2(a-bx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-3),x]
```

```
[Out] (5*a*x - 3*b*x^3)/(8*a^2*(a - b*x^2)^2) + (3*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(8*a^(5/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a-bx^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a - b*x^2)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a - b*x^2)^(-3), x]
```

fricas [A] time = 0.50, size = 188, normalized size = 2.94

$$\left[-\frac{6\,ab^2x^3 - 10\,a^2bx - 3\,(b^2x^4 - 2\,abx^2 + a^2)\sqrt{ab}\,\log\left(\frac{bx^2+2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{16\,(a^3b^3x^4 - 2\,a^4b^2x^2 + a^5b)}, -\frac{3\,ab^2x^3 - 5\,a^2bx + 3\,(b^2x^4 - 2\,abx^2 + a^2)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{8\,(a^3b^3x^4 - 2\,a^4b^2x^2 + a^5b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(6*a*b^2*x^3 - 10*a^2*b*x - 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)*
log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^3*b^3*x^4 - 2*a^4*b^2*x^2
+ a^5*b), -1/8*(3*a*b^2*x^3 - 5*a^2*b*x + 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqr
t(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^3*b^3*x^4 - 2*a^4*b^2*x^2 + a^5*b)]
```

giac [A] time = 0.62, size = 49, normalized size = 0.77

$$-\frac{3 \arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8 \sqrt{-ab}\, a^2} - \frac{3 bx^3 - 5 ax}{8 \left(bx^2 - a\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/8*(3*b*x^3 - 5*a*x)/((b*x^
2 - a)^2*a^2)
```

maple [A] time = 0.00, size = 61, normalized size = 0.95

$$\frac{x}{4 \left(b\, x^2 - a\right)^2 a} + \frac{-\frac{3x}{8\left(b\, x^2-a\right)a} + \frac{3 \operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab}\; a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^3,x)
```

```
[Out] 1/4*x/a/(b*x^2-a)^2+3/4/a*(-1/2/(b*x^2-a)/a*x+1/2/a/(a*b)^(1/2)*arctanh(1/(
a*b)^(1/2)*b*x))
```

maxima [A] time = 2.99, size = 73, normalized size = 1.14

$$-\frac{3 bx^3 - 5 ax}{8 \left(a^2b^2x^4 - 2 a^3bx^2 + a^4\right)} - \frac{3 \log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16 \sqrt{ab}\, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b*x^3 - 5*a*x)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 3/16*log((b*x -
sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

mupad [B] time = 4.60, size = 55, normalized size = 0.86

$$\frac{\frac{5\,x}{8\,a}-\frac{3\,b\,x^3}{8\,a^2}}{a^2-2\,a\,b\,x^2+b^2\,x^4}+\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^3,x)
```

```
[Out] ((5*x)/(8*a) - (3*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 - 2*a*b*x^2) + (3*atanh((b
^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

sympy [A] time = 0.33, size = 99, normalized size = 1.55

$$-\frac{3\sqrt{\frac{1}{a^5b}}\log\left(-a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16}+\frac{3\sqrt{\frac{1}{a^5b}}\log\left(a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16}-\frac{-5ax+3bx^3}{8a^4-16a^3bx^2+8a^2b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(1/(a**5*b))*log(-a**3*sqrt(1/(a**5*b)) + x)/16 + 3*sqrt(1/(a**5*b))
*log(a**3*sqrt(1/(a**5*b)) + x)/16 - (-5*a*x + 3*b*x**3)/(8*a**4 - 16*a**3*
b*x**2 + 8*a**2*b**2*x**4)
```

3.243 $\int \frac{1}{x(a-bx^2)^3} \, dx$

Optimal. Leaf size=57

$$-\frac{\log\left(a-bx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{1}{2a^2\left(a-bx^2\right)}+\frac{1}{4a\left(a-bx^2\right)^2}$$

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{1}{2a^2\left(a-bx^2\right)}-\frac{\log\left(a-bx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{1}{4a\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)^3),x]
```

```
[Out] 1/(4*a*(a - b*x^2)^2) + 1/(2*a^2*(a - b*x^2)) + Log[x]/a^3 - Log[a - b*x^2]
/(2*a^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a-bx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a-bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^3x} + \frac{b}{a(a-bx)^3} + \frac{b}{a^2(a-bx)^2} + \frac{b}{a^3(a-bx)}\right)dx, x, x^2\right)$$
$$= \frac{1}{4a\left(a-bx^2\right)^2} + \frac{1}{2a^2\left(a-bx^2\right)} + \frac{\log(x)}{a^3} - \frac{\log\left(a-bx^2\right)}{2a^3}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.79

$$\frac{\frac{a\left(3a-2bx^2\right)}{\left(a-bx^2\right)^2} - 2\log\left(a-bx^2\right) + 4\log(x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^3),x]
```

```
[Out] ((a*(3*a - 2*b*x^2))/(a - b*x^2)^2 + 4*Log[x] - 2*Log[a - b*x^2])/(4*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a-bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a - b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a - b*x^2)^3), x]
```

fricas [A] time = 0.48, size = 92, normalized size = 1.61

$$-\frac{2\,abx^2 - 3\,a^2 + 2\left(b^2x^4 - 2\,abx^2 + a^2\right)\log\left(bx^2 - a\right) - 4\left(b^2x^4 - 2\,abx^2 + a^2\right)\log(x)}{4\left(a^3b^2x^4 - 2\,a^4bx^2 + a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*x^2 - 3*a^2 + 2*(b^2*x^4 - 2*a*b*x^2 + a^2)*log(b*x^2 - a) - 4*
(b^2*x^4 - 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 - 2*a^4*b*x^2 + a^5)
```

giac [A] time = 0.63, size = 63, normalized size = 1.11

$$\frac{\log\left(x^2\right)}{2\,a^3} - \frac{\log\left(\left|bx^2 - a\right|\right)}{2\,a^3} + \frac{3\,b^2x^4 - 8\,abx^2 + 6\,a^2}{4\left(bx^2 - a\right)^2 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^3 - 1/2*log(abs(b*x^2 - a))/a^3 + 1/4*(3*b^2*x^4 - 8*a*b*x^2
 + 6*a^2)/((b*x^2 - a)^2*a^3)
```

maple [A] time = 0.01, size = 55, normalized size = 0.96

$$\frac{1}{4\left(b\,x^2 - a\right)^2 a} - \frac{1}{2\left(b\,x^2 - a\right)a^2} + \frac{\ln(x)}{a^3} - \frac{\ln\left(b\,x^2 - a\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^3,x)
```

```
[Out] 1/a^3*ln(x)-1/2/a^3*ln(b*x^2-a)+1/4/a/(b*x^2-a)^2-1/2/a^2/(b*x^2-a)
```

maxima [A] time = 1.37, size = 62, normalized size = 1.09

$$-\frac{2\,bx^2 - 3\,a}{4\left(a^2b^2x^4 - 2\,a^3bx^2 + a^4\right)} - \frac{\log\left(bx^2 - a\right)}{2\,a^3} + \frac{\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 - 3*a)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 - a)
/a^3 + 1/2*log(x^2)/a^3
```

mupad [B] time = 0.06, size = 57, normalized size = 1.00

$$\frac{\ln(x)}{a^3} + \frac{\frac{3}{4\,a} - \frac{b\,x^2}{2\,a^2}}{a^2 - 2\,a\,b\,x^2 + b^2\,x^4} - \frac{\ln\left(a - b\,x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)^3),x)
```

```
[Out] log(x)/a^3 + (3/(4*a) - (b*x^2)/(2*a^2))/(a^2 + b^2*x^4 - 2*a*b*x^2) - log(
a - b*x^2)/(2*a^3)
```

sympy [A] time = 0.42, size = 56, normalized size = 0.98

$$-\frac{-3a+2bx^2}{4a^4-8a^3bx^2+4a^2b^2x^4}+\frac{\log(x)}{a^3}-\frac{\log\left(-\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**3,x)
```

```
[Out] -(-3*a + 2*b*x**2)/(4*a**4 - 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + log(x)/a**
3 - log(-a/b + x**2)/(2*a**3)
```

3.244 $\int \frac{1}{x^2(a-bx^2)^3} dx$

Optimal. Leaf size=78

$$\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{1}{4ax\left(a-bx^2\right)^2}$$

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {290, 325, 208}

$$\frac{5}{8a^2x\left(a-bx^2\right)}+\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15}{8a^3x}+\frac{1}{4ax\left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^3),x]
```

```
[Out] -15/(8*a^3*x) + 1/(4*a*x*(a - b*x^2)^2) + 5/(8*a^2*x*(a - b*x^2)) + (15*Sqr
t[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a-bx^2\right)^3} \, dx = \frac{1}{4ax \left(a-bx^2\right)^2} + \frac{5 \int \frac{1}{x^2 \left(a-bx^2\right)^2} \, dx}{4a}$$
$$= \frac{1}{4ax \left(a-bx^2\right)^2} + \frac{5}{8a^2 x \left(a-bx^2\right)} + \frac{15 \int \frac{1}{x^2 \left(a-bx^2\right)} \, dx}{8a^2}$$
$$= -\frac{15}{8a^3 x} + \frac{1}{4ax \left(a-bx^2\right)^2} + \frac{5}{8a^2 x \left(a-bx^2\right)} + \frac{(15b) \int \frac{1}{a-bx^2} \, dx}{8a^3}$$
$$= -\frac{15}{8a^3 x} + \frac{1}{4ax \left(a-bx^2\right)^2} + \frac{5}{8a^2 x \left(a-bx^2\right)} + \frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.88

$$\frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}} + \frac{-8a^2 + 25abx^2 - 15b^2x^4}{8a^3 x \left(a-bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^3),x]
```

```
[Out] (-8*a^2 + 25*a*b*x^2 - 15*b^2*x^4)/(8*a^3*x*(a - b*x^2)^2) + (15*Sqrt[b]*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a-bx^2\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a - b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a - b*x^2)^3), x]
```

fricas [A] time = 0.58, size = 202, normalized size = 2.59

$$\left[-\frac{30\,b^2x^4-50\,abx^2-15\left(b^2x^5-2\,abx^3+a^2x\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)+16\,a^2}{16\left(a^3b^2x^5-2\,a^4bx^3+a^5x\right)},-\frac{15\,b^2x^4-25\,abx^2+15\left(b^2x^5-2\,abx^3+a^2x\right)\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right)+8\,a^2}{8\left(a^3b^2x^5-2\,a^4bx^3+a^5x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 - 50*a*b*x^2 - 15*(b^2*x^5 - 2*a*b*x^3 + a^2*x)*sqrt(b/a
)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)) + 16*a^2)/(a^3*b^2*x^5 - 2
*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 - 25*a*b*x^2 + 15*(b^2*x^5 - 2*a*b*x^
3 + a^2*x)*sqrt(-b/a)*arctan(x*sqrt(-b/a)) + 8*a^2)/(a^3*b^2*x^5 - 2*a^4*b*
x^3 + a^5*x)]
```

giac [A] time = 0.63, size = 61, normalized size = 0.78

$$-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\,\sqrt{-ab}\,a^3}-\frac{7\,b^2x^3-9\,abx}{8\left(bx^2-a\right)^2a^3}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3) - 1/8*(7*b^2*x^3 - 9*a*b*x)
/((b*x^2 - a)^2*a^3) - 1/(a^3*x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.72

$$-\frac{\left(-\frac{15\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}+\frac{\frac{7}{8}b\,x^3-\frac{9}{8}ax}{\left(b\,x^2-a\right)^2}\right)b}{a^3}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^3,x)
```

```
[Out] -1/a^3/x-1/a^3*b*((7/8*b*x^3-9/8*a*x)/(b*x^2-a)^2-15/8/(a*b)^(1/2)*arctanh(
1/(a*b)^(1/2)*b*x))
```

maxima [A] time = 2.91, size = 86, normalized size = 1.10

$$-\frac{15\,b^2x^4-25\,abx^2+8\,a^2}{8\left(a^3b^2x^5-2\,a^4bx^3+a^5x\right)}-\frac{15\,b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*b^2*x^4 - 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 - 2*a^4*b*x^3 + a^5*x)
- 15/16*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^3)
```

mupad [B] time = 4.60, size = 66, normalized size = 0.85

$$\frac{15\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{\frac{1}{a}-\frac{25\,b\,x^2}{8\,a^2}+\frac{15\,b^2\,x^4}{8\,a^3}}{a^2\,x-2\,a\,b\,x^3+b^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^3),x)
```

```
[Out] (15*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2)) - (1/a - (25*b*x^2)/(8*
a^2) + (15*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 - 2*a*b*x^3)
```

sympy [A] time = 0.43, size = 107, normalized size = 1.37

$$-\frac{15\sqrt{\frac{b}{a^7}}\log\left(-\frac{a^4\sqrt{\frac{b}{a^7}}}{b}+x\right)}{16}+\frac{15\sqrt{\frac{b}{a^7}}\log\left(\frac{a^4\sqrt{\frac{b}{a^7}}}{b}+x\right)}{16}-\frac{8a^2-25abx^2+15b^2x^4}{8a^5x-16a^4bx^3+8a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**3,x)
```

```
[Out] -15*sqrt(b/a**7)*log(-a**4*sqrt(b/a**7)/b + x)/16 + 15*sqrt(b/a**7)*log(a**
4*sqrt(b/a**7)/b + x)/16 - (8*a**2 - 25*a*b*x**2 + 15*b**2*x**4)/(8*a**5*x
- 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

3.245 $\int \frac{1}{x^3(a-bx^2)^3} dx$

Optimal. Leaf size=69

$$-\frac{3b\log\left(a-bx^2\right)}{2a^4}+\frac{3b\log(x)}{a^4}+\frac{b}{a^3\left(a-bx^2\right)}-\frac{1}{2a^3x^2}+\frac{b}{4a^2\left(a-bx^2\right)^2}$$

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{b}{a^3\left(a-bx^2\right)}+\frac{b}{4a^2\left(a-bx^2\right)^2}-\frac{3b\log\left(a-bx^2\right)}{2a^4}+\frac{3b\log(x)}{a^4}-\frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^3),x]
```

```
[Out] -1/(2*a^3*x^2) + b/(4*a^2*(a - b*x^2)^2) + b/(a^3*(a - b*x^2)) + (3*b*Log[x
])/a^4 - (3*b*Log[a - b*x^2])/(2*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a-bx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a-bx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{a^2(a-bx)^3}+\frac{2b^2}{a^3(a-bx)^2}+\frac{3b^2}{a^4(a-bx)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{2a^3x^2}+\frac{b}{4a^2\left(a-bx^2\right)^2}+\frac{b}{a^3\left(a-bx^2\right)}+\frac{3b\log(x)}{a^4}-\frac{3b\log\left(a-bx^2\right)}{2a^4}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.87

$$\frac{\frac{a\left(-2a^2+9abx^2-6b^2x^4\right)}{\left(ax-bx^3\right)^2}-6b\log\left(a-bx^2\right)+12b\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^3),x]
```

```
[Out] ((a*(-2*a^2 + 9*a*b*x^2 - 6*b^2*x^4))/(a*x - b*x^3)^2 + 12*b*Log[x] - 6*b*L
og[a - b*x^2])/(4*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a-bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a - b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a - b*x^2)^3), x]
```

fricas [A] time = 0.61, size = 121, normalized size = 1.75

$$-\frac{6\,ab^2x^4-9\,a^2bx^2+2\,a^3+6\left(b^3x^6-2\,ab^2x^4+a^2bx^2\right)\log\left(bx^2-a\right)-12\left(b^3x^6-2\,ab^2x^4+a^2bx^2\right)\log(x)}{4\left(a^4b^2x^6-2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3 + 6*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 - a) - 12*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 - 2*a^5*b*x^4 + a^6*x^2)
```

giac [A] time = 0.63, size = 84, normalized size = 1.22

$$\frac{3\,b\log\left(x^2\right)}{2\,a^4} - \frac{3\,b\log\left(\left|bx^2-a\right|\right)}{2\,a^4} + \frac{9\,b^3x^4 - 22\,ab^2x^2 + 14\,a^2b}{4\left(bx^2-a\right)^2a^4} - \frac{3\,bx^2+a}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*b*log(x^2)/a^4 - 3/2*b*log(abs(b*x^2 - a))/a^4 + 1/4*(9*b^3*x^4 - 22*a*
b^2*x^2 + 14*a^2*b)/((b*x^2 - a)^2*a^4) - 1/2*(3*b*x^2 + a)/(a^4*x^2)
```

maple [A] time = 0.01, size = 68, normalized size = 0.99

$$\frac{b}{4\left(b\,x^2-a\right)^2a^2} - \frac{b}{\left(b\,x^2-a\right)a^3} + \frac{3b\ln(x)}{a^4} - \frac{3b\ln\left(b\,x^2-a\right)}{2a^4} - \frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^3,x)
```

```
[Out] -1/2/a^3/x^2+3/a^4*b*ln(x)-3/2/a^4*b*ln(b*x^2-a)+1/4/a^2*b/(b*x^2-a)^2-1/a^
3*b/(b*x^2-a)
```

maxima [A] time = 1.31, size = 79, normalized size = 1.14

$$-\frac{6\,b^2x^4 - 9\,abx^2 + 2\,a^2}{4\left(a^3b^2x^6 - 2\,a^4bx^4 + a^5x^2\right)} - \frac{3\,b\log\left(bx^2-a\right)}{2\,a^4} + \frac{3\,b\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*x^4 - 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 - 2*a^4*b*x^4 + a^5*x^2)
- 3/2*b*log(b*x^2 - a)/a^4 + 3/2*b*log(x^2)/a^4
```

mupad [B] time = 4.61, size = 76, normalized size = 1.10

$$\frac{3\,b\,\ln(x)}{a^4} - \frac{3\,b\,\ln\left(a-b\,x^2\right)}{2\,a^4} - \frac{\frac{1}{2\,a} - \frac{9\,b\,x^2}{4\,a^2} + \frac{3\,b^2\,x^4}{2\,a^3}}{a^2\,x^2 - 2\,a\,b\,x^4 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)^3),x)
```

```
[Out] (3*b*log(x))/a^4 - (3*b*log(a - b*x^2))/(2*a^4) - (1/(2*a) - (9*b*x^2)/(4*a
^2) + (3*b^2*x^4)/(2*a^3))/(a^2*x^2 + b^2*x^6 - 2*a*b*x^4)
```

sympy [A] time = 0.51, size = 78, normalized size = 1.13

$$-\frac{2a^2 - 9abx^2 + 6b^2x^4}{4a^5x^2 - 8a^4bx^4 + 4a^3b^2x^6} + \frac{3b\log(x)}{a^4} - \frac{3b\log\left(-\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**3,x)
```

```
[Out] -(2*a**2 - 9*a*b*x**2 + 6*b**2*x**4)/(4*a**5*x**2 - 8*a**4*b*x**4 + 4*a**3*
b**2*x**6) + 3*b*log(x)/a**4 - 3*b*log(-a/b + x**2)/(2*a**4)
```

3.246 $\int \frac{x^3}{\left(a-bx^2\right)^5}\,dx$

Optimal. Leaf size=36

$$\frac{a}{8b^2\left(a-bx^2\right)^4}-\frac{1}{6b^2\left(a-bx^2\right)^3}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 43}

$$\frac{a}{8b^2\left(a-bx^2\right)^4}-\frac{1}{6b^2\left(a-bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2)^5,x]
```

```
[Out] a/(8*b^2*(a - b*x^2)^4) - 1/(6*b^2*(a - b*x^2)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a-bx^2\right)^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a-bx)^5}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a}{b(a-bx)^5}-\frac{1}{b(a-bx)^4}\right)dx,x,x^2\right)\\ &= \frac{a}{8b^2\left(a-bx^2\right)^4}-\frac{1}{6b^2\left(a-bx^2\right)^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.69

$$-\frac{a-4bx^2}{24b^2\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2)^5,x]
[Out] -1/24*(a - 4*b*x^2)/(b^2*(a - b*x^2)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a-bx^2\right)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a - b*x^2)^5,x]
[Out] IntegrateAlgebraic[x^3/(a - b*x^2)^5, x]
```

fricas [A] time = 0.63, size = 60, normalized size = 1.67

$$\frac{4\,bx^2-a}{24\left(b^6x^8-4\,ab^5x^6+6\,a^2b^4x^4-4\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="fricas")
[Out] 1/24*(4*b*x^2 - a)/(b^6*x^8 - 4*a*b^5*x^6 + 6*a^2*b^4*x^4 - 4*a^3*b^3*x^2 +
 a^4*b^2)
```

giac [A] time = 0.59, size = 39, normalized size = 1.08

$$\frac{\frac{4}{\left(bx^2-a\right)^3b}+\frac{3\,a}{\left(bx^2-a\right)^4b}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="giac")
[Out] 1/24*(4/((b*x^2 - a)^3*b) + 3*a/((b*x^2 - a)^4*b))/b
```

maple [A] time = 0.01, size = 35, normalized size = 0.97

$$\frac{a}{8\left(b\,x^{2}-a\right)^{4}b^{2}}+\frac{1}{6\left(b\,x^{2}-a\right)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^2+a)^5,x)
```

```
[Out] 1/6/b^2/(b*x^2-a)^3+1/8*a/b^2/(b*x^2-a)^4
```

maxima [A] time = 1.36, size = 60, normalized size = 1.67

$$\frac{4\,bx^{2}-a}{24\left(b^{6}x^{8}-4\,ab^{5}x^{6}+6\,a^{2}b^{4}x^{4}-4\,a^{3}b^{3}x^{2}+a^{4}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(4*b*x^2 - a)/(b^6*x^8 - 4*a*b^5*x^6 + 6*a^2*b^4*x^4 - 4*a^3*b^3*x^2 +
 a^4*b^2)
```

mupad [B] time = 4.60, size = 59, normalized size = 1.64

$$-\frac{\frac{a}{24\,b^{2}}-\frac{x^{2}}{6\,b}}{a^{4}-4\,a^{3}\,b\,x^{2}+6\,a^{2}\,b^{2}\,x^{4}-4\,a\,b^{3}\,x^{6}+b^{4}\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2)^5,x)
```

```
[Out] -(a/(24*b^2) - x^2/(6*b))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^
2*b^2*x^4)
```

sympy [B] time = 0.44, size = 60, normalized size = 1.67

$$-\frac{a-4bx^{2}}{24a^{4}b^{2}-96a^{3}b^{3}x^{2}+144a^{2}b^{4}x^{4}-96ab^{5}x^{6}+24b^{6}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**2+a)**5,x)
```

```
[Out] -(a - 4*b*x**2)/(24*a**4*b**2 - 96*a**3*b**3*x**2 + 144*a**2*b**4*x**4 - 96
*a*b**5*x**6 + 24*b**6*x**8)
```

3.247 $\int \frac{x^2}{(a-bx^2)^5} dx$

Optimal. Leaf size=109

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}-\frac{5x}{128a^3b\left(a-bx^2\right)}-\frac{5x}{192a^2b\left(a-bx^2\right)^2}-\frac{x}{48ab\left(a-bx^2\right)^3}+\frac{x}{8b\left(a-bx^2\right)^4}$$

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {288, 199, 208}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}-\frac{5x}{128a^3b\left(a-bx^2\right)}-\frac{5x}{192a^2b\left(a-bx^2\right)^2}-\frac{x}{48ab\left(a-bx^2\right)^3}+\frac{x}{8b\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2)^5,x]
```

```
[Out] x/(8*b*(a - b*x^2)^4) - x/(48*a*b*(a - b*x^2)^3) - (5*x)/(192*a^2*b*(a - b*
x^2)^2) - (5*x)/(128*a^3*b*(a - b*x^2)) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(128*a^(7/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^2\right)^5}\,dx = \frac{x}{8b\left(a-bx^2\right)^4}-\frac{\int \frac{1}{\left(a-bx^2\right)^4}\,dx}{8b}$$

$$= \frac{x}{8b\left(a-bx^2\right)^4}-\frac{x}{48ab\left(a-bx^2\right)^3}-\frac{5\int \frac{1}{\left(a-bx^2\right)^3}\,dx}{48ab}$$

$$= \frac{x}{8b\left(a-bx^2\right)^4}-\frac{x}{48ab\left(a-bx^2\right)^3}-\frac{5x}{192a^2b\left(a-bx^2\right)^2}-\frac{5\int \frac{1}{\left(a-bx^2\right)^2}\,dx}{64a^2b}$$

$$= \frac{x}{8b\left(a-bx^2\right)^4}-\frac{x}{48ab\left(a-bx^2\right)^3}-\frac{5x}{192a^2b\left(a-bx^2\right)^2}-\frac{5x}{128a^3b\left(a-bx^2\right)}-\frac{5\int \frac{1}{a-bx^2}\,dx}{128a^3b}$$

$$= \frac{x}{8b\left(a-bx^2\right)^4}-\frac{x}{48ab\left(a-bx^2\right)^3}-\frac{5x}{192a^2b\left(a-bx^2\right)^2}-\frac{5x}{128a^3b\left(a-bx^2\right)}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 0.74

$$\frac{15a^3x+73a^2bx^3-55ab^2x^5+15b^3x^7}{384a^3b\left(a-bx^2\right)^4}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2)^5,x]
```

```
[Out] (15*a^3*x + 73*a^2*b*x^3 - 55*a*b^2*x^5 + 15*b^3*x^7)/(384*a^3*b*(a - b*x^2
)^4) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a-bx^2\right)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a - b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a - b*x^2)^5, x]
```

fricas [A] time = 0.54, size = 324, normalized size = 2.97

$$\left[\frac{30\,ab^{4}x^{7}-110\,a^{2}b^{3}x^{5}+146\,a^{3}b^{2}x^{3}+30\,a^{4}bx+15\,\left(b^{4}x^{8}-4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}-4\,a^{3}bx^{2}+a^{4}\right)\sqrt{ab}\log\left(\frac{bx^{2}-2\,\sqrt{ab}x+a}{bx^{2}-a}\right)}{768\,\left(a^{4}b^{6}x^{8}-4\,a^{5}b^{5}x^{6}+6\,a^{6}b^{4}x^{4}-4\,a^{7}b^{3}x^{2}+a^{8}b^{2}\right)},\ \frac{15\,ab^{4}x^{7}-55\,a^{2}b^{3}x^{5}+73\,a^{3}b^{2}x^{3}+15\,a^{4}bx+15\,\left(b^{4}x^{8}-4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}-4\,a^{3}bx^{2}+a^{4}\right)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{384\,\left(a^{4}b^{6}x^{8}-4\,a^{5}b^{5}x^{6}+6\,a^{6}b^{4}x^{4}-4\,a^{7}b^{3}x^{2}+a^{8}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 - 110*a^2*b^3*x^5 + 146*a^3*b^2*x^3 + 30*a^4*b*x + 15*
(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(a*b)*log((
b*x^2 - 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^4*b^6*x^8 - 4*a^5*b^5*x^6 + 6*a
^6*b^4*x^4 - 4*a^7*b^3*x^2 + a^8*b^2), 1/384*(15*a*b^4*x^7 - 55*a^2*b^3*x^5
 + 73*a^3*b^2*x^3 + 15*a^4*b*x + 15*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4
- 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^4*b^6*x^8 - 4*a^
5*b^5*x^6 + 6*a^6*b^4*x^4 - 4*a^7*b^3*x^2 + a^8*b^2)]
```

giac [A] time = 0.63, size = 77, normalized size = 0.71

$$\frac{5\,\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\,\sqrt{-ab}\,a^{3}b}+\frac{15\,b^{3}x^{7}-55\,ab^{2}x^{5}+73\,a^{2}bx^{3}+15\,a^{3}x}{384\,\left(bx^{2}-a\right)^{4}a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 5/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3*b) + 1/384*(15*b^3*x^7 - 55*a*
b^2*x^5 + 73*a^2*b*x^3 + 15*a^3*x)/((b*x^2 - a)^4*a^3*b)
```

maple [A] time = 0.01, size = 72, normalized size = 0.66

$$-\frac{5\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}\,a^{3}b}-\frac{-\frac{5b^{2}x^{7}}{128a^{3}}+\frac{55b\,x^{5}}{384a^{2}}-\frac{73x^{3}}{384a}-\frac{5x}{128b}}{\left(b\,x^{2}-a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^5,x)
```

```
[Out] -(-5/128/a^3*b^2*x^7+55/384/a^2*b*x^5-73/384/a*x^3-5/128/b*x)/(b*x^2-a)^4-5
/128/a^3/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 124, normalized size = 1.14

$$\frac{15\,b^{3}x^{7}-55\,ab^{2}x^{5}+73\,a^{2}bx^{3}+15\,a^{3}x}{384\left(a^{3}b^{5}x^{8}-4\,a^{4}b^{4}x^{6}+6\,a^{5}b^{3}x^{4}-4\,a^{6}b^{2}x^{2}+a^{7}b\right)}+\frac{5\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/384*(15*b^3*x^7 - 55*a*b^2*x^5 + 73*a^2*b*x^3 + 15*a^3*x)/(a^3*b^5*x^8 -
4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 - 4*a^6*b^2*x^2 + a^7*b) + 5/256*log((b*x - s
qrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^3*b)
```

mupad [B] time = 4.76, size = 96, normalized size = 0.88

$$\frac{\frac{5\,x}{128\,b}+\frac{73\,x^{3}}{384\,a}-\frac{55\,b\,x^{5}}{384\,a^{2}}+\frac{5\,b^{2}\,x^{7}}{128\,a^{3}}}{a^{4}-4\,a^{3}\,b\,x^{2}+6\,a^{2}\,b^{2}\,x^{4}-4\,a\,b^{3}\,x^{6}+b^{4}\,x^{8}}-\frac{5\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{7/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2)^5,x)
```

```
[Out] ((5*x)/(128*b) + (73*x^3)/(384*a) - (55*b*x^5)/(384*a^2) + (5*b^2*x^7)/(128
*a^3))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4) - (5*ata
nh((b^(1/2)*x)/a^(1/2)))/(128*a^(7/2)*b^(3/2))
```

sympy [A] time = 0.50, size = 160, normalized size = 1.47

$$\frac{5\sqrt{\frac{1}{a^{7}b^{3}}}\log\left(-a^{4}b\sqrt{\frac{1}{a^{7}b^{3}}}+x\right)}{256}-\frac{5\sqrt{\frac{1}{a^{7}b^{3}}}\log\left(a^{4}b\sqrt{\frac{1}{a^{7}b^{3}}}+x\right)}{256}-\frac{-15a^{3}x-73a^{2}bx^{3}+55ab^{2}x^{5}-15b^{3}x^{7}}{384a^{7}b-1536a^{6}b^{2}x^{2}+2304a^{5}b^{3}x^{4}-1536a^{4}b^{4}x^{6}+384a^{3}b^{5}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**5,x)
```

```
[Out] 5*sqrt(1/(a**7*b**3))*log(-a**4*b*sqrt(1/(a**7*b**3)) + x)/256 - 5*sqrt(1/(
a**7*b**3))*log(a**4*b*sqrt(1/(a**7*b**3)) + x)/256 - (-15*a**3*x - 73*a**2
*b*x**3 + 55*a*b**2*x**5 - 15*b**3*x**7)/(384*a**7*b - 1536*a**6*b**2*x**2
+ 2304*a**5*b**3*x**4 - 1536*a**4*b**4*x**6 + 384*a**3*b**5*x**8)
```

3.248 $\int \frac{x}{(a-bx^2)^5} \, dx$

Optimal. Leaf size=17

$$\frac{1}{8b(a-bx^2)^4}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {261}

$$\frac{1}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2)^5,x]
```

```
[Out] 1/(8*b*(a - b*x^2)^4)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a-bx^2)^5} \, dx = \frac{1}{8b(a-bx^2)^4}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2)^5,x]
```

```
[Out] 1/(8*b*(a - b*x^2)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a-bx^2\right)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a - b*x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x/(a - b*x^2)^5, x]
```

fricas [B] time = 0.53, size = 48, normalized size = 2.82

$$\frac{1}{8\left(b^5x^8-4\,ab^4x^6+6\,a^2b^3x^4-4\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8/(b^5*x^8 - 4*a*b^4*x^6 + 6*a^2*b^3*x^4 - 4*a^3*b^2*x^2 + a^4*b)
```

giac [A] time = 0.62, size = 16, normalized size = 0.94

$$\frac{1}{8\left(bx^2-a\right)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/8/((b*x^2 - a)^4*b)
```

maple [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{8\left(b\,x^2-a\right)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^5,x)
```

```
[Out] 1/8/b/(b*x^2-a)^4
```

maxima [A] time = 1.31, size = 16, normalized size = 0.94

$$\frac{1}{8\left(bx^2-a\right)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] 1/8/((b*x^2 - a)^4*b)
```

mupad [B] time = 4.69, size = 48, normalized size = 2.82

$$\frac{1}{8\,a^4\,b-32\,a^3\,b^2\,x^2+48\,a^2\,b^3\,x^4-32\,a\,b^4\,x^6+8\,b^5\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2)^5,x)
```

```
[Out] 1/(8*a^4*b + 8*b^5*x^8 - 32*a*b^4*x^6 - 32*a^3*b^2*x^2 + 48*a^2*b^3*x^4)
```

sympy [B] time = 0.40, size = 49, normalized size = 2.88

$$\frac{1}{8a^4b-32a^3b^2x^2+48a^2b^3x^4-32ab^4x^6+8b^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**5,x)
```

```
[Out] 1/(8*a**4*b - 32*a**3*b**2*x**2 + 48*a**2*b**3*x**4 - 32*a*b**4*x**6 + 8*b*
*5*x**8)
```

3.249 $\int \frac{1}{(a-bx^2)^5} dx$

Optimal. Leaf size=100

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}+\frac{35x}{128a^4\left(a-bx^2\right)}+\frac{35x}{192a^3\left(a-bx^2\right)^2}+\frac{7x}{48a^2\left(a-bx^2\right)^3}+\frac{x}{8a\left(a-bx^2\right)^4}$$

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {199, 208}

$$\frac{35x}{128a^4\left(a-bx^2\right)}+\frac{35x}{192a^3\left(a-bx^2\right)^2}+\frac{7x}{48a^2\left(a-bx^2\right)^3}+\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}+\frac{x}{8a\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(-5),x]
```

```
[Out] x/(8*a*(a - b*x^2)^4) + (7*x)/(48*a^2*(a - b*x^2)^3) + (35*x)/(192*a^3*(a -
 b*x^2)^2) + (35*x)/(128*a^4*(a - b*x^2)) + (35*ArcTanh[(Sqrt[b]*x)/Sqrt[a]
])/(128*a^(9/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^2\right)^5}\,dx = \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7\int \frac{1}{\left(a-bx^2\right)^4}\,dx}{8a}$$

$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35\int \frac{1}{\left(a-bx^2\right)^3}\,dx}{48a^2}$$

$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35x}{192a^3\left(a-bx^2\right)^2} + \frac{35\int \frac{1}{\left(a-bx^2\right)^2}\,dx}{64a^3}$$

$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35x}{192a^3\left(a-bx^2\right)^2} + \frac{35x}{128a^4\left(a-bx^2\right)} + \frac{35\int \frac{1}{a-bx^2}\,dx}{128a^4}$$

$$= \frac{x}{8a\left(a-bx^2\right)^4} + \frac{7x}{48a^2\left(a-bx^2\right)^3} + \frac{35x}{192a^3\left(a-bx^2\right)^2} + \frac{35x}{128a^4\left(a-bx^2\right)} + \frac{35\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.79

$$\frac{\frac{\sqrt{a}\,x\left(279a^3-511a^2bx^2+385ab^2x^4-105b^3x^6\right)}{\left(a-bx^2\right)^4} + \frac{105\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}}{384a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(-5),x]
```

```
[Out] ((Sqrt[a]*x*(279*a^3 - 511*a^2*b*x^2 + 385*a*b^2*x^4 - 105*b^3*x^6))/(a - b
*x^2)^4 + (105*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(384*a^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a-bx^2\right)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a - b*x^2)^(-5),x]
```

[Out] IntegrateAlgebraic[(a - b*x^2)^(-5), x]

fricas [A] time = 0.56, size = 320, normalized size = 3.20

$$\left[-\frac{210\,ab^4x^7 - 770\,a^2b^3x^5 + 1022\,a^3b^2x^3 - 558\,a^4bx - 105\left(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4\right)\sqrt{ab}\,\log\left(\frac{bx^2+2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{768\left(a^5b^5x^8 - 4\,a^6b^4x^6 + 6\,a^7b^3x^4 - 4\,a^8b^2x^2 + a^9b\right)}, -\frac{105\,ab^4x^7 - 385\,a^2b^3x^5 + 511\,a^3b^2x^3 - 279\,a^4bx + 105\left(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{384\left(a^5b^5x^8 - 4\,a^6b^4x^6 + 6\,a^7b^3x^4 - 4\,a^8b^2x^2 + a^9b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^5,x, algorithm="fricas")

[Out] [-1/768*(210*a*b^4*x^7 - 770*a^2*b^3*x^5 + 1022*a^3*b^2*x^3 - 558*a^4*b*x - 105*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^5*b^5*x^8 - 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 - 4*a^8*b^2*x^2 + a^9*b), -1/384*(105*a*b^4*x^7 - 385*a^2*b^3*x^5 + 511*a^3*b^2*x^3 - 279*a^4*b*x + 105*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^5*b^5*x^8 - 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 - 4*a^8*b^2*x^2 + a^9*b)]

giac [A] time = 0.62, size = 71, normalized size = 0.71

$$-\frac{35\,\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\,\sqrt{-ab}\,a^4} - \frac{105\,b^3x^7 - 385\,ab^2x^5 + 511\,a^2bx^3 - 279\,a^3x}{384\left(bx^2-a\right)^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^5,x, algorithm="giac")

[Out] -35/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^4) - 1/384*(105*b^3*x^7 - 385*a*b^2*x^5 + 511*a^2*b*x^3 - 279*a^3*x)/((b*x^2 - a)^4*a^4)

maple [A] time = 0.00, size = 107, normalized size = 1.07

$$\frac{x}{8\left(b\,x^2-a\right)^4a} + \frac{-\frac{7x}{48\left(b\,x^2-a\right)^3a} - \frac{35\left(-\frac{x}{4\left(b\,x^2-a\right)^2a} - \frac{3\left(-\frac{x}{2\left(b\,x^2-a\right)a} + \frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}\right)}{4a}\right)}{48a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b*x^2+a)^5,x)

```
[Out] 1/8*x/a/(b*x^2-a)^4+7/8/a*(-1/6*x/a/(b*x^2-a)^3-5/6/a*(-1/4/(b*x^2-a)^2/a*x
-3/4/a*(-1/2/(b*x^2-a)/a*x+1/2/a/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x))))
```

maxima [A] time = 2.90, size = 117, normalized size = 1.17

$$-\frac{105\,b^3x^7-385\,ab^2x^5+511\,a^2bx^3-279\,a^3x}{384\left(a^4b^4x^8-4\,a^5b^3x^6+6\,a^6b^2x^4-4\,a^7bx^2+a^8\right)}-\frac{35\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/384*(105*b^3*x^7 - 385*a*b^2*x^5 + 511*a^2*b*x^3 - 279*a^3*x)/(a^4*b^4*x
^8 - 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 - 4*a^7*b*x^2 + a^8) - 35/256*log((b*x -
 sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^4)
```

mupad [B] time = 4.79, size = 99, normalized size = 0.99

$$\frac{\frac{93\,x}{128\,a}-\frac{511\,b\,x^3}{384\,a^2}+\frac{385\,b^2\,x^5}{384\,a^3}-\frac{35\,b^3\,x^7}{128\,a^4}}{a^4-4\,a^3\,b\,x^2+6\,a^2\,b^2\,x^4-4\,a\,b^3\,x^6+b^4\,x^8}+\frac{35\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{9/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^5,x)
```

```
[Out] ((93*x)/(128*a) - (511*b*x^3)/(384*a^2) + (385*b^2*x^5)/(384*a^3) - (35*b^3
*x^7)/(128*a^4))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4
) + (35*atanh((b^(1/2)*x)/a^(1/2)))/(128*a^(9/2)*b^(1/2))
```

sympy [A] time = 0.52, size = 146, normalized size = 1.46

$$-\frac{35\sqrt{\frac{1}{a^9b}}\,\log\left(-a^5\sqrt{\frac{1}{a^9b}}+x\right)}{256}+\frac{35\sqrt{\frac{1}{a^9b}}\,\log\left(a^5\sqrt{\frac{1}{a^9b}}+x\right)}{256}-\frac{-279a^3x+511a^2bx^3-385ab^2x^5+105b^3x^7}{384a^8-1536a^7bx^2+2304a^6b^2x^4-1536a^5b^3x^6+384a^4b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**5,x)
```

```
[Out] -35*sqrt(1/(a**9*b))*log(-a**5*sqrt(1/(a**9*b)) + x)/256 + 35*sqrt(1/(a**9*
b))*log(a**5*sqrt(1/(a**9*b)) + x)/256 - (-279*a**3*x + 511*a**2*b*x**3 - 3
85*a*b**2*x**5 + 105*b**3*x**7)/(384*a**8 - 1536*a**7*b*x**2 + 2304*a**6*b*
*2*x**4 - 1536*a**5*b**3*x**6 + 384*a**4*b**4*x**8)
```

3.250 $\int \frac{1}{x\left(a-bx^2\right)^5}\,dx$

Optimal. Leaf size=91

$$-\frac{\log\left(a-bx^2\right)}{2a^5}+\frac{\log(x)}{a^5}+\frac{1}{2a^4\left(a-bx^2\right)}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{6a^2\left(a-bx^2\right)^3}+\frac{1}{8a\left(a-bx^2\right)^4}$$

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{1}{2a^4\left(a-bx^2\right)}+\frac{1}{4a^3\left(a-bx^2\right)^2}+\frac{1}{6a^2\left(a-bx^2\right)^3}-\frac{\log\left(a-bx^2\right)}{2a^5}+\frac{\log(x)}{a^5}+\frac{1}{8a\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2)^5),x]
```

```
[Out] 1/(8*a*(a - b*x^2)^4) + 1/(6*a^2*(a - b*x^2)^3) + 1/(4*a^3*(a - b*x^2)^2) +
 1/(2*a^4*(a - b*x^2)) + Log[x]/a^5 - Log[a - b*x^2]/(2*a^5)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a-bx^2\right)^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a-bx)^5}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^5x} + \frac{b}{a(a-bx)^5} + \frac{b}{a^2(a-bx)^4} + \frac{b}{a^3(a-bx)^3} + \frac{b}{a^4(a-bx)^2} + \frac{b}{a^5(a-bx)}\right) d$$
$$= \frac{1}{8a\left(a-bx^2\right)^4} + \frac{1}{6a^2\left(a-bx^2\right)^3} + \frac{1}{4a^3\left(a-bx^2\right)^2} + \frac{1}{2a^4\left(a-bx^2\right)} + \frac{\log(x)}{a^5} - \frac{\log\left(a-bx^2\right)}{2a^5}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.74

$$\frac{\frac{a\left(25a^3-52a^2bx^2+42ab^2x^4-12b^3x^6\right)}{\left(a-bx^2\right)^4} - 12\log\left(a-bx^2\right) + 24\log(x)}{24a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2)^5),x]
```

```
[Out] ((a*(25*a^3 - 52*a^2*b*x^2 + 42*a*b^2*x^4 - 12*b^3*x^6))/(a - b*x^2)^4 + 24
*Log[x] - 12*Log[a - b*x^2])/(24*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a-bx^2\right)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a - b*x^2)^5),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a - b*x^2)^5), x]
```

fricas [B] time = 0.54, size = 180, normalized size = 1.98

$$-\frac{12\,ab^3x^6 - 42\,a^2b^2x^4 + 52\,a^3bx^2 - 25\,a^4 + 12\left(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4\right)\log\left(bx^2-a\right) - 24\left(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4\right)\log(x)}{24\left(a^5b^4x^8 - 4\,a^6b^3x^6 + 6\,a^7b^2x^4 - 4\,a^8bx^2 + a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a*b^3*x^6 - 42*a^2*b^2*x^4 + 52*a^3*b*x^2 - 25*a^4 + 12*(b^4*x^8
- 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*log(b*x^2 - a) - 24*(b^4
```

```
*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*log(x))/(a^5*b^4*x^
8 - 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 - 4*a^8*b*x^2 + a^9)
```

giac [A] time = 0.61, size = 85, normalized size = 0.93

$$\frac{\log\left(x^2\right)}{2\,a^5} - \frac{\log\left(\left|bx^2-a\right|\right)}{2\,a^5} + \frac{25\,b^4x^8 - 112\,ab^3x^6 + 192\,a^2b^2x^4 - 152\,a^3bx^2 + 50\,a^4}{24\left(bx^2-a\right)^4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^5 - 1/2*log(abs(b*x^2 - a))/a^5 + 1/24*(25*b^4*x^8 - 112*a*b
^3*x^6 + 192*a^2*b^2*x^4 - 152*a^3*b*x^2 + 50*a^4)/((b*x^2 - a)^4*a^5)
```

maple [A] time = 0.01, size = 87, normalized size = 0.96

$$\frac{1}{8\left(b\,x^2-a\right)^4a} - \frac{1}{6\left(b\,x^2-a\right)^3a^2} + \frac{1}{4\left(b\,x^2-a\right)^2a^3} - \frac{1}{2\left(b\,x^2-a\right)a^4} + \frac{\ln(x)}{a^5} - \frac{\ln\left(b\,x^2-a\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^2+a)^5,x)
```

```
[Out] ln(x)/a^5-1/2/a^5*ln(b*x^2-a)-1/6/a^2/(b*x^2-a)^3+1/4/a^3/(b*x^2-a)^2+1/8/a
/(b*x^2-a)^4-1/2/a^4/(b*x^2-a)
```

maxima [A] time = 1.39, size = 106, normalized size = 1.16

$$-\frac{12\,b^3x^6 - 42\,ab^2x^4 + 52\,a^2bx^2 - 25\,a^3}{24\left(a^4b^4x^8 - 4\,a^5b^3x^6 + 6\,a^6b^2x^4 - 4\,a^7bx^2 + a^8\right)} - \frac{\log\left(bx^2-a\right)}{2\,a^5} + \frac{\log\left(x^2\right)}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*b^3*x^6 - 42*a*b^2*x^4 + 52*a^2*b*x^2 - 25*a^3)/(a^4*b^4*x^8 - 4*
a^5*b^3*x^6 + 6*a^6*b^2*x^4 - 4*a^7*b*x^2 + a^8) - 1/2*log(b*x^2 - a)/a^5 +
 1/2*log(x^2)/a^5
```

mupad [B] time = 5.24, size = 101, normalized size = 1.11

$$\frac{\ln(x)}{a^5} + \frac{\frac{25}{24\,a} - \frac{13\,b\,x^2}{6\,a^2} + \frac{7\,b^2\,x^4}{4\,a^3} - \frac{b^3\,x^6}{2\,a^4}}{a^4 - 4\,a^3\,b\,x^2 + 6\,a^2\,b^2\,x^4 - 4\,a\,b^3\,x^6 + b^4\,x^8} - \frac{\ln\left(a-b\,x^2\right)}{2\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2)^5),x)
```

```
[Out] log(x)/a^5 + (25/(24*a) - (13*b*x^2)/(6*a^2) + (7*b^2*x^4)/(4*a^3) - (b^3*x
^6)/(2*a^4))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4) -
log(a - b*x^2)/(2*a^5)
```

sympy [A] time = 0.64, size = 104, normalized size = 1.14

$$-\frac{-25a^3 + 52a^2bx^2 - 42ab^2x^4 + 12b^3x^6}{24a^8 - 96a^7bx^2 + 144a^6b^2x^4 - 96a^5b^3x^6 + 24a^4b^4x^8} + \frac{\log(x)}{a^5} - \frac{\log\left(-\frac{a}{b} + x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**2+a)**5,x)
```

```
[Out] -(-25*a**3 + 52*a**2*b*x**2 - 42*a*b**2*x**4 + 12*b**3*x**6)/(24*a**8 - 96*
a**7*b*x**2 + 144*a**6*b**2*x**4 - 96*a**5*b**3*x**6 + 24*a**4*b**4*x**8) +
 log(x)/a**5 - log(-a/b + x**2)/(2*a**5)
```

3.251 $\int \frac{1}{x^2(a-bx^2)^5}\,dx$

Optimal. Leaf size=118

$$\frac{315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}}-\frac{315}{128a^5x}+\frac{105}{128a^4x\left(a-bx^2\right)}+\frac{21}{64a^3x\left(a-bx^2\right)^2}+\frac{3}{16a^2x\left(a-bx^2\right)^3}+\frac{1}{8ax\left(a-bx^2\right)^4}$$

Rubi [A] time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {290, 325, 208}

$$\frac{105}{128a^4x\left(a-bx^2\right)}+\frac{21}{64a^3x\left(a-bx^2\right)^2}+\frac{3}{16a^2x\left(a-bx^2\right)^3}+\frac{315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}}-\frac{315}{128a^5x}+\frac{1}{8ax\left(a-bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2)^5),x]
```

```
[Out] -315/(128*a^5*x) + 1/(8*a*x*(a - b*x^2)^4) + 3/(16*a^2*x*(a - b*x^2)^3) + 2
1/(64*a^3*x*(a - b*x^2)^2) + 105/(128*a^4*x*(a - b*x^2)) + (315*Sqrt[b]*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^2\right)^5}\,dx = \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{9\int \frac{1}{x^2\left(a-bx^2\right)^4}\,dx}{8a}$$

$$= \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21\int \frac{1}{x^2\left(a-bx^2\right)^3}\,dx}{16a^2}$$

$$= \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105\int \frac{1}{x^2\left(a-bx^2\right)^2}\,dx}{64a^3}$$

$$= \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105}{128a^4x\left(a-bx^2\right)} + \frac{315\int \frac{1}{x^2(a-b}}{128a^4}$$

$$= -\frac{315}{128a^5x} + \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105}{128a^4x\left(a-bx^2\right)} +$$

$$= -\frac{315}{128a^5x} + \frac{1}{8ax\left(a-bx^2\right)^4} + \frac{3}{16a^2x\left(a-bx^2\right)^3} + \frac{21}{64a^3x\left(a-bx^2\right)^2} + \frac{105}{128a^4x\left(a-bx^2\right)} +$$

Mathematica [A] time = 0.06, size = 92, normalized size = 0.78

$$\frac{\frac{\sqrt{a}\left(-128a^4+837a^3bx^2-1533a^2b^2x^4+1155ab^3x^6-315b^4x^8\right)}{x\left(a-bx^2\right)^4} + 315\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2)^5),x]
```

```
[Out] ((Sqrt[a]*(-128*a^4 + 837*a^3*b*x^2 - 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 - 3
15*b^4*x^8))/(x*(a - b*x^2)^4) + 315*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/
(128*a^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a-bx^2\right)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a - b*x^2)^5),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a - b*x^2)^5), x]
```

fricas [A] time = 0.56, size = 334, normalized size = 2.83

$$\left[-\frac{630\,b^4x^8 - 2310\,ab^3x^6 + 3066\,a^2b^2x^4 - 1674\,a^3bx^2 + 256\,a^4 - 315\left(b^4x^9 - 4\,ab^3x^7 + 6\,a^2b^2x^5 - 4\,a^3bx^3 + a^4x\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)}{256\left(a^5b^4x^9 - 4\,a^6b^3x^7 + 6\,a^7b^2x^5 - 4\,a^8bx^3 + a^9x\right)}, -\frac{315\,b^4x^8 - 1155\,ab^3x^6 + 1533\,a^2b^2x^4 - 837\,a^3bx^2 + 128\,a^4 + 315\left(b^4x^9 - 4\,ab^3x^7 + 6\,a^2b^2x^5 - 4\,a^3bx^3 + a^4x\right)\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right)}{128\left(a^5b^4x^9 - 4\,a^6b^3x^7 + 6\,a^7b^2x^5 - 4\,a^8bx^3 + a^9x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/256*(630*b^4*x^8 - 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 - 1674*a^3*b*x^2 +
 256*a^4 - 315*(b^4*x^9 - 4*a*b^3*x^7 + 6*a^2*b^2*x^5 - 4*a^3*b*x^3 + a^4*x
)*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)))/(a^5*b^4*x^9 -
4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^9*x), -1/128*(315*b^4*x^8 -
 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 - 837*a^3*b*x^2 + 128*a^4 + 315*(b^4*x^9
 - 4*a*b^3*x^7 + 6*a^2*b^2*x^5 - 4*a^3*b*x^3 + a^4*x)*sqrt(-b/a)*arctan(x*s
qrt(-b/a)))/(a^5*b^4*x^9 - 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^
9*x)]
```

giac [A] time = 0.63, size = 83, normalized size = 0.70

$$-\frac{315\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\,\sqrt{-ab}\,a^5} - \frac{1}{a^5x} - \frac{187\,b^4x^7 - 643\,ab^3x^5 + 765\,a^2b^2x^3 - 325\,a^3bx}{128\left(bx^2 - a\right)^4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] -315/128*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^5) - 1/(a^5*x) - 1/128*(187
*b^4*x^7 - 643*a*b^3*x^5 + 765*a^2*b^2*x^3 - 325*a^3*b*x)/((b*x^2 - a)^4*a^
5)
```

maple [A] time = 0.02, size = 78, normalized size = 0.66

$$-\frac{\left(-\frac{315\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}} + \frac{\frac{187}{128}b^3x^7 - \frac{643}{128}a\,b^2x^5 + \frac{765}{128}a^2b\,x^3 - \frac{325}{128}a^3x}{\left(b\,x^2-a\right)^4}\right)b}{a^5} - \frac{1}{a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^2+a)^5,x)
```

```
[Out] -1/a^5/x-1/a^5*b*((187/128*b^3*x^7-643/128*a*b^2*x^5+765/128*a^2*b*x^3-325/
128*a^3*x)/(b*x^2-a)^4-315/128/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x))
```

maxima [A] time = 2.90, size = 130, normalized size = 1.10

$$-\frac{315\,b^4x^8-1155\,ab^3x^6+1533\,a^2b^2x^4-837\,a^3bx^2+128\,a^4}{128\,\left(a^5b^4x^9-4\,a^6b^3x^7+6\,a^7b^2x^5-4\,a^8bx^3+a^9x\right)}-\frac{315\,b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/128*(315*b^4*x^8 - 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 - 837*a^3*b*x^2 + 1
28*a^4)/(a^5*b^4*x^9 - 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^9*x)
 - 315/256*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^5)
```

mupad [B] time = 5.17, size = 110, normalized size = 0.93

$$\frac{315\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{11/2}}-\frac{\frac{1}{a}-\frac{837\,b\,x^2}{128\,a^2}+\frac{1533\,b^2\,x^4}{128\,a^3}-\frac{1155\,b^3\,x^6}{128\,a^4}+\frac{315\,b^4\,x^8}{128\,a^5}}{a^4\,x-4\,a^3\,b\,x^3+6\,a^2\,b^2\,x^5-4\,a\,b^3\,x^7+b^4\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2)^5),x)
```

```
[Out] (315*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(128*a^(11/2)) - (1/a - (837*b*x^2
)/(128*a^2) + (1533*b^2*x^4)/(128*a^3) - (1155*b^3*x^6)/(128*a^4) + (315*b^
4*x^8)/(128*a^5))/(a^4*x + b^4*x^9 - 4*a^3*b*x^3 - 4*a*b^3*x^7 + 6*a^2*b^2*
x^5)
```

sympy [A] time = 0.66, size = 155, normalized size = 1.31

$$-\frac{315\sqrt{\frac{b}{a^{11}}}\log\left(-\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256}+\frac{315\sqrt{\frac{b}{a^{11}}}\log\left(\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256}-\frac{128a^4-837a^3bx^2+1533a^2b^2x^4-1155ab^3x^6+315b^4x^8}{128a^9x-512a^8bx^3+768a^7b^2x^5-512a^6b^3x^7+128a^5b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**2+a)**5,x)
```

```
[Out] -315*sqrt(b/a**11)*log(-a**6*sqrt(b/a**11)/b + x)/256 + 315*sqrt(b/a**11)*l
og(a**6*sqrt(b/a**11)/b + x)/256 - (128*a**4 - 837*a**3*b*x**2 + 1533*a**2*
b**2*x**4 - 1155*a*b**3*x**6 + 315*b**4*x**8)/(128*a**9*x - 512*a**8*b*x**3
 + 768*a**7*b**2*x**5 - 512*a**6*b**3*x**7 + 128*a**5*b**4*x**9)
```

3.252 $\int \frac{1}{x^3(a-bx^2)^5} dx$

Optimal. Leaf size=106

$$-\frac{5b\log\left(a-bx^2\right)}{2a^6}+\frac{5b\log(x)}{a^6}+\frac{2b}{a^5\left(a-bx^2\right)}-\frac{1}{2a^5x^2}+\frac{3b}{4a^4\left(a-bx^2\right)^2}+\frac{b}{3a^3\left(a-bx^2\right)^3}+\frac{b}{8a^2\left(a-bx^2\right)^4}$$

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 44}

$$\frac{2b}{a^5\left(a-bx^2\right)}+\frac{3b}{4a^4\left(a-bx^2\right)^2}+\frac{b}{3a^3\left(a-bx^2\right)^3}+\frac{b}{8a^2\left(a-bx^2\right)^4}-\frac{5b\log\left(a-bx^2\right)}{2a^6}+\frac{5b\log(x)}{a^6}-\frac{1}{2a^5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2)^5),x]
```

```
[Out] -1/(2*a^5*x^2) + b/(8*a^2*(a - b*x^2)^4) + b/(3*a^3*(a - b*x^2)^3) + (3*b)/
(4*a^4*(a - b*x^2)^2) + (2*b)/(a^5*(a - b*x^2)) + (5*b*Log[x])/a^6 - (5*b*L
og[a - b*x^2])/(2*a^6)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a-bx^2\right)^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a-bx)^5}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^5x^2} + \frac{5b}{a^6x} + \frac{b^2}{a^2(a-bx)^5} + \frac{2b^2}{a^3(a-bx)^4} + \frac{3b^2}{a^4(a-bx)^3} + \frac{4b^2}{a^5(a-bx)^2} + \frac{}{a^6(}\right.\right.$$

$$= -\frac{1}{2a^5x^2} + \frac{b}{8a^2\left(a-bx^2\right)^4} + \frac{b}{3a^3\left(a-bx^2\right)^3} + \frac{3b}{4a^4\left(a-bx^2\right)^2} + \frac{2b}{a^5\left(a-bx^2\right)} + \frac{5b\log(x)}{a^6} -$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.78

$$\frac{\frac{a\left(-12a^4+125a^3bx^2-260a^2b^2x^4+210ab^3x^6-60b^4x^8\right)}{x^2\left(a-bx^2\right)^4} - 60b\log\left(a-bx^2\right) + 120b\log(x)}{24a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2)^5),x]
```

```
[Out] ((a*(-12*a^4 + 125*a^3*b*x^2 - 260*a^2*b^2*x^4 + 210*a*b^3*x^6 - 60*b^4*x^8
))/(x^2*(a - b*x^2)^4) + 120*b*Log[x] - 60*b*Log[a - b*x^2])/(24*a^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a-bx^2\right)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a - b*x^2)^5),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a - b*x^2)^5), x]
```

fricas [B] time = 0.69, size = 209, normalized size = 1.97

$$-\frac{60\,ab^4x^8 - 210\,a^2b^3x^6 + 260\,a^3b^2x^4 - 125\,a^4bx^2 + 12\,a^5 + 60\left(b^5x^{10} - 4\,ab^4x^8 + 6\,a^2b^3x^6 - 4\,a^3b^2x^4 + a^4bx^2\right)\log\left(bx^2-a\right) - 120\left(b^5x^{10} - 4\,ab^4x^8 + 6\,a^2b^3x^6 - 4\,a^3b^2x^4 + a^4bx^2\right)\log(x)}{24\left(a^6b^4x^{10} - 4\,a^7b^3x^8 + 6\,a^8b^2x^6 - 4\,a^9bx^4 + a^{10}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(60*a*b^4*x^8 - 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 - 125*a^4*b*x^2 + 1
2*a^5 + 60*(b^5*x^10 - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b^2*x^4 + a^4*b*
```

```
x^2)*log(b*x^2 - a) - 120*(b^5*x^10 - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b
^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^4*x^10 - 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 -
 4*a^9*b*x^4 + a^10*x^2)
```

giac [A] time = 0.64, size = 106, normalized size = 1.00

$$\frac{5\,b\log\left(x^2\right)}{2\,a^6}-\frac{5\,b\log\left(\left|bx^2-a\right|\right)}{2\,a^6}-\frac{5\,bx^2+a}{2\,a^6x^2}+\frac{125\,b^5x^8-548\,ab^4x^6+912\,a^2b^3x^4-688\,a^3b^2x^2+202\,a^4b}{24\left(bx^2-a\right)^4a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] 5/2*b*log(x^2)/a^6 - 5/2*b*log(abs(b*x^2 - a))/a^6 - 1/2*(5*b*x^2 + a)/(a^6
*x^2) + 1/24*(125*b^5*x^8 - 548*a*b^4*x^6 + 912*a^2*b^3*x^4 - 688*a^3*b^2*x
^2 + 202*a^4*b)/((b*x^2 - a)^4*a^6)
```

maple [A] time = 0.02, size = 102, normalized size = 0.96

$$\frac{b}{8\left(b\,x^2-a\right)^4a^2}-\frac{b}{3\left(b\,x^2-a\right)^3a^3}+\frac{3b}{4\left(b\,x^2-a\right)^2a^4}-\frac{2b}{\left(b\,x^2-a\right)a^5}+\frac{5b\ln(x)}{a^6}-\frac{5b\ln\left(b\,x^2-a\right)}{2a^6}-\frac{1}{2a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^2+a)^5,x)
```

```
[Out] -1/2/a^5/x^2+5*b*ln(x)/a^6-1/3/a^3*b/(b*x^2-a)^3-5/2/a^6*b*ln(b*x^2-a)+3/4/
a^4*b/(b*x^2-a)^2+1/8/a^2*b/(b*x^2-a)^4-2/a^5*b/(b*x^2-a)
```

maxima [A] time = 1.35, size = 123, normalized size = 1.16

$$-\frac{60\,b^4x^8-210\,ab^3x^6+260\,a^2b^2x^4-125\,a^3bx^2+12\,a^4}{24\left(a^5b^4x^{10}-4\,a^6b^3x^8+6\,a^7b^2x^6-4\,a^8bx^4+a^9x^2\right)}-\frac{5\,b\log\left(bx^2-a\right)}{2\,a^6}+\frac{5\,b\log\left(x^2\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(60*b^4*x^8 - 210*a*b^3*x^6 + 260*a^2*b^2*x^4 - 125*a^3*b*x^2 + 12*a^
4)/(a^5*b^4*x^10 - 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 - 4*a^8*b*x^4 + a^9*x^2) -
 5/2*b*log(b*x^2 - a)/a^6 + 5/2*b*log(x^2)/a^6
```

mupad [B] time = 5.35, size = 120, normalized size = 1.13

$$\frac{5\,b\,\ln(x)}{a^6}-\frac{5\,b\,\ln\left(a-b\,x^2\right)}{2\,a^6}-\frac{\frac{1}{2\,a}-\frac{125\,b\,x^2}{24\,a^2}+\frac{65\,b^2\,x^4}{6\,a^3}-\frac{35\,b^3\,x^6}{4\,a^4}+\frac{5\,b^4\,x^8}{2\,a^5}}{a^4\,x^2-4\,a^3\,b\,x^4+6\,a^2\,b^2\,x^6-4\,a\,b^3\,x^8+b^4\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2)^5),x)
```

```
[Out] (5*b*log(x))/a^6 - (5*b*log(a - b*x^2))/(2*a^6) - (1/(2*a) - (125*b*x^2)/(2
4*a^2) + (65*b^2*x^4)/(6*a^3) - (35*b^3*x^6)/(4*a^4) + (5*b^4*x^8)/(2*a^5))
/(a^4*x^2 + b^4*x^10 - 4*a^3*b*x^4 - 4*a*b^3*x^8 + 6*a^2*b^2*x^6)
```

sympy [A] time = 0.76, size = 126, normalized size = 1.19

$$-\frac{12a^4 - 125a^3bx^2 + 260a^2b^2x^4 - 210ab^3x^6 + 60b^4x^8}{24a^9x^2 - 96a^8bx^4 + 144a^7b^2x^6 - 96a^6b^3x^8 + 24a^5b^4x^{10}} + \frac{5b\log(x)}{a^6} - \frac{5b\log\left(-\frac{a}{b} + x^2\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**2+a)**5,x)
```

```
[Out] -(12*a**4 - 125*a**3*b*x**2 + 260*a**2*b**2*x**4 - 210*a*b**3*x**6 + 60*b**
4*x**8)/(24*a**9*x**2 - 96*a**8*b*x**4 + 144*a**7*b**2*x**6 - 96*a**6*b**3*
x**8 + 24*a**5*b**4*x**10) + 5*b*log(x)/a**6 - 5*b*log(-a/b + x**2)/(2*a**6
)
```

3.253 $\int \frac{1}{x(1+bx^2)}\,dx$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{2}\log\left(bx^2+1\right)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\log(x) - \frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x^2)),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(1+bx^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+bx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{1+bx}\,dx, x, x^2\right)$$
$$= \log(x) - \frac{1}{2}\log\left(1+bx^2\right)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x^2)),x]
[Out] Log[x] - Log[1 + b*x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + b*x^2)),x]
[Out] IntegrateAlgebraic[1/(x*(1 + b*x^2)), x]
```

fricas [A] time = 0.55, size = 13, normalized size = 0.87

$$-\frac{1}{2}\log\left(bx^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="fricas")
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

giac [A] time = 0.62, size = 18, normalized size = 1.20

$$\frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x) - \frac{\ln\left(b\,x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+1),x)
```

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

maxima [A] time = 1.35, size = 17, normalized size = 1.13

$$-\frac{1}{2}\,\log\left(bx^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + 1) + 1/2*log(x^2)
```

mupad [B] time = 5.11, size = 14, normalized size = 0.93

$$\ln(x) - \frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + 1)),x)
```

```
[Out] log(x) - log((3*b*x^2)/2 + 3/2)/2
```

sympy [A] time = 0.13, size = 12, normalized size = 0.80

$$\log(x) - \frac{\log\left(x^2+\frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+1),x)
```

```
[Out] log(x) - log(x**2 + 1/b)/2
```

3.254 $\int \frac{1}{x(-1+bx^2)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x^2)),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(-1+bx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(-1+bx)}\,dx, x, x^2\right) \\
&= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)\right) + \frac{1}{2} b\,\text{Subst}\left(\int \frac{1}{-1+bx}\,dx, x, x^2\right) \\
&= -\log(x) + \frac{1}{2}\log\left(1-bx^2\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{2}\log\left(1-bx^2\right) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x^2)),x]
[Out] -Log[x] + Log[1 - b*x^2]/2
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-1+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(-1 + b*x^2)),x]
[Out] IntegrateAlgebraic[1/(x*(-1 + b*x^2)), x]
```

fricas [**A**] time = 0.45, size = 15, normalized size = 0.83

$$\frac{1}{2}\,\log\left(bx^2-1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="fricas")
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

giac [**A**] time = 0.62, size = 18, normalized size = 1.00

$$-\frac{1}{2}\,\log\left(x^2\right) + \frac{1}{2}\,\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

maple [A] time = 0.00, size = 16, normalized size = 0.89

$$-\ln(x)+\frac{\ln\left(b\,x^{2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2-1),x)
```

```
[Out] -ln(x)+1/2*ln(b*x^2-1)
```

maxima [A] time = 1.37, size = 17, normalized size = 0.94

$$\frac{1}{2}\,\log\left(bx^{2}-1\right)-\frac{1}{2}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 - 1) - 1/2*log(x^2)
```

mupad [B] time = 0.06, size = 16, normalized size = 0.89

$$\frac{\ln\left(\frac{3}{2}-\frac{3\,b\,x^{2}}{2}\right)}{2}-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 - 1)),x)
```

```
[Out] log(3/2 - (3*b*x^2)/2)/2 - log(x)
```

sympy [A] time = 0.14, size = 12, normalized size = 0.67

$$-\log\left(x\right)+\frac{\log\left(x^{2}-\frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2-1),x)
```

```
[Out] -log(x) + log(x**2 - 1/b)/2
```

3.255 $\int \frac{1}{x^3(1+bx^2)}\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + b*x^2)),x]
```

```
[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(1+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+bx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right)dx, x, x^2\right)\\ &= -\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + b*x^2)),x]
[Out] -1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\left(1+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 + b*x^2)),x]
[Out] IntegrateAlgebraic[1/(x^3*(1 + b*x^2)), x]
```

fricas [A] time = 0.64, size = 28, normalized size = 1.08

$$\frac{bx^2\log\left(bx^2+1\right)-2\,bx^2\log(x)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="fricas")
[Out] 1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

giac [A] time = 0.63, size = 32, normalized size = 1.23

$$-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2+1\right|\right)+\frac{bx^2-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="giac")
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

maple [A] time = 0.01, size = 23, normalized size = 0.88

$$-b\ln(x)+\frac{b\ln\left(b\,x^2+1\right)}{2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+1),x)
```

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

maxima [A] time = 1.34, size = 24, normalized size = 0.92

$$\frac{1}{2}\,b\log\left(bx^2+1\right)-\frac{1}{2}\,b\log\left(x^2\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + 1) - 1/2*b*log(x^2) - 1/2/x^2
```

mupad [B] time = 0.06, size = 22, normalized size = 0.85

$$\frac{b\,\ln\left(b\,x^2+1\right)}{2}-b\,\ln(x)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + 1)),x)
```

```
[Out] (b*log(b*x^2 + 1))/2 - b*log(x) - 1/(2*x^2)
```

sympy [A] time = 0.20, size = 22, normalized size = 0.85

$$-b\log\left(x\right)+\frac{b\log\left(x^2+\frac{1}{b}\right)}{2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+1),x)
```

```
[Out] -b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

3.256 $\int \frac{1}{x^3(-1+bx^2)}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-1 + b*x^2)),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(-1+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-1+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{-1+bx}\right)\,dx,x,x^2\right)\\
&= \frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-1 + b*x^2)),x]
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\left(-1+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(-1 + b*x^2)),x]
[Out] IntegrateAlgebraic[1/(x^3*(-1 + b*x^2)), x]
```

fricas [A] time = 0.57, size = 28, normalized size = 1.04

$$\frac{bx^2\log\left(bx^2-1\right)-2\,bx^2\log(x)+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="fricas")
[Out] 1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

giac [A] time = 0.59, size = 32, normalized size = 1.19

$$-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2-1\right|\right)+\frac{bx^2+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="giac")
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

maple [A] time = 0.00, size = 23, normalized size = 0.85

$$-b\ln(x)+\frac{b\ln\left(b\,x^2-1\right)}{2}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2-1),x)
```

```
[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2-1)
```

maxima [A] time = 1.34, size = 24, normalized size = 0.89

$$\frac{1}{2}\,b\log\left(bx^2-1\right)-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2-1),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 - 1) - 1/2*b*log(x^2) + 1/2/x^2
```

mupad [B] time = 0.06, size = 22, normalized size = 0.81

$$\frac{b\,\ln\left(b\,x^2-1\right)}{2}-b\,\ln(x)+\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 - 1)),x)
```

```
[Out] (b*log(b*x^2 - 1))/2 - b*log(x) + 1/(2*x^2)
```

sympy [A] time = 0.21, size = 22, normalized size = 0.81

$$-b\log\left(x\right)+\frac{b\log\left(x^2-\frac{1}{b}\right)}{2}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2-1),x)
```

```
[Out] -b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

3.257 $\int \frac{1}{-1+a+ax^2} dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a + a*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*x)/Sqrt[1 - a]]/Sqrt[(1 - a)*a])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{-1+a+ax^2} dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.93

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-1}}\right)}{\sqrt{a-1}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a + a*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[-1 + a]]/(Sqrt[-1 + a]*Sqrt[a])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-1+a+ax^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + a + a*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(-1 + a + a*x^2)^(-1), x]
```

fricas [A] time = 0.53, size = 82, normalized size = 2.73

$$\left[-\frac{\sqrt{-a^2+a}\log\left(\frac{ax^2-2\sqrt{-a^2+a}\,x-a+1}{ax^2+a-1}\right)}{2\left(a^2-a\right)}, \frac{\arctan\left(\frac{\sqrt{a^2-a}\,x}{a-1}\right)}{\sqrt{a^2-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + a)*log((a*x^2 - 2*sqrt(-a^2 + a)*x - a + 1)/(a*x^2 + a -
1))/(a^2 - a), arctan(sqrt(a^2 - a)*x/(a - 1))/sqrt(a^2 - a)]
```

giac [A] time = 0.58, size = 23, normalized size = 0.77

$$\frac{\arctan\left(\frac{ax}{\sqrt{a^2-a}}\right)}{\sqrt{a^2-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="giac")
```

```
[Out] arctan(a*x/sqrt(a^2 - a))/sqrt(a^2 - a)
```

maple [A] time = 0.01, size = 20, normalized size = 0.67

$$\frac{\arctan\left(\frac{ax}{\sqrt{(a-1)a}}\right)}{\sqrt{(a-1)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+a-1),x)
```

```
[Out] 1/((a-1)*a)^(1/2)*arctan(x*a/((a-1)*a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a-1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive or negative?
```

mupad [B] time = 0.17, size = 23, normalized size = 0.77

$$\frac{\operatorname{atan}\left(\frac{a\,x}{\sqrt{a^2-a}}\right)}{\sqrt{a^2-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*x^2 - 1),x)
```

```
[Out] atan((a*x)/(a^2 - a)^(1/2))/(a^2 - a)^(1/2)
```

sympy [B] time = 0.17, size = 83, normalized size = 2.77

$$-\frac{\sqrt{-\frac{1}{a(a-1)}}\log\left(-a\sqrt{-\frac{1}{a(a-1)}}+x+\sqrt{-\frac{1}{a(a-1)}}\right)}{2}+\frac{\sqrt{-\frac{1}{a(a-1)}}\log\left(a\sqrt{-\frac{1}{a(a-1)}}+x-\sqrt{-\frac{1}{a(a-1)}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+a-1),x)
```

```
[Out] -sqrt(-1/(a*(a - 1)))*log(-a*sqrt(-1/(a*(a - 1))) + x + sqrt(-1/(a*(a - 1))
))/2 + sqrt(-1/(a*(a - 1)))*log(a*sqrt(-1/(a*(a - 1))) + x - sqrt(-1/(a*(a
- 1))))/2
```

3.258 $\int \frac{1}{-c-d+(c-d)x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {208}

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(-c - d + (c - d)*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(Sqrt[c - d]*x)/Sqrt[c + d]]/(Sqrt[c - d]*Sqrt[c + d]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{-c-d+(c-d)x^2}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{c-d}\,x}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.19

$$\frac{\tan^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{-c-d}}\right)}{\sqrt{-c-d}\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-c - d + (c - d)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c - d]*x)/Sqrt[-c - d]]/(Sqrt[-c - d]*Sqrt[c - d])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-c-d+(c-d)x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-c - d + (c - d)*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(-c - d + (c - d)*x^2)^(-1), x]
```

fricas [A] time = 0.52, size = 102, normalized size = 2.76

$$\left[\frac{\log\left(\frac{(c-d)x^2-2\sqrt{c^2-d^2}\,x+c+d}{(c-d)x^2-c-d}\right)}{2\sqrt{c^2-d^2}}, \frac{\sqrt{-c^2+d^2}\arctan\left(\frac{\sqrt{-c^2+d^2}\,x}{c+d}\right)}{c^2-d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(((c - d)*x^2 - 2*sqrt(c^2 - d^2)*x + c + d)/((c - d)*x^2 - c - d))
/sqrt(c^2 - d^2), sqrt(-c^2 + d^2)*arctan(sqrt(-c^2 + d^2)*x/(c + d))/(c^2
- d^2)]
```

giac [A] time = 0.59, size = 33, normalized size = 0.89

$$\frac{\arctan\left(\frac{cx-dx}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="giac")
```

```
[Out] arctan((c*x - d*x)/sqrt(-c^2 + d^2))/sqrt(-c^2 + d^2)
```

maple [A] time = 0.01, size = 33, normalized size = 0.89

$$-\frac{\operatorname{arctanh}\left(\frac{(c-d)x}{\sqrt{(c+d)(c-d)}}\right)}{\sqrt{(c+d)(c-d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c-d+(c-d)*x^2),x)
```

[Out] -1/((c+d)*(c-d))^(1/2)*arctanh((c-d)*x/((c+d)*(c-d))^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c-d+(c-d)*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more details)Is 4*c^2-4*d^2 positive or negative?

mupad [B] time = 0.29, size = 29, normalized size = 0.78

$$-\frac{\mathrm{atanh}\left(\frac{x\,\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c+d}\,\sqrt{c-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(c + d - x^2*(c - d)),x)

[Out] -atanh((x*(c - d)^(1/2))/(c + d)^(1/2))/((c + d)^(1/2)*(c - d)^(1/2))

sympy [B] time = 0.23, size = 87, normalized size = 2.35

$$\frac{\sqrt{\frac{1}{(c-d)(c+d)}}\log\left(-c\sqrt{\frac{1}{(c-d)(c+d)}}-d\sqrt{\frac{1}{(c-d)(c+d)}}+x\right)}{2}-\frac{\sqrt{\frac{1}{(c-d)(c+d)}}\log\left(c\sqrt{\frac{1}{(c-d)(c+d)}}+d\sqrt{\frac{1}{(c-d)(c+d)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c-d+(c-d)*x**2),x)

[Out] sqrt(1/((c - d)*(c + d)))*log(-c*sqrt(1/((c - d)*(c + d))) - d*sqrt(1/((c -
d)*(c + d))) + x)/2 - sqrt(1/((c - d)*(c + d)))*log(c*sqrt(1/((c - d)*(c +
d))) + d*sqrt(1/((c - d)*(c + d))) + x)/2

3.259 $\int \frac{1}{x(1+bx^2)^2} \, dx$

Optimal. Leaf size=28

$$\frac{1}{2(bx^2+1)} - \frac{1}{2}\log\left(bx^2+1\right) + \log(x)$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2(bx^2+1)} - \frac{1}{2}\log\left(bx^2+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x^2)^2),x]
```

```
[Out] 1/(2*(1 + b*x^2)) + Log[x] - Log[1 + b*x^2]/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+bx^2\right)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+bx)^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x} - \frac{b}{(1+bx)^2} - \frac{b}{1+bx}\right) dx, x, x^2\right) \\
&= \frac{1}{2\left(1+bx^2\right)} + \log(x) - \frac{1}{2}\log\left(1+bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.89

$$\frac{1}{2bx^2+2}-\frac{1}{2}\log\left(bx^2+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x^2)^2),x]
[Out] (2 + 2*b*x^2)^(-1) + Log[x] - Log[1 + b*x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + b*x^2)^2),x]
[Out] IntegrateAlgebraic[1/(x*(1 + b*x^2)^2), x]
```

fricas [A] time = 0.45, size = 40, normalized size = 1.43

$$-\frac{\left(bx^2+1\right)\log\left(bx^2+1\right)-2\left(bx^2+1\right)\log(x)-1}{2\left(bx^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*((b*x^2 + 1)*log(b*x^2 + 1) - 2*(b*x^2 + 1)*log(x) - 1)/(b*x^2 + 1)
```

giac [A] time = 0.63, size = 36, normalized size = 1.29

$$\frac{bx^2+2}{2\left(bx^2+1\right)}+\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="giac")
[Out] 1/2*(b*x^2 + 2)/(b*x^2 + 1) + 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

maple [A] time = 0.01, size = 25, normalized size = 0.89

$$\ln(x)-\frac{\ln\left(b\,x^2+1\right)}{2}+\frac{1}{2b\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+1)^2,x)
```

```
[Out] 1/2/(b*x^2+1)+ln(x)-1/2*ln(b*x^2+1)
```

maxima [A] time = 1.35, size = 28, normalized size = 1.00

$$\frac{1}{2\left(bx^2+1\right)}-\frac{1}{2}\log\left(bx^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(b*x^2 + 1) - 1/2*log(b*x^2 + 1) + 1/2*log(x^2)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.86

$$\ln(x)-\frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2}+\frac{1}{2\,\left(b\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + 1)^2),x)
```

```
[Out] log(x) - log((3*b*x^2)/2 + 3/2)/2 + 1/(2*(b*x^2 + 1))
```

sympy [A] time = 0.20, size = 22, normalized size = 0.79

$$\log\left(x\right)-\frac{\log\left(x^2+\frac{1}{b}\right)}{2}+\frac{1}{2bx^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+1)**2,x)
```

```
[Out] log(x) - log(x**2 + 1/b)/2 + 1/(2*b*x**2 + 2)
```

3.260 $\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx$

Optimal. Leaf size=30

$$\frac{1}{2\left(1-bx^2\right)}-\frac{1}{2}\log\left(1-bx^2\right)+\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{2\left(1-bx^2\right)}-\frac{1}{2}\log\left(1-bx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x^2)^2),x]
```

```
[Out] 1/(2*(1 - b*x^2)) + Log[x] - Log[1 - b*x^2]/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(-1+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x}+\frac{b}{(-1+bx)^2}-\frac{b}{-1+bx}\right)dx,x,x^2\right)\\
&= \frac{1}{2\left(1-bx^2\right)}+\log(x)-\frac{1}{2}\log\left(1-bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.87

$$\frac{1}{2-2bx^2}-\frac{1}{2}\log\left(1-bx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x^2)^2),x]
[Out] (2 - 2*b*x^2)^(-1) + Log[x] - Log[1 - b*x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(-1 + b*x^2)^2),x]
[Out] IntegrateAlgebraic[1/(x*(-1 + b*x^2)^2), x]
```

fricas [A] time = 0.47, size = 40, normalized size = 1.33

$$-\frac{\left(bx^2-1\right)\log\left(bx^2-1\right)-2\left(bx^2-1\right)\log(x)+1}{2\left(bx^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="fricas")
[Out] -1/2*((b*x^2 - 1)*log(b*x^2 - 1) - 2*(b*x^2 - 1)*log(x) + 1)/(b*x^2 - 1)
```

giac [A] time = 0.63, size = 36, normalized size = 1.20

$$\frac{bx^2-2}{2\left(bx^2-1\right)}+\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="giac")
[Out] 1/2*(b*x^2 - 2)/(b*x^2 - 1) + 1/2*log(x^2) - 1/2*log(abs(b*x^2 - 1))
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$\ln(x)-\frac{\ln\left(b\,x^2-1\right)}{2}-\frac{1}{2\left(b\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2-1)^2,x)
```

```
[Out] ln(x)-1/2/(b*x^2-1)-1/2*ln(b*x^2-1)
```

maxima [A] time = 1.35, size = 28, normalized size = 0.93

$$-\frac{1}{2\left(bx^2-1\right)}-\frac{1}{2}\,\log\left(bx^2-1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(b*x^2 - 1) - 1/2*log(b*x^2 - 1) + 1/2*log(x^2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$\ln(x)-\frac{\ln\left(\frac{3\,b\,x^2}{2}-\frac{3}{2}\right)}{2}-\frac{1}{2\,\left(b\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 - 1)^2),x)
```

```
[Out] log(x) - log((3*b*x^2)/2 - 3/2)/2 - 1/(2*(b*x^2 - 1))
```

sympy [A] time = 0.21, size = 22, normalized size = 0.73

$$\log\left(x\right)-\frac{\log\left(x^2-\frac{1}{b}\right)}{2}-\frac{1}{2bx^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2-1)**2,x)
```

```
[Out] log(x) - log(x**2 - 1/b)/2 - 1/(2*b*x**2 - 2)
```

3.261 $\int \frac{1}{a+(b-ac)x^2}\,dx$

Optimal. Leaf size=34

$$\frac{\tan^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+(b-ac)x^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b-ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.06

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{ac-b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{ac-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[-b + a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[-b + a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + (b - ac)x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + (b - a*c)*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + (b - a*c)*x^2)^(-1), x]
```

fricas [A] time = 0.60, size = 106, normalized size = 3.12

$$\left[\frac{\log\left(\frac{(ac-b)x^2+2\sqrt{a^2c-ab}\,x+a}{(ac-b)x^2-a}\right)}{2\sqrt{a^2c-ab}}, -\frac{\sqrt{-a^2c+ab}\,\arctan\left(\frac{\sqrt{-a^2c+ab}\,x}{a}\right)}{a^2c-ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(((a*c - b)*x^2 + 2*sqrt(a^2*c - a*b)*x + a)/((a*c - b)*x^2 - a))/s
qrt(a^2*c - a*b), -sqrt(-a^2*c + a*b)*arctan(sqrt(-a^2*c + a*b)*x/a)/(a^2*c
 - a*b)]
```

giac [A] time = 0.63, size = 37, normalized size = 1.09

$$-\frac{\arctan\left(\frac{acx-bx}{\sqrt{-a^2c+ab}}\right)}{\sqrt{-a^2c+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="giac")
```

```
[Out] -arctan((a*c*x - b*x)/sqrt(-a^2*c + a*b))/sqrt(-a^2*c + a*b)
```

maple [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{\operatorname{arctanh}\left(\frac{(ac-b)x}{\sqrt{(ac-b)a}}\right)}{\sqrt{(ac-b)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(-a*c+b)*x^2),x)
```

```
[Out] 1/(a*(a*c-b))^(1/2)*arctanh((a*c-b)*x/(a*(a*c-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*c-b>0)', see `assume?` for more d
etails)Is a*c-b positive or negative?
```

mupad [B] time = 5.14, size = 38, normalized size = 1.12

$$-\frac{\operatorname{atanh}\left(\frac{x\,\left(2\,b-2\,a\,c\right)}{2\,\sqrt{a}\,\sqrt{a\,c-b}}\right)}{\sqrt{a}\,\sqrt{a\,c-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + x^2*(b - a*c)),x)
```

```
[Out] -atanh((x*(2*b - 2*a*c))/(2*a^(1/2)*(a*c - b)^(1/2)))/(a^(1/2)*(a*c - b)^(1
/2))
```

sympy [B] time = 0.25, size = 60, normalized size = 1.76

$$-\frac{\sqrt{\frac{1}{a(ac-b)}}\log\left(-a\sqrt{\frac{1}{a(ac-b)}}+x\right)}{2}+\frac{\sqrt{\frac{1}{a(ac-b)}}\log\left(a\sqrt{\frac{1}{a(ac-b)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(-a*c+b)*x**2),x)
```

```
[Out] -sqrt(1/(a*(a*c - b)))*log(-a*sqrt(1/(a*(a*c - b))) + x)/2 + sqrt(1/(a*(a*c
 - b)))*log(a*sqrt(1/(a*(a*c - b))) + x)/2
```

3.262 $\int \frac{1}{a-(b-ac)x^2}\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - (b - a*c)*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-(b-ac)x^2}\,dx=\frac{\tanh^{-1}\left(\frac{\sqrt{b-ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ac-b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{ac-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - (b - a*c)*x^2)^(-1),x]
```

[Out] ArcTan[(Sqrt[-b + a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[-b + a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a-(b-ac)x^2}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a - (b - a*c)*x^2)^(-1),x]

[Out] IntegrateAlgebraic[(a - (b - a*c)*x^2)^(-1), x]

fricas [A] time = 0.84, size = 105, normalized size = 3.09

$$\left[-\frac{\sqrt{-a^2c+ab}\,\log\left(\frac{(ac-b)x^2-2\,\sqrt{-a^2c+ab}\,x-a}{(ac-b)x^2+a}\right)}{2\left(a^2c-ab\right)},\ \frac{\arctan\left(\frac{\sqrt{a^2c-ab}\,x}{a}\right)}{\sqrt{a^2c-ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a^2*c + a*b)*log(((a*c - b)*x^2 - 2*sqrt(-a^2*c + a*b)*x - a)/((a*c - b)*x^2 + a))/(a^2*c - a*b), arctan(sqrt(a^2*c - a*b)*x/a)/sqrt(a^2*c - a*b)]

giac [A] time = 0.62, size = 36, normalized size = 1.06

$$\frac{\arctan\left(\frac{acx-bx}{\sqrt{a^2c-ab}}\right)}{\sqrt{a^2c-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="giac")

[Out] arctan((a*c*x - b*x)/sqrt(a^2*c - a*b))/sqrt(a^2*c - a*b)

maple [A] time = 0.00, size = 34, normalized size = 1.00

$$\frac{\arctan\left(\frac{(ac-b)x}{\sqrt{(ac-b)a}}\right)}{\sqrt{(ac-b)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-(-a*c+b)*x^2),x)
```

```
[Out] 1/((a*c-b)*a)^(1/2)*arctan((a*c-b)/((a*c-b)*a)^(1/2)*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*c-b>0)', see `assume?` for more d
etails)Is a*c-b positive or negative?
```

mupad [B] time = 4.64, size = 38, normalized size = 1.12

$$-\frac{\mathrm{atan}\left(\frac{x\,\left(2\,b-2\,a\,c\right)}{2\,\sqrt{a^2\,c-a\,b}}\right)}{\sqrt{a^2\,c-a\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - x^2*(b - a*c)),x)
```

```
[Out] -atan((x*(2*b - 2*a*c))/(2*(a^2*c - a*b)^(1/2)))/(a^2*c - a*b)^(1/2)
```

sympy [B] time = 0.23, size = 66, normalized size = 1.94

$$-\frac{\sqrt{-\frac{1}{a(ac-b)}}\log\left(-a\sqrt{-\frac{1}{a(ac-b)}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a(ac-b)}}\log\left(a\sqrt{-\frac{1}{a(ac-b)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(-a*c+b)*x**2),x)
```

```
[Out] -sqrt(-1/(a*(a*c - b)))*log(-a*sqrt(-1/(a*(a*c - b))) + x)/2 + sqrt(-1/(a*(
a*c - b)))*log(a*sqrt(-1/(a*(a*c - b))) + x)/2
```

3.263 $\int \frac{1}{c(a-d)-(b-c)x^2}\,dx$

Optimal. Leaf size=50

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\,\sqrt{a-d}}\right)}{\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}$$

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\,\sqrt{a-d}}\right)}{\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[b - c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[b - c]*Sqrt[c]*Sqrt[a
- d])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx = \frac{\tanh^{-1}\left(\frac{\sqrt{b-c}\,x}{\sqrt{c}\,\sqrt{a-d}}\right)}{\sqrt{b-c}\,\sqrt{c}\,\sqrt{a-d}}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{c-b}}{\sqrt{c}\,\sqrt{a-d}}\right)}{\sqrt{c}\,\sqrt{a-d}\,\sqrt{c-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[-b + c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[c]*Sqrt[-b + c]*Sqrt[a
 - d])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(c*(a - d) - (b - c)*x^2)^(-1), x]
```

fricas [B] time = 0.51, size = 182, normalized size = 3.64

$$\left[\frac{\log\left(\frac{(b-c)x^2+ac-cd+2\sqrt{abc-ac^2-(bc-c^2)d}\,x}{(b-c)x^2-ac+cd}\right)}{2\sqrt{abc-ac^2-(bc-c^2)d}},\ \frac{\sqrt{-abc+ac^2+(bc-c^2)d}\,\arctan\left(-\frac{\sqrt{-abc+ac^2+(bc-c^2)d}\,x}{ac-cd}\right)}{abc-ac^2-(bc-c^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(((b - c)*x^2 + a*c - c*d + 2*sqrt(a*b*c - a*c^2 - (b*c - c^2)*d)*x
)/((b - c)*x^2 - a*c + c*d))/sqrt(a*b*c - a*c^2 - (b*c - c^2)*d), sqrt(-a*b
*c + a*c^2 + (b*c - c^2)*d)*arctan(-sqrt(-a*b*c + a*c^2 + (b*c - c^2)*d)*x/
(a*c - c*d))/(a*b*c - a*c^2 - (b*c - c^2)*d)]
```

giac [A] time = 0.64, size = 58, normalized size = 1.16

$$-\frac{\arctan\left(\frac{bx-cx}{\sqrt{-abc+ac^2+bcd-c^2d}}\right)}{\sqrt{-abc+ac^2+bcd-c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="giac")
```

```
[Out] -arctan((b*x - c*x)/sqrt(-a*b*c + a*c^2 + b*c*d - c^2*d))/sqrt(-a*b*c + a*c
^2 + b*c*d - c^2*d)
```

maple [A] time = 0.01, size = 38, normalized size = 0.76

$$\frac{\operatorname{arctanh}\left(\frac{(b-c)x}{\sqrt{(a-d)(b-c)c}}\right)}{\sqrt{(a-d)\,(b-c)\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a-d)-(b-c)*x^2),x)
```

```
[Out] 1/(c*(a-d)*(b-c))^(1/2)*arctanh((b-c)*x/(c*(a-d)*(b-c))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((c-b)*(d-a)>0)', see `assume?` for
more details)Is (c-b)*(d-a) positive or negative?
```

mupad [B] time = 4.88, size = 46, normalized size = 0.92

$$\frac{\operatorname{atanh}\left(\frac{x\,(2\,b-2\,c)}{2\,\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}\right)}{\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a - d) - x^2*(b - c)),x)
```

```
[Out] atanh((x*(2*b - 2*c))/(2*c^(1/2)*(a - d)^(1/2)*(b - c)^(1/2)))/(c^(1/2)*(a
- d)^(1/2)*(b - c)^(1/2))
```

sympy [B] time = 0.32, size = 104, normalized size = 2.08

$$-\frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(-ac\sqrt{\frac{1}{c(a-d)(b-c)}}+cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2}+\frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(ac\sqrt{\frac{1}{c(a-d)(b-c)}}-cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(a-d)-(b-c)*x**2),x)
```

```
[Out] -sqrt(1/(c*(a - d)*(b - c)))*log(-a*c*sqrt(1/(c*(a - d)*(b - c))) + c*d*sqr
t(1/(c*(a - d)*(b - c))) + x)/2 + sqrt(1/(c*(a - d)*(b - c)))*log(a*c*sqrt(
1/(c*(a - d)*(b - c))) - c*d*sqrt(1/(c*(a - d)*(b - c))) + x)/2
```

3.264 $\int x^{7/2}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{9}ax^{9/2}+\frac{2}{13}bx^{13/2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{9}ax^{9/2}+\frac{2}{13}bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(9/2))/9 + (2*b*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)\,dx&=\int\left(ax^{7/2}+bx^{11/2}\right)\,dx\\&=\frac{2}{9}ax^{9/2}+\frac{2}{13}bx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{9}ax^{9/2}+\frac{2}{13}bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(9/2))/9 + (2*b*x^(13/2))/13
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{117}\left(13ax^{9/2}+9bx^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^2),x]
[Out] (2*(13*a*x^(9/2) + 9*b*x^(13/2)))/117
```

fricas [A] time = 0.47, size = 18, normalized size = 0.86

$$\frac{2}{117}\left(9\,bx^6+13\,ax^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="fricas")
[Out] 2/117*(9*b*x^6 + 13*a*x^4)*sqrt(x)
```

giac [A] time = 0.63, size = 13, normalized size = 0.62

$$\frac{2}{13}\,bx^{\frac{13}{2}}+\frac{2}{9}\,ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="giac")
[Out] 2/13*b*x^(13/2) + 2/9*a*x^(9/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(9b\,x^2+13a\right)x^{\frac{9}{2}}}{117}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a),x)
[Out] 2/117*x^(9/2)*(9*b*x^2+13*a)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.62

$$\frac{2}{13}\,bx^{\frac{13}{2}}+\frac{2}{9}\,ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a),x, algorithm="maxima")
[Out] 2/13*b*x^(13/2) + 2/9*a*x^(9/2)
```

mupad [B] time = 4.50, size = 15, normalized size = 0.71

$$\frac{2\,x^{9/2}\,\left(9\,b\,x^2+13\,a\right)}{117}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2),x)
[Out] (2*x^(9/2)*(13*a + 9*b*x^2))/117
```

sympy [A] time = 5.45, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{9}{2}}}{9}+\frac{2bx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a),x)
[Out] 2*a*x**(9/2)/9 + 2*b*x**(13/2)/13
```

3.265 $\int x^{5/2} \left(a+bx^2\right) dx$

Optimal. Leaf size=21

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)dx&=\int\left(ax^{5/2}+bx^{9/2}\right)dx\\&=\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{77}\left(11ax^{7/2}+7bx^{11/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2),x]
[Out] (2*(11*a*x^(7/2) + 7*b*x^(11/2)))/77
```

fricas [A] time = 0.66, size = 18, normalized size = 0.86

$$\frac{2}{77}\left(7\,bx^5+11\,ax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="fricas")
[Out] 2/77*(7*b*x^5 + 11*a*x^3)*sqrt(x)
```

giac [A] time = 0.60, size = 13, normalized size = 0.62

$$\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="giac")
[Out] 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(7b\,x^2+11a\right)x^{\frac{7}{2}}}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a),x)
[Out] 2/77*x^(7/2)*(7*b*x^2+11*a)
```

maxima [A] time = 1.29, size = 13, normalized size = 0.62

$$\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{7/2}\,\left(7\,b\,x^2+11\,a\right)}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2),x)
```

```
[Out] (2*x^(7/2)*(11*a + 7*b*x^2))/77
```

sympy [A] time = 2.41, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(11/2)/11
```

3.266 $\int x^{3/2} \left(a+bx^2\right) \, dx$

Optimal. Leaf size=21

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2} \left(a+bx^2\right) \, dx &= \int \left(ax^{3/2}+bx^{7/2}\right) \, dx\\ &= \frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{45}\left(9ax^{5/2}+5bx^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2),x]
[Out] (2*(9*a*x^(5/2) + 5*b*x^(9/2)))/45
```

fricas [A] time = 0.64, size = 18, normalized size = 0.86

$$\frac{2}{45}\left(5\,bx^4+9\,ax^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="fricas")
[Out] 2/45*(5*b*x^4 + 9*a*x^2)*sqrt(x)
```

giac [A] time = 0.63, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="giac")
[Out] 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(5b\,x^2+9a\right)x^{\frac{5}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a),x)
[Out] 2/45*x^(5/2)*(5*b*x^2+9*a)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

mupad [B] time = 0.02, size = 15, normalized size = 0.71

$$\frac{2\,x^{5/2}\,\left(5\,b\,x^2+9\,a\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2),x)
```

```
[Out] (2*x^(5/2)*(9*a + 5*b*x^2))/45
```

sympy [A] time = 0.95, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(9/2)/9
```

3.267 $\int \sqrt{x} \left(a+bx^2\right) \, dx$

Optimal. Leaf size=21

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x} \left(a+bx^2\right) \, dx &= \int \left(a\sqrt{x}+bx^{5/2}\right) \, dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{21}\left(7ax^{3/2}+3bx^{7/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2),x]
[Out] (2*(7*a*x^(3/2) + 3*b*x^(7/2)))/21
```

fricas [A] time = 0.53, size = 16, normalized size = 0.76

$$\frac{2}{21}\left(3\,bx^3+7\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*b*x^3 + 7*a*x)*sqrt(x)
```

giac [A] time = 0.61, size = 13, normalized size = 0.62

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="giac")
[Out] 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(3b\,x^2+7a\right)x^{\frac{3}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*x^(1/2),x)
[Out] 2/21*x^(3/2)*(3*b*x^2+7*a)
```

maxima [A] time = 1.31, size = 13, normalized size = 0.62

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{3/2}\,\left(3\,b\,x^2+7\,a\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2),x)
```

```
[Out] (2*x^(3/2)*(7*a + 3*b*x^2))/21
```

sympy [A] time = 1.30, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{3}{2}}}{3} + \frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*x**(1/2),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(7/2)/7
```

3.268 $\int \frac{a+bx^2}{\sqrt{x}} dx$

Optimal. Leaf size=19

$$2a\sqrt{x} + \frac{2}{5}bx^{5/2}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$2a\sqrt{x} + \frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\sqrt{x}}\,dx &= \int \left(\frac{a}{\sqrt{x}} + bx^{3/2}\right) dx\\ &= 2a\sqrt{x} + \frac{2}{5}bx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$2a\sqrt{x} + \frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5
```

IntegrateAlgebraic [A] time = 0.01, size = 20, normalized size = 1.05

$$\frac{2}{5}\left(5a\sqrt{x}+bx^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/Sqrt[x],x]
[Out] (2*(5*a*Sqrt[x] + b*x^(5/2)))/5
```

fricas [A] time = 0.57, size = 14, normalized size = 0.74

$$\frac{2}{5}\left(bx^2+5\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="fricas")
[Out] 2/5*(b*x^2 + 5*a)*sqrt(x)
```

giac [A] time = 0.63, size = 13, normalized size = 0.68

$$\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="giac")
[Out] 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

maple [A] time = 0.00, size = 15, normalized size = 0.79

$$\frac{2\left(b\,x^2+5a\right)\sqrt{x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(1/2),x)
[Out] 2/5*x^(1/2)*(b*x^2+5*a)
```

maxima [A] time = 1.30, size = 13, normalized size = 0.68

$$\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.74

$$\frac{2\sqrt{x}\,\left(b\,x^2+5\,a\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(5*a + b*x^2))/5
```

sympy [A] time = 0.23, size = 17, normalized size = 0.89

$$2a\sqrt{x}+\frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(5/2)/5
```

3.269 $\int \frac{a+bx^2}{x^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{3/2}}\,dx &= \int \left(\frac{a}{x^{3/2}}+b\sqrt{x}\right)dx\\ &= -\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 0.95

$$\frac{2\left(bx^2-3a\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/x^(3/2),x]
[Out] (2*(-3*a + b*x^2))/(3*Sqrt[x])
```

fricas [A] time = 0.55, size = 14, normalized size = 0.74

$$\frac{2\left(bx^2-3\,a\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="fricas")
[Out] 2/3*(b*x^2 - 3*a)/sqrt(x)
```

giac [A] time = 0.59, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="giac")
[Out] 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{2\left(-b\,x^2+3a\right)}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(3/2),x)
[Out] -2/3*(-b*x^2+3*a)/x^(1/2)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{6\,a-2\,b\,x^2}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(3/2),x)
```

```
[Out] -(6*a - 2*b*x^2)/(3*x^(1/2))
```

sympy [A] time = 0.38, size = 17, normalized size = 0.89

$$-\frac{2a}{\sqrt{x}}+\frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*x**(3/2)/3
```

3.270 $\int \frac{a+bx^2}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{\sqrt{x}}\right)dx\\ &= -\frac{2a}{3x^{3/2}}+2b\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2\left(3bx^2-a\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/x^(5/2),x]
[Out] (2*(-a + 3*b*x^2))/(3*x^(3/2))
```

fricas [A] time = 0.45, size = 15, normalized size = 0.79

$$\frac{2\left(3\,bx^2-a\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="fricas")
[Out] 2/3*(3*b*x^2 - a)/x^(3/2)
```

giac [A] time = 0.60, size = 13, normalized size = 0.68

$$2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="giac")
[Out] 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{2\left(-3b\,x^2+a\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(5/2),x)
[Out] -2/3*(-3*b*x^2+a)/x^(3/2)
```

maxima [A] time = 1.32, size = 13, normalized size = 0.68

$$2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{2\,a-6\,b\,x^2}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(5/2),x)
```

```
[Out] -(2*a - 6*b*x^2)/(3*x^(3/2))
```

sympy [A] time = 0.56, size = 17, normalized size = 0.89

$$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) + 2*b*sqrt(x)
```

3.271 $\int \frac{a+bx^2}{x^{7/2}} \, dx$

Optimal. Leaf size=19

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^{7/2}} \, dx &= \int \left(\frac{a}{x^{7/2}}+\frac{b}{x^{3/2}}\right) \, dx\\ &= -\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.01, size = 17, normalized size = 0.89

$$-\frac{2\left(a+5bx^2\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/x^(7/2),x]
[Out] (-2*(a + 5*b*x^2))/(5*x^(5/2))
```

fricas [A] time = 0.53, size = 13, normalized size = 0.68

$$-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="fricas")
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

giac [A] time = 0.59, size = 13, normalized size = 0.68

$$-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="giac")
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{2\left(5b\,x^2+a\right)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^(7/2),x)
[Out] -2/5*(5*b*x^2+a)/x^(5/2)
```

maxima [**A**] time = 1.31, size = 13, normalized size = 0.68

$$-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*b*x^2 + a)/x^(5/2)
```

mupad [**B**] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{10\,b\,x^2+2\,a}{5\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/x^(7/2),x)
```

```
[Out] -(2*a + 10*b*x^2)/(5*x^(5/2))
```

sympy [**A**] time = 1.24, size = 19, normalized size = 1.00

$$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**(7/2),x)
```

```
[Out] -2*a/(5*x**(5/2)) - 2*b/sqrt(x)
```

3.272 $\int x^{7/2}\left(a+bx^2\right)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(9/2))/9 + (4*a*b*x^(13/2))/13 + (2*b^2*x^(17/2))/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\,dx &= \int \left(a^2x^{7/2}+2abx^{11/2}+b^2x^{15/2}\right)dx\\ &= \frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{9/2}\left(221a^2+306abx^2+117b^2x^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(9/2)*(221*a^2 + 306*a*b*x^2 + 117*b^2*x^4))/1989
```

IntegrateAlgebraic [A] time = 0.02, size = 34, normalized size = 0.94

$$\frac{2\left(221a^2x^{9/2}+306abx^{13/2}+117b^2x^{17/2}\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^2)^2,x]
[Out] (2*(221*a^2*x^(9/2) + 306*a*b*x^(13/2) + 117*b^2*x^(17/2)))/1989
```

fricas [A] time = 0.65, size = 29, normalized size = 0.81

$$\frac{2}{1989}\left(117\,b^2x^8+306\,abx^6+221\,a^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 2/1989*(117*b^2*x^8 + 306*a*b*x^6 + 221*a^2*x^4)*sqrt(x)
```

giac [A] time = 0.62, size = 24, normalized size = 0.67

$$\frac{2}{17}\,b^2x^{\frac{17}{2}}+\frac{4}{13}\,abx^{\frac{13}{2}}+\frac{2}{9}\,a^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="giac")
[Out] 2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(117b^2x^4+306ab\,x^2+221a^2\right)x^{\frac{9}{2}}}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2,x)
[Out] 2/1989*x^(9/2)*(117*b^2*x^4+306*a*b*x^2+221*a^2)
```

maxima [A] time = 1.36, size = 24, normalized size = 0.67

$$\frac{2}{17}\,b^2x^{\frac{17}{2}}+\frac{4}{13}\,abx^{\frac{13}{2}}+\frac{2}{9}\,a^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.69

$$x^{9/2}\left(\frac{2\,a^2}{9}+\frac{4\,a\,b\,x^2}{13}+\frac{2\,b^2\,x^4}{17}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^2,x)
```

```
[Out] x^(9/2)*((2*a^2)/9 + (2*b^2*x^4)/17 + (4*a*b*x^2)/13)
```

sympy [A] time = 10.60, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{9}{2}}}{9}+\frac{4abx^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(9/2)/9 + 4*a*b*x**(13/2)/13 + 2*b**2*x**(17/2)/17
```

3.273 $\int x^{5/2} \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*b^2*x^(15/2))/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\,dx &= \int \left(a^2x^{5/2}+2abx^{9/2}+b^2x^{13/2}\right)dx\\ &= \frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{7/2}\left(165a^2+210abx^2+77b^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(7/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

IntegrateAlgebraic [A] time = 0.02, size = 34, normalized size = 0.94

$$\frac{2\left(165a^2x^{7/2}+210abx^{11/2}+77b^2x^{15/2}\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2)^2,x]
[Out] (2*(165*a^2*x^(7/2) + 210*a*b*x^(11/2) + 77*b^2*x^(15/2)))/1155
```

fricas [A] time = 0.64, size = 29, normalized size = 0.81

$$\frac{2}{1155}\left(77\,b^2x^7+210\,abx^5+165\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 2/1155*(77*b^2*x^7 + 210*a*b*x^5 + 165*a^2*x^3)*sqrt(x)
```

giac [A] time = 0.63, size = 24, normalized size = 0.67

$$\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="giac")
[Out] 2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(77b^2x^4+210ab\,x^2+165a^2\right)x^{\frac{7}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2,x)
[Out] 2/1155*x^(7/2)*(77*b^2*x^4+210*a*b*x^2+165*a^2)
```

maxima [A] time = 1.26, size = 24, normalized size = 0.67

$$\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{7/2}\,\left(165\,a^2+210\,a\,b\,x^2+77\,b^2\,x^4\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^2,x)
```

```
[Out] (2*x^(7/2)*(165*a^2 + 77*b^2*x^4 + 210*a*b*x^2))/1155
```

sympy [A] time = 5.51, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{2b^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 2*b**2*x**(15/2)/15
```

3.274 $\int x^{3/2} \left(a + bx^2\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{5}a^2x^{5/2} + \frac{4}{9}abx^{9/2} + \frac{2}{13}b^2x^{13/2}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{5}a^2x^{5/2} + \frac{4}{9}abx^{9/2} + \frac{2}{13}b^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(13/2))/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int x^{3/2} \left(a + bx^2\right)^2 \, dx &= \int \left(a^2x^{3/2} + 2abx^{7/2} + b^2x^{11/2}\right) dx \\ &= \frac{2}{5}a^2x^{5/2} + \frac{4}{9}abx^{9/2} + \frac{2}{13}b^2x^{13/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2}{585}x^{5/2}\left(117a^2 + 130abx^2 + 45b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(5/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

IntegrateAlgebraic [A] time = 0.02, size = 34, normalized size = 0.94

$$\frac{2}{585}\left(117a^2x^{5/2}+130abx^{9/2}+45b^2x^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2)^2,x]
[Out] (2*(117*a^2*x^(5/2) + 130*a*b*x^(9/2) + 45*b^2*x^(13/2)))/585
```

fricas [A] time = 0.50, size = 29, normalized size = 0.81

$$\frac{2}{585}\left(45\,b^2x^6+130\,abx^4+117\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="fricas")
[Out] 2/585*(45*b^2*x^6 + 130*a*b*x^4 + 117*a^2*x^2)*sqrt(x)
```

giac [A] time = 0.63, size = 24, normalized size = 0.67

$$\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="giac")
[Out] 2/13*b^2*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(45b^2x^4+130ab\,x^2+117a^2\right)x^{\frac{5}{2}}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2,x)
[Out] 2/585*x^(5/2)*(45*b^2*x^4+130*a*b*x^2+117*a^2)
```

maxima [A] time = 1.36, size = 24, normalized size = 0.67

$$\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/13*b^2*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{5/2}\,\left(117\,a^2+130\,a\,b\,x^2+45\,b^2\,x^4\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^2,x)
```

```
[Out] (2*x^(5/2)*(117*a^2 + 45*b^2*x^4 + 130*a*b*x^2))/585
```

sympy [A] time = 2.55, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2)/13
```

3.275 $\int \sqrt{x} \left(a+bx^2\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(11/2))/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2 \, dx &= \int \left(a^2\sqrt{x}+2abx^{5/2}+b^2x^{9/2}\right) dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2}{231}x^{3/2}\left(77a^2+66abx^2+21b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

IntegrateAlgebraic [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{2}{231}\left(77a^2x^{3/2}+66abx^{7/2}+21b^2x^{11/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2)^2,x]
[Out] (2*(77*a^2*x^(3/2) + 66*a*b*x^(7/2) + 21*b^2*x^(11/2)))/231
```

fricas [A] time = 0.64, size = 27, normalized size = 0.75

$$\frac{2}{231}\left(21\,b^2x^5+66\,abx^3+77\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="fricas")
[Out] 2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(x)
```

giac [A] time = 0.58, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="giac")
[Out] 2/11*b^2*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(21b^2x^4+66ab\,x^2+77a^2\right)x^{\frac{3}{2}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2),x)
[Out] 2/231*x^(3/2)*(21*b^2*x^4+66*a*b*x^2+77*a^2)
```

maxima [A] time = 1.34, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2),x, algorithm="maxima")
[Out] 2/11*b^2*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{3/2}\,\left(77\,a^2+66\,a\,b\,x^2+21\,b^2\,x^4\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^2,x)
[Out] (2*x^(3/2)*(77*a^2 + 21*b^2*x^4 + 66*a*b*x^2))/231
```

sympy [A] time = 1.70, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2),x)
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(11/2)/11
```

3.276 $\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=34

$$2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}b^2x^{9/2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/Sqrt[x],x]
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*b^2*x^(9/2))/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}}+2abx^{3/2}+b^2x^{7/2}\right)dx\\ &= 2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}b^2x^{9/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{2}{45}\sqrt{x}\left(45a^2+18abx^2+5b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/Sqrt[x],x]
[Out] (2*Sqrt[x]*(45*a^2 + 18*a*b*x^2 + 5*b^2*x^4))/45
```

IntegrateAlgebraic [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{2}{45}\left(45a^2\sqrt{x}+18abx^{5/2}+5b^2x^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/Sqrt[x],x]
[Out] (2*(45*a^2*Sqrt[x] + 18*a*b*x^(5/2) + 5*b^2*x^(9/2)))/45
```

fricas [A] time = 0.62, size = 26, normalized size = 0.76

$$\frac{2}{45}\left(5\,b^2x^4+18\,abx^2+45\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(x)
```

giac [A] time = 0.63, size = 24, normalized size = 0.71

$$\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="giac")
[Out] 2/9*b^2*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.79

$$\frac{2\left(5b^2x^4+18ab\,x^2+45a^2\right)\sqrt{x}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(1/2),x)
[Out] 2/45*x^(1/2)*(5*b^2*x^4+18*a*b*x^2+45*a^2)
```

maxima [A] time = 1.30, size = 24, normalized size = 0.71

$$\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

mupad [B] time = 0.03, size = 26, normalized size = 0.76

$$\frac{2\sqrt{x}\left(45\,a^2+18\,a\,b\,x^2+5\,b^2\,x^4\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(45*a^2 + 5*b^2*x^4 + 18*a*b*x^2))/45
```

sympy [A] time = 0.75, size = 32, normalized size = 0.94

$$2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9
```

3.277 $\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(7/2))/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2}{x^{3/2}}+2ab\sqrt{x}+b^2x^{5/2}\right)dx\\ &= -\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{2\left(-21a^2+14abx^2+3b^2x^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(3/2),x]
```

```
[Out] (2*(-21*a^2 + 14*a*b*x^2 + 3*b^2*x^4))/(21*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 0.88

$$\frac{2\left(-21a^2+14abx^2+3b^2x^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^(3/2),x]
[Out] (2*(-21*a^2 + 14*a*b*x^2 + 3*b^2*x^4))/(21*Sqrt[x])
```

fricas [A] time = 0.56, size = 26, normalized size = 0.76

$$\frac{2\left(3\,b^2x^4+14\,abx^2-21\,a^2\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
[Out] 2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)/sqrt(x)
```

giac [A] time = 0.63, size = 24, normalized size = 0.71

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="giac")
[Out] 2/7*b^2*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.79

$$-\frac{2\left(-3b^2x^4-14ab\,x^2+21a^2\right)}{21\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2),x)
[Out] -2/21*(-3*b^2*x^4-14*a*b*x^2+21*a^2)/x^(1/2)
```

maxima [A] time = 1.33, size = 24, normalized size = 0.71

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^2*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.76

$$\frac{-42\,a^2+28\,a\,b\,x^2+6\,b^2\,x^4}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(3/2),x)
```

```
[Out] (6*b^2*x^4 - 42*a^2 + 28*a*b*x^2)/(21*x^(1/2))
```

sympy [A] time = 0.94, size = 32, normalized size = 0.94

$$-\frac{2a^2}{\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(7/2)/7
```

3.278 $\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*b^2*x^(5/2))/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{\sqrt{x}}+b^2x^{3/2}\right)dx\\ &= -\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{2\left(-5a^2+30abx^2+3b^2x^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(5/2),x]
```

[Out] (2*(-5*a^2 + 30*a*b*x^2 + 3*b^2*x^4))/(15*x^(3/2))

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 0.88

$$\frac{2\left(-5a^2+30abx^2+3b^2x^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2)^2/x^(5/2),x]

[Out] (2*(-5*a^2 + 30*a*b*x^2 + 3*b^2*x^4))/(15*x^(3/2))

fricas [A] time = 0.62, size = 26, normalized size = 0.76

$$\frac{2\left(3\,b^2x^4+30\,abx^2-5\,a^2\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="fricas")

[Out] 2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)/x^(3/2)

giac [A] time = 0.64, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="giac")

[Out] 2/5*b^2*x^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)

maple [A] time = 0.00, size = 27, normalized size = 0.79

$$-\frac{2\left(-3b^2x^4-30ab\,x^2+5a^2\right)}{15x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^2/x^(5/2),x)

[Out] -2/15*(-3*b^2*x^4-30*a*b*x^2+5*a^2)/x^(3/2)

maxima [**A**] time = 1.35, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*b^2*x^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

mupad [**B**] time = 0.04, size = 26, normalized size = 0.76

$$\frac{-10\,a^2 + 60\,a\,b\,x^2 + 6\,b^2\,x^4}{15\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(5/2),x)
```

```
[Out] (6*b^2*x^4 - 10*a^2 + 60*a*b*x^2)/(15*x^(3/2))
```

sympy [**A**] time = 1.11, size = 32, normalized size = 0.94

$$-\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5
```

3.279 $\int \frac{(a+bx^2)^2}{x^{7/2}} \, dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/x^(7/2),x]
```

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{7/2}} \, dx &= \int \left(\frac{a^2}{x^{7/2}}+\frac{2ab}{x^{3/2}}+b^2\sqrt{x}\right) dx\\ &= -\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{2\left(-3a^2-30abx^2+5b^2x^4\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/x^(7/2),x]
```

```
[Out] (2*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 0.88

$$\frac{2\left(-3a^2-30abx^2+5b^2x^4\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/x^(7/2),x]
[Out] (2*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*x^(5/2))
```

fricas [A] time = 0.48, size = 26, normalized size = 0.76

$$\frac{2\left(5\,b^2x^4-30\,abx^2-3\,a^2\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="fricas")
[Out] 2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)/x^(5/2)
```

giac [A] time = 0.60, size = 25, normalized size = 0.74

$$\frac{2}{3}\,b^2x^{\frac{3}{2}}-\frac{2\left(10\,abx^2+a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="giac")
[Out] 2/3*b^2*x^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.79

$$-\frac{2\left(-5b^2x^4+30ab\,x^2+3a^2\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2),x)
[Out] -2/15*(-5*b^2*x^4+30*a*b*x^2+3*a^2)/x^(5/2)
```

maxima [A] time = 1.31, size = 25, normalized size = 0.74

$$\frac{2}{3}\,b^2x^{\frac{3}{2}} - \frac{2\left(10\,abx^2 + a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^2*x^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

mupad [B] time = 0.03, size = 26, normalized size = 0.76

$$-\frac{6\,a^2 + 60\,a\,b\,x^2 - 10\,b^2\,x^4}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/x^(7/2),x)
```

```
[Out] -(6*a^2 - 10*b^2*x^4 + 60*a*b*x^2)/(15*x^(5/2))
```

sympy [A] time = 1.71, size = 32, normalized size = 0.94

$$-\frac{2a^2}{5x^{\frac{5}{2}}} - \frac{4ab}{\sqrt{x}} + \frac{2b^2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2),x)
```

```
[Out] -2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 2*b**2*x**(3/2)/3
```

3.280 $\int x^{7/2}\left(a+bx^2\right)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{9}a^3x^{9/2}+\frac{6}{13}a^2bx^{13/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{13}a^2bx^{13/2}+\frac{2}{9}a^3x^{9/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(9/2))/9 + (6*a^2*b*x^(13/2))/13 + (6*a*b^2*x^(17/2))/17 + (2*b^3*
x^(21/2))/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^3\,dx&=\int\left(a^3x^{7/2}+3a^2bx^{11/2}+3ab^2x^{15/2}+b^3x^{19/2}\right)dx\\&=\frac{2}{9}a^3x^{9/2}+\frac{6}{13}a^2bx^{13/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{9/2}\left(1547a^3+3213a^2bx^2+2457ab^2x^4+663b^3x^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(9/2)*(1547*a^3 + 3213*a^2*b*x^2 + 2457*a*b^2*x^4 + 663*b^3*x^6))/1392
3
```

IntegrateAlgebraic [A] time = 0.02, size = 47, normalized size = 0.92

$$\frac{2\left(1547a^3x^{9/2}+3213a^2bx^{13/2}+2457ab^2x^{17/2}+663b^3x^{21/2}\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*(1547*a^3*x^(9/2) + 3213*a^2*b*x^(13/2) + 2457*a*b^2*x^(17/2) + 663*b^3*
x^(21/2)))/13923
```

fricas [A] time = 0.63, size = 40, normalized size = 0.78

$$\frac{2}{13923}\left(663\,b^3x^{10}+2457\,ab^2x^8+3213\,a^2bx^6+1547\,a^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^3*x^10 + 2457*a*b^2*x^8 + 3213*a^2*b*x^6 + 1547*a^3*x^4)*sqr
t(x)
```

giac [A] time = 0.61, size = 35, normalized size = 0.69

$$\frac{2}{21}\,b^3x^{\frac{21}{2}}+\frac{6}{17}\,ab^2x^{\frac{17}{2}}+\frac{6}{13}\,a^2bx^{\frac{13}{2}}+\frac{2}{9}\,a^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/21*b^3*x^(21/2) + 6/17*a*b^2*x^(17/2) + 6/13*a^2*b*x^(13/2) + 2/9*a^3*x^(
9/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(663b^3x^6+2457a\,b^2x^4+3213a^2b\,x^2+1547a^3\right)x^{\frac{9}{2}}}{13923}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^3,x)
```

```
[Out] 2/13923*x^(9/2)*(663*b^3*x^6+2457*a*b^2*x^4+3213*a^2*b*x^2+1547*a^3)
```

maxima [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{21}\,b^3x^{\frac{21}{2}}+\frac{6}{17}\,ab^2x^{\frac{17}{2}}+\frac{6}{13}\,a^2bx^{\frac{13}{2}}+\frac{2}{9}\,a^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/21*b^3*x^(21/2) + 6/17*a*b^2*x^(17/2) + 6/13*a^2*b*x^(13/2) + 2/9*a^3*x^(
9/2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{9/2}}{9}+\frac{2\,b^3\,x^{21/2}}{21}+\frac{6\,a^2\,b\,x^{13/2}}{13}+\frac{6\,a\,b^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(9/2))/9 + (2*b^3*x^(21/2))/21 + (6*a^2*b*x^(13/2))/13 + (6*a*b^2*
x^(17/2))/17
```

sympy [A] time = 19.69, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{9}{2}}}{9}+\frac{6a^2bx^{\frac{13}{2}}}{13}+\frac{6ab^2x^{\frac{17}{2}}}{17}+\frac{2b^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(9/2)/9 + 6*a**2*b*x**(13/2)/13 + 6*a*b**2*x**(17/2)/17 + 2*b**3*
x**(21/2)/21
```

3.281 $\int x^{5/2} \left(a + bx^2\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{7}a^3x^{7/2} + \frac{6}{11}a^2bx^{11/2} + \frac{2}{5}ab^2x^{15/2} + \frac{2}{19}b^3x^{19/2}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{11}a^2bx^{11/2} + \frac{2}{7}a^3x^{7/2} + \frac{2}{5}ab^2x^{15/2} + \frac{2}{19}b^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*b^2*x^(15/2))/5 + (2*b^3*x
^(19/2))/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2} \left(a + bx^2\right)^3 \, dx &= \int \left(a^3x^{5/2} + 3a^2bx^{9/2} + 3ab^2x^{13/2} + b^3x^{17/2}\right) dx \\ &= \frac{2}{7}a^3x^{7/2} + \frac{6}{11}a^2bx^{11/2} + \frac{2}{5}ab^2x^{15/2} + \frac{2}{19}b^3x^{19/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{7/2}\left(1045a^3 + 1995a^2bx^2 + 1463ab^2x^4 + 385b^3x^6\right)}{7315}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(7/2)*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b^2*x^4 + 385*b^3*x^6))/7315
```

IntegrateAlgebraic [A] time = 0.02, size = 47, normalized size = 0.92

$$\frac{2\left(1045a^3x^{7/2}+1995a^2bx^{11/2}+1463ab^2x^{15/2}+385b^3x^{19/2}\right)}{7315}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*(1045*a^3*x^(7/2) + 1995*a^2*b*x^(11/2) + 1463*a*b^2*x^(15/2) + 385*b^3*
x^(19/2)))/7315
```

fricas [A] time = 0.53, size = 40, normalized size = 0.78

$$\frac{2}{7315}\left(385\,b^3x^9+1463\,ab^2x^7+1995\,a^2bx^5+1045\,a^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^3*x^9 + 1463*a*b^2*x^7 + 1995*a^2*b*x^5 + 1045*a^3*x^3)*sqrt(
x)
```

giac [A] time = 0.63, size = 35, normalized size = 0.69

$$\frac{2}{19}\,b^3x^{\frac{19}{2}}+\frac{2}{5}\,ab^2x^{\frac{15}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/19*b^3*x^(19/2) + 2/5*a*b^2*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7
/2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3\right)x^{\frac{7}{2}}}{7315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^3,x)
```

```
[Out] 2/7315*x^(7/2)*(385*b^3*x^6+1463*a*b^2*x^4+1995*a^2*b*x^2+1045*a^3)
```

maximā [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{19}\,b^3x^{\frac{19}{2}} + \frac{2}{5}\,ab^2x^{\frac{15}{2}} + \frac{6}{11}\,a^2bx^{\frac{11}{2}} + \frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/19*b^3*x^(19/2) + 2/5*a*b^2*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7
/2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{7/2}}{7} + \frac{2\,b^3\,x^{19/2}}{19} + \frac{6\,a^2\,b\,x^{11/2}}{11} + \frac{2\,a\,b^2\,x^{15/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*b^3*x^(19/2))/19 + (6*a^2*b*x^(11/2))/11 + (2*a*b^2*
x^(15/2))/5
```

sympy [A] time = 10.62, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{7}{2}}}{7} + \frac{6a^2bx^{\frac{11}{2}}}{11} + \frac{2ab^2x^{\frac{15}{2}}}{5} + \frac{2b^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a*b**2*x**(15/2)/5 + 2*b**3*x
**(19/2)/19
```

3.282 $\int x^{3/2} \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}ab^2x^{13/2}+\frac{2}{17}b^3x^{17/2}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{2}{3}a^2bx^{9/2}+\frac{2}{5}a^3x^{5/2}+\frac{6}{13}ab^2x^{13/2}+\frac{2}{17}b^3x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(13/2))/13 + (2*b^3*x^
(17/2))/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^3\,dx &= \int \left(a^3x^{3/2}+3a^2bx^{7/2}+3ab^2x^{11/2}+b^3x^{15/2}\right)dx\\ &= \frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}ab^2x^{13/2}+\frac{2}{17}b^3x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{5/2}\left(663a^3+1105a^2bx^2+765ab^2x^4+195b^3x^6\right)}{3315}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^3,x]
```

[Out] (2*x^(5/2)*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2*x^4 + 195*b^3*x^6))/3315

IntegrateAlgebraic [A] time = 0.02, size = 47, normalized size = 0.92

$$\frac{2\left(663a^3x^{5/2}+1105a^2bx^{9/2}+765ab^2x^{13/2}+195b^3x^{17/2}\right)}{3315}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2)^3,x]

[Out] (2*(663*a^3*x^(5/2) + 1105*a^2*b*x^(9/2) + 765*a*b^2*x^(13/2) + 195*b^3*x^(
17/2)))/3315

fricas [A] time = 0.80, size = 40, normalized size = 0.78

$$\frac{2}{3315}\left(195\,b^3x^8+765\,ab^2x^6+1105\,a^2bx^4+663\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="fricas")

[Out] 2/3315*(195*b^3*x^8 + 765*a*b^2*x^6 + 1105*a^2*b*x^4 + 663*a^3*x^2)*sqrt(x)

giac [A] time = 0.63, size = 35, normalized size = 0.69

$$\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="giac")

[Out] 2/17*b^3*x^(17/2) + 6/13*a*b^2*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/
2)

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$\frac{2\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)x^{\frac{5}{2}}}{3315}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(b*x^2+a)^3,x)

[Out] 2/3315*x^(5/2)*(195*b^3*x^6+765*a*b^2*x^4+1105*a^2*b*x^2+663*a^3)

maxima [A] time = 1.37, size = 35, normalized size = 0.69

$$\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/17*b^3*x^(17/2) + 6/13*a*b^2*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/
2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,b^3\,x^{17/2}}{17}+\frac{2\,a^2\,b\,x^{9/2}}{3}+\frac{6\,a\,b^2\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*b^3*x^(17/2))/17 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^
(13/2))/13
```

sympy [A] time = 5.73, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{6ab^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(13/2)/13 + 2*b**3*x*
*(17/2)/17
```

3.283 $\int \sqrt{x} \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(15/2))/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x} \left(a+bx^2\right)^3 \, dx &= \int \left(a^3\sqrt{x}+3a^2bx^{5/2}+3ab^2x^{9/2}+b^3x^{13/2}\right) dx\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{3/2}\left(385a^3+495a^2bx^2+315ab^2x^4+77b^3x^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*(385*a^3 + 495*a^2*b*x^2 + 315*a*b^2*x^4 + 77*b^3*x^6))/1155
```

IntegrateAlgebraic [A] time = 0.02, size = 47, normalized size = 0.92

$$\frac{2\left(385a^3x^{3/2}+495a^2bx^{7/2}+315ab^2x^{11/2}+77b^3x^{15/2}\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2)^3,x]
```

```
[Out] (2*(385*a^3*x^(3/2) + 495*a^2*b*x^(7/2) + 315*a*b^2*x^(11/2) + 77*b^3*x^(15
/2)))/1155
```

fricas [A] time = 0.82, size = 38, normalized size = 0.75

$$\frac{2}{1155}\left(77\,b^3x^7+315\,ab^2x^5+495\,a^2bx^3+385\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*sqrt(x)
```

giac [A] time = 0.58, size = 35, normalized size = 0.69

$$\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*b^3*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/
2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)x^{\frac{3}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*x^(1/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*b^3*x^6+315*a*b^2*x^4+495*a^2*b*x^2+385*a^3)
```

maxima [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{15}\,b^{3}x^{\frac{15}{2}}+\frac{6}{11}\,ab^{2}x^{\frac{11}{2}}+\frac{6}{7}\,a^{2}bx^{\frac{7}{2}}+\frac{2}{3}\,a^{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^3*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/
2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^{3}\,x^{3/2}}{3}+\frac{2\,b^{3}\,x^{15/2}}{15}+\frac{6\,a^{2}\,b\,x^{7/2}}{7}+\frac{6\,a\,b^{2}\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^3,x)
```

```
[Out] (2*a^3*x^(3/2))/3 + (2*b^3*x^(15/2))/15 + (6*a^2*b*x^(7/2))/7 + (6*a*b^2*x^
(11/2))/11
```

sympy [A] time = 2.17, size = 49, normalized size = 0.96

$$\frac{2a^{3}x^{\frac{3}{2}}}{3}+\frac{6a^{2}bx^{\frac{7}{2}}}{7}+\frac{6ab^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{3}x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*x**(1/2),x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x*
*(15/2)/15
```

3.284 $\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=49

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\frac{6}{5}a^2bx^{5/2}+2a^3\sqrt{x}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(13/2)
)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2bx^{3/2}+3ab^2x^{7/2}+b^3x^{11/2}\right)dx\\ &= 2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.84

$$\frac{2}{195}\sqrt{x}\left(195a^3+117a^2bx^2+65ab^2x^4+15b^3x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/Sqrt[x],x]
```

[Out] (2*Sqrt[x]*(195*a^3 + 117*a^2*b*x^2 + 65*a*b^2*x^4 + 15*b^3*x^6))/195

IntegrateAlgebraic [A] time = 0.02, size = 47, normalized size = 0.96

$$\frac{2}{195}\left(195a^3\sqrt{x}+117a^2bx^{5/2}+65ab^2x^{9/2}+15b^3x^{13/2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2)^3/Sqrt[x],x]

[Out] (2*(195*a^3*Sqrt[x] + 117*a^2*b*x^(5/2) + 65*a*b^2*x^(9/2) + 15*b^3*x^(13/2
)))/195

fricas [A] time = 0.56, size = 37, normalized size = 0.76

$$\frac{2}{195}\left(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="fricas")

[Out] 2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(x)

giac [A] time = 0.59, size = 35, normalized size = 0.71

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="giac")

[Out] 2/13*b^3*x^(13/2) + 2/3*a*b^2*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{2\left(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3\right)\sqrt{x}}{195}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^3/x^(1/2),x)

[Out] 2/195*x^(1/2)*(15*b^3*x^6+65*a*b^2*x^4+117*a^2*b*x^2+195*a^3)

maxima [A] time = 1.34, size = 35, normalized size = 0.71

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*b^3*x^(13/2) + 2/3*a*b^2*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.71

$$2\,a^3\,\sqrt{x}+\frac{2\,b^3\,x^{13/2}}{13}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{2\,a\,b^2\,x^{9/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(1/2),x)
```

```
[Out] 2*a^3*x^(1/2) + (2*b^3*x^(13/2))/13 + (6*a^2*b*x^(5/2))/5 + (2*a*b^2*x^(9/2
))/3
```

sympy [A] time = 2.10, size = 48, normalized size = 0.98

$$2a^3\sqrt{x}+\frac{6a^2bx^{\frac{5}{2}}}{5}+\frac{2ab^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(13/
2)/13
```

3.285 $\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$2a^2bx^{3/2}-\frac{2a^3}{\sqrt{x}}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(11/2))
/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^3}{x^{3/2}}+3a^2b\sqrt{x}+3ab^2x^{5/2}+b^3x^{9/2}\right)dx\\ &= -\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.87

$$\frac{2\left(-77a^3+77a^2bx^2+33ab^2x^4+7b^3x^6\right)}{77\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(3/2),x]
[Out] (2*(-77*a^3 + 77*a^2*b*x^2 + 33*a*b^2*x^4 + 7*b^3*x^6))/(77*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 0.87

$$\frac{2\left(-77a^3+77a^2bx^2+33ab^2x^4+7b^3x^6\right)}{77\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^(3/2),x]
[Out] (2*(-77*a^3 + 77*a^2*b*x^2 + 33*a*b^2*x^4 + 7*b^3*x^6))/(77*Sqrt[x])
```

fricas [A] time = 0.46, size = 37, normalized size = 0.79

$$\frac{2\left(7\,b^3x^6+33\,ab^2x^4+77\,a^2bx^2-77\,a^3\right)}{77\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="fricas")
[Out] 2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)/sqrt(x)
```

giac [A] time = 0.58, size = 35, normalized size = 0.74

$$\frac{2}{11}\,b^3x^{\frac{11}{2}}+\frac{6}{7}\,ab^2x^{\frac{7}{2}}+2\,a^2bx^{\frac{3}{2}}-\frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="giac")
[Out] 2/11*b^3*x^(11/2) + 6/7*a*b^2*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

maple [A] time = 0.01, size = 38, normalized size = 0.81

$$-\frac{2\left(-7b^3x^6-33a\,b^2x^4-77a^2b\,x^2+77a^3\right)}{77\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(3/2),x)
[Out] -2/77*(-7*b^3*x^6-33*a*b^2*x^4-77*a^2*b*x^2+77*a^3)/x^(1/2)
```

maxima [A] time = 1.39, size = 35, normalized size = 0.74

$$\frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + 2\,a^2bx^{\frac{3}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^3*x^(11/2) + 6/7*a*b^2*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{2\,b^3\,x^{11/2}}{11} - \frac{2\,a^3}{\sqrt{x}} + 2\,a^2\,b\,x^{3/2} + \frac{6\,a\,b^2\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(3/2),x)
```

```
[Out] (2*b^3*x^(11/2))/11 - (2*a^3)/x^(1/2) + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(7/2))
/7
```

sympy [A] time = 2.31, size = 46, normalized size = 0.98

$$-\frac{2a^3}{\sqrt{x}} + 2a^2bx^{\frac{3}{2}} + \frac{6ab^2x^{\frac{7}{2}}}{7} + \frac{2b^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(7/2)/7 + 2*b**3*x**(11/2
)/11
```

3.286 $\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{9}b^3x^{9/2}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$6a^2b\sqrt{x}-\frac{2a^3}{3x^{3/2}}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(9/
2))/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{\sqrt{x}}+3ab^2x^{3/2}+b^3x^{7/2}\right)dx\\ &= -\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{9}b^3x^{9/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.84

$$\frac{2\left(-15a^3+135a^2bx^2+27ab^2x^4+5b^3x^6\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(5/2),x]
[Out] (2*(-15*a^3 + 135*a^2*b*x^2 + 27*a*b^2*x^4 + 5*b^3*x^6))/(45*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 0.84

$$\frac{2\left(-15a^3+135a^2bx^2+27ab^2x^4+5b^3x^6\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^(5/2),x]
[Out] (2*(-15*a^3 + 135*a^2*b*x^2 + 27*a*b^2*x^4 + 5*b^3*x^6))/(45*x^(3/2))
```

fricas [A] time = 0.67, size = 37, normalized size = 0.76

$$\frac{2\left(5\,b^3x^6+27\,ab^2x^4+135\,a^2bx^2-15\,a^3\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="fricas")
[Out] 2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)/x^(3/2)
```

giac [A] time = 0.58, size = 35, normalized size = 0.71

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="giac")
[Out] 2/9*b^3*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$-\frac{2\left(-5b^3x^6-27a\,b^2x^4-135a^2b\,x^2+15a^3\right)}{45x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(5/2),x)
[Out] -2/45*(-5*b^3*x^6-27*a*b^2*x^4-135*a^2*b*x^2+15*a^3)/x^(3/2)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.71

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.71

$$\frac{2\,b^3\,x^{9/2}}{9}-\frac{2\,a^3}{3\,x^{3/2}}+6\,a^2\,b\,\sqrt{x}+\frac{6\,a\,b^2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(5/2),x)
```

```
[Out] (2*b^3*x^(9/2))/9 - (2*a^3)/(3*x^(3/2)) + 6*a^2*b*x^(1/2) + (6*a*b^2*x^(5/2
))/5
```

sympy [A] time = 2.80, size = 48, normalized size = 0.98

$$-\frac{2a^3}{3x^{\frac{3}{2}}}+6a^2b\sqrt{x}+\frac{6ab^2x^{\frac{5}{2}}}{5}+\frac{2b^3x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(
9/2)/9
```

3.287 $\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2ab^2x^{3/2}+\frac{2}{7}b^3x^{7/2}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{5x^{5/2}}+2ab^2x^{3/2}+\frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(7/2)
)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx &= \int \left(\frac{a^3}{x^{7/2}}+\frac{3a^2b}{x^{3/2}}+3ab^2\sqrt{x}+b^3x^{5/2}\right)dx\\ &= -\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2ab^2x^{3/2}+\frac{2}{7}b^3x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.87

$$\frac{2\left(-7a^3-105a^2bx^2+35ab^2x^4+5b^3x^6\right)}{35x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3/x^(7/2),x]
[Out] (2*(-7*a^3 - 105*a^2*b*x^2 + 35*a*b^2*x^4 + 5*b^3*x^6))/(35*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 0.87

$$\frac{2\left(-7a^3-105a^2bx^2+35ab^2x^4+5b^3x^6\right)}{35x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^3/x^(7/2),x]
[Out] (2*(-7*a^3 - 105*a^2*b*x^2 + 35*a*b^2*x^4 + 5*b^3*x^6))/(35*x^(5/2))
```

fricas [A] time = 0.70, size = 37, normalized size = 0.79

$$\frac{2\left(5\,b^3x^6+35\,ab^2x^4-105\,a^2bx^2-7\,a^3\right)}{35\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="fricas")
[Out] 2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)/x^(5/2)
```

giac [A] time = 0.61, size = 36, normalized size = 0.77

$$\frac{2}{7}\,b^3x^{\frac{7}{2}}+2\,ab^2x^{\frac{3}{2}}-\frac{2\left(15\,a^2bx^2+a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="giac")
[Out] 2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

maple [A] time = 0.00, size = 38, normalized size = 0.81

$$-\frac{2\left(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3\right)}{35x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3/x^(7/2),x)
[Out] -2/35*(-5*b^3*x^6-35*a*b^2*x^4+105*a^2*b*x^2+7*a^3)/x^(5/2)
```

maxima [A] time = 1.35, size = 36, normalized size = 0.77

$$\frac{2}{7}\,b^3x^{\frac{7}{2}} + 2\,ab^2x^{\frac{3}{2}} - \frac{2\left(15\,a^2bx^2 + a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

mupad [B] time = 0.04, size = 37, normalized size = 0.79

$$-\frac{14\,a^3 + 210\,a^2\,b\,x^2 - 70\,a\,b^2\,x^4 - 10\,b^3\,x^6}{35\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3/x^(7/2),x)
```

```
[Out] -(14*a^3 - 10*b^3*x^6 + 210*a^2*b*x^2 - 70*a*b^2*x^4)/(35*x^(5/2))
```

sympy [A] time = 3.86, size = 46, normalized size = 0.98

$$-\frac{2a^3}{5x^{\frac{5}{2}}} - \frac{6a^2b}{\sqrt{x}} + 2ab^2x^{\frac{3}{2}} + \frac{2b^3x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3/x**(7/2),x)
```

```
[Out] -2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(7/
2)/7
```

3.288 $\int \frac{x^{7/2}}{a+bx^2}\,dx$

Optimal. Leaf size=215

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}}+$$

Rubi [A] time = 0.20, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{9/4}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2),x]
```

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(5/2))/(5*b) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4)) + (a^(5/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{a+bx^2}\,dx = \frac{2x^{5/2}}{5b} - \frac{a\int \frac{x^{3/2}}{a+bx^2}\,dx}{b}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{b^2}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \cdots\right.}{2b^{5/2}}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} - \frac{a^{5/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}} + \frac{a^{5/4}\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{\cdots}\right.}{2\sqrt{2}\,b^{9/4}}$$

$$= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} - \frac{a^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}} + \frac{a^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}} - \frac{a^{5/4}\log\left(\sqrt{a} - \sqrt{2}\cdots\right.}{2\sqrt{2}\cdots}$$

Mathematica [A] time = 0.07, size = 203, normalized size = 0.94

$$\frac{-5\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-10\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+10\sqrt{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)-40a\sqrt[4]{b}\sqrt{x}+8b^{5/4}x^{5/2}}{20b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2),x]
```

```
[Out] (-40*a*b^(1/4)*Sqrt[x] + 8*b^(5/4)*x^(5/2) - 10*Sqrt[2]*a^(5/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 10*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)] - 5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(20*b^(9/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 134, normalized size = 0.62

$$-\frac{a^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{a^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{9/4}}+\frac{2\sqrt{x}\left(bx^2-5a\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x]*(-5*a + b*x^2))/(5*b^2) - (a^(5/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1
/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(9/4)) + (a^(5/4
)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2
]*b^(9/4))
```

fricas [A] time = 0.60, size = 170, normalized size = 0.79

$$\frac{20\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac14}\arctan\left(-\frac{ab^7\sqrt{x}\left(-\frac{a^5}{b^9}\right)^{\frac34}-\sqrt{b^4\sqrt{-\frac{a^5}{b^9}}+a^2x}\,b^7\left(-\frac{a^5}{b^9}\right)^{\frac34}}{a^5}\right)+5\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac14}\log\left(b^2\left(-\frac{a^5}{b^9}\right)^{\frac14}+a\sqrt{x}\right)-5\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac14}\log\left(-b^2\left(-\frac{a^5}{b^9}\right)^{\frac14}+a\sqrt{x}\right)+4\left(bx^2-5\,a\right)\sqrt{x}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*(20*b^2*(-a^5/b^9)^(1/4)*arctan(-(a*b^7*sqrt(x)*(-a^5/b^9)^(3/4) - sqr
t(b^4*sqrt(-a^5/b^9) + a^2*x)*b^7*(-a^5/b^9)^(3/4))/a^5) + 5*b^2*(-a^5/b^9)
^(1/4)*log(b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) - 5*b^2*(-a^5/b^9)^(1/4)*log(-
b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) + 4*(b*x^2 - 5*a)*sqrt(x))/b^2
```

giac [A] time = 0.62, size = 196, normalized size = 0.91

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac14}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(ab^3\right)^{\frac14}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(ab^3\right)^{\frac14}a\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^3}-\frac{\sqrt{2}\left(ab^3\right)^{\frac14}a\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^3}+\frac{2\left(b^4x^{\frac52}-5\,ab^3\sqrt{x}\right)}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/b^3 + 1/2*sqrt(2)*(a*b^3)^(1/4)*a*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)*(a*b^3)^(1/4
)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 - 1/4*sqrt(2)*(a*b
^3)^(1/4)*a*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 2/5*(b^
4*x^(5/2) - 5*a*b^3*sqrt(x))/b^5
```

maple [A] time = 0.01, size = 152, normalized size = 0.71

$$\frac{2x^{\frac{5}{2}}}{5b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^{2}}-\frac{2a\sqrt{x}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a),x)
```

```
[Out] 2/5/b*x^(5/2)-2*a/b^2*x^(1/2)+1/4*a/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
)+1/2*a/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*a
/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.08, size = 194, normalized size = 0.90

$$\frac{2\left(bx^{\frac{5}{2}}-5\,a\sqrt{x}\right)}{5\,b^{2}}+\frac{\frac{2\,\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x^(5/2) - 5*a*sqrt(x))/b^2 + 1/4*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqr
t(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b))
+ sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a)
)/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/b^(1/4))/b^2
```

mupad [B] time = 4.49, size = 67, normalized size = 0.31

$$\frac{2\,x^{5/2}}{5\,b}-\frac{2\,a\,\sqrt{x}}{b^2}-\frac{{\left(-a\right)}^{5/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{9/4}}+\frac{{\left(-a\right)}^{5/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}}\right)\,1{}\mathrm{i}}{b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(5/2))/(5*b) - (2*a*x^(1/2))/b^2 - ((-a)^(5/4)*atan((b^(1/4)*x^(1/2))/
(-a)^(1/4)))/b^(9/4) + ((-a)^(5/4)*atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*1i
)/b^(9/4)
```

sympy [A] time = 50.31, size = 192, normalized size = 0.89

$$\begin{cases} \tilde{\infty}x^{\frac{5}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } a = 0 \\ \frac{2x^{\frac{9}{2}}}{9a} & \text{for } b = 0 \\ -\frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2} + \frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2} - \frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b^2} - \frac{2a\sqrt{x}}{b^2} + \frac{2x^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*b), Eq(a, 0))
, (2*x**(9/2)/(9*a), Eq(b, 0)), (-(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*log(-(-
1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) + (-1)**(1/4)*a**(5/4)*
(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) - (-
1)**(1/4)*a**(5/4)*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(
1/4)))/b**2 - 2*a*sqrt(x)/b**2 + 2*x**(5/2)/(5*b), True))
```

3.289 $\int \frac{x^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=204

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{a}{}$$

Rubi [A] time = 0.15, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{7/4}}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2))/(3*b) + (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(7/4)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(Sqrt[2]*b^(7/4)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{a+bx^2}\,dx}{b}$$

$$= \frac{2x^{3/2}}{3b} - \frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= \frac{2x^{3/2}}{3b} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}}$$

$$= \frac{2x^{3/2}}{3b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{a^{3/4}\,\text{Subs}}{}$$

$$= \frac{2x^{3/2}}{3b} - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}} + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}} - \frac{a^{3/4}}{}$$

$$= \frac{2x^{3/2}}{3b} + \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}} - \frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}} - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\,b^{7/4}}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.38

$$\frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{b^{7/4}} - \frac{(-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{b^{7/4}} + \frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2))/(3*b) + ((-a)^(3/4)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/b^(7/
4) - ((-a)^(3/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/b^(7/4)
```

IntegrateAlgebraic [A] time = 0.18, size = 124, normalized size = 0.61

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{7/4}} + \frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{7/4}} + \frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2))/(3*b) + (a^(3/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x
)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(7/4)) + (a^(3/4)*ArcTanh[(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(7/4))
```

fricas [A] time = 0.63, size = 165, normalized size = 0.81

$$\frac{12\,b\left(-\frac{a^3}{b^7}\right)^{\frac14}\arctan\left(-\frac{a^2b^2\sqrt{x}\left(-\frac{a^3}{b^7}\right)^{\frac14}-\sqrt{-a^3b^3\sqrt{-\frac{a^3}{b^7}}+a^4x}\,b^2\left(-\frac{a^3}{b^7}\right)^{\frac14}}{a^3}\right)-3\,b\left(-\frac{a^3}{b^7}\right)^{\frac14}\log\left(b^5\left(-\frac{a^3}{b^7}\right)^{\frac34}+a^2\sqrt{x}\right)+3\,b\left(-\frac{a^3}{b^7}\right)^{\frac14}\log\left(-b^5\left(-\frac{a^3}{b^7}\right)^{\frac34}+a^2\sqrt{x}\right)+4\,x^{\frac32}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*b*(-a^3/b^7)^(1/4)*arctan(-(a^2*b^2*sqrt(x)*(-a^3/b^7)^(1/4) - sqrt
(-a^3*b^3*sqrt(-a^3/b^7) + a^4*x)*b^2*(-a^3/b^7)^(1/4))/a^3) - 3*b*(-a^3/b^
7)^(1/4)*log(b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) + 3*b*(-a^3/b^7)^(1/4)*log
(-b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) + 4*x^(3/2))/b
```

giac [A] time = 0.65, size = 178, normalized size = 0.87

$$\frac{2\,x^{\frac32}}{3\,b}-\frac{\sqrt{2}\left(ab^3\right)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{2\,b^4}-\frac{\sqrt{2}\left(ab^3\right)^{\frac34}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{2\,b^4}+\frac{\sqrt{2}\left(ab^3\right)^{\frac34}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^4}-\frac{\sqrt{2}\left(ab^3\right)^{\frac34}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/b - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*
(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/4*sq
rt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4
```

maple [A] time = 0.00, size = 143, normalized size = 0.70

$$\frac{2x^{\frac32}}{3b}-\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac14}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac14}b^2}-\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac14}b^2}-\frac{\sqrt{2}\,a\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac14}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a),x)
```

```
[Out] 2/3/b*x^(3/2)-1/4*a/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/
2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*a/b^2/(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*a/b^2/(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 186, normalized size = 0.91

$$-\frac{a\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,b}+\frac{2\,x^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b +
 2/3*x^(3/2)/b
```

mupad [B] time = 0.09, size = 54, normalized size = 0.26

$$\frac{2\,x^{3/2}}{3\,b}+\frac{{\left(-a\right)}^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{7/4}}-\frac{{\left(-a\right)}^{3/4}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(3/2))/(3*b) + ((-a)^(3/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(7/4)
 - ((-a)^(3/4)*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(7/4)
```

sympy [A] time = 15.29, size = 180, normalized size = 0.88

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a=0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b=0 \\ \frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2\sqrt[4]{\frac{1}{b}}} - \frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2\sqrt[4]{\frac{1}{b}}} - \frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b^2\sqrt[4]{\frac{1}{b}}} + \frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b), Eq(a, 0))
, (2*x**(7/2)/(7*a), Eq(b, 0)), ((-1)**(3/4)*a**(3/4)*log(-(-1)**(1/4)*a**(
1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2*(1/b)**(1/4)) - (-1)**(3/4)*a**(3/4)*l
og((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2*(1/b)**(1/4)) - (-1
)**(3/4)*a**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(b**2*(
1/b)**(1/4)) + 2*x**(3/2)/(3*b), True))
```

3.290 $\int \frac{x^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=202

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\ \cdots}{}$$

Rubi [A] time = 0.16, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{5/4}}+\frac{2\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x])/b + (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*b^(5/4)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*b^(5/4)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{a+bx^2}\,dx = \frac{2\sqrt{x}}{b} - \frac{a\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}} + \frac{\sqrt[4]{a}\ \text{S}}{}$$
$$= \frac{2\sqrt{x}}{b} + \frac{\sqrt[4]{a}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{a}\,\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{a}\ \text{S}}{}$$
$$= \frac{2\sqrt{x}}{b} + \frac{\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{a}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}} + \frac{\sqrt[4]{a}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\,b^{5/4}}$$

Mathematica [A] time = 0.04, size = 189, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+8\sqrt[4]{b}\sqrt{x}}{4b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (8*b^(1/4)*Sqrt[x] + 2*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)] - 2*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 + Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x] - Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(4*b^(5/4))
```

IntegrateAlgebraic [A] time = 0.17, size = 123, normalized size = 0.61

$$\frac{\sqrt[4]{a}\,\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{5/4}} + \frac{2\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x])/b + (a^(1/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(S
qrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(5/4)) - (a^(1/4)*ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(5/4))
```

fricas [A] time = 0.50, size = 124, normalized size = 0.61

$$-\frac{4\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^2\sqrt{-\frac{a}{b^5}}+x}\,b^4\left(-\frac{a}{b^5}\right)^{\frac{3}{4}}-b^4\sqrt{x}\left(-\frac{a}{b^5}\right)^{\frac{3}{4}}}{a}\right)+b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\sqrt{x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*(-a/b^5)^(1/4)*arctan((sqrt(b^2*sqrt(-a/b^5) + x)*b^4*(-a/b^5)^(3
/4) - b^4*sqrt(x)*(-a/b^5)^(3/4))/a) + b*(-a/b^5)^(1/4)*log(b*(-a/b^5)^(1/4
) + sqrt(x)) - b*(-a/b^5)^(1/4)*log(-b*(-a/b^5)^(1/4) + sqrt(x)) - 4*sqrt(x
))/b
```

giac [A] time = 0.65, size = 178, normalized size = 0.88

$$-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^2}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^2}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/b^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^2 - 1/4*sqrt(2)*(a*b^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 1/4*sqrt(2)*(a*b^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 2*sqrt(x)/b
```

maple [A] time = 0.01, size = 140, normalized size = 0.69

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a),x)
```

```
[Out] 2*x^(1/2)/b-1/4/b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/
b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2/b*(a/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/b*(a/b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.90, size = 185, normalized size = 0.92

$$-\frac{\frac{2\,\sqrt{2}\,\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,a^{\frac{1}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\,a^{\frac{1}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt
(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/b + 2*sqrt(x)
/b
```

mupad [B] time = 0.09, size = 55, normalized size = 0.27

$$\frac{2\,\sqrt{x}}{b}-\frac{{\left(-a\right)}^{1/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{5/4}}-\frac{{\left(-a\right)}^{1/4}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2),x)
```

```
[Out] (2*x^(1/2))/b - ((-a)^(1/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(5/4) - (
(-a)^(1/4)*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(5/4)
```

sympy [A] time = 5.89, size = 172, normalized size = 0.85

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for}\: a = 0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for}\: b = 0 \\ \frac{\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b}-\frac{\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b}+\frac{\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{b}+\frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b, Eq(a, 0)), (2*x
**(5/2)/(5*a), Eq(b, 0)), ((-1)**(1/4)*a**(1/4)*(1/b)**(1/4)*log(-(-1)**(1/
4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) - (-1)**(1/4)*a**(1/4)*(1/b)**(1/
4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) + (-1)**(1/4)*a**
(1/4)*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/b + 2*
sqrt(x)/b, True))
```

3.291 $\int \frac{\sqrt{x}}{a+bx^2}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}$$

Rubi [A] time = 0.14, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) +
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1
/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(1/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2 \cdots}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a} \cdots}{\sqrt[4]{b}}} \cdots\right.}{2\sqrt{2}\sqrt[4]{a} \cdots}$$

$$= \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x \cdots\right.}{\sqrt{2}\sqrt[4]{a}\,b \cdots}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}} - \frac{\log\left(\sqrt{a} \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.28

$$\frac{a\left(\tan^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)\right)}{(-a)^{5/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] (a*(ArcTan[(a*b^(1/4)*Sqrt[x])/(-a)^(5/4)] + ArcTanh[(b^(1/4)*Sqrt[x])/(-a)
^(1/4)]))/((-a)^(5/4)*b^(3/4))
```

IntegrateAlgebraic [A] time = 0.15, size = 114, normalized size = 0.59

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x^2),x]
```

```
[Out] -(ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x
]]/(Sqrt[2]*a^(1/4)*b^(3/4))) - ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(
Sqrt[a] + Sqrt[b]*x)]/(Sqrt[2]*a^(1/4)*b^(3/4))
```

fricas [A] time = 0.53, size = 126, normalized size = 0.66

$$-2\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-ab\sqrt{-\frac{1}{ab^3}}+x}\,b\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}-b\sqrt{x}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*(-1/(a*b^3))^(1/4)*arctan(sqrt(-a*b*sqrt(-1/(a*b^3)) + x)*b*(-1/(a*b^3))
^(1/4) - b*sqrt(x)*(-1/(a*b^3))^(1/4)) + 1/2*(-1/(a*b^3))^(1/4)*log(a*b^2*(
-1/(a*b^3))^(3/4) + sqrt(x)) - 1/2*(-1/(a*b^3))^(1/4)*log(-a*b^2*(-1/(a*b^3
))^(3/4) + sqrt(x))
```

giac [A] time = 0.60, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(a*b^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/4*sqrt(2
)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3)
```

maple [A] time = 0.01, size = 132, normalized size = 0.69

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/4/b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x
+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2/b/(a/b)^(1/4)*2^(1/2)*arctan
```

```
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.98, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4a^{\frac{1}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + 1/4*sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
```

mupad [B] time = 0.07, size = 38, normalized size = 0.20

$$\frac{\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)-\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{(-a)^{1/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2),x)
```

```
[Out] (atan((b^(1/4)*x^(1/2))/(-a)^(1/4)) - atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/
((-a)^(1/4)*b^(3/4))
```

sympy [A] time = 3.17, size = 165, normalized size = 0.86

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a=0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b=0 \\ -\frac{(-1)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}\,b\sqrt[4]{\frac{1}{b}}}+\frac{(-1)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}\,b\sqrt[4]{\frac{1}{b}}}+\frac{(-1)^{\frac{3}{4}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{\sqrt[4]{a}\,b\sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
2*x**(3/2)/(3*a), Eq(b, 0)), (-(-1)**(3/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)*
*(1/4) + sqrt(x))/(2*a**(1/4)*b*(1/b)**(1/4)) + (-1)**(3/4)*log((-1)**(1/4)
*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)*b*(1/b)**(1/4)) + (-1)**(3/4)
*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(1/4)*b*(1/b)**(1/4)
), True))
```

3.292 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=192

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}}$$

Rubi [A] time = 0.15, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/4)) -
 Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(3
/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(
2*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{\cdots}{-\frac{\sqrt{a}}{\sqrt{b}}\cdots}\right.}{2\cdots}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-1-\cdots}\right.}{\cdots}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\text{lo}\cdots}{\cdots}$$

Mathematica [A] time = 0.03, size = 146, normalized size = 0.76

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(3/4)*b^(1/4))
```

IntegrateAlgebraic [A] time = 0.15, size = 113, normalized size = 0.59

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}} - \frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] -(ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x
]]/(Sqrt[2]*a^(3/4)*b^(1/4))) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(
Sqrt[a] + Sqrt[b]*x)]/(Sqrt[2]*a^(3/4)*b^(1/4))
```

fricas [A] time = 0.64, size = 126, normalized size = 0.66

$$2\left(-\frac{1}{a^{3}b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^{2}\sqrt{-\frac{1}{a^{3}b}}+x}\,a^{2}b\left(-\frac{1}{a^{3}b}\right)^{\frac{3}{4}}-a^{2}b\sqrt{x}\left(-\frac{1}{a^{3}b}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{a^{3}b}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^{3}b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{a^{3}b}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^{3}b}\right)^{\frac{1}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(a^3*b))^(1/4)*arctan(sqrt(a^2*sqrt(-1/(a^3*b)) + x)*a^2*b*(-1/(a^3*b
))^(3/4) - a^2*b*sqrt(x)*(-1/(a^3*b))^(3/4)) + 1/2*(-1/(a^3*b))^(1/4)*log(a
*(-1/(a^3*b))^(1/4) + sqrt(x)) - 1/2*(-1/(a^3*b))^(1/4)*log(-a*(-1/(a^3*b))
^(1/4) + sqrt(x))
```

giac [A] time = 0.63, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab}-\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b) + 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b) + 1/4*sqrt(2)*(a*b^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b) - 1/4*sqrt(2)*(a*b
^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b)
```

maple [A] time = 0.00, size = 132, normalized size = 0.69

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/x^(1/2),x)
```

```
[Out] 1/4*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x
-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*(a/b)^(1/4)/a*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4a^{\frac{3}{4}}b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/4*sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 1/4*sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4))
```

mupad [B] time = 0.08, size = 37, normalized size = 0.19

$$-\frac{\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)+\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{(-a)^{3/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)),x)
```

```
[Out] -(atan((b^(1/4)*x^(1/2))/(-a)^(1/4)) + atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))
/((-a)^(3/4)*b^(1/4))
```

sympy [A] time = 5.94, size = 160, normalized size = 0.83

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for}\: a = 0 \\ \frac{2\sqrt{x}}{a} & \text{for}\: b = 0 \\ -\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}}+\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}}-\frac{\sqrt[4]{-1}\sqrt[4]{\frac{1}{b}}\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0)
), (2*sqrt(x)/a, Eq(b, 0)), (-(-1)**(1/4)*(1/b)**(1/4)*log(-(-1)**(1/4)*a**
(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) + (-1)**(1/4)*(1/b)**(1/4)*log((
-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) - (-1)**(1/4)*(1/b
)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/a**(3/4), True))
```

3.293 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{}}{}$$

Rubi [A] time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{5/4}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] -2/(a*Sqrt[x]) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*a^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(5/4)) - (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx = -\frac{2}{a\sqrt{x}} - \frac{b\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a}$$

$$= -\frac{2}{a\sqrt{x}} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{a\sqrt{x}} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{a\sqrt{x}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a} - \frac{\sqrt[4]{b}\ \text{S}}{}$$

$$= -\frac{2}{a\sqrt{x}} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}} + \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}}$$

$$= -\frac{2}{a\sqrt{x}} + \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}\,a^{5/4}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{bx^2}{a}\right)}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)),x]
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^2)/a)])/(a*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.19, size = 122, normalized size = 0.60

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{5/4}} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{5/4}} - \frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] -2/(a*Sqrt[x]) + (b^(1/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(
Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(5/4)) + (b^(1/4)*ArcTanh[(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(5/4))
```

fricas [A] time = 0.55, size = 142, normalized size = 0.70

$$\frac{4\,ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab\sqrt{x}\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}-\sqrt{-a^3b\sqrt{-\frac{b}{a^5}}+b^2x}\,a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}}{b}\right)-ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+b\sqrt{x}\right)+ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(-a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+b\sqrt{x}\right)-4\sqrt{x}}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x*(-b/a^5)^(1/4)*arctan(-(a*b*sqrt(x)*(-b/a^5)^(1/4) - sqrt(-a^3*b
*sqrt(-b/a^5) + b^2*x)*a*(-b/a^5)^(1/4))/b) - a*x*(-b/a^5)^(1/4)*log(a^4*(-
b/a^5)^(3/4) + b*sqrt(x)) + a*x*(-b/a^5)^(1/4)*log(-a^4*(-b/a^5)^(3/4) + b*
sqrt(x)) - 4*sqrt(x))/(a*x)
```

giac [A] time = 0.62, size = 190, normalized size = 0.94

$$-\frac{2}{a\sqrt{x}}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^2}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2/(a*sqrt(x)) - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) - 1/2*sqrt(2)*(a*b^3)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) +
 1/4*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^2*b^2) - 1/4*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^2*b^2)
```

maple [A] time = 0.01, size = 140, normalized size = 0.69

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a),x)
```

```
[Out] -2/a/x^(1/2)-1/4/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2/a/(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/a/(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 186, normalized size = 0.92

$$-\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a -
 2/(a*sqrt(x))
```

mupad [B] time = 4.50, size = 54, normalized size = 0.27

$$\frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{5/4}}-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{5/4}}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(5/4) - ((-b)^(1/4)*atan
(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(5/4) - 2/(a*x^(1/2))
```

sympy [A] time = 12.16, size = 170, normalized size = 0.84

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{2}{a\sqrt{x}} + \frac{(-1)^{\frac{3}{4}} \log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}} - \frac{(-1)^{\frac{3}{4}} \log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}} - \frac{(-1)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)
), (-2/(a*sqrt(x)), Eq(b, 0)), (-2/(a*sqrt(x)) + (-1)**(3/4)*log(-(-1)**(1/
4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*(1/b)**(1/4)) - (-1)**(3/4)
*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*(1/b)**(1/4))
 - (-1)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(5/4)*
(1/b)**(1/4)), True))
```

3.294 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=204

$$\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}}{}$$

Rubi [A] time = 0.16, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*a^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*a^(7/4)) + (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)) - (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx = -\frac{2}{3ax^{3/2}} - \frac{b\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right.}{2a^{3/2}}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{b^{3/4}\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}} - \frac{b^{3/4}\log\left(\sqrt{a} + \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x} + \sqrt{b}\,x\right.}{2\sqrt{2}\,a^{7/4}}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{b^{3/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}} - \frac{b^{3/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}} + \frac{b^{3/4}\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\right.}{2\sqrt{2}\,a^{7}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.14

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{bx^2}{a}\right)}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)),x]
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -((b*x^2)/a)])/(3*a*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 125, normalized size = 0.61

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}} - \frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{7/4}} - \frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{7/4}} - \frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (b^(3/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)
/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(7/4)) - (b^(3/4)*ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(7/4))
```

fricas [A] time = 0.57, size = 167, normalized size = 0.82

$$-\frac{12\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^5 b\sqrt{x}\left(-\frac{b^3}{a^7}\right)^{\frac{3}{4}}-\sqrt{a^4\sqrt{-\frac{b^3}{a^7}}+b^2x}\,a^5\left(-\frac{b^3}{a^7}\right)^{\frac{3}{4}}}{b^3}\right)+3\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}+b\sqrt{x}\right)-3\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(-a^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}+b\sqrt{x}\right)+4\sqrt{x}}{6\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*x^2*(-b^3/a^7)^(1/4)*arctan(-(a^5*b*sqrt(x)*(-b^3/a^7)^(3/4) - s
qrt(a^4*sqrt(-b^3/a^7) + b^2*x)*a^5*(-b^3/a^7)^(3/4))/b^3) + 3*a*x^2*(-b^3/
a^7)^(1/4)*log(a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) - 3*a*x^2*(-b^3/a^7)^(1/4)
*log(-a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) + 4*sqrt(x))/(a*x^2)
```

giac [A] time = 0.64, size = 178, normalized size = 0.87

$$-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2}-\frac{2}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/a^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^2 - 1/4*sqrt(2)*(a*b^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^2 + 1/4*sqrt(2)*(a*b^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^2 - 2/3/(a*x^(3/2)
)
```

maple [A] time = 0.01, size = 143, normalized size = 0.70

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a^2}-\frac{2}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/4/a^2*b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2
))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2/a^2*b*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/a^2*b*(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-2/3/a/x^(3/2)
```

maxima [**A**] time = 3.13, size = 187, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\,a}-\frac{2}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*
b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)
*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/a - 2
/3/(a*x^(3/2))
```

mupad [**B**] time = 0.09, size = 53, normalized size = 0.26

$$\frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{7/4}}-\frac{2}{3\,a\,x^{3/2}}+\frac{(-b)^{3/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(7/4) - 2/(3*a*x^(3/2)) +
 ((-b)^(3/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(7/4)
```

sympy [A] time = 30.40, size = 178, normalized size = 0.87

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } a = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for } b = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} + \frac{\sqrt[4]{-1}\, b \sqrt[4]{\frac{1}{b}} \log\left(-\sqrt[4]{-1}\, \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2a^{\frac{7}{4}}} - \frac{\sqrt[4]{-1}\, b \sqrt[4]{\frac{1}{b}} \log\left(\sqrt[4]{-1}\, \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2a^{\frac{7}{4}}} + \frac{\sqrt[4]{-1}\, b \sqrt[4]{\frac{1}{b}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{7}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (-2/(3*a*x**(3/2)), Eq(b, 0)), (-2/(3*a*x**(3/2)) + (-1)**(1/4)*b*(1/b)*
*(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)) - (-1
)**(1/4)*b*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2
*a**(7/4)) + (-1)**(1/4)*b*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*
(1/b)**(1/4)))/a**(7/4), True))
```

3.295 $\int \frac{1}{x^{7/2}(a+bx^2)}\,dx$

Optimal. Leaf size=215

$$\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}}+\frac{b^{5/4}}{\ldots}$$

Rubi [A] time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{9/4}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)) - (b^(5/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx = -\frac{2}{5ax^{5/2}} - \frac{b\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{a}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{b^2\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a^2}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right.}{a^2}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x\right.}{2a^2}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{b^{5/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}} - \frac{b^{5/4}\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}\,a^{9/4}}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{b^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}} + \frac{b^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}} + \frac{b^{5/4}\log\left(\sqrt{a}\right.}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)),x]
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -((b*x^2)/a)])/(5*a*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 134, normalized size = 0.62

$$-\frac{b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}} - \frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{9/4}} - \frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{9/4}} - \frac{2\left(a-5bx^2\right)}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a - 5*b*x^2))/(5*a^2*x^(5/2)) - (b^(5/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1
/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(9/4)) - (b^(5/4
)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2
]*a^(9/4))
```

fricas [A] time = 0.69, size = 193, normalized size = 0.90

$$-\frac{20\,a^{2}x^{3}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^{2}b^{4}\sqrt{x}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{1}{4}}-\sqrt{-a^{5}b^{5}\sqrt{-\frac{b^{5}}{a^{9}}}+b^{8}x}\,a^{2}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{1}{4}}}{b^{5}}\right)-5\,a^{2}x^{3}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{1}{4}}\log\left(a^{7}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{3}{4}}+b^{4}\sqrt{x}\right)+5\,a^{2}x^{3}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{1}{4}}\log\left(-a^{7}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{3}{4}}+b^{4}\sqrt{x}\right)-4\left(5\,bx^{2}-a\right)\sqrt{x}}{10\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*x^3*(-b^5/a^9)^(1/4)*arctan(-(a^2*b^4*sqrt(x)*(-b^5/a^9)^(1/4
) - sqrt(-a^5*b^5*sqrt(-b^5/a^9) + b^8*x)*a^2*(-b^5/a^9)^(1/4))/b^5) - 5*a^
2*x^3*(-b^5/a^9)^(1/4)*log(a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) + 5*a^2*x^3*
(-b^5/a^9)^(1/4)*log(-a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) - 4*(5*b*x^2 - a)
*sqrt(x))/(a^2*x^3)
```

giac [A] time = 0.62, size = 200, normalized size = 0.93

$$\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{3}b}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{3}b}-\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^{3}b}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^{3}b}+\frac{2\left(5\,bx^{2}-a\right)}{5\,a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a^3*b) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) - 1/4*sqrt(2)*(a*b^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) + 1/4*sqrt(2
)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) +
 2/5*(5*b*x^2 - a)/(a^2*x^(5/2))
```

maple [A] time = 0.01, size = 152, normalized size = 0.71

$$\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{\sqrt{2}\,b\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{2b}{a^{2}\sqrt{x}}-\frac{2}{5a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a),x)
```

```
[Out] -2/5/a/x^(5/2)+2*b/a^2/x^(1/2)+1/4*b/a^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1
/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)
))+1/2*b/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*
b/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 198, normalized size = 0.92

$$\frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4a^2}+\frac{2\left(5bx^2-a\right)}{5a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^
2 + 2/5*(5*b*x^2 - a)/(a^2*x^(5/2))
```

mupad [B] time = 4.48, size = 66, normalized size = 0.31

$$\frac{(-b)^{5/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{9/4}}-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{9/4}}-\frac{\frac{2}{5a}-\frac{2bx^2}{a^2}}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(5/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(9/4) - ((-b)^(5/4)*atan
(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(9/4) - (2/(5*a) - (2*b*x^2)/a^2)/x^(5/2)
```

sympy [A] time = 109.16, size = 190, normalized size = 0.88

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{9bx^{\frac{9}{2}}} & \text{for } a = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for } b = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} + \frac{2b}{a^2\sqrt{x}} - \frac{(-1)^{\frac{3}{4}} b \log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2a^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}} + \frac{(-1)^{\frac{3}{4}} b \log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2a^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}} + \frac{(-1)^{\frac{3}{4}} b \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq(a, 0)
), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + 2*b/(a**2*sqrt(x)) -
 (-1)**(3/4)*b*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(9/4
)*(1/b)**(1/4)) + (-1)**(3/4)*b*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqr
t(x))/(2*a**(9/4)*(1/b)**(1/4)) + (-1)**(3/4)*b*atan((-1)**(3/4)*sqrt(x)/(a
**(1/4)*(1/b)**(1/4)))/(a**(9/4)*(1/b)**(1/4)), True))
```

3.296 $\int \frac{x^{7/2}}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=230

$$\frac{5\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}}-$$

Rubi [A] time = 0.17, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}b^{9/4}}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt{x}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2)^2,x]
```

```
[Out] (5*Sqrt[x])/(2*b^2) - x^(5/2)/(2*b*(a + b*x^2)) + (5*a^(1/4)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*a^(1/4)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(9/4)) + (5*a^(1/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(9/4))
 - (5*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{\left(a+bx^2\right)^2}\,dx &= -\frac{x^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\int \frac{x^{3/2}}{a+bx^2}\,dx}{4b}\\
&= \frac{5\sqrt{x}}{2b^2}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}-\frac{(5a)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4b^2}\\
&= \frac{5\sqrt{x}}{2b^2}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}-\frac{(5a)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2b^2}\\
&= \frac{5\sqrt{x}}{2b^2}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4b^2}-\frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx\right.}{4b^2}\\
&= \frac{5\sqrt{x}}{2b^2}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}-\frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^{5/2}}-\frac{\left(5\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}}{}}\right.}{8b^{5/2}}\\
&= \frac{5\sqrt{x}}{2b^2}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt[4]{a}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{}\right.}{8\sqrt{2}\,b^{9/4}}\\
&= \frac{5\sqrt{x}}{2b^2}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{a}\,\log\left(\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 221, normalized size = 0.96

$$\frac{\frac{32b^{5/4}x^{5/2}}{a+bx^2}+\frac{40a\sqrt[4]{b}\sqrt{x}}{a+bx^2}+5\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-5\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+10\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-10\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2)^2,x]
```

```
[Out] ((40*a*b^(1/4)*Sqrt[x])/(a + b*x^2) + (32*b^(5/4)*x^(5/2))/(a + b*x^2) + 10
*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 10*Sqrt[2]
```

```
*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 5*Sqrt[2]*a^(1/4)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 5*Sqrt[2]*a^(1
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(16*b^(9/4)
)
```

IntegrateAlgebraic [A] time = 0.33, size = 151, normalized size = 0.66

$$\frac{5\sqrt[4]{a}\ \tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\ \sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\ \sqrt[4]{a}}}{\sqrt{x}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\ \tanh^{-1}\left(\frac{\sqrt{2}\ \sqrt[4]{a}\ \sqrt[4]{b}\ \sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,b^{9/4}}+\frac{5a\sqrt{x}\ +4bx^{5/2}}{2b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(a + b*x^2)^2,x]
```

```
[Out] (5*a*Sqrt[x] + 4*b*x^(5/2))/(2*b^2*(a + b*x^2)) + (5*a^(1/4)*ArcTan[(a^(1/4
)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*b
^(9/4)) - (5*a^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + S
qrt[b]*x)])/(4*Sqrt[2]*b^(9/4))
```

fricas [A] time = 0.78, size = 192, normalized size = 0.83

$$-\frac{20\left(b^{3}x^{2}+ab^{2}\right)\left(-\frac{a}{b^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^{4}\sqrt{-\frac{a}{b^{9}}}+x}\,b^{7}\left(-\frac{a}{b^{9}}\right)^{\frac{3}{4}}-b^{7}\sqrt{x}\left(-\frac{a}{b^{9}}\right)^{\frac{3}{4}}}{a}\right)+5\left(b^{3}x^{2}+ab^{2}\right)\left(-\frac{a}{b^{9}}\right)^{\frac{1}{4}}\log\left(5\,b^{2}\left(-\frac{a}{b^{9}}\right)^{\frac{1}{4}}+5\sqrt{x}\right)-5\left(b^{3}x^{2}+ab^{2}\right)\left(-\frac{a}{b^{9}}\right)^{\frac{1}{4}}\log\left(-5\,b^{2}\left(-\frac{a}{b^{9}}\right)^{\frac{1}{4}}+5\sqrt{x}\right)-4\left(4\,bx^{2}+5\,a\right)\sqrt{x}}{8\left(b^{3}x^{2}+ab^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*(b^3*x^2 + a*b^2)*(-a/b^9)^(1/4)*arctan((sqrt(b^4*sqrt(-a/b^9) + x
)*b^7*(-a/b^9)^(3/4) - b^7*sqrt(x)*(-a/b^9)^(3/4))/a) + 5*(b^3*x^2 + a*b^2)
*(-a/b^9)^(1/4)*log(5*b^2*(-a/b^9)^(1/4) + 5*sqrt(x)) - 5*(b^3*x^2 + a*b^2)
*(-a/b^9)^(1/4)*log(-5*b^2*(-a/b^9)^(1/4) + 5*sqrt(x)) - 4*(4*b*x^2 + 5*a)*
sqrt(x))/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.65, size = 196, normalized size = 0.85

$$-\frac{5\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^{3}}-\frac{5\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^{3}}-\frac{5\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^{3}}+\frac{5\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^{3}}+\frac{a\sqrt{x}}{2\left(bx^{2}+a\right)b^{2}}+\frac{2\sqrt{x}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/b^3 - 5/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 - 5/16*sqrt(2)*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 5/16*sqrt(2)*(a*b^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 1/2*a*sqrt(x
)/((b*x^2 + a)*b^2) + 2*sqrt(x)/b^2
```

maple [A] time = 0.01, size = 158, normalized size = 0.69

$$\frac{a\sqrt{x}}{2\left(b\,x^2+a\right)b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\ \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\ \sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\ \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16b^2}+\frac{2\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] 2*x^(1/2)/b^2+1/2*a/b^2*x^(1/2)/(b*x^2+a)-5/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((
x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(
a/b)^(1/2)))-5/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)-5/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.02, size = 206, normalized size = 0.90

$$\frac{a\sqrt{x}}{2\left(b^3x^2+ab^2\right)}-\frac{5\left(\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}\right)}{16\,b^2}+\frac{2\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*sqrt(x)/(b^3*x^2 + a*b^2) - 5/16*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2
)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt
(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*
b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) +
 sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))
/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/b^(1/4))/b^2 + 2*sqrt(x)/b^2
```

mupad [B] time = 4.51, size = 80, normalized size = 0.35

$$\frac{2\sqrt{x}}{b^2}-\frac{5\,{(-a)}^{1/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{4\,b^{9/4}}+\frac{a\,\sqrt{x}}{2\,\left(b^3\,x^2+a\,b^2\right)}+\frac{{(-a)}^{1/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{(-a)}^{1/4}}\right)\,5{}\mathrm{i}}{4\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 - (5*(-a)^(1/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*b^(9
/4)) + ((-a)^(1/4)*atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*5i)/(4*b^(9/4)) +
(a*x^(1/2))/(2*(a*b^2 + b^3*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.297 $\int \frac{x^{5/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=218

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{3\tan^{-1}\left(\right.}{4\sqrt{2}}$$

Rubi [A] time = 0.15, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{x^{3/2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] -x^(3/2)/(2*b*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)^2}\,dx = -\frac{x^{3/2}}{2b\left(a+bx^2\right)} + \frac{3\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4b}$$
$$= -\frac{x^{3/2}}{2b\left(a+bx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$
$$= -\frac{x^{3/2}}{2b\left(a+bx^2\right)} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}}$$
$$= -\frac{x^{3/2}}{2b\left(a+bx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^2}$$
$$= -\frac{x^{3/2}}{2b\left(a+bx^2\right)} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}} - \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}$$
$$= -\frac{x^{3/2}}{2b\left(a+bx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.20

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{a} - \frac{1}{a+bx^2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*(-(a + b*x^2)^(-1) + Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)
]/a))/b
```

IntegrateAlgebraic [A] time = 0.33, size = 139, normalized size = 0.64

$$-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{x^{3/2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*x^(3/2)/(b*(a + b*x^2)) - (3*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1
/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(1
/4)*b^(7/4))
```

fricas [A] time = 0.59, size = 185, normalized size = 0.85

$$-\frac{12\left(b^2x^2+ab\right)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-ab^3\sqrt{-\frac{1}{ab^7}}+x}\,b^2\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}-b^2\sqrt{x}\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\right)-3\left(b^2x^2+ab\right)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\log\left(ab^5\left(-\frac{1}{ab^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)+3\left(b^2x^2+ab\right)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\log\left(-ab^5\left(-\frac{1}{ab^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)+4\,x^{\frac{3}{2}}}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*arctan(sqrt(-a*b^3*sqrt(-1/(a*b
^7)) + x)*b^2*(-1/(a*b^7))^(1/4) - b^2*sqrt(x)*(-1/(a*b^7))^(1/4)) - 3*(b^2
*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(a*b^5*(-1/(a*b^7))^(3/4) + sqrt(x)) + 3*
(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(-a*b^5*(-1/(a*b^7))^(3/4) + sqrt(x))
 + 4*x^(3/2))/(b^2*x^2 + a*b)
```

giac [A] time = 0.62, size = 199, normalized size = 0.91

$$-\frac{x^{\frac{3}{2}}}{2\left(bx^2+a\right)b}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^4}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^4}-\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^4}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^(3/2)/((b*x^2 + a)*b) + 3/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 3/8*sqrt(2)*(a*b^
3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))
/(a*b^4) - 3/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a*b^4) + 3/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/(a*b^4)
```

maple [A] time = 0.01, size = 149, normalized size = 0.68

$$-\frac{x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)b}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{3\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*x^(3/2)/b/(b*x^2+a)+3/16/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+3/8/
b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8/b^2/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.99, size = 195, normalized size = 0.89

$$-\frac{x^{\frac{3}{2}}}{2\left(b^{2}x^{2}+ab\right)}+\frac{3\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x^(3/2)/(b^2*x^2 + a*b) + 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
- sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/
4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sq
rt(a))/(a^(1/4)*b^(3/4)))/b
```

mupad [B] time = 0.08, size = 64, normalized size = 0.29

$$\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{4\,{(-a)}^{1/4}\,b^{7/4}}-\frac{x^{3/2}}{2\,b\,\left(b\,x^2+a\right)}-\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{4\,{(-a)}^{1/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2)^2,x)
```

```
[Out] (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(1/4)*b^(7/4)) - x^(3/2)/(2*
b*(a + b*x^2)) - (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(1/4)*b^(7
/4))
```

sympy [A] time = 153.06, size = 595, normalized size = 2.73

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{b^2\sqrt{x}} & \text{for } a = 0 \\ \frac{2x^{\frac{7}{2}}}{7a^2} & \text{for } b = 0 \\ -\frac{4\sqrt[4]{-1}\sqrt[4]{a}bx^{\frac{3}{2}}\sqrt[4]{\frac{1}{b}}}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^2\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\sqrt[4]{a}b^3x^2\sqrt[4]{\frac{1}{b}}} + \frac{3a\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^2\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\sqrt[4]{a}b^3x^2\sqrt[4]{\frac{1}{b}}} - \frac{3a\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^2\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\sqrt[4]{a}b^3x^2\sqrt[4]{\frac{1}{b}}} - \frac{6a\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^2\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\sqrt[4]{a}b^3x^2\sqrt[4]{\frac{1}{b}}} + \frac{3bx^2\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^2\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\sqrt[4]{a}b^3x^2\sqrt[4]{\frac{1}{b}}} - \frac{3bx^2\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^2\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\sqrt[4]{a}b^3x^2\sqrt[4]{\frac{1}{b}}} - \frac{6bx^2\,\mathrm{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{8\sqrt[4]{-1}a^{\frac{5}{4}}b^2\sqrt[4]{\frac{1}{b}}+8\sqrt[4]{-1}\sqrt[4]{a}b^3x^2\sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b**2*sqrt(x)), Eq(a, 0))
, (2*x**(7/2)/(7*a**2), Eq(b, 0)), (-4*(-1)**(1/4)*a**(1/4)*b*x**(3/2)*(1/b
)**(1/4)/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)
*b**3*x**2*(1/b)**(1/4)) + 3*a*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqr
t(x))/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b*
*3*x**2*(1/b)**(1/4)) - 3*a*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x)
)/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b**3*x
**2*(1/b)**(1/4)) - 6*a*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(
8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b**3*x**2
*(1/b)**(1/4)) + 3*b*x**2*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))
/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b**3*x*
*2*(1/b)**(1/4)) - 3*b*x**2*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x)
)/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b**3*x
**2*(1/b)**(1/4)) - 6*b*x**2*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4
)))/(8*(-1)**(1/4)*a**(5/4)*b**2*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(1/4)*b**3
*x**2*(1/b)**(1/4)), True))
```

3.298 $\int \frac{x^{3/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\ldots}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,a^{3/4}\ldots}$$

Rubi [A] time = 0.15, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] -Sqrt[x]/(2*b*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
/(4*Sqrt[2]*a^(3/4)*b^(5/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^2}\,dx = -\frac{\sqrt{x}}{2b\left(a+bx^2\right)} + \frac{\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4b}$$

$$= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{a}\,b} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{a}\,b}$$

$$= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{a}\,b^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{a}\,b^{3/2}} -$$

$$= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}} +$$

$$= -\frac{\sqrt{x}}{2b\left(a+bx^2\right)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{8\sqrt{2}\,a^{3/4}b^{5/4}}$$

Mathematica [A] time = 0.10, size = 198, normalized size = 0.91

$$\frac{-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-\frac{8\sqrt[4]{b}\sqrt{x}}{a+bx^2}}{16b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((-8*b^(1/4)*Sqrt[x])/(a + b*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/a^(3/4) - (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/a^(3/4) + (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/a^(3/4))/(16*b^(5/4))
```

IntegrateAlgebraic [A] time = 0.31, size = 139, normalized size = 0.64

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*Sqrt[x]/(b*(a + b*x^2)) - ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)
*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]]/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ArcTanh[(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]/(4*Sqrt[2]*a^(3/4)*b^(5
/4))
```

fricas [A] time = 0.64, size = 187, normalized size = 0.86

$$\frac{4\left(b^2x^2+ab\right)\left(-\frac{1}{a^3b^5}\right)^{\frac14}\arctan\left(\sqrt{a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x}\,a^2b^4\left(-\frac{1}{a^3b^5}\right)^{\frac34}-a^2b^4\sqrt{x}\left(-\frac{1}{a^3b^5}\right)^{\frac34}\right)+\left(b^2x^2+ab\right)\left(-\frac{1}{a^3b^5}\right)^{\frac14}\log\left(ab\left(-\frac{1}{a^3b^5}\right)^{\frac14}+\sqrt{x}\right)-\left(b^2x^2+ab\right)\left(-\frac{1}{a^3b^5}\right)^{\frac14}\log\left(-ab\left(-\frac{1}{a^3b^5}\right)^{\frac14}+\sqrt{x}\right)-4\sqrt{x}}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*arctan(sqrt(a^2*b^2*sqrt(-1/(a^
3*b^5)) + x)*a^2*b^4*(-1/(a^3*b^5))^(3/4) - a^2*b^4*sqrt(x)*(-1/(a^3*b^5))^
(3/4)) + (b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(a*b*(-1/(a^3*b^5))^(1/4)
+ sqrt(x)) - (b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(-a*b*(-1/(a^3*b^5))^(
1/4) + sqrt(x)) - 4*sqrt(x))/(b^2*x^2 + a*b)
```

giac [A] time = 0.65, size = 199, normalized size = 0.91

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac14}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{8ab^2}+\frac{\sqrt{2}\left(ab^3\right)^{\frac14}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{8ab^2}+\frac{\sqrt{2}\left(ab^3\right)^{\frac14}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{16ab^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac14}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{16ab^2}-\frac{\sqrt{x}}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) + 1/16*sqrt(2)*(a*b^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2) - 1/16*sqrt
```

```
(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2)
 - 1/2*sqrt(x)/((b*x^2 + a)*b)
```

maple [A] time = 0.01, size = 158, normalized size = 0.72

$$-\frac{\sqrt{x}}{2\left(b\,x^{2}+a\right)b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*x^(1/2)/b/(b*x^2+a)+1/16/b*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/8/
b*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/8/b*(a/b)^(
1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.88, size = 195, normalized size = 0.89

$$\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,b}-\frac{\sqrt{x}}{2\left(b^{2}x^{2}+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b - 1
/2*sqrt(x)/(b^2*x^2 + a*b)
```

mupad [B] time = 4.65, size = 64, normalized size = 0.29

$$-\frac{\sqrt{x}}{2\,b\,\left(b\,x^2+a\right)}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}-\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{3/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2)^2,x)
```

```
[Out] - x^(1/2)/(2*b*(a + b*x^2)) - atan((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(3
/4)*b^(5/4)) - atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(3/4)*b^(5/4))
```

sympy [A] time = 78.16, size = 440, normalized size = 2.02

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{3b^{2}x^{\frac{3}{2}}} & \text{for } a = 0 \\ \frac{2x^{\frac{5}{2}}}{5a^{2}} & \text{for } b = 0 \\ -\frac{\sqrt[4]{-1}\, a^{\frac{5}{4}} \sqrt[4]{\frac{1}{b}} \log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{8a^{2}b + 8ab^{2}x^{2}} + \frac{\sqrt[4]{-1}\, a^{\frac{5}{4}} \sqrt[4]{\frac{1}{b}} \log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{8a^{2}b + 8ab^{2}x^{2}} - \frac{2\sqrt[4]{-1}\, a^{\frac{5}{4}} \sqrt[4]{\frac{1}{b}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{8a^{2}b + 8ab^{2}x^{2}} - \frac{\sqrt[4]{-1}\sqrt[4]{a}\, bx^{2} \sqrt[4]{\frac{1}{b}} \log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{8a^{2}b + 8ab^{2}x^{2}} + \frac{\sqrt[4]{-1}\sqrt[4]{a}\, bx^{2} \sqrt[4]{\frac{1}{b}} \log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{8a^{2}b + 8ab^{2}x^{2}} - \frac{2\sqrt[4]{-1}\sqrt[4]{a}\, bx^{2} \sqrt[4]{\frac{1}{b}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{8a^{2}b + 8ab^{2}x^{2}} - \frac{4a\sqrt{x}}{8a^{2}b + 8ab^{2}x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**2*x**(3/2)), Eq(a,
 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0)), (-(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*
log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**2*b + 8*a*b**2*x**2
) + (-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4)
 + sqrt(x))/(8*a**2*b + 8*a*b**2*x**2) - 2*(-1)**(1/4)*a**(5/4)*(1/b)**(1/4
)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**2*b + 8*a*b**2*x*
*2) - (-1)**(1/4)*a**(1/4)*b*x**2*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1
/b)**(1/4) + sqrt(x))/(8*a**2*b + 8*a*b**2*x**2) + (-1)**(1/4)*a**(1/4)*b*x
**2*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**2*b
 + 8*a*b**2*x**2) - 2*(-1)**(1/4)*a**(1/4)*b*x**2*(1/b)**(1/4)*atan((-1)**(
3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**2*b + 8*a*b**2*x**2) - 4*a*sqrt
(x)/(8*a**2*b + 8*a*b**2*x**2), True))
```

3.299 $\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=218

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,a^{5/4}b^{3}}$$

Rubi [A] time = 0.15, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{x^{3/2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2)^2,x]
```

```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(
4*Sqrt[2]*a^(5/4)*b^(3/4)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(4*Sqrt[2]*a^(5/4)*b^(3/4)) + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(8*Sqrt[2]*a^(5/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx = \frac{x^{3/2}}{2a\left(a+bx^2\right)} + \frac{\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a}$$

$$= \frac{x^{3/2}}{2a\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a}$$

$$= \frac{x^{3/2}}{2a\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a\sqrt{b}}$$

$$= \frac{x^{3/2}}{2a\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8ab} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8ab} + \text{S}\ldots$$

$$= \frac{x^{3/2}}{2a\left(a+bx^2\right)} + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}} + \text{Su}\ldots$$

$$= \frac{x^{3/2}}{2a\left(a+bx^2\right)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\ldots\right.}{8\sqrt{2}\,a^{5/4}b^{3/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^2)
```

IntegrateAlgebraic [A] time = 0.31, size = 139, normalized size = 0.64

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{x^{3/2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x^2)^2,x]
```

```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)
/(Sqrt[2]*a^(1/4)))/Sqrt[x]]/(4*Sqrt[2]*a^(5/4)*b^(3/4)) - ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]/(4*Sqrt[2]*a^(5/4)*b^(3/4)
)
```

fricas [A] time = 0.68, size = 182, normalized size = 0.83

$$-\frac{4\left(abx^{2}+a^{2}\right)\left(-\frac{1}{a^{5}b^{3}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^{3}b\sqrt{-\frac{1}{a^{5}b^{3}}}+x}\,ab\left(-\frac{1}{a^{5}b^{3}}\right)^{\frac{1}{4}}-ab\sqrt{x}\left(-\frac{1}{a^{5}b^{3}}\right)^{\frac{1}{4}}\right)-\left(abx^{2}+a^{2}\right)\left(-\frac{1}{a^{5}b^{3}}\right)^{\frac{1}{4}}\log\left(a^{4}b^{2}\left(-\frac{1}{a^{5}b^{3}}\right)^{\frac{3}{4}}+\sqrt{x}\right)+\left(abx^{2}+a^{2}\right)\left(-\frac{1}{a^{5}b^{3}}\right)^{\frac{1}{4}}\log\left(-a^{4}b^{2}\left(-\frac{1}{a^{5}b^{3}}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4x^{\frac{3}{2}}}{8\left(abx^{2}+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*arctan(sqrt(-a^3*b*sqrt(-1/(a^
5*b^3)) + x)*a*b*(-1/(a^5*b^3))^(1/4) - a*b*sqrt(x)*(-1/(a^5*b^3))^(1/4)) -
 (a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*log(a^4*b^2*(-1/(a^5*b^3))^(3/4) + sq
rt(x)) + (a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*log(-a^4*b^2*(-1/(a^5*b^3))^(
3/4) + sqrt(x)) - 4*x^(3/2))/(a*b*x^2 + a^2)
```

giac [A] time = 0.62, size = 199, normalized size = 0.91

$$\frac{x^{\frac{3}{2}}}{2\left(bx^{2}+a\right)a}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{2}b^{3}}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{2}b^{3}}-\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^{2}b^{3}}+\frac{\sqrt{2}\left(ab^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^(3/2)/((b*x^2 + a)*a) + 1/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/8*sqrt(2)*(a*b
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(a^2*b^3) - 1/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(a^2*b^3) + 1/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

maple [A] time = 0.01, size = 158, normalized size = 0.72

$$\frac{x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)a}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x^(3/2)/a/(b*x^2+a)+1/16/a/b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/8/a
/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/8/a/b/(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.02, size = 194, normalized size = 0.89

$$\frac{x^{\frac{3}{2}}}{2\left(abx^{2}+a^{2}\right)}+\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^(3/2)/(a*b*x^2 + a^2) + 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sq
rt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) -
 sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4
)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/(a^(1/4)*b^(3/4)))/a
```

mupad [B] time = 0.09, size = 64, normalized size = 0.29

$$\frac{x^{3/2}}{2\,a\,\left(b\,x^2+a\right)}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{5/4}\,b^{3/4}}+\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{4\,{\left(-a\right)}^{5/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2)^2,x)
```

```
[Out] x^(3/2)/(2*a*(a + b*x^2)) - atan((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(5/4
)*b^(3/4)) + atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(5/4)*b^(3/4))
```

sympy [A] time = 50.14, size = 578, normalized size = 2.65

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{5 b^{2} x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 a^{2}} & \text{for}\: b = 0 \\ \frac{4 \sqrt[4]{-1} \sqrt[4]{a} b x^{\frac{3}{2}} \sqrt[4]{\frac{1}{b}}}{8 \sqrt[4]{-1} a^{\frac{9}{4}} b \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{5}{4}} b^{2} x^{2} \sqrt[4]{\frac{1}{b}}} + \frac{a \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 \sqrt[4]{-1} a^{\frac{9}{4}} b \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{5}{4}} b^{2} x^{2} \sqrt[4]{\frac{1}{b}}} - \frac{a \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 \sqrt[4]{-1} a^{\frac{9}{4}} b \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{5}{4}} b^{2} x^{2} \sqrt[4]{\frac{1}{b}}} - \frac{2 a \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{8 \sqrt[4]{-1} a^{\frac{9}{4}} b \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{5}{4}} b^{2} x^{2} \sqrt[4]{\frac{1}{b}}} + \frac{b x^{2} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 \sqrt[4]{-1} a^{\frac{9}{4}} b \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{5}{4}} b^{2} x^{2} \sqrt[4]{\frac{1}{b}}} - \frac{b x^{2} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 \sqrt[4]{-1} a^{\frac{9}{4}} b \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{5}{4}} b^{2} x^{2} \sqrt[4]{\frac{1}{b}}} - \frac{2 b x^{2} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{8 \sqrt[4]{-1} a^{\frac{9}{4}} b \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{5}{4}} b^{2} x^{2} \sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a,
 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0)), (4*(-1)**(1/4)*a**(1/4)*b*x**(3/2)*(
1/b)**(1/4)/(8*(-1)**(1/4)*a**(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)
*b**2*x**2*(1/b)**(1/4)) + a*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(
x))/(8*(-1)**(1/4)*a**(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x*
*2*(1/b)**(1/4)) - a*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*(-
1)**(1/4)*a**(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x**2*(1/b)*
*(1/4)) - 2*a*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*(-1)**(1
/4)*a**(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x**2*(1/b)**(1/4)
) + b*x**2*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*(-1)**(1/4)
*a**(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x**2*(1/b)**(1/4)) -
 b*x**2*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*(-1)**(1/4)*a**
(9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x**2*(1/b)**(1/4)) - 2*b
*x**2*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*(-1)**(1/4)*a**(
9/4)*b*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(5/4)*b**2*x**2*(1/b)**(1/4)), True)
)
```

3.300 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\tan^{-1}\left(\right.}{4\sqrt{}}$$

Rubi [A] time = 0.15, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx = \frac{\sqrt{x}}{2a\left(a+bx^2\right)} + \frac{3\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4a}$$

$$= \frac{\sqrt{x}}{2a\left(a+bx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a}$$

$$= \frac{\sqrt{x}}{2a\left(a+bx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{3/2}}$$

$$= \frac{\sqrt{x}}{2a\left(a+bx^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^{3/2}\sqrt{b}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{\right.}{8a^{3/2}\sqrt{b}}$$

$$= \frac{\sqrt{x}}{2a\left(a+bx^2\right)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b\right.}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}$$

$$= \frac{\sqrt{x}}{2a\left(a+bx^2\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{8\sqrt{2}\,a^{7/4}}$$

Mathematica [A] time = 0.10, size = 199, normalized size = 0.91

$$\frac{\frac{8a^{3/4}\sqrt{x}}{a+bx^2} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{b}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}}{16a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] ((8*a^(3/4)*Sqrt[x])/(a + b*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/b^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/b^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/b^(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/b^(1/4))/(16*a^(7/4))
```

IntegrateAlgebraic [A] time = 0.30, size = 139, normalized size = 0.64

$$-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x^2)) - (3*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)
*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*a^(7/4)*b^(1/4)) + (3*ArcTanh[(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(7/4)
*b^(1/4))
```

fricas [A] time = 0.78, size = 179, normalized size = 0.82

$$\frac{12\left(abx^2+a^2\right)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4\sqrt{-\frac{1}{a^7b}}+x}\,a^5b\left(-\frac{1}{a^7b}\right)^{\frac{3}{4}}-a^5b\sqrt{x}\left(-\frac{1}{a^7b}\right)^{\frac{3}{4}}\right)+3\left(abx^2+a^2\right)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\left(abx^2+a^2\right)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(-a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\sqrt{x}}{8\left(abx^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(12*(a*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*arctan(sqrt(a^4*sqrt(-1/(a^7*b))
 + x)*a^5*b*(-1/(a^7*b))^(3/4) - a^5*b*sqrt(x)*(-1/(a^7*b))^(3/4)) + 3*(a*b
*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(a^2*(-1/(a^7*b))^(1/4) + sqrt(x)) - 3*(a
*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(-a^2*(-1/(a^7*b))^(1/4) + sqrt(x)) + 4
*sqrt(x))/(a*b*x^2 + a^2)
```

giac [A] time = 0.65, size = 199, normalized size = 0.91

$$\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^2b}-\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^2b}+\frac{\sqrt{x}}{2\left(bx^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(
x))/(a/b)^(1/4))/(a^2*b) + 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 3/16*sqrt(2)*(a*b^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 3/16*sqrt
```

```
(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b)
 + 1/2*sqrt(x)/((b*x^2 + a)*a)
```

maple [A] time = 0.01, size = 149, normalized size = 0.68

$$\frac{\sqrt{x}}{2\left(b\,x^{2}+a\right)a}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)/a/(b*x^2+a)+3/16/a^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+3/8/a
^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8/a^2*(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.05, size = 194, normalized size = 0.89

$$\frac{3\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{16\,a}+\frac{\sqrt{x}}{2\left(abx^{2}+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a + 1
/2*sqrt(x)/(a*b*x^2 + a^2)
```

mupad [B] time = 0.09, size = 64, normalized size = 0.29

$$\frac{\sqrt{x}}{2\,a\,\left(b\,x^{2}+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{7/4}\,b^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{7/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)^2),x)
```

```
[Out] x^(1/2)/(2*a*(a + b*x^2)) + (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^
(7/4)*b^(1/4)) + (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(7/4)*b^(1
/4))
```

sympy [A] time = 75.79, size = 434, normalized size = 1.99

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{7 b^{2} x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ \frac{2 \sqrt{x}}{a^{2}} & \text{for}\: b = 0 \\ - \frac{3 \sqrt[4]{-1} a^{\frac{5}{4}} \sqrt[4]{\frac{1}{b}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} + \frac{3 \sqrt[4]{-1} a^{\frac{5}{4}} \sqrt[4]{\frac{1}{b}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} - \frac{6 \sqrt[4]{-1} a^{\frac{5}{4}} \sqrt[4]{\frac{1}{b}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} - \frac{3 \sqrt[4]{-1} \sqrt[4]{a} b x^{2} \sqrt[4]{\frac{1}{b}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} + \frac{3 \sqrt[4]{-1} \sqrt[4]{a} b x^{2} \sqrt[4]{\frac{1}{b}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} - \frac{6 \sqrt[4]{-1} \sqrt[4]{a} b x^{2} \sqrt[4]{\frac{1}{b}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{8 a^{3} + 8 a^{2} b x^{2}} + \frac{4 a \sqrt{x}}{8 a^{3} + 8 a^{2} b x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**2*x**(7/2)), Eq(a,
 0)), (2*sqrt(x)/a**2, Eq(b, 0)), (-3*(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*log
(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**3 + 8*a**2*b*x**2) + 3
*(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) +
sqrt(x))/(8*a**3 + 8*a**2*b*x**2) - 6*(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)*ata
n((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**3 + 8*a**2*b*x**2) - 3
*(-1)**(1/4)*a**(1/4)*b*x**2*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**
(1/4) + sqrt(x))/(8*a**3 + 8*a**2*b*x**2) + 3*(-1)**(1/4)*a**(1/4)*b*x**2*(
1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**3 + 8*a*
*2*b*x**2) - 6*(-1)**(1/4)*a**(1/4)*b*x**2*(1/b)**(1/4)*atan((-1)**(3/4)*sq
rt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**3 + 8*a**2*b*x**2) + 4*a*sqrt(x)/(8*a*
*3 + 8*a**2*b*x**2), True))
```

3.301 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}}$$

Rubi [A] time = 0.19, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}}-\frac{5\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{9/4}}-\frac{5}{2a^2\sqrt{x}}+\frac{1}{2a\sqrt{x}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -5/(2*a^2*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x^2)) + (5*b^(1/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)) - (5*b^(1/4)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)) - (5*b^(1/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)
) + (5*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} + \frac{5\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{4a}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{(5b)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^2}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{(5b)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a^2}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} + \frac{\left(5\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^2} - \frac{\left(5\sqrt{b}\right)\mathrm{Subst}\left(\int\right.}{4}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^2} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}}{}}\right.}{8a}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} - \frac{5\sqrt[4]{b}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}} + \frac{5\sqrt[4]{b}\,\log\left(\sqrt{a}\right.}{}$$

$$= -\frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}\left(a+bx^2\right)} + \frac{5\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}} - \frac{5\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^2),x]
[Out] (-2*Hypergeometric2F1[-1/4, 2, 3/4, -((b*x^2)/a)])/(a^2*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.34, size = 149, normalized size = 0.65

$$\frac{5\sqrt[4]{b}\,\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{4\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{9/4}}+\frac{-4a-5bx^2}{2a^2\sqrt{x}\,\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] (-4*a - 5*b*x^2)/(2*a^2*Sqrt[x]*(a + b*x^2)) + (5*b^(1/4)*ArcTan[(a^(1/4)/(
Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*a^(9
/4)) + (5*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)])/(4*Sqrt[2]*a^(9/4))
```

fricas [A] time = 0.57, size = 208, normalized size = 0.90

$$\frac{20\left(a^2bx^3+a^3x\right)\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,a^2b\sqrt{x}\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^5b\sqrt{-\frac{b}{a^9}}+15625\,b^2x}\,a^2\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}}{125\,b}\right)-5\left(a^2bx^3+a^3x\right)\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\log\left(125\,a^7\left(-\frac{b}{a^9}\right)^{\frac{3}{4}}+125\,b\sqrt{x}\right)+5\left(a^2bx^3+a^3x\right)\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\log\left(-125\,a^7\left(-\frac{b}{a^9}\right)^{\frac{3}{4}}+125\,b\sqrt{x}\right)-4\left(5\,bx^2+4\,a\right)\sqrt{x}}{8\left(a^2bx^3+a^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(20*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*arctan(-1/125*(125*a^2*b*sqrt(x)
*(-b/a^9)^(1/4) - sqrt(-15625*a^5*b*sqrt(-b/a^9) + 15625*b^2*x)*a^2*(-b/a^9
)^(1/4))/b) - 5*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*log(125*a^7*(-b/a^9)^(3/
4) + 125*b*sqrt(x)) + 5*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*log(-125*a^7*(-b
/a^9)^(3/4) + 125*b*sqrt(x)) - 4*(5*b*x^2 + 4*a)*sqrt(x))/(a^2*b*x^3 + a^3*
x)
```

giac [A] time = 0.62, size = 210, normalized size = 0.91

$$-\frac{5\,bx^2+4\,a}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^2}-\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^2}-\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^2}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b^2}-\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*b*x^2 + 4*a)/((b*x^(5/2) + a*sqrt(x))*a^2) - 5/8*sqrt(2)*(a*b^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*
b^2) - 5/8*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
```

```
 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 5/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 5/16*sqrt(2)*(a*b^3)^(3/4
)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2)
```

maple [A] time = 0.02, size = 158, normalized size = 0.69

$$-\frac{b\,x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{5\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{2}{a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/a^2/x^(1/2)-1/2/a^2*b*x^(3/2)/(b*x^2+a)-5/16/a^2/(a/b)^(1/4)*2^(1/2)*ln(
(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2)))-5/8/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)-5/8/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.98, size = 208, normalized size = 0.90

$$-\frac{5\,bx^{2}+4\,a}{2\left(a^{2}bx^{\frac{5}{2}}+a^{3}\sqrt{x}\right)}-\frac{5\,b\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(5*b*x^2 + 4*a)/(a^2*b*x^(5/2) + a^3*sqrt(x)) - 5/16*b*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2
```

mupad [B] time = 0.08, size = 77, normalized size = 0.33

$$\frac{5\,(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{9/4}}-\frac{5\,(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{9/4}}-\frac{\frac{2}{a}+\frac{5\,b\,x^{2}}{2\,a^{2}}}{a\,\sqrt{x}+b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)^2),x)
```

```
[Out] (5*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(9/4)) - (5*(-b)^(1
/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(9/4)) - (2/a + (5*b*x^2)/(2*a
^2))/(a*x^(1/2) + b*x^(5/2))
```

sympy [A] time = 150.18, size = 700, normalized size = 3.04

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{9 b^{2} x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{a^{2} \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{16 \sqrt[4]{-1} a^{\frac{5}{4}} \sqrt[4]{\frac{1}{b}}}{8 \sqrt[4]{-1} a^{\frac{13}{4}} \sqrt{x} \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{9}{4}} b x^{\frac{5}{2}} \sqrt[4]{\frac{1}{b}}} - \frac{20 \sqrt[4]{-1} \sqrt[4]{a} b x^{2} \sqrt[4]{\frac{1}{b}}}{8 \sqrt[4]{-1} a^{\frac{13}{4}} \sqrt{x} \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{9}{4}} b x^{\frac{5}{2}} \sqrt[4]{\frac{1}{b}}} - \frac{5 a \sqrt{x} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 \sqrt[4]{-1} a^{\frac{13}{4}} \sqrt{x} \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{9}{4}} b x^{\frac{5}{2}} \sqrt[4]{\frac{1}{b}}} + \frac{5 a \sqrt{x} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 \sqrt[4]{-1} a^{\frac{13}{4}} \sqrt{x} \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{9}{4}} b x^{\frac{5}{2}} \sqrt[4]{\frac{1}{b}}} + \frac{10 a \sqrt{x} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{8 \sqrt[4]{-1} a^{\frac{13}{4}} \sqrt{x} \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{9}{4}} b x^{\frac{5}{2}} \sqrt[4]{\frac{1}{b}}} - \frac{5 b x^{\frac{5}{2}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 \sqrt[4]{-1} a^{\frac{13}{4}} \sqrt{x} \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{9}{4}} b x^{\frac{5}{2}} \sqrt[4]{\frac{1}{b}}} + \frac{5 b x^{\frac{5}{2}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 \sqrt[4]{-1} a^{\frac{13}{4}} \sqrt{x} \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{9}{4}} b x^{\frac{5}{2}} \sqrt[4]{\frac{1}{b}}} + \frac{10 b x^{\frac{5}{2}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{8 \sqrt[4]{-1} a^{\frac{13}{4}} \sqrt{x} \sqrt[4]{\frac{1}{b}} + 8 \sqrt[4]{-1} a^{\frac{9}{4}} b x^{\frac{5}{2}} \sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b**2*x**(9/2)), Eq(a,
 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-16*(-1)**(1/4)*a**(5/4)*(1/b)**(1/4)
/(8*(-1)**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(9/4)*b*x
**(5/2)*(1/b)**(1/4)) - 20*(-1)**(1/4)*a**(1/4)*b*x**2*(1/b)**(1/4)/(8*(-1)
**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(9/4)*b*x**(5/2)*
(1/b)**(1/4)) - 5*a*sqrt(x)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x
))/(8*(-1)**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(9/4)*b
*x**(5/2)*(1/b)**(1/4)) + 5*a*sqrt(x)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4)
 + sqrt(x))/(8*(-1)**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-1)**(1/4)*a
**(9/4)*b*x**(5/2)*(1/b)**(1/4)) + 10*a*sqrt(x)*atan((-1)**(3/4)*sqrt(x)/(a
**(1/4)*(1/b)**(1/4)))/(8*(-1)**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-
1)**(1/4)*a**(9/4)*b*x**(5/2)*(1/b)**(1/4)) - 5*b*x**(5/2)*log(-(-1)**(1/4)
*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*(-1)**(1/4)*a**(13/4)*sqrt(x)*(1/b)**(
1/4) + 8*(-1)**(1/4)*a**(9/4)*b*x**(5/2)*(1/b)**(1/4)) + 5*b*x**(5/2)*log((
-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*(-1)**(1/4)*a**(13/4)*sqrt(x
)*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(9/4)*b*x**(5/2)*(1/b)**(1/4)) + 10*b*x**
(5/2)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*(-1)**(1/4)*a**(
13/4)*sqrt(x)*(1/b)**(1/4) + 8*(-1)**(1/4)*a**(9/4)*b*x**(5/2)*(1/b)**(1/4)
), True))
```

3.302 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=230

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}-$$

Rubi [A] time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{11/4}}-\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -7/(6*a^2*x^(3/2)) + 1/(2*a*x^(3/2)*(a + b*x^2)) + (7*b^(3/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)) - (7*b^(3/4)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)) + (7*b^(3/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11
/4)) - (7*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^{3/2}\left(a+bx^2\right)}+\frac{7\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx}{4a}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}-\frac{(7b)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4a^2}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}-\frac{(7b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a^2}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}-\frac{(7b)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^{5/2}}-\frac{(7b)\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{}}{a+bx}\right.}{4a^{5/2}}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}-\frac{\left(7\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^{5/2}}-\frac{\left(7\sqrt{b}\right)\text{Subs}}{}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}+\frac{7b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{11/4}}-\frac{7b^{3/4}\log\left(\sqrt{a}\right.}{}$$

$$= -\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}}-\frac{7b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.13

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^2),x]
[Out] (-2*Hypergeometric2F1[-3/4, 2, 1/4, -((b*x^2)/a)])/(3*a^2*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.34, size = 149, normalized size = 0.65

$$\frac{7b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{4\sqrt{2}\,a^{11/4}}-\frac{7b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{11/4}}+\frac{-4a-7bx^2}{6a^2x^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] (-4*a - 7*b*x^2)/(6*a^2*x^(3/2)*(a + b*x^2)) + (7*b^(3/4)*ArcTan[(a^(1/4)/(
Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(4*Sqrt[2]*a^(1
1/4)) - (7*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqr
t[b]*x)])/(4*Sqrt[2]*a^(11/4))
```

fricas [A] time = 0.53, size = 228, normalized size = 0.99

$$-\frac{84\left(a^2bx^4+a^3x^2\right)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^8b\sqrt{x}\left(-\frac{b^3}{a^{11}}\right)^{\frac{3}{4}}-\sqrt{a^6\sqrt{-\frac{b^3}{a^{11}}}+b^2x}\,a^8\left(-\frac{b^3}{a^{11}}\right)^{\frac{3}{4}}}{b^3}\right)+21\left(a^2bx^4+a^3x^2\right)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(7a^3\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}+7b\sqrt{x}\right)-21\left(a^2bx^4+a^3x^2\right)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(-7a^3\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}+7b\sqrt{x}\right)+4\left(7bx^2+4a\right)\sqrt{x}}{24\left(a^2bx^4+a^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(84*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*arctan(-(a^8*b*sqrt(x)*(-
b^3/a^11)^(3/4) - sqrt(a^6*sqrt(-b^3/a^11) + b^2*x)*a^8*(-b^3/a^11)^(3/4))/
b^3) + 21*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*log(7*a^3*(-b^3/a^11)^(1/
4) + 7*b*sqrt(x)) - 21*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*log(-7*a^3*(
-b^3/a^11)^(1/4) + 7*b*sqrt(x)) + 4*(7*b*x^2 + 4*a)*sqrt(x))/(a^2*b*x^4 + a
^3*x^2)
```

giac [A] time = 0.64, size = 196, normalized size = 0.85

$$-\frac{7\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^3}-\frac{7\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^3}-\frac{7\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^3}+\frac{7\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^3}-\frac{b\sqrt{x}}{2\left(bx^2+a\right)a^2}-\frac{2}{3a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/a^3 - 7/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^3 - 7/16*sqrt(2)*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^3 + 7/16*sqrt(2)*(a*b^3)
```

```
^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^3 - 1/2*b*sqrt(x
)/((b*x^2 + a)*a^2) - 2/3/(a^2*x^(3/2))
```

maple [A] time = 0.02, size = 161, normalized size = 0.70

$$-\frac{b\sqrt{x}}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^{3}}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^{3}}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^{3}}-\frac{2}{3a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/3/a^2/x^(3/2)-1/2/a^2*b*x^(1/2)/(b*x^2+a)-7/16/a^3*b*(a/b)^(1/4)*2^(1/2)
*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2)))-7/8/a^3*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)+1)-7/8/a^3*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
-1)
```

maxima [A] time = 3.06, size = 209, normalized size = 0.91

$$-\frac{7\,bx^{2}+4\,a}{6\left(a^{2}bx^{\frac{7}{2}}+a^{3}x^{\frac{3}{2}}\right)}-\frac{7\left(\frac{2\,\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)}{16\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(7*b*x^2 + 4*a)/(a^2*b*x^(7/2) + a^3*x^(3/2)) - 7/16*(2*sqrt(2)*b*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/a^2
```

mupad [B] time = 4.68, size = 77, normalized size = 0.33

$$\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{11/4}}-\frac{\frac{2}{3\,a}+\frac{7\,b\,x^2}{6\,a^2}}{a\,x^{3/2}+b\,x^{7/2}}+\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)^2),x)
```

```
[Out] (7*(-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(11/4)) - (2/(3*a) +
 (7*b*x^2)/(6*a^2))/(a*x^(3/2) + b*x^(7/2)) + (7*(-b)^(3/4)*atanh(((-b)^(1/
4)*x^(1/2))/a^(1/4)))/(4*a^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.303 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=243

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{9b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}}+$$

Rubi [A] time = 0.19, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{9b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}}+\frac{9b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{13/4}}+\frac{9b}{2a^3\sqrt{x}}-\frac{9}{10a^2x^{5/2}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] -9/(10*a^2*x^(5/2)) + (9*b)/(2*a^3*Sqrt[x]) + 1/(2*a*x^(5/2)*(a + b*x^2)) -
 (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13
/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]
*a^(13/4)) + (9*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(8*Sqrt[2]*a^(13/4)) - (9*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx = \frac{1}{2ax^{5/2}\left(a+bx^2\right)}+\frac{9\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{4a}$$

$$= -\frac{9}{10a^2x^{5/2}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}-\frac{(9b)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{4a^2}$$

$$= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}+\frac{\left(9b^2\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^3}$$

$$= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}+\frac{\left(9b^2\right)\operatorname{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a^3}$$

$$= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}-\frac{\left(9b^{3/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^3}+\frac{\left(9b^{3/2}\right.}{}$$

$$= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}+\frac{(9b)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^3}+\frac{(9b}{}$$

$$= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}+\frac{9b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{9b}{}$$

$$= -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}}+\frac{9b^{5/4}\tan^{-1}\left(1+\right.}{4\sqrt{2}\,a^{1}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 2, -1/4, -((b*x^2)/a)])/(5*a^2*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.41, size = 160, normalized size = 0.66

$$-\frac{9b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{4\sqrt{2}\,a^{13/4}}-\frac{9b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{13/4}}+\frac{-4a^2+36abx^2+45b^2x^4}{10a^3x^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] (-4*a^2 + 36*a*b*x^2 + 45*b^2*x^4)/(10*a^3*x^(5/2)*(a + b*x^2)) - (9*b^(5/4
)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x
]])/(4*Sqrt[2]*a^(13/4)) - (9*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(13/4))
```

fricas [A] time = 0.63, size = 251, normalized size = 1.03

$$-\frac{180\left(a^3bx^5+a^4x^3\right)\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729\,a^3b^4\sqrt{x}\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}-\sqrt{-531441\,a^7b^5\sqrt{-\frac{b^5}{a^{13}}}+531441\,b^8x}\,a^3\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}}{729\,b^5}\right)-45\left(a^3bx^5+a^4x^3\right)\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}\log\left(729\,a^{10}\left(-\frac{b^5}{a^{13}}\right)^{\frac{3}{4}}+729\,b^4\sqrt{x}\right)+45\left(a^3bx^5+a^4x^3\right)\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}\log\left(-729\,a^{10}\left(-\frac{b^5}{a^{13}}\right)^{\frac{3}{4}}+729\,b^4\sqrt{x}\right)-4\left(45\,b^2x^4+36\,abx^2-4\,a^2\right)\sqrt{x}}{40\left(a^3bx^5+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(180*(a^3*b*x^5 + a^4*x^3)*(-b^5/a^13)^(1/4)*arctan(-1/729*(729*a^3*b
^4*sqrt(x)*(-b^5/a^13)^(1/4) - sqrt(-531441*a^7*b^5*sqrt(-b^5/a^13) + 53144
1*b^8*x)*a^3*(-b^5/a^13)^(1/4))/b^5) - 45*(a^3*b*x^5 + a^4*x^3)*(-b^5/a^13)
^(1/4)*log(729*a^10*(-b^5/a^13)^(3/4) + 729*b^4*sqrt(x)) + 45*(a^3*b*x^5 +
a^4*x^3)*(-b^5/a^13)^(1/4)*log(-729*a^10*(-b^5/a^13)^(3/4) + 729*b^4*sqrt(x
)) - 4*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)*sqrt(x))/(a^3*b*x^5 + a^4*x^3)
```

giac [A] time = 0.66, size = 220, normalized size = 0.91

$$\frac{b^2x^{\frac{3}{2}}}{2\left(bx^2+a\right)a^3}+\frac{9\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b}+\frac{9\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b}-\frac{9\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^4b}+\frac{9\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^4b}+\frac{2\left(10\,bx^2-a\right)}{5\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^(3/2)/((b*x^2 + a)*a^3) + 9/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 9/8*sqrt(2)*
```

```
(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(
1/4))/(a^4*b) - 9/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(a^4*b) + 9/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) + 2/5*(10*b*x^2 - a)/(a^3*x^(5/2))
```

maple [A] time = 0.02, size = 172, normalized size = 0.71

$$\frac{b^{2} x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{9\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}+\frac{9\sqrt{2}\;b\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}+\frac{9\sqrt{2}\;b\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}+\frac{4b}{a^{3}\sqrt{x}}-\frac{2}{5a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] -2/5/a^2/x^(5/2)+4/a^3*b/x^(1/2)+1/2*b^2/a^3*x^(3/2)/(b*x^2+a)+9/16*b/a^3/(
a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^
(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+9/8*b/a^3/(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)+9/8*b/a^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.04, size = 221, normalized size = 0.91

$$\frac{45\,b^{2}x^{4}+36\,abx^{2}-4\,a^{2}}{10\left(a^{3}bx^{\frac{9}{2}}+a^{4}x^{\frac{5}{2}}\right)}+\frac{9\,b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)/(a^3*b*x^(9/2) + a^4*x^(5/2)) + 9/16
*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3
```

mupad [B] time = 4.69, size = 87, normalized size = 0.36

$$\frac{\frac{18\,b\,x^2}{5\,a^2}-\frac{2}{5\,a}+\frac{9\,b^2\,x^4}{2\,a^3}}{a\,x^{5/2}+b\,x^{9/2}}-\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{13/4}}+\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)^2),x)
```

```
[Out] ((18*b*x^2)/(5*a^2) - 2/(5*a) + (9*b^2*x^4)/(2*a^3))/(a*x^(5/2) + b*x^(9/2)
) - (9*(-b)^(5/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(13/4)) + (9*(-b
)^(5/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.304 $\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\tan^{-1}(}{32}$$

Rubi [A] time = 0.18, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{3/4}b^{9/4}}-\frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)}-\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2)^3,x]
```

```
[Out] -x^(5/2)/(4*b*(a + b*x^2)^2) - (5*Sqrt[x])/(16*b^2*(a + b*x^2)) - (5*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4)) + (5*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(9/4))
 - (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[
2]*a^(3/4)*b^(9/4)) + (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}+\frac{5\int \frac{x^{3/2}}{\left(a+bx^2\right)^2}\,dx}{8b}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}-\frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)}+\frac{5\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32b^2}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}-\frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)}+\frac{5\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16b^2}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}-\frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)}+\frac{5\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32\sqrt{a}\,b^2}+\frac{5\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,d\right.}{32\sqrt{a}\,b^2}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}-\frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)}+\frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{a}\,b^{5/2}}+\frac{5\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+\sqrt{}}\right.}{64\sqrt{}}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}-\frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)}-\frac{5\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{5\log\left(\sqrt{a}+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= -\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}-\frac{5\sqrt{x}}{16b^2\left(a+bx^2\right)}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{5\log}{}$$

Mathematica [A] time = 0.11, size = 242, normalized size = 1.01

$$\frac{-\frac{15\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}}+\frac{15\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}}-\frac{30\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{30\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-\frac{256b^{5/4}x^{5/2}}{\left(a+bx^2\right)^2}+\frac{40\sqrt[4]{b}\sqrt{x}}{a+bx^2}-\frac{160a\sqrt[4]{b}\sqrt{x}}{\left(a+bx^2\right)^2}}{384b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2)^3,x]
```

```
[Out] ((-160*a*b^(1/4)*Sqrt[x])/(a + b*x^2)^2 - (256*b^(5/4)*x^(5/2))/(a + b*x^2)
^2 + (40*b^(1/4)*Sqrt[x])/(a + b*x^2) - (30*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (30*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*S
```

```
qrt[x])/a^(1/4)])/a^(3/4) - (15*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (15*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4))/(384*b^(9/4))
```

IntegrateAlgebraic [A] time = 0.47, size = 151, normalized size = 0.63

$$-\frac{5\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{-5a\sqrt{x}-9bx^{5/2}}{16b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(a + b*x^2)^3,x]
```

```
[Out] (-5*a*Sqrt[x] - 9*b*x^(5/2))/(16*b^2*(a + b*x^2)^2) - (5*ArcTan[(a^(1/4)/(S
qrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*a^(3
/4)*b^(9/4)) + (5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)])/(32*Sqrt[2]*a^(3/4)*b^(9/4))
```

fricas [A] time = 0.64, size = 254, normalized size = 1.06

$$\frac{20\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x}\,a^2b^7\left(-\frac{1}{a^3b^9}\right)^{\frac{3}{4}}-a^2b^7\sqrt{x}\left(-\frac{1}{a^3b^9}\right)^{\frac{3}{4}}\right)+5\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}+\sqrt{x}\right)-5\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\left(9\,bx^2+5\,a\right)\sqrt{x}}{64\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b^9))^(1/4)*arctan(sqrt
(a^2*b^4*sqrt(-1/(a^3*b^9)) + x)*a^2*b^7*(-1/(a^3*b^9))^(3/4) - a^2*b^7*sqr
t(x)*(-1/(a^3*b^9))^(3/4)) + 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b
^9))^(1/4)*log(a*b^2*(-1/(a^3*b^9))^(1/4) + sqrt(x)) - 5*(b^4*x^4 + 2*a*b^3
*x^2 + a^2*b^2)*(-1/(a^3*b^9))^(1/4)*log(-a*b^2*(-1/(a^3*b^9))^(1/4) + sqrt
(x)) - 4*(9*b*x^2 + 5*a)*sqrt(x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.65, size = 209, normalized size = 0.87

$$\frac{5\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^3}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^3}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^3}-\frac{5\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^3}-\frac{9\,bx^{\frac{5}{2}}+5\,a\sqrt{x}}{16\left(bx^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a*b^3) + 5/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) + 5/128*sqrt(2)*(a*b
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) - 5/128*
sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*
b^3) - 1/16*(9*b*x^(5/2) + 5*a*sqrt(x))/((b*x^2 + a)^2*b^2)
```

maple [A] time = 0.01, size = 170, normalized size = 0.71

$$\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a\,b^{2}}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a\,b^{2}}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a\,b^{2}}+\frac{-\frac{9x^{\frac{5}{2}}}{16b}-\frac{5a\sqrt{x}}{16b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(-9/32/b*x^(5/2)-5/32*a/b^2*x^(1/2))/(b*x^2+a)^2+5/128/b^2*(a/b)^(1/4)/a*
2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/
2)*x^(1/2)+(a/b)^(1/2)))+5/64/b^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)+5/64/b^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)-1)
```

maxima [A] time = 3.09, size = 218, normalized size = 0.91

$$-\frac{9\,bx^{\frac{5}{2}}+5\,a\sqrt{x}}{16\left(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2}\right)}+\frac{5\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(9*b*x^(5/2) + 5*a*sqrt(x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 5/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2
```

mupad [B] time = 4.69, size = 87, normalized size = 0.36

$$-\frac{\frac{9\,x^{5/2}}{16\,b}+\frac{5\,a\,\sqrt{x}}{16\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{32\,{(-a)}^{3/4}\,b^{9/4}}-\frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)}{32\,{(-a)}^{3/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2)^3,x)
```

```
[Out] - ((9*x^(5/2))/(16*b) + (5*a*x^(1/2))/(16*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
 - (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(3/4)*b^(9/4)) - (5*atan
h((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(3/4)*b^(9/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.305 $\int \frac{x^{5/2}}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=242

$$\frac{3 \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x}+\sqrt{a}+\sqrt{b} x\right)}{64 \sqrt{2} a^{5/4} b^{7/4}}-\frac{3 \log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x}+\sqrt{a}+\sqrt{b} x\right)}{64 \sqrt{2} a^{5/4} b^{7/4}}-\frac{3 \tan ^{-1}\left(1-\frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}}\right)}{32 \sqrt{2} a^{5/4} b^{7/4}}+\frac{3 \tan ^{-1}\left(\cdots\right.}{32 \sqrt{2} \cdots}$$

Rubi [A] time = 0.17, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3 \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x}+\sqrt{a}+\sqrt{b} x\right)}{64 \sqrt{2} a^{5/4} b^{7/4}}-\frac{3 \log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x}+\sqrt{a}+\sqrt{b} x\right)}{64 \sqrt{2} a^{5/4} b^{7/4}}-\frac{3 \tan ^{-1}\left(1-\frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}}\right)}{32 \sqrt{2} a^{5/4} b^{7/4}}+\frac{3 \tan ^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}}+1\right)}{32 \sqrt{2} a^{5/4} b^{7/4}}+\frac{3 x^{3/2}}{16 a b \left(a+b x^2\right)}-\frac{x^{3/2}}{4 b \left(a+b x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] -x^(3/2)/(4*b*(a + b*x^2)^2) + (3*x^(3/2))/(16*a*b*(a + b*x^2)) - (3*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4)) + (3*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(7/4))
 + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[
2]*a^(5/4)*b^(7/4)) - (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)^3}\,dx = -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx}{8b}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}+\frac{3\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32ab}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16ab}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}-\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32ab^{3/2}}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,d\right.}{32ab^{3/2}}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{64ab^2}+\frac{3\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+}\right.}{64}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}+\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{5/4}b^{7/4}}-\frac{3\log\left(\sqrt{a}+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= -\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}+\frac{3x^{3/2}}{16ab\left(a+bx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{3\log}{}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.19

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{a^2}-\frac{1}{\left(a+bx^2\right)^2}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*(-(a + b*x^2)^(-2) + Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)
]/a^2))/(5*b)
```

IntegrateAlgebraic [A] time = 0.45, size = 154, normalized size = 0.64

$$-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{3bx^{7/2}-ax^{3/2}}{16ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x^2)^3,x]
```

```
[Out] (-(a*x^(3/2)) + 3*b*x^(7/2))/(16*a*b*(a + b*x^2)^2) - (3*ArcTan[(a^(1/4)/(S
qrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*a^(5
/4)*b^(7/4)) - (3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)])/(32*Sqrt[2]*a^(5/4)*b^(7/4))
```

fricas [A] time = 0.68, size = 260, normalized size = 1.07

$$-\frac{12\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x}\,ab^2\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}-ab^2\sqrt{x}\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\right)-3\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\log\left(a^4b^5\left(-\frac{1}{a^5b^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)+3\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\log\left(-a^4b^5\left(-\frac{1}{a^5b^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4\left(3bx^3-ax\right)\sqrt{x}}{64\left(ab^3x^4+2a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^5*b^7))^(1/4)*arctan(s
qrt(-a^3*b^3*sqrt(-1/(a^5*b^7)) + x)*a*b^2*(-1/(a^5*b^7))^(1/4) - a*b^2*sqr
t(x)*(-1/(a^5*b^7))^(1/4)) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^5
*b^7))^(1/4)*log(a^4*b^5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) + 3*(a*b^3*x^4 + 2
*a^2*b^2*x^2 + a^3*b)*(-1/(a^5*b^7))^(1/4)*log(-a^4*b^5*(-1/(a^5*b^7))^(3/4
) + sqrt(x)) - 4*(3*b*x^3 - a*x)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*
b)
```

giac [A] time = 0.66, size = 212, normalized size = 0.88

$$\frac{3\,bx^{\frac{7}{2}}-ax^{\frac{3}{2}}}{16\left(bx^2+a\right)^2ab}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4}-\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^4}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*b*x^(7/2) - a*x^(3/2))/((b*x^2 + a)^2*a*b) + 3/64*sqrt(2)*(a*b^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2
*b^4) + 3/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) - 3/128*sqrt(2)*(a*b^3)^(3/4)*log(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 3/128*sqrt(2)*(a*b^3)^
(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4)
```

maple [A] time = 0.02, size = 169, normalized size = 0.70

$$\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}} a\,b^{2}}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}} a\,b^{2}}+\frac{3\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}} a\,b^{2}}+\frac{\frac{3x^{\frac{7}{2}}}{16a}-\frac{x^{\frac{3}{2}}}{16b}}{\left(b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(3/32/a*x^(7/2)-1/32/b*x^(3/2))/(b*x^2+a)^2+3/128/b^2/a/(a/b)^(1/4)*2^(1/
2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^
(1/2)+(a/b)^(1/2)))+3/64/b^2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)+3/64/b^2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)-1)
```

maxima [A] time = 2.97, size = 222, normalized size = 0.92

$$\frac{3\,bx^{\frac{7}{2}}-ax^{\frac{3}{2}}}{16\left(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*b*x^(7/2) - a*x^(3/2))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b)
```

mupad [B] time = 0.08, size = 85, normalized size = 0.35

$$\frac{\frac{3\,x^{7/2}}{16\,a}-\frac{x^{3/2}}{16\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{5/4}\,b^{7/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{5/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2)^3,x)
```

```
[Out] ((3*x^(7/2))/(16*a) - x^(3/2)/(16*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (3*atan
((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(5/4)*b^(7/4)) + (3*atanh((b^(1/4)
*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(5/4)*b^(7/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.306 $\int \frac{x^{3/2}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{3\tan^{-1}\left(\right.}{32\sqrt{}}$$

Rubi [A] time = 0.16, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] -Sqrt[x]/(4*b*(a + b*x^2)^2) + Sqrt[x]/(16*a*b*(a + b*x^2)) - (3*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) + (3*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(5/4)) - (
3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a
^(7/4)*b^(5/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(64*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^3}\,dx = -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx}{8b}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}+\frac{3\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32ab}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16ab}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^{3/2}b}+\frac{3\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,d\ldots\right.}{32a^{3/2}b}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{64a^{3/2}b^{3/2}}+\frac{3\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+\ldots}\right.}{64a\ldots}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{3\log\left(\sqrt{a}+\sqrt{2}\ldots\right.}{64\sqrt{2}\ldots}$$

$$= -\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}+\frac{\sqrt{x}}{16ab\left(a+bx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{3\log\ldots}{}$$

Mathematica [A] time = 0.11, size = 223, normalized size = 0.92

$$\frac{-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{7/4}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{7/4}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+\frac{8\sqrt[4]{b}\sqrt{x}}{a^2+abx^2}-\frac{32\sqrt[4]{b}\sqrt{x}}{\left(a+bx^2\right)^2}}{128b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] ((-32*b^(1/4)*Sqrt[x])/(a + b*x^2)^2 + (8*b^(1/4)*Sqrt[x])/(a^2 + a*b*x^2)
- (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) + (6*Sq
rt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) - (3*Sqrt[2]*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3*Sqr
t[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4))/(
128*b^(5/4))
```

IntegrateAlgebraic [A] time = 0.44, size = 153, normalized size = 0.63

$$-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{bx^{5/2}-3a\sqrt{x}}{16ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*Sqrt[x] + b*x^(5/2))/(16*a*b*(a + b*x^2)^2) - (3*ArcTan[(a^(1/4)/(Sqr
t[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*a^(7/4
)*b^(5/4)) + (3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b
]*x)])/(32*Sqrt[2]*a^(7/4)*b^(5/4))
```

fricas [A] time = 0.77, size = 257, normalized size = 1.06

$$\frac{12\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x}\,a^5b^4\left(-\frac{1}{a^7b^5}\right)^{\frac{3}{4}}-a^5b^4\sqrt{x}\left(-\frac{1}{a^7b^5}\right)^{\frac{3}{4}}\right)+3\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}\log\left(a^2b\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\left(ab^3x^4+2a^2b^2x^2+a^3b\right)\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}\log\left(-a^2b\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\left(bx^2-3a\right)\sqrt{x}}{64\left(ab^3x^4+2a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^(1/4)*arctan(sq
rt(a^4*b^2*sqrt(-1/(a^7*b^5)) + x)*a^5*b^4*(-1/(a^7*b^5))^(3/4) - a^5*b^4*s
qrt(x)*(-1/(a^7*b^5))^(3/4)) + 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a
^7*b^5))^(1/4)*log(a^2*b*(-1/(a^7*b^5))^(1/4) + sqrt(x)) - 3*(a*b^3*x^4 + 2
*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^(1/4)*log(-a^2*b*(-1/(a^7*b^5))^(1/4)
+ sqrt(x)) + 4*(b*x^2 - 3*a)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.66, size = 211, normalized size = 0.87

$$\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^2b^2}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^2b^2}+\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^2b^2}-\frac{3\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^2b^2}+\frac{bx^{\frac{5}{2}}-3a\sqrt{x}}{16\left(bx^2+a\right)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a^2*b^2) + 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 3/128*sqrt(2)*
(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) -
3/128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b
))/(a^2*b^2) + 1/16*(b*x^(5/2) - 3*a*sqrt(x))/((b*x^2 + a)^2*a*b)
```

maple [A] time = 0.02, size = 169, normalized size = 0.70

$$\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^2b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^2b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a^2b}+\frac{\frac{x^{\frac{5}{2}}}{16a}-\frac{3\sqrt{x}}{16b}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32/a*x^(5/2)-3/32/b*x^(1/2))/(b*x^2+a)^2+3/128/b/a^2*(a/b)^(1/4)*2^(1/
2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^
(1/2)+(a/b)^(1/2)))+3/64/b/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)+3/64/b/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)-1)
```

maxima [A] time = 2.89, size = 221, normalized size = 0.91

$$\frac{bx^{\frac{5}{2}}-3\,a\sqrt{x}}{16\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)}+\frac{3\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(b*x^(5/2) - 3*a*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

mupad [B] time = 4.67, size = 85, normalized size = 0.35

$$\frac{\frac{x^{5/2}}{16\,a}-\frac{3\,\sqrt{x}}{16\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{7/4}\,b^{5/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{7/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2)^3,x)
```

```
[Out] (x^(5/2)/(16*a) - (3*x^(1/2))/(16*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (3*atan
((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(7/4)*b^(5/4)) + (3*atanh((b^(1/4)
*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(7/4)*b^(5/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.307 $\int \frac{\sqrt{x}}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=239

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5\tan^{-1}\left(\cdots\right.}{32\sqrt{2}\cdots}$$

Rubi [A] time = 0.16, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5x^{3/2}}{16a^2\left(a+bx^2\right)}+\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2)^3,x]
```

```
[Out] x^(3/2)/(4*a*(a + b*x^2)^2) + (5*x^(3/2))/(16*a^2*(a + b*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4))
+ (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2
]*a^(9/4)*b^(3/4)) - (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)^3}\,dx = \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} + \frac{5\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^2}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^2\sqrt{b}} + \frac{5\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx\right.}{32a^2\sqrt{b}}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^2b} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}}{}}\right.}{64a}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} + \frac{5\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{3/4}} - \frac{5\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{}\right.}{64\sqrt{2}\,a^{}}$$

$$= \frac{x^{3/2}}{4a\left(a+bx^2\right)^2} + \frac{5x^{3/2}}{16a^2\left(a+bx^2\right)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}} + \frac{5\log(}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2)^3,x]
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)])/(3*a^3)
```

IntegrateAlgebraic [A] time = 0.27, size = 149, normalized size = 0.62

$$-\frac{5\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{x^{3/2}\left(9a+5bx^2\right)}{16a^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x^2)^3,x]
```

```
[Out] (x^(3/2)*(9*a + 5*b*x^2))/(16*a^2*(a + b*x^2)^2) - (5*ArcTan[(a^(1/4)/(Sqrt
[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*a^(9/4)
*b^(3/4)) - (5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]
*x)])/(32*Sqrt[2]*a^(9/4)*b^(3/4))
```

fricas [A] time = 0.64, size = 250, normalized size = 1.05

$$-\frac{20\left(a^2b^2x^4+2a^3bx^2+a^4\right)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-a^5b\sqrt{-\frac{1}{a^9b^3}}+x}\,a^2b\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}-a^2b\sqrt{x}\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\right)-5\left(a^2b^2x^4+2a^3bx^2+a^4\right)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\log\left(a^7b^2\left(-\frac{1}{a^9b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)+5\left(a^2b^2x^4+2a^3bx^2+a^4\right)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\log\left(-a^7b^2\left(-\frac{1}{a^9b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4\left(5bx^3+9ax\right)\sqrt{x}}{64\left(a^2b^2x^4+2a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(20*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^9*b^3))^(1/4)*arctan(sqr
t(-a^5*b*sqrt(-1/(a^9*b^3)) + x)*a^2*b*(-1/(a^9*b^3))^(1/4) - a^2*b*sqrt(x)
*(-1/(a^9*b^3))^(1/4)) - 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^9*b^3))
^(1/4)*log(a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt(x)) + 5*(a^2*b^2*x^4 + 2*a^3
*b*x^2 + a^4)*(-1/(a^9*b^3))^(1/4)*log(-a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt
(x)) - 4*(5*b*x^3 + 9*a*x)*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

giac [A] time = 0.65, size = 209, normalized size = 0.87

$$\frac{5bx^{\frac{7}{2}}+9ax^{\frac{3}{2}}}{16\left(bx^2+a\right)^2a^2}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^3b^3}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^3b^3}-\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^3b^3}+\frac{5\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/((b*x^2 + a)^2*a^2) + 5/64*sqrt(2)*(a*b^3)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a
^3*b^3) + 5/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 5/128*sqrt(2)*(a*b^3)^(3/4)*log(sq
```

```
rt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 5/128*sqrt(2)*(a*b^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

maple [A] time = 0.01, size = 175, normalized size = 0.73

$$\frac{x^{\frac{3}{2}}}{4\left(b\,x^{2}+a\right)^{2} a}+\frac{5x^{\frac{3}{2}}}{16\left(b\,x^{2}+a\right)a^{2}}+\frac{5\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{5\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}+\frac{5\sqrt{2}\;\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^3,x)
```

```
[Out] 1/4*x^(3/2)/a/(b*x^2+a)^2+5/16*x^(3/2)/a^2/(b*x^2+a)+5/128/a^2/b/(a/b)^(1/4
)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2)))+5/64/a^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)+5/64/a^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1)
```

maxima [A] time = 2.99, size = 217, normalized size = 0.91

$$\frac{5\,bx^{\frac{7}{2}}+9\,ax^{\frac{3}{2}}}{16\left(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\right)}+\frac{5\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 5/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2
```

mupad [B] time = 0.08, size = 86, normalized size = 0.36

$$\frac{\frac{9\,x^{3/2}}{16\,a}+\frac{5\,b\,x^{7/2}}{16\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{9/4}\,b^{3/4}}-\frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{9/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2)^3,x)
```

```
[Out] ((9*x^(3/2))/(16*a) + (5*b*x^(7/2))/(16*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) +
 (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(9/4)*b^(3/4)) - (5*atanh(
(b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(9/4)*b^(3/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.308 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\tan}{3}$$

Rubi [A] time = 0.17, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)}-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*a*(a + b*x^2)^2) + (7*Sqrt[x])/(16*a^2*(a + b*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(1/
4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*S
qrt[2]*a^(11/4)*b^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^3}\,dx = \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32a^2}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^{5/2}} + \frac{21\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{}}{a+bx}\right.}{32a^{5/}}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} + \frac{21\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^{5/2}\sqrt{b}} + \frac{21\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}} + \frac{21\log\left(\sqrt{a}+\right.}{64}$$

$$= \frac{\sqrt{x}}{4a\left(a+bx^2\right)^2} + \frac{7\sqrt{x}}{16a^2\left(a+bx^2\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}} -$$

Mathematica [A] time = 0.08, size = 220, normalized size = 0.92

$$\frac{\frac{32a^{7/4}\sqrt{x}}{\left(a+bx^2\right)^2} + \frac{56a^{3/4}\sqrt{x}}{a+bx^2} - \frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{b}} + \frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{b}} - \frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}}{128a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((32*a^(7/4)*Sqrt[x])/(a + b*x^2)^2 + (56*a^(3/4)*Sqrt[x])/(a + b*x^2) - (4
2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (42*Sqrt
[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (21*Sqrt[2]*Lo
```

```
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (21*Sqr
t[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4))/(
128*a^(11/4))
```

IntegrateAlgebraic [A] time = 0.27, size = 149, normalized size = 0.62

$$-\frac{21\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{21\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{\sqrt{x}\left(11a+7bx^2\right)}{16a^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] (Sqrt[x]*(11*a + 7*b*x^2))/(16*a^2*(a + b*x^2)^2) - (21*ArcTan[(a^(1/4)/(Sq
rt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*a^(11
/4)*b^(1/4)) + (21*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqr
t[b]*x)])/(32*Sqrt[2]*a^(11/4)*b^(1/4))
```

fricas [A] time = 0.54, size = 241, normalized size = 1.01

$$\frac{84\left(a^2b^2x^4+2a^3bx^2+a^4\right)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^6\sqrt{-\frac{1}{a^{11}b}}+x}\,a^8b\left(-\frac{1}{a^{11}b}\right)^{\frac{3}{4}}-a^8b\sqrt{x}\left(-\frac{1}{a^{11}b}\right)^{\frac{3}{4}}\right)+21\left(a^2b^2x^4+2a^3bx^2+a^4\right)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-21\left(a^2b^2x^4+2a^3bx^2+a^4\right)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\log\left(-a^3\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\left(7bx^2+11a\right)\sqrt{x}}{64\left(a^2b^2x^4+2a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(84*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*arctan(sqrt(
a^6*sqrt(-1/(a^11*b)) + x)*a^8*b*(-1/(a^11*b))^(3/4) - a^8*b*sqrt(x)*(-1/(a
^11*b))^(3/4)) + 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*l
og(a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) - 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4
)*(-1/(a^11*b))^(1/4)*log(-a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) + 4*(7*b*x^2
+ 11*a)*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

giac [A] time = 0.65, size = 209, normalized size = 0.87

$$\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^3b}+\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^3b}+\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^3b}-\frac{21\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^3b}+\frac{7bx^{\frac{5}{2}}+11a\sqrt{x}}{16\left(bx^2+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/(a^3*b) + 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 21/128*sqrt(2)*(
a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) - 21/
128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^3*b) + 1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/((b*x^2 + a)^2*a^2)
```

maple [A] time = 0.01, size = 166, normalized size = 0.69

$$\frac{\sqrt{x}}{4\left(b\,x^{2}+a\right)^{2} a}+\frac{7\sqrt{x}}{16\left(b\,x^{2}+a\right) a^{2}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\;\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\;\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\;\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^3/x^(1/2),x)
```

```
[Out] 1/4*x^(1/2)/a/(b*x^2+a)^2+7/16*x^(1/2)/a^2/(b*x^2+a)+21/128/a^3*(a/b)^(1/4)
*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2)))+21/64/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)+21/64/a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)-1)
```

maxima [A] time = 3.02, size = 217, normalized size = 0.91

$$\frac{7\,bx^{\frac{5}{2}}+11\,a\sqrt{x}}{16\left(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\right)}+\frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 21/12
8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1
/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqrt(2)
*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^2
```

mupad [B] time = 4.67, size = 86, normalized size = 0.36

$$\frac{\frac{11\,\sqrt{x}}{16\,a}+\frac{7\,b\,x^{5/2}}{16\,a^{2}}}{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}-\frac{21\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{11/4}\,b^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{32\,{\left(-a\right)}^{11/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)^3),x)
```

```
[Out] ((11*x^(1/2))/(16*a) + (7*b*x^(5/2))/(16*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
- (21*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(11/4)*b^(1/4)) - (21*at
anh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(11/4)*b^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

3.309 $\int \frac{1}{x^{3/2}(a+bx^2)^3}\,dx$

Optimal. Leaf size=251

$$-\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{13/4}}+\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{32\sqrt{2}\,a^{13/4}}$$

Rubi [A] time = 0.19, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{13/4}}+\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{13/4}}-\frac{45\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{13/4}}-\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] -45/(16*a^3*Sqrt[x]) + 1/(4*a*Sqrt[x]*(a + b*x^2)^2) + 9/(16*a^2*Sqrt[x]*(a
 + b*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32
*Sqrt[2]*a^(13/4)) - (45*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)])/(32*Sqrt[2]*a^(13/4)) - (45*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4)) + (45*b^(1/4)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^3}\,dx = \frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}+\frac{45\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{32a^2}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{(45b)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{(45b)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}+\frac{\left(45\sqrt{b}\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\right.}{32a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{45\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\right.}{64a^3}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}-\frac{45\sqrt[4]{b}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{64\sqrt{2}\,a^{13/4}}$$

$$= -\frac{45}{16a^3\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}+\frac{9}{16a^2\sqrt{x}\left(a+bx^2\right)}+\frac{45\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{13/4}}-\frac{45}{}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-\frac{bx^2}{a}\right)}{a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 3, 3/4, -((b*x^2)/a)])/(a^3*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.48, size = 160, normalized size = 0.64

$$\frac{45\sqrt[4]{b}\,\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{32\sqrt{2}\,a^{13/4}}+\frac{45\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{13/4}}+\frac{-32a^2-81abx^2-45b^2x^4}{16a^3\sqrt{x}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] (-32*a^2 - 81*a*b*x^2 - 45*b^2*x^4)/(16*a^3*Sqrt[x]*(a + b*x^2)^2) + (45*b^
(1/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sq
rt[x]])/(32*Sqrt[2]*a^(13/4)) + (45*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(32*Sqrt[2]*a^(13/4))
```

fricas [A] time = 0.46, size = 263, normalized size = 1.05

$$\frac{180\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\,a^3b\sqrt{x}\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,a^7b\sqrt{-\frac{b}{a^{13}}}+8303765625\,b^2x}\,a^3\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}}{91125\,b}\right)-45\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}\log\left(91125\,a^{10}\left(-\frac{b}{a^{13}}\right)^{\frac{3}{4}}+91125\,b\sqrt{x}\right)+45\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}\log\left(-91125\,a^{10}\left(-\frac{b}{a^{13}}\right)^{\frac{3}{4}}+91125\,b\sqrt{x}\right)-4\left(45\,b^2x^4+81\,abx^2+32\,a^2\right)\sqrt{x}}{64\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(180*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-b/a^13)^(1/4)*arctan(-1/911
25*(91125*a^3*b*sqrt(x)*(-b/a^13)^(1/4) - sqrt(-8303765625*a^7*b*sqrt(-b/a^
13) + 8303765625*b^2*x)*a^3*(-b/a^13)^(1/4))/b) - 45*(a^3*b^2*x^5 + 2*a^4*b
*x^3 + a^5*x)*(-b/a^13)^(1/4)*log(91125*a^10*(-b/a^13)^(3/4) + 91125*b*sqrt
(x)) + 45*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-b/a^13)^(1/4)*log(-91125*a^
10*(-b/a^13)^(3/4) + 91125*b*sqrt(x)) - 4*(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2
)*sqrt(x))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
```

giac [A] time = 0.65, size = 220, normalized size = 0.88

$$-\frac{2}{a^3\sqrt{x}}-\frac{45\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^2}-\frac{45\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^2}+\frac{45\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^2}-\frac{45\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^2}-\frac{13\,b^2x^{\frac{7}{2}}+17\,abx^{\frac{3}{2}}}{16\left(bx^2+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -2/(a^3*sqrt(x)) - 45/64*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 45/64*sqrt(2)*(a*b^3)^(3/
4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*
b^2) + 45/128*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + s
qrt(a/b))/(a^4*b^2) - 45/128*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*(13*b^2*x^(7/2) + 17*a*b*x^(3/2)
)/((b*x^2 + a)^2*a^3)
```

maple [A] time = 0.02, size = 178, normalized size = 0.71

$$-\frac{13 b^{2} x^{\frac{7}{2}}}{16 \left(b x^{2}+a\right)^{2} a^{3}}-\frac{17 b x^{\frac{3}{2}}}{16 \left(b x^{2}+a\right)^{2} a^{2}}-\frac{45 \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}-\frac{45 \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}-\frac{45 \sqrt{2}\, \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}-\frac{2}{a^{3} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^3,x)
```

```
[Out] -2/a^3/x^(1/2)-13/16/a^3*b^2/(b*x^2+a)^2*x^(7/2)-17/16/a^2*b/(b*x^2+a)^2*x^
(3/2)-45/128/a^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b
)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-45/64/a^3/(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-45/64/a^3/(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.00, size = 230, normalized size = 0.92

$$-\frac{45 \, b^{2} x^{4} + 81 \, a b x^{2} + 32 \, a^{2}}{16 \, {\left(a^{3} b^{2} x^{\frac{9}{2}} + 2 \, a^{4} b x^{\frac{5}{2}} + a^{5} \sqrt{x}\right)}} - \frac{45 \, b {\left(\frac{2 \, \sqrt{2} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} + 2 \, \sqrt{b} \sqrt{x}\right)}}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}} + \frac{2 \, \sqrt{2} \arctan\left(-\frac{\sqrt{2} {\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} - 2 \, \sqrt{b} \sqrt{x}\right)}}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}} - \frac{\sqrt{2} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}}{128 \, a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2)/(a^3*b^2*x^(9/2) + 2*a^4*b*x^(5/2)
 + a^5*sqrt(x)) - 45/128*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(1/4)*b^(3/4)))/a^3
```

mupad [B] time = 0.10, size = 99, normalized size = 0.39

$$\frac{45\,{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{13/4}}-\frac{45\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{13/4}}-\frac{\frac{2}{a}+\frac{81\,b\,x^2}{16\,a^2}+\frac{45\,b^2\,x^4}{16\,a^3}}{a^2\,\sqrt{x}+b^2\,x^{9/2}+2\,a\,b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)^3),x)
```

```
[Out] (45*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(13/4)) - (45*(-b
)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(13/4)) - (2/a + (81*b*x^
2)/(16*a^2) + (45*b^2*x^4)/(16*a^3))/(a^2*x^(1/2) + b^2*x^(9/2) + 2*a*b*x^(
5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.310 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=251

$$\frac{77b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{15/4}}-\frac{77b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{15/4}}+\frac{77b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{}}{\sqrt[4]{a}}\right.}{32\sqrt{2}\,a^{15/4}}$$

Rubi [A] time = 0.18, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{77b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{15/4}}-\frac{77b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{15/4}}+\frac{77b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{15/4}}-\frac{77b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{15/4}}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] -77/(48*a^3*x^(3/2)) + 1/(4*a*x^(3/2)*(a + b*x^2)^2) + 11/(16*a^2*x^(3/2)*(
a + b*x^2)) + (77*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(3
2*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(32*Sqrt[2]*a^(15/4)) + (77*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx}{32a^2}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{(77b)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32a^3}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{(77b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16a^3}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{(77b)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^{7/2}}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}-\frac{\left(77\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,d\right.}{64a^{7/2}}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{64\sqrt{2}\,a^{15/4}}$$

$$= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{15/4}}-\frac{77}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-\frac{bx^2}{a}\right)}{3a^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 3, 1/4, -((b*x^2)/a)])/(3*a^3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.47, size = 160, normalized size = 0.64

$$\frac{77b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{32\sqrt{2}a^{15/4}}-\frac{77b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}a^{15/4}}+\frac{-32a^2-121abx^2-77b^2x^4}{48a^3x^{3/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] (-32*a^2 - 121*a*b*x^2 - 77*b^2*x^4)/(48*a^3*x^(3/2)*(a + b*x^2)^2) + (77*b
^(3/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/S
qrt[x]])/(32*Sqrt[2]*a^(15/4)) - (77*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(32*Sqrt[2]*a^(15/4))
```

fricas [A] time = 0.62, size = 283, normalized size = 1.13

$$-\frac{924\left(a^{3}b^{2}x^{6}+2a^{4}bx^{4}+a^{5}x^{2}\right)\left(-\frac{b^{3}}{a^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^{11}b\sqrt{x}\left(-\frac{b^{3}}{a^{15}}\right)^{\frac{3}{4}}-\sqrt{a^{8}\sqrt{-\frac{b^{3}}{a^{15}}}+b^{2}x}\,a^{11}\left(-\frac{b^{3}}{a^{15}}\right)^{\frac{3}{4}}}{b^{3}}\right)+231\left(a^{3}b^{2}x^{6}+2a^{4}bx^{4}+a^{5}x^{2}\right)\left(-\frac{b^{3}}{a^{15}}\right)^{\frac{1}{4}}\log\left(77a^{4}\left(-\frac{b^{3}}{a^{15}}\right)^{\frac{1}{4}}+77b\sqrt{x}\right)-231\left(a^{3}b^{2}x^{6}+2a^{4}bx^{4}+a^{5}x^{2}\right)\left(-\frac{b^{3}}{a^{15}}\right)^{\frac{1}{4}}\log\left(-77a^{4}\left(-\frac{b^{3}}{a^{15}}\right)^{\frac{1}{4}}+77b\sqrt{x}\right)+4\left(77b^{2}x^{4}+121abx^{2}+32a^{2}\right)\sqrt{x}}{192\left(a^{3}b^{2}x^{6}+2a^{4}bx^{4}+a^{5}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(924*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/a^15)^(1/4)*arctan(
-(a^11*b*sqrt(x)*(-b^3/a^15)^(3/4) - sqrt(a^8*sqrt(-b^3/a^15) + b^2*x)*a^11
*(-b^3/a^15)^(3/4))/b^3) + 231*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/
a^15)^(1/4)*log(77*a^4*(-b^3/a^15)^(1/4) + 77*b*sqrt(x)) - 231*(a^3*b^2*x^6
 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/a^15)^(1/4)*log(-77*a^4*(-b^3/a^15)^(1/4) +
 77*b*sqrt(x)) + 4*(77*b^2*x^4 + 121*a*b*x^2 + 32*a^2)*sqrt(x))/(a^3*b^2*x^
6 + 2*a^4*b*x^4 + a^5*x^2)
```

giac [A] time = 0.65, size = 208, normalized size = 0.83

$$-\frac{77\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^{4}}-\frac{77\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^{4}}-\frac{77\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^{4}}+\frac{77\sqrt{2}\left(ab^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^{4}}-\frac{15b^{2}x^{\frac{5}{2}}+19ab\sqrt{x}}{16\left(bx^{2}+a\right)^{2}a^{3}}-\frac{2}{3a^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/a^4 - 77/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^4 - 77/128*sqrt(2)*(a*b^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^4 + 77/128*sqrt(2)
*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^4 - 1/16
*(15*b^2*x^(5/2) + 19*a*b*sqrt(x))/((b*x^2 + a)^2*a^3) - 2/3/(a^3*x^(3/2))
```

maple [A]   time = 0.02, size = 181, normalized size = 0.72

$$-\frac{15b^2x^{\frac{5}{2}}}{16\left(b\,x^2+a\right)^2 a^3}-\frac{19b\sqrt{x}}{16\left(b\,x^2+a\right)^2 a^2}-\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^4}-\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^4}-\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a^4}-\frac{2}{3a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^3,x)
```

```
[Out] -2/3/a^3/x^(3/2)-15/16/a^3*b^2/(b*x^2+a)^2*x^(5/2)-19/16/a^2*b/(b*x^2+a)^2*
x^(1/2)-77/128/a^4*b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-77/64/a^4*b*(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-77/64/a^4*b*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A]   time = 2.97, size = 231, normalized size = 0.92

$$-\frac{77\,b^2x^4+121\,abx^2+32\,a^2}{48\left(a^3b^2x^{\frac{11}{2}}+2\,a^4bx^{\frac{7}{2}}+a^5x^{\frac{3}{2}}\right)}-\frac{77\left(\frac{2\,\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)}{128\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(77*b^2*x^4 + 121*a*b*x^2 + 32*a^2)/(a^3*b^2*x^(11/2) + 2*a^4*b*x^(7/
2) + a^5*x^(3/2)) - 77/128*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqr
t(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(
3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/a^(3/4))/a^3
```

mupad [B] time = 4.66, size = 99, normalized size = 0.39

$$\frac{77\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{15/4}}-\frac{\frac{2}{3\,a}+\frac{121\,b\,x^2}{48\,a^2}+\frac{77\,b^2\,x^4}{48\,a^3}}{a^2\,x^{3/2}+b^2\,x^{11/2}+2\,a\,b\,x^{7/2}}+\frac{77\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)^3),x)
```

```
[Out] (77*(-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(15/4)) - (2/(3*a)
 + (121*b*x^2)/(48*a^2) + (77*b^2*x^4)/(48*a^3))/(a^2*x^(3/2) + b^2*x^(11/2
) + 2*a*b*x^(7/2)) + (77*(-b)^(3/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(3
2*a^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.311 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=264

$$\frac{117b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{117b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{117b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{32\sqrt{2}a^{17/4}}$$

Rubi [A] time = 0.21, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{117b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{117b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{117b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}}+\frac{117b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{17/4}}+\frac{13}{16a^2x^{5/2}\left(a+bx^2\right)}+\frac{117b}{16a^4\sqrt{x}}-\frac{117}{80a^3x^{5/2}}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] -117/(80*a^3*x^(5/2)) + (117*b)/(16*a^4*Sqrt[x]) + 1/(4*a*x^(5/2)*(a + b*x^
2)^2) + 13/(16*a^2*x^(5/2)*(a + b*x^2)) - (117*b^(5/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (117*b^(5/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (117*b^(5/4)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(17
/4)) - (117*b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(64*Sqrt[2]*a^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^3}\,dx = \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx}{8a}$$

$$= \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{117\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{32a^2}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} - \frac{(117b)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{32a^3}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{\left(117b^2\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{\left(117b^2\right)\text{Subst}\left(\int \frac{x^2}{a+b\cdots}\right.}{16a^4}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} - \frac{\left(117b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}\cdots}{\cdots}\right.}{32a^4\cdots}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{(117b)\text{Subst}\left(\int \frac{\cdots}{\frac{\sqrt{a}}{\sqrt{b}}-\cdots}\right.}{64a\cdots}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} + \frac{117b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\cdots\right.}{64\sqrt{2}\cdots}$$

$$= -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}\left(a+bx^2\right)^2} + \frac{13}{16a^2x^{5/2}\left(a+bx^2\right)} - \frac{117b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right.}{32\sqrt{2}\,a^{17/4}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.11

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-\frac{bx^2}{a}\right)}{5a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 3, -1/4, -((b*x^2)/a)])/(5*a^3*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.48, size = 171, normalized size = 0.65

$$-\frac{117b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{32\sqrt{2}\,a^{17/4}}-\frac{117b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{17/4}}+\frac{-32a^3+416a^2bx^2+1053ab^2x^4+585b^3x^6}{80a^4x^{5/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] (-32*a^3 + 416*a^2*b*x^2 + 1053*a*b^2*x^4 + 585*b^3*x^6)/(80*a^4*x^(5/2)*(a
 + b*x^2)^2) - (117*b^(5/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)
/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(32*Sqrt[2]*a^(17/4)) - (117*b^(5/4)*ArcTanh[
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(32*Sqrt[2]*a^(17
/4))
```

fricas [A] time = 0.55, size = 306, normalized size = 1.16

$$-\frac{2340\left(a^4b^2x^7+2a^5bx^5+a^6x^3\right)\left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613a^4b^4\sqrt{x}\left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}}-\sqrt{-2565164201769a^9b^5\sqrt{-\frac{b^5}{a^{17}}}+2565164201769b^8x}\,a^4\left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}}}{1601613b^5}\right)-585\left(a^4b^2x^7+2a^5bx^5+a^6x^3\right)\left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}}\log\left(1601613a^{13}\left(-\frac{b^5}{a^{17}}\right)^{\frac{3}{4}}+1601613b^4\sqrt{x}\right)+585\left(a^4b^2x^7+2a^5bx^5+a^6x^3\right)\left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}}\log\left(-1601613a^{13}\left(-\frac{b^5}{a^{17}}\right)^{\frac{3}{4}}+1601613b^4\sqrt{x}\right)-4\left(585b^3x^6+1053ab^2x^4+416a^2bx^2-32a^3\right)\sqrt{x}}{320\left(a^4b^2x^7+2a^5bx^5+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(2340*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*arctan
(-1/1601613*(1601613*a^4*b^4*sqrt(x)*(-b^5/a^17)^(1/4) - sqrt(-256516420176
9*a^9*b^5*sqrt(-b^5/a^17) + 2565164201769*b^8*x)*a^4*(-b^5/a^17)^(1/4))/b^5
) - 585*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*log(1601613
*a^13*(-b^5/a^17)^(3/4) + 1601613*b^4*sqrt(x)) + 585*(a^4*b^2*x^7 + 2*a^5*b
*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*log(-1601613*a^13*(-b^5/a^17)^(3/4) + 160
1613*b^4*sqrt(x)) - 4*(585*b^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^
3)*sqrt(x))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

giac [A] time = 0.63, size = 232, normalized size = 0.88

$$\frac{117\,\sqrt{2}\,\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{5}b}+\frac{117\,\sqrt{2}\,\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{5}b}-\frac{117\,\sqrt{2}\,\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{5}b}+\frac{117\,\sqrt{2}\,\left(ab^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{5}b}+\frac{21\,b^{3}x^{\frac{7}{2}}+25\,ab^{2}x^{\frac{3}{2}}}{16\left(bx^{2}+a\right)^{2}a^{4}}+\frac{2\left(15\,bx^{2}-a\right)}{5\,a^{4}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/(a^5*b) + 117/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^5*b) - 117/128*sqrt(2
)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b) +
117/128*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a
/b))/(a^5*b) + 1/16*(21*b^3*x^(7/2) + 25*a*b^2*x^(3/2))/((b*x^2 + a)^2*a^4)
 + 2/5*(15*b*x^2 - a)/(a^4*x^(5/2))
```

maple [A] time = 0.02, size = 192, normalized size = 0.73

$$\frac{21 b^{3} x^{\frac{7}{2}}}{16 \left(b\,x^{2}+a\right)^{2} a^{4}}+\frac{25 b^{2} x^{\frac{3}{2}}}{16 \left(b\,x^{2}+a\right)^{2} a^{3}}+\frac{117 \sqrt{2}\; b \arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{4}}+\frac{117 \sqrt{2}\; b \arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{4}}+\frac{117 \sqrt{2}\; b \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{4}}+\frac{6 b}{a^{4} \sqrt{x}}-\frac{2}{5 a^{3} x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^3,x)
```

```
[Out] -2/5/a^3/x^(5/2)+6*b/a^4/x^(1/2)+21/16/a^4*b^3/(b*x^2+a)^2*x^(7/2)+25/16/a^
3*b^2/(b*x^2+a)^2*x^(3/2)+117/128/a^4*b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/
4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))
)+117/64/a^4*b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+11
7/64/a^4*b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 3.05, size = 243, normalized size = 0.92

$$\frac{585\,b^{3}x^{6}+1053\,ab^{2}x^{4}+416\,a^{2}bx^{2}-32\,a^{3}}{80\left(a^{4}b^{2}x^{\frac{13}{2}}+2\,a^{5}bx^{\frac{9}{2}}+a^{6}x^{\frac{5}{2}}\right)}+\frac{117\,b^{2}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/80*(585*b^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^3)/(a^4*b^2*x^(13
/2) + 2*a^5*b*x^(9/2) + a^6*x^(5/2)) + 117/128*b^2*(2*sqrt(2)*arctan(1/2*sq
```

```
rt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^
(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqr
t(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
 sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^4
```

mupad [B] time = 4.65, size = 109, normalized size = 0.41

$$\frac{\frac{26\,b\,x^2}{5\,a^2}-\frac{2}{5\,a}+\frac{1053\,b^2\,x^4}{80\,a^3}+\frac{117\,b^3\,x^6}{16\,a^4}}{a^2\,x^{5/2}+b^2\,x^{13/2}+2\,a\,b\,x^{9/2}}-\frac{117\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{17/4}}+\frac{117\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)^3),x)
```

```
[Out] ((26*b*x^2)/(5*a^2) - 2/(5*a) + (1053*b^2*x^4)/(80*a^3) + (117*b^3*x^6)/(16
*a^4))/(a^2*x^(5/2) + b^2*x^(13/2) + 2*a*b*x^(9/2)) - (117*(-b)^(5/4)*atan(
((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(17/4)) + (117*(-b)^(5/4)*atanh(((-b)^
(1/4)*x^(1/2))/a^(1/4)))/(32*a^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.312 $\int \frac{\sqrt{x}}{a-bx^2} \, dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {329, 298, 205, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x^2),x]
```

```
[Out] -(ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))) + ArcTanh[(b^(1/4)*S
qrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{a-bx^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{a-bx^4}\,dx,x,\sqrt{x}\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}-\sqrt{b}\,x^2}\,dx,x,\sqrt{x}\right)}{\sqrt{b}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}+\sqrt{b}\,x^2}\,dx,x,\sqrt{x}\right)}{\sqrt{b}}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x^2),x]
[Out] (-ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)] + ArcTanh[(b^(1/4)*Sqrt[x])/a^(1/4)])/(
a^(1/4)*b^(3/4))
```

IntegrateAlgebraic [A] time = 0.05, size = 58, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a - b*x^2),x]
[Out] -(ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))) + ArcTanh[(b^(1/4)*S
qrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4))
```

fricas [B] time = 0.65, size = 117, normalized size = 2.02

$$2\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{ab\sqrt{\frac{1}{ab^3}}+x}\,b\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}-b\sqrt{x}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2*(1/(a*b^3))^(1/4)*arctan(sqrt(a*b*sqrt(1/(a*b^3)) + x)*b*(1/(a*b^3))^(1/4
) - b*sqrt(x)*(1/(a*b^3))^(1/4)) + 1/2*(1/(a*b^3))^(1/4)*log(a*b^2*(1/(a*b^
3))^(3/4) + sqrt(x)) - 1/2*(1/(a*b^3))^(1/4)*log(-a*b^2*(1/(a*b^3))^(3/4) +
 sqrt(x))
```

giac [B] time = 0.63, size = 194, normalized size = 3.34

$$\frac{\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{3}}+\frac{\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{3}}-\frac{\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{-\frac{a}{b}}\right)}{4\,ab^{3}}+\frac{\sqrt{2}\left(-ab^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{-\frac{a}{b}}\right)}{4\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sqr
t(x))/(-a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(-a*b^3)^(3/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a/b)^(1/4) - 2*sqrt(x))/(-a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(-
a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(-a/b)^(1/4) + x + sqrt(-a/b))/(a*b^3) + 1
/4*sqrt(2)*(-a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(-a/b)^(1/4) + x + sqrt(-a/b
))/(a*b^3)
```

maple [A] time = 0.01, size = 66, normalized size = 1.14

$$-\frac{\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\ln\left(\frac{\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x^2+a),x)
```

```
[Out] -1/b/(a/b)^(1/4)*arctan(x^(1/2)/(a/b)^(1/4))+1/2/b/(a/b)^(1/4)*ln((x^(1/2)+
(a/b)^(1/4))/(x^(1/2)-(a/b)^(1/4)))
```

maxima [B] time = 2.97, size = 86, normalized size = 1.48

$$-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\log\left(\frac{\sqrt{b}\sqrt{x}-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sqrt{x}+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(b)*sqrt(x)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(
b)) - 1/2*log((sqrt(b)*sqrt(x) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sqrt(x) +
sqrt(sqrt(a)*sqrt(b))))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

mupad [B] time = 0.08, size = 33, normalized size = 0.57

$$-\frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{a^{1/4}}\right)-\operatorname{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{a^{1/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x^2),x)
```

```
[Out] -(atan((b^(1/4)*x^(1/2))/a^(1/4)) - atanh((b^(1/4)*x^(1/2))/a^(1/4)))/(a^(1
/4)*b^(3/4))
```

sympy [A] time = 3.10, size = 122, normalized size = 2.10

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{b\sqrt{x}} & \text{for}\: a = 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for}\: b = 0 \\ -\frac{\log\left(-\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}\,b\sqrt[4]{\frac{1}{b}}}+\frac{\log\left(\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{a}\,b\sqrt[4]{\frac{1}{b}}}-\frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{\sqrt[4]{a}\,b\sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x**2+a),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2/(b*sqrt(x)), Eq(a, 0)), (2
*x**(3/2)/(3*a), Eq(b, 0)), (-log(-a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(
1/4)*b*(1/b)**(1/4)) + log(a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)*b*(
1/b)**(1/4)) - atan(sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(1/4)*b*(1/b)**(1/
4)), True))
```

3.313 $\int \frac{x^{7/2}}{1+x^2} \, dx$

Optimal. Leaf size=108

$$\frac{2x^{5/2}}{5}-2\sqrt{x}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2x^{5/2}}{5}-2\sqrt{x}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2),x]
```

```
[Out] -2*Sqrt[x] + (2*x^(5/2))/5 - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{1+x^2}\,dx &= \frac{2x^{5/2}}{5} - \int \frac{x^{3/2}}{1+x^2}\,dx \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + \int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + 2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,\cdots\right.}{\sqrt{2}} \\
&= -2\sqrt{x} + \frac{2x^{5/2}}{5} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\cdots\right.}{2\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 108, normalized size = 1.00

$$
\frac{2x^{5/2}}{5} - 2\sqrt{x} - \frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}} + \frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2),x]
```

```
[Out] -2*Sqrt[x] + (2*x^(5/2))/5 - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.08, size = 66, normalized size = 0.61

$$
\frac{2}{5}\sqrt{x}\left(x^2-5\right) + \frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(1 + x^2),x]
```

[Out] (2*Sqrt[x]*(-5 + x^2))/5 + ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]/Sqrt[
2] + ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)]/Sqrt[2]

fricas [A] time = 0.62, size = 117, normalized size = 1.08

$$\frac{2}{5}\left(x^{2}-5\right)\sqrt{x}-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 - 5)*sqrt(x) - sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1
) - sqrt(2)*sqrt(x) - 1) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(
x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) +
4*x + 4) - 1/4*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

giac [A] time = 0.63, size = 84, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2*sqrt(x)
```

maple [A] time = 0.01, size = 72, normalized size = 0.67

$$\frac{2x^{\frac{5}{2}}}{5}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{2}+\frac{\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{4}-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1),x)
```

```
[Out] 2/5*x^(5/2)-2*x^(1/2)+1/4*2^(1/2)*ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(
1/2)))+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^(1/2))
*2^(1/2)
```

maxima [A] time = 3.05, size = 84, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2*sqrt(x)
```

mupad [B] time = 0.08, size = 47, normalized size = 0.44

$$\frac{2\,x^{5/2}}{5}-2\,\sqrt{x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2*x^(1/2) + (2*x^(5/2))/5
```

sympy [A] time = 4.15, size = 105, normalized size = 0.97

$$\frac{2x^{\frac{5}{2}}}{5}-2\sqrt{x}-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1),x)
```

```
[Out] 2*x**(5/2)/5 - 2*sqrt(x) - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sq
rt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1
)/2 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/2
```

3.314 $\int \frac{x^{5/2}}{1+x^2} \, dx$

Optimal. Leaf size=101

$$\frac{2x^{3/2}}{3}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2x^{3/2}}{3}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2),x]
```

```
[Out] (2*x^(3/2))/3 + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sq
rt[x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]
*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{1+x^2}\,dx &= \frac{2x^{3/2}}{3} - \int \frac{\sqrt{x}}{1+x^2}\,dx \\
&= \frac{2x^{3/2}}{3} - 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3} + \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) - \frac{\text{Subst}\left(\int\right.}{} \\
&= \frac{2x^{3/2}}{3} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\text{Subs}}{} \\
&= \frac{2x^{3/2}}{3} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.24

$$
-\frac{2}{3}x^{3/2}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-x^2\right)-1\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2),x]
```

```
[Out] (-2*x^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -x^2]))/3
```

IntegrateAlgebraic [A] time = 0.07, size = 62, normalized size = 0.61

$$
\frac{2x^{3/2}}{3} - \frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(1 + x^2),x]
```

```
[Out] (2*x^(3/2))/3 - ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]/Sqrt[2] + ArcTan
h[(Sqrt[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

fricas [A] time = 0.54, size = 110, normalized size = 1.09

$$
\frac{2}{3}x^{\frac{3}{2}} + \sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right) + \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right) + \frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right) - \frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2) + sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2
)*sqrt(x) - 1) + sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x +
 4) - sqrt(2)*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) -
 1/4*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

giac [A] time = 0.63, size = 79, normalized size = 0.78

$$\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

maple [A] time = 0.01, size = 67, normalized size = 0.66

$$\frac{2x^{\frac{3}{2}}}{3} - \frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{2} - \frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{2} - \frac{\sqrt{2}\ \ln\left(\frac{x-\sqrt{2}\ \sqrt{x}+1}{x+\sqrt{2}\ \sqrt{x}+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1),x)
```

```
[Out] 2/3*x^(3/2)-1/4*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))-1/2
*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 2.97, size = 79, normalized size = 0.78

$$\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)
*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

mupad [B] time = 0.04, size = 42, normalized size = 0.42

$$\frac{2\,x^{3/2}}{3}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2 + 1),x)
```

```
[Out] (2*x^(3/2))/3 - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) - 2
^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2)
```

sympy [A] time = 1.75, size = 99, normalized size = 0.98

$$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1),x)
```

```
[Out] 2*x**(3/2)/3 - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*
sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2
)*atan(sqrt(2)*sqrt(x) + 1)/2
```

3.315 $\int \frac{x^{3/2}}{1+x^2} dx$

Optimal. Leaf size=99

$$2\sqrt{x}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {321, 329, 211, 1165, 628, 1162, 617, 204}

$$2\sqrt{x}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2),x]
```

```
[Out] 2*Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[x
]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqr
t[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{1+x^2}\,dx &= 2\sqrt{x}-\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx\\ &= 2\sqrt{x}-2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\ &= 2\sqrt{x}-\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\ &= 2\sqrt{x}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{\text{Subst}\left(\right.}{}\\ &= 2\sqrt{x}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\text{Sub}}{}\\ &= 2\sqrt{x}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right.}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 99, normalized size = 1.00

$$2\sqrt{x}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2),x]
```

```
[Out] 2*Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[x
]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqr
t[x] + x]/(2*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.08, size = 61, normalized size = 0.62

$$2\sqrt{x}-\frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(1 + x^2),x]
```

```
[Out] 2*Sqrt[x] - ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]/Sqrt[2] - ArcTanh[(S
qrt[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

fricas [A] time = 0.62, size = 110, normalized size = 1.11

$$\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1)
 + sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*
sqrt(x) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 1/4*sqrt(2)*l
og(-4*sqrt(2)*sqrt(x) + 4*x + 4) + 2*sqrt(x)
```

giac [A] time = 0.60, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.68

$$-\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}\,-1\right)}{2}-\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}\,+1\right)}{2}-\frac{\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{4}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1),x)
```

```
[Out] 2*x^(1/2)-1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-1/4*2^(1/2)*ln((x+2^(1/2)*x
^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 2.96, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

mupad [B] time = 0.04, size = 42, normalized size = 0.42

$$2\sqrt{x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2 + 1),x)
```

```
[Out] 2*x^(1/2) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2^(1/
2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2)
```

sympy [A] time = 0.84, size = 97, normalized size = 0.98

$$2\sqrt{x}+\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1),x)
```

```
[Out] 2*sqrt(x) + sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqr
t(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*a
tan(sqrt(2)*sqrt(x) + 1)/2
```

3.316 $\int \frac{\sqrt{x}}{1+x^2}\,dx$

Optimal. Leaf size=92

$$\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2
] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]
/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{1+x^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}}{-1-\sqrt{}}\right.}{2}$$
$$= \frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \right.}{}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.24

$$\frac{2}{3}x^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2),x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -x^2])/3
```

IntegrateAlgebraic [A] time = 0.06, size = 53, normalized size = 0.58

$$\frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(1 + x^2),x]
```

```
[Out] ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]/Sqrt[2] - ArcTanh[(Sqrt[2]*Sqrt[
x])/(1 + x)]/Sqrt[2]
```

fricas [A] time = 0.66, size = 107, normalized size = 1.16

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1
) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)
*sqrt(x) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 1/4*sqrt(2)*
log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

giac [A] time = 0.63, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

maple [A] time = 0.01, size = 62, normalized size = 0.67

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{2}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{2}+\frac{\sqrt{2}\ \ln\left(\frac{x-\sqrt{2}\ \sqrt{x}+1}{x+\sqrt{2}\ \sqrt{x}+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+1/4*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/
(x+2^(1/2)*x^(1/2)+1))+1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 2.87, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

mupad [B] time = 0.04, size = 37, normalized size = 0.40

$$\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2)
```

sympy [A] time = 0.55, size = 90, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1),x)

[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2
```

3.317 $\int \frac{1}{\sqrt{x}\left(1+x^2\right)} dx$

Optimal. Leaf size=92

$$-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2
] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[x] + x]
/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{\text{Subst}\left(\int \cdots\right.}{\cdots}\\
&= -\frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}-\frac{\text{Subs}\cdots}{\cdots}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,\sqrt{x}+x\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 76, normalized size = 0.83

$$\frac{-\log\left(x-\sqrt{2}\sqrt{x}+1\right)+\log\left(x+\sqrt{2}\sqrt{x}+1\right)-2\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+2\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[x]] - Log[1 - S
qrt[2]*Sqrt[x] + x] + Log[1 + Sqrt[2]*Sqrt[x] + x])/(2*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.06, size = 52, normalized size = 0.57

$$\frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]/Sqrt[2] + ArcTanh[(Sqrt[2]*Sqrt[
x])/(1 + x)]/Sqrt[2]
```

fricas [A] time = 0.54, size = 107, normalized size = 1.16

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqrt(x) - 1
) - sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)
*sqrt(x) + 1) + 1/4*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 1/4*sqrt(2)*
log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

giac [A] time = 0.59, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

maple [A] time = 0.00, size = 62, normalized size = 0.67

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{2}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{2}+\frac{\sqrt{2}\ \ln\left(\frac{x+\sqrt{2}\ \sqrt{x}+1}{x-\sqrt{2}\ \sqrt{x}+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/x^(1/2),x)
```

```
[Out] 1/4*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+1/2*2^(1/2)*arc
tan(2^(1/2)*x^(1/2)-1)+1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 2.89, size = 74, normalized size = 0.80

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

mupad [B] time = 0.03, size = 37, normalized size = 0.40

$$\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \sqrt{x}\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^2 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2)
```

sympy [A] time = 0.73, size = 90, normalized size = 0.98

$$-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/x**(1/2),x)
```

```
[Out] -sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)
*sqrt(x) + 1)/2
```

3.318 $\int \frac{1}{x^{3/2}(1+x^2)}\,dx$

Optimal. Leaf size=99

$$-\frac{2}{\sqrt{x}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2}{\sqrt{x}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)),x]
```

```
[Out] -2/Sqrt[x] + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[
x]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sq
rt[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx &= -\frac{2}{\sqrt{x}}-\int \frac{\sqrt{x}}{1+x^2}\,dx\\ &= -\frac{2}{\sqrt{x}}-2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2}{\sqrt{x}}+\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2}{\sqrt{x}}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{\text{Su}}{}\\ &= -\frac{2}{\sqrt{x}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\\ &= -\frac{2}{\sqrt{x}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{}\right.}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 20, normalized size = 0.20

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-x^2\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -x^2])/Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.08, size = 60, normalized size = 0.61

$$-\frac{2}{\sqrt{x}}-\frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(1 + x^2)),x]
```

```
[Out] -2/Sqrt[x] - ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]/Sqrt[2] + ArcTanh[(
Sqrt[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

fricas [A] time = 0.57, size = 120, normalized size = 1.21

$$\frac{4\sqrt{2}x\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}x\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\sqrt{2}x\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-\sqrt{2}x\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\sqrt{x}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)*sqr
t(x) - 1) + 4*sqrt(2)*x*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x +
4) - sqrt(2)*sqrt(x) + 1) + sqrt(2)*x*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - sq
rt(2)*x*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*sqrt(x))/x
```

giac [A] time = 0.63, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.68

$$-\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{\sqrt{2}\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)}{4}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1),x)
```

```
[Out] -1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-1/4*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)
/(x+2^(1/2)*x^(1/2)+1))-1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/x^(1/2)
```

maxima [A] time = 2.99, size = 79, normalized size = 0.80

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

mupad [B] time = 0.04, size = 42, normalized size = 0.42

$$-\frac{2}{\sqrt{x}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^2 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) - 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) - 2/x^(1/2)
```

sympy [A] time = 1.35, size = 97, normalized size = 0.98

$$-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1),x)
```

```
[Out] -sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(2)
*sqrt(x) + 1)/2 - 2/sqrt(x)
```

3.319 $\int \frac{1}{x^{5/2}(1+x^2)}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{3x^{3/2}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2}{3x^{3/2}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)),x]
```

```
[Out] -2/(3*x^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*S
qrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2
]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx &= -\frac{2}{3x^{3/2}} - \int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx \\
&= -\frac{2}{3x^{3/2}} - 2\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2}{3x^{3/2}} - \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2}{3x^{3/2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \\
&= -\frac{2}{3x^{3/2}} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right.}{\sqrt{2}} \\
&= -\frac{2}{3x^{3/2}} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\right.}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.22

$$
-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-x^2\right)}{3x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -x^2])/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 63, normalized size = 0.62

$$
-\frac{2}{3x^{3/2}} - \frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(1 + x^2)),x]
```

```
[Out] -2/(3*x^(3/2)) - ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]/Sqrt[2] - ArcTa
nh[(Sqrt[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

fricas [A] time = 0.69, size = 129, normalized size = 1.28

$$\frac{12\sqrt{2}x^2\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}x^2\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-3\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+3\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\sqrt{x}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2)
*sqrt(x) - 1) + 12*sqrt(2)*x^2*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) +
 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*x^2*log(4*sqrt(2)*sqrt(x) + 4*
x + 4) + 3*sqrt(2)*x^2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*sqrt(x))/x^2
```

giac [A] time = 0.63, size = 79, normalized size = 0.78

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/3/x^(3/2)
```

maple [A] time = 0.01, size = 67, normalized size = 0.66

$$-\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{4}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1),x)
```

```
[Out] -1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-1/4*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)
/(x-2^(1/2)*x^(1/2)+1))-1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/3/x^(3/2)
```

maxima [A] time = 2.93, size = 79, normalized size = 0.78

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/3/x^(3/2)
```

mupad [B] time = 0.04, size = 42, normalized size = 0.42

$$-\frac{2}{3\,x^{3/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^2 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) - 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2/(3*x^(3/2))
```

sympy [A] time = 2.44, size = 99, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2 - 2/(3*x**(3/2))
```

3.320 $\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx$

Optimal. Leaf size=108

$$-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)),x]
```

```
[Out] -2/(5*x^(5/2)) + 2/Sqrt[x] - ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx = -\frac{2}{5x^{5/2}} - \int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} + \int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} + 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} - \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{}\right.$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\right.}{\sqrt{2}}$$
$$= -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{2\sqrt{2}} - \frac{\log(1}{}$$

Mathematica [**C**] time = 0.01, size = 22, normalized size = 0.20

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-x^2\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)),x]
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -x^2])/(5*x^(5/2))
```

IntegrateAlgebraic [**A**] time = 0.09, size = 69, normalized size = 0.64

$$\frac{2\left(5x^2-1\right)}{5x^{5/2}} + \frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(1 + x^2)),x]
```

```
[Out] (2*(-1 + 5*x^2))/(5*x^(5/2)) + ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]/S
qrt[2] - ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

fricas [A] time = 0.56, size = 136, normalized size = 1.26

$$-\frac{20 \sqrt{2} x^3 \arctan\left(\sqrt{2} \sqrt{\sqrt{2} \sqrt{x} + x + 1} - \sqrt{2} \sqrt{x} - 1\right) + 20 \sqrt{2} x^3 \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{-4 \sqrt{2} \sqrt{x} + 4 x + 4} - \sqrt{2} \sqrt{x} + 1\right) + 5 \sqrt{2} x^3 \log\left(4 \sqrt{2} \sqrt{x} + 4 x + 4\right) - 5 \sqrt{2} x^3 \log\left(-4 \sqrt{2} \sqrt{x} + 4 x + 4\right) - 8 \left(5 x^2 - 1\right) \sqrt{x}}{20 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/20*(20*sqrt(2)*x^3*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - sqrt(2
)*sqrt(x) - 1) + 20*sqrt(2)*x^3*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x)
+ 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*x^3*log(4*sqrt(2)*sqrt(x) + 4
*x + 4) - 5*sqrt(2)*x^3*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(5*x^2 - 1)*s
qrt(x))/x^3
```

giac [A] time = 0.63, size = 86, normalized size = 0.80

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + 2 \sqrt{x}\right)\right) + \frac{1}{2} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - 2 \sqrt{x}\right)\right) - \frac{1}{4} \sqrt{2} \log\left(\sqrt{2} \sqrt{x} + x + 1\right) + \frac{1}{4} \sqrt{2} \log\left(-\sqrt{2} \sqrt{x} + x + 1\right) + \frac{2 \left(5 x^2 - 1\right)}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2/5*(5*x^2 - 1)/x^(5/2)
```

maple [A] time = 0.01, size = 72, normalized size = 0.67

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{2} + \frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{2} + \frac{\sqrt{2}\ \ln\left(\frac{x-\sqrt{2}\ \sqrt{x}+1}{x+\sqrt{2}\ \sqrt{x}+1}\right)}{4} + \frac{2}{\sqrt{x}} - \frac{2}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1),x)
```

```
[Out] 1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+1/4*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/
(x+2^(1/2)*x^(1/2)+1))+1/2*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/5/x^(5/2)+2/
x^(1/2)
```

maxima [A] time = 3.05, size = 86, normalized size = 0.80

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + 2 \sqrt{x}\right)\right) + \frac{1}{2} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - 2 \sqrt{x}\right)\right) - \frac{1}{4} \sqrt{2} \log\left(\sqrt{2} \sqrt{x} + x + 1\right) + \frac{1}{4} \sqrt{2} \log\left(-\sqrt{2} \sqrt{x} + x + 1\right) + \frac{2 \left(5 x^2 - 1\right)}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x +
 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2/5*(5*x^2 - 1)/x^(5/2)
```

mupad [B] time = 0.04, size = 48, normalized size = 0.44

$$\frac{2\,x^2-\frac{2}{5}}{x^{5/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^2 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) + (2*x^2 - 2/5)/x^(5/2)
```

sympy [A] time = 5.46, size = 105, normalized size = 0.97

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}+\frac{2}{\sqrt{x}}-\frac{2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1),x)
```

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2)*
sqrt(x) + 1)/2 + 2/sqrt(x) - 2/(5*x**(5/2))
```

3.321 $\int \frac{x^{7/2}}{(1+x^2)^2} \, dx$

Optimal. Leaf size=122

$$-\frac{x^{5/2}}{2(x^2+1)}+\frac{5\sqrt{x}}{2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^{5/2}}{2(x^2+1)}+\frac{5\sqrt{x}}{2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2)^2,x]
```

```
[Out] (5*Sqrt[x])/2 - x^(5/2)/(2*(1 + x^2)) + (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*
Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (5*Log[1 - Sqrt[2]
*Sqrt[x] + x])/(8*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(1+x^2\right)^2}\,dx = -\frac{x^{5/2}}{2\left(1+x^2\right)}+\frac{5}{4}\int \frac{x^{3/2}}{1+x^2}\,dx$$

$$= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$

$$= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{2}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)$$

$$= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{5}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$

$$= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}-\frac{5}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{x}\right)-\frac{5}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x$$

$$= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\right.}{4\sqrt{2}}$$

$$= \frac{5\sqrt{x}}{2}-\frac{x^{5/2}}{2\left(1+x^2\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}$$

Mathematica [A] time = 0.05, size = 121, normalized size = 0.99

$$\frac{1}{16}\left(\frac{40\sqrt{x}}{x^2+1}+\frac{32x^{5/2}}{x^2+1}+5\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)-5\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)-10\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2)^2,x]
```

```
[Out] ((40*Sqrt[x])/(1 + x^2) + (32*x^(5/2))/(1 + x^2) + 10*Sqrt[2]*ArcTan[1 - Sq
rt[2]*Sqrt[x]] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]] + 5*Sqrt[2]*Log[1 -
 Sqrt[2]*Sqrt[x] + x] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/16
```

IntegrateAlgebraic [A] time = 0.16, size = 84, normalized size = 0.69

$$\frac{4x^{5/2}+5\sqrt{x}}{2\left(x^2+1\right)}-\frac{5\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{4\sqrt{2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(1 + x^2)^2,x]
```

```
[Out] (5*Sqrt[x] + 4*x^(5/2))/(2*(1 + x^2)) - (5*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2]
)/Sqrt[x]])/(4*Sqrt[2]) - (5*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(4*Sqrt[2]
)
```

fricas [A] time = 0.56, size = 148, normalized size = 1.21

$$\frac{20\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-5\sqrt{2}\left(x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+5\sqrt{2}\left(x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)+8\left(4x^2+5\right)\sqrt{x}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 5*sqrt(2)*(x^2 + 1)*log(4*sq
rt(2)*sqrt(x) + 4*x + 4) + 5*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x
 + 4) + 8*(4*x^2 + 5)*sqrt(x))/(x^2 + 1)
```

giac [A] time = 0.62, size = 91, normalized size = 0.75

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+2\sqrt{x}+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x) + 1/2*sqrt(x
)/(x^2 + 1)
```

maple [A] time = 0.01, size = 79, normalized size = 0.65

$$-\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{8}-\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{8}-\frac{5\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{16}+2\sqrt{x}+\frac{\sqrt{x}}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1)^2,x)
```

```
[Out] 2*x^(1/2)+1/2*x^(1/2)/(x^2+1)-5/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-5/16*2^
(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-5/8*2^(1/2)*arctan(2^
(1/2)*x^(1/2)+1)
```

maxima [A] time = 3.00, size = 91, normalized size = 0.75

$$-\frac{5}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right)-\frac{5}{8}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)-\frac{5}{16}\,\sqrt{2}\log\left(\sqrt{2}\,\sqrt{x}+x+1\right)+\frac{5}{16}\,\sqrt{2}\log\left(-\sqrt{2}\,\sqrt{x}+x+1\right)+2\,\sqrt{x}+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x) + 1/2*sqrt(x
)/(x^2 + 1)
```

mupad [B] time = 4.62, size = 55, normalized size = 0.45

$$\frac{\sqrt{x}}{2\left(x^2+1\right)}+2\,\sqrt{x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{8}-\frac{5}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{8}+\frac{5}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2 + 1)^2,x)
```

```
[Out] x^(1/2)/(2*(x^2 + 1)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/8 - 5
i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/8 + 5i/8) + 2*x^(1/2)
```

sympy [B] time = 8.34, size = 277, normalized size = 2.27

$$\frac{32x^{\frac{5}{2}}}{16x^2+16}+\frac{40\sqrt{x}}{16x^2+16}+\frac{5\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{5\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{10\sqrt{2}\,x^2\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{16x^2+16}-\frac{10\sqrt{2}\,x^2\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{16x^2+16}+\frac{5\sqrt{2}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{5\sqrt{2}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{10\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{16x^2+16}-\frac{10\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1)**2,x)
```

```
[Out] 32*x**(5/2)/(16*x**2 + 16) + 40*sqrt(x)/(16*x**2 + 16) + 5*sqrt(2)*x**2*log
(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 5*sqrt(2)*x**2*log(4*sqrt(2
)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 10*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x)
- 1)/(16*x**2 + 16) - 10*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 +
16) + 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 5*sqrt(2
)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 10*sqrt(2)*atan(sqrt(2)
*sqrt(x) - 1)/(16*x**2 + 16) - 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**
2 + 16)
```

3.322 $\int \frac{x^{5/2}}{(1+x^2)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2)^2,x]
```

```
[Out] -x^(3/2)/(2*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*A
rcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*Log[1 - Sqrt[2]*Sqrt[x] + x])/
(8*Sqrt[2]) - (3*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\left(1+x^2\right)^2}\,dx &= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}-\frac{3}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{3}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}+\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{3\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{\right.}}{4\sqrt{2}}\\
&= -\frac{x^{3/2}}{2\left(1+x^2\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{3\log\left(\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.27

$$
2x^{3/2}\left({}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-x^2\right)-\frac{1}{x^2+1}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2)^2,x]
```

```
[Out] 2*x^(3/2)*(-(1 + x^2)^(-1) + Hypergeometric2F1[3/4, 2, 7/4, -x^2])
```

IntegrateAlgebraic [A] time = 0.16, size = 74, normalized size = 0.65

$$
-\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{3\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{4\sqrt{2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{4\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(1 + x^2)^2,x]
```

```
[Out] -1/2*x^(3/2)/(1 + x^2) + (3*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]])/(4*
Sqrt[2]) - (3*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(4*Sqrt[2])
```

fricas [A] time = 0.71, size = 141, normalized size = 1.25

$$-\frac{12\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+3\sqrt{2}\left(x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-3\sqrt{2}\left(x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)+8x^{\frac{3}{2}}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt
(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 3*sqrt(2)*(x^2 + 1)*log(4*s
qrt(2)*sqrt(x) + 4*x + 4) - 3*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*
x + 4) + 8*x^(3/2))/(x^2 + 1)
```

giac [A] time = 0.60, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*x^(3/2)/(x^2 + 1)
```

maple [A] time = 0.01, size = 74, normalized size = 0.65

$$-\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{8}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{8}+\frac{3\sqrt{2}\,\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1)^2,x)
```

```
[Out] -1/2*x^(3/2)/(x^2+1)+3/16*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/
2)+1))+3/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)+3/8*2^(1/2)*arctan(2^(1/2)*x^(
1/2)-1)
```

maxima [A] time = 3.00, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*x^(3/2)/(x^2 + 1)
```

mupad [B] time = 4.62, size = 51, normalized size = 0.45

$$-\frac{x^{3/2}}{2\left(x^2+1\right)}+\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{8}-\frac{3}{8}\mathrm{i}\right)+\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{8}+\frac{3}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2 + 1)^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/8 - 3i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(3/8 + 3i/8) - x^(3/2)/(2*(x^2 + 1))
```

sympy [B] time = 5.31, size = 264, normalized size = 2.34

$$-\frac{8x^{\frac{3}{2}}}{16x^2+16}+\frac{3\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{3\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}+\frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1)**2,x)
```

```
[Out] -8*x**(3/2)/(16*x**2 + 16) + 3*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x +
4)/(16*x**2 + 16) - 3*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**
2 + 16) + 6*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*sqrt(
2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) + 3*sqrt(2)*log(-4*sqrt(2)
*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x
+ 4)/(16*x**2 + 16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) +
6*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

3.323 $\int \frac{x^{3/2}}{\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2)^2,x]
```

```
[Out] -Sqrt[x]/(2*(1 + x^2)) - ArcTan[1 - Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + ArcTan[1
 + Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) - Log[1 - Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
+ Log[1 + Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(1+x^2\right)^2}\,dx = -\frac{\sqrt{x}}{2\left(1+x^2\right)} + \frac{1}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$

$$= -\frac{\sqrt{x}}{2\left(1+x^2\right)} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{2\left(1+x^2\right)} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{2\left(1+x^2\right)} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{2\left(1+x^2\right)} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\ldots\right.}{4\sqrt{2}}$$

$$= -\frac{\sqrt{x}}{2\left(1+x^2\right)} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{\log\left(1+\ldots\right.}{8\ldots}$$

Mathematica [A] time = 0.05, size = 106, normalized size = 0.94

$$\frac{1}{16}\left(-\frac{8\sqrt{x}}{x^2+1} - \sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right) + \sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right) - 2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right) + 2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2)^2,x]
```

```
[Out] ((-8*Sqrt[x])/(1 + x^2) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[x]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] + x] + Sqrt[
2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/16
```

IntegrateAlgebraic [A] time = 0.16, size = 74, normalized size = 0.65

$$-\frac{\sqrt{x}}{2\left(x^2+1\right)} + \frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{4\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(1 + x^2)^2,x]
```

```
[Out] -1/2*Sqrt[x]/(1 + x^2) + ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]/(4*Sqrt
[2]) + ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)]/(4*Sqrt[2])
```

fricas [A] time = 0.77, size = 140, normalized size = 1.24

$$-\frac{4\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-\sqrt{2}\left(x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+\sqrt{2}\left(x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)+8\sqrt{x}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 4*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2
)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - sqrt(2)*(x^2 + 1)*log(4*sqrt(
2)*sqrt(x) + 4*x + 4) + sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
 + 8*sqrt(x))/(x^2 + 1)
```

giac [A] time = 0.63, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
```

maple [A] time = 0.01, size = 74, normalized size = 0.65

$$\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)}{8}+\frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{8}+\frac{\sqrt{2}\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{16}-\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1)^2,x)
```

```
[Out] -1/2/(x^2+1)*x^(1/2)+1/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)+1/8*2^(1/2)*arct
an(2^(1/2)*x^(1/2)-1)+1/16*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1
/2)+1))
```

maxima [A] time = 2.96, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
```

mupad [B] time = 4.68, size = 51, normalized size = 0.45

$$-\frac{\sqrt{x}}{2\left(x^2+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2 + 1)^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/8 + 1i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/8 - 1i/8) - x^(1/2)/(2*(x^2 + 1))
```

sympy [B] time = 3.39, size = 257, normalized size = 2.27

$$-\frac{8\sqrt{x}}{16x^2+16}-\frac{\sqrt{2}\,x^2\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{\sqrt{2}\,x^2\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}\,x^2\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}\,x^2\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{16x^2+16}-\frac{\sqrt{2}\log\left(-4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{\sqrt{2}\log\left(4\sqrt{2}\,\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}+1\right)}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1)**2,x)
```

```
[Out] -8*sqrt(x)/(16*x**2 + 16) - sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/
(16*x**2 + 16) + sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 1
6) + 2*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)*x*
*2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) - sqrt(2)*log(-4*sqrt(2)*sqrt(x
) + 4*x + 4)/(16*x**2 + 16) + sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**2 + 16) + 2*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)
*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

3.324 $\int \frac{\sqrt{x}}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=113

$$\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^{3/2}}{2\left(x^2+1\right)}+\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2)^2,x]
```

```
[Out] x^(3/2)/(2*(1 + x^2)) - ArcTan[1 - Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + ArcTan[1
+ Sqrt[2]*Sqrt[x]]/(4*Sqrt[2]) + Log[1 - Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2]) -
 Log[1 + Sqrt[2]*Sqrt[x] + x]/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\left(1+x^2\right)^2} dx &= \frac{x^{3/2}}{2\left(1+x^2\right)} + \frac{1}{4}\int \frac{\sqrt{x}}{1+x^2} dx \\
&= \frac{x^{3/2}}{2\left(1+x^2\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{x}\right) \\
&= \frac{x^{3/2}}{2\left(1+x^2\right)} - \frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{x}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{x}\right) \\
&= \frac{x^{3/2}}{2\left(1+x^2\right)} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{x}\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{x}\right) + \\
&= \frac{x^{3/2}}{2\left(1+x^2\right)} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} \\
&= \frac{x^{3/2}}{2\left(1+x^2\right)} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\right.}{8\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.19

$$\frac{2}{3}x^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2)^2,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -x^2])/3
```

IntegrateAlgebraic [A] time = 0.15, size = 74, normalized size = 0.65

$$\frac{x^{3/2}}{2\left(x^2+1\right)} + \frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{4\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(1 + x^2)^2,x]
```

```
[Out] x^(3/2)/(2*(1 + x^2)) + ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]]/(4*Sqrt[
2]) - ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)]/(4*Sqrt[2])
```

fricas [A] time = 0.62, size = 140, normalized size = 1.24

$$-\frac{4\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+4\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\sqrt{2}\left(x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-\sqrt{2}\left(x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8x^{\frac{3}{2}}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 4*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2
)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + sqrt(2)*(x^2 + 1)*log(4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
 - 8*x^(3/2))/(x^2 + 1)
```

giac [A] time = 0.63, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1)
```

maple [A] time = 0.01, size = 74, normalized size = 0.65

$$\frac{x^{\frac{3}{2}}}{2x^2+2}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}-1\right)}{8}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}+1\right)}{8}+\frac{\sqrt{2}\ \ln\left(\frac{x-\sqrt{2}\ \sqrt{x}+1}{x+\sqrt{2}\ \sqrt{x}+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)^2,x)
```

```
[Out] 1/2/(x^2+1)*x^(3/2)+1/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+1/16*2^(1/2)*ln((
x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+1/8*2^(1/2)*arctan(2^(1/2)*x^(1
/2)+1)
```

maxima [A] time = 2.94, size = 86, normalized size = 0.76

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1)
```

mupad [B] time = 0.04, size = 50, normalized size = 0.44

$$\frac{x^{3/2}}{2\left(x^2+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2 + 1)^2,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/8 - 1i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(1/8 + 1i/8) + x^(3/2)/(2*(x^2 + 1))
```

sympy [B] time = 2.17, size = 257, normalized size = 2.27

$$\frac{8x^{\frac{3}{2}}}{16x^2+16}+\frac{\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}+\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)**2,x)
```

```
[Out] 8*x**(3/2)/(16*x**2 + 16) + sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/
(16*x**2 + 16) - sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 1
6) + 2*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)*x*
*2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) + sqrt(2)*log(-4*sqrt(2)*sqrt(x
) + 4*x + 4)/(16*x**2 + 16) - sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**2 + 16) + 2*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)
*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

3.325 $\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (3*Ar
cTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (3*Log[1 - Sqrt[2]*Sqrt[x] + x])/(
8*Sqrt[2]) + (3*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx = \frac{\sqrt{x}}{2\left(1+x^2\right)} + \frac{3}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$
$$= \frac{\sqrt{x}}{2\left(1+x^2\right)} + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\sqrt{x}}{2\left(1+x^2\right)} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{3}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\sqrt{x}}{2\left(1+x^2\right)} + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\right.$$
$$= \frac{\sqrt{x}}{2\left(1+x^2\right)} - \frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{4\sqrt{2}}$$
$$= \frac{\sqrt{x}}{2\left(1+x^2\right)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}} + \frac{3\mathrm{l}}{}$$

Mathematica [A] time = 0.04, size = 107, normalized size = 0.95

$$\frac{1}{16}\left(\frac{8\sqrt{x}}{x^2+1} - 3\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right) + 3\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right) - 6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right) + 6\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)^2),x]
```

```
[Out] ((8*Sqrt[x])/(1 + x^2) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 6*Sqrt[2]*
ArcTan[1 + Sqrt[2]*Sqrt[x]] - 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] + x] + 3*Sq
rt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/16
```

IntegrateAlgebraic [A] time = 0.14, size = 74, normalized size = 0.65

$$\frac{\sqrt{x}}{2\left(x^2+1\right)} + \frac{3\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{4\sqrt{2}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(1 + x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*(1 + x^2)) + (3*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]])/(4*S
qrt[2]) + (3*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(4*Sqrt[2])
```

fricas [A] time = 0.55, size = 141, normalized size = 1.25

$$-\frac{12\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-3\sqrt{2}\left(x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+3\sqrt{2}\left(x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\sqrt{x}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt
(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*(x^2 + 1)*log(4*s
qrt(2)*sqrt(x) + 4*x + 4) + 3*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*
x + 4) - 8*sqrt(x))/(x^2 + 1)
```

giac [A] time = 0.60, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*sqrt(x)/(x^2 + 1)
```

maple [A] time = 0.01, size = 74, normalized size = 0.65

$$\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{8}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{8}+\frac{3\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{16}+\frac{\sqrt{x}}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/x^(1/2),x)
```

```
[Out] 1/2/(x^2+1)*x^(1/2)+3/16*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2
)+1))+3/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)+3/8*2^(1/2)*arctan(2^(1/2)*x^(1
/2)-1)
```

maxima [A] time = 2.84, size = 86, normalized size = 0.76

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) - 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*sqrt(x)/(x^2 + 1)
```

mupad [B] time = 4.66, size = 50, normalized size = 0.44

$$\frac{\sqrt{x}}{2\left(x^2+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{8}+\frac{3}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{8}-\frac{3}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^2 + 1)^2),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/8 + 3i/8) + 2^(1/2)*atan(2^(1
/2)*x^(1/2)*(1/2 + 1i/2))*(3/8 - 3i/8) + x^(1/2)/(2*(x^2 + 1))
```

sympy [B] time = 2.99, size = 264, normalized size = 2.34

$$\frac{8\sqrt{x}}{16x^2+16}-\frac{3\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{3\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}-\frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{6\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/x**(1/2),x)
```

```
[Out] 8*sqrt(x)/(16*x**2 + 16) - 3*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
/(16*x**2 + 16) + 3*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2
+ 16) + 6*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*sqrt(2)
*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) - 3*sqrt(2)*log(-4*sqrt(2)*s
qrt(x) + 4*x + 4)/(16*x**2 + 16) + 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x +
4)/(16*x**2 + 16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*
sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

3.326 $\int \frac{1}{x^{3/2}(1+x^2)^2}\,dx$

Optimal. Leaf size=122

$$\frac{1}{2\sqrt{x}\left(x^2+1\right)}-\frac{5}{2\sqrt{x}}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+\right.}{4\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{2\sqrt{x}\left(x^2+1\right)}-\frac{5}{2\sqrt{x}}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)^2),x]
```

```
[Out] -5/(2*Sqrt[x]) + 1/(2*Sqrt[x]*(1 + x^2)) + (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/
(4*Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (5*Log[1 - Sqrt
[2]*Sqrt[x] + x])/(8*Sqrt[2]) + (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx = \frac{1}{2\sqrt{x}\left(1+x^2\right)}+\frac{5}{4}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx$$

$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx$$

$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}+\frac{5}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{5}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{5}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+ \cdots$$

$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}-\frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{5\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}\left(1+x^2\right)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}\sqrt{x} \cdots\right.}{8\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 20, normalized size = 0.16

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-x^2\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 2, 3/4, -x^2])/Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.20, size = 81, normalized size = 0.66

$$\frac{-5x^2-4}{2\sqrt{x}\left(x^2+1\right)}-\frac{5\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{4\sqrt{2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(1 + x^2)^2),x]
```

```
[Out] (-4 - 5*x^2)/(2*Sqrt[x]*(1 + x^2)) - (5*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/S
qrt[x]])/(4*Sqrt[2]) + (5*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(4*Sqrt[2])
```

fricas [A] time = 0.74, size = 148, normalized size = 1.21

$$\frac{20\sqrt{2}\left(x^{3}+x\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+20\sqrt{2}\left(x^{3}+x\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+5\sqrt{2}\left(x^{3}+x\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-5\sqrt{2}\left(x^{3}+x\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\left(5x^{2}+4\right)\sqrt{x}}{16\left(x^{3}+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*sqrt(2)*(x^3 + x)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) - s
qrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^3 + x)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(
2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*(x^3 + x)*log(4*sq
rt(2)*sqrt(x) + 4*x + 4) - 5*sqrt(2)*(x^3 + x)*log(-4*sqrt(2)*sqrt(x) + 4*x
 + 4) - 8*(5*x^2 + 4)*sqrt(x))/(x^3 + x)
```

giac [A] time = 0.64, size = 92, normalized size = 0.75

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{5x^{2}+4}{2\left(x^{\frac{5}{2}}+\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) - 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*(5*x^2 + 4)/(x^(5/
2) + sqrt(x))
```

maple [A] time = 0.01, size = 79, normalized size = 0.65

$$-\frac{x^{\frac{3}{2}}}{2\left(x^{2}+1\right)}-\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{8}-\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{8}-\frac{5\sqrt{2}\,\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)}{16}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1)^2,x)
```

```
[Out] -1/2/(x^2+1)*x^(3/2)-5/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-5/8*2^(1/2)*arct
an(2^(1/2)*x^(1/2)-1)-5/16*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1
/2)+1))-2/x^(1/2)
```

maxima [A] time = 3.04, size = 92, normalized size = 0.75

$$-\frac{5}{8}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right)-\frac{5}{8}\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)+\frac{5}{16}\,\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt{x}+x+1\right)-\frac{5}{16}\,\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt{x}+x+1\right)-\frac{5\,x^{2}+4}{2\left(x^{\frac{5}{2}}+\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) - 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*(5*x^2 + 4)/(x^(5/
2) + sqrt(x))
```

mupad [B] time = 4.68, size = 55, normalized size = 0.45

$$-\frac{\frac{5\,x^2}{2}+2}{\sqrt{x}+x^{5/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{8}+\frac{5}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{8}-\frac{5}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^2 + 1)^2),x)
```

```
[Out] - ((5*x^2)/2 + 2)/(x^(1/2) + x^(5/2)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 -
 1i/2))*(5/8 - 5i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/8 + 5i
/8)
```

sympy [B] time = 4.56, size = 366, normalized size = 3.00

$$-\frac{5\sqrt{2}x^{\frac{5}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}+\frac{5\sqrt{2}x^{\frac{5}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}x^{\frac{5}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}x^{\frac{5}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{5\sqrt{2}\sqrt{x}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}+\frac{5\sqrt{2}\sqrt{x}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}\sqrt{x}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{10\sqrt{2}\sqrt{x}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{40x^2}{16x^{\frac{5}{2}}+16\sqrt{x}}-\frac{32}{16x^{\frac{5}{2}}+16\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1)**2,x)
```

```
[Out] -5*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5/2) + 16*sqr
t(x)) + 5*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5/2) +
16*sqrt(x)) - 10*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**(5/2) +
16*sqrt(x)) - 10*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**(5/2) +
16*sqrt(x)) - 5*sqrt(2)*sqrt(x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5
/2) + 16*sqrt(x)) + 5*sqrt(2)*sqrt(x)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**(5/2) + 16*sqrt(x)) - 10*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) - 1)/(16*x
**(5/2) + 16*sqrt(x)) - 10*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) + 1)/(16*x*
*(5/2) + 16*sqrt(x)) - 40*x**2/(16*x**(5/2) + 16*sqrt(x)) - 32/(16*x**(5/2)
 + 16*sqrt(x))
```

3.327 $\int \frac{1}{x^{5/2}\left(1+x^2\right)^2} dx$

Optimal. Leaf size=122

$$-\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(x^2+1\right)}+\frac{7\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{7\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}\sqrt{x}\right.}{4\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{2x^{3/2}\left(x^2+1\right)}-\frac{7}{6x^{3/2}}+\frac{7\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{7\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}+\frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{7\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)^2),x]
```

```
[Out] -7/(6*x^(3/2)) + 1/(2*x^(3/2)*(1 + x^2)) + (7*ArcTan[1 - Sqrt[2]*Sqrt[x]])/
(4*Sqrt[2]) - (7*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (7*Log[1 - Sqrt
[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (7*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^2}\,dx = \frac{1}{2x^{3/2}\left(1+x^2\right)}+\frac{7}{4}\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}-\frac{7}{4}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$
$$= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}-\frac{7}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}-\frac{7}{4}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{7}{4}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}-\frac{7}{8}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{7}{8}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x
$$
$$= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}+\frac{7\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{7\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{7\operatorname{Subst}\left(\int\right.}{}$$
$$= -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}\left(1+x^2\right)}+\frac{7\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{7\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{7\log\left(1-\sqrt{2}\right.}{8\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.18

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},2;\frac{1}{4};-x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)^2),x]
[Out] (-2*Hypergeometric2F1[-3/4, 2, 1/4, -x^2])/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 81, normalized size = 0.66

$$\frac{-7x^2-4}{6x^{3/2}\left(x^2+1\right)}-\frac{7\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{4\sqrt{2}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(1 + x^2)^2),x]
```

```
[Out] (-4 - 7*x^2)/(6*x^(3/2)*(1 + x^2)) - (7*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2])/S
qrt[x]])/(4*Sqrt[2]) - (7*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(4*Sqrt[2])
```

fricas [A] time = 0.63, size = 158, normalized size = 1.30

$$\frac{84\sqrt{2}\left(x^4+x^2\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+84\sqrt{2}\left(x^4+x^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-21\sqrt{2}\left(x^4+x^2\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+21\sqrt{2}\left(x^4+x^2\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\left(7x^2+4\right)\sqrt{x}}{48\left(x^4+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(84*sqrt(2)*(x^4 + x^2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1) -
 sqrt(2)*sqrt(x) - 1) + 84*sqrt(2)*(x^4 + x^2)*arctan(1/2*sqrt(2)*sqrt(-4*s
qrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 21*sqrt(2)*(x^4 + x^2)*l
og(4*sqrt(2)*sqrt(x) + 4*x + 4) + 21*sqrt(2)*(x^4 + x^2)*log(-4*sqrt(2)*sqr
t(x) + 4*x + 4) - 8*(7*x^2 + 4)*sqrt(x))/(x^4 + x^2)
```

giac [A] time = 0.63, size = 91, normalized size = 0.75

$$-\frac{7}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{7}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{7}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{7}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 7/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 7/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 7/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1)
- 2/3/x^(3/2)
```

maple [A] time = 0.01, size = 79, normalized size = 0.65

$$-\frac{7\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{8}-\frac{7\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{8}-\frac{7\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{16}-\frac{\sqrt{x}}{2\left(x^2+1\right)}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1)^2,x)
```

```
[Out] -1/2/(x^2+1)*x^(1/2)-7/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)-7/16*2^(1/2)*ln(
(x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-7/8*2^(1/2)*arctan(2^(1/2)*x^(
1/2)+1)-2/3/x^(3/2)
```

maxima [A] time = 2.94, size = 92, normalized size = 0.75

$$-\frac{7}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right)-\frac{7}{8}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)-\frac{7}{16}\,\sqrt{2}\log\left(\sqrt{2}\,\sqrt{x}+x+1\right)+\frac{7}{16}\,\sqrt{2}\log\left(-\sqrt{2}\,\sqrt{x}+x+1\right)-\frac{7\,x^2+4}{6\left(x^{\frac{7}{2}}+x^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -7/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 7/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 7/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 7/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/6*(7*x^2 + 4)/(x^(7/
2) + x^(3/2))
```

mupad [B] time = 0.08, size = 55, normalized size = 0.45

$$-\frac{\frac{7\,x^2}{6}+\frac{2}{3}}{x^{3/2}+x^{7/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{7}{8}-\frac{7}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{7}{8}+\frac{7}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^2 + 1)^2),x)
```

```
[Out] - ((7*x^2)/6 + 2/3)/(x^(3/2) + x^(7/2)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2
 - 1i/2))*(7/8 + 7i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(7/8 -
7i/8)
```

sympy [B] time = 8.33, size = 366, normalized size = 3.00

$$\frac{21\sqrt{2}x^{\frac{7}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{21\sqrt{2}x^{\frac{7}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}x^{\frac{7}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}x^{\frac{7}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}+\frac{21\sqrt{2}x^{\frac{3}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{21\sqrt{2}x^{\frac{3}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}x^{\frac{3}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{42\sqrt{2}x^{\frac{3}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{56x^2}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}-\frac{32}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1)**2,x)
```

```
[Out] 21*sqrt(2)*x**(7/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(48*x**(7/2) + 48*x**
(3/2)) - 21*sqrt(2)*x**(7/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(48*x**(7/2)
+ 48*x**(3/2)) - 42*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) - 1)/(48*x**(7/2)
 + 48*x**(3/2)) - 42*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) + 1)/(48*x**(7/2
) + 48*x**(3/2)) + 21*sqrt(2)*x**(3/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(4
8*x**(7/2) + 48*x**(3/2)) - 21*sqrt(2)*x**(3/2)*log(4*sqrt(2)*sqrt(x) + 4*x
 + 4)/(48*x**(7/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(x
) - 1)/(48*x**(7/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(
x) + 1)/(48*x**(7/2) + 48*x**(3/2)) - 56*x**2/(48*x**(7/2) + 48*x**(3/2)) -
 32/(48*x**(7/2) + 48*x**(3/2))
```

3.328 $\int \frac{1}{x^{7/2}\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=131

$$-\frac{9}{10x^{5/2}}+\frac{1}{2x^{5/2}\left(x^2+1\right)}+\frac{9}{2\sqrt{x}}+\frac{9\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\tan^{-1}\left(\right.}{4}$$

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{2x^{5/2}\left(x^2+1\right)}-\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{9\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{8\sqrt{2}}-\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)^2),x]
```

```
[Out] -9/(10*x^(5/2)) + 9/(2*Sqrt[x]) + 1/(2*x^(5/2)*(1 + x^2)) - (9*ArcTan[1 - S
qrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2])
+ (9*Log[1 - Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (9*Log[1 + Sqrt[2]*Sqrt[x]
 + x])/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx = \frac{1}{2x^{5/2}\left(1+x^2\right)}+\frac{9}{4}\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{9}{10x^{5/2}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}-\frac{9}{4}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx$$
$$= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}+\frac{9}{4}\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}+\frac{9}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}-\frac{9}{4}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{9}{4}\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx\right.$$
$$= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}+\frac{9}{8}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{9}{8}\,\mathrm{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}+\frac{9\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}-\frac{9\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{8\sqrt{2}}+\frac{9\,\mathrm{S}}{}$$
$$= -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}\left(1+x^2\right)}-\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\log\left(\right.}{}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.17

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},2;-\frac{1}{4};-x^2\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)^2),x]
[Out] (-2*Hypergeometric2F1[-5/4, 2, -1/4, -x^2])/(5*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.18, size = 86, normalized size = 0.66

$$\frac{45x^4+36x^2-4}{10x^{5/2}\left(x^2+1\right)}+\frac{9\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{4\sqrt{2}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(1 + x^2)^2),x]
```

```
[Out] (-4 + 36*x^2 + 45*x^4)/(10*x^(5/2)*(1 + x^2)) + (9*ArcTan[(-(1/Sqrt[2]) + x
/Sqrt[2])/Sqrt[x]])/(4*Sqrt[2]) - (9*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(4
*Sqrt[2])
```

fricas [A] time = 0.70, size = 163, normalized size = 1.24

$$-\frac{180\sqrt{2}\left(x^{5}+x^{3}\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+180\sqrt{2}\left(x^{5}+x^{3}\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+45\sqrt{2}\left(x^{5}+x^{3}\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-45\sqrt{2}\left(x^{5}+x^{3}\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\left(45x^{4}+36x^{2}-4\right)\sqrt{x}}{80\left(x^{5}+x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/80*(180*sqrt(2)*(x^5 + x^3)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) + x + 1)
 - sqrt(2)*sqrt(x) - 1) + 180*sqrt(2)*(x^5 + x^3)*arctan(1/2*sqrt(2)*sqrt(-
4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 45*sqrt(2)*(x^5 + x^3
)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 45*sqrt(2)*(x^5 + x^3)*log(-4*sqrt(2)*
sqrt(x) + 4*x + 4) - 8*(45*x^4 + 36*x^2 - 4)*sqrt(x))/(x^5 + x^3)
```

giac [A] time = 0.64, size = 98, normalized size = 0.75

$$\frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{9}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{x^{\frac{3}{2}}}{2\left(x^{2}+1\right)}+\frac{2\left(10x^{2}-1\right)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 9/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 9/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 9/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 9/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1) +
 2/5*(10*x^2 - 1)/x^(5/2)
```

maple [A] time = 0.02, size = 84, normalized size = 0.64

$$\frac{x^{\frac{3}{2}}}{2x^{2}+2}+\frac{9\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{8}+\frac{9\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{8}+\frac{9\sqrt{2}\,\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)}{16}+\frac{4}{\sqrt{x}}-\frac{2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1)^2,x)
```

```
[Out] 1/2/(x^2+1)*x^(3/2)+9/8*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+9/16*2^(1/2)*ln((
x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+9/8*2^(1/2)*arctan(2^(1/2)*x^(1
/2)+1)-2/5/x^(5/2)+4/x^(1/2)
```

maxima [A] time = 3.04, size = 97, normalized size = 0.74

$$\frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{9}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{45x^4+36x^2-4}{10\left(x^{\frac{9}{2}}+x^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 9/8*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 9/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x
+ 1) + 9/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/10*(45*x^4 + 36*x^2 -
 4)/(x^(9/2) + x^(5/2))
```

mupad [B] time = 0.07, size = 59, normalized size = 0.45

$$\frac{\frac{9x^4}{2}+\frac{18x^2}{5}-\frac{2}{5}}{x^{5/2}+x^{9/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{9}{8}-\frac{9}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{9}{8}+\frac{9}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^2 + 1)^2),x)
```

```
[Out] ((18*x^2)/5 + (9*x^4)/2 - 2/5)/(x^(5/2) + x^(9/2)) + 2^(1/2)*atan(2^(1/2)*x
^(1/2)*(1/2 - 1i/2))*(9/8 - 9i/8) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/
2))*(9/8 + 9i/8)
```

sympy [B] time = 18.73, size = 384, normalized size = 2.93

$$\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}-\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}x^{\frac{9}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}x^{\frac{9}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{45\sqrt{2}x^{\frac{5}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}-\frac{45\sqrt{2}x^{\frac{5}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}x^{\frac{5}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{90\sqrt{2}x^{\frac{5}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{360x^4}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}+\frac{288x^2}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}-\frac{32}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1)**2,x)
```

```
[Out] 45*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(80*x**(9/2) + 80*x**
(5/2)) - 45*sqrt(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(80*x**(9/2)
+ 80*x**(5/2)) + 90*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) - 1)/(80*x**(9/2)
 + 80*x**(5/2)) + 90*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) + 1)/(80*x**(9/2
) + 80*x**(5/2)) + 45*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(8
0*x**(9/2) + 80*x**(5/2)) - 45*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x
 + 4)/(80*x**(9/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x
) - 1)/(80*x**(9/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(
x) + 1)/(80*x**(9/2) + 80*x**(5/2)) + 360*x**4/(80*x**(9/2) + 80*x**(5/2))
+ 288*x**2/(80*x**(9/2) + 80*x**(5/2)) - 32/(80*x**(9/2) + 80*x**(5/2))
```

3.329 $\int \frac{x^{7/2}}{(1+x^2)^3} \, dx$

Optimal. Leaf size=129

$$-\frac{5\sqrt{x}}{16(x^2+1)}-\frac{x^{5/2}}{4(x^2+1)^2}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^{5/2}}{4(x^2+1)^2}-\frac{5\sqrt{x}}{16(x^2+1)}-\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(1 + x^2)^3,x]
```

```
[Out] -x^(5/2)/(4*(1 + x^2)^2) - (5*Sqrt[x])/(16*(1 + x^2)) - (5*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) -
(5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (5*Log[1 + Sqrt[2]*Sqrt[x]
+ x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{\left(1+x^2\right)^3}\,dx &= -\frac{x^{5/2}}{4\left(1+x^2\right)^2} + \frac{5}{8}\int \frac{x^{3/2}}{\left(1+x^2\right)^2}\,dx \\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2} - \frac{5\sqrt{x}}{16\left(1+x^2\right)} + \frac{5}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx \\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2} - \frac{5\sqrt{x}}{16\left(1+x^2\right)} + \frac{5}{16}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2} - \frac{5\sqrt{x}}{16\left(1+x^2\right)} + \frac{5}{32}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{5}{32}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\right. \\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2} - \frac{5\sqrt{x}}{16\left(1+x^2\right)} + \frac{5}{64}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{5}{64}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\right. \\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2} - \frac{5\sqrt{x}}{16\left(1+x^2\right)} - \frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{5\,\text{Subst}\left(\int\right.}{} \\
&= -\frac{x^{5/2}}{4\left(1+x^2\right)^2} - \frac{5\sqrt{x}}{16\left(1+x^2\right)} - \frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} - \frac{5\log\left(1-\sqrt{2}\right.}{64\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 135, normalized size = 1.05

$$\frac{1}{384}\left(\frac{40\sqrt{x}}{x^2+1} - \frac{160\sqrt{x}}{\left(x^2+1\right)^2} - \frac{256x^{5/2}}{\left(x^2+1\right)^2} - 15\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right) + 15\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right) - 30\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right) + 30\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(1 + x^2)^3,x]
```

```
[Out] ((-160*Sqrt[x])/(1 + x^2)^2 - (256*x^(5/2))/(1 + x^2)^2 + (40*Sqrt[x])/(1 +
 x^2) - 30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 30*Sqrt[2]*ArcTan[1 + Sqrt
[2]*Sqrt[x]] - 15*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] + x] + 15*Sqrt[2]*Log[1 +
 Sqrt[2]*Sqrt[x] + x])/384
```

IntegrateAlgebraic [A] time = 0.25, size = 81, normalized size = 0.63

$$-\frac{\sqrt{x}\left(9x^2+5\right)}{16\left(x^2+1\right)^2} + \frac{5\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{32\sqrt{2}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(1 + x^2)^3,x]
```

```
[Out] -1/16*(Sqrt[x]*(5 + 9*x^2))/(1 + x^2)^2 + (5*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[
2])/Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(32*Sqr
t[2])
```

fricas [A] time = 0.74, size = 173, normalized size = 1.34

$$-\frac{20 \sqrt{2} \left(x^{4}+2 x^{2}+1\right) \arctan\left(\sqrt{2} \sqrt{\sqrt{2} \sqrt{x}+x+1}-\sqrt{2} \sqrt{x}-1\right)+20 \sqrt{2} \left(x^{4}+2 x^{2}+1\right) \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{-4 \sqrt{2} \sqrt{x}+4 x+4}-\sqrt{2} \sqrt{x}+1\right)-5 \sqrt{2} \left(x^{4}+2 x^{2}+1\right) \log\left(4 \sqrt{2} \sqrt{x}+4 x+4\right)+5 \sqrt{2} \left(x^{4}+2 x^{2}+1\right) \log\left(-4 \sqrt{2} \sqrt{x}+4 x+4\right)+8 \left(9 x^{2}+5\right) \sqrt{x}}{128 \left(x^{4}+2 x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 5*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 5*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) + 8*(9*x^2 + 5)*sqrt(x))/(x^4 + 2*x
^2 + 1)
```

giac [A] time = 0.69, size = 94, normalized size = 0.73

$$\frac{5}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(\sqrt{2}+2 \sqrt{x}\right)\right)+\frac{5}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2}-2 \sqrt{x}\right)\right)+\frac{5}{128} \sqrt{2} \log\left(\sqrt{2} \sqrt{x}+x+1\right)-\frac{5}{128} \sqrt{2} \log\left(-\sqrt{2} \sqrt{x}+x+1\right)-\frac{9 x^{\frac{5}{2}}+5 \sqrt{x}}{16 \left(x^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(9*x^(5/2) + 5
*sqrt(x))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 82, normalized size = 0.64

$$\frac{5 \sqrt{2}\, \arctan\left(\sqrt{2}\, \sqrt{x}-1\right)}{64}+\frac{5 \sqrt{2}\, \arctan\left(\sqrt{2}\, \sqrt{x}+1\right)}{64}+\frac{5 \sqrt{2}\, \ln\left(\frac{x+\sqrt{2}\, \sqrt{x}+1}{x-\sqrt{2}\, \sqrt{x}+1}\right)}{128}+\frac{-\frac{9 x^{\frac{5}{2}}}{16}-\frac{5 \sqrt{x}}{16}}{\left(x^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2+1)^3,x)
```

```
[Out] 2*(-9/32*x^(5/2)-5/32*x^(1/2))/(x^2+1)^2+5/64*2^(1/2)*arctan(2^(1/2)*x^(1/2
)-1)+5/128*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+5/64*2^(
1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 3.01, size = 99, normalized size = 0.77

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{9x^{\frac{5}{2}}+5\sqrt{x}}{16\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(9*x^(5/2) + 5
*sqrt(x))/(x^4 + 2*x^2 + 1)
```

mupad [B] time = 4.73, size = 62, normalized size = 0.48

$$-\frac{\frac{5\sqrt{x}}{16}+\frac{9x^{5/2}}{16}}{x^4+2x^2+1}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{64}+\frac{5}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{64}-\frac{5}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/64 + 5i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/64 - 5i/64) - ((5*x^(1/2))/16 + (9*x^(5/2))/
16)/(2*x^2 + x^4 + 1)
```

sympy [B] time = 23.35, size = 481, normalized size = 3.73

$$-\frac{72x^{\frac{5}{2}}}{128x^4+256x^2+128}-\frac{40\sqrt{x}}{128x^4+256x^2+128}-\frac{5\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{5\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}x^4\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}x^4\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}-\frac{10\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{20\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{20\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}-\frac{5\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{5\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(x**2+1)**3,x)
```

```
[Out] -72*x**(5/2)/(128*x**4 + 256*x**2 + 128) - 40*sqrt(x)/(128*x**4 + 256*x**2
+ 128) - 5*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 5*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x
**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2
 + 128) - 10*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 10*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 25
```

```
6*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x
**2 + 128) - 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) + 5*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128)
+ 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

3.330 $\int \frac{x^{5/2}}{\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=129

$$\frac{3x^{3/2}}{16\left(x^2+1\right)}-\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\right.}{32\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {288, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3x^{3/2}}{16\left(x^2+1\right)}-\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(1 + x^2)^3,x]
```

```
[Out] -x^(3/2)/(4*(1 + x^2)^2) + (3*x^(3/2))/(16*(1 + x^2)) - (3*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) +
(3*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (3*Log[1 + Sqrt[2]*Sqrt[x]
+ x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(1+x^2\right)^3}\,dx = -\frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{3}{8}\int \frac{\sqrt{x}}{\left(1+x^2\right)^2}\,dx$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{3x^{3/2}}{16\left(1+x^2\right)}+\frac{3}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{3x^{3/2}}{16\left(1+x^2\right)}+\frac{3}{16}\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{3x^{3/2}}{16\left(1+x^2\right)}-\frac{3}{32}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\frac{3}{32}\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\right.$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{3x^{3/2}}{16\left(1+x^2\right)}+\frac{3}{64}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)+\frac{3}{64}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}\right.$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{3x^{3/2}}{16\left(1+x^2\right)}+\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{x^{3/2}}{4\left(1+x^2\right)^2}+\frac{3x^{3/2}}{16\left(1+x^2\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\log\left(1-\sqrt{2}\right.}{64\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.25

$$\frac{2}{5}x^{3/2}\left({}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-x^2\right)-\frac{1}{\left(x^2+1\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(1 + x^2)^3,x]
[Out] (2*x^(3/2)*(-(1 + x^2)^(-2) + Hypergeometric2F1[3/4, 3, 7/4, -x^2]))/5
```

IntegrateAlgebraic [A] time = 0.21, size = 81, normalized size = 0.63

$$\frac{\left(3x^2-1\right)x^{3/2}}{16\left(x^2+1\right)^2}+\frac{3\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{32\sqrt{2}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(1 + x^2)^3,x]
```

```
[Out] (x^(3/2)*(-1 + 3*x^2))/(16*(1 + x^2)^2) + (3*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[
2])/Sqrt[x]])/(32*Sqrt[2]) - (3*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(32*Sqr
t[2])
```

fricas [A] time = 0.93, size = 175, normalized size = 1.36

$$-\frac{12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+3\sqrt{2}\left(x^4+2x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-3\sqrt{2}\left(x^4+2x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\left(3x^3-x\right)\sqrt{x}}{128\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 3*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 3*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(3*x^3 - x)*sqrt(x))/(x^4 + 2*x
^2 + 1)
```

giac [A] time = 0.65, size = 94, normalized size = 0.73

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{3x^{\frac{7}{2}}-x^{\frac{3}{2}}}{16\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(3*x^(7/2) - x
^(3/2))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 82, normalized size = 0.64

$$\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}\,-1\right)}{64}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}\,+1\right)}{64}+\frac{3\sqrt{2}\,\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)}{128}+\frac{\frac{3x^{\frac{7}{2}}}{16}-\frac{x^{\frac{3}{2}}}{16}}{\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2+1)^3,x)
```

```
[Out] 2*(3/32*x^(7/2)-1/32*x^(3/2))/(x^2+1)^2+3/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)
-1)+3/128*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+3/64*2^(1
/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 2.91, size = 99, normalized size = 0.77

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{3}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{3x^{\frac{7}{2}}-x^{\frac{3}{2}}}{16\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(3*x^(7/2) - x
^(3/2))/(x^4 + 2*x^2 + 1)
```

mupad [B] time = 0.07, size = 62, normalized size = 0.48

$$-\frac{\frac{x^{3/2}}{16}-\frac{3x^{7/2}}{16}}{x^4+2x^2+1}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}+\frac{3}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/64 - 3i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(3/64 + 3i/64) - (x^(3/2)/16 - (3*x^(7/2))/16)/
(2*x^2 + x^4 + 1)
```

sympy [B] time = 15.49, size = 481, normalized size = 3.73

$$\frac{24x^{\frac{7}{2}}}{128x^4+256x^2+128}-\frac{8x^{\frac{3}{2}}}{128x^4+256x^2+128}+\frac{3\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{3\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}x^4\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}x^4\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{6\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{12\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{12\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}+\frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(x**2+1)**3,x)
```

```
[Out] 24*x**(7/2)/(128*x**4 + 256*x**2 + 128) - 8*x**(3/2)/(128*x**4 + 256*x**2 +
 128) + 3*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) - 3*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**
2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
128) + 6*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**
2 + 128) - 6*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**
```

```
2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
 128) + 3*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 12
8) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128) + 6*sq
rt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

3.331 $\int \frac{x^{3/2}}{\left(1+x^2\right)^3} \, dx$

Optimal. Leaf size=129

$$\frac{\sqrt{x}}{16\left(x^2+1\right)}-\frac{\sqrt{x}}{4\left(x^2+1\right)^2}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{x}}{16\left(x^2+1\right)}-\frac{\sqrt{x}}{4\left(x^2+1\right)^2}-\frac{3\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{3\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(1 + x^2)^3,x]
```

```
[Out] -Sqrt[x]/(4*(1 + x^2)^2) + Sqrt[x]/(16*(1 + x^2)) - (3*ArcTan[1 - Sqrt[2]*S
qrt[x]])/(32*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (3*L
og[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (3*Log[1 + Sqrt[2]*Sqrt[x] + x]
)/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(1+x^2\right)^3}\,dx = -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{1}{8}\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{16}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right)+\frac{3}{32}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\right.$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}+\frac{3}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right)+\frac{3}{64}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\right.$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}-\frac{3\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{x}}{4\left(1+x^2\right)^2}+\frac{\sqrt{x}}{16\left(1+x^2\right)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}\right.}{64\sqrt{2}}$$

Mathematica [A] time = 0.03, size = 121, normalized size = 0.94

$$\frac{1}{128}\left(\frac{8\sqrt{x}}{x^2+1}-\frac{32\sqrt{x}}{\left(x^2+1\right)^2}-3\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right)+3\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right)-6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)+6\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(1 + x^2)^3,x]
```

```
[Out] ((-32*Sqrt[x])/(1 + x^2)^2 + (8*Sqrt[x])/(1 + x^2) - 6*Sqrt[2]*ArcTan[1 - S
qrt[2]*Sqrt[x]] + 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]] - 3*Sqrt[2]*Log[1 -
 Sqrt[2]*Sqrt[x] + x] + 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/128
```

IntegrateAlgebraic [A] time = 0.21, size = 79, normalized size = 0.61

$$\frac{\sqrt{x}\left(x^2-3\right)}{16\left(x^2+1\right)^2}+\frac{3\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{32\sqrt{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(1 + x^2)^3,x]
```

```
[Out] (Sqrt[x]*(-3 + x^2))/(16*(1 + x^2)^2) + (3*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2]
)/Sqrt[x]])/(32*Sqrt[2]) + (3*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(32*Sqrt[
2])
```

fricas [A] time = 0.54, size = 171, normalized size = 1.33

$$-\frac{12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+12\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-3\sqrt{2}\left(x^4+2x^2+1\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+3\sqrt{2}\left(x^4+2x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\left(x^2-3\right)\sqrt{x}}{128\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 12*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 3*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 3*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(x^2 - 3)*sqrt(x))/(x^4 + 2*x^2
 + 1)
```

giac [A] time = 0.64, size = 92, normalized size = 0.71

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{x^{\frac{5}{2}}-3\sqrt{x}}{16\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(x^(5/2) - 3*s
qrt(x))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 82, normalized size = 0.64

$$\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{64}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{64}+\frac{3\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{128}+\frac{\frac{x^{\frac{5}{2}}}{16}-\frac{3\sqrt{x}}{16}}{\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2+1)^3,x)
```

```
[Out] 2*(1/32*x^(5/2)-3/32*x^(1/2))/(x^2+1)^2+3/128*2^(1/2)*ln((x+2^(1/2)*x^(1/2)
+1)/(x-2^(1/2)*x^(1/2)+1))+3/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)+3/64*2^(1
/2)*arctan(2^(1/2)*x^(1/2)-1)
```

maxima [A] time = 2.97, size = 97, normalized size = 0.75

$$\frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{3}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{x^{\frac{5}{2}}-3\sqrt{x}}{16\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(x^(5/2) - 3*s
qrt(x))/(x^4 + 2*x^2 + 1)
```

mupad [B] time = 4.69, size = 62, normalized size = 0.48

$$-\frac{\frac{3\sqrt{x}}{16}-\frac{x^{5/2}}{16}}{x^4+2x^2+1}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}+\frac{3}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/64 + 3i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(3/64 - 3i/64) - ((3*x^(1/2))/16 - x^(5/2)/16)/
(2*x^2 + x^4 + 1)
```

sympy [B] time = 9.81, size = 481, normalized size = 3.73

$$\frac{8x^{\frac{5}{2}}}{128x^4+256x^2+128}-\frac{24\sqrt{x}}{128x^4+256x^2+128}-\frac{3\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{3\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}x^4\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}x^4\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}-\frac{6\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{12\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{12\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}-\frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{6\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**2+1)**3,x)
```

```
[Out] 8*x**(5/2)/(128*x**4 + 256*x**2 + 128) - 24*sqrt(x)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**
2 + 128) + 3*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2
 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 1
28) - 6*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 6*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2
```

```
 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
128) - 3*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 1
28) + 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2 + 128
) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128) + 6*sqr
t(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

3.332 $\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx$

Optimal. Leaf size=129

$$\frac{5x^{3/2}}{16\left(x^2+1\right)}+\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{5x^{3/2}}{16\left(x^2+1\right)}+\frac{x^{3/2}}{4\left(x^2+1\right)^2}+\frac{5\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{5\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^2)^3,x]
```

```
[Out] x^(3/2)/(4*(1 + x^2)^2) + (5*x^(3/2))/(16*(1 + x^2)) - (5*ArcTan[1 - Sqrt[2
]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (
5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] +
 x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(1+x^2\right)^3}\,dx = \frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{5}{8}\int \frac{\sqrt{x}}{\left(1+x^2\right)^2}\,dx$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{5x^{3/2}}{16\left(1+x^2\right)} + \frac{5}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{5x^{3/2}}{16\left(1+x^2\right)} + \frac{5}{16}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{5x^{3/2}}{16\left(1+x^2\right)} - \frac{5}{32}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{5}{32}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right.$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{5x^{3/2}}{16\left(1+x^2\right)} + \frac{5}{64}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) + \frac{5}{64}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x\right.$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{5x^{3/2}}{16\left(1+x^2\right)} + \frac{5\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} - \frac{5\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{5\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{x^{3/2}}{4\left(1+x^2\right)^2} + \frac{5x^{3/2}}{16\left(1+x^2\right)} - \frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{5\log\left(1-\sqrt{2}\sqrt{x}\right.}{64\sqrt{2}}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.17

$$\frac{2}{3}x^{3/2}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^2)^3,x]
[Out] (2*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -x^2])/3
```

IntegrateAlgebraic [A] time = 0.12, size = 81, normalized size = 0.63

$$\frac{\left(5x^2+9\right)x^{3/2}}{16\left(x^2+1\right)^2} + \frac{5\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{32\sqrt{2}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(1 + x^2)^3,x]
```

```
[Out] (x^(3/2)*(9 + 5*x^2))/(16*(1 + x^2)^2) + (5*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2
])/Sqrt[x]])/(32*Sqrt[2]) - (5*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(32*Sqrt
[2])
```

fricas [A] time = 0.66, size = 175, normalized size = 1.36

$$-\frac{20\,\sqrt{2}\left(x^{4}+2\,x^{2}+1\right)\arctan\left(\sqrt{2}\,\sqrt{\sqrt{2}\,\sqrt{x}+x+1}-\sqrt{2}\,\sqrt{x}-1\right)+20\,\sqrt{2}\left(x^{4}+2\,x^{2}+1\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{-4\,\sqrt{2}\,\sqrt{x}+4\,x+4}-\sqrt{2}\,\sqrt{x}+1\right)+5\,\sqrt{2}\left(x^{4}+2\,x^{2}+1\right)\log\left(4\,\sqrt{2}\,\sqrt{x}+4\,x+4\right)-5\,\sqrt{2}\left(x^{4}+2\,x^{2}+1\right)\log\left(-4\,\sqrt{2}\,\sqrt{x}+4\,x+4\right)-8\left(5\,x^{3}+9\,x\right)\sqrt{x}}{128\left(x^{4}+2\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 20*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 5*sqrt(2)*
(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 5*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(5*x^3 + 9*x)*sqrt(x))/(x^4 + 2
*x^2 + 1)
```

giac [A] time = 0.59, size = 94, normalized size = 0.73

$$\frac{5}{64}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right)+\frac{5}{64}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)-\frac{5}{128}\,\sqrt{2}\log\left(\sqrt{2}\,\sqrt{x}+x+1\right)+\frac{5}{128}\,\sqrt{2}\log\left(-\sqrt{2}\,\sqrt{x}+x+1\right)+\frac{5\,x^{\frac{7}{2}}+9\,x^{\frac{3}{2}}}{16\left(x^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(5*x^(7/2) + 9
*x^(3/2))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 86, normalized size = 0.67

$$\frac{x^{\frac{3}{2}}}{4\left(x^{2}+1\right)^{2}}+\frac{5x^{\frac{3}{2}}}{16\left(x^{2}+1\right)}+\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{64}+\frac{5\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{64}+\frac{5\sqrt{2}\,\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)^3,x)
```

```
[Out] 1/4*x^(3/2)/(x^2+1)^2+5/16/(x^2+1)*x^(3/2)+5/128*2^(1/2)*ln((x-2^(1/2)*x^(1
/2)+1)/(x+2^(1/2)*x^(1/2)+1))+5/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+5/64*2
^(1/2)*arctan(2^(1/2)*x^(1/2)+1)
```

maxima [A] time = 3.02, size = 99, normalized size = 0.77

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{5x^{\frac{7}{2}}+9x^{\frac{3}{2}}}{16\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/128*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(5*x^(7/2) + 9
*x^(3/2))/(x^4 + 2*x^2 + 1)
```

mupad [B] time = 0.04, size = 61, normalized size = 0.47

$$\frac{\frac{9x^{3/2}}{16}+\frac{5x^{7/2}}{16}}{x^4+2x^2+1}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{64}-\frac{5}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{64}+\frac{5}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2 + 1)^3,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/64 - 5i/64) + 2^(1/2)*atan(2^
(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/64 + 5i/64) + ((9*x^(3/2))/16 + (5*x^(7/2))/
16)/(2*x^2 + x^4 + 1)
```

sympy [B] time = 6.38, size = 481, normalized size = 3.73

$$\frac{40x^{\frac{7}{2}}}{128x^4+256x^2+128}+\frac{72x^{\frac{3}{2}}}{128x^4+256x^2+128}+\frac{5\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{5\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}x^4\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}x^4\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{10\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{20\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{20\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}+\frac{5\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}-\frac{5\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)**3,x)
```

```
[Out] 40*x**(7/2)/(128*x**4 + 256*x**2 + 128) + 72*x**(3/2)/(128*x**4 + 256*x**2
+ 128) + 5*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) - 5*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x
**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) - 10*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 25
6*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x
**2 + 128) + 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x*
*2 + 128) - 5*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x**2
 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 128)
+ 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

3.333 $\int \frac{1}{\sqrt{x}\left(1+x^2\right)^3} dx$

Optimal. Leaf size=129

$$\frac{7\sqrt{x}}{16\left(x^2+1\right)}+\frac{\sqrt{x}}{4\left(x^2+1\right)^2}-\frac{21\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{21\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{21\tan^{-1}\left(\sqrt{\right.}}{32\sqrt{}}$$

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{7\sqrt{x}}{16\left(x^2+1\right)}+\frac{\sqrt{x}}{4\left(x^2+1\right)^2}-\frac{21\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{21\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{21\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*(1 + x^2)^2) + (7*Sqrt[x])/(16*(1 + x^2)) - (21*ArcTan[1 - Sqrt[
2]*Sqrt[x]])/(32*Sqrt[2]) + (21*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) -
 (21*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (21*Log[1 + Sqrt[2]*Sqrt[
x] + x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(1+x^2\right)^3}\,dx = \frac{\sqrt{x}}{4\left(1+x^2\right)^2} + \frac{7}{8}\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx$$

$$= \frac{\sqrt{x}}{4\left(1+x^2\right)^2} + \frac{7\sqrt{x}}{16\left(1+x^2\right)} + \frac{21}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$

$$= \frac{\sqrt{x}}{4\left(1+x^2\right)^2} + \frac{7\sqrt{x}}{16\left(1+x^2\right)} + \frac{21}{16}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}}{4\left(1+x^2\right)^2} + \frac{7\sqrt{x}}{16\left(1+x^2\right)} + \frac{21}{32}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{21}{32}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\vphantom{x}}\right.$$

$$= \frac{\sqrt{x}}{4\left(1+x^2\right)^2} + \frac{7\sqrt{x}}{16\left(1+x^2\right)} + \frac{21}{64}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{21}{64}\,\text{Subst}\left(\int \frac{}{1+\sqrt{2}}\right.$$

$$= \frac{\sqrt{x}}{4\left(1+x^2\right)^2} + \frac{7\sqrt{x}}{16\left(1+x^2\right)} - \frac{21\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{21\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}} + \frac{21\,\text{Subs}}{}$$

$$= \frac{\sqrt{x}}{4\left(1+x^2\right)^2} + \frac{7\sqrt{x}}{16\left(1+x^2\right)} - \frac{21\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{21\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} - \frac{21\log\left(1-\right.}{64}$$

Mathematica [A] time = 0.03, size = 121, normalized size = 0.94

$$\frac{1}{128}\left(\frac{56\sqrt{x}}{x^2+1} + \frac{32\sqrt{x}}{\left(x^2+1\right)^2} - 21\sqrt{2}\log\left(x-\sqrt{2}\sqrt{x}+1\right) + 21\sqrt{2}\log\left(x+\sqrt{2}\sqrt{x}+1\right) - 42\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right) + 42\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + x^2)^3),x]
```

```
[Out] ((32*Sqrt[x])/(1 + x^2)^2 + (56*Sqrt[x])/(1 + x^2) - 42*Sqrt[2]*ArcTan[1 -
Sqrt[2]*Sqrt[x]] + 42*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]] - 21*Sqrt[2]*Log[
1 - Sqrt[2]*Sqrt[x] + x] + 21*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/128
```

IntegrateAlgebraic [A] time = 0.12, size = 81, normalized size = 0.63

$$\frac{\sqrt{x}\left(7x^2+11\right)}{16\left(x^2+1\right)^2} + \frac{21\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{32\sqrt{2}} + \frac{21\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(1 + x^2)^3),x]
```

```
[Out] (Sqrt[x]*(11 + 7*x^2))/(16*(1 + x^2)^2) + (21*ArcTan[(-(1/Sqrt[2]) + x/Sqrt
[2])/Sqrt[x]])/(32*Sqrt[2]) + (21*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/(32*S
qrt[2])
```

fricas [A] time = 0.99, size = 173, normalized size = 1.34

$$-\frac{84 \sqrt{2} \left(x^{4}+2 x^{2}+1\right) \arctan \left(\sqrt{2} \sqrt{\sqrt{2} \sqrt{x}+x+1}-\sqrt{2} \sqrt{x}-1\right)+84 \sqrt{2} \left(x^{4}+2 x^{2}+1\right) \arctan \left(\frac{1}{2} \sqrt{2} \sqrt{-4 \sqrt{2} \sqrt{x}+4 x+4}-\sqrt{2} \sqrt{x}+1\right)-21 \sqrt{2} \left(x^{4}+2 x^{2}+1\right) \log \left(4 \sqrt{2} \sqrt{x}+4 x+4\right)+21 \sqrt{2} \left(x^{4}+2 x^{2}+1\right) \log \left(-4 \sqrt{2} \sqrt{x}+4 x+4\right)-8 \left(7 x^{2}+11\right) \sqrt{x}}{128 \left(x^{4}+2 x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/128*(84*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 84*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(1/2*sqr
t(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 21*sqrt(2)
*(x^4 + 2*x^2 + 1)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 21*sqrt(2)*(x^4 + 2*x
^2 + 1)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(7*x^2 + 11)*sqrt(x))/(x^4 +
2*x^2 + 1)
```

giac [A] time = 0.61, size = 94, normalized size = 0.73

$$\frac{21}{64} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(\sqrt{2}+2 \sqrt{x}\right)\right)+\frac{21}{64} \sqrt{2} \arctan \left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2}-2 \sqrt{x}\right)\right)+\frac{21}{128} \sqrt{2} \log \left(\sqrt{2} \sqrt{x}+x+1\right)-\frac{21}{128} \sqrt{2} \log \left(-\sqrt{2} \sqrt{x}+x+1\right)+\frac{7 x^{\frac{5}{2}}+11 \sqrt{x}}{16 \left(x^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 21/64*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 21/128*sqrt(2)*log(sqrt(2)*sqrt(x
) + x + 1) - 21/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(7*x^(5/2)
 + 11*sqrt(x))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 86, normalized size = 0.67

$$\frac{21 \sqrt{2}\, \arctan \left(\sqrt{2}\, \sqrt{x}-1\right)}{64}+\frac{21 \sqrt{2}\, \arctan \left(\sqrt{2}\, \sqrt{x}+1\right)}{64}+\frac{21 \sqrt{2}\, \ln \left(\frac{x+\sqrt{2}\, \sqrt{x}+1}{x-\sqrt{2}\, \sqrt{x}+1}\right)}{128}+\frac{\sqrt{x}}{4 \left(x^{2}+1\right)^{2}}+\frac{7 \sqrt{x}}{16 \left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3/x^(1/2),x)
```

```
[Out] 1/4*x^(1/2)/(x^2+1)^2+7/16/(x^2+1)*x^(1/2)+21/128*2^(1/2)*ln((x+2^(1/2)*x^(
1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+21/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)+21/6
4*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)
```

maxima [A] time = 2.90, size = 99, normalized size = 0.77

$$\frac{21}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{21}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{21}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{21}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{7x^{\frac{5}{2}}+11\sqrt{x}}{16\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 21/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 21/64*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 21/128*sqrt(2)*log(sqrt(2)*sqrt(x
) + x + 1) - 21/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(7*x^(5/2)
 + 11*sqrt(x))/(x^4 + 2*x^2 + 1)
```

mupad [B] time = 4.75, size = 61, normalized size = 0.47

$$\frac{\frac{11\sqrt{x}}{16}+\frac{7x^{5/2}}{16}}{x^4+2x^2+1}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{21}{64}+\frac{21}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{21}{64}-\frac{21}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^2 + 1)^3),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(21/64 + 21i/64) + 2^(1/2)*atan(
2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(21/64 - 21i/64) + ((11*x^(1/2))/16 + (7*x^(5
/2))/16)/(2*x^2 + x^4 + 1)
```

sympy [B] time = 8.46, size = 481, normalized size = 3.73

$$\frac{56x^{\frac{5}{2}}}{128x^4+256x^2+128}+\frac{88\sqrt{x}}{128x^4+256x^2+128}-\frac{21\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{21\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{42\sqrt{2}x^4\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{42\sqrt{2}x^4\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}-\frac{42\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{42\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{84\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{84\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}-\frac{21\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{21\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{42\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{42\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/x**(1/2),x)
```

```
[Out] 56*x**(5/2)/(128*x**4 + 256*x**2 + 128) + 88*sqrt(x)/(128*x**4 + 256*x**2 +
 128) - 21*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 21*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 2
56*x**2 + 128) + 42*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*
x**2 + 128) + 42*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**
2 + 128) - 42*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 42*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
+ 256*x**2 + 128) + 84*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 2
```

```
56*x**2 + 128) + 84*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*
x**2 + 128) - 21*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) + 21*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 42*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 + 12
8) + 42*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

3.334 $\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=138

$$\frac{9}{16\sqrt{x}\left(x^2+1\right)}+\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}-\frac{45}{16\sqrt{x}}-\frac{45\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right.}{32\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{16\sqrt{x}\left(x^2+1\right)}+\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}-\frac{45}{16\sqrt{x}}-\frac{45\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{45\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{45\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(1 + x^2)^3),x]
```

```
[Out] -45/(16*Sqrt[x]) + 1/(4*Sqrt[x]*(1 + x^2)^2) + 9/(16*Sqrt[x]*(1 + x^2)) + (
45*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (45*ArcTan[1 + Sqrt[2]*Sqrt[
x]])/(32*Sqrt[2]) - (45*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) + (45*Lo
g[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx &= \frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{8}\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx\\
&= \frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}+\frac{45}{32}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}-\frac{45}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}-\frac{45}{16}\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}+\frac{45}{32}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{45}{32}\operatorname{Subst}\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}-\frac{45}{64}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{45}{64}\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}-\frac{45\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}+\frac{45\log\left(1+\sqrt{2}\sqrt{}\right.}{64\sqrt{2}}\\
&= -\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}+\frac{45\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{45\tan^{-1}\left(1+\sqrt{2}\sqrt{}\right.}{32\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 20, normalized size = 0.14

$$
-\frac{2\,{}_2F_1\left(-\frac{1}{4},3;\frac{3}{4};-x^2\right)}{\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(1 + x^2)^3),x]

[Out] (-2*Hypergeometric2F1[-1/4, 3, 3/4, -x^2])/Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.27, size = 86, normalized size = 0.62

$$\frac{-45x^4-81x^2-32}{16\sqrt{x}\,(x^2+1)^2}-\frac{45\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{32\sqrt{2}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(1 + x^2)^3),x]
```

```
[Out] (-32 - 81*x^2 - 45*x^4)/(16*Sqrt[x]*(1 + x^2)^2) - (45*ArcTan[(-(1/Sqrt[2])
 + x/Sqrt[2])/Sqrt[x]])/(32*Sqrt[2]) + (45*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x
)])/(32*Sqrt[2])
```

fricas [A] time = 0.74, size = 178, normalized size = 1.29

$$\frac{180\sqrt{2}(x^5+2x^3+x)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+180\sqrt{2}(x^5+2x^3+x)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+45\sqrt{2}(x^5+2x^3+x)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-45\sqrt{2}(x^5+2x^3+x)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8(45x^4+81x^2+32)\sqrt{x}}{128(x^5+2x^3+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(180*sqrt(2)*(x^5 + 2*x^3 + x)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x) +
x + 1) - sqrt(2)*sqrt(x) - 1) + 180*sqrt(2)*(x^5 + 2*x^3 + x)*arctan(1/2*sq
rt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 45*sqrt(2
)*(x^5 + 2*x^3 + x)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 45*sqrt(2)*(x^5 + 2*
x^3 + x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(45*x^4 + 81*x^2 + 32)*sqrt(
x))/(x^5 + 2*x^3 + x)
```

giac [A] time = 0.63, size = 99, normalized size = 0.72

$$-\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{45}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{45}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{\sqrt{x}}-\frac{13x^{\frac{7}{2}}+17x^{\frac{3}{2}}}{16(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -45/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 45/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 45/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) - 45/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x) - 1/
16*(13*x^(7/2) + 17*x^(3/2))/(x^2 + 1)^2
```

maple [A] time = 0.01, size = 87, normalized size = 0.63

$$-\frac{45\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{64}-\frac{45\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{64}-\frac{45\sqrt{2}\,\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)}{128}-\frac{2}{\sqrt{x}}-\frac{2\left(\frac{13x^{\frac{7}{2}}}{32}+\frac{17x^{\frac{3}{2}}}{32}\right)}{(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^2+1)^3,x)
```

```
[Out] -2*(13/32*x^(7/2)+17/32*x^(3/2))/(x^2+1)^2-45/64*2^(1/2)*arctan(2^(1/2)*x^(
1/2)-1)-45/128*2^(1/2)*ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))-45/6
4*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/x^(1/2)
```

maxima [A] time = 2.98, size = 102, normalized size = 0.74

$$-\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{45}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{45}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{45x^4+81x^2+32}{16\left(x^{\frac{9}{2}}+2x^{\frac{5}{2}}+\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -45/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 45/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 45/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) - 45/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(45*x^4 +
 81*x^2 + 32)/(x^(9/2) + 2*x^(5/2) + sqrt(x))
```

mupad [B] time = 4.72, size = 65, normalized size = 0.47

$$-\frac{\frac{45x^4}{16}+\frac{81x^2}{16}+2}{\sqrt{x}+2x^{5/2}+x^{9/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{45}{64}+\frac{45}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{45}{64}-\frac{45}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^2 + 1)^3),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(45/64 - 45i/64) - 2^(1/2)*ata
n(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(45/64 + 45i/64) - ((81*x^2)/16 + (45*x^4)/
16 + 2)/(x^(1/2) + 2*x^(5/2) + x^(9/2))
```

sympy [B] time = 12.63, size = 653, normalized size = 4.73

$$-\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}+\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}x^{\frac{5}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}+\frac{90\sqrt{2}x^{\frac{5}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{180\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{180\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{45\sqrt{2}\sqrt{x}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}+\frac{45\sqrt{2}\sqrt{x}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}\sqrt{x}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{90\sqrt{2}\sqrt{x}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{360x^4}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{648x^2}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}-\frac{256}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(x**2+1)**3,x)
```

```
[Out] -45*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256*
x**(5/2) + 128*sqrt(x)) + 45*sqrt(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x +
 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(2)*x**(9/2)*atan(
sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(
2)*x**(9/2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sq
```

```
rt(x)) - 90*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**(9/2
) + 256*x**(5/2) + 128*sqrt(x)) + 90*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 180*sqrt(2)*x**(5
/2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) -
 180*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**(5/2
) + 128*sqrt(x)) - 45*sqrt(2)*sqrt(x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(12
8*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) + 45*sqrt(2)*sqrt(x)*log(4*sqrt(2)
*sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(2
)*sqrt(x)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt
(x)) - 90*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**
(5/2) + 128*sqrt(x)) - 360*x**4/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x))
 - 648*x**2/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 256/(128*x**(9/2)
 + 256*x**(5/2) + 128*sqrt(x))
```

3.335 $\int \frac{1}{x^{5/2}(1+x^2)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{77}{48x^{3/2}}+\frac{11}{16x^{3/2}\left(x^2+1\right)}+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}+\frac{77\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{77\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{77\tan^{-1}\left(1-\sqrt{2}\right.}{32\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{11}{16x^{3/2}\left(x^2+1\right)}-\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}+\frac{77\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{77\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}+\frac{77\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{77\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(1 + x^2)^3),x]
```

```
[Out] -77/(48*x^(3/2)) + 1/(4*x^(3/2)*(1 + x^2)^2) + 11/(16*x^(3/2)*(1 + x^2)) +
(77*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) - (77*ArcTan[1 + Sqrt[2]*Sqrt
[x]])/(32*Sqrt[2]) + (77*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (77*L
og[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^3}\,dx = \frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{8}\int \frac{1}{x^{5/2}\left(1+x^2\right)^2}\,dx$$

$$= \frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}+\frac{77}{32}\int \frac{1}{x^{5/2}\left(1+x^2\right)}\,dx$$

$$= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}-\frac{77}{32}\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx$$

$$= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}-\frac{77}{16}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}-\frac{77}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)-\frac{77}{32}\text{Subst}$$

$$= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}-\frac{77}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{x}\right)-\frac{77}{64}$$

$$= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}+\frac{77\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{77\log\left(1+\sqrt{2}\right.}{64\sqrt{2}}$$

$$= -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)}+\frac{77\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{77\tan^{-1}\left(1+\sqrt{2}\right.}{32\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.16

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},3;\frac{1}{4};-x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(1 + x^2)^3),x]
[Out] (-2*Hypergeometric2F1[-3/4, 3, 1/4, -x^2])/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.26, size = 86, normalized size = 0.62

$$\frac{-77x^4-121x^2-32}{48x^{3/2}\left(x^2+1\right)^2}-\frac{77\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{32\sqrt{2}}-\frac{77\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(1 + x^2)^3),x]
```

```
[Out] (-32 - 121*x^2 - 77*x^4)/(48*x^(3/2)*(1 + x^2)^2) - (77*ArcTan[(-(1/Sqrt[2]
) + x/Sqrt[2])/Sqrt[x]])/(32*Sqrt[2]) - (77*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 +
x)])/(32*Sqrt[2])
```

fricas [B] time = 0.71, size = 188, normalized size = 1.36

$$\frac{924\sqrt{2}\left(x^6+2x^4+x^2\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+924\sqrt{2}\left(x^6+2x^4+x^2\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)-231\sqrt{2}\left(x^6+2x^4+x^2\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)+231\sqrt{2}\left(x^6+2x^4+x^2\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\left(77x^4+121x^2+32\right)\sqrt{x}}{384\left(x^6+2x^4+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(924*sqrt(2)*(x^6 + 2*x^4 + x^2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x)
+ x + 1) - sqrt(2)*sqrt(x) - 1) + 924*sqrt(2)*(x^6 + 2*x^4 + x^2)*arctan(1/
2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) - 231*s
qrt(2)*(x^6 + 2*x^4 + x^2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) + 231*sqrt(2)*(
x^6 + 2*x^4 + x^2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(77*x^4 + 121*x^2
+ 32)*sqrt(x))/(x^6 + 2*x^4 + x^2)
```

giac [A] time = 0.62, size = 99, normalized size = 0.72

$$-\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{77}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{77}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{15x^{\frac{5}{2}}+19\sqrt{x}}{16\left(x^2+1\right)^2}-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 77/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 77/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) + 77/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(15*x^(5/
2) + 19*sqrt(x))/(x^2 + 1)^2 - 2/3/x^(3/2)
```

maple [A] time = 0.02, size = 87, normalized size = 0.63

$$-\frac{77\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}-1\right)}{64}-\frac{77\sqrt{2}\,\arctan\left(\sqrt{2}\,\sqrt{x}+1\right)}{64}-\frac{77\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{128}-\frac{2}{3x^{\frac{3}{2}}}-\frac{2\left(\frac{15x^{\frac{5}{2}}}{32}+\frac{19\sqrt{x}}{32}\right)}{\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^2+1)^3,x)
```

```
[Out] -2*(15/32*x^(5/2)+19/32*x^(1/2))/(x^2+1)^2-77/64*2^(1/2)*arctan(2^(1/2)*x^(
1/2)-1)-77/128*2^(1/2)*ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))-77/6
4*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/3/x^(3/2)
```

maxima [A] time = 2.85, size = 102, normalized size = 0.74

$$-\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{77}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{77}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{77x^4+121x^2+32}{48\left(x^{\frac{11}{2}}+2x^{\frac{7}{2}}+x^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -77/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 77/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 77/128*sqrt(2)*log(sqrt(2)*sqrt(
x) + x + 1) + 77/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/48*(77*x^4 +
 121*x^2 + 32)/(x^(11/2) + 2*x^(7/2) + x^(3/2))
```

mupad [B] time = 4.72, size = 65, normalized size = 0.47

$$-\frac{\frac{77x^4}{48}+\frac{121x^2}{48}+\frac{2}{3}}{x^{3/2}+2x^{7/2}+x^{11/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{77}{64}-\frac{77}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{77}{64}+\frac{77}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^2 + 1)^3),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(77/64 + 77i/64) - 2^(1/2)*ata
n(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(77/64 - 77i/64) - ((121*x^2)/48 + (77*x^4)
/48 + 2/3)/(x^(3/2) + 2*x^(7/2) + x^(11/2))
```

sympy [B] time = 22.42, size = 653, normalized size = 4.73

$$\frac{231\sqrt{2}x^{\frac{11}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{231\sqrt{2}x^{\frac{11}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}x^{\frac{11}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}x^{\frac{11}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}+\frac{462\sqrt{2}x^{\frac{7}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}x^{\frac{7}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{924\sqrt{2}x^{\frac{7}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{924\sqrt{2}x^{\frac{7}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}+\frac{231\sqrt{2}x^{\frac{3}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{231\sqrt{2}x^{\frac{3}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}x^{\frac{3}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{462\sqrt{2}x^{\frac{3}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{616x^4}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{968x^2}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}-\frac{256}{384x^{\frac{11}{2}}+768x^{\frac{7}{2}}+384x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(x**2+1)**3,x)
```

```
[Out] 231*sqrt(2)*x**(11/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 76
8*x**(7/2) + 384*x**(3/2)) - 231*sqrt(2)*x**(11/2)*log(4*sqrt(2)*sqrt(x) +
4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(11
/2)*atan(sqrt(2)*sqrt(x) - 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2))
 - 462*sqrt(2)*x**(11/2)*atan(sqrt(2)*sqrt(x) + 1)/(384*x**(11/2) + 768*x**
```

```
(7/2) + 384*x**(3/2)) + 462*sqrt(2)*x**(7/2)*log(-4*sqrt(2)*sqrt(x) + 4*x +
 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(7/2)*lo
g(4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)
) - 924*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) - 1)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) - 924*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) + 1)/(384
*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) + 231*sqrt(2)*x**(3/2)*log(-4*sqr
t(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 231
*sqrt(2)*x**(3/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**
(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqrt(x) - 1)/(384
*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(3/2)*atan(sqrt(
2)*sqrt(x) + 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 616*x**4/(3
84*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 968*x**2/(384*x**(11/2) + 768
*x**(7/2) + 384*x**(3/2)) - 256/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2
))
```

3.336 $\int \frac{1}{x^{7/2}(1+x^2)^3}\,dx$

Optimal. Leaf size=147

$$-\frac{117}{80x^{5/2}}+\frac{13}{16x^{5/2}\left(x^2+1\right)}+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}+\frac{117}{16\sqrt{x}}+\frac{117\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\tan^{-1}}{}$$

Rubi [A] time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13}{16x^{5/2}\left(x^2+1\right)}+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}-\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{117\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{64\sqrt{2}}-\frac{117\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{117\tan^{-1}\left(\sqrt{2}\sqrt{x}+1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(1 + x^2)^3),x]
```

```
[Out] -117/(80*x^(5/2)) + 117/(16*Sqrt[x]) + 1/(4*x^(5/2)*(1 + x^2)^2) + 13/(16*x
^(5/2)*(1 + x^2)) - (117*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (117*A
rcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (117*Log[1 - Sqrt[2]*Sqrt[x] + x
])/(64*Sqrt[2]) - (117*Log[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}\left(1+x^2\right)^3}\,dx &= \frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{8}\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx\\
&= \frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{32}\int \frac{1}{x^{7/2}\left(1+x^2\right)}\,dx\\
&= -\frac{117}{80x^{5/2}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}-\frac{117}{32}\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{32}\int \frac{\sqrt{x}}{1+x^2}\,dx\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{16}\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}-\frac{117}{32}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{x}\right)+\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117}{64}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x\right.\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}+\frac{117\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{64\sqrt{2}}-\frac{117\log}{}\\
&= -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)}-\frac{117\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{117\tan^{-}}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.15

$$
-\frac{2\,{}_2F_1\left(-\frac{5}{4},3;-\frac{1}{4};-x^2\right)}{5x^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(1 + x^2)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 3, -1/4, -x^2])/(5*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.24, size = 91, normalized size = 0.62

$$\frac{585x^6+1053x^4+416x^2-32}{80x^{5/2}\left(x^2+1\right)^2}+\frac{117\tan^{-1}\left(\frac{\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x}}\right)}{32\sqrt{2}}-\frac{117\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{x+1}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(1 + x^2)^3),x]
```

```
[Out] (-32 + 416*x^2 + 1053*x^4 + 585*x^6)/(80*x^(5/2)*(1 + x^2)^2) + (117*ArcTan
[(-(1/Sqrt[2]) + x/Sqrt[2])/Sqrt[x]])/(32*Sqrt[2]) - (117*ArcTanh[(Sqrt[2]*
Sqrt[x])/(1 + x)])/(32*Sqrt[2])
```

fricas [A] time = 0.61, size = 193, normalized size = 1.31

$$-\frac{2340\sqrt{2}\left(x^7+2x^5+x^3\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)+2340\sqrt{2}\left(x^7+2x^5+x^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+585\sqrt{2}\left(x^7+2x^5+x^3\right)\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-585\sqrt{2}\left(x^7+2x^5+x^3\right)\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)-8\left(585x^6+1053x^4+416x^2-32\right)\sqrt{x}}{640\left(x^7+2x^5+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/640*(2340*sqrt(2)*(x^7 + 2*x^5 + x^3)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x
) + x + 1) - sqrt(2)*sqrt(x) - 1) + 2340*sqrt(2)*(x^7 + 2*x^5 + x^3)*arctan
(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 58
5*sqrt(2)*(x^7 + 2*x^5 + x^3)*log(4*sqrt(2)*sqrt(x) + 4*x + 4) - 585*sqrt(2
)*(x^7 + 2*x^5 + x^3)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4) - 8*(585*x^6 + 1053
*x^4 + 416*x^2 - 32)*sqrt(x))/(x^7 + 2*x^5 + x^3)
```

giac [A] time = 0.64, size = 106, normalized size = 0.72

$$\frac{117}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{117}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{117}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{117}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{21x^{\frac{7}{2}}+25x^{\frac{3}{2}}}{16\left(x^2+1\right)^2}+\frac{2\left(15x^2-1\right)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 117/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 117/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) + 117/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(21*x^
(7/2) + 25*x^(3/2))/(x^2 + 1)^2 + 2/5*(15*x^2 - 1)/x^(5/2)
```

maple [A] time = 0.01, size = 92, normalized size = 0.63

$$\frac{117\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ -1\right)}{64}+\frac{117\sqrt{2}\ \arctan\left(\sqrt{2}\ \sqrt{x}\ +1\right)}{64}+\frac{117\sqrt{2}\ \ln\left(\frac{x-\sqrt{2}\ \sqrt{x}+1}{x+\sqrt{2}\ \sqrt{x}+1}\right)}{128}+\frac{6}{\sqrt{x}}-\frac{2}{5x^{\frac{5}{2}}}+\frac{\frac{21x^{\frac{7}{2}}}{16}+\frac{25x^{\frac{3}{2}}}{16}}{\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^2+1)^3,x)
```

```
[Out] 2*(21/32*x^(7/2)+25/32*x^(3/2))/(x^2+1)^2+117/128*2^(1/2)*ln((x-2^(1/2)*x^(
1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+117/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)-1)+117
/64*2^(1/2)*arctan(2^(1/2)*x^(1/2)+1)-2/5/x^(5/2)+6/x^(1/2)
```

maxima [A] time = 2.95, size = 107, normalized size = 0.73

$$\frac{117}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{117}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{117}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{117}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+\frac{585x^6+1053x^4+416x^2-32}{80\left(x^{\frac{13}{2}}+2x^{\frac{9}{2}}+x^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 117/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 117/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 117/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) + 117/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/80*(585*x
^6 + 1053*x^4 + 416*x^2 - 32)/(x^(13/2) + 2*x^(9/2) + x^(5/2))
```

mupad [B] time = 0.07, size = 69, normalized size = 0.47

$$\frac{\frac{117x^6}{16}+\frac{1053x^4}{80}+\frac{26x^2}{5}-\frac{2}{5}}{x^{5/2}+2x^{9/2}+x^{13/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{117}{64}-\frac{117}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{117}{64}+\frac{117}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^2 + 1)^3),x)
```

```
[Out] ((26*x^2)/5 + (1053*x^4)/80 + (117*x^6)/16 - 2/5)/(x^(5/2) + 2*x^(9/2) + x^
(13/2)) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(117/64 - 117i/64) + 2
^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(117/64 + 117i/64)
```

sympy [B] time = 51.16, size = 678, normalized size = 4.61

$$\frac{585\sqrt{2}x^{\frac{13}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}-\frac{585\sqrt{2}x^{\frac{13}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}x^{\frac{13}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}x^{\frac{13}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}x^{\frac{9}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}-\frac{1170\sqrt{2}x^{\frac{9}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{2340\sqrt{2}x^{\frac{9}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{2340\sqrt{2}x^{\frac{9}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{585\sqrt{2}x^{\frac{5}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}-\frac{585\sqrt{2}x^{\frac{5}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}x^{\frac{5}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{1170\sqrt{2}x^{\frac{5}{2}}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{4680x^6}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{8424x^4}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}+\frac{3328x^2}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}-\frac{256}{640x^{\frac{13}{2}}+1280x^{\frac{9}{2}}+640x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**2+1)**3,x)
```

```
[Out] 585*sqrt(2)*x**(13/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 12
80*x**(9/2) + 640*x**(5/2)) - 585*sqrt(2)*x**(13/2)*log(4*sqrt(2)*sqrt(x) +
 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**
(13/2)*atan(sqrt(2)*sqrt(x) - 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5
/2)) + 1170*sqrt(2)*x**(13/2)*atan(sqrt(2)*sqrt(x) + 1)/(640*x**(13/2) + 12
```

```
80*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x)
+ 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) - 1170*sqrt(2)*x*
*(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 64
0*x**(5/2)) + 2340*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) - 1)/(640*x**(13/2
) + 1280*x**(9/2) + 640*x**(5/2)) + 2340*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt
(x) + 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 585*sqrt(2)*x**(5
/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*
x**(5/2)) - 585*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(
13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**(5/2)*atan(sqrt(2)*
sqrt(x) - 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*
x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**
(5/2)) + 4680*x**6/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 8424*x*
*4/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 3328*x**2/(640*x**(13/2
) + 1280*x**(9/2) + 640*x**(5/2)) - 256/(640*x**(13/2) + 1280*x**(9/2) + 64
0*x**(5/2))
```

3.337 $\int \frac{\sqrt{x}}{1-x^2} dx$

Optimal. Leaf size=15

$$\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {329, 298, 203, 206}

$$\tanh^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 - x^2),x]
```

```
[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{1-x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\
&= -\tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.00, size = 15, normalized size = 1.00

$$\tanh^{-1}\left(\sqrt{x}\right) - \tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 - x^2),x]
[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

IntegrateAlgebraic [**A**] time = 0.02, size = 15, normalized size = 1.00

$$\tanh^{-1}\left(\sqrt{x}\right) - \tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(1 - x^2),x]
[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

fricas [**B**] time = 0.81, size = 23, normalized size = 1.53

$$-\arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(\sqrt{x}+1\right) - \frac{1}{2}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="fricas")
[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

giac [**B**] time = 0.62, size = 24, normalized size = 1.60

$$-\arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(\sqrt{x}+1\right) - \frac{1}{2}\,\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="giac")
[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

maple [B] time = 0.01, size = 24, normalized size = 1.60

$$-\arctan\left(\sqrt{x}\right)-\frac{\ln\left(\sqrt{x}-1\right)}{2}+\frac{\ln\left(\sqrt{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-x^2+1),x)
[Out] -1/2*ln(-1+x^(1/2))+1/2*ln(1+x^(1/2))-arctan(x^(1/2))
```

maxima [B] time = 2.97, size = 23, normalized size = 1.53

$$-\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^2+1),x, algorithm="maxima")
[Out] -arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.73

$$\operatorname{atanh}\left(\sqrt{x}\right)-\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(x^2 - 1),x)
[Out] atanh(x^(1/2)) - atan(x^(1/2))
```

sympy [B] time = 0.29, size = 26, normalized size = 1.73

$$-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}-\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-x**2+1),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 - atan(sqrt(x))
```

3.338 $\int \frac{x^{2/3}}{1+x^2} \, dx$

Optimal. Leaf size=73

$$-\frac{1}{2}\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x}}{x^{2/3}+1}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right)+\frac{1}{2}\tan^{-1}\left(2\sqrt[3]{x}+\sqrt{3}\right)+\tan^{-1}\left(\sqrt[3]{x}\right)$$

Rubi [A] time = 0.26, antiderivative size = 100, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {329, 295, 634, 618, 204, 628, 203}

$$\frac{1}{4}\sqrt{3}\log\left(x^{2/3}-\sqrt{3}\sqrt[3]{x}+1\right)-\frac{1}{4}\sqrt{3}\log\left(x^{2/3}+\sqrt{3}\sqrt[3]{x}+1\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right)+\frac{1}{2}\tan^{-1}\left(2\sqrt[3]{x}+\sqrt{3}\right)+\tan^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(1 + x^2),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x^(1/3)]/2 + ArcTan[Sqrt[3] + 2*x^(1/3)]/2 + ArcTan[x^(
1/3)] + (Sqrt[3]*Log[1 - Sqrt[3]*x^(1/3) + x^(2/3)])/4 - (Sqrt[3]*Log[1 + S
qrt[3]*x^(1/3) + x^(2/3)])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{2/3}}{1+x^2}\,dx = 3\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \sqrt[3]{x}\right)$$

$$= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[3]{x}\right) + \text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \sqrt[3]{x}\right) + \text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx,\right.$$

$$= \tan^{-1}\left(\sqrt[3]{x}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \sqrt[3]{x}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \sqrt[3]{x}\right) + \frac{1}{4}$$

$$= \tan^{-1}\left(\sqrt[3]{x}\right) + \frac{1}{4}\sqrt{3}\log\left(1-\sqrt{3}\sqrt[3]{x}+x^{2/3}\right) - \frac{1}{4}\sqrt{3}\log\left(1+\sqrt{3}\sqrt[3]{x}+x^{2/3}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x}\right.$$

$$= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right) + \frac{1}{2}\tan^{-1}\left(\sqrt{3}+2\sqrt[3]{x}\right) + \tan^{-1}\left(\sqrt[3]{x}\right) + \frac{1}{4}\sqrt{3}\log\left(1-\sqrt{3}\sqrt[3]{x}+x^{2/3}\right) - \frac{1}{4}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.30

$$\frac{3}{5}x^{5/3}\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(1 + x^2),x]
```

```
[Out] (3*x^(5/3)*Hypergeometric2F1[5/6, 1, 11/6, -x^2])/5
```

IntegrateAlgebraic [C] time = 0.13, size = 79, normalized size = 1.08

$$\tan^{-1}\left(\sqrt[3]{x}\right)+\frac{1}{2}\left(1-i\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt[3]{x}\right)+\frac{1}{2}\left(1+i\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)/(1 + x^2),x]
```

```
[Out] ArcTan[x^(1/3)] + ((1 - I*Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^(1/3))/2])/2 +
 ((1 + I*Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^(1/3))/2])/2
```

fricas [B] time = 0.83, size = 108, normalized size = 1.48

$$-\frac{1}{4}\sqrt{3}\log\left(16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16\right)+\frac{1}{4}\sqrt{3}\log\left(-16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16\right)-\arctan\left(\sqrt{3}+\frac{1}{2}\sqrt{-16\sqrt{3}x^{\frac{1}{3}}+16x^{\frac{2}{3}}+16}-2x^{\frac{1}{3}}\right)-\arctan\left(-\sqrt{3}+2\sqrt{\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1}-2x^{\frac{1}{3}}\right)+\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*log(16*sqrt(3)*x^(1/3) + 16*x^(2/3) + 16) + 1/4*sqrt(3)*log(-1
6*sqrt(3)*x^(1/3) + 16*x^(2/3) + 16) - arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(3
)*x^(1/3) + 16*x^(2/3) + 16) - 2*x^(1/3)) - arctan(-sqrt(3) + 2*sqrt(sqrt(3
)*x^(1/3) + x^(2/3) + 1) - 2*x^(1/3)) + arctan(x^(1/3))
```

giac [A] time = 0.63, size = 68, normalized size = 0.93

$$-\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2x^{\frac{1}{3}}\right)+\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)*
x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sqrt
(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

maple [A] time = 0.05, size = 69, normalized size = 0.95

$$\arctan\left(x^{\frac{1}{3}}\right)+\frac{\arctan\left(2x^{\frac{1}{3}}-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x^{\frac{1}{3}}+\sqrt{3}\right)}{2}+\frac{\sqrt{3}\,\ln\left(x^{\frac{2}{3}}-\sqrt{3}\,x^{\frac{1}{3}}+1\right)}{4}-\frac{\sqrt{3}\,\ln\left(x^{\frac{2}{3}}+\sqrt{3}\,x^{\frac{1}{3}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(x^2+1),x)
```

```
[Out] arctan(x^(1/3))+1/2*arctan(2*x^(1/3)-3^(1/2))+1/2*arctan(2*x^(1/3)+3^(1/2))
+1/4*ln(1+x^(2/3)-x^(1/3)*3^(1/2))*3^(1/2)-1/4*ln(1+x^(2/3)+x^(1/3)*3^(1/2)
)*3^(1/2)
```

maxima [A] time = 2.97, size = 68, normalized size = 0.93

$$-\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2x^{\frac{1}{3}}\right)+\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)*
x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sqrt
(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

mupad [B] time = 4.77, size = 57, normalized size = 0.78

$$\mathrm{atan}\left(x^{1/3}\right)-\mathrm{atan}\left(\frac{486\,x^{1/3}}{-243+\sqrt{3}\,243\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\mathrm{atan}\left(\frac{486\,x^{1/3}}{243+\sqrt{3}\,243\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(x^2 + 1),x)
```

```
[Out] atan(x^(1/3)) - atan((486*x^(1/3))/(3^(1/2)*243i - 243))*((3^(1/2)*1i)/2 +
1/2) - atan((486*x^(1/3))/(3^(1/2)*243i + 243))*((3^(1/2)*1i)/2 - 1/2)
```

sympy [A] time = 1.78, size = 94, normalized size = 1.29

$$\frac{\sqrt{3}\log\left(4x^{\frac{2}{3}}-4\sqrt{3}\sqrt[3]{x}+4\right)}{4}-\frac{\sqrt{3}\log\left(4x^{\frac{2}{3}}+4\sqrt{3}\sqrt[3]{x}+4\right)}{4}+\mathrm{atan}\left(\sqrt[3]{x}\right)+\frac{\mathrm{atan}\left(2\sqrt[3]{x}-\sqrt{3}\right)}{2}+\frac{\mathrm{atan}\left(2\sqrt[3]{x}+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(x**2+1),x)
```

```
[Out] sqrt(3)*log(4*x**(2/3) - 4*sqrt(3)*x**(1/3) + 4)/4 - sqrt(3)*log(4*x**(2/3)
 + 4*sqrt(3)*x**(1/3) + 4)/4 + atan(x**(1/3)) + atan(2*x**(1/3) - sqrt(3))/
2 + atan(2*x**(1/3) + sqrt(3))/2
```

3.339 $\int x^m \left(a+bx^2\right)^5 \, dx$

Optimal. Leaf size=97

$$\frac{a^5x^{m+1}}{m+1}+\frac{5a^4bx^{m+3}}{m+3}+\frac{10a^3b^2x^{m+5}}{m+5}+\frac{10a^2b^3x^{m+7}}{m+7}+\frac{5ab^4x^{m+9}}{m+9}+\frac{b^5x^{m+11}}{m+11}$$

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{10a^3b^2x^{m+5}}{m+5}+\frac{10a^2b^3x^{m+7}}{m+7}+\frac{5a^4bx^{m+3}}{m+3}+\frac{a^5x^{m+1}}{m+1}+\frac{5ab^4x^{m+9}}{m+9}+\frac{b^5x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^5,x]
```

```
[Out] (a^5*x^(1 + m))/(1 + m) + (5*a^4*b*x^(3 + m))/(3 + m) + (10*a^3*b^2*x^(5 +
m))/(5 + m) + (10*a^2*b^3*x^(7 + m))/(7 + m) + (5*a*b^4*x^(9 + m))/(9 + m)
+ (b^5*x^(11 + m))/(11 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^5 \, dx &= \int \left(a^5x^m+5a^4bx^{2+m}+10a^3b^2x^{4+m}+10a^2b^3x^{6+m}+5ab^4x^{8+m}+b^5x^{10+m}\right) dx\\ &= \frac{a^5x^{1+m}}{1+m}+\frac{5a^4bx^{3+m}}{3+m}+\frac{10a^3b^2x^{5+m}}{5+m}+\frac{10a^2b^3x^{7+m}}{7+m}+\frac{5ab^4x^{9+m}}{9+m}+\frac{b^5x^{11+m}}{11+m}\end{aligned}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.91

$$x^{m+1}\left(\frac{a^5}{m+1}+\frac{5a^4bx^2}{m+3}+\frac{10a^3b^2x^4}{m+5}+\frac{10a^2b^3x^6}{m+7}+\frac{5ab^4x^8}{m+9}+\frac{b^5x^{10}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^5,x]
```

```
[Out] x^(1 + m)*(a^5/(1 + m) + (5*a^4*b*x^2)/(3 + m) + (10*a^3*b^2*x^4)/(5 + m) +
 (10*a^2*b^3*x^6)/(7 + m) + (5*a*b^4*x^8)/(9 + m) + (b^5*x^10)/(11 + m))
```

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx^2\right)^5 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2)^5,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2)^5, x]
```

fricas [B] time = 0.84, size = 367, normalized size = 3.78

$$\frac{\left(\left(b^5m^5+25\,b^5m^4+230\,b^5m^3+950\,b^5m^2+1689\,b^5m+945\,b^5\right)x^{11}+5\left(ab^4m^5+27\,ab^4m^4+262\,ab^4m^3+1122\,ab^4m^2+2041\,ab^4m+1155\,ab^4\right)x^9+10\left(a^2b^3m^5+29\,a^2b^3m^4+302\,a^2b^3m^3+1366\,a^2b^3m^2+2577\,a^2b^3m+1485\,a^2b^3\right)x^7+10\left(a^3b^2m^5+31\,a^3b^2m^4+350\,a^3b^2m^3+1730\,a^3b^2m^2+3489\,a^3b^2m+2079\,a^3b^2\right)x^5+5\left(a^4bm^5+33\,a^4bm^4+406\,a^4bm^3+2262\,a^4bm^2+5353\,a^4bm+3465\,a^4b\right)x^3+\left(a^5m^5+35\,a^5m^4+470\,a^5m^3+3010\,a^5m^2+9129\,a^5m+10395\,a^5\right)x\right)x^m}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m + 945*b^5)*
x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 1122*a*b^4*m^2 + 2041*
a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 + 29*a^2*b^3*m^4 + 302*a^2*b^3*
m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b^2*m
^5 + 31*a^3*b^2*m^4 + 350*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^2*m +
 2079*a^3*b^2)*x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 2262*a^4
*b*m^2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 470*a^5*m
^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*x^m/(m^6 + 36*m^5 + 505*m^4
+ 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

giac [B] time = 0.69, size = 540, normalized size = 5.57

$$\frac{b^5m^5x^{11}x^m+25\,b^5m^4x^{11}x^m+5\,ab^4m^5x^9x^m+230\,b^5m^3x^{11}x^m+135\,ab^4m^4x^9x^m+950\,b^5m^2x^{11}x^m+10\,a^2b^3m^5x^7x^m+1310\,ab^4m^3x^9x^m+1689\,b^5mx^{11}x^m+290\,a^2b^3m^4x^7x^m+5610\,ab^4m^2x^9x^m+945\,b^5x^{11}x^m+10\,a^3b^2m^5x^5x^m+3020\,a^2b^3m^3x^7x^m+10205\,ab^4mx^9x^m+310\,a^3b^2m^4x^5x^m+13660\,a^2b^3m^2x^7x^m+5775\,ab^4x^9x^m+5\,a^4bm^5x^3x^m+3500\,a^3b^2m^3x^5x^m+25770\,a^2b^3mx^7x^m+165\,a^4bm^4x^3x^m+17300\,a^3b^2m^2x^5x^m+14850\,a^2b^3x^7x^m+a^5m^5xx^m+2030\,a^4bm^3x^3x^m+34890\,a^3b^2mx^5x^m+35\,a^5m^4xx^m+11310\,a^4bm^2x^3x^m+20790\,a^3b^2x^5x^m+470\,a^5m^3xx^m+26765\,a^4bmx^3x^m+3010\,a^5m^2xx^m+17325\,a^4bx^3x^m+9129\,a^5mxx^m+10395\,a^5xx^m}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="giac")
```

```
[Out] (b^5*m^5*x^11*x^m + 25*b^5*m^4*x^11*x^m + 5*a*b^4*m^5*x^9*x^m + 230*b^5*m^3
*x^11*x^m + 135*a*b^4*m^4*x^9*x^m + 950*b^5*m^2*x^11*x^m + 10*a^2*b^3*m^5*x
^7*x^m + 1310*a*b^4*m^3*x^9*x^m + 1689*b^5*m*x^11*x^m + 290*a^2*b^3*m^4*x^7
*x^m + 5610*a*b^4*m^2*x^9*x^m + 945*b^5*x^11*x^m + 10*a^3*b^2*m^5*x^5*x^m +
 3020*a^2*b^3*m^3*x^7*x^m + 10205*a*b^4*m*x^9*x^m + 310*a^3*b^2*m^4*x^5*x^m
 + 13660*a^2*b^3*m^2*x^7*x^m + 5775*a*b^4*x^9*x^m + 5*a^4*b*m^5*x^3*x^m + 3
500*a^3*b^2*m^3*x^5*x^m + 25770*a^2*b^3*m*x^7*x^m + 165*a^4*b*m^4*x^3*x^m +
```

```
 17300*a^3*b^2*m^2*x^5*x^m + 14850*a^2*b^3*x^7*x^m + a^5*m^5*x*x^m + 2030*a
^4*b*m^3*x^3*x^m + 34890*a^3*b^2*m*x^5*x^m + 35*a^5*m^4*x*x^m + 11310*a^4*b
*m^2*x^3*x^m + 20790*a^3*b^2*x^5*x^m + 470*a^5*m^3*x*x^m + 26765*a^4*b*m*x^
3*x^m + 3010*a^5*m^2*x*x^m + 17325*a^4*b*x^3*x^m + 9129*a^5*m*x*x^m + 10395
*a^5*x*x^m)/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 1039
5)
```

maple [B] time = 0.01, size = 432, normalized size = 4.45

$$\frac{\left(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+1689b^5m\,x^{10}+290a^2b^3m^4x^6+5610a\,b^4m^2x^8+945b^5x^{10}+10a^3b^2m^5x^4+3020a^2b^3m^3x^6+10205a\,b^4m\,x^8+310a^3b^2m^4x^4+13660a^2b^3m^2x^6+5775a\,b^4x^8+5a^4b\,m^5x^2+3500a^3b^2m^3x^4+25770a^2b^3m\,x^6+165a^4b\,m^4x^2+17300a^3b^2m^2x^4+14850a^2b^3x^6+a^5m^5+2030a^4b\,m^3x^2+34890a^3b^2m\,x^4+35a^5m^4+11310a^4b\,m^2x^2+20790a^3b^2x^4+470a^5m^3+26765a^4bm\,x^2+3010a^5m^2+17325a^4b\,x^2+9129a^5m+10395a^5\right)x^{m+1}}{(m+11)(m+9)(m+7)(m+5)(m+3)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^5,x)
```

```
[Out] x^(m+1)*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+135*
a*b^4*m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*m^3*x^8+1689*b
^5*m*x^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+945*b^5*x^10+10*a^3*b^2*m^
5*x^4+3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*x^8+310*a^3*b^2*m^4*x^4+13660*a^2*
b^3*m^2*x^6+5775*a*b^4*x^8+5*a^4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b
^3*m*x^6+165*a^4*b*m^4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+
2030*a^4*b*m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+20790
*a^3*b^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*b*x^2+912
9*a^5*m+10395*a^5)/(11+m)/(9+m)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.33, size = 97, normalized size = 1.00

$$\frac{b^5x^{m+11}}{m+11}+\frac{5\,ab^4x^{m+9}}{m+9}+\frac{10\,a^2b^3x^{m+7}}{m+7}+\frac{10\,a^3b^2x^{m+5}}{m+5}+\frac{5\,a^4bx^{m+3}}{m+3}+\frac{a^5x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^5,x, algorithm="maxima")
```

```
[Out] b^5*x^(m + 11)/(m + 11) + 5*a*b^4*x^(m + 9)/(m + 9) + 10*a^2*b^3*x^(m + 7)/
(m + 7) + 10*a^3*b^2*x^(m + 5)/(m + 5) + 5*a^4*b*x^(m + 3)/(m + 3) + a^5*x^
(m + 1)/(m + 1)
```

mupad [B] time = 5.11, size = 389, normalized size = 4.01

$$\frac{a^5\,x\,x^m\,\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{b^5\,x^m\,x^{11}\,\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{5\,a\,b^4\,x^m\,x^9\,\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{5\,a^4\,b\,x^m\,x^3\,\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{10\,a^2\,b^3\,x^m\,x^7\,\left(m^5+29\,m^4+302\,m^3+1366\,m^2+2577\,m+1485\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{10\,a^3\,b^2\,x^m\,x^5\,\left(m^5+31\,m^4+350\,m^3+1730\,m^2+3489\,m+2079\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^5,x)
```

```
[Out] (a^5*x*x^m*(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395))/(19524*m +
 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (b^5*x^m*x^11*(16
```

```
89*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))/(19524*m + 12139*m^2 + 3480
*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (5*a*b^4*x^m*x^9*(2041*m + 1122*m^
2 + 262*m^3 + 27*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m
^4 + 36*m^5 + m^6 + 10395) + (5*a^4*b*x^m*x^3*(5353*m + 2262*m^2 + 406*m^3
+ 33*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5
+ m^6 + 10395) + (10*a^2*b^3*x^m*x^7*(2577*m + 1366*m^2 + 302*m^3 + 29*m^4
+ m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 1
0395) + (10*a^3*b^2*x^m*x^5*(3489*m + 1730*m^2 + 350*m^3 + 31*m^4 + m^5 + 2
079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395)
```

sympy [A] time = 4.50, size = 1999, normalized size = 20.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**5,x)
```

```
[Out] Piecewise((-a**5/(10*x**10) - 5*a**4*b/(8*x**8) - 5*a**3*b**2/(3*x**6) - 5*
a**2*b**3/(2*x**4) - 5*a*b**4/(2*x**2) + b**5*log(x), Eq(m, -11)), (-a**5/(
8*x**8) - 5*a**4*b/(6*x**6) - 5*a**3*b**2/(2*x**4) - 5*a**2*b**3/x**2 + 5*a
*b**4*log(x) + b**5*x**2/2, Eq(m, -9)), (-a**5/(6*x**6) - 5*a**4*b/(4*x**4)
 - 5*a**3*b**2/x**2 + 10*a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4,
Eq(m, -7)), (-a**5/(4*x**4) - 5*a**4*b/(2*x**2) + 10*a**3*b**2*log(x) + 5*a
**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6, Eq(m, -5)), (-a**5/(2*x**2)
+ 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 + 5*a*b**4*x**6/6
 + b**5*x**8/8, Eq(m, -3)), (a**5*log(x) + 5*a**4*b*x**2/2 + 5*a**3*b**2*x*
*4/2 + 5*a**2*b**3*x**6/3 + 5*a*b**4*x**8/8 + b**5*x**10/10, Eq(m, -1)), (a
**5*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 35*a**5*m**4*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 470*a**5*m**3*x*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3010*a**5*m**2*x*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 9129
*a**5*m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 10395*a**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 5*a**4*b*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 165*a**4*b*m**4*x**3*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
2030*a**4*b*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 11310*a**4*b*m**2*x**3*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 26765*a**4*b*m*x**3*x**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1
7325*a**4*b*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 10*a**3*b**2*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4
+ 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 310*a**3*b**2*m**4*x**5*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35
```

```
00*a**3*b**2*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 17300*a**3*b**2*m**2*x**5*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 34890*a**3*b**2*m*x*
*5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 20790*a**3*b**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 10*a**2*b**3*m**5*x**7*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 290*a**2*b**3*m**4*
x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 3020*a**2*b**3*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 13660*a**2*b**3*m**2*x**7*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25770*a**2
*b**3*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 195
24*m + 10395) + 14850*a**2*b**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 5*a*b**4*m**5*x**9*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 135*a*b**4*m*
*4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 1310*a*b**4*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 5610*a*b**4*m**2*x**9*x**m/(m**6 + 36*m
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10205*a*b**4*m
*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
10395) + 5775*a*b**4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + b**5*m**5*x**11*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25*b**5*m**4*x**11*x**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 230*b
**5*m**3*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 950*b**5*m**2*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 1689*b**5*m*x**11*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 945*b**5*x**1
1*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5), True))
```

3.340 $\int x^m \left(a+bx^2\right)^4 dx$

Optimal. Leaf size=79

$$\frac{a^4x^{m+1}}{m+1}+\frac{4a^3bx^{m+3}}{m+3}+\frac{6a^2b^2x^{m+5}}{m+5}+\frac{4ab^3x^{m+7}}{m+7}+\frac{b^4x^{m+9}}{m+9}$$

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{6a^2b^2x^{m+5}}{m+5}+\frac{4a^3bx^{m+3}}{m+3}+\frac{a^4x^{m+1}}{m+1}+\frac{4ab^3x^{m+7}}{m+7}+\frac{b^4x^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^4,x]
```

```
[Out] (a^4*x^(1 + m))/(1 + m) + (4*a^3*b*x^(3 + m))/(3 + m) + (6*a^2*b^2*x^(5 + m
))/(5 + m) + (4*a*b^3*x^(7 + m))/(7 + m) + (b^4*x^(9 + m))/(9 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^4 dx &= \int \left(a^4x^m+4a^3bx^{2+m}+6a^2b^2x^{4+m}+4ab^3x^{6+m}+b^4x^{8+m}\right) dx\\ &= \frac{a^4x^{1+m}}{1+m}+\frac{4a^3bx^{3+m}}{3+m}+\frac{6a^2b^2x^{5+m}}{5+m}+\frac{4ab^3x^{7+m}}{7+m}+\frac{b^4x^{9+m}}{9+m}\end{aligned}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.91

$$x^{m+1}\left(\frac{a^4}{m+1}+\frac{4a^3bx^2}{m+3}+\frac{6a^2b^2x^4}{m+5}+\frac{4ab^3x^6}{m+7}+\frac{b^4x^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^4,x]
```

```
[Out] x^(1 + m)*(a^4/(1 + m) + (4*a^3*b*x^2)/(3 + m) + (6*a^2*b^2*x^4)/(5 + m) +
(4*a*b^3*x^6)/(7 + m) + (b^4*x^8)/(9 + m))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^m \left(a + bx^2\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2)^4,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2)^4, x]
```

fricas [B] time = 0.69, size = 251, normalized size = 3.18

$$\frac{\left(\left(b^4 m^4 + 16\, b^4 m^3 + 86\, b^4 m^2 + 176\, b^4 m + 105\, b^4\right) x^9 + 4\left(a b^3 m^4 + 18\, a b^3 m^3 + 104\, a b^3 m^2 + 222\, a b^3 m + 135\, a b^3\right) x^7 + 6\left(a^2 b^2 m^4 + 20\, a^2 b^2 m^3 + 130\, a^2 b^2 m^2 + 300\, a^2 b^2 m + 189\, a^2 b^2\right) x^5 + 4\left(a^3 b m^4 + 22\, a^3 b m^3 + 164\, a^3 b m^2 + 458\, a^3 b m + 315\, a^3 b\right) x^3 + \left(a^4 m^4 + 24\, a^4 m^3 + 206\, a^4 m^2 + 744\, a^4 m + 945\, a^4\right) x\right) x^m}{m^5 + 25\, m^4 + 230\, m^3 + 950\, m^2 + 1689\, m + 945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] ((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 + 4*(a*b^3*m
^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a*b^3)*x^7 + 6*(a^2*b
^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 300*a^2*b^2*m + 189*a^2*b^2)*x^
5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 164*a^3*b*m^2 + 458*a^3*b*m + 315*a^3*b)*
x^3 + (a^4*m^4 + 24*a^4*m^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*x^m/(m^
5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [B] time = 0.61, size = 365, normalized size = 4.62

$$\frac{b^4 m^4 x^9 x^m + 16\, b^4 m^3 x^9 x^m + 4\, a b^3 m^4 x^7 x^m + 86\, b^4 m^2 x^9 x^m + 72\, a b^3 m^3 x^7 x^m + 176\, b^4 m x^9 x^m + 6\, a^2 b^2 m^4 x^5 x^m + 416\, a b^3 m^2 x^7 x^m + 105\, b^4 x^9 x^m + 120\, a^2 b^2 m^3 x^5 x^m + 888\, a b^3 m x^7 x^m + 4\, a^3 b m^4 x^3 x^m + 780\, a^2 b^2 m^2 x^5 x^m + 540\, a b^3 x^7 x^m + 88\, a^3 b m^3 x^3 x^m + 1800\, a^2 b^2 m x^5 x^m + a^4 m^4 x x^m + 656\, a^3 b m^2 x^3 x^m + 1134\, a^2 b^2 x^5 x^m + 24\, a^4 m^3 x x^m + 1832\, a^3 b m x^3 x^m + 206\, a^4 m^2 x x^m + 1260\, a^3 b x^3 x^m + 744\, a^4 m x x^m + 945\, a^4 x x^m}{m^5 + 25\, m^4 + 230\, m^3 + 950\, m^2 + 1689\, m + 945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] (b^4*m^4*x^9*x^m + 16*b^4*m^3*x^9*x^m + 4*a*b^3*m^4*x^7*x^m + 86*b^4*m^2*x^
9*x^m + 72*a*b^3*m^3*x^7*x^m + 176*b^4*m*x^9*x^m + 6*a^2*b^2*m^4*x^5*x^m +
416*a*b^3*m^2*x^7*x^m + 105*b^4*x^9*x^m + 120*a^2*b^2*m^3*x^5*x^m + 888*a*b
^3*m*x^7*x^m + 4*a^3*b*m^4*x^3*x^m + 780*a^2*b^2*m^2*x^5*x^m + 540*a*b^3*x^
7*x^m + 88*a^3*b*m^3*x^3*x^m + 1800*a^2*b^2*m*x^5*x^m + a^4*m^4*x*x^m + 656
*a^3*b*m^2*x^3*x^m + 1134*a^2*b^2*x^5*x^m + 24*a^4*m^3*x*x^m + 1832*a^3*b*m
*x^3*x^m + 206*a^4*m^2*x*x^m + 1260*a^3*b*x^3*x^m + 744*a^4*m*x*x^m + 945*a
^4*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

maple [B] time = 0.01, size = 291, normalized size = 3.68

$$\frac{\left(b^4 m^4 x^8 + 16 b^4 m^3 x^8 + 4 a\, b^3 m^4 x^6 + 86 b^4 m^2 x^8 + 72 a\, b^3 m^3 x^6 + 176 b^4 m\, x^8 + 6 a^2 b^2 m^4 x^4 + 416 a\, b^3 m^2 x^6 + 105 b^4 x^8 + 120 a^2 b^2 m^3 x^4 + 888 a\, b^3 m\, x^6 + 4 a^3 b\, m^4 x^2 + 780 a^2 b^2 m^2 x^4 + 540 a\, b^3 x^6 + 88 a^3 b\, m^3 x^2 + 1800 a^2 b^2 m\, x^4 + a^4 m^4 + 656 a^3 b\, m^2 x^2 + 1134 a^2 b^2 x^4 + 24 a^4 m^3 + 1832 a^3 b m\, x^2 + 206 a^4 m^2 + 1260 a^3 b\, x^2 + 744 a^4 m + 945 a^4\right) x^{m+1}}{\left(m + 9\right) \left(m + 7\right) \left(m + 5\right) \left(m + 3\right) \left(m + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^4,x)
```

```
[Out] x^(m+1)*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^8+72*a*b^3
*m^3*x^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*x^6+105*b^4*x^8+120*
a^2*b^2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x^2+780*a^2*b^2*m^2*x^4+540*a*b
^3*x^6+88*a^3*b*m^3*x^2+1800*a^2*b^2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+1134*a
^2*b^2*x^4+24*a^4*m^3+1832*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*a^4*m
+945*a^4)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.34, size = 79, normalized size = 1.00

$$\frac{b^4x^{m+9}}{m+9}+\frac{4\,ab^3x^{m+7}}{m+7}+\frac{6\,a^2b^2x^{m+5}}{m+5}+\frac{4\,a^3bx^{m+3}}{m+3}+\frac{a^4x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] b^4*x^(m + 9)/(m + 9) + 4*a*b^3*x^(m + 7)/(m + 7) + 6*a^2*b^2*x^(m + 5)/(m
+ 5) + 4*a^3*b*x^(m + 3)/(m + 3) + a^4*x^(m + 1)/(m + 1)
```

mupad [B] time = 4.99, size = 272, normalized size = 3.44

$$\frac{a^4\,x\,x^m\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{b^4\,x^m\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{6\,a^2\,b^2\,x^m\,x^5\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{4\,a\,b^3\,x^m\,x^7\,\left(m^4+18\,m^3+104\,m^2+222\,m+135\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{4\,a^3\,b\,x^m\,x^3\,\left(m^4+22\,m^3+164\,m^2+458\,m+315\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^4,x)
```

```
[Out] (a^4*x*x^m*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 230*
m^3 + 25*m^4 + m^5 + 945) + (b^4*x^m*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 1
05))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (6*a^2*b^2*x^m*x^5
*(300*m + 130*m^2 + 20*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m
^4 + m^5 + 945) + (4*a*b^3*x^m*x^7*(222*m + 104*m^2 + 18*m^3 + m^4 + 135))/
(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (4*a^3*b*x^m*x^3*(458*m
 + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^
5 + 945)
```

sympy [A] time = 2.85, size = 1221, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**4,x)
```

```
[Out] Piecewise((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*a*
b**3/x**2 + b**4*log(x), Eq(m, -9)), (-a**4/(6*x**6) - a**3*b/x**4 - 3*a**2
*b**2/x**2 + 4*a*b**3*log(x) + b**4*x**2/2, Eq(m, -7)), (-a**4/(4*x**4) - 2
*a**3*b/x**2 + 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4, Eq(m, -5))
, (-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4 + b**4
*x**6/6, Eq(m, -3)), (a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*
a*b**3*x**6/3 + b**4*x**8/8, Eq(m, -1)), (a**4*m**4*x*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 24*a**4*m**3*x*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 206*a**4*m**2*x*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*a**4*m*x*x**m/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 945*a**4*x*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a**3*b*m**4*x**3*x**m/(m**5 + 25*m
**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*m**3*x**3*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*m**2*x**3*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1832*a**3*b*m*x**3
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1260*a**3*b*x
**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 6*a**2*b**
2*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 12
0*a**2*b**2*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 780*a**2*b**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
+ 1689*m + 945) + 1800*a**2*b**2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 1134*a**2*b**2*x**5*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 4*a*b**3*m**4*x**7*x**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 72*a*b**3*m**3*x**7*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 416*a*b**3*m**2*x**7*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 888*a*b**3*m*x**7*x**m/(m
**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 540*a*b**3*x**7*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**4*m**4*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*b**4*m**3*x**9*
x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**4*m**2*x
**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*b**4*m
*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*b**4
*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

3.341 $\int x^m \left(a+bx^2\right)^3 \, dx$

Optimal. Leaf size=61

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+3}}{m+3}+\frac{3ab^2x^{m+5}}{m+5}+\frac{b^3x^{m+7}}{m+7}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3a^2bx^{m+3}}{m+3}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+5}}{m+5}+\frac{b^3x^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(3 + m))/(3 + m) + (3*a*b^2*x^(5 + m))
/(5 + m) + (b^3*x^(7 + m))/(7 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^3 \, dx &= \int \left(a^3x^m+3a^2bx^{2+m}+3ab^2x^{4+m}+b^3x^{6+m}\right) dx\\ &= \frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{3+m}}{3+m}+\frac{3ab^2x^{5+m}}{5+m}+\frac{b^3x^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.92

$$x^{m+1}\left(\frac{a^3}{m+1}+\frac{3a^2bx^2}{m+3}+\frac{3ab^2x^4}{m+5}+\frac{b^3x^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x^2)/(3 + m) + (3*a*b^2*x^4)/(5 + m) + (b
^3*x^6)/(7 + m))
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2)^3, x]
```

fricas [B] time = 0.91, size = 157, normalized size = 2.57

$$\frac{\left(\left(b^3m^3+9\,b^3m^2+23\,b^3m+15\,b^3\right)x^7+3\left(ab^2m^3+11\,ab^2m^2+31\,ab^2m+21\,ab^2\right)x^5+3\left(a^2bm^3+13\,a^2bm^2+47\,a^2bm+35\,a^2b\right)x^3+\left(a^3m^3+15\,a^3m^2+71\,a^3m+105\,a^3\right)x\right)x^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 11*a*b^2*m^
2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*b*m^2 + 47*a^2*b*m +
 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a^3*m + 105*a^3)*x)*x^m/(m^4 +
16*m^3 + 86*m^2 + 176*m + 105)
```

giac [B] time = 0.66, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^7x^m+9\,b^3m^2x^7x^m+3\,ab^2m^3x^5x^m+23\,b^3mx^7x^m+33\,ab^2m^2x^5x^m+15\,b^3x^7x^m+3\,a^2bm^3x^3x^m+93\,ab^2mx^5x^m+39\,a^2bm^2x^3x^m+63\,ab^2x^5x^m+a^3m^3xx^m+141\,a^2bmx^3x^m+15\,a^3m^2xx^m+105\,a^2bx^3x^m+71\,a^3mxx^m+105\,a^3xx^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^7*x^m + 9*b^3*m^2*x^7*x^m + 3*a*b^2*m^3*x^5*x^m + 23*b^3*m*x^7*x
^m + 33*a*b^2*m^2*x^5*x^m + 15*b^3*x^7*x^m + 3*a^2*b*m^3*x^3*x^m + 93*a*b^2
*m*x^5*x^m + 39*a^2*b*m^2*x^3*x^m + 63*a*b^2*x^5*x^m + a^3*m^3*x*x^m + 141*
a^2*b*m*x^3*x^m + 15*a^3*m^2*x*x^m + 105*a^2*b*x^3*x^m + 71*a^3*m*x*x^m + 1
05*a^3*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

maple [B] time = 0.01, size = 178, normalized size = 2.92

$$\frac{\left(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23b^3m\,x^6+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93a\,b^2m\,x^4+39a^2b\,m^2x^2+63a\,b^2x^4+a^3m^3+141a^2bm\,x^2+15a^3m^2+105a^2b\,x^2+71a^3m+105a^3\right)x^{m+1}}{(m+7)\,(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^3,x)
```

```
[Out] x^(m+1)*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^
2*x^4+15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x
^4+a^3*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)/(m+7)
/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.34, size = 61, normalized size = 1.00

$$\frac{b^3x^{m+7}}{m+7}+\frac{3\,ab^2x^{m+5}}{m+5}+\frac{3\,a^2bx^{m+3}}{m+3}+\frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 7)/(m + 7) + 3*a*b^2*x^(m + 5)/(m + 5) + 3*a^2*b*x^(m + 3)/(m +
3) + a^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 4.79, size = 167, normalized size = 2.74

$$x^m\,\left(\frac{a^3\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b^3\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{3\,a\,b^2\,x^5\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{3\,a^2\,b\,x^3\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^3,x)
```

```
[Out] x^m*((a^3*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^3 + m^4 + 1
05) + (b^3*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 +
105) + (3*a*b^2*x^5*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 + 16*m^3 +
m^4 + 105) + (3*a^2*b*x^3*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16*
m^3 + m^4 + 105))
```

sympy [A] time = 1.66, size = 683, normalized size = 11.20

$$\begin{cases} -\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{2x^2}+b^3\log(x) & \text{for } m=-7 \\ -\frac{a^3}{4x^4}-\frac{3a^2b}{2x^2}+3ab^2\log(x)+\frac{b^3x^2}{2} & \text{for } m=-5 \\ -\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3ab^2x^2}{2}+\frac{b^3x^4}{4} & \text{for } m=-3 \\ a^3\log(x)+\frac{3a^2bx^2}{2}+\frac{3ab^2x^4}{4}+\frac{b^3x^6}{6} & \text{for } m=-1 \\ \frac{a^3m^3xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{15a^3m^2xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{71a^3mxx^m}{m^4+16m^3+86m^2+176m+105}+\frac{105a^3xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{3a^2bm^3x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{39a^2bm^2x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{141a^2bmx^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{105a^2bx^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{3ab^2m^3x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{33ab^2m^2x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{93ab^2mx^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{63ab^2x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{b^3m^3x^7x^m}{m^4+16m^3+86m^2+176m+105}+\frac{9b^3m^2x^7x^m}{m^4+16m^3+86m^2+176m+105}+\frac{23b^3mx^7x^m}{m^4+16m^3+86m^2+176m+105}+\frac{15b^3x^7x^m}{m^4+16m^3+86m^2+176m+105} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a*b**2/(2*x**2) + b**3*lo
g(x), Eq(m, -7)), (-a**3/(4*x**4) - 3*a**2*b/(2*x**2) + 3*a*b**2*log(x) + b
**3*x**2/2, Eq(m, -5)), (-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2
 + b**3*x**4/4, Eq(m, -3)), (a**3*log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x**4/
4 + b**3*x**6/6, Eq(m, -1)), (a**3*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 +
176*m + 105) + 15*a**3*m**2*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
```

```
 + 71*a**3*m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*a**3*x*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 3*a**2*b*m**3*x**3*x**m/(m**
4 + 16*m**3 + 86*m**2 + 176*m + 105) + 39*a**2*b*m**2*x**3*x**m/(m**4 + 16*
m**3 + 86*m**2 + 176*m + 105) + 141*a**2*b*m*x**3*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 105*a**2*b*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 3*a*b**2*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105
) + 33*a*b**2*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 93*
a*b**2*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 63*a*b**2*x**
5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b**3*m**3*x**7*x**m/(m**4
 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*b**3*m**2*x**7*x**m/(m**4 + 16*m**3
 + 86*m**2 + 176*m + 105) + 23*b**3*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 +
 176*m + 105) + 15*b**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105),
 True))
```

3.342 $\int x^m \left(a+bx^2\right)^2 dx$

Optimal. Leaf size=43

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+3}}{m+3}+\frac{b^2x^{m+5}}{m+5}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+3}}{m+3}+\frac{b^2x^{m+5}}{m+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(3 + m))/(3 + m) + (b^2*x^(5 + m))/(5 +
m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^2 dx &= \int \left(a^2x^m+2abx^{2+m}+b^2x^{4+m}\right) dx\\ &= \frac{a^2x^{1+m}}{1+m}+\frac{2abx^{3+m}}{3+m}+\frac{b^2x^{5+m}}{5+m}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2}{m+1}+\frac{2abx^2}{m+3}+\frac{b^2x^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^m \left(a + bx^2\right)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2)^2, x]
```

fricas [A] time = 0.93, size = 85, normalized size = 1.98

$$\frac{\left(\left(b^2m^2 + 4\,b^2m + 3\,b^2\right)x^5 + 2\left(abm^2 + 6\,abm + 5\,ab\right)x^3 + \left(a^2m^2 + 8\,a^2m + 15\,a^2\right)x\right)x^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^2
*m^2 + 8*a^2*m + 15*a^2)*x)*x^m/(m^3 + 9*m^2 + 23*m + 15)
```

giac [B] time = 0.65, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^5x^m + 4\,b^2mx^5x^m + 2\,abm^2x^3x^m + 3\,b^2x^5x^m + 12\,abmx^3x^m + a^2m^2xx^m + 10\,abx^3x^m + 8\,a^2mxx^m + 15\,a^2xx^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^5*x^m + 4*b^2*m*x^5*x^m + 2*a*b*m^2*x^3*x^m + 3*b^2*x^5*x^m + 12
*a*b*m*x^3*x^m + a^2*m^2*x*x^m + 10*a*b*x^3*x^m + 8*a^2*m*x*x^m + 15*a^2*x*
x^m)/(m^3 + 9*m^2 + 23*m + 15)
```

maple [B] time = 0.01, size = 93, normalized size = 2.16

$$\frac{\left(b^2m^2x^4 + 4b^2m\,x^4 + 2ab\,m^2x^2 + 3b^2x^4 + 12abm\,x^2 + a^2m^2 + 10ab\,x^2 + 8a^2m + 15a^2\right)x^{m+1}}{(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2,x)
```

```
[Out] x^(m+1)*(b^2*m^2*x^4+4*b^2*m*x^4+2*a*b*m^2*x^2+3*b^2*x^4+12*a*b*m*x^2+a^2*m
^2+10*a*b*x^2+8*a^2*m+15*a^2)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.31, size = 43, normalized size = 1.00

$$\frac{b^2x^{m+5}}{m+5}+\frac{2\,abx^{m+3}}{m+3}+\frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 5)/(m + 5) + 2*a*b*x^(m + 3)/(m + 3) + a^2*x^(m + 1)/(m + 1)
```

mupad [B] time = 4.75, size = 93, normalized size = 2.16

$$x^m\left(\frac{a^2\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}+\frac{b^2\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15}+\frac{2\,a\,b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^2,x)
```

```
[Out] x^m*((a^2*x*(8*m + m^2 + 15))/(23*m + 9*m^2 + m^3 + 15) + (b^2*x^5*(4*m + m
^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (2*a*b*x^3*(6*m + m^2 + 5))/(23*m + 9*
m^2 + m^3 + 15))
```

sympy [A] time = 0.94, size = 306, normalized size = 7.12

$$\begin{cases} -\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x) & \text{for}\ m=-5 \\ -\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2} & \text{for}\ m=-3 \\ a^2\log(x)+abx^2+\frac{b^2x^4}{4} & \text{for}\ m=-1 \\ \frac{a^2m^2xx^m}{m^3+9m^2+23m+15}+\frac{8a^2mxx^m}{m^3+9m^2+23m+15}+\frac{15a^2xx^m}{m^3+9m^2+23m+15}+\frac{2abm^2x^3x^m}{m^3+9m^2+23m+15}+\frac{12abmx^3x^m}{m^3+9m^2+23m+15}+\frac{10abx^3x^m}{m^3+9m^2+23m+15}+\frac{b^2m^2x^5x^m}{m^3+9m^2+23m+15}+\frac{4b^2mx^5x^m}{m^3+9m^2+23m+15}+\frac{3b^2x^5x^m}{m^3+9m^2+23m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x), Eq(m, -5)), (-a**2/(2*x
**2) + 2*a*b*log(x) + b**2*x**2/2, Eq(m, -3)), (a**2*log(x) + a*b*x**2 + b*
*2*x**4/4, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8*a*
*2*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a**2*x*x**m/(m**3 + 9*m**2 + 2
3*m + 15) + 2*a*b*m**2*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 12*a*b*m*x**
3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 10*a*b*x**3*x**m/(m**3 + 9*m**2 + 23*m
 + 15) + b**2*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*m*x**5*x*
*m/(m**3 + 9*m**2 + 23*m + 15) + 3*b**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 1
5), True))
```

3.343 $\int x^m\left(a+bx^2\right)\,dx$

Optimal. Leaf size=25

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+3}}{m+3}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)\,dx&=\int\left(ax^m+bx^{2+m}\right)\,dx\\&=\frac{ax^{1+m}}{1+m}+\frac{bx^{3+m}}{3+m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^m \left(a + bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2),x]
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2), x]
```

fricas [A] time = 0.87, size = 33, normalized size = 1.32

$$\frac{\left((bm+b)x^3 + (am+3\,a)x\right)x^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="fricas")
[Out] ((b*m + b)*x^3 + (a*m + 3*a)*x)*x^m/(m^2 + 4*m + 3)
```

giac [A] time = 0.69, size = 43, normalized size = 1.72

$$\frac{bmx^3x^m + bx^3x^m + amxx^m + 3\,axx^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="giac")
[Out] (b*m*x^3*x^m + b*x^3*x^m + a*m*x*x^m + 3*a*x*x^m)/(m^2 + 4*m + 3)
```

maple [A] time = 0.00, size = 35, normalized size = 1.40

$$\frac{\left(bm\,x^2 + b\,x^2 + am + 3a\right)x^{m+1}}{(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a),x)
[Out] x^(m+1)*(b*m*x^2+b*x^2+a*m+3*a)/(m+3)/(m+1)
```

maxima [A] time = 1.30, size = 25, normalized size = 1.00

$$\frac{bx^{m+3}}{m+3} + \frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a),x, algorithm="maxima")
```

```
[Out] b*x^(m + 3)/(m + 3) + a*x^(m + 1)/(m + 1)
```

mupad [B] time = 4.80, size = 34, normalized size = 1.36

$$\frac{x^{m+1}\,\left(3\,a+a\,m+b\,x^2+b\,m\,x^2\right)}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2),x)
```

```
[Out] (x^(m + 1)*(3*a + a*m + b*x^2 + b*m*x^2))/(4*m + m^2 + 3)
```

sympy [A] time = 0.41, size = 94, normalized size = 3.76

$$\begin{cases} -\frac{a}{2x^2} + b\log{(x)} & \text{for}\: m = -3 \\ a\log{(x)} + \frac{bx^2}{2} & \text{for}\: m = -1 \\ \frac{amxx^m}{m^2+4m+3} + \frac{3axx^m}{m^2+4m+3} + \frac{bmx^3x^m}{m^2+4m+3} + \frac{bx^3x^m}{m^2+4m+3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a),x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*log(x), Eq(m, -3)), (a*log(x) + b*x**2/2, Eq(m,
-1)), (a*m*x*x**m/(m**2 + 4*m + 3) + 3*a*x*x**m/(m**2 + 4*m + 3) + b*m*x**3
*x**m/(m**2 + 4*m + 3) + b*x**3*x**m/(m**2 + 4*m + 3), True))
```

3.344 $\int x^7\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{3a^2\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{\left(a+bx^2\right)^{9/2}}{9b^4}-\frac{3a\left(a+bx^2\right)^{7/2}}{7b^4}$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{5/2}}{5b^4}-\frac{a^3\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{\left(a+bx^2\right)^{9/2}}{9b^4}-\frac{3a\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[a + b*x^2],x]
```

```
[Out] -(a^3*(a + b*x^2)^(3/2))/(3*b^4) + (3*a^2*(a + b*x^2)^(5/2))/(5*b^4) - (3*a
*(a + b*x^2)^(7/2))/(7*b^4) + (a + b*x^2)^(9/2)/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\sqrt{a+bx^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x^3\sqrt{a+bx}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx,x,x^2\right)\\&=-\frac{a^3\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{3a^2\left(a+bx^2\right)^{5/2}}{5b^4}-\frac{3a\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{\left(a+bx^2\right)^{9/2}}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{3/2}\left(-16a^3+24a^2bx^2-30ab^2x^4+35b^3x^6\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[a + b*x^2],x]
[Out] ((a + b*x^2)^(3/2)*(-16*a^3 + 24*a^2*b*x^2 - 30*a*b^2*x^4 + 35*b^3*x^6))/(3
15*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 61, normalized size = 0.76

$$\frac{\sqrt{a+bx^2}\left(-16a^4+8a^3bx^2-6a^2b^2x^4+5ab^3x^6+35b^4x^8\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(-16*a^4 + 8*a^3*b*x^2 - 6*a^2*b^2*x^4 + 5*a*b^3*x^6 + 35*
b^4*x^8))/(315*b^4)
```

fricas [A] time = 1.01, size = 57, normalized size = 0.71

$$\frac{\left(35\,b^4x^8+5\,ab^3x^6-6\,a^2b^2x^4+8\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/315*(35*b^4*x^8 + 5*a*b^3*x^6 - 6*a^2*b^2*x^4 + 8*a^3*b*x^2 - 16*a^4)*sqr
t(b*x^2 + a)/b^4
```

giac [A] time = 0.60, size = 57, normalized size = 0.71

$$\frac{35\left(bx^2+a\right)^{\frac{9}{2}}-135\left(bx^2+a\right)^{\frac{7}{2}}a+189\left(bx^2+a\right)^{\frac{5}{2}}a^2-105\left(bx^2+a\right)^{\frac{3}{2}}a^3}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/315*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5/
2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-35b^{3}x^{6}+30a\,b^{2}x^{4}-24a^{2}b\,x^{2}+16a^{3}\right)}{315b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(1/2),x)
```

```
[Out] -1/315*(b*x^2+a)^(3/2)*(-35*b^3*x^6+30*a*b^2*x^4-24*a^2*b*x^2+16*a^3)/b^4
```

maxima [A] time = 1.36, size = 73, normalized size = 0.91

$$\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}x^{6}}{9\,b}-\frac{2\left(bx^{2}+a\right)^{\frac{3}{2}}ax^{4}}{21\,b^{2}}+\frac{8\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}x^{2}}{105\,b^{3}}-\frac{16\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}}{315\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(3/2)*x^6/b - 2/21*(b*x^2 + a)^(3/2)*a*x^4/b^2 + 8/105*(b*x
^2 + a)^(3/2)*a^2*x^2/b^3 - 16/315*(b*x^2 + a)^(3/2)*a^3/b^4
```

mupad [B] time = 4.59, size = 55, normalized size = 0.69

$$\sqrt{b\,x^{2}+a}\,\left(\frac{x^{8}}{9}-\frac{16\,a^{4}}{315\,b^{4}}+\frac{a\,x^{6}}{63\,b}-\frac{2\,a^{2}\,x^{4}}{105\,b^{2}}+\frac{8\,a^{3}\,x^{2}}{315\,b^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*(x^8/9 - (16*a^4)/(315*b^4) + (a*x^6)/(63*b) - (2*a^2*x^4
)/(105*b^2) + (8*a^3*x^2)/(315*b^3))
```

sympy [A] time = 1.42, size = 110, normalized size = 1.38

$$\begin{cases} -\frac{16a^{4}\sqrt{a+bx^{2}}}{315b^{4}}+\frac{8a^{3}x^{2}\sqrt{a+bx^{2}}}{315b^{3}}-\frac{2a^{2}x^{4}\sqrt{a+bx^{2}}}{105b^{2}}+\frac{ax^{6}\sqrt{a+bx^{2}}}{63b}+\frac{x^{8}\sqrt{a+bx^{2}}}{9} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^{8}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**4*sqrt(a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x*
*2)/(315*b**3) - 2*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a +
b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, True)
)
```

3.345 $\int x^5\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{2a\left(a+bx^2\right)^{5/2}}{5b^3}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{2a\left(a+bx^2\right)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(a + b*x^2)^(3/2))/(3*b^3) - (2*a*(a + b*x^2)^(5/2))/(5*b^3) + (a + b*
x^2)^(7/2)/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5\sqrt{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{a^2\left(a+bx^2\right)^{3/2}}{3b^3}-\frac{2a\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{\left(a+bx^2\right)^{7/2}}{7b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{3/2}\left(8a^2-12abx^2+15b^2x^4\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2],x]
```

```
[Out] ((a + b*x^2)^(3/2)*(8*a^2 - 12*a*b*x^2 + 15*b^2*x^4))/(105*b^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 50, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\left(8a^3-4a^2bx^2+3ab^2x^4+15b^3x^6\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^3 - 4*a^2*b*x^2 + 3*a*b^2*x^4 + 15*b^3*x^6))/(105*b^3
)
```

fricas [A] time = 1.35, size = 46, normalized size = 0.78

$$\frac{\left(15\,b^3x^6+3\,ab^2x^4-4\,a^2bx^2+8\,a^3\right)\sqrt{bx^2+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*x^6 + 3*a*b^2*x^4 - 4*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a)/b^3
```

giac [A] time = 0.59, size = 43, normalized size = 0.73

$$\frac{15\left(bx^2+a\right)^{\frac{7}{2}}-42\left(bx^2+a\right)^{\frac{5}{2}}a+35\left(bx^2+a\right)^{\frac{3}{2}}a^2}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2)
*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(15b^{2}x^{4}-12ab\,x^{2}+8a^{2}\right)}{105b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/105*(b*x^2+a)^(3/2)*(15*b^2*x^4-12*a*b*x^2+8*a^2)/b^3
```

maxima [A] time = 1.32, size = 53, normalized size = 0.90

$$\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}x^{4}}{7\,b}-\frac{4\left(bx^{2}+a\right)^{\frac{3}{2}}ax^{2}}{35\,b^{2}}+\frac{8\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(3/2)*x^4/b - 4/35*(b*x^2 + a)^(3/2)*a*x^2/b^2 + 8/105*(b*x
^2 + a)^(3/2)*a^2/b^3
```

mupad [B] time = 4.66, size = 44, normalized size = 0.75

$$\sqrt{b\,x^{2}+a}\,\left(\frac{x^{6}}{7}+\frac{8\,a^{3}}{105\,b^{3}}+\frac{a\,x^{4}}{35\,b}-\frac{4\,a^{2}\,x^{2}}{105\,b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*(x^6/7 + (8*a^3)/(105*b^3) + (a*x^4)/(35*b) - (4*a^2*x^2)
/(105*b^2))
```

sympy [A] time = 0.70, size = 87, normalized size = 1.47

$$\begin{cases}\frac{8a^{3}\sqrt{a+bx^{2}}}{105b^{3}}-\frac{4a^{2}x^{2}\sqrt{a+bx^{2}}}{105b^{2}}+\frac{ax^{4}\sqrt{a+bx^{2}}}{35b}+\frac{x^{6}\sqrt{a+bx^{2}}}{7} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^{6}}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2
)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne
(b, 0)), (sqrt(a)*x**6/6, True))
```

3.346 $\int x^3\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=38

$$\frac{(a+bx^2)^{5/2}}{5b^2}-\frac{a(a+bx^2)^{3/2}}{3b^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{(a+bx^2)^{5/2}}{5b^2}-\frac{a(a+bx^2)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2],x]
```

```
[Out] -(a*(a + b*x^2)^(3/2))/(3*b^2) + (a + b*x^2)^(5/2)/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\sqrt{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx,x,x^2\right)\\ &= -\frac{a(a+bx^2)^{3/2}}{3b^2}+\frac{(a+bx^2)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{3/2}\left(3bx^2-2a\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2],x]
[Out] ((a + b*x^2)^(3/2)*(-2*a + 3*b*x^2))/(15*b^2)
```

IntegrateAlgebraic [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\left(-2a^2+abx^2+3b^2x^4\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(-2*a^2 + a*b*x^2 + 3*b^2*x^4))/(15*b^2)
```

fricas [A] time = 1.19, size = 34, normalized size = 0.89

$$\frac{\left(3\,b^2x^4+abx^2-2\,a^2\right)\sqrt{bx^2+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*b^2*x^4 + a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a)/b^2
```

giac [A] time = 0.60, size = 29, normalized size = 0.76

$$\frac{3\left(bx^2+a\right)^{\frac{5}{2}}-5\left(bx^2+a\right)^{\frac{3}{2}}a}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/15*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(-3b\,x^2+2a\right)}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/2),x)
```

```
[Out] -1/15*(b*x^2+a)^(3/2)*(-3*b*x^2+2*a)/b^2
```

maxima [A] time = 1.34, size = 33, normalized size = 0.87

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}x^2}{5\,b}-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}a}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2 + a)^(3/2)*x^2/b - 2/15*(b*x^2 + a)^(3/2)*a/b^2
```

mupad [B] time = 4.65, size = 33, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{x^4}{5}-\frac{2\,a^2}{15\,b^2}+\frac{a\,x^2}{15\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*(x^4/5 - (2*a^2)/(15*b^2) + (a*x^2)/(15*b))
```

sympy [A] time = 0.32, size = 63, normalized size = 1.66

$$\begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15
*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

3.347 $\int x\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{3/2}}{3b}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2],x]
```

```
[Out] (a + b*x^2)^(3/2)/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{a+bx^2}\,dx=\frac{\left(a+bx^2\right)^{3/2}}{3b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2],x]
```

```
[Out] (a + b*x^2)^(3/2)/(3*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a + b*x^2],x]
[Out] (a + b*x^2)^(3/2)/(3*b)
```

fricas [A] time = 0.96, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

giac [A] time = 0.60, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/2),x)
[Out] 1/3*(b*x^2+a)^(3/2)/b
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)/b
```

mupad [B] time = 4.63, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^2+a\right)^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(3/2)/(3*b)
```

sympy [A] time = 0.17, size = 39, normalized size = 2.17

$$\begin{cases} \frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0 \\ \frac{\sqrt{a}\,x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (
sqrt(a)*x**2/2, True))
```

3.348 $\int \frac{\sqrt{a+bx^2}}{x} \, dx$

Optimal. Leaf size=37

$$\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$\sqrt{a+bx^2}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x,x]
```

```
[Out] Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= \sqrt{a+bx^2} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \sqrt{a+bx^2} + \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= \sqrt{a+bx^2} - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\sqrt{a+bx^2} - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x,x]
[Out] Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.03, size = 37, normalized size = 1.00

$$\sqrt{a+bx^2} - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/x,x]
[Out] Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

fricas [A] time = 0.99, size = 77, normalized size = 2.08

$$\left[\frac{1}{2}\sqrt{a}\log\left(-\frac{bx^2 - 2\sqrt{bx^2+a}\sqrt{a} + 2a}{x^2}\right) + \sqrt{bx^2+a}, \sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) + \sqrt{bx^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(b*x
^2 + a), sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + sqrt(b*x^2 + a)]
```

giac [A] time = 0.59, size = 33, normalized size = 0.89

$$\frac{a \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a)
```

maple [A] time = 0.00, size = 39, normalized size = 1.05

$$-\sqrt{a}\, \ln\left(\frac{2a + 2\sqrt{b\,x^2 + a}\,\sqrt{a}}{x}\right) + \sqrt{b\,x^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x,x)
```

```
[Out] (b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

maxima [A] time = 1.36, size = 27, normalized size = 0.73

$$-\sqrt{a}\, \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right) + \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)
```

mupad [B] time = 4.66, size = 29, normalized size = 0.78

$$\sqrt{b\,x^2 + a} - \sqrt{a}\, \operatorname{atanh}\left(\frac{\sqrt{b\,x^2 + a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x,x)
```

```
[Out] (a + b*x^2)^(1/2) - a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2))
```

sympy [A] time = 1.41, size = 56, normalized size = 1.51

$$-\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{a}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) +
sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

3.349 $\int \frac{\sqrt{a+bx^2}}{x^3}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{a+bx^2}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{\sqrt{a+bx^2}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^3,x]
```

```
[Out] -Sqrt[a + b*x^2]/(2*x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{2x^2} + \frac{1}{4}b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{2x^2} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= -\frac{\sqrt{a+bx^2}}{2x^2} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 1.26

$$-\frac{bx^2\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+a+bx^2}{2x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^3,x]
```

```
[Out] -1/2*(a + b*x^2 + b*x^2*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(
x^2*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.07, size = 47, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{2x^2} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*x^2]/x^2 - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

fricas [A] time = 1.70, size = 106, normalized size = 2.26

$$\left[\frac{\sqrt{a}bx^2\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}\,a}{4\,ax^2}, \frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,a}{2\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2
 + a)) - sqrt(b*x^2 + a)*a)/(a*x^2)]
```

giac [A] time = 0.61, size = 46, normalized size = 0.98

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx^2+a}\,b}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x^2 + a)*b/x^2)
/b
```

maple [A] time = 0.00, size = 63, normalized size = 1.34

$$-\frac{b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2\sqrt{a}}+\frac{\sqrt{b\,x^2+a}\;b}{2a}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(3/2)-1/2/a^(1/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))
/x)+1/2/a*b*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.37, size = 51, normalized size = 1.09

$$-\frac{b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,\sqrt{a}}+\frac{\sqrt{bx^2+a}\,b}{2\,a}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*sqrt(b*x^2 + a)*b/a - 1/
2*(b*x^2 + a)^(3/2)/(a*x^2)
```

mupad [B] time = 4.80, size = 35, normalized size = 0.74

$$-\frac{\sqrt{b\,x^2+a}}{2\,x^2}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^3,x)
```

```
[Out] - (a + b*x^2)^(1/2)/(2*x^2) - (b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/
2))
```

sympy [A] time = 1.93, size = 42, normalized size = 0.89

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - b*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(
a))
```

3.350 $\int \frac{\sqrt{a+bx^2}}{x^5}\,dx$

Optimal. Leaf size=71

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b\sqrt{a+bx^2}}{8ax^2}-\frac{\sqrt{a+bx^2}}{4x^4}$$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b\sqrt{a+bx^2}}{8ax^2}-\frac{\sqrt{a+bx^2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^5,x]
```

```
[Out] -Sqrt[a + b*x^2]/(4*x^4) - (b*Sqrt[a + b*x^2])/(8*a*x^2) + (b^2*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} + \frac{1}{8}b\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{16a}\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{8a}\\
&= -\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.55

$$
-\frac{b^2\left(a+bx^2\right)^{3/2}\ {}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx^2}{a}+1\right)}{3a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^5,x]
```

```
[Out] -1/3*(b^2*(a + b*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^2)/a])/
a^3
```

IntegrateAlgebraic [A] time = 0.09, size = 62, normalized size = 0.87

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}} + \frac{(-2a - bx^2)\sqrt{a + bx^2}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/x^5,x]
```

```
[Out] ((-2*a - b*x^2)*Sqrt[a + b*x^2])/(8*a*x^4) + (b^2*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(8*a^(3/2))
```

fricas [A] time = 0.64, size = 131, normalized size = 1.85

$$\left[\frac{\sqrt{a}\, b^2 x^4 \log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right) - 2\left(abx^2 + 2\,a^2\right)\sqrt{bx^2 + a}}{16\,a^2x^4}, -\frac{\sqrt{-a}\, b^2 x^4 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) + \left(abx^2 + 2\,a^2\right)\sqrt{bx^2 + a}}{8\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(a)*b^2*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
- 2*(a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a^2*x^4), -1/8*(sqrt(-a)*b^2*x^4*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) + (a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a^2*x^
4)]
```

giac [A] time = 0.60, size = 72, normalized size = 1.01

$$-\frac{\frac{b^3 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} + \frac{\left(bx^2+a\right)^{\frac{3}{2}} b^3 + \sqrt{bx^2+a}\,ab^3}{ab^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*(b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x^2 + a)^(3/2
)*b^3 + sqrt(b*x^2 + a)*a*b^3)/(a*b^2*x^4))/b
```

maple [A] time = 0.01, size = 85, normalized size = 1.20

$$\frac{b^{2} \ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{2}+a}\;b^{2}}{8a^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} b}{8a^{2}x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(3/2)+1/8/a^2*b/x^2*(b*x^2+a)^(3/2)+1/8/a^(3/2)*b^2*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/8/a^2*b^2*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.32, size = 73, normalized size = 1.03

$$\frac{b^{2} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^{2}+a}\,b^{2}}{8\,a^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}} b}{8\,a^{2}x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/8*sqrt(b*x^2 + a)*b^2/a^2
 + 1/8*(b*x^2 + a)^(3/2)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(3/2)/(a*x^4)
```

mupad [B] time = 4.91, size = 54, normalized size = 0.76

$$\frac{b^{2} \operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{8\,a^{3/2}}-\frac{\sqrt{b\,x^{2}+a}}{8\,x^{4}}-\frac{\left(b\,x^{2}+a\right)^{3/2}}{8\,a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^5,x)
```

```
[Out] (b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (a + b*x^2)^(1/2)/(8*x
^4) - (a + b*x^2)^(3/2)/(8*a*x^4)
```

sympy [A] time = 3.73, size = 92, normalized size = 1.30

$$-\frac{a}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3\sqrt{b}}{8x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{b^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^{2}}+1}}+\frac{b^{2} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] -a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(8*x**3*sqrt(a/(b*x**2
) + 1)) - b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + b**2*asinh(sqrt(a)/(sqrt(
b)*x))/(8*a**(3/2))
```

3.351 $\int \frac{\sqrt{a+bx^2}}{x^7} dx$

Optimal. Leaf size=95

$$-\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{b^2\sqrt{a+bx^2}}{16a^2x^2}-\frac{\sqrt{a+bx^2}}{6x^6}-\frac{b\sqrt{a+bx^2}}{24ax^4}$$

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2\sqrt{a+bx^2}}{16a^2x^2}-\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}-\frac{b\sqrt{a+bx^2}}{24ax^4}-\frac{\sqrt{a+bx^2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^7,x]
```

```
[Out] -Sqrt[a + b*x^2]/(6*x^6) - (b*Sqrt[a + b*x^2])/(24*a*x^4) + (b^2*Sqrt[a + b
*x^2])/(16*a^2*x^2) - (b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} + \frac{1}{12} b \operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} - \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{16a}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} + \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{32a^2}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{16a^2}\\
&= -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} - \frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.41

$$
\frac{b^3\left(a+bx^2\right)^{3/2} {}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx^2}{a}+1\right)}{3a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^7,x]
```

```
[Out] (b^3*(a + b*x^2)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x^2)/a])/(3*a^
4)
```

IntegrateAlgebraic [A] time = 0.12, size = 73, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(-8a^2-2abx^2+3b^2x^4\right)}{48a^2x^6}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2 - 2*a*b*x^2 + 3*b^2*x^4))/(48*a^2*x^6) - (b^3*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

fricas [A] time = 1.52, size = 157, normalized size = 1.65

$$\left[\frac{3\sqrt{a}b^3x^6\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(3ab^2x^4-2a^2bx^2-8a^3\right)\sqrt{bx^2+a}}{96a^3x^6},\frac{3\sqrt{-a}b^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3ab^2x^4-2a^2bx^2-8a^3\right)\sqrt{bx^2+a}}{48a^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(a)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(3*a*b^2*x^4 - 2*a^2*b*x^2 - 8*a^3)*sqrt(b*x^2 + a))/(a^3*x^6), 1/48*
(3*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b^2*x^4 - 2*a^2
*b*x^2 - 8*a^3)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

giac [A] time = 0.68, size = 92, normalized size = 0.97

$$\frac{\frac{3b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}b^4-8\left(bx^2+a\right)^{\frac{3}{2}}ab^4-3\sqrt{bx^2+a}a^2b^4}{a^2b^3x^6}}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a
)^(5/2)*b^4 - 8*(b*x^2 + a)^(3/2)*a*b^4 - 3*sqrt(b*x^2 + a)*a^2*b^4)/(a^2*b
^3*x^6))/b
```

maple [A] time = 0.01, size = 105, normalized size = 1.11

$$-\frac{b^{3}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{5}{2}}}+\frac{\sqrt{b\,x^{2}+a}\,b^{3}}{16a^{3}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{2}}{16a^{3}x^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}b}{8a^{2}x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(3/2)+1/8/a^2*b/x^4*(b*x^2+a)^(3/2)-1/16/a^3*b^2/x^2*(
b*x^2+a)^(3/2)-1/16/a^(5/2)*b^3*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+1/16/
a^3*b^3*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.34, size = 93, normalized size = 0.98

$$-\frac{b^{3}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{5}{2}}}+\frac{\sqrt{bx^{2}+a}\,b^{3}}{16\,a^{3}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}{16\,a^{3}x^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}b}{8\,a^{2}x^{4}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}}{6\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/16*sqrt(b*x^2 + a)*b^3/
a^3 - 1/16*(b*x^2 + a)^(3/2)*b^2/(a^3*x^2) + 1/8*(b*x^2 + a)^(3/2)*b/(a^2*x
^4) - 1/6*(b*x^2 + a)^(3/2)/(a*x^6)
```

mupad [B] time = 4.94, size = 74, normalized size = 0.78

$$\frac{\left(b\,x^2+a\right)^{5/2}}{16\,a^2\,x^6}-\frac{\left(b\,x^2+a\right)^{3/2}}{6\,a\,x^6}-\frac{\sqrt{b\,x^2+a}}{16\,x^6}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}}{16\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^7,x)
```

```
[Out] (b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (a + b*x^2)^(1
/2)/(16*x^6) - (a + b*x^2)^(3/2)/(6*a*x^6) + (a + b*x^2)^(5/2)/(16*a^2*x^6)
```

sympy [A] time = 5.91, size = 117, normalized size = 1.23

$$-\frac{a}{6\sqrt{b}\,x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5\sqrt{b}}{24x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{b^{\frac{3}{2}}}{48ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{b^{\frac{5}{2}}}{16a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] -a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*sqrt(b)/(24*x**5*sqrt(a/(b*x**
2) + 1)) + b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(16*a**2*x*
sqrt(a/(b*x**2) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2))
```

3.352 $\int x^4\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=94

$$\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}$$

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^2],x]
```

```
[Out] -(a^2*x*Sqrt[a + b*x^2])/(16*b^2) + (a*x^3*Sqrt[a + b*x^2])/(24*b) + (x^5*S
qrt[a + b*x^2])/6 + (a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^4\sqrt{a+bx^2}\,dx &= \frac{1}{6}x^5\sqrt{a+bx^2}+\frac{1}{6}a\int\frac{x^4}{\sqrt{a+bx^2}}\,dx\\
&= \frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{1}{6}x^5\sqrt{a+bx^2}-\frac{a^2\int\frac{x^2}{\sqrt{a+bx^2}}\,dx}{8b}\\
&= -\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{a^3\int\frac{1}{\sqrt{a+bx^2}}\,dx}{16b^2}\\
&= -\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{a^3\,\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{16b^2}\\
&= -\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.82

$$
\frac{a^3\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{16b^{5/2}}+\sqrt{a+bx^2}\left(-\frac{a^2x}{16b^2}+\frac{ax^3}{24b}+\frac{x^5}{6}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]*(-1/16*(a^2*x)/b^2 + (a*x^3)/(24*b) + x^5/6) + (a^3*Log[b*x
 + Sqrt[b]*Sqrt[a + b*x^2]])/(16*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 74, normalized size = 0.79

$$
\frac{\sqrt{a+bx^2}\left(-3a^2x+2abx^3+8b^2x^5\right)}{48b^2}-\frac{a^3\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2*x + 2*a*b*x^3 + 8*b^2*x^5))/(48*b^2) - (a^3*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(5/2))
```

fricas [A] time = 1.10, size = 146, normalized size = 1.55

$$\left[\frac{3\,a^{3}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,\left(8\,b^{3}x^{5}+2\,ab^{2}x^{3}-3\,a^{2}bx\right)\sqrt{bx^{2}+a}}{96\,b^{3}},-\frac{3\,a^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-\left(8\,b^{3}x^{5}+2\,ab^{2}x^{3}-3\,a^{2}bx\right)\sqrt{bx^{2}+a}}{48\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8
*b^3*x^5 + 2*a*b^2*x^3 - 3*a^2*b*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*a^3*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 2*a*b^2*x^3 - 3*a^2*
b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.69, size = 64, normalized size = 0.68

$$\frac{1}{48}\left(2\left(4\,x^{2}+\frac{a}{b}\right)x^{2}-\frac{3\,a^{2}}{b^{2}}\right)\sqrt{bx^{2}+a}\,x-\frac{a^{3}\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2 + a/b)*x^2 - 3*a^2/b^2)*sqrt(b*x^2 + a)*x - 1/16*a^3*log(abs
(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 77, normalized size = 0.82

$$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}x^{3}}{6b}+\frac{a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{5}{2}}}+\frac{\sqrt{b\,x^{2}+a}\,a^{2}x}{16b^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}ax}{8b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*x^3*(b*x^2+a)^(3/2)/b-1/8*a/b^2*x*(b*x^2+a)^(3/2)+1/16*a^2*x*(b*x^2+a)^
(1/2)/b^2+1/16*a^3/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.31, size = 69, normalized size = 0.73

$$\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}x^{3}}{6\,b}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}ax}{8\,b^{2}}+\frac{\sqrt{bx^{2}+a}\,a^{2}x}{16\,b^{2}}+\frac{a^{3}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(3/2)*x^3/b - 1/8*(b*x^2 + a)^(3/2)*a*x/b^2 + 1/16*sqrt(b*x
^2 + a)*a^2*x/b^2 + 1/16*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \sqrt{b x^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(1/2), x)
```

sympy [A] time = 5.77, size = 117, normalized size = 1.24

$$-\frac{a^{\frac{5}{2}} x}{16 b^{2} \sqrt{1 + \frac{b x^{2}}{a}}} - \frac{a^{\frac{3}{2}} x^{3}}{48 b \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{5 \sqrt{a} x^{5}}{24 \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{a^{3} \operatorname{asinh}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{16 b^{\frac{5}{2}}} + \frac{b x^{7}}{6 \sqrt{a} \sqrt{1 + \frac{b x^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - a**(3/2)*x**3/(48*b*sqrt(1 + b*x
**2/a)) + 5*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*b**(5/2)) + b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.353 $\int x^2\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=70

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{1}{4}x^3\sqrt{a+bx^2}+\frac{ax\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^2],x]
```

```
[Out] (a*x*Sqrt[a + b*x^2])/(8*b) + (x^3*Sqrt[a + b*x^2])/4 - (a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{a+bx^2}\,dx &= \frac{1}{4}x^3\sqrt{a+bx^2}+\frac{1}{4}a\int\frac{x^2}{\sqrt{a+bx^2}}\,dx\\
&= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\
&= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\,\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b}\\
&= \frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.91

$$\sqrt{a+bx^2}\left(\frac{ax}{8b}+\frac{x^3}{4}\right)-\frac{a^2\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]*((a*x)/(8*b) + x^3/4) - (a^2*Log[b*x + Sqrt[b]*Sqrt[a + b*x
^2]])/(8*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 62, normalized size = 0.89

$$\frac{a^2\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{3/2}}+\frac{\sqrt{a+bx^2}\left(ax+2bx^3\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(a*x + 2*b*x^3))/(8*b) + (a^2*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(8*b^(3/2))
```

fricas [A] time = 0.91, size = 119, normalized size = 1.70

$$\left[\frac{a^2\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,b^2x^3+abx\right)\sqrt{bx^2+a}}{16\,b^2},\ \frac{a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(2\,b^2x^3+abx\right)\sqrt{bx^2+a}}{8\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*b
^2*x^3 + a*b*x)*sqrt(b*x^2 + a))/b^2, 1/8*(a^2*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (2*b^2*x^3 + a*b*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.71, size = 50, normalized size = 0.71

$$\frac{1}{8}\,\sqrt{bx^2+a}\left(2\,x^2+\frac{a}{b}\right)x+\frac{a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*x^2 + a/b)*x + 1/8*a^2*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

maple [A] time = 0.00, size = 57, normalized size = 0.81

$$-\frac{a^2\ln\left(\sqrt{b}\ x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\ ax}{8b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\,x}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)/b-1/8*a*x*(b*x^2+a)^(1/2)/b-1/8*a^2/b^(3/2)*ln(b^(1/2
)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.36, size = 49, normalized size = 0.70

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}x}{4\,b}-\frac{\sqrt{bx^2+a}\,ax}{8\,b}-\frac{a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*x/b - 1/8*sqrt(b*x^2 + a)*a*x/b - 1/8*a^2*arcsinh(b*x
/sqrt(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sqrt{b\,x^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/2), x)
```

sympy [A] time = 3.53, size = 92, normalized size = 1.31

$$\frac{a^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{bx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*sqrt(a)*x**3/(8*sqrt(1 + b*x**2/a))
 - a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + b*x**5/(4*sqrt(a)*sqrt(1 +
b*x**2/a))
```

3.354 $\int \sqrt{a+bx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^2}\,dx &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2],x]
[Out] (x*Sqrt[a + b*x^2])/2 + (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

IntegrateAlgebraic [**A**] time = 0.04, size = 48, normalized size = 1.04

$$\frac{1}{2}x\sqrt{a+bx^2}-\frac{a\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2],x]
[Out] (x*Sqrt[a + b*x^2])/2 - (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

fricas [**A**] time = 0.73, size = 94, normalized size = 2.04

$$\left[\frac{2\sqrt{bx^2+a}\,bx+a\sqrt{b}\,\log\left(-2\,bx^2-2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b}, \frac{\sqrt{bx^2+a}\,bx-a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)))/b]
```

giac [A] time = 0.66, size = 37, normalized size = 0.80

$$\frac{1}{2}\sqrt{bx^2+a}\,x-\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

maple [A] time = 0.00, size = 36, normalized size = 0.78

$$\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}+\frac{\sqrt{b\,x^2+a}\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.33, size = 28, normalized size = 0.61

$$\frac{1}{2}\sqrt{bx^2+a}\,x+\frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x + 1/2*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 4.67, size = 35, normalized size = 0.76

$$\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2),x)
```

```
[Out] (x*(a + b*x^2)^(1/2))/2 + (a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2)
)
```

sympy [A] time = 1.83, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

3.355 $\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=42

$$\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 206}

$$\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^2]/x) + Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^2}\,dx = -\frac{\sqrt{a+bx^2}}{x} + b\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= -\frac{\sqrt{a+bx^2}}{x} + b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= -\frac{\sqrt{a+bx^2}}{x} + \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Mathematica [A] time = 0.07, size = 63, normalized size = 1.50

$$-\frac{-\sqrt{a}\,\sqrt{b}\,x\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+a+bx^2}{x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^2,x]
```

```
[Out] -((a + b*x^2 - Sqrt[a]*Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sq
rt[a]])/(x*Sqrt[a + b*x^2]))
```

IntegrateAlgebraic [A] time = 0.06, size = 45, normalized size = 1.07

$$-\frac{\sqrt{a+bx^2}}{x} - \sqrt{b}\,\log\left(\sqrt{a+bx^2} - \sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^2]/x) - Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

fricas [A] time = 0.58, size = 88, normalized size = 2.10

$$\left[\frac{\sqrt{b}\,x\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right) - 2\,\sqrt{bx^2+a}}{2\,x}, -\frac{\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right) + \sqrt{bx^2+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*sqrt(b*
x^2 + a))/x, -(sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + sqrt(b*x^2 +
 a))/x]
```

giac [A] time = 0.65, size = 57, normalized size = 1.36

$$-\frac{1}{2}\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2a\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a*sqrt(b)/((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.00, size = 54, normalized size = 1.29

$$\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)+\frac{\sqrt{b\,x^2+a}\,bx}{a}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(3/2)+1/a*b*x*(b*x^2+a)^(1/2)+b^(1/2)*ln(b^(1/2)*x+(b*x^2+
a)^(1/2))
```

maxima [A] time = 1.37, size = 28, normalized size = 0.67

$$\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\sqrt{bx^2+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] sqrt(b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)/x
```

mupad [B] time = 4.82, size = 56, normalized size = 1.33

$$-\frac{\sqrt{b\,x^2+a}}{x}-\frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^2,x)
```

```
[Out] - (a + b*x^2)^(1/2)/x - (b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(
1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

sympy [A] time = 1.42, size = 56, normalized size = 1.33

$$-\frac{\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{bx}{\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(a)/(x*sqrt(1 + b*x**2/a)) + sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - b*x/(s
qrt(a)*sqrt(1 + b*x**2/a))
```

3.356 $\int \frac{\sqrt{a+bx^2}}{x^4} \, dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^4,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(3*a*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^4} \, dx = -\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{\left(a+bx^2\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^4,x]
```

```
[Out] -1/3*(a + b*x^2)^(3/2)/(a*x^3)
```

IntegrateAlgebraic [A] time = 0.06, size = 31, normalized size = 1.48

$$\frac{\left(-a-bx^2\right)\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/x^4,x]
```

```
[Out] ((-a - b*x^2)*Sqrt[a + b*x^2])/(3*a*x^3)
```

fricas [A] time = 0.89, size = 17, normalized size = 0.81

$$-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(b*x^2 + a)^(3/2)/(a*x^3)
```

giac [B] time = 0.68, size = 59, normalized size = 2.81

$$\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4 b^{\frac{3}{2}}+a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) + a^2*b^(3/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*(b*x^2+a)^(3/2)/a/x^3
```

maxima [A] time = 1.32, size = 17, normalized size = 0.81

$$-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/3*(b*x^2 + a)^(3/2)/(a*x^3)
```

mupad [B] time = 4.57, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^2+a\right)^{3/2}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^4,x)
[Out] -(a + b*x^2)^(3/2)/(3*a*x^3)
```

sympy [B] time = 0.71, size = 42, normalized size = 2.00

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**4,x)
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a
)
```

3.357 $\int \frac{\sqrt{a+bx^2}}{x^6}\,dx$

Optimal. Leaf size=44

$$\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^6,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(5*a*x^5) + (2*b*(a + b*x^2)^(3/2))/(15*a^2*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^6}\,dx &= -\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}-\frac{(2b)\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{5a}\\ &= -\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}+\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{(a+bx^2)^{3/2}(2bx^2-3a)}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^6,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-3*a + 2*b*x^2))/(15*a^2*x^5)
```

IntegrateAlgebraic [A] time = 0.07, size = 42, normalized size = 0.95

$$\frac{\sqrt{a+bx^2}\,(-3a^2-abx^2+2b^2x^4)}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2 - a*b*x^2 + 2*b^2*x^4))/(15*a^2*x^5)
```

fricas [A] time = 0.94, size = 38, normalized size = 0.86

$$\frac{(2\,b^2x^4-abx^2-3\,a^2)\sqrt{bx^2+a}}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(2*b^2*x^4 - a*b*x^2 - 3*a^2)*sqrt(b*x^2 + a)/(a^2*x^5)
```

giac [B] time = 0.68, size = 112, normalized size = 2.55

$$\frac{4\left(15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6 b^{\frac{5}{2}}+5\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4 ab^{\frac{5}{2}}+5\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 a^2b^{\frac{5}{2}}-a^3b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2) + 5*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a*b^(5/2) + 5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2) - a^3*b
^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(-2b\,x^2+3a\right)}{15a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^6,x)
```

```
[Out] -1/15*(b*x^2+a)^(3/2)*(-2*b*x^2+3*a)/a^2/x^5
```

maxima [A] time = 1.28, size = 36, normalized size = 0.82

$$\frac{2\left(bx^2+a\right)^{\frac{3}{2}}b}{15\,a^2x^3}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^2 + a)^(3/2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(3/2)/(a*x^5)
```

mupad [B] time = 4.77, size = 37, normalized size = 0.84

$$-\frac{\sqrt{b\,x^2+a}\,\left(3\,a^2+a\,b\,x^2-2\,b^2\,x^4\right)}{15\,a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^6,x)
```

```
[Out] -((a + b*x^2)^(1/2)*(3*a^2 - 2*b^2*x^4 + a*b*x^2))/(15*a^2*x^5)
```

sympy [A] time = 0.91, size = 68, normalized size = 1.55

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*
a*x**2) + 2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2)
```

3.358 $\int \frac{\sqrt{a+bx^2}}{x^8}\,dx$

Optimal. Leaf size=68

$$-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^3}+\frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^3}+\frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^8,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(7*a*x^7) + (4*b*(a + b*x^2)^(3/2))/(35*a^2*x^5) - (8*b^
2*(a + b*x^2)^(3/2))/(105*a^3*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^8}\,dx = -\frac{\left(a+bx^2\right)^{3/2}}{7ax^7} - \frac{(4b)\int \frac{\sqrt{a+bx^2}}{x^6}\,dx}{7a}$$
$$= -\frac{\left(a+bx^2\right)^{3/2}}{7ax^7} + \frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5} + \frac{\left(8b^2\right)\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{35a^2}$$
$$= -\frac{\left(a+bx^2\right)^{3/2}}{7ax^7} + \frac{4b\left(a+bx^2\right)^{3/2}}{35a^2x^5} - \frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^3}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^2\right)^{3/2}\left(15a^2-12abx^2+8b^2x^4\right)}{105a^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^8,x]
[Out] -1/105*((a + b*x^2)^(3/2)*(15*a^2 - 12*a*b*x^2 + 8*b^2*x^4))/(a^3*x^7)
```

IntegrateAlgebraic [A] time = 0.08, size = 53, normalized size = 0.78

$$\frac{\sqrt{a+bx^2}\left(-15a^3-3a^2bx^2+4ab^2x^4-8b^3x^6\right)}{105a^3x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/x^8,x]
[Out] (Sqrt[a + b*x^2]*(-15*a^3 - 3*a^2*b*x^2 + 4*a*b^2*x^4 - 8*b^3*x^6))/(105*a^
3*x^7)
```

fricas [A] time = 1.12, size = 49, normalized size = 0.72

$$-\frac{\left(8\,b^3x^6-4\,ab^2x^4+3\,a^2bx^2+15\,a^3\right)\sqrt{bx^2+a}}{105\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="fricas")
[Out] -1/105*(8*b^3*x^6 - 4*a*b^2*x^4 + 3*a^2*b*x^2 + 15*a^3)*sqrt(b*x^2 + a)/(a^
3*x^7)
```

giac [B] time = 0.65, size = 138, normalized size = 2.03

$$\frac{16\left(70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8b^{\frac{7}{2}}+35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{7}{2}}+21\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{7}{2}}-7\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^3b^{\frac{7}{2}}+a^4b^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(7/2) + 35*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a*b^(7/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(7/2) - 7
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(7/2) + a^4*b^(7/2))/((sqrt(b)*x - s
qrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(8b^2x^4-12ab\,x^2+15a^2\right)}{105a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^8,x)
```

```
[Out] -1/105*(b*x^2+a)^(3/2)*(8*b^2*x^4-12*a*b*x^2+15*a^2)/a^3/x^7
```

maxima [A] time = 1.32, size = 56, normalized size = 0.82

$$-\frac{8\left(bx^2+a\right)^{\frac{3}{2}}b^2}{105\,a^3x^3}+\frac{4\left(bx^2+a\right)^{\frac{3}{2}}b}{35\,a^2x^5}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] -8/105*(b*x^2 + a)^(3/2)*b^2/(a^3*x^3) + 4/35*(b*x^2 + a)^(3/2)*b/(a^2*x^5)
 - 1/7*(b*x^2 + a)^(3/2)/(a*x^7)
```

mupad [B] time = 4.71, size = 73, normalized size = 1.07

$$\frac{4\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3}-\frac{b\,\sqrt{b\,x^2+a}}{35\,a\,x^5}-\frac{\sqrt{b\,x^2+a}}{7\,x^7}-\frac{8\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^8,x)
```

```
[Out] (4*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^3) - (b*(a + b*x^2)^(1/2))/(35*a*x^5)
- (a + b*x^2)^(1/2)/(7*x^7) - (8*b^3*(a + b*x^2)^(1/2))/(105*a^3*x)
```

sympy [B] time = 1.30, size = 359, normalized size = 5.28

$$-\frac{15a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33a^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17a^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{3a^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{12ab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{8b^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**8,x)
```

```
[Out] -15*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*
x**8 + 105*a**3*b**6*x**10) - 33*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*a**3*b*
*(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8
+ 105*a**3*b**6*x**10) - 3*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a*
*5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*a*b**(17/2)*x
**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**
3*b**6*x**10) - 8*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
+ 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
```

3.359 $\int \frac{\sqrt{a+bx^2}}{x^{10}} \, dx$

Optimal. Leaf size=92

$$\frac{16b^3 (a+bx^2)^{3/2}}{315a^4x^3} - \frac{8b^2 (a+bx^2)^{3/2}}{105a^3x^5} + \frac{2b (a+bx^2)^{3/2}}{21a^2x^7} - \frac{(a+bx^2)^{3/2}}{9ax^9}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{16b^3 (a+bx^2)^{3/2}}{315a^4x^3} - \frac{8b^2 (a+bx^2)^{3/2}}{105a^3x^5} + \frac{2b (a+bx^2)^{3/2}}{21a^2x^7} - \frac{(a+bx^2)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/x^10,x]
```

```
[Out] -(a + b*x^2)^(3/2)/(9*a*x^9) + (2*b*(a + b*x^2)^(3/2))/(21*a^2*x^7) - (8*b^
2*(a + b*x^2)^(3/2))/(105*a^3*x^5) + (16*b^3*(a + b*x^2)^(3/2))/(315*a^4*x^
3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx = -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9} - \frac{(2b)\int \frac{\sqrt{a+bx^2}}{x^8}\,dx}{3a}$$
$$= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9} + \frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7} + \frac{\left(8b^2\right)\int \frac{\sqrt{a+bx^2}}{x^6}\,dx}{21a^2}$$
$$= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9} + \frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7} - \frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^5} - \frac{\left(16b^3\right)\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{105a^3}$$
$$= -\frac{\left(a+bx^2\right)^{3/2}}{9ax^9} + \frac{2b\left(a+bx^2\right)^{3/2}}{21a^2x^7} - \frac{8b^2\left(a+bx^2\right)^{3/2}}{105a^3x^5} + \frac{16b^3\left(a+bx^2\right)^{3/2}}{315a^4x^3}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{3/2}\left(-35a^3+30a^2bx^2-24ab^2x^4+16b^3x^6\right)}{315a^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/x^10,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-35*a^3 + 30*a^2*b*x^2 - 24*a*b^2*x^4 + 16*b^3*x^6))/(3
15*a^4*x^9)
```

IntegrateAlgebraic [A] time = 0.09, size = 64, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\left(-35a^4-5a^3bx^2+6a^2b^2x^4-8ab^3x^6+16b^4x^8\right)}{315a^4x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/x^10,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-35*a^4 - 5*a^3*b*x^2 + 6*a^2*b^2*x^4 - 8*a*b^3*x^6 + 16*
b^4*x^8))/(315*a^4*x^9)
```

fricas [A] time = 0.97, size = 60, normalized size = 0.65

$$\frac{\left(16\,b^4x^8-8\,ab^3x^6+6\,a^2b^2x^4-5\,a^3bx^2-35\,a^4\right)\sqrt{bx^2+a}}{315\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(16*b^4*x^8 - 8*a*b^3*x^6 + 6*a^2*b^2*x^4 - 5*a^3*b*x^2 - 35*a^4)*sqr
t(b*x^2 + a)/(a^4*x^9)
```

giac [B] time = 0.70, size = 166, normalized size = 1.80

$$\frac{32\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{9}{2}}+189\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{9}{2}}+84\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^2b^{\frac{9}{2}}-36\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^3b^{\frac{9}{2}}+9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^4b^{\frac{9}{2}}-a^5b^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(9/2) + 189*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a*b^(9/2) + 84*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(9/2)
- 36*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(9/2) + 9*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a^4*b^(9/2) - a^5*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^
9
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(-16b^3x^6+24a\,b^2x^4-30a^2b\,x^2+35a^3\right)}{315a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(3/2)*(-16*b^3*x^6+24*a*b^2*x^4-30*a^2*b*x^2+35*a^3)/x^9/a
^4
```

maxima [A] time = 1.34, size = 76, normalized size = 0.83

$$\frac{16\left(bx^2+a\right)^{\frac{3}{2}}b^3}{315\,a^4x^3}-\frac{8\left(bx^2+a\right)^{\frac{3}{2}}b^2}{105\,a^3x^5}+\frac{2\left(bx^2+a\right)^{\frac{3}{2}}b}{21\,a^2x^7}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] 16/315*(b*x^2 + a)^(3/2)*b^3/(a^4*x^3) - 8/105*(b*x^2 + a)^(3/2)*b^2/(a^3*x
^5) + 2/21*(b*x^2 + a)^(3/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(3/2)/(a*x^9)
```

mupad [B] time = 4.95, size = 93, normalized size = 1.01

$$\frac{2\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^5}-\frac{b\,\sqrt{b\,x^2+a}}{63\,a\,x^7}-\frac{\sqrt{b\,x^2+a}}{9\,x^9}-\frac{8\,b^3\,\sqrt{b\,x^2+a}}{315\,a^3\,x^3}+\frac{16\,b^4\,\sqrt{b\,x^2+a}}{315\,a^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/x^10,x)
```

```
[Out] (2*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^5) - (b*(a + b*x^2)^(1/2))/(63*a*x^7)
- (a + b*x^2)^(1/2)/(9*x^9) - (8*b^3*(a + b*x^2)^(1/2))/(315*a^3*x^3) + (16
*b^4*(a + b*x^2)^(1/2))/(315*a^4*x)
```

sympy [B] time = 1.74, size = 575, normalized size = 6.25

$$-\frac{35a^7b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{110a^6b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{114a^5b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{40a^4b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{5a^3b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{30a^2b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{40ab^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{16b^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**10,x)
```

```
[Out] -35*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**1
0*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*a**6*b**(21/2)
*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*a**5*b**(23/2)*x**4*sqrt(a/
(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x
**12 + 315*a**4*b**12*x**14) - 40*a**4*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/
(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**
4*b**12*x**14) + 5*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*
x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14)
+ 30*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a*
*6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 40*a*b**(31
/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 16*b**(33/2)*x**14*sqrt(a/(
b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x*
*12 + 315*a**4*b**12*x**14)
```

3.360 $\int x^7 \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=80

$$-\frac{a^3 \left(a+bx^2\right)^{5/2}}{5b^4}+\frac{3a^2 \left(a+bx^2\right)^{7/2}}{7b^4}+\frac{\left(a+bx^2\right)^{11/2}}{11b^4}-\frac{a \left(a+bx^2\right)^{9/2}}{3b^4}$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2 \left(a+bx^2\right)^{7/2}}{7b^4}-\frac{a^3 \left(a+bx^2\right)^{5/2}}{5b^4}+\frac{\left(a+bx^2\right)^{11/2}}{11b^4}-\frac{a \left(a+bx^2\right)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^(5/2))/(5*b^4) + (3*a^2*(a + b*x^2)^(7/2))/(7*b^4) - (a*(
a + b*x^2)^(9/2))/(3*b^4) + (a + b*x^2)^(11/2)/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^2\right)^{3/2} \, dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{3/2} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right) dx, x, x^2\right)\\
&= -\frac{a^3 \left(a+bx^2\right)^{5/2}}{5b^4}+\frac{3a^2 \left(a+bx^2\right)^{7/2}}{7b^4}-\frac{a \left(a+bx^2\right)^{9/2}}{3b^4}+\frac{\left(a+bx^2\right)^{11/2}}{11b^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{5/2}\left(-16a^3+40a^2bx^2-70ab^2x^4+105b^3x^6\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-16*a^3 + 40*a^2*b*x^2 - 70*a*b^2*x^4 + 105*b^3*x^6))/(
1155*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 72, normalized size = 0.90

$$\frac{\sqrt{a+bx^2}\left(-16a^5+8a^4bx^2-6a^3b^2x^4+5a^2b^3x^6+140ab^4x^8+105b^5x^{10}\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^5 + 8*a^4*b*x^2 - 6*a^3*b^2*x^4 + 5*a^2*b^3*x^6 + 1
40*a*b^4*x^8 + 105*b^5*x^10))/(1155*b^4)
```

fricas [A] time = 1.04, size = 68, normalized size = 0.85

$$\frac{\left(105\,b^5x^{10}+140\,ab^4x^8+5\,a^2b^3x^6-6\,a^3b^2x^4+8\,a^4bx^2-16\,a^5\right)\sqrt{bx^2+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*b^5*x^10 + 140*a*b^4*x^8 + 5*a^2*b^3*x^6 - 6*a^3*b^2*x^4 + 8*a^
4*b*x^2 - 16*a^5)*sqrt(b*x^2 + a)/b^4
```

giac [A] time = 0.68, size = 57, normalized size = 0.71

$$\frac{105\left(bx^2+a\right)^{\frac{11}{2}}-385\left(bx^2+a\right)^{\frac{9}{2}}a+495\left(bx^2+a\right)^{\frac{7}{2}}a^2-231\left(bx^2+a\right)^{\frac{5}{2}}a^3}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1155*(105*(b*x^2 + a)^(11/2) - 385*(b*x^2 + a)^(9/2)*a + 495*(b*x^2 + a)^
(7/2)*a^2 - 231*(b*x^2 + a)^(5/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(-105 b^{3} x^{6}+70 a\,b^{2} x^{4}-40 a^{2} b\,x^{2}+16 a^{3}\right)}{1155 b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(3/2),x)
[Out] -1/1155*(b*x^2+a)^(5/2)*(-105*b^3*x^6+70*a*b^2*x^4-40*a^2*b*x^2+16*a^3)/b^4
```

maxima [A] time = 1.33, size = 73, normalized size = 0.91

$$\frac{\left(bx^{2}+a\right)^{\frac{5}{2}} x^{6}}{11\,b}-\frac{2\left(bx^{2}+a\right)^{\frac{5}{2}} a x^{4}}{33\,b^{2}}+\frac{8\left(bx^{2}+a\right)^{\frac{5}{2}} a^{2} x^{2}}{231\,b^{3}}-\frac{16\left(bx^{2}+a\right)^{\frac{5}{2}} a^{3}}{1155\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] 1/11*(b*x^2 + a)^(5/2)*x^6/b - 2/33*(b*x^2 + a)^(5/2)*a*x^4/b^2 + 8/231*(b*
x^2 + a)^(5/2)*a^2*x^2/b^3 - 16/1155*(b*x^2 + a)^(5/2)*a^3/b^4
```

mupad [B] time = 4.60, size = 64, normalized size = 0.80

$$\sqrt{b\,x^2+a}\,\left(\frac{4\,a\,x^8}{33}+\frac{b\,x^{10}}{11}-\frac{16\,a^5}{1155\,b^4}+\frac{a^2\,x^6}{231\,b}-\frac{2\,a^3\,x^4}{385\,b^2}+\frac{8\,a^4\,x^2}{1155\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(3/2),x)
[Out] (a + b*x^2)^(1/2)*((4*a*x^8)/33 + (b*x^10)/11 - (16*a^5)/(1155*b^4) + (a^2*
x^6)/(231*b) - (2*a^3*x^4)/(385*b^2) + (8*a^4*x^2)/(1155*b^3))
```

sympy [A] time = 3.81, size = 133, normalized size = 1.66

$$\begin{cases} -\frac{16a^{5}\sqrt{a+bx^{2}}}{1155b^{4}}+\frac{8a^{4}x^{2}\sqrt{a+bx^{2}}}{1155b^{3}}-\frac{2a^{3}x^{4}\sqrt{a+bx^{2}}}{385b^{2}}+\frac{a^{2}x^{6}\sqrt{a+bx^{2}}}{231b}+\frac{4ax^{8}\sqrt{a+bx^{2}}}{33}+\frac{bx^{10}\sqrt{a+bx^{2}}}{11} & \text{for}\ b\neq 0\\ \frac{a^{\frac{3}{2}}x^{8}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-16*a**5*sqrt(a + b*x**2)/(1155*b**4) + 8*a**4*x**2*sqrt(a + b*x
**2)/(1155*b**3) - 2*a**3*x**4*sqrt(a + b*x**2)/(385*b**2) + a**2*x**6*sqrt
(a + b*x**2)/(231*b) + 4*a*x**8*sqrt(a + b*x**2)/33 + b*x**10*sqrt(a + b*x*
*2)/11, Ne(b, 0)), (a**(3/2)*x**8/8, True))
```

3.361 $\int x^5 \left(a+bx^2\right)^{3/2} \, dx$

Optimal. Leaf size=59

$$\frac{a^2 \left(a+bx^2\right)^{5/2}}{5b^3}+\frac{\left(a+bx^2\right)^{9/2}}{9b^3}-\frac{2a \left(a+bx^2\right)^{7/2}}{7b^3}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^{5/2}}{5b^3}+\frac{\left(a+bx^2\right)^{9/2}}{9b^3}-\frac{2a \left(a+bx^2\right)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(3/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(5/2))/(5*b^3) - (2*a*(a + b*x^2)^(7/2))/(7*b^3) + (a + b*
x^2)^(9/2)/(9*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^{3/2} \, dx &= \frac{1}{2} \text{Subst}\left(\int x^2 (a+bx)^{3/2} \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{a^2 (a+bx)^{3/2}}{b^2}-\frac{2a (a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right) dx, x, x^2\right)\\
&= \frac{a^2 \left(a+bx^2\right)^{5/2}}{5b^3}-\frac{2a \left(a+bx^2\right)^{7/2}}{7b^3}+\frac{\left(a+bx^2\right)^{9/2}}{9b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{5/2}\left(8a^2-20abx^2+35b^2x^4\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(8*a^2 - 20*a*b*x^2 + 35*b^2*x^4))/(315*b^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 61, normalized size = 1.03

$$\frac{\sqrt{a+bx^2}\left(8a^4-4a^3bx^2+3a^2b^2x^4+50ab^3x^6+35b^4x^8\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^4 - 4*a^3*b*x^2 + 3*a^2*b^2*x^4 + 50*a*b^3*x^6 + 35*b
^4*x^8))/(315*b^3)
```

fricas [A] time = 1.06, size = 57, normalized size = 0.97

$$\frac{\left(35\,b^4x^8+50\,ab^3x^6+3\,a^2b^2x^4-4\,a^3bx^2+8\,a^4\right)\sqrt{bx^2+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*x^8 + 50*a*b^3*x^6 + 3*a^2*b^2*x^4 - 4*a^3*b*x^2 + 8*a^4)*sqr
t(b*x^2 + a)/b^3
```

giac [A] time = 0.64, size = 43, normalized size = 0.73

$$\frac{35\left(bx^2+a\right)^{\frac{9}{2}}-90\left(bx^2+a\right)^{\frac{7}{2}}a+63\left(bx^2+a\right)^{\frac{5}{2}}a^2}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(b*x^2 + a)^(9/2) - 90*(b*x^2 + a)^(7/2)*a + 63*(b*x^2 + a)^(5/2)
*a^2)/b^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(35b^2x^4-20ab\,x^2+8a^2\right)}{315b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(5/2)*(35*b^2*x^4-20*a*b*x^2+8*a^2)/b^3
```

maxima [A] time = 1.32, size = 53, normalized size = 0.90

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}x^4}{9\,b}-\frac{4\left(bx^2+a\right)^{\frac{5}{2}}ax^2}{63\,b^2}+\frac{8\left(bx^2+a\right)^{\frac{5}{2}}a^2}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(5/2)*x^4/b - 4/63*(b*x^2 + a)^(5/2)*a*x^2/b^2 + 8/315*(b*x
^2 + a)^(5/2)*a^2/b^3
```

mupad [B] time = 4.66, size = 53, normalized size = 0.90

$$\sqrt{b\,x^2+a}\,\left(\frac{10\,a\,x^6}{63}+\frac{b\,x^8}{9}+\frac{8\,a^4}{315\,b^3}+\frac{a^2\,x^4}{105\,b}-\frac{4\,a^3\,x^2}{315\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((10*a*x^6)/63 + (b*x^8)/9 + (8*a^4)/(315*b^3) + (a^2*x^4
)/(105*b) - (4*a^3*x^2)/(315*b^2))
```

sympy [A] time = 2.24, size = 109, normalized size = 1.85

$$\begin{cases} \frac{8a^4\sqrt{a+bx^2}}{315b^3}-\frac{4a^3x^2\sqrt{a+bx^2}}{315b^2}+\frac{a^2x^4\sqrt{a+bx^2}}{105b}+\frac{10ax^6\sqrt{a+bx^2}}{63}+\frac{bx^8\sqrt{a+bx^2}}{9} & \text{for}\ b\neq 0\\ \frac{a^{\frac{3}{2}}x^6}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((8*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*a**3*x**2*sqrt(a + b*x**2
)/(315*b**2) + a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*a*x**6*sqrt(a + b*x*
*2)/63 + b*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(3/2)*x**6/6, True))
```

3.362 $\int x^3 \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{7/2}}{7b^2}-\frac{a\left(a+bx^2\right)^{5/2}}{5b^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{7/2}}{7b^2}-\frac{a\left(a+bx^2\right)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(3/2),x]
```

```
[Out] -(a*(a + b*x^2)^(5/2))/(5*b^2) + (a + b*x^2)^(7/2)/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{3/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^{3/2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right) dx, x, x^2\right)\\ &= -\frac{a\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{\left(a+bx^2\right)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{5/2}\left(5bx^2-2a\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(3/2),x]
[Out] ((a + b*x^2)^(5/2)*(-2*a + 5*b*x^2))/(35*b^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 49, normalized size = 1.29

$$\frac{\sqrt{a+bx^2}\left(-2a^3+a^2bx^2+8ab^2x^4+5b^3x^6\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^(3/2),x]
[Out] (Sqrt[a + b*x^2]*(-2*a^3 + a^2*b*x^2 + 8*a*b^2*x^4 + 5*b^3*x^6))/(35*b^2)
```

fricas [A] time = 0.86, size = 45, normalized size = 1.18

$$\frac{\left(5\,b^3x^6+8\,ab^2x^4+a^2bx^2-2\,a^3\right)\sqrt{bx^2+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] 1/35*(5*b^3*x^6 + 8*a*b^2*x^4 + a^2*b*x^2 - 2*a^3)*sqrt(b*x^2 + a)/b^2
```

giac [A] time = 0.65, size = 29, normalized size = 0.76

$$\frac{5\left(bx^2+a\right)^{\frac{7}{2}}-7\left(bx^2+a\right)^{\frac{5}{2}}a}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] 1/35*(5*(b*x^2 + a)^(7/2) - 7*(b*x^2 + a)^(5/2)*a)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(-5b\,x^2+2a\right)}{35b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(3/2),x)
```

```
[Out] -1/35*(b*x^2+a)^(5/2)*(-5*b*x^2+2*a)/b^2
```

maxima [A] time = 1.33, size = 33, normalized size = 0.87

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}x^2}{7\,b}-\frac{2\left(bx^2+a\right)^{\frac{5}{2}}a}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(5/2)*x^2/b - 2/35*(b*x^2 + a)^(5/2)*a/b^2
```

mupad [B] time = 4.64, size = 42, normalized size = 1.11

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,a\,x^4}{35}+\frac{b\,x^6}{7}-\frac{2\,a^3}{35\,b^2}+\frac{a^2\,x^2}{35\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a*x^4)/35 + (b*x^6)/7 - (2*a^3)/(35*b^2) + (a^2*x^2)/
(35*b))
```

sympy [A] time = 1.13, size = 85, normalized size = 2.24

$$\begin{cases} -\frac{2a^3\sqrt{a+bx^2}}{35b^2}+\frac{a^2x^2\sqrt{a+bx^2}}{35b}+\frac{8ax^4\sqrt{a+bx^2}}{35}+\frac{bx^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0\\ \frac{a^{\frac{3}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-2*a**3*sqrt(a + b*x**2)/(35*b**2) + a**2*x**2*sqrt(a + b*x**2)/
(35*b) + 8*a*x**4*sqrt(a + b*x**2)/35 + b*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)
), (a**(3/2)*x**4/4, True))
```

3.363 $\int x\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(3/2),x]
```

```
[Out] (a + b*x^2)^(5/2)/(5*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{3/2}\,dx=\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(3/2),x]
```

```
[Out] (a + b*x^2)^(5/2)/(5*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^(3/2),x]
[Out] (a + b*x^2)^(5/2)/(5*b)
```

fricas [B] time = 0.78, size = 32, normalized size = 1.78

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{bx^2+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] 1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/b
```

giac [A] time = 0.67, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] 1/5*(b*x^2 + a)^(5/2)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(3/2),x)
[Out] 1/5*(b*x^2+a)^(5/2)/b
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2 + a)^(5/2)/b
```

mupad [B] time = 4.57, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^2+a\right)^{5/2}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(5/2)/(5*b)
```

sympy [A] time = 0.59, size = 61, normalized size = 3.39

$$\begin{cases} \frac{a^2\sqrt{a+bx^2}}{5b}+\frac{2ax^2\sqrt{a+bx^2}}{5}+\frac{bx^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{a^{\frac{3}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((a**2*sqrt(a + b*x**2)/(5*b) + 2*a*x**2*sqrt(a + b*x**2)/5 + b*x*
*4*sqrt(a + b*x**2)/5, Ne(b, 0)), (a**(3/2)*x**2/2, True))
```

3.364 $\int \frac{(a+bx^2)^{3/2}}{x}\,dx$

Optimal. Leaf size=54

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$a^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a\sqrt{a+bx^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x,x]
```

```
[Out] a*Sqrt[a + b*x^2] + (a + b*x^2)^(3/2)/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{3}\left(a+bx^2\right)^{3/2} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= a\sqrt{a+bx^2} + \frac{1}{3}\left(a+bx^2\right)^{3/2} + \frac{1}{2}a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= a\sqrt{a+bx^2} + \frac{1}{3}\left(a+bx^2\right)^{3/2} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= a\sqrt{a+bx^2} + \frac{1}{3}\left(a+bx^2\right)^{3/2} - a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.93

$$\frac{1}{3}\sqrt{a+bx^2}\left(4a+bx^2\right) - a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*a + b*x^2))/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```

IntegrateAlgebraic [A] time = 0.04, size = 50, normalized size = 0.93

$$\frac{1}{3}\sqrt{a+bx^2}\left(4a+bx^2\right) - a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*a + b*x^2))/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```

fricas [A] time = 0.74, size = 100, normalized size = 1.85

$$\left[\frac{1}{2}\, a^{\frac{3}{2}} \log\left(-\frac{bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{a} + 2\,a}{x^2}\right) + \frac{1}{3}\left(bx^2 + 4\,a\right)\sqrt{bx^2+a},\ \sqrt{-a}\,a \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) + \frac{1}{3}\left(bx^2 + 4\,a\right)\sqrt{bx^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(3/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/3*(b*x
^2 + 4*a)*sqrt(b*x^2 + a), sqrt(-a)*a*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 1/
3*(b*x^2 + 4*a)*sqrt(b*x^2 + a)]
```

giac [A] time = 0.63, size = 48, normalized size = 0.89

$$\frac{a^2 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{3}\left(bx^2 + a\right)^{\frac{3}{2}} + \sqrt{bx^2+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*(b*x^2 + a)^(3/2) + sqr
t(b*x^2 + a)*a
```

maple [A] time = 0.01, size = 52, normalized size = 0.96

$$-a^{\frac{3}{2}} \ln\left(\frac{2a + 2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right) + \sqrt{b\,x^2+a}\;a + \frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/3*(b*x^2+a)^(3/2)-ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)*a^(3/2)+a*(b*x^2+
a)^(1/2)
```

maxima [A] time = 1.28, size = 40, normalized size = 0.74

$$-a^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right) + \frac{1}{3}\left(bx^2 + a\right)^{\frac{3}{2}} + \sqrt{bx^2+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2) + sqrt(b*x^2
 + a)*a
```

mupad [B] time = 4.70, size = 42, normalized size = 0.78

$$a\sqrt{b\,x^2+a}-a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+\frac{\left(b\,x^2+a\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x,x)
```

```
[Out] a*(a + b*x^2)^(1/2) - a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + (a + b*x^2
)^(3/2)/3
```

sympy [A] time = 1.92, size = 78, normalized size = 1.44

$$\frac{4a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}{3}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^2}{a}\right)}{2}-a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)+\frac{\sqrt{a}\,bx^2\sqrt{1+\frac{bx^2}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x,x)
```

```
[Out] 4*a**(3/2)*sqrt(1 + b*x**2/a)/3 + a**(3/2)*log(b*x**2/a)/2 - a**(3/2)*log(s
qrt(1 + b*x**2/a) + 1) + sqrt(a)*b*x**2*sqrt(1 + b*x**2/a)/3
```

3.365 $\int \frac{(a+bx^2)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^2)^{3/2}}{2x^2}+\frac{3}{2}b\sqrt{a+bx^2}-\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{(a+bx^2)^{3/2}}{2x^2}+\frac{3}{2}b\sqrt{a+bx^2}-\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^3,x]
```

```
[Out] (3*b*Sqrt[a + b*x^2])/2 - (a + b*x^2)^(3/2)/(2*x^2) - (3*Sqrt[a]*b*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{1}{4}(3b)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{2}b\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{1}{4}(3ab)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{3}{2}b\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{1}{2}(3a)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)\\
&= \frac{3}{2}b\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{3/2}}{2x^2}-\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.59

$$\frac{b\left(a+bx^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^2)/a])/(5*a^2)
```

IntegrateAlgebraic [A] time = 0.08, size = 57, normalized size = 0.90

$$\frac{\sqrt{a+bx^2}\left(2bx^2-a\right)}{2x^2}-\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-a + 2*b*x^2))/(2*x^2) - (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/2
```

fricas [A] time = 0.62, size = 119, normalized size = 1.89

$$\left[\frac{3\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(2bx^2-a\right)\sqrt{bx^2+a}}{4x^2},\frac{3\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2bx^2-a\right)\sqrt{bx^2+a}}{2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(2*b*x^2 - a)*sqrt(b*x^2 + a))/x^2, 1/2*(3*sqrt(-a)*b*x^2*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) + (2*b*x^2 - a)*sqrt(b*x^2 + a))/x^2]
```

giac [A] time = 0.68, size = 63, normalized size = 1.00

$$\frac{\frac{3ab^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx^2+a}\,b^2-\frac{\sqrt{bx^2+a}\,ab}{x^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x^2 + a)*
b^2 - sqrt(b*x^2 + a)*a*b/x^2)/b
```

maple [A] time = 0.01, size = 75, normalized size = 1.19

$$-\frac{3\sqrt{a}\,b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2}+\frac{3\sqrt{b\,x^2+a}\,b}{2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}b}{2a}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(5/2)+1/2/a*b*(b*x^2+a)^(3/2)-3/2*a^(1/2)*b*ln((2*a+2*
(b*x^2+a)^(1/2)*a^(1/2))/x)+3/2*b*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.34, size = 63, normalized size = 1.00

$$-\frac{3}{2}\sqrt{a}\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{3}{2}\sqrt{bx^2+a}\,b+\frac{\left(bx^2+a\right)^{\frac{3}{2}}b}{2\,a}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*b + 1/2*
(b*x^2 + a)^(3/2)*b/a - 1/2*(b*x^2 + a)^(5/2)/(a*x^2)
```

mupad [B] time = 4.82, size = 47, normalized size = 0.75

$$b\sqrt{b\,x^2+a}-\frac{a\sqrt{b\,x^2+a}}{2\,x^2}-\frac{3\sqrt{a}\;b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^3,x)
```

```
[Out] b*(a + b*x^2)^(1/2) - (a*(a + b*x^2)^(1/2))/(2*x^2) - (3*a^(1/2)*b*atanh((a
 + b*x^2)^(1/2)/a^(1/2)))/2
```

sympy [A] time = 2.33, size = 88, normalized size = 1.40

$$-\frac{3\sqrt{a}\,b\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{a^2}{2\sqrt{b}\,x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{a\sqrt{b}}{2x\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - a**2/(2*sqrt(b)*x**3*sqrt(a/(b*
x**2) + 1)) + a*sqrt(b)/(2*x*sqrt(a/(b*x**2) + 1)) + b**(3/2)*x/sqrt(a/(b*x
**2) + 1)
```

3.366 $\int \frac{(a+bx^2)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=68

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{(a+bx^2)^{3/2}}{4x^4}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{(a+bx^2)^{3/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^5,x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2])/(8*x^2) - (a + b*x^2)^(3/2)/(4*x^4) - (3*b^2*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{4x^4}+\frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}+\frac{1}{16}\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}+\frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= -\frac{3b\sqrt{a+bx^2}}{8x^2}-\frac{\left(a+bx^2\right)^{3/2}}{4x^4}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 1.12

$$-\frac{2a^2+3b^2x^4\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+7abx^2+5b^2x^4}{8x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^5,x]
```

```
[Out] -1/8*(2*a^2 + 7*a*b*x^2 + 5*b^2*x^4 + 3*b^2*x^4*Sqrt[1 + (b*x^2)/a]*ArcTanh
[Sqrt[1 + (b*x^2)/a]])/(x^4*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.09, size = 59, normalized size = 0.87

$$\frac{\left(-2a-5bx^2\right)\sqrt{a+bx^2}}{8x^4}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x^5,x]
```

```
[Out] ((-2*a - 5*b*x^2)*Sqrt[a + b*x^2])/(8*x^4) - (3*b^2*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/(8*Sqrt[a])
```

fricas [A] time = 0.82, size = 136, normalized size = 2.00

$$\left[\frac{3\,\sqrt{a}\,b^2x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(5\,abx^2+2\,a^2\right)\sqrt{bx^2+a}}{16\,ax^4},\ \frac{3\,\sqrt{-a}\,b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(5\,abx^2+2\,a^2\right)\sqrt{bx^2+a}}{8\,ax^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 2*(5*a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a*x^4), 1/8*(3*sqrt(-a)*b^2*x^4
*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (5*a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a
*x^4)]
```

giac [A] time = 0.65, size = 70, normalized size = 1.03

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}b^3-3\,\sqrt{bx^2+a}\,ab^3}{b^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (5*(b*x^2 + a)^(3/2)
*b^3 - 3*sqrt(b*x^2 + a)*a*b^3)/(b^2*x^4))/b
```

maple [A] time = 0.01, size = 102, normalized size = 1.50

$$-\frac{3b^2\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{8\sqrt{a}}+\frac{3\sqrt{b\,x^2+a}\,b^2}{8a}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}b^2}{8a^2}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}b}{8a^2x^2}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(5/2)-1/8/a^2*b/x^2*(b*x^2+a)^(5/2)+1/8/a^2*b^2*(b*x^2
+a)^(3/2)-3/8/a^(1/2)*b^2*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/8/a*b^2*(
b*x^2+a)^(1/2)
```

maxima [A] time = 1.37, size = 90, normalized size = 1.32

$$-\frac{3\,b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,\sqrt{a}}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}b^2}{8\,a^2}+\frac{3\,\sqrt{bx^2+a}\,b^2}{8\,a}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}b}{8\,a^2x^2}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -3/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*(b*x^2 + a)^(3/2)*b^2/
a^2 + 3/8*sqrt(b*x^2 + a)*b^2/a - 1/8*(b*x^2 + a)^(5/2)*b/(a^2*x^2) - 1/4*(
b*x^2 + a)^(5/2)/(a*x^4)
```

mupad [B] time = 4.91, size = 52, normalized size = 0.76

$$\frac{3\,a\,\sqrt{b\,x^2+a}}{8\,x^4}-\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}}-\frac{5\,{\left(b\,x^2+a\right)}^{3/2}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^5,x)
```

```
[Out] (3*a*(a + b*x^2)^(1/2))/(8*x^4) - (3*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/
(8*a^(1/2)) - (5*(a + b*x^2)^(3/2))/(8*x^4)
```

sympy [A] time = 2.98, size = 71, normalized size = 1.04

$$-\frac{a\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{4x^3}-\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{8x}-\frac{3b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**5,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(4*x**3) - 5*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(8*x) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a))
```

3.367 $\int \frac{(a+bx^2)^{3/2}}{x^7} dx$

Optimal. Leaf size=92

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{(a+bx^2)^{3/2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{8x^4}$$

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} - \frac{b\sqrt{a+bx^2}}{8x^4} - \frac{(a+bx^2)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^7,x]
```

```
[Out] -(b*Sqrt[a + b*x^2])/(8*x^4) - (b^2*Sqrt[a + b*x^2])/(16*a*x^2) - (a + b*x^
2)^(3/2)/(6*x^6) + (b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{6x^6}+\frac{1}{4}b\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4}-\frac{\left(a+bx^2\right)^{3/2}}{6x^6}+\frac{1}{16}b^2\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4}-\frac{b^2\sqrt{a+bx^2}}{16ax^2}-\frac{\left(a+bx^2\right)^{3/2}}{6x^6}-\frac{b^3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{32a}\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4}-\frac{b^2\sqrt{a+bx^2}}{16ax^2}-\frac{\left(a+bx^2\right)^{3/2}}{6x^6}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{16a}\\
&= -\frac{b\sqrt{a+bx^2}}{8x^4}-\frac{b^2\sqrt{a+bx^2}}{16ax^2}-\frac{\left(a+bx^2\right)^{3/2}}{6x^6}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.42

$$\frac{b^3\left(a+bx^2\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx^2}{a}+1\right)}{5a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^7,x]
```

```
[Out] (b^3*(a + b*x^2)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x^2)/a])/(5*a^
4)
```

IntegrateAlgebraic [A] time = 0.10, size = 73, normalized size = 0.79

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{\sqrt{a+bx^2}\left(-8a^2-14abx^2-3b^2x^4\right)}{48ax^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2 - 14*a*b*x^2 - 3*b^2*x^4))/(48*a*x^6) + (b^3*ArcTa
nh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

fricas [A] time = 0.71, size = 157, normalized size = 1.71

$$\left[\frac{3\sqrt{a}b^3x^6\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(3ab^2x^4+14a^2bx^2+8a^3\right)\sqrt{bx^2+a}}{96a^2x^6},-\frac{3\sqrt{-a}b^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3ab^2x^4+14a^2bx^2+8a^3\right)\sqrt{bx^2+a}}{48a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(a)*b^3*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 2*(3*a*b^2*x^4 + 14*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a^2*x^6), -1/4
8*(3*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b^2*x^4 + 14*
a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a^2*x^6)]
```

giac [A] time = 0.64, size = 92, normalized size = 1.00

$$-\frac{\frac{3b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}b^4+8\left(bx^2+a\right)^{\frac{3}{2}}ab^4-3\sqrt{bx^2+a}a^2b^4}{ab^3x^6}}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x^2 + a)
^(5/2)*b^4 + 8*(b*x^2 + a)^(3/2)*a*b^4 - 3*sqrt(b*x^2 + a)*a^2*b^4)/(a*b^3*
x^6))/b
```

maple [A] time = 0.01, size = 122, normalized size = 1.33

$$\frac{b^{3}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{2}+a}\,b^{3}}{16a^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{3}}{48a^{3}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}b^{2}}{48a^{3}x^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}b}{24a^{2}x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6a\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(5/2)+1/24/a^2*b/x^4*(b*x^2+a)^(5/2)+1/48/a^3*b^2/x^2*
(b*x^2+a)^(5/2)-1/48/a^3*b^3*(b*x^2+a)^(3/2)+1/16/a^(3/2)*b^3*ln((2*a+2*(b*
x^2+a)^(1/2)*a^(1/2))/x)-1/16/a^2*b^3*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 110, normalized size = 1.20

$$\frac{b^{3}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{3}{2}}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}}{48\,a^{3}}-\frac{\sqrt{bx^{2}+a}\,b^{3}}{16\,a^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}b^{2}}{48\,a^{3}x^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}b}{24\,a^{2}x^{4}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}}{6\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/48*(b*x^2 + a)^(3/2)*b^3
/a^3 - 1/16*sqrt(b*x^2 + a)*b^3/a^2 + 1/48*(b*x^2 + a)^(5/2)*b^2/(a^3*x^2)
+ 1/24*(b*x^2 + a)^(5/2)*b/(a^2*x^4) - 1/6*(b*x^2 + a)^(5/2)/(a*x^6)
```

mupad [B] time = 4.94, size = 72, normalized size = 0.78

$$\frac{a\,\sqrt{b\,x^{2}+a}}{16\,x^{6}}-\frac{\left(b\,x^{2}+a\right)^{3/2}}{6\,x^{6}}-\frac{\left(b\,x^{2}+a\right)^{5/2}}{16\,a\,x^{6}}-\frac{b^{3}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}}{16\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^7,x)
```

```
[Out] (a*(a + b*x^2)^(1/2))/(16*x^6) - (b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*
1i)/(16*a^(3/2)) - (a + b*x^2)^(3/2)/(6*x^6) - (a + b*x^2)^(5/2)/(16*a*x^6)
```

sympy [A] time = 5.33, size = 119, normalized size = 1.29

$$-\frac{a^{2}}{6\sqrt{b}\,x^{7}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{11a\sqrt{b}}{24x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{17b^{\frac{3}{2}}}{48x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{b^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^{2}}+1}}+\frac{b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**7,x)
```

```
[Out] -a**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*a*sqrt(b)/(24*x**5*sqrt(a/
(b*x**2) + 1)) - 17*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - b**(5/2)/(16*
a*x*sqrt(a/(b*x**2) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2))
```

3.368 $\int \frac{\left(a+bx^2\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=116

$$-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}-\frac{\left(a+bx^2\right)^{3/2}}{8x^8}-\frac{b\sqrt{a+bx^2}}{16x^6}$$

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{3b^3\sqrt{a+bx^2}}{128a^2x^2}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}-\frac{b^2\sqrt{a+bx^2}}{64ax^4}-\frac{b\sqrt{a+bx^2}}{16x^6}-\frac{\left(a+bx^2\right)^{3/2}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^9,x]
```

```
[Out] -(b*Sqrt[a + b*x^2])/(16*x^6) - (b^2*Sqrt[a + b*x^2])/(64*a*x^4) + (3*b^3*S
qrt[a + b*x^2])/(128*a^2*x^2) - (a + b*x^2)^(3/2)/(8*x^8) - (3*b^4*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^9}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}}{8x^8} + \frac{1}{16}(3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{\left(a+bx^2\right)^{3/2}}{8x^8} + \frac{1}{32}b^2\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} - \frac{\left(a+bx^2\right)^{3/2}}{8x^8} - \frac{\left(3b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{128a}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} + \frac{3b^3\sqrt{a+bx^2}}{128a^2x^2} - \frac{\left(a+bx^2\right)^{3/2}}{8x^8} + \frac{\left(3b^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x\right.}{256a^2}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} + \frac{3b^3\sqrt{a+bx^2}}{128a^2x^2} - \frac{\left(a+bx^2\right)^{3/2}}{8x^8} + \frac{\left(3b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.}}{128a^2}\\
&= -\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} + \frac{3b^3\sqrt{a+bx^2}}{128a^2x^2} - \frac{\left(a+bx^2\right)^{3/2}}{8x^8} - \frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.34

$$-\frac{b^4 \left(a+b x^2\right)^{5/2} \, _2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{b x^2}{a}+1\right)}{5 a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^9,x]
```

```
[Out] -1/5*(b^4*(a + b*x^2)^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + (b*x^2)/a])/
a^5
```

IntegrateAlgebraic [A] time = 0.13, size = 84, normalized size = 0.72

$$\frac{\sqrt{a+b x^2} \left(-16 a^3-24 a^2 b x^2-2 a b^2 x^4+3 b^3 x^6\right)}{128 a^2 x^8}-\frac{3 b^4 \tanh ^{-1}\left(\frac{\sqrt{a+b x^2}}{\sqrt{a}}\right)}{128 a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^3 - 24*a^2*b*x^2 - 2*a*b^2*x^4 + 3*b^3*x^6))/(128*a
^2*x^8) - (3*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

fricas [A] time = 0.91, size = 179, normalized size = 1.54

$$\left[\frac{3 \sqrt{a} b^{4} x^{8} \log\left(-\frac{b x^{2} - 2 \sqrt{b x^{2} + a} \sqrt{a} + 2 a}{x^{2}}\right) + 2 \left(3 a b^{3} x^{6} - 2 a^{2} b^{2} x^{4} - 24 a^{3} b x^{2} - 16 a^{4}\right) \sqrt{b x^{2} + a}}{256 a^{3} x^{8}}, \frac{3 \sqrt{-a} b^{4} x^{8} \arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^{2} + a}}\right) + \left(3 a b^{3} x^{6} - 2 a^{2} b^{2} x^{4} - 24 a^{3} b x^{2} - 16 a^{4}\right) \sqrt{b x^{2} + a}}{128 a^{3} x^{8}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/256*(3*sqrt(a)*b^4*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(3*a*b^3*x^6 - 2*a^2*b^2*x^4 - 24*a^3*b*x^2 - 16*a^4)*sqrt(b*x^2 + a
))/(a^3*x^8), 1/128*(3*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) +
(3*a*b^3*x^6 - 2*a^2*b^2*x^4 - 24*a^3*b*x^2 - 16*a^4)*sqrt(b*x^2 + a))/(a^3
*x^8)]
```

giac [A] time = 0.65, size = 109, normalized size = 0.94

$$\frac{\frac{3 b^{5} \arctan\left(\frac{\sqrt{b x^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a} a^{2}}+\frac{3 \left(b x^{2}+a\right)^{\frac{7}{2}} b^{5}-11 \left(b x^{2}+a\right)^{\frac{5}{2}} a b^{5}-11 \left(b x^{2}+a\right)^{\frac{3}{2}} a^{2} b^{5}+3 \sqrt{b x^{2}+a} a^{3} b^{5}}{a^{2} b^{4} x^{8}}}{128 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/128*(3*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 +
a)^(7/2)*b^5 - 11*(b*x^2 + a)^(5/2)*a*b^5 - 11*(b*x^2 + a)^(3/2)*a^2*b^5 +
3*sqrt(b*x^2 + a)*a^3*b^5)/(a^2*b^4*x^8))/b
```

maple [A] time = 0.02, size = 142, normalized size = 1.22

$$-\frac{3 b^{4} \ln \left(\frac{2 a+2 \sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{128 a^{\frac{5}{2}}}+\frac{3 \sqrt{b\,x^{2}+a}\,b^{4}}{128 a^{3}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} b^{4}}{128 a^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} b^{3}}{128 a^{4} x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} b^{2}}{64 a^{3} x^{4}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} b}{16 a^{2} x^{6}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{8 a\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^9,x)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(5/2)+1/16/a^2*b/x^6*(b*x^2+a)^(5/2)-1/64/a^3*b^2/x^4*
(b*x^2+a)^(5/2)-1/128/a^4*b^3/x^2*(b*x^2+a)^(5/2)+1/128/a^4*b^4*(b*x^2+a)^(
3/2)-3/128/a^(5/2)*b^4*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/128/a^3*b^4*
(b*x^2+a)^(1/2)
```

maxima [A] time = 1.38, size = 130, normalized size = 1.12

$$-\frac{3\,b^{4}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{5}{2}}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}} b^{4}}{128\,a^{4}}+\frac{3\,\sqrt{bx^{2}+a}\,b^{4}}{128\,a^{3}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}} b^{3}}{128\,a^{4} x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}} b^{2}}{64\,a^{3} x^{4}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}} b}{16\,a^{2} x^{6}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}}{8\,a x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -3/128*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/128*(b*x^2 + a)^(3/2)*
b^4/a^4 + 3/128*sqrt(b*x^2 + a)*b^4/a^3 - 1/128*(b*x^2 + a)^(5/2)*b^3/(a^4*
x^2) - 1/64*(b*x^2 + a)^(5/2)*b^2/(a^3*x^4) + 1/16*(b*x^2 + a)^(5/2)*b/(a^2
*x^6) - 1/8*(b*x^2 + a)^(5/2)/(a*x^8)
```

mupad [B] time = 5.21, size = 89, normalized size = 0.77

$$\frac{3\,a\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{11\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^8}-\frac{11\,{\left(b\,x^2+a\right)}^{5/2}}{128\,a\,x^8}+\frac{3\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^2\,x^8}+\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{128\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^9,x)
```

[Out] (b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) - (11*(a + b*x^2)^(3/2))/(128*x^8) + (3*a*(a + b*x^2)^(1/2))/(128*x^8) - (11*(a + b*x^2)^(5/2))/(128*a*x^8) + (3*(a + b*x^2)^(7/2))/(128*a^2*x^8)

sympy [A] time = 8.26, size = 148, normalized size = 1.28

$$-\frac{a^2}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{5a\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{13b^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{5}{2}}}{128ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{7}{2}}}{128a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**9,x)
```

```
[Out] -a**2/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 5*a*sqrt(b)/(16*x**7*sqrt(a/(b*x**2) + 1)) - 13*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(128*a*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(7/2)/(128*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(5/2))
```

3.369 $\int x^4 \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=115

$$\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}-\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{16}ax^5\sqrt{a+bx^2}$$

Rubi [A] time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{16}ax^5\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(3/2),x]
```

```
[Out] (-3*a^3*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*x^3*Sqrt[a + b*x^2])/(64*b) + (
a*x^5*Sqrt[a + b*x^2])/16 + (x^5*(a + b*x^2)^(3/2))/8 + (3*a^4*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^{3/2}dx &= \frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{8}(3a)\int x^4\sqrt{a+bx^2}\,dx\\ &= \frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{1}{16}a^2\int\frac{x^4}{\sqrt{a+bx^2}}\,dx\\ &= \frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}-\frac{\left(3a^3\right)\int\frac{x^2}{\sqrt{a+bx^2}}\,dx}{64b}\\ &= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{\left(3a^4\right)\int\frac{1}{\sqrt{a+b}}}{128b^2}\\ &= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{\left(3a^4\right)\text{Subst}\Big(}{}\\ &= -\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{3a^4\tanh^{-1}\Big(}{128b^{5/}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 94, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\left(\frac{3a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}x\left(-3a^3+2a^2bx^2+24ab^2x^4+16b^3x^6\right)\right)}{128b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-3*a^3 + 2*a^2*b*x^2 + 24*a*b^2*x^4 + 16*b^3*x
^6) + (3*a^(7/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(128*b
^(5/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 85, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-3a^3x+2a^2bx^3+24ab^2x^5+16b^3x^7\right)}{128b^2}-\frac{3a^4\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{128b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^3*x + 2*a^2*b*x^3 + 24*a*b^2*x^5 + 16*b^3*x^7))/(128
*b^2) - (3*a^4*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(5/2))
```

fricas [A] time = 0.93, size = 168, normalized size = 1.46

$$\left[\frac{3\,a^4\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(16\,b^4x^7+24\,ab^3x^5+2\,a^2b^2x^3-3\,a^3bx\right)\sqrt{bx^2+a}}{256\,b^3},-\frac{3\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(16\,b^4x^7+24\,ab^3x^5+2\,a^2b^2x^3-3\,a^3bx\right)\sqrt{bx^2+a}}{128\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
16*b^4*x^7 + 24*a*b^3*x^5 + 2*a^2*b^2*x^3 - 3*a^3*b*x)*sqrt(b*x^2 + a))/b^3
, -1/128*(3*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (16*b^4*x^7 +
 24*a*b^3*x^5 + 2*a^2*b^2*x^3 - 3*a^3*b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.66, size = 76, normalized size = 0.66

$$\frac{1}{128}\left(2\left(4\left(2\,bx^2+3\,a\right)x^2+\frac{a^2}{b}\right)x^2-\frac{3\,a^3}{b^2}\right)\sqrt{bx^2+a}\,x-\frac{3\,a^4\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*(2*(4*(2*b*x^2 + 3*a)*x^2 + a^2/b)*x^2 - 3*a^3/b^2)*sqrt(b*x^2 + a)*x
 - 3/128*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 95, normalized size = 0.83

$$\frac{3a^4\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{128b^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^2+a}\,a^3x}{128b^2}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}x^3}{8b}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}a^2x}{64b^2}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}ax}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/8*x^3*(b*x^2+a)^(5/2)/b-1/16*a/b^2*x*(b*x^2+a)^(5/2)+1/64*a^2/b^2*x*(b*x^
2+a)^(3/2)+3/128*a^3*x*(b*x^2+a)^(1/2)/b^2+3/128*a^4/b^(5/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))
```

maxima [A] time = 1.36, size = 87, normalized size = 0.76

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}x^3}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}ax}{16\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}a^2x}{64\,b^2}+\frac{3\,\sqrt{bx^2+a}\,a^3x}{128\,b^2}+\frac{3\,a^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*x^3/b - 1/16*(b*x^2 + a)^(5/2)*a*x/b^2 + 1/64*(b*x^2
+ a)^(3/2)*a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*a^3*x/b^2 + 3/128*a^4*arcsinh(
b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(3/2), x)
```

sympy [A] time = 8.20, size = 148, normalized size = 1.29

$$-\frac{3a^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13a^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}\,bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^4\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{b^2x^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/2),x)
```

```
[Out] -3*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - a**(5/2)*x**3/(128*b*sqrt(1 +
 b*x**2/a)) + 13*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*sqrt(a)*b*x**7/(
16*sqrt(1 + b*x**2/a)) + 3*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) + b
**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.370 $\int x^2 \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=91

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(3/2),x]
```

```
[Out] (a^2*x*Sqrt[a + b*x^2])/(16*b) + (a*x^3*Sqrt[a + b*x^2])/8 + (x^3*(a + b*x^
2)^(3/2))/6 - (a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+bx^2\right)^{3/2}\,dx &= \frac{1}{6}x^3\left(a+bx^2\right)^{3/2}+\frac{1}{2}a\int x^2\sqrt{a+bx^2}\,dx\\
&= \frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}a^2\int\frac{x^2}{\sqrt{a+bx^2}}\,dx\\
&= \frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\int\frac{1}{\sqrt{a+bx^2}}\,dx}{16b}\\
&= \frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{16b}\\
&= \frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 83, normalized size = 0.91

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(3a^2+14abx^2+8b^2x^4\right)-\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(3*a^2 + 14*a*b*x^2 + 8*b^2*x^4) - (3*a^(5/2)*A
rcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(48*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 74, normalized size = 0.81

$$\frac{a^3\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{3/2}}+\frac{\sqrt{a+bx^2}\left(3a^2x+14abx^3+8b^2x^5\right)}{48b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(3*a^2*x + 14*a*b*x^3 + 8*b^2*x^5))/(48*b) + (a^3*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(3/2))
```

fricas [A] time = 0.57, size = 145, normalized size = 1.59

$$\left[\frac{3\,a^3\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8\,b^3x^5+14\,ab^2x^3+3\,a^2bx\right)\sqrt{bx^2+a}}{96\,b^2},\ \frac{3\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(8\,b^3x^5+14\,ab^2x^3+3\,a^2bx\right)\sqrt{bx^2+a}}{48\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8
*b^3*x^5 + 14*a*b^2*x^3 + 3*a^2*b*x)*sqrt(b*x^2 + a))/b^2, 1/48*(3*a^3*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*b^3*x^5 + 14*a*b^2*x^3 + 3*a^2
*b*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.64, size = 63, normalized size = 0.69

$$\frac{1}{48}\left(2\left(4\,bx^2+7\,a\right)x^2+\frac{3\,a^2}{b}\right)\sqrt{bx^2+a}\,x+\frac{a^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2 + 7*a)*x^2 + 3*a^2/b)*sqrt(b*x^2 + a)*x + 1/16*a^3*log(abs
(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.00, size = 75, normalized size = 0.82

$$-\frac{a^3\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}\;a^2x}{16b}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}ax}{24b}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}x}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)/b-1/24*a/b*x*(b*x^2+a)^(3/2)-1/16*a^2*x*(b*x^2+a)^(1/
2)/b-1/16*a^3/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.33, size = 67, normalized size = 0.74

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}x}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}ax}{24\,b}-\frac{\sqrt{bx^2+a}\,a^2x}{16\,b}-\frac{a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*x/b - 1/24*(b*x^2 + a)^(3/2)*a*x/b - 1/16*sqrt(b*x^2
+ a)*a^2*x/b - 1/16*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(3/2), x)
```

sympy [A] time = 5.21, size = 119, normalized size = 1.31

$$\frac{a^{\frac{5}{2}} x}{16 b \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{17 a^{\frac{3}{2}} x^{3}}{48 \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{11 \sqrt{a} b x^{5}}{24 \sqrt{1 + \frac{b x^{2}}{a}}} - \frac{a^{3} \operatorname{asinh}{\left(\frac{\sqrt{b} x}{\sqrt{a}} \right)}}{16 b^{\frac{3}{2}}} + \frac{b^{2} x^{7}}{6 \sqrt{a} \sqrt{1 + \frac{b x^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/2),x)
```

```
[Out] a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*a**(3/2)*x**3/(48*sqrt(1 + b*x**2
/a)) + 11*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*b**(3/2)) + b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.371 $\int \left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=65

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)^{3/2}\,dx &= \frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{4}(3a)\int \sqrt{a+bx^2}\,dx\\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 65, normalized size = 1.00

$$
\frac{1}{8}\sqrt{a+bx^2}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}+5ax+2bx^3\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(5*a*x + 2*b*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/8
```

IntegrateAlgebraic [A] time = 0.07, size = 60, normalized size = 0.92

$$
\frac{1}{8}\sqrt{a+bx^2}\left(5ax+2bx^3\right)-\frac{3a^2\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(5*a*x + 2*b*x^3))/8 - (3*a^2*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(8*Sqrt[b])
```

fricas [A] time = 0.86, size = 124, normalized size = 1.91

$$
\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,b^2x^3+5\,abx\right)\sqrt{bx^2+a}}{16\,b},-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2x^3+5\,abx\right)\sqrt{bx^2+a}}{8\,b}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.63, size = 49, normalized size = 0.75

$$\frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}\,x-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.00, size = 51, normalized size = 0.78

$$\frac{3a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}+\frac{3\sqrt{b\,x^2+a}\,ax}{8}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)+3/8*a*x*(b*x^2+a)^(1/2)+3/8*a^2/b^(1/2)*ln(b^(1/2)*x+
(b*x^2+a)^(1/2))
```

maxima [A] time = 1.31, size = 43, normalized size = 0.66

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{bx^2+a}\,ax+\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*x + 3/8*sqrt(b*x^2 + a)*a*x + 3/8*a^2*arcsinh(b*x/sqr
t(a*b))/sqrt(b)
```

mupad [B] time = 4.61, size = 37, normalized size = 0.57

$$\frac{x\left(b\,x^2+a\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2),x)
```

```
[Out] (x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(3/2)
```

sympy [A] time = 2.90, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8}+\frac{\sqrt{a}\,bx^3\sqrt{1+\frac{bx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 + 3
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

3.372 $\int \frac{(a+bx^2)^{3/2}}{x^2} dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^2)^{3/2}}{x}+\frac{3}{2}bx\sqrt{a+bx^2}+\frac{3}{2}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$-\frac{(a+bx^2)^{3/2}}{x}+\frac{3}{2}bx\sqrt{a+bx^2}+\frac{3}{2}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^2,x]
```

```
[Out] (3*b*x*Sqrt[a + b*x^2])/2 - (a + b*x^2)^(3/2)/x + (3*a*Sqrt[b]*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx &= -\frac{(a+bx^2)^{3/2}}{x} + (3b)\int \sqrt{a+bx^2}\,dx \\
&= \frac{3}{2}bx\sqrt{a+bx^2} - \frac{(a+bx^2)^{3/2}}{x} + \frac{1}{2}(3ab)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{3}{2}bx\sqrt{a+bx^2} - \frac{(a+bx^2)^{3/2}}{x} + \frac{1}{2}(3ab)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\
&= \frac{3}{2}bx\sqrt{a+bx^2} - \frac{(a+bx^2)^{3/2}}{x} + \frac{3}{2}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.79

$$-\frac{a\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^2)/a)])/(x*S
qrt[1 + (b*x^2)/a]))
```

IntegrateAlgebraic [A] time = 0.11, size = 59, normalized size = 0.94

$$\frac{(bx^2-2a)\sqrt{a+bx^2}}{2x} - \frac{3}{2}a\sqrt{b}\,\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x^2,x]
```

```
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(2*x) - (3*a*Sqrt[b]*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/2
```

fricas [A] time = 0.92, size = 112, normalized size = 1.78

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\sqrt{bx^2+a}\left(bx^2-2\,a\right)}{4\,x},-\frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\left(bx^2-2\,a\right)}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqr
t(b*x^2 + a)*(b*x^2 - 2*a))/x, -1/2*(3*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - sqrt(b*x^2 + a)*(b*x^2 - 2*a))/x]
```

giac [A] time = 0.67, size = 73, normalized size = 1.16

$$\frac{1}{2}\,\sqrt{bx^2+a}\,bx-\frac{3}{4}\,a\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^2\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b*x - 3/4*a*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2
) + 2*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.00, size = 69, normalized size = 1.10

$$\frac{3a\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2}+\frac{3\sqrt{b\,x^2+a}\,bx}{2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}bx}{a}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(5/2)+1/a*b*x*(b*x^2+a)^(3/2)+3/2*b*x*(b*x^2+a)^(1/2)+3/2*
a*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.34, size = 43, normalized size = 0.68

$$\frac{3}{2}\,\sqrt{bx^2+a}\,bx+\frac{3}{2}\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*b*x + 3/2*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a
)^(3/2)/x
```

mupad [B] time = 5.15, size = 40, normalized size = 0.63

$$-\frac{\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^2,x)
```

```
[Out] -((a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a
 + 1)^(3/2))
```

sympy [A] time = 2.37, size = 88, normalized size = 1.40

$$-\frac{a^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,bx}{2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}+\frac{b^2x^3}{2\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -a**(3/2)/(x*sqrt(1 + b*x**2/a)) - sqrt(a)*b*x/(2*sqrt(1 + b*x**2/a)) + 3*a
*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2 + b**2*x**3/(2*sqrt(a)*sqrt(1 + b*x**2/
a))
```

3.373 $\int \frac{(a+bx^2)^{3/2}}{x^4} dx$

Optimal. Leaf size=61

$$b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 206}

$$b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^4,x]
```

```
[Out] -((b*Sqrt[a + b*x^2])/x) - (a + b*x^2)^(3/2)/(3*x^3) + b^(3/2)*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^4}\,dx &= -\frac{\left(a+bx^2\right)^{3/2}}{3x^3}+b\int \frac{\sqrt{a+bx^2}}{x^2}\,dx \\
&= -\frac{b\sqrt{a+bx^2}}{x}-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}+b^2\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= -\frac{b\sqrt{a+bx^2}}{x}-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}+b^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right) \\
&= -\frac{b\sqrt{a+bx^2}}{x}-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}+b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.85

$$
-\frac{a\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^4,x]
```

```
[Out] -1/3*(a*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^2)/a)])/
(x^3*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.10, size = 57, normalized size = 0.93

$$
\frac{\left(-a-4bx^2\right)\sqrt{a+bx^2}}{3x^3}-b^{3/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x^4,x]
```

```
[Out] ((-a - 4*b*x^2)*Sqrt[a + b*x^2])/(3*x^3) - b^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]]
```

fricas [A] time = 0.95, size = 112, normalized size = 1.84

$$
\left[\frac{3\,b^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(4\,bx^2+a\right)\sqrt{bx^2+a}}{6\,x^3},-\frac{3\,\sqrt{-b}\,bx^3\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(4\,bx^2+a\right)\sqrt{bx^2+a}}{3\,x^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(4*
b*x^2 + a)*sqrt(b*x^2 + a))/x^3, -1/3*(3*sqrt(-b)*b*x^3*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (4*b*x^2 + a)*sqrt(b*x^2 + a))/x^3]
```

giac [B] time = 0.71, size = 114, normalized size = 1.87

$$-\frac{1}{2}\,b^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{4\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4ab^{\frac{3}{2}}-3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{3}{2}}+2\,a^3b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 4/3*(3*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a*b^(3/2) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2) +
 2*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.00, size = 92, normalized size = 1.51

$$b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)+\frac{\sqrt{b\,x^2+a}\,b^2x}{a}+\frac{2\left(b\,x^2+a\right)^{\frac{3}{2}}b^2x}{3a^2}-\frac{2\left(b\,x^2+a\right)^{\frac{5}{2}}b}{3a^2x}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^4,x)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(5/2)-2/3/a^2*b/x*(b*x^2+a)^(5/2)+2/3/a^2*b^2*x*(b*x^2
+a)^(3/2)+1/a*b^2*x*(b*x^2+a)^(1/2)+b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 66, normalized size = 1.08

$$\frac{\sqrt{bx^2+a}\,b^2x}{a}+b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}b}{3\,ax}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*b^2*x/a + b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 2/3*(b*x^2 + a)^
(3/2)*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)/(a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^4,x)
```

```
[Out] int((a + b*x^2)^(3/2)/x^4, x)
```

sympy [A] time = 2.03, size = 78, normalized size = 1.28

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}-\frac{b^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**4,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - 4*b**(3/2)*sqrt(a/(b*x**2) + 1)/
3 - b**(3/2)*log(a/(b*x**2))/2 + b**(3/2)*log(sqrt(a/(b*x**2) + 1) + 1)
```

3.374 $\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^6,x]
```

```
[Out] -(a + b*x^2)^(5/2)/(5*a*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx = -\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^6,x]
```

```
[Out] -1/5*(a + b*x^2)^(5/2)/(a*x^5)
```

IntegrateAlgebraic [A] time = 0.08, size = 42, normalized size = 2.00

$$\frac{\sqrt{a+bx^2}\left(-a^2-2abx^2-b^2x^4\right)}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-a^2 - 2*a*b*x^2 - b^2*x^4))/(5*a*x^5)
```

fricas [B] time = 0.82, size = 35, normalized size = 1.67

$$-\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{bx^2+a}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/(a*x^5)
```

giac [B] time = 0.64, size = 86, normalized size = 4.10

$$\frac{2\left(5\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8b^{\frac{5}{2}}+10\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{5}{2}}+a^4b^{\frac{5}{2}}\right)}{5\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2) + 10*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(5/2) + a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^6,x)
```

```
[Out] -1/5*(b*x^2+a)^(5/2)/a/x^5
```

maxima [A] time = 1.34, size = 17, normalized size = 0.81

$$-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(b*x^2 + a)^(5/2)/(a*x^5)
```

mupad [B] time = 5.29, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^2+a\right)^{5/2}}{5\,a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^6,x)
```

```
[Out] -(a + b*x^2)^(5/2)/(5*a*x^5)
```

sympy [B] time = 0.91, size = 68, normalized size = 3.24

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2}-\frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**6,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(5*x**2) - b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a)
```

3.375 $\int \frac{(a+bx^2)^{3/2}}{x^8} dx$

Optimal. Leaf size=44

$$\frac{2b(a+bx^2)^{5/2}}{35a^2x^5} - \frac{(a+bx^2)^{5/2}}{7ax^7}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b(a+bx^2)^{5/2}}{35a^2x^5} - \frac{(a+bx^2)^{5/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^8,x]
```

```
[Out] -(a + b*x^2)^(5/2)/(7*a*x^7) + (2*b*(a + b*x^2)^(5/2))/(35*a^2*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{x^8} dx &= -\frac{(a+bx^2)^{5/2}}{7ax^7} - \frac{(2b)\int \frac{(a+bx^2)^{3/2}}{x^6} dx}{7a} \\ &= -\frac{(a+bx^2)^{5/2}}{7ax^7} + \frac{2b(a+bx^2)^{5/2}}{35a^2x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^2\right)^{5/2}\left(2bx^2-5a\right)}{35a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^8,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-5*a + 2*b*x^2))/(35*a^2*x^7)
```

IntegrateAlgebraic [A] time = 0.09, size = 53, normalized size = 1.20

$$\frac{\sqrt{a+bx^2}\left(-5a^3-8a^2bx^2-ab^2x^4+2b^3x^6\right)}{35a^2x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x^8,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-5*a^3 - 8*a^2*b*x^2 - a*b^2*x^4 + 2*b^3*x^6))/(35*a^2*x^
7)
```

fricas [A] time = 0.84, size = 49, normalized size = 1.11

$$\frac{\left(2\,b^3x^6-ab^2x^4-8\,a^2bx^2-5\,a^3\right)\sqrt{bx^2+a}}{35\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(2*b^3*x^6 - a*b^2*x^4 - 8*a^2*b*x^2 - 5*a^3)*sqrt(b*x^2 + a)/(a^2*x^7
)
```

giac [B] time = 0.69, size = 166, normalized size = 3.77

$$\frac{4\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{7}{2}}+35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{7}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^2b^{\frac{7}{2}}+14\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^3b^{\frac{7}{2}}+7\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^4b^{\frac{7}{2}}-a^5b^{\frac{7}{2}}\right)}{35\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 4/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2) + 35*(sqrt(b)*x - sqrt(b*
x^2 + a))^8*a*b^(7/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2) + 14
```

```
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2) + 7*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a^4*b^(7/2) - a^5*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-2b\,x^{2}+5a\right)}{35a^{2}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^8,x)
```

```
[Out] -1/35*(b*x^2+a)^(5/2)*(-2*b*x^2+5*a)/x^7/a^2
```

maxima [A] time = 1.39, size = 36, normalized size = 0.82

$$\frac{2\left(bx^{2}+a\right)^{\frac{5}{2}}b}{35\,a^{2}x^{5}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}}{7\,ax^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] 2/35*(b*x^2 + a)^(5/2)*b/(a^2*x^5) - 1/7*(b*x^2 + a)^(5/2)/(a*x^7)
```

mupad [B] time = 5.65, size = 71, normalized size = 1.61

$$\frac{2\,b^{3}\,\sqrt{b\,x^{2}+a}}{35\,a^{2}\,x}-\frac{8\,b\,\sqrt{b\,x^{2}+a}}{35\,x^{5}}-\frac{b^{2}\,\sqrt{b\,x^{2}+a}}{35\,a\,x^{3}}-\frac{a\,\sqrt{b\,x^{2}+a}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^8,x)
```

```
[Out] (2*b^3*(a + b*x^2)^(1/2))/(35*a^2*x) - (8*b*(a + b*x^2)^(1/2))/(35*x^5) - (
b^2*(a + b*x^2)^(1/2))/(35*a*x^3) - (a*(a + b*x^2)^(1/2))/(7*x^7)
```

sympy [B] time = 1.18, size = 94, normalized size = 2.14

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{7x^{6}}-\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{35x^{4}}-\frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{35ax^{2}}+\frac{2b^{\frac{7}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**8,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 8*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(35*x**4) - b**(5/2)*sqrt(a/(b*x**2) + 1)/(35*a*x**2) + 2*b**(7/2)*sqrt(a/(
b*x**2) + 1)/(35*a**2)
```

3.376 $\int \frac{(a+bx^2)^{3/2}}{x^{10}} dx$

Optimal. Leaf size=68

$$-\frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5}+\frac{4b(a+bx^2)^{5/2}}{63a^2x^7}-\frac{(a+bx^2)^{5/2}}{9ax^9}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5}+\frac{4b(a+bx^2)^{5/2}}{63a^2x^7}-\frac{(a+bx^2)^{5/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^10,x]
```

```
[Out] -(a + b*x^2)^(5/2)/(9*a*x^9) + (4*b*(a + b*x^2)^(5/2))/(63*a^2*x^7) - (8*b^
2*(a + b*x^2)^(5/2))/(315*a^3*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x^{10}}\,dx = -\frac{\left(a+bx^2\right)^{5/2}}{9ax^9} - \frac{(4b)\int \frac{\left(a+bx^2\right)^{3/2}}{x^8}\,dx}{9a}$$
$$= -\frac{\left(a+bx^2\right)^{5/2}}{9ax^9} + \frac{4b\left(a+bx^2\right)^{5/2}}{63a^2x^7} + \frac{\left(8b^2\right)\int \frac{\left(a+bx^2\right)^{3/2}}{x^6}\,dx}{63a^2}$$
$$= -\frac{\left(a+bx^2\right)^{5/2}}{9ax^9} + \frac{4b\left(a+bx^2\right)^{5/2}}{63a^2x^7} - \frac{8b^2\left(a+bx^2\right)^{5/2}}{315a^3x^5}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^2\right)^{5/2}\left(35a^2-20abx^2+8b^2x^4\right)}{315a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^10,x]
```

```
[Out] -1/315*((a + b*x^2)^(5/2)*(35*a^2 - 20*a*b*x^2 + 8*b^2*x^4))/(a^3*x^9)
```

IntegrateAlgebraic [A] time = 0.10, size = 64, normalized size = 0.94

$$\frac{\sqrt{a+bx^2}\left(-35a^4-50a^3bx^2-3a^2b^2x^4+4ab^3x^6-8b^4x^8\right)}{315a^3x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x^10,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-35*a^4 - 50*a^3*b*x^2 - 3*a^2*b^2*x^4 + 4*a*b^3*x^6 - 8*
b^4*x^8))/(315*a^3*x^9)
```

fricas [A] time = 0.72, size = 60, normalized size = 0.88

$$-\frac{\left(8\,b^4x^8-4\,ab^3x^6+3\,a^2b^2x^4+50\,a^3bx^2+35\,a^4\right)\sqrt{bx^2+a}}{315\,a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(8*b^4*x^8 - 4*a*b^3*x^6 + 3*a^2*b^2*x^4 + 50*a^3*b*x^2 + 35*a^4)*sq
rt(b*x^2 + a)/(a^3*x^9)
```

giac [B] time = 0.64, size = 192, normalized size = 2.82

$$\frac{16\left(210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}b^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}ab^{\frac{9}{2}}+441\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^2b^{\frac{9}{2}}+126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^3b^{\frac{9}{2}}+36\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^4b^{\frac{9}{2}}-9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^5b^{\frac{9}{2}}+a^6b^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(9/2) + 315*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a*b^(9/2) + 441*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(9/2
) + 126*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(9/2) + 36*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*a^4*b^(9/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(9/2) +
 a^6*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

maple [A] time = 0.00, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(8b^2x^4-20ab\,x^2+35a^2\right)}{315a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(5/2)*(8*b^2*x^4-20*a*b*x^2+35*a^2)/x^9/a^3
```

maxima [A] time = 1.37, size = 56, normalized size = 0.82

$$-\frac{8\left(bx^2+a\right)^{\frac{5}{2}}b^2}{315\,a^3x^5}+\frac{4\left(bx^2+a\right)^{\frac{5}{2}}b}{63\,a^2x^7}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] -8/315*(b*x^2 + a)^(5/2)*b^2/(a^3*x^5) + 4/63*(b*x^2 + a)^(5/2)*b/(a^2*x^7)
 - 1/9*(b*x^2 + a)^(5/2)/(a*x^9)
```

mupad [B] time = 5.71, size = 91, normalized size = 1.34

$$\frac{4\,b^3\,\sqrt{b\,x^2+a}}{315\,a^2\,x^3}-\frac{10\,b\,\sqrt{b\,x^2+a}}{63\,x^7}-\frac{b^2\,\sqrt{b\,x^2+a}}{105\,a\,x^5}-\frac{a\,\sqrt{b\,x^2+a}}{9\,x^9}-\frac{8\,b^4\,\sqrt{b\,x^2+a}}{315\,a^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^10,x)
```

```
[Out] (4*b^3*(a + b*x^2)^(1/2))/(315*a^2*x^3) - (10*b*(a + b*x^2)^(1/2))/(63*x^7)
 - (b^2*(a + b*x^2)^(1/2))/(105*a*x^5) - (a*(a + b*x^2)^(1/2))/(9*x^9) - (8
*b^4*(a + b*x^2)^(1/2))/(315*a^3*x)
```

sympy [B] time = 1.61, size = 420, normalized size = 6.18

$$-\frac{35a^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{120a^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{138a^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{52a^3b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{3a^2b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{12ab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}-\frac{8b^{\frac{21}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**10,x)
```

```
[Out] -35*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*
x**10 + 315*a**3*b**6*x**12) - 120*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)
/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 138*a**
4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x
**10 + 315*a**3*b**6*x**12) - 52*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(
315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 3*a**2*b*
*(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10
 + 315*a**3*b**6*x**12) - 12*a*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a*
*5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 8*b**(21/2)*x**
12*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**
3*b**6*x**12)
```

3.377 $\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=92

$$\frac{16b^3(a+bx^2)^{5/2}}{1155a^4x^5}-\frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7}+\frac{2b(a+bx^2)^{5/2}}{33a^2x^9}-\frac{(a+bx^2)^{5/2}}{11ax^{11}}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{16b^3(a+bx^2)^{5/2}}{1155a^4x^5}-\frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7}+\frac{2b(a+bx^2)^{5/2}}{33a^2x^9}-\frac{(a+bx^2)^{5/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/x^12,x]
```

```
[Out] -(a + b*x^2)^(5/2)/(11*a*x^11) + (2*b*(a + b*x^2)^(5/2))/(33*a^2*x^9) - (8*
b^2*(a + b*x^2)^(5/2))/(231*a^3*x^7) + (16*b^3*(a + b*x^2)^(5/2))/(1155*a^4
*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx = -\frac{(a+bx^2)^{5/2}}{11ax^{11}} - \frac{(6b)\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx}{11a}$$
$$= -\frac{(a+bx^2)^{5/2}}{11ax^{11}} + \frac{2b(a+bx^2)^{5/2}}{33a^2x^9} + \frac{(8b^2)\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx}{33a^2}$$
$$= -\frac{(a+bx^2)^{5/2}}{11ax^{11}} + \frac{2b(a+bx^2)^{5/2}}{33a^2x^9} - \frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7} - \frac{(16b^3)\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx}{231a^3}$$
$$= -\frac{(a+bx^2)^{5/2}}{11ax^{11}} + \frac{2b(a+bx^2)^{5/2}}{33a^2x^9} - \frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7} + \frac{16b^3(a+bx^2)^{5/2}}{1155a^4x^5}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.58

$$\frac{(a+bx^2)^{5/2}(-105a^3+70a^2bx^2-40ab^2x^4+16b^3x^6)}{1155a^4x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/x^12,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-105*a^3 + 70*a^2*b*x^2 - 40*a*b^2*x^4 + 16*b^3*x^6))/(
1155*a^4*x^11)
```

IntegrateAlgebraic [A] time = 0.11, size = 75, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\,(-105a^5-140a^4bx^2-5a^3b^2x^4+6a^2b^3x^6-8ab^4x^8+16b^5x^{10})}{1155a^4x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/x^12,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-105*a^5 - 140*a^4*b*x^2 - 5*a^3*b^2*x^4 + 6*a^2*b^3*x^6
- 8*a*b^4*x^8 + 16*b^5*x^10))/(1155*a^4*x^11)
```

fricas [A] time = 0.86, size = 71, normalized size = 0.77

$$\frac{(16\,b^5x^{10}-8\,ab^4x^8+6\,a^2b^3x^6-5\,a^3b^2x^4-140\,a^4bx^2-105\,a^5)\sqrt{bx^2+a}}{1155\,a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] 1/1155*(16*b^5*x^10 - 8*a*b^4*x^8 + 6*a^2*b^3*x^6 - 5*a^3*b^2*x^4 - 140*a^4
*b*x^2 - 105*a^5)*sqrt(b*x^2 + a)/(a^4*x^11)
```

giac [B] time = 0.69, size = 220, normalized size = 2.39

$$\frac{32\left(1155\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{11}{2}}+2079\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{11}{2}}+2541\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{11}{2}}+825\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^3b^{\frac{11}{2}}+165\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^4b^{\frac{11}{2}}-55\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^5b^{\frac{11}{2}}+11\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^6b^{\frac{11}{2}}-a^7b^{\frac{11}{2}}\right)}{1155\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] 32/1155*(1155*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(11/2) + 2079*(sqrt(b)*x -
 sqrt(b*x^2 + a))^12*a*b^(11/2) + 2541*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2
*b^(11/2) + 825*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(11/2) + 165*(sqrt(b)
*x - sqrt(b*x^2 + a))^6*a^4*b^(11/2) - 55*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a
^5*b^(11/2) + 11*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(11/2) - a^7*b^(11/2
))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^11
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(-16b^3x^6+40a\,b^2x^4-70a^2b\,x^2+105a^3\right)}{1155a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^12,x)
```

```
[Out] -1/1155*(b*x^2+a)^(5/2)*(-16*b^3*x^6+40*a*b^2*x^4-70*a^2*b*x^2+105*a^3)/x^1
1/a^4
```

maxima [A] time = 1.42, size = 76, normalized size = 0.83

$$\frac{16\left(bx^2+a\right)^{\frac{5}{2}}b^3}{1155\,a^4x^5}-\frac{8\left(bx^2+a\right)^{\frac{5}{2}}b^2}{231\,a^3x^7}+\frac{2\left(bx^2+a\right)^{\frac{5}{2}}b}{33\,a^2x^9}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] 16/1155*(b*x^2 + a)^(5/2)*b^3/(a^4*x^5) - 8/231*(b*x^2 + a)^(5/2)*b^2/(a^3*
x^7) + 2/33*(b*x^2 + a)^(5/2)*b/(a^2*x^9) - 1/11*(b*x^2 + a)^(5/2)/(a*x^11)
```

mupad [B] time = 5.78, size = 111, normalized size = 1.21

$$\frac{2\,b^3\,\sqrt{b\,x^2+a}}{385\,a^2\,x^5}-\frac{4\,b\,\sqrt{b\,x^2+a}}{33\,x^9}-\frac{b^2\,\sqrt{b\,x^2+a}}{231\,a\,x^7}-\frac{a\,\sqrt{b\,x^2+a}}{11\,x^{11}}-\frac{8\,b^4\,\sqrt{b\,x^2+a}}{1155\,a^3\,x^3}+\frac{16\,b^5\,\sqrt{b\,x^2+a}}{1155\,a^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/x^12,x)
```

```
[Out] (2*b^3*(a + b*x^2)^(1/2))/(385*a^2*x^5) - (4*b*(a + b*x^2)^(1/2))/(33*x^9)
- (b^2*(a + b*x^2)^(1/2))/(231*a*x^7) - (a*(a + b*x^2)^(1/2))/(11*x^11) - (
8*b^4*(a + b*x^2)^(1/2))/(1155*a^3*x^3) + (16*b^5*(a + b*x^2)^(1/2))/(1155*
a^4*x)
```

sympy [B] time = 2.12, size = 648, normalized size = 7.04

$$-\frac{105a^8b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}-\frac{455a^7b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}-\frac{740a^6b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}-\frac{534a^5b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}-\frac{145a^4b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}+\frac{5a^3b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}+\frac{30a^2b^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}+\frac{40ab^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}+\frac{16b^{\frac{35}{2}}x^{16}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**12,x)
```

```
[Out] -105*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*
b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 455*a**7*b**
(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x*
*12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 740*a**6*b**(23/2)*x
**4*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 34
65*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 534*a**5*b**(25/2)*x**6*sqrt
(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*
b**11*x**14 + 1155*a**4*b**12*x**16) - 145*a**4*b**(27/2)*x**8*sqrt(a/(b*x*
*2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x*
*14 + 1155*a**4*b**12*x**16) + 5*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/
(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 115
5*a**4*b**12*x**16) + 30*a**2*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(1155*a*
*7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b
**12*x**16) + 40*a*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**
10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16)
 + 16*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**
6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16)
```

3.378 $\int x^7 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{a^2\left(a+bx^2\right)^{9/2}}{3b^4}+\frac{\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{3a\left(a+bx^2\right)^{11/2}}{11b^4}$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{9/2}}{3b^4}-\frac{a^3\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{3a\left(a+bx^2\right)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^(7/2))/(7*b^4) + (a^2*(a + b*x^2)^(9/2))/(3*b^4) - (3*a*(
a + b*x^2)^(11/2))/(11*b^4) + (a + b*x^2)^(13/2)/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{5/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{5/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{a^3\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{a^2\left(a+bx^2\right)^{9/2}}{3b^4}-\frac{3a\left(a+bx^2\right)^{11/2}}{11b^4}+\frac{\left(a+bx^2\right)^{13/2}}{13b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{7/2}\left(-16a^3+56a^2bx^2-126ab^2x^4+231b^3x^6\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-16*a^3 + 56*a^2*b*x^2 - 126*a*b^2*x^4 + 231*b^3*x^6))/
(3003*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{7/2}\left(-16a^3+56a^2bx^2-126ab^2x^4+231b^3x^6\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-16*a^3 + 56*a^2*b*x^2 - 126*a*b^2*x^4 + 231*b^3*x^6))/
(3003*b^4)
```

fricas [A] time = 0.81, size = 79, normalized size = 0.99

$$\frac{\left(231\,b^6x^{12}+567\,ab^5x^{10}+371\,a^2b^4x^8+5\,a^3b^3x^6-6\,a^4b^2x^4+8\,a^5bx^2-16\,a^6\right)\sqrt{bx^2+a}}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(231*b^6*x^12 + 567*a*b^5*x^10 + 371*a^2*b^4*x^8 + 5*a^3*b^3*x^6 - 6
*a^4*b^2*x^4 + 8*a^5*b*x^2 - 16*a^6)*sqrt(b*x^2 + a)/b^4
```

giac [A] time = 0.66, size = 57, normalized size = 0.71

$$\frac{231\left(bx^2+a\right)^{\frac{13}{2}}-819\left(bx^2+a\right)^{\frac{11}{2}}a+1001\left(bx^2+a\right)^{\frac{9}{2}}a^2-429\left(bx^2+a\right)^{\frac{7}{2}}a^3}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3003*(231*(b*x^2 + a)^(13/2) - 819*(b*x^2 + a)^(11/2)*a + 1001*(b*x^2 + a
)^(9/2)*a^2 - 429*(b*x^2 + a)^(7/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}\left(-231b^3x^6+126a\,b^2x^4-56a^2b\,x^2+16a^3\right)}{3003b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3003*(b*x^2+a)^(7/2)*(-231*b^3*x^6+126*a*b^2*x^4-56*a^2*b*x^2+16*a^3)/b^
4
```

maxima [A] time = 1.33, size = 73, normalized size = 0.91

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}x^6}{13\,b}-\frac{6\left(bx^2+a\right)^{\frac{7}{2}}ax^4}{143\,b^2}+\frac{8\left(bx^2+a\right)^{\frac{7}{2}}a^2x^2}{429\,b^3}-\frac{16\left(bx^2+a\right)^{\frac{7}{2}}a^3}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^2 + a)^(7/2)*x^6/b - 6/143*(b*x^2 + a)^(7/2)*a*x^4/b^2 + 8/429*(b
*x^2 + a)^(7/2)*a^2*x^2/b^3 - 16/3003*(b*x^2 + a)^(7/2)*a^3/b^4
```

mupad [B] time = 4.86, size = 75, normalized size = 0.94

$$\sqrt{b\,x^2+a}\,\left(\frac{53\,a^2\,x^8}{429}-\frac{16\,a^6}{3003\,b^4}+\frac{b^2\,x^{12}}{13}+\frac{5\,a^3\,x^6}{3003\,b}-\frac{2\,a^4\,x^4}{1001\,b^2}+\frac{8\,a^5\,x^2}{3003\,b^3}+\frac{27\,a\,b\,x^{10}}{143}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((53*a^2*x^8)/429 - (16*a^6)/(3003*b^4) + (b^2*x^12)/13 +
 (5*a^3*x^6)/(3003*b) - (2*a^4*x^4)/(1001*b^2) + (8*a^5*x^2)/(3003*b^3) + (
27*a*b*x^10)/143)
```

sympy [A] time = 9.13, size = 158, normalized size = 1.98

$$\begin{cases} -\frac{16a^6\sqrt{a+bx^2}}{3003b^4}+\frac{8a^5x^2\sqrt{a+bx^2}}{3003b^3}-\frac{2a^4x^4\sqrt{a+bx^2}}{1001b^2}+\frac{5a^3x^6\sqrt{a+bx^2}}{3003b}+\frac{53a^2x^8\sqrt{a+bx^2}}{429}+\frac{27abx^{10}\sqrt{a+bx^2}}{143}+\frac{b^2x^{12}\sqrt{a+bx^2}}{13} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-16*a**6*sqrt(a + b*x**2)/(3003*b**4) + 8*a**5*x**2*sqrt(a + b*x
**2)/(3003*b**3) - 2*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2) + 5*a**3*x**6*s
qrt(a + b*x**2)/(3003*b) + 53*a**2*x**8*sqrt(a + b*x**2)/429 + 27*a*b*x**10
*sqrt(a + b*x**2)/143 + b**2*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(5/2
)*x**8/8, True))
```

3.379 $\int x^5 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=59

$$\frac{a^2 \left(a+bx^2\right)^{7/2}}{7b^3}+\frac{\left(a+bx^2\right)^{11/2}}{11b^3}-\frac{2a \left(a+bx^2\right)^{9/2}}{9b^3}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^{7/2}}{7b^3}+\frac{\left(a+bx^2\right)^{11/2}}{11b^3}-\frac{2a \left(a+bx^2\right)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(7/2))/(7*b^3) - (2*a*(a + b*x^2)^(9/2))/(9*b^3) + (a + b*
x^2)^(11/2)/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^{5/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^{5/2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}\right) dx, x, x^2\right)\\
&= \frac{a^2 \left(a+bx^2\right)^{7/2}}{7b^3}-\frac{2a \left(a+bx^2\right)^{9/2}}{9b^3}+\frac{\left(a+bx^2\right)^{11/2}}{11b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{7/2}\left(8a^2-28abx^2+63b^2x^4\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(5/2),x]
[Out] ((a + b*x^2)^(7/2)*(8*a^2 - 28*a*b*x^2 + 63*b^2*x^4))/(693*b^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{7/2}\left(8a^2-28abx^2+63b^2x^4\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^(5/2),x]
[Out] ((a + b*x^2)^(7/2)*(8*a^2 - 28*a*b*x^2 + 63*b^2*x^4))/(693*b^3)
```

fricas [A] time = 0.84, size = 68, normalized size = 1.15

$$\frac{\left(63\,b^5x^{10}+161\,ab^4x^8+113\,a^2b^3x^6+3\,a^3b^2x^4-4\,a^4bx^2+8\,a^5\right)\sqrt{bx^2+a}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/693*(63*b^5*x^10 + 161*a*b^4*x^8 + 113*a^2*b^3*x^6 + 3*a^3*b^2*x^4 - 4*a^
4*b*x^2 + 8*a^5)*sqrt(b*x^2 + a)/b^3
```

giac [A] time = 0.92, size = 43, normalized size = 0.73

$$\frac{63\left(bx^2+a\right)^{\frac{11}{2}}-154\left(bx^2+a\right)^{\frac{9}{2}}a+99\left(bx^2+a\right)^{\frac{7}{2}}a^2}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/693*(63*(b*x^2 + a)^(11/2) - 154*(b*x^2 + a)^(9/2)*a + 99*(b*x^2 + a)^(7/
2)*a^2)/b^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(63b^{2}x^{4}-28ab\,x^{2}+8a^{2}\right)}{693b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/693*(b*x^2+a)^(7/2)*(63*b^2*x^4-28*a*b*x^2+8*a^2)/b^3
```

maxima [A] time = 1.36, size = 53, normalized size = 0.90

$$\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}x^{4}}{11\,b}-\frac{4\left(bx^{2}+a\right)^{\frac{7}{2}}ax^{2}}{99\,b^{2}}+\frac{8\left(bx^{2}+a\right)^{\frac{7}{2}}a^{2}}{693\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(7/2)*x^4/b - 4/99*(b*x^2 + a)^(7/2)*a*x^2/b^2 + 8/693*(b*
x^2 + a)^(7/2)*a^2/b^3
```

mupad [B] time = 4.75, size = 64, normalized size = 1.08

$$\sqrt{b\,x^{2}+a}\,\left(\frac{8\,a^{5}}{693\,b^{3}}+\frac{113\,a^{2}\,x^{6}}{693}+\frac{b^{2}\,x^{10}}{11}+\frac{a^{3}\,x^{4}}{231\,b}-\frac{4\,a^{4}\,x^{2}}{693\,b^{2}}+\frac{23\,a\,b\,x^{8}}{99}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^5)/(693*b^3) + (113*a^2*x^6)/693 + (b^2*x^10)/11 +
(a^3*x^4)/(231*b) - (4*a^4*x^2)/(693*b^2) + (23*a*b*x^8)/99)
```

sympy [A] time = 6.28, size = 133, normalized size = 2.25

$$\begin{cases} \frac{8a^{5}\sqrt{a+bx^{2}}}{693b^{3}}-\frac{4a^{4}x^{2}\sqrt{a+bx^{2}}}{693b^{2}}+\frac{a^{3}x^{4}\sqrt{a+bx^{2}}}{231b}+\frac{113a^{2}x^{6}\sqrt{a+bx^{2}}}{693}+\frac{23abx^{8}\sqrt{a+bx^{2}}}{99}+\frac{b^{2}x^{10}\sqrt{a+bx^{2}}}{11} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{2}}x^{6}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*a**4*x**2*sqrt(a + b*x**2
)/(693*b**2) + a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*a**2*x**6*sqrt(a +
b*x**2)/693 + 23*a*b*x**8*sqrt(a + b*x**2)/99 + b**2*x**10*sqrt(a + b*x**2)
/11, Ne(b, 0)), (a**(5/2)*x**6/6, True))
```

3.380 $\int x^3 \left(a+bx^2\right)^{5/2} \, dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{9/2}}{9b^2}-\frac{a\left(a+bx^2\right)^{7/2}}{7b^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{9/2}}{9b^2}-\frac{a\left(a+bx^2\right)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(5/2),x]
```

```
[Out] -(a*(a + b*x^2)^(7/2))/(7*b^2) + (a + b*x^2)^(9/2)/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{5/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{5/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^{5/2}}{b}+\frac{(a+bx)^{7/2}}{b}\right)dx,x,x^2\right)\\ &= -\frac{a\left(a+bx^2\right)^{7/2}}{7b^2}+\frac{\left(a+bx^2\right)^{9/2}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{7/2}\left(7bx^2-2a\right)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(5/2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-2*a + 7*b*x^2))/(63*b^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 60, normalized size = 1.58

$$\frac{\sqrt{a+bx^2}\left(-2a^4+a^3bx^2+15a^2b^2x^4+19ab^3x^6+7b^4x^8\right)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a^4 + a^3*b*x^2 + 15*a^2*b^2*x^4 + 19*a*b^3*x^6 + 7*b^
4*x^8))/(63*b^2)
```

fricas [A] time = 0.91, size = 56, normalized size = 1.47

$$\frac{\left(7\,b^4x^8+19\,ab^3x^6+15\,a^2b^2x^4+a^3bx^2-2\,a^4\right)\sqrt{bx^2+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^4*x^8 + 19*a*b^3*x^6 + 15*a^2*b^2*x^4 + a^3*b*x^2 - 2*a^4)*sqrt(b
*x^2 + a)/b^2
```

giac [A] time = 0.99, size = 29, normalized size = 0.76

$$\frac{7\left(bx^2+a\right)^{\frac{9}{2}}-9\left(bx^2+a\right)^{\frac{7}{2}}a}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/63*(7*(b*x^2 + a)^(9/2) - 9*(b*x^2 + a)^(7/2)*a)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(-7b\,x^{2}+2a\right)}{63b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(5/2),x)
```

```
[Out] -1/63*(b*x^2+a)^(7/2)*(-7*b*x^2+2*a)/b^2
```

maxima [A] time = 1.37, size = 33, normalized size = 0.87

$$\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}x^{2}}{9\,b}-\frac{2\left(bx^{2}+a\right)^{\frac{7}{2}}a}{63\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(7/2)*x^2/b - 2/63*(b*x^2 + a)^(7/2)*a/b^2
```

mupad [B] time = 4.74, size = 53, normalized size = 1.39

$$\sqrt{b\,x^{2}+a}\,\left(\frac{5\,a^{2}\,x^{4}}{21}-\frac{2\,a^{4}}{63\,b^{2}}+\frac{b^{2}\,x^{8}}{9}+\frac{a^{3}\,x^{2}}{63\,b}+\frac{19\,a\,b\,x^{6}}{63}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((5*a^2*x^4)/21 - (2*a^4)/(63*b^2) + (b^2*x^8)/9 + (a^3*x
^2)/(63*b) + (19*a*b*x^6)/63)
```

sympy [A] time = 3.83, size = 109, normalized size = 2.87

$$\begin{cases} -\frac{2a^{4}\sqrt{a+bx^{2}}}{63b^{2}}+\frac{a^{3}x^{2}\sqrt{a+bx^{2}}}{63b}+\frac{5a^{2}x^{4}\sqrt{a+bx^{2}}}{21}+\frac{19abx^{6}\sqrt{a+bx^{2}}}{63}+\frac{b^{2}x^{8}\sqrt{a+bx^{2}}}{9} & \text{for}\; b\neq 0\\ \frac{a^{\frac{5}{2}}x^{4}}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*a**4*sqrt(a + b*x**2)/(63*b**2) + a**3*x**2*sqrt(a + b*x**2)/
(63*b) + 5*a**2*x**4*sqrt(a + b*x**2)/21 + 19*a*b*x**6*sqrt(a + b*x**2)/63
+ b**2*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

3.381 $\int x\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(5/2),x]
```

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{5/2}\,dx=\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(5/2),x]
```

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^(5/2),x]
```

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

fricas [B] time = 0.96, size = 43, normalized size = 2.39

$$\frac{\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{bx^2+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/b
```

giac [A] time = 1.20, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/7*(b*x^2 + a)^(7/2)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/7*(b*x^2+a)^(7/2)/b
```

maxima [A] time = 1.34, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(7/2)/b
```

mupad [B] time = 4.60, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^2+a\right)^{7/2}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(7/2)/(7*b)
```

sympy [A] time = 2.10, size = 85, normalized size = 4.72

$$\begin{cases} \frac{a^3\sqrt{a+bx^2}}{7b}+\frac{3a^2x^2\sqrt{a+bx^2}}{7}+\frac{3abx^4\sqrt{a+bx^2}}{7}+\frac{b^2x^6\sqrt{a+bx^2}}{7} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((a**3*sqrt(a + b*x**2)/(7*b) + 3*a**2*x**2*sqrt(a + b*x**2)/7 + 3
*a*b*x**4*sqrt(a + b*x**2)/7 + b**2*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (a*
*(5/2)*x**2/2, True))
```

3.382 $\int \frac{(a+bx^2)^{5/2}}{x}\,dx$

Optimal. Leaf size=72

$$a^{5/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a^2\sqrt{a+bx^2}+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}$$

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$a^2\sqrt{a+bx^2}+a^{5/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{3}a\left(a+bx^2\right)^{3/2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x,x]
```

```
[Out] a^2*Sqrt[a + b*x^2] + (a*(a + b*x^2)^(3/2))/3 + (a + b*x^2)^(5/2)/5 - a^(5/
2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{5}\left(a+bx^2\right)^{5/2} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{3}a\left(a+bx^2\right)^{3/2} + \frac{1}{5}\left(a+bx^2\right)^{5/2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= a^2\sqrt{a+bx^2} + \frac{1}{3}a\left(a+bx^2\right)^{3/2} + \frac{1}{5}\left(a+bx^2\right)^{5/2} + \frac{1}{2}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= a^2\sqrt{a+bx^2} + \frac{1}{3}a\left(a+bx^2\right)^{3/2} + \frac{1}{5}\left(a+bx^2\right)^{5/2} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= a^2\sqrt{a+bx^2} + \frac{1}{3}a\left(a+bx^2\right)^{3/2} + \frac{1}{5}\left(a+bx^2\right)^{5/2} - a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.86

$$\frac{1}{15}\sqrt{a+bx^2}\left(23a^2+11abx^2+3b^2x^4\right)-a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(23*a^2 + 11*a*b*x^2 + 3*b^2*x^4))/15 - a^(5/2)*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.05, size = 62, normalized size = 0.86

$$\frac{1}{15}\sqrt{a+bx^2}\left(23a^2+11abx^2+3b^2x^4\right)-a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(23*a^2 + 11*a*b*x^2 + 3*b^2*x^4))/15 - a^(5/2)*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]]
```

fricas [A] time = 0.96, size = 126, normalized size = 1.75

$$\left[\frac{1}{2}\,a^{\frac{5}{2}}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+\frac{1}{15}\left(3\,b^2x^4+11\,abx^2+23\,a^2\right)\sqrt{bx^2+a}\,,\ \sqrt{-a}\,a^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\frac{1}{15}\left(3\,b^2x^4+11\,abx^2+23\,a^2\right)\sqrt{bx^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(5/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/15*(3*
b^2*x^4 + 11*a*b*x^2 + 23*a^2)*sqrt(b*x^2 + a), sqrt(-a)*a^2*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) + 1/15*(3*b^2*x^4 + 11*a*b*x^2 + 23*a^2)*sqrt(b*x^2 + a)
]
```

giac [A] time = 1.19, size = 62, normalized size = 0.86

$$\frac{a^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a+\sqrt{bx^2+a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/5*(b*x^2 + a)^(5/2) + 1/3
*(b*x^2 + a)^(3/2)*a + sqrt(b*x^2 + a)*a^2
```

maple [A] time = 0.00, size = 66, normalized size = 0.92

$$-a^{\frac{5}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^2+a}\,a^2+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}a}{3}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x,x)
```

```
[Out] 1/5*(b*x^2+a)^(5/2)+1/3*a*(b*x^2+a)^(3/2)-a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)
*a^(1/2))/x)+a^2*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 54, normalized size = 0.75

$$-a^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right) + \frac{1}{5}\left(bx^2 + a\right)^{\frac{5}{2}} + \frac{1}{3}\left(bx^2 + a\right)^{\frac{3}{2}} a + \sqrt{bx^2 + a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2) + 1/3*(b*x^2
 + a)^(3/2)*a + sqrt(b*x^2 + a)*a^2
```

mupad [B] time = 4.68, size = 59, normalized size = 0.82

$$\frac{a\left(b\,x^2 + a\right)^{3/2}}{3} + \frac{\left(b\,x^2 + a\right)^{5/2}}{5} + a^2\,\sqrt{b\,x^2 + a} + a^{5/2}\,\operatorname{atan}\left(\frac{\sqrt{b\,x^2 + a}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x,x)
```

```
[Out] a^(5/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (a*(a + b*x^2)^(3/2))/3 +
 (a + b*x^2)^(5/2)/5 + a^2*(a + b*x^2)^(1/2)
```

sympy [A] time = 3.56, size = 105, normalized size = 1.46

$$\frac{23a^{\frac{5}{2}}\sqrt{1 + \frac{bx^2}{a}}}{15} + \frac{a^{\frac{5}{2}}\log\left(\frac{bx^2}{a}\right)}{2} - a^{\frac{5}{2}}\log\left(\sqrt{1 + \frac{bx^2}{a}} + 1\right) + \frac{11a^{\frac{3}{2}}bx^2\sqrt{1 + \frac{bx^2}{a}}}{15} + \frac{\sqrt{a}\,b^2x^4\sqrt{1 + \frac{bx^2}{a}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x,x)
```

```
[Out] 23*a**(5/2)*sqrt(1 + b*x**2/a)/15 + a**(5/2)*log(b*x**2/a)/2 - a**(5/2)*log
(sqrt(1 + b*x**2/a) + 1) + 11*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a)/15 + sqrt(
a)*b**2*x**4*sqrt(1 + b*x**2/a)/5
```

3.383 $\int \frac{(a+bx^2)^{5/2}}{x^3} \, dx$

Optimal. Leaf size=80

$$-\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}+\frac{5}{2}ab\sqrt{a+bx^2}$$

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{5/2}}{2x^2}+\frac{5}{6}b\left(a+bx^2\right)^{3/2}+\frac{5}{2}ab\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^3,x]
```

```
[Out] (5*a*b*Sqrt[a + b*x^2])/2 + (5*b*(a + b*x^2)^(3/2))/6 - (a + b*x^2)^(5/2)/(
2*x^2) - (5*a^(3/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{2x^2} + \frac{1}{4}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{5}{6}b\left(a+bx^2\right)^{3/2} - \frac{\left(a+bx^2\right)^{5/2}}{2x^2} + \frac{1}{4}(5ab)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= \frac{5}{2}ab\sqrt{a+bx^2} + \frac{5}{6}b\left(a+bx^2\right)^{3/2} - \frac{\left(a+bx^2\right)^{5/2}}{2x^2} + \frac{1}{4}\left(5a^2b\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{5}{2}ab\sqrt{a+bx^2} + \frac{5}{6}b\left(a+bx^2\right)^{3/2} - \frac{\left(a+bx^2\right)^{5/2}}{2x^2} + \frac{1}{2}\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= \frac{5}{2}ab\sqrt{a+bx^2} + \frac{5}{6}b\left(a+bx^2\right)^{3/2} - \frac{\left(a+bx^2\right)^{5/2}}{2x^2} - \frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.46

$$\frac{b\left(a+bx^2\right)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + (b*x^2)/a])/(7*a^2)
```

IntegrateAlgebraic [A] time = 0.09, size = 68, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\left(-3a^2+14abx^2+2b^2x^4\right)}{6x^2}-\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2 + 14*a*b*x^2 + 2*b^2*x^4))/(6*x^2) - (5*a^(3/2)*b*
ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

fricas [A] time = 0.70, size = 142, normalized size = 1.78

$$\left[\frac{15\,a^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,b^2x^4+14\,abx^2-3\,a^2\right)\sqrt{bx^2+a}}{12\,x^2},\frac{15\,\sqrt{-a}\,abx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2\,b^2x^4+14\,abx^2-3\,a^2\right)\sqrt{bx^2+a}}{6\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(15*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(2*b^2*x^4 + 14*a*b*x^2 - 3*a^2)*sqrt(b*x^2 + a))/x^2, 1/6*(15*sqrt(-a
)*a*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*b^2*x^4 + 14*a*b*x^2 - 3*a^
2)*sqrt(b*x^2 + a))/x^2]
```

giac [A] time = 1.13, size = 82, normalized size = 1.02

$$\frac{\frac{15\,a^2b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\left(bx^2+a\right)^{\frac{3}{2}}b^2+12\,\sqrt{bx^2+a}\,ab^2-\frac{3\,\sqrt{bx^2+a}\,a^2b}{x^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^2*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*(b*x^2 + a)^(
3/2)*b^2 + 12*sqrt(b*x^2 + a)*a*b^2 - 3*sqrt(b*x^2 + a)*a^2*b/x^2)/b
```

maple [A] time = 0.01, size = 88, normalized size = 1.10

$$-\frac{5a^{\frac{3}{2}}b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2}+\frac{5\sqrt{b\,x^2+a}\;ab}{2}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}b}{6}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}b}{2a}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(7/2)+1/2/a*b*(b*x^2+a)^(5/2)+5/6*b*(b*x^2+a)^(3/2)-5/
2*a^(3/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+5/2*a*b*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.38, size = 76, normalized size = 0.95

$$-\frac{5}{2}\,a^{\frac{3}{2}}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{5}{6}\left(bx^2+a\right)^{\frac{3}{2}}b+\frac{\left(bx^2+a\right)^{\frac{5}{2}}b}{2\,a}+\frac{5}{2}\,\sqrt{bx^2+a}\,ab-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -5/2*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 5/6*(b*x^2 + a)^(3/2)*b + 1/
2*(b*x^2 + a)^(5/2)*b/a + 5/2*sqrt(b*x^2 + a)*a*b - 1/2*(b*x^2 + a)^(7/2)/(
a*x^2)
```

mupad [B] time = 4.95, size = 66, normalized size = 0.82

$$\frac{b\left(b\,x^2+a\right)^{3/2}}{3}-\frac{a^2\,\sqrt{b\,x^2+a}}{2\,x^2}+2\,a\,b\,\sqrt{b\,x^2+a}+\frac{a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^3,x)
```

```
[Out] (b*(a + b*x^2)^(3/2))/3 - (a^2*(a + b*x^2)^(1/2))/(2*x^2) + (a^(3/2)*b*atan
(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2 + 2*a*b*(a + b*x^2)^(1/2)
```

sympy [A] time = 3.22, size = 112, normalized size = 1.40

$$-\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}}{2x^2}+\frac{7a^{\frac{3}{2}}b\sqrt{1+\frac{bx^2}{a}}}{3}+\frac{5a^{\frac{3}{2}}b\log\left(\frac{bx^2}{a}\right)}{4}-\frac{5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2}+\frac{\sqrt{a}\,b^2x^2\sqrt{1+\frac{bx^2}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -a**(5/2)*sqrt(1 + b*x**2/a)/(2*x**2) + 7*a**(3/2)*b*sqrt(1 + b*x**2/a)/3 +
 5*a**(3/2)*b*log(b*x**2/a)/4 - 5*a**(3/2)*b*log(sqrt(1 + b*x**2/a) + 1)/2
+ sqrt(a)*b**2*x**2*sqrt(1 + b*x**2/a)/3
```

3.384 $\int \frac{(a+bx^2)^{5/2}}{x^5} \, dx$

Optimal. Leaf size=86

$$\frac{15}{8}b^2\sqrt{a+bx^2}-\frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{5b\left(a+bx^2\right)^{3/2}}{8x^2}-\frac{\left(a+bx^2\right)^{5/2}}{4x^4}$$

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$\frac{15}{8}b^2\sqrt{a+bx^2}-\frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{5/2}}{4x^4}-\frac{5b\left(a+bx^2\right)^{3/2}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^5,x]
```

```
[Out] (15*b^2*Sqrt[a + b*x^2])/8 - (5*b*(a + b*x^2)^(3/2))/(8*x^2) - (a + b*x^2)^
(5/2)/(4*x^4) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{4x^4}+\frac{1}{8}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{5b\left(a+bx^2\right)^{3/2}}{8x^2}-\frac{\left(a+bx^2\right)^{5/2}}{4x^4}+\frac{1}{16}\left(15b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= \frac{15}{8}b^2\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{8x^2}-\frac{\left(a+bx^2\right)^{5/2}}{4x^4}+\frac{1}{16}\left(15ab^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{15}{8}b^2\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{8x^2}-\frac{\left(a+bx^2\right)^{5/2}}{4x^4}+\frac{1}{8}(15ab)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)\\
&= \frac{15}{8}b^2\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{8x^2}-\frac{\left(a+bx^2\right)^{5/2}}{4x^4}-\frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.45

$$
-\frac{b^2\left(a+bx^2\right)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^3}
$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(5/2)/x^5,x]

[Out] -1/7*(b^2*(a + b*x^2)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 + (b*x^2)/a])/a^3

IntegrateAlgebraic [A] time = 0.10, size = 70, normalized size = 0.81

$$\frac{\sqrt{a+bx^2}\left(-2a^2-9abx^2+8b^2x^4\right)}{8x^4}-\frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^5,x]

[Out] (Sqrt[a + b*x^2]*(-2*a^2 - 9*a*b*x^2 + 8*b^2*x^4))/(8*x^4) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8

fricas [A] time = 0.91, size = 145, normalized size = 1.69

$$\left[\frac{15\sqrt{a}\,b^2x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(8b^2x^4-9abx^2-2a^2\right)\sqrt{bx^2+a}}{16x^4},\ \frac{15\sqrt{-a}\,b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(8b^2x^4-9abx^2-2a^2\right)\sqrt{bx^2+a}}{8x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="fricas")

[Out] [1/16*(15*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*b^2*x^4 - 9*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^4, 1/8*(15*sqrt(-a)*b^2*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*b^2*x^4 - 9*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^4]

giac [A] time = 1.16, size = 88, normalized size = 1.02

$$\frac{\frac{15ab^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\sqrt{bx^2+a}\,b^3-\frac{9\left(bx^2+a\right)^{\frac{3}{2}}ab^3-7\sqrt{bx^2+a}\,a^2b^3}{b^2x^4}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="giac")

[Out] 1/8*(15*a*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x^2 + a)*b^3 - (9*(b*x^2 + a)^(3/2)*a*b^3 - 7*sqrt(b*x^2 + a)*a^2*b^3)/(b^2*x^4))/b

maple [A] time = 0.01, size = 116, normalized size = 1.35

$$-\frac{15\sqrt{a}\, b^{2} \ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8}+\frac{15\sqrt{b\,x^{2}+a}\, b^{2}}{8}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}} b^{2}}{8a}+\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{2}} b^{2}}{8a^{2}}-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{2}} b}{8a^{2}x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{4a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(7/2)-3/8/a^2*b/x^2*(b*x^2+a)^(7/2)+3/8/a^2*b^2*(b*x^2
+a)^(5/2)+5/8/a*b^2*(b*x^2+a)^(3/2)-15/8*a^(1/2)*b^2*ln((2*a+2*(b*x^2+a)^(1
/2)*a^(1/2))/x)+15/8*b^2*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.35, size = 104, normalized size = 1.21

$$-\frac{15}{8}\sqrt{a}\, b^{2} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{15}{8}\sqrt{bx^{2}+a}\, b^{2}+\frac{3\left(bx^{2}+a\right)^{\frac{5}{2}} b^{2}}{8\,a^{2}}+\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}} b^{2}}{8\,a}-\frac{3\left(bx^{2}+a\right)^{\frac{7}{2}} b}{8\,a^{2}x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -15/8*sqrt(a)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) + 15/8*sqrt(b*x^2 + a)*b^2
+ 3/8*(b*x^2 + a)^(5/2)*b^2/a^2 + 5/8*(b*x^2 + a)^(3/2)*b^2/a - 3/8*(b*x^2
+ a)^(7/2)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(7/2)/(a*x^4)
```

mupad [B] time = 5.01, size = 71, normalized size = 0.83

$$b^{2}\sqrt{b\,x^{2}+a}-\frac{9\,a\left(b\,x^{2}+a\right)^{3/2}}{8\,x^{4}}+\frac{7\,a^{2}\sqrt{b\,x^{2}+a}}{8\,x^{4}}+\frac{\sqrt{a}\,b^{2}\operatorname{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1\mathrm{i}}{\sqrt{a}}\right)15\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^5,x)
```

```
[Out] b^2*(a + b*x^2)^(1/2) + (a^(1/2)*b^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1
5i)/8 - (9*a*(a + b*x^2)^(3/2))/(8*x^4) + (7*a^2*(a + b*x^2)^(1/2))/(8*x^4)
```

sympy [A] time = 3.73, size = 117, normalized size = 1.36

$$-\frac{15\sqrt{a}\,b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8}-\frac{a^{3}}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{11a^{2}\sqrt{b}}{8x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{b^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**5,x)
```

```
[Out] -15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**3/(4*sqrt(b)*x**5*sqrt(a
/(b*x**2) + 1)) - 11*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - a*b**(3/2
)/(8*x*sqrt(a/(b*x**2) + 1)) + b**(5/2)*x/sqrt(a/(b*x**2) + 1)
```

3.385 $\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=89

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{(a+bx^2)^{5/2}}{6x^6}-\frac{5b(a+bx^2)^{3/2}}{24x^4}$$

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{5b(a+bx^2)^{3/2}}{24x^4}-\frac{(a+bx^2)^{5/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^7,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x^2])/(16*x^2) - (5*b*(a + b*x^2)^(3/2))/(24*x^4) - (a +
 b*x^2)^(5/2)/(6*x^6) - (5*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a
])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{5/2}}{6x^6}+\frac{1}{12}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{5b\left(a+bx^2\right)^{3/2}}{24x^4}-\frac{\left(a+bx^2\right)^{5/2}}{6x^6}+\frac{1}{16}\left(5b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{24x^4}-\frac{\left(a+bx^2\right)^{5/2}}{6x^6}+\frac{1}{32}\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{24x^4}-\frac{\left(a+bx^2\right)^{5/2}}{6x^6}+\frac{1}{16}\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\right.\\
&= -\frac{5b^2\sqrt{a+bx^2}}{16x^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{24x^4}-\frac{\left(a+bx^2\right)^{5/2}}{6x^6}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 87, normalized size = 0.98

$$-\frac{8a^3+34a^2bx^2+15b^3x^6\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+59ab^2x^4+33b^3x^6}{48x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^7,x]
```

```
[Out] -1/48*(8*a^3 + 34*a^2*b*x^2 + 59*a*b^2*x^4 + 33*b^3*x^6 + 15*b^3*x^6*Sqrt[1
 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(x^6*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.12, size = 70, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\left(-8a^2-26abx^2-33b^2x^4\right)}{48x^6}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2 - 26*a*b*x^2 - 33*b^2*x^4))/(48*x^6) - (5*b^3*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a])
```

fricas [A] time = 0.99, size = 158, normalized size = 1.78

$$\left[\frac{15\sqrt{a}b^3x^6\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(33ab^2x^4+26a^2bx^2+8a^3\right)\sqrt{bx^2+a}}{96ax^6}, \frac{15\sqrt{-a}b^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(33ab^2x^4+26a^2bx^2+8a^3\right)\sqrt{bx^2+a}}{48ax^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*sqrt(a)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) - 2*(33*a*b^2*x^4 + 26*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a*x^6), 1/48
*(15*sqrt(-a)*b^3*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (33*a*b^2*x^4 + 26
*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a*x^6)]
```

giac [A] time = 1.22, size = 87, normalized size = 0.98

$$\frac{\frac{15b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{33\left(bx^2+a\right)^{\frac{5}{2}}b^4-40\left(bx^2+a\right)^{\frac{3}{2}}ab^4+15\sqrt{bx^2+a}a^2b^4}{b^3x^6}}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(15*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (33*(b*x^2 + a)^(5
/2)*b^4 - 40*(b*x^2 + a)^(3/2)*a*b^4 + 15*sqrt(b*x^2 + a)*a^2*b^4)/(b^3*x^6
))/b
```

maple [A] time = 0.01, size = 139, normalized size = 1.56

$$-\frac{5b^3\ln\left(\frac{2a+2\sqrt{bx^2+a}\sqrt{a}}{x}\right)}{16\sqrt{a}}+\frac{5\sqrt{bx^2+a}\,b^3}{16a}+\frac{5\left(bx^2+a\right)^{\frac{3}{2}}b^3}{48a^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}b^3}{16a^3}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}b^2}{16a^3x^2}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}b}{24a^2x^4}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{6a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(7/2)-1/24/a^2*b/x^4*(b*x^2+a)^(7/2)-1/16/a^3*b^2/x^2*
(b*x^2+a)^(7/2)+1/16/a^3*b^3*(b*x^2+a)^(5/2)+5/48/a^2*b^3*(b*x^2+a)^(3/2)-5
/16/a^(1/2)*b^3*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+5/16/a*b^3*(b*x^2+a)^
(1/2)
```

maxima [A] time = 1.47, size = 127, normalized size = 1.43

$$-\frac{5\,b^{3}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,\sqrt{a}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}b^{3}}{16\,a^{3}}+\frac{5\,\left(bx^{2}+a\right)^{\frac{3}{2}}b^{3}}{48\,a^{2}}+\frac{5\,\sqrt{bx^{2}+a}\,b^{3}}{16\,a}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}{16\,a^{3}x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}b}{24\,a^{2}x^{4}}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{6\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] -5/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/16*(b*x^2 + a)^(5/2)*b^
3/a^3 + 5/48*(b*x^2 + a)^(3/2)*b^3/a^2 + 5/16*sqrt(b*x^2 + a)*b^3/a - 1/16*
(b*x^2 + a)^(7/2)*b^2/(a^3*x^2) - 1/24*(b*x^2 + a)^(7/2)*b/(a^2*x^4) - 1/6*
(b*x^2 + a)^(7/2)/(a*x^6)
```

mupad [B] time = 5.20, size = 72, normalized size = 0.81

$$\frac{5\,a\,\left(b\,x^2+a\right)^{3/2}}{6\,x^6}-\frac{11\,\left(b\,x^2+a\right)^{5/2}}{16\,x^6}-\frac{5\,a^2\,\sqrt{b\,x^2+a}}{16\,x^6}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^7,x)
```

```
[Out] (b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(1/2)) - (11*(a + b*x^2
)^(5/2))/(16*x^6) + (5*a*(a + b*x^2)^(3/2))/(6*x^6) - (5*a^2*(a + b*x^2)^(1
/2))/(16*x^6)
```

sympy [A] time = 4.53, size = 99, normalized size = 1.11

$$-\frac{a^{2}\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{6x^{5}}-\frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{24x^{3}}-\frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{16x}-\frac{5b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**7,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(6*x**5) - 13*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(24*x**3) - 11*b**(5/2)*sqrt(a/(b*x**2) + 1)/(16*x) - 5*b**3*asinh(sq
rt(a)/(sqrt(b)*x))/(16*sqrt(a))
```

3.386 $\int \frac{(a+bx^2)^{5/2}}{x^9} \, dx$

Optimal. Leaf size=113

$$\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{(a+bx^2)^{5/2}}{8x^8} - \frac{5b(a+bx^2)^{3/2}}{48x^6}$$

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b(a+bx^2)^{3/2}}{48x^6} - \frac{(a+bx^2)^{5/2}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^9,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x^2])/(64*x^4) - (5*b^3*Sqrt[a + b*x^2])/(128*a*x^2) - (
5*b*(a + b*x^2)^(3/2))/(48*x^6) - (a + b*x^2)^(5/2)/(8*x^8) + (5*b^4*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{5/2}}{8x^8} + \frac{1}{16}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8} + \frac{1}{32}\left(5b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8} + \frac{1}{128}\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8} - \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx \ldots\right.}{256a}$$

$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx \ldots\right.}{128a}$$

$$= -\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b\left(a+bx^2\right)^{3/2}}{48x^6} - \frac{\left(a+bx^2\right)^{5/2}}{8x^8} + \frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.35

$$-\frac{b^4\left(a+bx^2\right)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^9,x]
```

```
[Out] -1/7*(b^4*(a + b*x^2)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, 1 + (b*x^2)/a])/
a^5
```

IntegrateAlgebraic [A] time = 0.13, size = 84, normalized size = 0.74

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}+\frac{\sqrt{a+bx^2}\left(-48a^3-136a^2bx^2-118ab^2x^4-15b^3x^6\right)}{384ax^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3 - 136*a^2*b*x^2 - 118*a*b^2*x^4 - 15*b^3*x^6))/(3
84*a*x^8) + (5*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

fricas [A] time = 0.98, size = 179, normalized size = 1.58

$$\left[\frac{15\sqrt{a}b^4x^8\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(15ab^3x^6+118a^2b^2x^4+136a^3bx^2+48a^4\right)\sqrt{bx^2+a}}{768a^2x^8},-\frac{15\sqrt{-a}b^4x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(15ab^3x^6+118a^2b^2x^4+136a^3bx^2+48a^4\right)\sqrt{bx^2+a}}{384a^2x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/768*(15*sqrt(a)*b^4*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) - 2*(15*a*b^3*x^6 + 118*a^2*b^2*x^4 + 136*a^3*b*x^2 + 48*a^4)*sqrt(b*x^
2 + a))/(a^2*x^8), -1/384*(15*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqrt(b*x^2 +
 a)) + (15*a*b^3*x^6 + 118*a^2*b^2*x^4 + 136*a^3*b*x^2 + 48*a^4)*sqrt(b*x^2
 + a))/(a^2*x^8)]
```

giac [A] time = 1.11, size = 109, normalized size = 0.96

$$-\frac{\frac{15\,b^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{15\left(bx^2+a\right)^{\frac{7}{2}}b^5+73\left(bx^2+a\right)^{\frac{5}{2}}ab^5-55\left(bx^2+a\right)^{\frac{3}{2}}a^2b^5+15\sqrt{bx^2+a}\,a^3b^5}{ab^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/384*(15*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x^2 +
 a)^(7/2)*b^5 + 73*(b*x^2 + a)^(5/2)*a*b^5 - 55*(b*x^2 + a)^(3/2)*a^2*b^5 +
 15*sqrt(b*x^2 + a)*a^3*b^5)/(a*b^4*x^8))/b
```

maple [A] time = 0.02, size = 159, normalized size = 1.41

$$\frac{5b^4\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{128a^{\frac{3}{2}}}-\frac{5\sqrt{b\,x^2+a}\,b^4}{128a^2}-\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}b^4}{384a^3}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}b^4}{128a^4}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}b^3}{128a^4x^2}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}{192a^3x^4}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}b}{48a^2x^6}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{8a\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^9,x)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(7/2)+1/48/a^2*b/x^6*(b*x^2+a)^(7/2)+1/192/a^3*b^2/x^4
*(b*x^2+a)^(7/2)+1/128/a^4*b^3/x^2*(b*x^2+a)^(7/2)-1/128/a^4*b^4*(b*x^2+a)^
(5/2)-5/384/a^3*b^4*(b*x^2+a)^(3/2)+5/128/a^(3/2)*b^4*ln((2*a+2*(b*x^2+a)^(
1/2)*a^(1/2))/x)-5/128/a^2*b^4*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.38, size = 147, normalized size = 1.30

$$\frac{5\,b^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{3}{2}}}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}b^4}{128\,a^4}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}b^4}{384\,a^3}-\frac{5\sqrt{bx^2+a}\,b^4}{128\,a^2}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}b^3}{128\,a^4x^2}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}b^2}{192\,a^3x^4}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}b}{48\,a^2x^6}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 5/128*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/128*(b*x^2 + a)^(5/2)*b
^4/a^4 - 5/384*(b*x^2 + a)^(3/2)*b^4/a^3 - 5/128*sqrt(b*x^2 + a)*b^4/a^2 +
1/128*(b*x^2 + a)^(7/2)*b^3/(a^4*x^2) + 1/192*(b*x^2 + a)^(7/2)*b^2/(a^3*x^
4) + 1/48*(b*x^2 + a)^(7/2)*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(7/2)/(a*x^8)
```

mupad [B] time = 5.43, size = 89, normalized size = 0.79

$$\frac{55\,a\,{\left(b\,x^2+a\right)}^{3/2}}{384\,x^8}-\frac{73\,{\left(b\,x^2+a\right)}^{5/2}}{384\,x^8}-\frac{5\,a^2\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{5\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^8}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{128\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^9,x)
```

```
[Out] (55*a*(a + b*x^2)^(3/2))/(384*x^8) - (b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/
2))*5i)/(128*a^(3/2)) - (73*(a + b*x^2)^(5/2))/(384*x^8) - (5*a^2*(a + b*x^
2)^(1/2))/(128*x^8) - (5*(a + b*x^2)^(7/2))/(128*a*x^8)
```

sympy [A] time = 7.58, size = 150, normalized size = 1.33

$$-\frac{a^3}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{23a^2\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{127ab^{\frac{3}{2}}}{192x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{133b^{\frac{5}{2}}}{384x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{5b^{\frac{7}{2}}}{128ax\sqrt{\frac{a}{bx^2}+1}}+\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**9,x)
```

```
[Out] -a**3/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 23*a**2*sqrt(b)/(48*x**7*sqrt
(a/(b*x**2) + 1)) - 127*a*b**(3/2)/(192*x**5*sqrt(a/(b*x**2) + 1)) - 133*b*
*(5/2)/(384*x**3*sqrt(a/(b*x**2) + 1)) - 5*b**(7/2)/(128*a*x*sqrt(a/(b*x**2
) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(3/2))
```

3.387 $\int \frac{(a+bx^2)^{5/2}}{x^{11}} dx$

Optimal. Leaf size=137

$$-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}+\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}-\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{(a+bx^2)^{5/2}}{10x^{10}}-\frac{b(a+bx^2)^{3/2}}{16x^8}$$

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{3b^4\sqrt{a+bx^2}}{256a^2x^2}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}-\frac{b^3\sqrt{a+bx^2}}{128ax^4}-\frac{b^2\sqrt{a+bx^2}}{32x^6}-\frac{b(a+bx^2)^{3/2}}{16x^8}-\frac{(a+bx^2)^{5/2}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^11,x]
```

```
[Out] -(b^2*Sqrt[a + b*x^2])/(32*x^6) - (b^3*Sqrt[a + b*x^2])/(128*a*x^4) + (3*b^
4*Sqrt[a + b*x^2])/(256*a^2*x^2) - (b*(a + b*x^2)^(3/2))/(16*x^8) - (a + b*
x^2)^(5/2)/(10*x^10) - (3*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(5/2
))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2)^{5/2}}{10x^{10}} + \frac{1}{4}b\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} + \frac{1}{32}(3b^2)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} + \frac{1}{64}b^3\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} - \frac{(3b^4)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x\right.}{256a}$$

$$= -\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} + \frac{3b^4\sqrt{a+bx^2}}{256a^2x^2} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} + \frac{(3b^5)\,\text{Subst}}{}$$

$$= -\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} + \frac{3b^4\sqrt{a+bx^2}}{256a^2x^2} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} + \frac{(3b^4)\,\text{Subst}}{}$$

$$= -\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} + \frac{3b^4\sqrt{a+bx^2}}{256a^2x^2} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} - \frac{3b^5\tanh^{-1}(}{256a^5}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.28

$$\frac{b^5(a+bx^2)^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};\frac{bx^2}{a}+1\right)}{7a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^11,x]
```

```
[Out] (b^5*(a + b*x^2)^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, 1 + (b*x^2)/a])/(7*a^
6)
```

IntegrateAlgebraic [A] time = 0.16, size = 95, normalized size = 0.69

$$\frac{\sqrt{a+bx^2}\left(-128a^4-336a^3bx^2-248a^2b^2x^4-10ab^3x^6+15b^4x^8\right)}{1280a^2x^{10}} - \frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-128*a^4 - 336*a^3*b*x^2 - 248*a^2*b^2*x^4 - 10*a*b^3*x^6
 + 15*b^4*x^8))/(1280*a^2*x^10) - (3*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(256*a^(5/2))
```

fricas [A] time = 1.04, size = 201, normalized size = 1.47

$$\left[\frac{15\,\sqrt{a}\,b^5x^{10}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(15\,ab^4x^8-10\,a^2b^3x^6-248\,a^3b^2x^4-336\,a^4bx^2-128\,a^5\right)\sqrt{bx^2+a}}{2560\,a^3x^{10}},\ \frac{15\,\sqrt{-a}\,b^5x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(15\,ab^4x^8-10\,a^2b^3x^6-248\,a^3b^2x^4-336\,a^4bx^2-128\,a^5\right)\sqrt{bx^2+a}}{1280\,a^3x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*sqrt(a)*b^5*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)
/x^2) + 2*(15*a*b^4*x^8 - 10*a^2*b^3*x^6 - 248*a^3*b^2*x^4 - 336*a^4*b*x^2
- 128*a^5)*sqrt(b*x^2 + a))/(a^3*x^10), 1/1280*(15*sqrt(-a)*b^5*x^10*arctan
(sqrt(-a)/sqrt(b*x^2 + a)) + (15*a*b^4*x^8 - 10*a^2*b^3*x^6 - 248*a^3*b^2*x
^4 - 336*a^4*b*x^2 - 128*a^5)*sqrt(b*x^2 + a))/(a^3*x^10)]
```

giac [A] time = 1.16, size = 126, normalized size = 0.92

$$\frac{\frac{15\,b^6\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{15\left(bx^2+a\right)^{\frac{9}{2}}b^6-70\left(bx^2+a\right)^{\frac{7}{2}}ab^6-128\left(bx^2+a\right)^{\frac{5}{2}}a^2b^6+70\left(bx^2+a\right)^{\frac{3}{2}}a^3b^6-15\,\sqrt{bx^2+a}\,a^4b^6}{a^2b^5x^{10}}}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*(15*b^6*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (15*(b*x^2
 + a)^(9/2)*b^6 - 70*(b*x^2 + a)^(7/2)*a*b^6 - 128*(b*x^2 + a)^(5/2)*a^2*b^
6 + 70*(b*x^2 + a)^(3/2)*a^3*b^6 - 15*sqrt(b*x^2 + a)*a^4*b^6)/(a^2*b^5*x^1
0))/b
```

maple [A] time = 0.04, size = 179, normalized size = 1.31

$$-\frac{3b^5\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{256a^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^2+a}\,b^5}{256a^3}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}b^5}{256a^4}+\frac{3\left(b\,x^2+a\right)^{\frac{5}{2}}b^5}{1280a^5}-\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}b^4}{1280a^5x^2}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}b^3}{640a^4x^4}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}{160a^3x^6}+\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}b}{80a^2x^8}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{10a\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^11,x)
```

```
[Out] -1/10/a/x^10*(b*x^2+a)^(7/2)+3/80/a^2*b/x^8*(b*x^2+a)^(7/2)-1/160/a^3*b^2/x
^6*(b*x^2+a)^(7/2)-1/640/a^4*b^3/x^4*(b*x^2+a)^(7/2)-3/1280/a^5*b^4/x^2*(b*
x^2+a)^(7/2)+3/1280/a^5*b^5*(b*x^2+a)^(5/2)+1/256/a^4*b^5*(b*x^2+a)^(3/2)-3
/256/a^(5/2)*b^5*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/256/a^3*b^5*(b*x^2
+a)^(1/2)
```

maxima [A] time = 1.44, size = 167, normalized size = 1.22

$$-\frac{3\,b^5\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\,a^{\frac{5}{2}}}+\frac{3\,\left(bx^2+a\right)^{\frac{5}{2}}b^5}{1280\,a^5}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}b^5}{256\,a^4}+\frac{3\,\sqrt{bx^2+a}\,b^5}{256\,a^3}-\frac{3\,\left(bx^2+a\right)^{\frac{7}{2}}b^4}{1280\,a^5x^2}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}b^3}{640\,a^4x^4}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}b^2}{160\,a^3x^6}+\frac{3\,\left(bx^2+a\right)^{\frac{7}{2}}b}{80\,a^2x^8}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] -3/256*b^5*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/1280*(b*x^2 + a)^(5/2)
*b^5/a^5 + 1/256*(b*x^2 + a)^(3/2)*b^5/a^4 + 3/256*sqrt(b*x^2 + a)*b^5/a^3
- 3/1280*(b*x^2 + a)^(7/2)*b^4/(a^5*x^2) - 1/640*(b*x^2 + a)^(7/2)*b^3/(a^4
*x^4) - 1/160*(b*x^2 + a)^(7/2)*b^2/(a^3*x^6) + 3/80*(b*x^2 + a)^(7/2)*b/(a
^2*x^8) - 1/10*(b*x^2 + a)^(7/2)/(a*x^10)
```

mupad [B] time = 5.68, size = 106, normalized size = 0.77

$$\frac{7\,a\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^{10}}-\frac{{\left(b\,x^2+a\right)}^{5/2}}{10\,x^{10}}-\frac{3\,a^2\,\sqrt{b\,x^2+a}}{256\,x^{10}}-\frac{7\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^{10}}+\frac{3\,{\left(b\,x^2+a\right)}^{9/2}}{256\,a^2\,x^{10}}+\frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{256\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^11,x)
```

```
[Out] (b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(256*a^(5/2)) - (a + b*x^2)^(
5/2)/(10*x^10) + (7*a*(a + b*x^2)^(3/2))/(128*x^10) - (3*a^2*(a + b*x^2)^(1
/2))/(256*x^10) - (7*(a + b*x^2)^(7/2))/(128*a*x^10) + (3*(a + b*x^2)^(9/2)
)/(256*a^2*x^10)
```

sympy [A] time = 11.54, size = 175, normalized size = 1.28

$$-\frac{a^3}{10\sqrt{b}\,x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{29a^2\sqrt{b}}{80x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{73ab^{\frac{3}{2}}}{160x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{129b^{\frac{5}{2}}}{640x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{7}{2}}}{256ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{9}{2}}}{256a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3b^5\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{256a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**11,x)
```

```
[Out] -a**3/(10*sqrt(b)*x**11*sqrt(a/(b*x**2) + 1)) - 29*a**2*sqrt(b)/(80*x**9*sq
rt(a/(b*x**2) + 1)) - 73*a*b**(3/2)/(160*x**7*sqrt(a/(b*x**2) + 1)) - 129*b
```

```
**(5/2)/(640*x**5*sqrt(a/(b*x**2) + 1)) + b**(7/2)/(256*a*x**3*sqrt(a/(b*x*
*2) + 1)) + 3*b**(9/2)/(256*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**5*asinh(sqr
t(a)/(sqrt(b)*x))/(256*a**(5/2))
```

3.388 $\int x^4 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=136

$$\frac{3a^5 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}$$

Rubi [A] time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{3a^5 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(5/2),x]
```

```
[Out] (-3*a^4*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*x^3*Sqrt[a + b*x^2])/(128*b) +
(a^2*x^5*Sqrt[a + b*x^2])/32 + (a*x^5*(a + b*x^2)^(3/2))/16 + (x^5*(a + b*x
^2)^(5/2))/10 + (3*a^5*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{5/2}\,dx=\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{2}a\int x^4\left(a+bx^2\right)^{3/2}\,dx$$
$$=\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^4\sqrt{a+bx^2}\,dx$$
$$=\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{1}{32}a^3\int\frac{x^4}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}-\frac{\left(3a^4\right)\int}{12}$$
$$=-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5(a+$$
$$=-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5(a+$$
$$=-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5(a+$$

Mathematica [A] time = 0.15, size = 105, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(\frac{15a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}x\left(-15a^4+10a^3bx^2+248a^2b^2x^4+336ab^3x^6+128b^4x^8\right)\right)}{1280b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-15*a^4 + 10*a^3*b*x^2 + 248*a^2*b^2*x^4 + 336
*a*b^3*x^6 + 128*b^4*x^8) + (15*a^(9/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[
1 + (b*x^2)/a]))/(1280*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 96, normalized size = 0.71

$$\frac{\sqrt{a+bx^2}\,\left(-15a^4x+10a^3bx^3+248a^2b^2x^5+336ab^3x^7+128b^4x^9\right)}{1280b^2}-\frac{3a^5\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{256b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-15*a^4*x + 10*a^3*b*x^3 + 248*a^2*b^2*x^5 + 336*a*b^3*x^
7 + 128*b^4*x^9))/(1280*b^2) - (3*a^5*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/
(256*b^(5/2))
```

fricas [A] time = 0.97, size = 190, normalized size = 1.40

$$\left[\frac{15\,a^5\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(128\,b^5x^9+336\,ab^4x^7+248\,a^2b^3x^5+10\,a^3b^2x^3-15\,a^4bx\right)\sqrt{bx^2+a}}{2560\,b^3},-\frac{15\,a^5\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(128\,b^5x^9+336\,ab^4x^7+248\,a^2b^3x^5+10\,a^3b^2x^3-15\,a^4bx\right)\sqrt{bx^2+a}}{1280\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(15*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(128*b^5*x^9 + 336*a*b^4*x^7 + 248*a^2*b^3*x^5 + 10*a^3*b^2*x^3 - 15*a^4*b
*x)*sqrt(b*x^2 + a))/b^3, -1/1280*(15*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (128*b^5*x^9 + 336*a*b^4*x^7 + 248*a^2*b^3*x^5 + 10*a^3*b^2*x^
3 - 15*a^4*b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 1.09, size = 91, normalized size = 0.67

$$-\frac{3\,a^5\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}+\frac{1}{1280}\left(2\left(4\left(2\left(8\,b^2x^2+21\,ab\right)x^2+31\,a^2\right)x^2+\frac{5\,a^3}{b}\right)x^2-\frac{15\,a^4}{b^2}\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -3/256*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/1280*(2*(4*(2
*(8*b^2*x^2 + 21*a*b)*x^2 + 31*a^2)*x^2 + 5*a^3/b)*x^2 - 15*a^4/b^2)*sqrt(b
*x^2 + a)*x
```

maple [A] time = 0.01, size = 113, normalized size = 0.83

$$\frac{3a^5\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{256b^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^2+a}\,a^4x}{256b^2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}a^3x}{128b^2}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}x^3}{10b}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}a^2x}{160b^2}-\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}ax}{80b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/10*x^3*(b*x^2+a)^(7/2)/b-3/80*a/b^2*x*(b*x^2+a)^(7/2)+1/160*a^2/b^2*x*(b*
x^2+a)^(5/2)+1/128*a^3/b^2*x*(b*x^2+a)^(3/2)+3/256*a^4*x*(b*x^2+a)^(1/2)/b^
2+3/256*a^5/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.26, size = 105, normalized size = 0.77

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}x^3}{10\,b}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}ax}{80\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}a^2x}{160\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}a^3x}{128\,b^2}+\frac{3\sqrt{bx^2+a}\,a^4x}{256\,b^2}+\frac{3\,a^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*x^3/b - 3/80*(b*x^2 + a)^(7/2)*a*x/b^2 + 1/160*(b*x^
2 + a)^(5/2)*a^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*a^3*x/b^2 + 3/256*sqrt(b*x
^2 + a)*a^4*x/b^2 + 3/256*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(b\,x^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(5/2), x)
```

sympy [A] time = 11.11, size = 175, normalized size = 1.29

$$-\frac{3a^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{129a^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{73a^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{29\sqrt{a}\,b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^5\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}+\frac{b^3x^{11}}{10\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(5/2),x)
```

```
[Out] -3*a**(9/2)*x/(256*b**2*sqrt(1 + b*x**2/a)) - a**(7/2)*x**3/(256*b*sqrt(1 +
 b*x**2/a)) + 129*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*a**(3/2)*b*x*
*7/(160*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a))
+ 3*a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + b**3*x**11/(10*sqrt(a)*s
qrt(1 + b*x**2/a))
```

3.389 $\int x^2 \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=112

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}$$

Rubi [A] time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^3*x*Sqrt[a + b*x^2])/(128*b) + (5*a^2*x^3*Sqrt[a + b*x^2])/64 + (5*a*x
^3*(a + b*x^2)^(3/2))/48 + (x^3*(a + b*x^2)^(5/2))/8 - (5*a^4*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^{5/2}dx &= \frac{1}{8}x^3\left(a+bx^2\right)^{5/2}+\frac{1}{8}(5a)\int x^2\left(a+bx^2\right)^{3/2}dx\\ &= \frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^2\right)\int x^2\sqrt{a+bx^2}\,dx\\ &= \frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}+\frac{1}{64}\left(5a^3\right)\int\frac{x^2}{\sqrt{a+bx^2}}dx\\ &= \frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}-\frac{\left(5a^4\right)\int\frac{}{\sqrt{}}}{128}\\ &= \frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}-\frac{\left(5a^4\right)\text{Sub}}{}\\ &= \frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}-\frac{5a^4\tanh^{}}{128}\end{aligned}$$

Mathematica [A] time = 0.14, size = 94, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(15a^3+118a^2bx^2+136ab^2x^4+48b^3x^6\right)-\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{384b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^3 + 118*a^2*b*x^2 + 136*a*b^2*x^4 + 48*b^
3*x^6) - (15*a^(7/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(3
84*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 85, normalized size = 0.76

$$\frac{5a^4\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{128b^{3/2}}+\frac{\sqrt{a+bx^2}\left(15a^3x+118a^2bx^3+136ab^2x^5+48b^3x^7\right)}{384b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(15*a^3*x + 118*a^2*b*x^3 + 136*a*b^2*x^5 + 48*b^3*x^7))/(
384*b) + (5*a^4*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

fricas [A] time = 0.89, size = 167, normalized size = 1.49

$$\left[\frac{15\,a^{4}\sqrt{b}\,\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,\left(48\,b^{4}x^{7}+136\,ab^{3}x^{5}+118\,a^{2}b^{2}x^{3}+15\,a^{3}bx\right)\sqrt{bx^{2}+a}}{768\,b^{2}},\ \frac{15\,a^{4}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(48\,b^{4}x^{7}+136\,ab^{3}x^{5}+118\,a^{2}b^{2}x^{3}+15\,a^{3}bx\right)\sqrt{bx^{2}+a}}{384\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(48*b^4*x^7 + 136*a*b^3*x^5 + 118*a^2*b^2*x^3 + 15*a^3*b*x)*sqrt(b*x^2 + a)
)/b^2, 1/384*(15*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (48*b^4*
x^7 + 136*a*b^3*x^5 + 118*a^2*b^2*x^3 + 15*a^3*b*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 1.08, size = 77, normalized size = 0.69

$$\frac{5\,a^{4}\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{384}\left(2\left(4\left(6\,b^{2}x^{2}+17\,ab\right)x^{2}+59\,a^{2}\right)x^{2}+\frac{15\,a^{3}}{b}\right)\sqrt{bx^{2}+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/384*(2*(4*(6*b
^2*x^2 + 17*a*b)*x^2 + 59*a^2)*x^2 + 15*a^3/b)*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.01, size = 93, normalized size = 0.83

$$-\frac{5a^{4}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{3}{2}}}-\frac{5\sqrt{b\,x^{2}+a}\;a^{3}x}{128b}-\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}x}{192b}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}ax}{48b}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}x}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(5/2),x)
```

```
[Out] 1/8*x*(b*x^2+a)^(7/2)/b-1/48*a/b*x*(b*x^2+a)^(5/2)-5/192*a^2/b*x*(b*x^2+a)^
(3/2)-5/128*a^3*x*(b*x^2+a)^(1/2)/b-5/128*a^4/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))
```

maxima [A] time = 1.34, size = 85, normalized size = 0.76

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}x}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}ax}{48\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}a^2x}{192\,b}-\frac{5\,\sqrt{bx^2+a}\,a^3x}{128\,b}-\frac{5\,a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(7/2)*x/b - 1/48*(b*x^2 + a)^(5/2)*a*x/b - 5/192*(b*x^2 + a
)^(3/2)*a^2*x/b - 5/128*sqrt(b*x^2 + a)*a^3*x/b - 5/128*a^4*arcsinh(b*x/sqr
t(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(5/2), x)
```

sympy [A] time = 7.22, size = 150, normalized size = 1.34

$$\frac{5a^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127a^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}\,b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^4\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{b^3x^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(5/2),x)
```

```
[Out] 5*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*a**(5/2)*x**3/(384*sqrt(1 + b
*x**2/a)) + 127*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*sqrt(a)*b**2*
x**7/(48*sqrt(1 + b*x**2/a)) - 5*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2
)) + b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.390 $\int \left(a+bx^2\right)^{5/2} dx$

Optimal. Leaf size=84

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*(a + b*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^{5/2} dx &= \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{6}(5a)\int \left(a+bx^2\right)^{3/2} dx \\
&= \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{8}\left(5a^2\right)\int \sqrt{a+bx^2}\, dx \\
&= \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+bx^2}} dx \\
&= \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x,\right. \\
&= \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax\left(a+bx^2\right)^{3/2} + \frac{1}{6}x\left(a+bx^2\right)^{5/2} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 76, normalized size = 0.90

$$\frac{1}{48}\sqrt{a+bx^2}\left(\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}} + 33a^2x + 26abx^3 + 8b^2x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5 + (15*a^(5/2)*ArcSinh[(
Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/48
```

IntegrateAlgebraic [A] time = 0.09, size = 71, normalized size = 0.85

$$\frac{1}{48}\sqrt{a+bx^2}\left(33a^2x + 26abx^3 + 8b^2x^5\right) - \frac{5a^3\log\left(\sqrt{a+bx^2} - \sqrt{b}x\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5))/48 - (5*a^3*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

fricas [A] time = 1.02, size = 146, normalized size = 1.74

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\,\sqrt{b}x - a\right) + 2\left(8\,b^3x^5 + 26\,ab^2x^3 + 33\,a^2bx\right)\sqrt{bx^2+a}}{96\,b}, -\frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - \left(8\,b^3x^5 + 26\,ab^2x^3 + 33\,a^2bx\right)\sqrt{bx^2+a}}{48\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 + 33*
a^2*b*x)*sqrt(b*x^2 + a))/b]
```

giac [**A**] time = 1.23, size = 63, normalized size = 0.75

$$-\frac{5\,a^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*x
^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

maple [**A**] time = 0.00, size = 66, normalized size = 0.79

$$\frac{5a^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}+\frac{5\sqrt{b\,x^2+a}\,a^2x}{16}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}ax}{24}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2),x)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)+5/24*a*x*(b*x^2+a)^(3/2)+5/16*a^2*x*(b*x^2+a)^(1/2)+5
/16*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [**A**] time = 1.32, size = 58, normalized size = 0.69

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}x+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ax+\frac{5}{16}\sqrt{bx^2+a}\,a^2x+\frac{5\,a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*x + 5/24*(b*x^2 + a)^(3/2)*a*x + 5/16*sqrt(b*x^2 + a)
*a^2*x + 5/16*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 4.46, size = 37, normalized size = 0.44

$$\frac{x\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2),x)
```

```
[Out] (x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(5/2)
```

sympy [A] time = 4.21, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}\,b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a)
/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqrt(
a))/(16*sqrt(b))
```

3.391 $\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=83

$$\frac{15}{8}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{5/2}}{x}+\frac{5}{4}bx\left(a+bx^2\right)^{3/2}+\frac{15}{8}abx\sqrt{a+bx^2}$$

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{15}{8}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{5/2}}{x}+\frac{5}{4}bx\left(a+bx^2\right)^{3/2}+\frac{15}{8}abx\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^2,x]
```

```
[Out] (15*a*b*x*Sqrt[a + b*x^2])/8 + (5*b*x*(a + b*x^2)^(3/2))/4 - (a + b*x^2)^(5
/2)/x + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx &= -\frac{(a+bx^2)^{5/2}}{x} + (5b)\int (a+bx^2)^{3/2}\,dx \\
&= \frac{5}{4}bx\left(a+bx^2\right)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{1}{4}(15ab)\int \sqrt{a+bx^2}\,dx \\
&= \frac{15}{8}abx\sqrt{a+bx^2} + \frac{5}{4}bx\left(a+bx^2\right)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{15}{8}abx\sqrt{a+bx^2} + \frac{5}{4}bx\left(a+bx^2\right)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{1}{8}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a}}\right. \\
&= \frac{15}{8}abx\sqrt{a+bx^2} + \frac{5}{4}bx\left(a+bx^2\right)^{3/2} - \frac{(a+bx^2)^{5/2}}{x} + \frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.63

$$-\frac{a^2\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((b*x^2)/a)])/(x
*Sqrt[1 + (b*x^2)/a]))
```

IntegrateAlgebraic [A] time = 0.11, size = 73, normalized size = 0.88

$$\frac{\sqrt{a+bx^2}\left(-8a^2+9abx^2+2b^2x^4\right)}{8x} - \frac{15}{8}a^2\sqrt{b}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2 + 9*a*b*x^2 + 2*b^2*x^4))/(8*x) - (15*a^2*Sqrt[b]*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/8
```

fricas [A] time = 0.77, size = 140, normalized size = 1.69

$$\left[\frac{15\,a^{2}\sqrt{b}\,x\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,\left(2\,b^{2}x^{4}+9\,abx^{2}-8\,a^{2}\right)\sqrt{bx^{2}+a}}{16\,x},-\frac{15\,a^{2}\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-\left(2\,b^{2}x^{4}+9\,abx^{2}-8\,a^{2}\right)\sqrt{bx^{2}+a}}{8\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/16*(15*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*sqrt(b*x^2 + a))/x, -1/8*(15*a^2*sqrt(-b)*
x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*sqrt
(b*x^2 + a))/x]
```

giac [A] time = 1.11, size = 87, normalized size = 1.05

$$-\frac{15}{16}\,a^{2}\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}\right)+\frac{2\,a^{3}\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}-a}+\frac{1}{8}\,\left(2\,b^{2}x^{2}+9\,ab\right)\sqrt{bx^{2}+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] -15/16*a^2*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^3*sqrt(b)/((s
qrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*b^2*x^2 + 9*a*b)*sqrt(b*x^2 + a
)*x
```

maple [A] time = 0.00, size = 85, normalized size = 1.02

$$\frac{15a^{2}\sqrt{b}\;\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{8}+\frac{15\sqrt{b\,x^{2}+a}\;abx}{8}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}bx}{4}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}bx}{a}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(7/2)+1/a*b*x*(b*x^2+a)^(5/2)+5/4*b*x*(b*x^2+a)^(3/2)+15/8
*a*b*x*(b*x^2+a)^(1/2)+15/8*a^2*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 59, normalized size = 0.71

$$\frac{5}{4}\left(bx^{2}+a\right)^{\frac{3}{2}}bx+\frac{15}{8}\sqrt{bx^{2}+a}\,abx+\frac{15}{8}\,a^{2}\sqrt{b}\;\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*b*x + 15/8*sqrt(b*x^2 + a)*a*b*x + 15/8*a^2*sqrt(b)*a
rcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(5/2)/x
```

mupad [B] time = 5.05, size = 40, normalized size = 0.48

$$-\frac{\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^2,x)
```

```
[Out] -((a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a
 + 1)^(5/2))
```

sympy [A] time = 3.63, size = 117, normalized size = 1.41

$$-\frac{a^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}\,b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\sqrt{b}\ \operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{b^3x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] -a**(5/2)/(x*sqrt(1 + b*x**2/a)) + a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 11
*sqrt(a)*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*a**2*sqrt(b)*asinh(sqrt(b)*x
/sqrt(a))/8 + b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.392 $\int \frac{(a+bx^2)^{5/2}}{x^4} \, dx$

Optimal. Leaf size=86

$$\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{3x}-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}$$

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{5}{2}b^2x\sqrt{a+bx^2}+\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}-\frac{5b\left(a+bx^2\right)^{3/2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^4,x]
```

```
[Out] (5*b^2*x*Sqrt[a + b*x^2])/2 - (5*b*(a + b*x^2)^(3/2))/(3*x) - (a + b*x^2)^(
5/2)/(3*x^3) + (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}}{x^4}\,dx &= -\frac{\left(a+bx^2\right)^{5/2}}{3x^3}+\frac{1}{3}(5b)\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx\\
&= -\frac{5b\left(a+bx^2\right)^{3/2}}{3x}-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}+\left(5b^2\right)\int \sqrt{a+bx^2}\,dx\\
&= \frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{3x}-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{3x}-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}+\frac{1}{2}\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+}}\right.\\
&= \frac{5}{2}b^2x\sqrt{a+bx^2}-\frac{5b\left(a+bx^2\right)^{3/2}}{3x}-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}+\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.63

$$
-\frac{a^2\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^4,x]
```

```
[Out] -1/3*(a^2*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -3/2, -1/2, -((b*x^2)/a)]
)/(x^3*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.13, size = 71, normalized size = 0.83

$$
\frac{\sqrt{a+bx^2}\left(-2a^2-14abx^2+3b^2x^4\right)}{6x^3}-\frac{5}{2}ab^{3/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a^2 - 14*a*b*x^2 + 3*b^2*x^4))/(6*x^3) - (5*a*b^(3/2)*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/2
```

fricas [A] time = 0.93, size = 141, normalized size = 1.64

$$\left[\frac{15\,ab^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(3\,b^2x^4-14\,abx^2-2\,a^2\right)\sqrt{bx^2+a}}{12\,x^3},\,-\frac{15\,a\sqrt{-b}\,bx^3\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(3\,b^2x^4-14\,abx^2-2\,a^2\right)\sqrt{bx^2+a}}{6\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(15*a*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(3*b^2*x^4 - 14*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^3, -1/6*(15*a*sqrt(-b)
*b*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*b^2*x^4 - 14*a*b*x^2 - 2*a^2
)*sqrt(b*x^2 + a))/x^3]
```

giac [A] time = 1.12, size = 132, normalized size = 1.53

$$\frac{1}{2}\,\sqrt{bx^2+a}\,b^2x-\frac{5}{4}\,ab^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(9\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{3}{2}}-12\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^3b^{\frac{3}{2}}+7\,a^4b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^2*x - 5/4*a*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))
^2) + 2/3*(9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2) - 12*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a^3*b^(3/2) + 7*a^4*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^3
```

maple [A] time = 0.01, size = 110, normalized size = 1.28

$$\frac{5a\,b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2}+\frac{5\sqrt{b\,x^2+a}\,b^2x}{2}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}b^2x}{3a}+\frac{4\left(b\,x^2+a\right)^{\frac{5}{2}}b^2x}{3a^2}-\frac{4\left(b\,x^2+a\right)^{\frac{7}{2}}b}{3a^2x}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^4,x)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(7/2)-4/3/a^2*b/x*(b*x^2+a)^(7/2)+4/3/a^2*b^2*x*(b*x^2
+a)^(5/2)+5/3/a*b^2*x*(b*x^2+a)^(3/2)+5/2*b^2*x*(b*x^2+a)^(1/2)+5/2*a*b^(3/
2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 84, normalized size = 0.98

$$\frac{5}{2}\sqrt{bx^2+a}\,b^2x+\frac{5\left(bx^2+a\right)^{\frac{3}{2}}b^2x}{3\,a}+\frac{5}{2}\,ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{4\left(bx^2+a\right)^{\frac{5}{2}}b}{3\,ax}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(b*x^2 + a)*b^2*x + 5/3*(b*x^2 + a)^(3/2)*b^2*x/a + 5/2*a*b^(3/2)*a
rcsinh(b*x/sqrt(a*b)) - 4/3*(b*x^2 + a)^(5/2)*b/(a*x) - 1/3*(b*x^2 + a)^(7/
2)/(a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^4,x)
```

```
[Out] int((a + b*x^2)^(5/2)/x^4, x)
```

sympy [A] time = 3.16, size = 112, normalized size = 1.30

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{7ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}-\frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{4}+\frac{5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)}{2}+\frac{b^{\frac{5}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**4,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - 7*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/3 - 5*a*b**(3/2)*log(a/(b*x**2))/4 + 5*a*b**(3/2)*log(sqrt(a/(b*x**2)
+ 1) + 1)/2 + b**(5/2)*x**2*sqrt(a/(b*x**2) + 1)/2
```

3.393 $\int \frac{(a+bx^2)^{5/2}}{x^6} dx$

Optimal. Leaf size=82

$$b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b^2\sqrt{a+bx^2}}{x}-\frac{(a+bx^2)^{5/2}}{5x^5}-\frac{b(a+bx^2)^{3/2}}{3x^3}$$

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 206}

$$-\frac{b^2\sqrt{a+bx^2}}{x}+b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b(a+bx^2)^{3/2}}{3x^3}-\frac{(a+bx^2)^{5/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^6,x]
```

```
[Out] -((b^2*Sqrt[a + b*x^2])/x) - (b*(a + b*x^2)^(3/2))/(3*x^3) - (a + b*x^2)^(5
/2)/(5*x^5) + b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{5/2}}{x^6}\,dx &= -\frac{\left(a+bx^2\right)^{5/2}}{5x^5}+b\int \frac{\left(a+bx^2\right)^{3/2}}{x^4}\,dx\\
&= -\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}+b^2\int \frac{\sqrt{a+bx^2}}{x^2}\,dx\\
&= -\frac{b^2\sqrt{a+bx^2}}{x}-\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}+b^3\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= -\frac{b^2\sqrt{a+bx^2}}{x}-\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}+b^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= -\frac{b^2\sqrt{a+bx^2}}{x}-\frac{b\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}+b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.66

$$-\frac{a^2\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^6,x]
```

```
[Out] -1/5*(a^2*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/2, -5/2, -3/2, -((b*x^2)/a)]
)/(x^5*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.14, size = 68, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\left(-3a^2-11abx^2-23b^2x^4\right)}{15x^5}-b^{5/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2 - 11*a*b*x^2 - 23*b^2*x^4))/(15*x^5) - b^(5/2)*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

fricas [A] time = 0.85, size = 140, normalized size = 1.71

$$\left[\frac{15\,b^{\frac{5}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(23\,b^2x^4+11\,abx^2+3\,a^2\right)\sqrt{bx^2+a}}{30\,x^5},\,-\frac{15\,\sqrt{-b}\,b^2x^5\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(23\,b^2x^4+11\,abx^2+3\,a^2\right)\sqrt{bx^2+a}}{15\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(
23*b^2*x^4 + 11*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a))/x^5, -1/15*(15*sqrt(-b)*b
^2*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (23*b^2*x^4 + 11*a*b*x^2 + 3*a^
2)*sqrt(b*x^2 + a))/x^5]
```

giac [B] time = 1.14, size = 168, normalized size = 2.05

$$-\frac{1}{2}b^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8ab^{\frac{5}{2}}-90\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2b^{\frac{5}{2}}+140\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^3b^{\frac{5}{2}}-70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^4b^{\frac{5}{2}}+23a^5b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(45*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*a*b^(5/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(5/2
) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(5/2) - 70*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*a^4*b^(5/2) + 23*a^5*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)^5
```

maple [A] time = 0.01, size = 130, normalized size = 1.59

$$b^{\frac{5}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)+\frac{\sqrt{b\,x^2+a}\,b^3x}{a}+\frac{2\left(b\,x^2+a\right)^{\frac{3}{2}}b^3x}{3a^2}+\frac{8\left(b\,x^2+a\right)^{\frac{5}{2}}b^3x}{15a^3}-\frac{8\left(b\,x^2+a\right)^{\frac{7}{2}}b^2}{15a^3x}-\frac{2\left(b\,x^2+a\right)^{\frac{7}{2}}b}{15a^2x^3}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^6,x)
```

```
[Out] -1/5/a/x^5*(b*x^2+a)^(7/2)-2/15/a^2*b/x^3*(b*x^2+a)^(7/2)-8/15/a^3*b^2/x*(b
*x^2+a)^(7/2)+8/15/a^3*b^3*x*(b*x^2+a)^(5/2)+2/3/a^2*b^3*x*(b*x^2+a)^(3/2)+
1/a*b^3*x*(b*x^2+a)^(1/2)+b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.40, size = 104, normalized size = 1.27

$$\frac{2\left(bx^2+a\right)^{\frac{3}{2}}b^3x}{3\,a^2}+\frac{\sqrt{bx^2+a}\,b^3x}{a}+b^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{8\left(bx^2+a\right)^{\frac{5}{2}}b^2}{15\,a^2x}-\frac{2\left(bx^2+a\right)^{\frac{7}{2}}b}{15\,a^2x^3}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x^2 + a)^(3/2)*b^3*x/a^2 + sqrt(b*x^2 + a)*b^3*x/a + b^(5/2)*arcsinh
(b*x/sqrt(a*b)) - 8/15*(b*x^2 + a)^(5/2)*b^2/(a^2*x) - 2/15*(b*x^2 + a)^(7/
2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(7/2)/(a*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^6,x)
```

```
[Out] int((a + b*x^2)^(5/2)/x^6, x)
```

sympy [A] time = 3.66, size = 105, normalized size = 1.28

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{11ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{23b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15}-\frac{b^{\frac{5}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{5}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**6,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 11*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(15*x**2) - 23*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 - b**(5/2)*log(a/(b*x
**2))/2 + b**(5/2)*log(sqrt(a/(b*x**2) + 1) + 1)
```

3.394 $\int \frac{(a+bx^2)^{5/2}}{x^8} dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^8,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(7*a*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^8} dx = -\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^8,x]
```

```
[Out] -1/7*(a + b*x^2)^(7/2)/(a*x^7)
```

IntegrateAlgebraic [B] time = 0.11, size = 53, normalized size = 2.52

$$\frac{\sqrt{a+bx^2}\left(-a^3-3a^2bx^2-3ab^2x^4-b^3x^6\right)}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^8,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-a^3 - 3*a^2*b*x^2 - 3*a*b^2*x^4 - b^3*x^6))/(7*a*x^7)
```

fricas [B] time = 0.56, size = 46, normalized size = 2.19

$$-\frac{\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{bx^2+a}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/(a*x^7)
```

giac [B] time = 0.99, size = 113, normalized size = 5.38

$$\frac{2\left(7\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}b^{\frac{7}{2}}+35\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}a^2b^{\frac{7}{2}}+21\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}a^4b^{\frac{7}{2}}+a^6b^{\frac{7}{2}}\right)}{7\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 2/7*(7*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(7/2) + 35*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*a^2*b^(7/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(7/2) + a^
6*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{7a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^8,x)
```

```
[Out] -1/7*(b*x^2+a)^(7/2)/a/x^7
```

maxima [A] time = 1.41, size = 17, normalized size = 0.81

$$-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/7*(b*x^2 + a)^(7/2)/(a*x^7)
```

mupad [B] time = 5.07, size = 71, normalized size = 3.38

$$-\frac{a^2\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{3\,b^2\,\sqrt{b\,x^2+a}}{7\,x^3}-\frac{b^3\,\sqrt{b\,x^2+a}}{7\,a\,x}-\frac{3\,a\,b\,\sqrt{b\,x^2+a}}{7\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^8,x)
```

```
[Out] - (a^2*(a + b*x^2)^(1/2))/(7*x^7) - (3*b^2*(a + b*x^2)^(1/2))/(7*x^3) - (b^
3*(a + b*x^2)^(1/2))/(7*a*x) - (3*a*b*(a + b*x^2)^(1/2))/(7*x^5)
```

sympy [B] time = 1.26, size = 95, normalized size = 4.52

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6}-\frac{3ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{7x^4}-\frac{3b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{7x^2}-\frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**8,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 3*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/(7*x**4) - 3*b**(5/2)*sqrt(a/(b*x**2) + 1)/(7*x**2) - b**(7/2)*sqrt(a/
(b*x**2) + 1)/(7*a)
```

3.395 $\int \frac{\left(a+bx^2\right)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=44

$$\frac{2b\left(a+bx^2\right)^{7/2}}{63a^2x^7}-\frac{\left(a+bx^2\right)^{7/2}}{9ax^9}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b\left(a+bx^2\right)^{7/2}}{63a^2x^7}-\frac{\left(a+bx^2\right)^{7/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^10,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(9*a*x^9) + (2*b*(a + b*x^2)^(7/2))/(63*a^2*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{5/2}}{x^{10}}\,dx &= -\frac{\left(a+bx^2\right)^{7/2}}{9ax^9}-\frac{(2b)\int \frac{\left(a+bx^2\right)^{5/2}}{x^8}\,dx}{9a}\\ &= -\frac{\left(a+bx^2\right)^{7/2}}{9ax^9}+\frac{2b\left(a+bx^2\right)^{7/2}}{63a^2x^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^2\right)^{7/2}\left(2bx^2-7a\right)}{63a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^10,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-7*a + 2*b*x^2))/(63*a^2*x^9)
```

IntegrateAlgebraic [A] time = 0.12, size = 64, normalized size = 1.45

$$\frac{\sqrt{a+bx^2}\left(-7a^4-19a^3bx^2-15a^2b^2x^4-ab^3x^6+2b^4x^8\right)}{63a^2x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^10,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-7*a^4 - 19*a^3*b*x^2 - 15*a^2*b^2*x^4 - a*b^3*x^6 + 2*b^
4*x^8))/(63*a^2*x^9)
```

fricas [A] time = 0.96, size = 60, normalized size = 1.36

$$\frac{\left(2\,b^4x^8-ab^3x^6-15\,a^2b^2x^4-19\,a^3bx^2-7\,a^4\right)\sqrt{bx^2+a}}{63\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(2*b^4*x^8 - a*b^3*x^6 - 15*a^2*b^2*x^4 - 19*a^3*b*x^2 - 7*a^4)*sqrt(b
*x^2 + a)/(a^2*x^9)
```

giac [B] time = 1.14, size = 220, normalized size = 5.00

$$\frac{4\left(63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{9}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{9}{2}}+189\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^3b^{\frac{9}{2}}+189\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^4b^{\frac{9}{2}}+27\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^5b^{\frac{9}{2}}+9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^6b^{\frac{9}{2}}-a^7b^{\frac{9}{2}}\right)}{63\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 4/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(9/2) + 105*(sqrt(b)*x - sqrt(b
*x^2 + a))^12*a*b^(9/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2)
+ 189*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2) + 189*(sqrt(b)*x - sqrt(b
```

```
*x^2 + a))^6*a^4*b^(9/2) + 27*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/2) +
 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2) - a^7*b^(9/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^9
```

maple [A] time = 0.01, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(-2b\,x^{2}+7a\right)}{63a^{2}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^10,x)
```

```
[Out] -1/63*(b*x^2+a)^(7/2)*(-2*b*x^2+7*a)/x^9/a^2
```

maxima [A] time = 1.53, size = 36, normalized size = 0.82

$$\frac{2\left(bx^{2}+a\right)^{\frac{7}{2}}b}{63\,a^{2}x^{7}}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{9\,ax^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 2/63*(b*x^2 + a)^(7/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(7/2)/(a*x^9)
```

mupad [B] time = 5.37, size = 91, normalized size = 2.07

$$\frac{2\,b^{4}\,\sqrt{b\,x^{2}+a}}{63\,a^{2}\,x}-\frac{5\,b^{2}\,\sqrt{b\,x^{2}+a}}{21\,x^{5}}-\frac{b^{3}\,\sqrt{b\,x^{2}+a}}{63\,a\,x^{3}}-\frac{a^{2}\,\sqrt{b\,x^{2}+a}}{9\,x^{9}}-\frac{19\,a\,b\,\sqrt{b\,x^{2}+a}}{63\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^10,x)
```

```
[Out] (2*b^4*(a + b*x^2)^(1/2))/(63*a^2*x) - (5*b^2*(a + b*x^2)^(1/2))/(21*x^5) -
 (b^3*(a + b*x^2)^(1/2))/(63*a*x^3) - (a^2*(a + b*x^2)^(1/2))/(9*x^9) - (19
*a*b*(a + b*x^2)^(1/2))/(63*x^7)
```

sympy [B] time = 1.59, size = 121, normalized size = 2.75

$$-\frac{a^{2}\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{9x^{8}}-\frac{19ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{63x^{6}}-\frac{5b^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{21x^{4}}-\frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{63ax^{2}}+\frac{2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{63a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**10,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(9*x**8) - 19*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(63*x**6) - 5*b**(5/2)*sqrt(a/(b*x**2) + 1)/(21*x**4) - b**(7/2)*sqrt
(a/(b*x**2) + 1)/(63*a*x**2) + 2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(63*a**2)
```

3.396 $\int \frac{\left(a+bx^2\right)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=68

$$-\frac{8b^2\left(a+bx^2\right)^{7/2}}{693a^3x^7}+\frac{4b\left(a+bx^2\right)^{7/2}}{99a^2x^9}-\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2\left(a+bx^2\right)^{7/2}}{693a^3x^7}+\frac{4b\left(a+bx^2\right)^{7/2}}{99a^2x^9}-\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^12,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(11*a*x^11) + (4*b*(a + b*x^2)^(7/2))/(99*a^2*x^9) - (8*
b^2*(a + b*x^2)^(7/2))/(693*a^3*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx = -\frac{(a+bx^2)^{7/2}}{11ax^{11}} - \frac{(4b)\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx}{11a}$$
$$= -\frac{(a+bx^2)^{7/2}}{11ax^{11}} + \frac{4b(a+bx^2)^{7/2}}{99a^2x^9} + \frac{(8b^2)\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx}{99a^2}$$
$$= -\frac{(a+bx^2)^{7/2}}{11ax^{11}} + \frac{4b(a+bx^2)^{7/2}}{99a^2x^9} - \frac{8b^2(a+bx^2)^{7/2}}{693a^3x^7}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{(a+bx^2)^{7/2}(63a^2-28abx^2+8b^2x^4)}{693a^3x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^12,x]
```

```
[Out] -1/693*((a + b*x^2)^(7/2)*(63*a^2 - 28*a*b*x^2 + 8*b^2*x^4))/(a^3*x^11)
```

IntegrateAlgebraic [A] time = 0.13, size = 75, normalized size = 1.10

$$\frac{\sqrt{a+bx^2}\,(-63a^5-161a^4bx^2-113a^3b^2x^4-3a^2b^3x^6+4ab^4x^8-8b^5x^{10})}{693a^3x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^12,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-63*a^5 - 161*a^4*b*x^2 - 113*a^3*b^2*x^4 - 3*a^2*b^3*x^6
 + 4*a*b^4*x^8 - 8*b^5*x^10))/(693*a^3*x^11)
```

fricas [A] time = 0.66, size = 71, normalized size = 1.04

$$-\frac{(8\,b^5x^{10}-4\,ab^4x^8+3\,a^2b^3x^6+113\,a^3b^2x^4+161\,a^4bx^2+63\,a^5)\sqrt{bx^2+a}}{693\,a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(8*b^5*x^10 - 4*a*b^4*x^8 + 3*a^2*b^3*x^6 + 113*a^3*b^2*x^4 + 161*a^
4*b*x^2 + 63*a^5)*sqrt(b*x^2 + a)/(a^3*x^11)
```

giac [B] time = 1.16, size = 246, normalized size = 3.62

$$\frac{16 \left(462 \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16} b^{\frac{11}{2}}+1155 \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14} a b^{\frac{11}{2}}+2541 \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12} a^2 b^{\frac{11}{2}}+2079 \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10} a^3 b^{\frac{11}{2}}+1485 \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8} a^4 b^{\frac{11}{2}}+297 \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6} a^5 b^{\frac{11}{2}}+55 \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4} a^6 b^{\frac{11}{2}}-11 \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2} a^7 b^{\frac{11}{2}}+a^8 b^{\frac{11}{2}}\right)}{693 \left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 16/693*(462*(sqrt(b)*x - sqrt(b*x^2 + a))^16*b^(11/2) + 1155*(sqrt(b)*x - s
qrt(b*x^2 + a))^14*a*b^(11/2) + 2541*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b
^(11/2) + 2079*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(11/2) + 1485*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*a^4*b^(11/2) + 297*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*a^5*b^(11/2) + 55*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(11/2) - 11*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a^7*b^(11/2) + a^8*b^(11/2))/((sqrt(b)*x - sqrt(b
*x^2 + a))^2 - a)^11
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}} \left(8b^2x^4-28ab\,x^2+63a^2\right)}{693a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^12,x)
```

```
[Out] -1/693*(b*x^2+a)^(7/2)*(8*b^2*x^4-28*a*b*x^2+63*a^2)/x^11/a^3
```

maxima [A] time = 1.41, size = 56, normalized size = 0.82

$$-\frac{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}{693\,a^3x^7}+\frac{4\left(bx^2+a\right)^{\frac{7}{2}}b}{99\,a^2x^9}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] -8/693*(b*x^2 + a)^(7/2)*b^2/(a^3*x^7) + 4/99*(b*x^2 + a)^(7/2)*b/(a^2*x^9)
 - 1/11*(b*x^2 + a)^(7/2)/(a*x^11)
```

mupad [B] time = 5.62, size = 111, normalized size = 1.63

$$\frac{4\,b^4\,\sqrt{b\,x^2+a}}{693\,a^2\,x^3}-\frac{113\,b^2\,\sqrt{b\,x^2+a}}{693\,x^7}-\frac{b^3\,\sqrt{b\,x^2+a}}{231\,a\,x^5}-\frac{a^2\,\sqrt{b\,x^2+a}}{11\,x^{11}}-\frac{8\,b^5\,\sqrt{b\,x^2+a}}{693\,a^3\,x}-\frac{23\,a\,b\,\sqrt{b\,x^2+a}}{99\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^12,x)
```

```
[Out] (4*b^4*(a + b*x^2)^(1/2))/(693*a^2*x^3) - (113*b^2*(a + b*x^2)^(1/2))/(693*
x^7) - (b^3*(a + b*x^2)^(1/2))/(231*a*x^5) - (a^2*(a + b*x^2)^(1/2))/(11*x^
11) - (8*b^5*(a + b*x^2)^(1/2))/(693*a^3*x) - (23*a*b*(a + b*x^2)^(1/2))/(9
9*x^9)
```

sympy [B] time = 2.16, size = 481, normalized size = 7.07

$$-\frac{63a^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^2\left(693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}\right)}-\frac{287a^6b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{498a^5b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{390a^4b^{\frac{15}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{115a^3b^{\frac{17}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{3a^2b^{\frac{19}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{12ab^{\frac{21}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}-\frac{8b^{\frac{23}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**12,x)
```

```
[Out] -63*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(x**2*(693*a**5*b**4*x**8 + 1386*a**
4*b**5*x**10 + 693*a**3*b**6*x**12)) - 287*a**6*b**(11/2)*sqrt(a/(b*x**2) +
 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 498
*a**5*b**(13/2)*x**2*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b
**5*x**10 + 693*a**3*b**6*x**12) - 390*a**4*b**(15/2)*x**4*sqrt(a/(b*x**2)
+ 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 11
5*a**3*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*
b**5*x**10 + 693*a**3*b**6*x**12) - 3*a**2*b**(19/2)*x**8*sqrt(a/(b*x**2) +
 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 12*
a*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5
*x**10 + 693*a**3*b**6*x**12) - 8*b**(23/2)*x**12*sqrt(a/(b*x**2) + 1)/(693
*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12)
```

3.397 $\int \frac{(a+bx^2)^{5/2}}{x^{14}} dx$

Optimal. Leaf size=92

$$\frac{16b^3 (a+bx^2)^{7/2}}{3003a^4x^7} - \frac{8b^2 (a+bx^2)^{7/2}}{429a^3x^9} + \frac{6b (a+bx^2)^{7/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{16b^3 (a+bx^2)^{7/2}}{3003a^4x^7} - \frac{8b^2 (a+bx^2)^{7/2}}{429a^3x^9} + \frac{6b (a+bx^2)^{7/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^14,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(13*a*x^13) + (6*b*(a + b*x^2)^(7/2))/(143*a^2*x^11) - (
8*b^2*(a + b*x^2)^(7/2))/(429*a^3*x^9) + (16*b^3*(a + b*x^2)^(7/2))/(3003*a
^4*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^{14}}\,dx = -\frac{\left(a+bx^2\right)^{7/2}}{13ax^{13}} - \frac{(6b)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{12}}\,dx}{13a}$$
$$= -\frac{\left(a+bx^2\right)^{7/2}}{13ax^{13}} + \frac{6b\left(a+bx^2\right)^{7/2}}{143a^2x^{11}} + \frac{\left(24b^2\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{10}}\,dx}{143a^2}$$
$$= -\frac{\left(a+bx^2\right)^{7/2}}{13ax^{13}} + \frac{6b\left(a+bx^2\right)^{7/2}}{143a^2x^{11}} - \frac{8b^2\left(a+bx^2\right)^{7/2}}{429a^3x^9} - \frac{\left(16b^3\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^8}\,dx}{429a^3}$$
$$= -\frac{\left(a+bx^2\right)^{7/2}}{13ax^{13}} + \frac{6b\left(a+bx^2\right)^{7/2}}{143a^2x^{11}} - \frac{8b^2\left(a+bx^2\right)^{7/2}}{429a^3x^9} + \frac{16b^3\left(a+bx^2\right)^{7/2}}{3003a^4x^7}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{7/2}\left(-231a^3+126a^2bx^2-56ab^2x^4+16b^3x^6\right)}{3003a^4x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^14,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-231*a^3 + 126*a^2*b*x^2 - 56*a*b^2*x^4 + 16*b^3*x^6))/
(3003*a^4*x^13)
```

IntegrateAlgebraic [A] time = 0.15, size = 86, normalized size = 0.93

$$\frac{\sqrt{a+bx^2}\left(-231a^6-567a^5bx^2-371a^4b^2x^4-5a^3b^3x^6+6a^2b^4x^8-8ab^5x^{10}+16b^6x^{12}\right)}{3003a^4x^{13}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^14,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-231*a^6 - 567*a^5*b*x^2 - 371*a^4*b^2*x^4 - 5*a^3*b^3*x^
6 + 6*a^2*b^4*x^8 - 8*a*b^5*x^10 + 16*b^6*x^12))/(3003*a^4*x^13)
```

fricas [A] time = 1.20, size = 82, normalized size = 0.89

$$\frac{\left(16\,b^6x^{12}-8\,ab^5x^{10}+6\,a^2b^4x^8-5\,a^3b^3x^6-371\,a^4b^2x^4-567\,a^5bx^2-231\,a^6\right)\sqrt{bx^2+a}}{3003\,a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/3003*(16*b^6*x^12 - 8*a*b^5*x^10 + 6*a^2*b^4*x^8 - 5*a^3*b^3*x^6 - 371*a^
4*b^2*x^4 - 567*a^5*b*x^2 - 231*a^6)*sqrt(b*x^2 + a)/(a^4*x^13)
```

giac [B] time = 1.18, size = 274, normalized size = 2.98

$$\frac{32\left(3003\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}b^{\frac{13}{2}}+9009\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}ab^{\frac{13}{2}}+18018\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^2b^{\frac{13}{2}}+16302\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^3b^{\frac{13}{2}}+10296\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^4b^{\frac{13}{2}}+2288\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^5b^{\frac{13}{2}}+286\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^6b^{\frac{13}{2}}-78\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^7b^{\frac{13}{2}}+13\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^8b^{\frac{13}{2}}-a^9b^{\frac{13}{2}}\right)}{3003\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] 32/3003*(3003*(sqrt(b)*x - sqrt(b*x^2 + a))^18*b^(13/2) + 9009*(sqrt(b)*x -
 sqrt(b*x^2 + a))^16*a*b^(13/2) + 18018*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^
2*b^(13/2) + 16302*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(13/2) + 10296*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(13/2) + 2288*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^5*b^(13/2) + 286*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(13/2) - 7
8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(13/2) + 13*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^8*b^(13/2) - a^9*b^(13/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
^13
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}\left(-16b^3x^6+56a\,b^2x^4-126a^2b\,x^2+231a^3\right)}{3003a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^14,x)
```

```
[Out] -1/3003*(b*x^2+a)^(7/2)*(-16*b^3*x^6+56*a*b^2*x^4-126*a^2*b*x^2+231*a^3)/x^
13/a^4
```

maxima [A] time = 1.39, size = 76, normalized size = 0.83

$$\frac{16\left(bx^2+a\right)^{\frac{7}{2}}b^3}{3003\,a^4x^7}-\frac{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}{429\,a^3x^9}+\frac{6\left(bx^2+a\right)^{\frac{7}{2}}b}{143\,a^2x^{11}}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}}{13\,ax^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] 16/3003*(b*x^2 + a)^(7/2)*b^3/(a^4*x^7) - 8/429*(b*x^2 + a)^(7/2)*b^2/(a^3*
x^9) + 6/143*(b*x^2 + a)^(7/2)*b/(a^2*x^11) - 1/13*(b*x^2 + a)^(7/2)/(a*x^1
3)
```

mupad [B] time = 5.98, size = 131, normalized size = 1.42

$$\frac{2\,b^4\,\sqrt{b\,x^2+a}}{1001\,a^2\,x^5}-\frac{53\,b^2\,\sqrt{b\,x^2+a}}{429\,x^9}-\frac{5\,b^3\,\sqrt{b\,x^2+a}}{3003\,a\,x^7}-\frac{a^2\,\sqrt{b\,x^2+a}}{13\,x^{13}}-\frac{8\,b^5\,\sqrt{b\,x^2+a}}{3003\,a^3\,x^3}+\frac{16\,b^6\,\sqrt{b\,x^2+a}}{3003\,a^4\,x}-\frac{27\,a\,b\,\sqrt{b\,x^2+a}}{143\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^14,x)
```

```
[Out] (2*b^4*(a + b*x^2)^(1/2))/(1001*a^2*x^5) - (53*b^2*(a + b*x^2)^(1/2))/(429*
x^9) - (5*b^3*(a + b*x^2)^(1/2))/(3003*a*x^7) - (a^2*(a + b*x^2)^(1/2))/(13
*x^13) - (8*b^5*(a + b*x^2)^(1/2))/(3003*a^3*x^3) + (16*b^6*(a + b*x^2)^(1/
2))/(3003*a^4*x) - (27*a*b*(a + b*x^2)^(1/2))/(143*x^11)
```

sympy [B] time = 2.79, size = 721, normalized size = 7.84

$$-\frac{231a^9b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{1260a^8b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{2765a^7b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{3050a^6b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{1689a^5b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}-\frac{376a^4b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}+\frac{5a^3b^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}+\frac{30a^2b^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}+\frac{40ab^{\frac{35}{2}}x^{16}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}+\frac{16b^{\frac{37}{2}}x^{18}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**14,x)
```

```
[Out] -231*a**9*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*
b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1260*a**8*b*
*(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x
**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 2765*a**7*b**(23/2)
*x**4*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 +
9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 3050*a**6*b**(25/2)*x**6*s
qrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a*
*5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1689*a**5*b**(27/2)*x**8*sqrt(a/(
b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**1
1*x**16 + 3003*a**4*b**12*x**18) - 376*a**4*b**(29/2)*x**10*sqrt(a/(b*x**2)
 + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16
 + 3003*a**4*b**12*x**18) + 5*a**3*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(30
03*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a
**4*b**12*x**18) + 30*a**2*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(3003*a**7*
b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**1
2*x**18) + 40*a*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12
+ 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) +
16*b**(37/2)*x**18*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b
**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18)
```

3.398 $\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=116

$$-\frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7}+\frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9}-\frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}}+\frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}}-\frac{(a+bx^2)^{7/2}}{15ax^{15}}$$

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7}+\frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9}-\frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}}+\frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}}-\frac{(a+bx^2)^{7/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^16,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(15*a*x^15) + (8*b*(a + b*x^2)^(7/2))/(195*a^2*x^13) - (
16*b^2*(a + b*x^2)^(7/2))/(715*a^3*x^11) + (64*b^3*(a + b*x^2)^(7/2))/(6435
*a^4*x^9) - (128*b^4*(a + b*x^2)^(7/2))/(45045*a^5*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx = -\frac{(a+bx^2)^{7/2}}{15ax^{15}} - \frac{(8b)\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx}{15a}$$
$$= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} + \frac{(16b^2)\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx}{65a^2}$$
$$= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} - \frac{(64b^3)\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx}{715a^3}$$
$$= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} + \frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9} + \frac{(128b^4)\int \frac{(a+bx^2)}{x^8}}{6435a^4}$$
$$= -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} + \frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9} - \frac{128b^4(a+bx^2)^{7/}}{45045a^5x^7}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.55

$$-\frac{(a+bx^2)^{7/2}(3003a^4-1848a^3bx^2+1008a^2b^2x^4-448ab^3x^6+128b^4x^8)}{45045a^5x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^16,x]
```

```
[Out] -1/45045*((a + b*x^2)^(7/2)*(3003*a^4 - 1848*a^3*b*x^2 + 1008*a^2*b^2*x^4 -
 448*a*b^3*x^6 + 128*b^4*x^8))/(a^5*x^15)
```

IntegrateAlgebraic [A] time = 0.16, size = 97, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(-3003a^7-7161a^6bx^2-4473a^5b^2x^4-35a^4b^3x^6+40a^3b^4x^8-48a^2b^5x^{10}+64ab^6x^{12}-128b^7x^{14}\right)}{45045a^5x^{15}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^16,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3003*a^7 - 7161*a^6*b*x^2 - 4473*a^5*b^2*x^4 - 35*a^4*b^
3*x^6 + 40*a^3*b^4*x^8 - 48*a^2*b^5*x^10 + 64*a*b^6*x^12 - 128*b^7*x^14))/(
45045*a^5*x^15)
```

fricas [A] time = 1.44, size = 93, normalized size = 0.80

$$-\frac{\left(128\,b^7x^{14}-64\,ab^6x^{12}+48\,a^2b^5x^{10}-40\,a^3b^4x^8+35\,a^4b^3x^6+4473\,a^5b^2x^4+7161\,a^6bx^2+3003\,a^7\right)\sqrt{bx^2+a}}{45045\,a^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(128*b^7*x^14 - 64*a*b^6*x^12 + 48*a^2*b^5*x^10 - 40*a^3*b^4*x^8 +
 35*a^4*b^3*x^6 + 4473*a^5*b^2*x^4 + 7161*a^6*b*x^2 + 3003*a^7)*sqrt(b*x^2
+ a)/(a^5*x^15)
```

giac [B] time = 1.23, size = 300, normalized size = 2.59

$$\frac{256 \left(18018 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{20} b^{\frac{15}{2}} + 60060 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{18} a b^{\frac{15}{2}} + 115830 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{16} a^2 b^{\frac{15}{2}} + 109395 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{14} a^3 b^{\frac{15}{2}} + 65065 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{12} a^4 b^{\frac{15}{2}} + 15015 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{10} a^5 b^{\frac{15}{2}} + 1365 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{8} a^6 b^{\frac{15}{2}} - 455 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{6} a^7 b^{\frac{15}{2}} + 105 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{4} a^8 b^{\frac{15}{2}} - 15 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{2} a^9 b^{\frac{15}{2}} + a^{10} b^{\frac{15}{2}}\right)}{45045 \left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 - a\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] 256/45045*(18018*(sqrt(b)*x - sqrt(b*x^2 + a))^20*b^(15/2) + 60060*(sqrt(b)
*x - sqrt(b*x^2 + a))^18*a*b^(15/2) + 115830*(sqrt(b)*x - sqrt(b*x^2 + a))^
16*a^2*b^(15/2) + 109395*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^3*b^(15/2) + 65
065*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(15/2) + 15015*(sqrt(b)*x - sqrt
(b*x^2 + a))^10*a^5*b^(15/2) + 1365*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(
15/2) - 455*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^7*b^(15/2) + 105*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*a^8*b^(15/2) - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^9*b
^(15/2) + a^10*b^(15/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^15
```

maple [A] time = 0.01, size = 61, normalized size = 0.53

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}} \left(128 b^4 x^8 - 448 a\,b^3 x^6 + 1008 a^2 b^2 x^4 - 1848 a^3 b\,x^2 + 3003 a^4\right)}{45045 a^5 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^16,x)
```

```
[Out] -1/45045*(b*x^2+a)^(7/2)*(128*b^4*x^8-448*a*b^3*x^6+1008*a^2*b^2*x^4-1848*a
^3*b*x^2+3003*a^4)/x^15/a^5
```

maxima [A] time = 1.45, size = 96, normalized size = 0.83

$$-\frac{128 \left(bx^2+a\right)^{\frac{7}{2}} b^4}{45045\,a^5 x^7} + \frac{64 \left(bx^2+a\right)^{\frac{7}{2}} b^3}{6435\,a^4 x^9} - \frac{16 \left(bx^2+a\right)^{\frac{7}{2}} b^2}{715\,a^3 x^{11}} + \frac{8 \left(bx^2+a\right)^{\frac{7}{2}} b}{195\,a^2 x^{13}} - \frac{\left(bx^2+a\right)^{\frac{7}{2}}}{15\,a x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] -128/45045*(b*x^2 + a)^(7/2)*b^4/(a^5*x^7) + 64/6435*(b*x^2 + a)^(7/2)*b^3/
(a^4*x^9) - 16/715*(b*x^2 + a)^(7/2)*b^2/(a^3*x^11) + 8/195*(b*x^2 + a)^(7/
2)*b/(a^2*x^13) - 1/15*(b*x^2 + a)^(7/2)/(a*x^15)
```

mupad [B] time = 6.37, size = 151, normalized size = 1.30

$$\frac{8\,b^4\,\sqrt{b\,x^2+a}}{9009\,a^2\,x^7}-\frac{71\,b^2\,\sqrt{b\,x^2+a}}{715\,x^{11}}-\frac{b^3\,\sqrt{b\,x^2+a}}{1287\,a\,x^9}-\frac{a^2\,\sqrt{b\,x^2+a}}{15\,x^{15}}-\frac{16\,b^5\,\sqrt{b\,x^2+a}}{15015\,a^3\,x^5}+\frac{64\,b^6\,\sqrt{b\,x^2+a}}{45045\,a^4\,x^3}-\frac{128\,b^7\,\sqrt{b\,x^2+a}}{45045\,a^5\,x}-\frac{31\,a\,b\,\sqrt{b\,x^2+a}}{195\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^16,x)
```

```
[Out] (8*b^4*(a + b*x^2)^(1/2))/(9009*a^2*x^7) - (71*b^2*(a + b*x^2)^(1/2))/(715*
x^11) - (b^3*(a + b*x^2)^(1/2))/(1287*a*x^9) - (a^2*(a + b*x^2)^(1/2))/(15*
x^15) - (16*b^5*(a + b*x^2)^(1/2))/(15015*a^3*x^5) + (64*b^6*(a + b*x^2)^(1
/2))/(45045*a^4*x^3) - (128*b^7*(a + b*x^2)^(1/2))/(45045*a^5*x) - (31*a*b*
(a + b*x^2)^(1/2))/(195*x^13)
```

sympy [B] time = 3.49, size = 1012, normalized size = 8.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**16,x)
```

```
[Out] -3003*a**11*b**(33/2)*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180
*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 450
45*a**5*b**20*x**22) - 19173*a**10*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(450
45*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 1
80180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 51135*a**9*b**(37/2)*x**
4*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 +
270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22)
 - 72905*a**8*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 +
 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**2
0 + 45045*a**5*b**20*x**22) - 58585*a**7*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1
)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**
18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 25151*a**6*b**(43/
2)*x**10*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x
**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20
*x**22) - 4501*a**5*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*
x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**
19*x**20 + 45045*a**5*b**20*x**22) - 35*a**4*b**(47/2)*x**14*sqrt(a/(b*x**2
) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**1
8*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 280*a**3*b**(
49/2)*x**16*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**1
```

```
7*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b*
*20*x**22) - 560*a**2*b**(51/2)*x**18*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**1
6*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b
**19*x**20 + 45045*a**5*b**20*x**22) - 448*a*b**(53/2)*x**20*sqrt(a/(b*x**2
) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**1
8*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 128*b**(55/2)
*x**22*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**
16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x
**22)
```

3.399 $\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=140

$$\frac{256b^5(a+bx^2)^{7/2}}{153153a^6x^7}-\frac{128b^4(a+bx^2)^{7/2}}{21879a^5x^9}+\frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}}-\frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}}+\frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}}-\frac{(a+bx^2)^{7/2}}{17ax^{17}}$$

Rubi [A] time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{256b^5(a+bx^2)^{7/2}}{153153a^6x^7}-\frac{128b^4(a+bx^2)^{7/2}}{21879a^5x^9}+\frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}}-\frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}}+\frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}}-\frac{(a+bx^2)^{7/2}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/x^18,x]
```

```
[Out] -(a + b*x^2)^(7/2)/(17*a*x^17) + (2*b*(a + b*x^2)^(7/2))/(51*a^2*x^15) - (1
6*b^2*(a + b*x^2)^(7/2))/(663*a^3*x^13) + (32*b^3*(a + b*x^2)^(7/2))/(2431*
a^4*x^11) - (128*b^4*(a + b*x^2)^(7/2))/(21879*a^5*x^9) + (256*b^5*(a + b*x
^2)^(7/2))/(153153*a^6*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^{18}}\,dx = -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} - \frac{(10b)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{16}}\,dx}{17a}$$

$$= -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}} + \frac{\left(16b^2\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{14}}\,dx}{51a^2}$$

$$= -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}} - \frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}} - \frac{\left(32b^3\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^{12}}\,dx}{221a^3}$$

$$= -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}} - \frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}} + \frac{32b^3\left(a+bx^2\right)^{7/2}}{2431a^4x^{11}} + \frac{\left(128b^4\right)\int \frac{(a+bx}{x^{10}}}{2431a^4}$$

$$= -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}} - \frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}} + \frac{32b^3\left(a+bx^2\right)^{7/2}}{2431a^4x^{11}} - \frac{128b^4\left(a+bx^2\right)}{21879a^5x^9}$$

$$= -\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{7/2}}{51a^2x^{15}} - \frac{16b^2\left(a+bx^2\right)^{7/2}}{663a^3x^{13}} + \frac{32b^3\left(a+bx^2\right)^{7/2}}{2431a^4x^{11}} - \frac{128b^4\left(a+bx^2\right)}{21879a^5x^9}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 0.54

$$\frac{\left(a+bx^2\right)^{7/2}\left(-9009a^5+6006a^4bx^2-3696a^3b^2x^4+2016a^2b^3x^6-896ab^4x^8+256b^5x^{10}\right)}{153153a^6x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/x^18,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-9009*a^5 + 6006*a^4*b*x^2 - 3696*a^3*b^2*x^4 + 2016*a^
2*b^3*x^6 - 896*a*b^4*x^8 + 256*b^5*x^10))/(153153*a^6*x^17)
```

IntegrateAlgebraic [A] time = 0.17, size = 108, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(-9009a^8-21021a^7bx^2-12705a^6b^2x^4-63a^5b^3x^6+70a^4b^4x^8-80a^3b^5x^{10}+96a^2b^6x^{12}-128ab^7x^{14}+256b^8x^{16}\right)}{153153a^6x^{17}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/x^18,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-9009*a^8 - 21021*a^7*b*x^2 - 12705*a^6*b^2*x^4 - 63*a^5*
b^3*x^6 + 70*a^4*b^4*x^8 - 80*a^3*b^5*x^10 + 96*a^2*b^6*x^12 - 128*a*b^7*x^
14 + 256*b^8*x^16))/(153153*a^6*x^17)
```

fricas [A] time = 1.47, size = 104, normalized size = 0.74

$$\frac{\left(256 \, b^{8} x^{16} - 128 \, a b^{7} x^{14} + 96 \, a^{2} b^{6} x^{12} - 80 \, a^{3} b^{5} x^{10} + 70 \, a^{4} b^{4} x^{8} - 63 \, a^{5} b^{3} x^{6} - 12705 \, a^{6} b^{2} x^{4} - 21021 \, a^{7} b x^{2} - 9009 \, a^{8}\right)} \sqrt{b x^{2} + a}}{153153 \, a^{6} x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] 1/153153*(256*b^8*x^16 - 128*a*b^7*x^14 + 96*a^2*b^6*x^12 - 80*a^3*b^5*x^10
 + 70*a^4*b^4*x^8 - 63*a^5*b^3*x^6 - 12705*a^6*b^2*x^4 - 21021*a^7*b*x^2 -
9009*a^8)*sqrt(b*x^2 + a)/(a^6*x^17)
```

giac [B] time = 1.10, size = 328, normalized size = 2.34

$$\frac{512 \, \left(102102 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{22} b^{\frac{17}{2}} + 364650 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{20} a b^{\frac{17}{2}} + 692835 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{18} a^{2} b^{\frac{17}{2}} + 668525 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{16} a^{3} b^{\frac{17}{2}} + 384098 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{14} a^{4} b^{\frac{17}{2}} + 89726 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{12} a^{5} b^{\frac{17}{2}} + 6188 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{10} a^{6} b^{\frac{17}{2}} - 2380 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{8} a^{7} b^{\frac{17}{2}} + 680 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{6} a^{8} b^{\frac{17}{2}} - 136 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{4} a^{9} b^{\frac{17}{2}} + 17 \, \left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)^{2} a^{10} b^{\frac{17}{2}} - a^{11} b^{\frac{17}{2}}\right)}}{153153 \, {\left({\left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)}^{2} - a\right)}^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] 512/153153*(102102*(sqrt(b)*x - sqrt(b*x^2 + a))^22*b^(17/2) + 364650*(sqrt
(b)*x - sqrt(b*x^2 + a))^20*a*b^(17/2) + 692835*(sqrt(b)*x - sqrt(b*x^2 + a
))^18*a^2*b^(17/2) + 668525*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^3*b^(17/2) +
 384098*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*b^(17/2) + 89726*(sqrt(b)*x -
sqrt(b*x^2 + a))^12*a^5*b^(17/2) + 6188*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^
6*b^(17/2) - 2380*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*b^(17/2) + 680*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*a^8*b^(17/2) - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^
4*a^9*b^(17/2) + 17*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*b^(17/2) - a^11*b^
(17/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^17
```

maple [A] time = 0.01, size = 72, normalized size = 0.51

$$-\frac{\left(b \,x^{2}+a \right)^{\frac{7}{2}} \left(-256 b^{5} x^{10}+896 a \,b^{4} x^{8}-2016 a^{2} b^{3} x^{6}+3696 a^{3} b^{2} x^{4}-6006 a^{4} b \,x^{2}+9009 a^{5}\right)}{153153 a^{6} x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^18,x)
```

```
[Out] -1/153153*(b*x^2+a)^(7/2)*(-256*b^5*x^10+896*a*b^4*x^8-2016*a^2*b^3*x^6+369
6*a^3*b^2*x^4-6006*a^4*b*x^2+9009*a^5)/x^17/a^6
```

maxima [A] time = 1.54, size = 116, normalized size = 0.83

$$\frac{256 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} b^{5}}{153153 \, a^{6} x^{7}} - \frac{128 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} b^{4}}{21879 \, a^{5} x^{9}} + \frac{32 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} b^{3}}{2431 \, a^{4} x^{11}} - \frac{16 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} b^{2}}{663 \, a^{3} x^{13}} + \frac{2 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} b}{51 \, a^{2} x^{15}} - \frac{{\left(b x^{2} + a\right)}^{\frac{7}{2}}}{17 \, a x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] 256/153153*(b*x^2 + a)^(7/2)*b^5/(a^6*x^7) - 128/21879*(b*x^2 + a)^(7/2)*b^
4/(a^5*x^9) + 32/2431*(b*x^2 + a)^(7/2)*b^3/(a^4*x^11) - 16/663*(b*x^2 + a)
^(7/2)*b^2/(a^3*x^13) + 2/51*(b*x^2 + a)^(7/2)*b/(a^2*x^15) - 1/17*(b*x^2 +
 a)^(7/2)/(a*x^17)
```

mupad [B] time = 6.87, size = 171, normalized size = 1.22

$$\frac{10\,b^4\,\sqrt{b\,x^2+a}}{21879\,a^2\,x^9}-\frac{55\,b^2\,\sqrt{b\,x^2+a}}{663\,x^{13}}-\frac{b^3\,\sqrt{b\,x^2+a}}{2431\,a\,x^{11}}-\frac{a^2\,\sqrt{b\,x^2+a}}{17\,x^{17}}-\frac{80\,b^5\,\sqrt{b\,x^2+a}}{153153\,a^3\,x^7}+\frac{32\,b^6\,\sqrt{b\,x^2+a}}{51051\,a^4\,x^5}-\frac{128\,b^7\,\sqrt{b\,x^2+a}}{153153\,a^5\,x^3}+\frac{256\,b^8\,\sqrt{b\,x^2+a}}{153153\,a^6\,x}-\frac{7\,a\,b\,\sqrt{b\,x^2+a}}{51\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/x^18,x)
```

```
[Out] (10*b^4*(a + b*x^2)^(1/2))/(21879*a^2*x^9) - (55*b^2*(a + b*x^2)^(1/2))/(66
3*x^13) - (b^3*(a + b*x^2)^(1/2))/(2431*a*x^11) - (a^2*(a + b*x^2)^(1/2))/(
17*x^17) - (80*b^5*(a + b*x^2)^(1/2))/(153153*a^3*x^7) + (32*b^6*(a + b*x^2
)^(1/2))/(51051*a^4*x^5) - (128*b^7*(a + b*x^2)^(1/2))/(153153*a^5*x^3) + (
256*b^8*(a + b*x^2)^(1/2))/(153153*a^6*x) - (7*a*b*(a + b*x^2)^(1/2))/(51*x
^15)
```

sympy [B] time = 4.42, size = 1346, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**18,x)
```

```
[Out] -9009*a**13*b**(51/2)*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 7657
65*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22
+ 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 66066*a**12*b**(53/2
)*x**2*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*
x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b
**29*x**24 + 153153*a**6*b**30*x**26) - 207900*a**11*b**(55/2)*x**4*sqrt(a/
(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 153153
0*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 1
53153*a**6*b**30*x**26) - 363888*a**10*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/
(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x
**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**
30*x**26) - 382550*a**9*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(153153*a**11*b
**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*
a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 241
524*a**8*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 7
65765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**
```

```
22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 84780*a**7*b**(63
/2)*x**12*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**
26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**
7*b**29*x**24 + 153153*a**6*b**30*x**26) - 12768*a**6*b**(65/2)*x**14*sqrt(
a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531
530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 +
 153153*a**6*b**30*x**26) + 63*a**5*b**(67/2)*x**16*sqrt(a/(b*x**2) + 1)/(1
53153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**
20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30
*x**26) + 630*a**4*b**(69/2)*x**18*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25
*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8
*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) + 1680*a*
*3*b**(71/2)*x**20*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*
a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 7
65765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) + 2016*a**2*b**(73/2)*x**
22*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**1
8 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29
*x**24 + 153153*a**6*b**30*x**26) + 1152*a*b**(75/2)*x**24*sqrt(a/(b*x**2)
+ 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b*
*27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**
6*b**30*x**26) + 256*b**(77/2)*x**26*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**
25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a*
*8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26)
```

3.400 $\int x^{15} \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=161

$$-\frac{a^7 \left(a+bx^2\right)^{11/2}}{11b^8}+\frac{7a^6 \left(a+bx^2\right)^{13/2}}{13b^8}-\frac{7a^5 \left(a+bx^2\right)^{15/2}}{5b^8}+\frac{35a^4 \left(a+bx^2\right)^{17/2}}{17b^8}-\frac{35a^3 \left(a+bx^2\right)^{19/2}}{19b^8}+\frac{a^2 \left(a+bx^2\right)^{21/2}}{b^8}$$

Rubi [A] time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^2\right)^{21/2}}{b^8}-\frac{35a^3 \left(a+bx^2\right)^{19/2}}{19b^8}+\frac{35a^4 \left(a+bx^2\right)^{17/2}}{17b^8}-\frac{7a^5 \left(a+bx^2\right)^{15/2}}{5b^8}+\frac{7a^6 \left(a+bx^2\right)^{13/2}}{13b^8}-\frac{a^7 \left(a+bx^2\right)^{11/2}}{11b^8}+\frac{\left(a+bx^2\right)^{25/2}}{25b^8}-\frac{7a \left(a+bx^2\right)^{23/2}}{23b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a^7*(a + b*x^2)^(11/2))/(11*b^8) + (7*a^6*(a + b*x^2)^(13/2))/(13*b^8) -
(7*a^5*(a + b*x^2)^(15/2))/(5*b^8) + (35*a^4*(a + b*x^2)^(17/2))/(17*b^8) -
 (35*a^3*(a + b*x^2)^(19/2))/(19*b^8) + (a^2*(a + b*x^2)^(21/2))/b^8 - (7*a
*(a + b*x^2)^(23/2))/(23*b^8) + (a + b*x^2)^(25/2)/(25*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{15}\left(a+bx^2\right)^{9/2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int x^7(a+bx)^{9/2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{a^7(a+bx)^{9/2}}{b^7}+\frac{7a^6(a+bx)^{11/2}}{b^7}-\frac{21a^5(a+bx)^{13/2}}{b^7}+\frac{35a^4(a+bx)^{15/2}}{b^7}-\right.\right.$$
$$=-\frac{a^7\left(a+bx^2\right)^{11/2}}{11b^8}+\frac{7a^6\left(a+bx^2\right)^{13/2}}{13b^8}-\frac{7a^5\left(a+bx^2\right)^{15/2}}{5b^8}+\frac{35a^4\left(a+bx^2\right)^{17/2}}{17b^8}-\frac{35a^3\left(a\right.}{1}$$

Mathematica [A] time = 0.06, size = 94, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{11/2}\left(-2048a^7+11264a^6bx^2-36608a^5b^2x^4+91520a^4b^3x^6-194480a^3b^4x^8+369512a^2b^5x^{10}-646646ab^6x^{12}+1062347b^7x^{14}\right)}{26558675b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-2048*a^7 + 11264*a^6*b*x^2 - 36608*a^5*b^2*x^4 + 9152
0*a^4*b^3*x^6 - 194480*a^3*b^4*x^8 + 369512*a^2*b^5*x^10 - 646646*a*b^6*x^1
2 + 1062347*b^7*x^14))/(26558675*b^8)
```

IntegrateAlgebraic [A] time = 0.06, size = 149, normalized size = 0.93

$$\frac{\sqrt{a+bx^2}\left(-2048a^{12}+1024a^{11}bx^2-768a^{10}b^2x^4+640a^9b^3x^6-560a^8b^4x^8+504a^7b^5x^{10}-462a^6b^6x^{12}+429a^5b^7x^{14}+1659515a^4b^8x^{16}+5810090a^3b^9x^{18}+7759752a^2b^{10}x^{20}+4665089ab^{11}x^{22}+1062347b^{12}x^{24}\right)}{26558675b^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^15*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2048*a^12 + 1024*a^11*b*x^2 - 768*a^10*b^2*x^4 + 640*a^9
*b^3*x^6 - 560*a^8*b^4*x^8 + 504*a^7*b^5*x^10 - 462*a^6*b^6*x^12 + 429*a^5*
b^7*x^14 + 1659515*a^4*b^8*x^16 + 5810090*a^3*b^9*x^18 + 7759752*a^2*b^10*x
^20 + 4665089*a*b^11*x^22 + 1062347*b^12*x^24))/(26558675*b^8)
```

fricas [A] time = 1.04, size = 145, normalized size = 0.90

$$\frac{\left(1062347\,b^{12}x^{24}+4665089\,ab^{11}x^{22}+7759752\,a^2b^{10}x^{20}+5810090\,a^3b^9x^{18}+1659515\,a^4b^8x^{16}+429\,a^5b^7x^{14}-462\,a^6b^6x^{12}+504\,a^7b^5x^{10}-560\,a^8b^4x^8+640\,a^9b^3x^6-768\,a^{10}b^2x^4+1024\,a^{11}bx^2-2048\,a^{12}\right)\sqrt{bx^2+a}}{26558675\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/26558675*(1062347*b^12*x^24 + 4665089*a*b^11*x^22 + 7759752*a^2*b^10*x^20
 + 5810090*a^3*b^9*x^18 + 1659515*a^4*b^8*x^16 + 429*a^5*b^7*x^14 - 462*a^6
```

```
*b^6*x^12 + 504*a^7*b^5*x^10 - 560*a^8*b^4*x^8 + 640*a^9*b^3*x^6 - 768*a^10
*b^2*x^4 + 1024*a^11*b*x^2 - 2048*a^12)*sqrt(b*x^2 + a)/b^8
```

giac [A] time = 1.07, size = 113, normalized size = 0.70

$$\frac{1062347\left(bx^2+a\right)^{\frac{25}{2}}-8083075\left(bx^2+a\right)^{\frac{23}{2}}a+26558675\left(bx^2+a\right)^{\frac{21}{2}}a^2-48923875\left(bx^2+a\right)^{\frac{19}{2}}a^3+54679625\left(bx^2+a\right)^{\frac{17}{2}}a^4-37182145\left(bx^2+a\right)^{\frac{15}{2}}a^5+14300825\left(bx^2+a\right)^{\frac{13}{2}}a^6-2414425\left(bx^2+a\right)^{\frac{11}{2}}a^7}{26558675\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/26558675*(1062347*(b*x^2 + a)^(25/2) - 8083075*(b*x^2 + a)^(23/2)*a + 265
58675*(b*x^2 + a)^(21/2)*a^2 - 48923875*(b*x^2 + a)^(19/2)*a^3 + 54679625*(
b*x^2 + a)^(17/2)*a^4 - 37182145*(b*x^2 + a)^(15/2)*a^5 + 14300825*(b*x^2 +
 a)^(13/2)*a^6 - 2414425*(b*x^2 + a)^(11/2)*a^7)/b^8
```

maple [A] time = 0.01, size = 91, normalized size = 0.57

$$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(-1062347x^{14}b^7+646646a\,x^{12}b^6-369512a^2x^{10}b^5+194480a^3x^8b^4-91520a^4x^6b^3+36608a^5x^4b^2-11264a^6x^2b+2048a^7\right)}{26558675b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/26558675*(b*x^2+a)^(11/2)*(-1062347*b^7*x^14+646646*a*b^6*x^12-369512*a^
2*b^5*x^10+194480*a^3*b^4*x^8-91520*a^4*b^3*x^6+36608*a^5*b^2*x^4-11264*a^6
*b*x^2+2048*a^7)/b^8
```

maxima [A] time = 1.47, size = 153, normalized size = 0.95

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^{14}}{25\,b}-\frac{14\left(bx^2+a\right)^{\frac{11}{2}}ax^{12}}{575\,b^2}+\frac{8\left(bx^2+a\right)^{\frac{11}{2}}a^2x^{10}}{575\,b^3}-\frac{16\left(bx^2+a\right)^{\frac{11}{2}}a^3x^8}{2185\,b^4}+\frac{128\left(bx^2+a\right)^{\frac{11}{2}}a^4x^6}{37145\,b^5}-\frac{256\left(bx^2+a\right)^{\frac{11}{2}}a^5x^4}{185725\,b^6}+\frac{1024\left(bx^2+a\right)^{\frac{11}{2}}a^6x^2}{2414425\,b^7}-\frac{2048\left(bx^2+a\right)^{\frac{11}{2}}a^7}{26558675\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/25*(b*x^2 + a)^(11/2)*x^14/b - 14/575*(b*x^2 + a)^(11/2)*a*x^12/b^2 + 8/5
75*(b*x^2 + a)^(11/2)*a^2*x^10/b^3 - 16/2185*(b*x^2 + a)^(11/2)*a^3*x^8/b^4
 + 128/37145*(b*x^2 + a)^(11/2)*a^4*x^6/b^5 - 256/185725*(b*x^2 + a)^(11/2)
*a^5*x^4/b^6 + 1024/2414425*(b*x^2 + a)^(11/2)*a^6*x^2/b^7 - 2048/26558675*
(b*x^2 + a)^(11/2)*a^7/b^8
```

mupad [B] time = 4.84, size = 141, normalized size = 0.88

$$\sqrt{b\,x^2+a}\,\left(\frac{2321\,a^4\,x^{16}}{37145}-\frac{2048\,a^{12}}{26558675\,b^8}+\frac{b^4\,x^{24}}{25}+\frac{478\,a^3\,b\,x^{18}}{2185}+\frac{101\,a\,b^3\,x^{22}}{575}+\frac{3\,a^5\,x^{14}}{185725\,b}-\frac{42\,a^6\,x^{12}}{2414425\,b^2}+\frac{504\,a^7\,x^{10}}{26558675\,b^3}-\frac{112\,a^8\,x^8}{5311735\,b^4}+\frac{128\,a^9\,x^6}{5311735\,b^5}-\frac{768\,a^{10}\,x^4}{26558675\,b^6}+\frac{1024\,a^{11}\,x^2}{26558675\,b^7}+\frac{168\,a^2\,b^2\,x^{20}}{575}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((2321*a^4*x^16)/37145 - (2048*a^12)/(26558675*b^8) + (b^
4*x^24)/25 + (478*a^3*b*x^18)/2185 + (101*a*b^3*x^22)/575 + (3*a^5*x^14)/(1
85725*b) - (42*a^6*x^12)/(2414425*b^2) + (504*a^7*x^10)/(26558675*b^3) - (1
12*a^8*x^8)/(5311735*b^4) + (128*a^9*x^6)/(5311735*b^5) - (768*a^10*x^4)/(2
6558675*b^6) + (1024*a^11*x^2)/(26558675*b^7) + (168*a^2*b^2*x^20)/575)
```

sympy [A] time = 103.13, size = 301, normalized size = 1.87

$$\begin{cases} -\frac{2048a^{12}\sqrt{a+bx^2}}{26558675b^8} + \frac{1024a^{11}x^2\sqrt{a+bx^2}}{26558675b^7} - \frac{768a^{10}x^4\sqrt{a+bx^2}}{26558675b^6} + \frac{128a^9x^6\sqrt{a+bx^2}}{5311735b^5} - \frac{112a^8x^8\sqrt{a+bx^2}}{5311735b^4} + \frac{504a^7x^{10}\sqrt{a+bx^2}}{26558675b^3} - \frac{42a^6x^{12}\sqrt{a+bx^2}}{2414425b^2} + \frac{3a^5x^{14}\sqrt{a+bx^2}}{185725b} + \frac{2321a^4x^{16}\sqrt{a+bx^2}}{37145} + \frac{478a^3bx^{18}\sqrt{a+bx^2}}{2185} + \frac{168a^2b^2x^{20}\sqrt{a+bx^2}}{575} + \frac{101ab^3x^{22}\sqrt{a+bx^2}}{575} + \frac{b^4x^{24}\sqrt{a+bx^2}}{25} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^{16}}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-2048*a**12*sqrt(a + b*x**2)/(26558675*b**8) + 1024*a**11*x**2*s
qrt(a + b*x**2)/(26558675*b**7) - 768*a**10*x**4*sqrt(a + b*x**2)/(26558675
*b**6) + 128*a**9*x**6*sqrt(a + b*x**2)/(5311735*b**5) - 112*a**8*x**8*sqrt
(a + b*x**2)/(5311735*b**4) + 504*a**7*x**10*sqrt(a + b*x**2)/(26558675*b**
3) - 42*a**6*x**12*sqrt(a + b*x**2)/(2414425*b**2) + 3*a**5*x**14*sqrt(a +
b*x**2)/(185725*b) + 2321*a**4*x**16*sqrt(a + b*x**2)/37145 + 478*a**3*b*x*
*18*sqrt(a + b*x**2)/2185 + 168*a**2*b**2*x**20*sqrt(a + b*x**2)/575 + 101*
a*b**3*x**22*sqrt(a + b*x**2)/575 + b**4*x**24*sqrt(a + b*x**2)/25, Ne(b, 0
)), (a**(9/2)*x**16/16, True))
```

3.401 $\int x^{13} \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=140

$$\frac{a^6 \left(a+bx^2\right)^{11/2}}{11b^7}-\frac{6a^5 \left(a+bx^2\right)^{13/2}}{13b^7}+\frac{a^4 \left(a+bx^2\right)^{15/2}}{b^7}-\frac{20a^3 \left(a+bx^2\right)^{17/2}}{17b^7}+\frac{15a^2 \left(a+bx^2\right)^{19/2}}{19b^7}+\frac{\left(a+bx^2\right)^{23/2}}{23b^7}-\frac{2a}{}$$

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{15a^2 \left(a+bx^2\right)^{19/2}}{19b^7}-\frac{20a^3 \left(a+bx^2\right)^{17/2}}{17b^7}+\frac{a^4 \left(a+bx^2\right)^{15/2}}{b^7}-\frac{6a^5 \left(a+bx^2\right)^{13/2}}{13b^7}+\frac{a^6 \left(a+bx^2\right)^{11/2}}{11b^7}+\frac{\left(a+bx^2\right)^{23/2}}{23b^7}-\frac{2a \left(a+bx^2\right)^{21/2}}{7b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^6*(a + b*x^2)^(11/2))/(11*b^7) - (6*a^5*(a + b*x^2)^(13/2))/(13*b^7) + (
a^4*(a + b*x^2)^(15/2))/b^7 - (20*a^3*(a + b*x^2)^(17/2))/(17*b^7) + (15*a^
2*(a + b*x^2)^(19/2))/(19*b^7) - (2*a*(a + b*x^2)^(21/2))/(7*b^7) + (a + b*
x^2)^(23/2)/(23*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{13}\left(a+bx^2\right)^{9/2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int x^6(a+bx)^{9/2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{a^6(a+bx)^{9/2}}{b^6} - \frac{6a^5(a+bx)^{11/2}}{b^6} + \frac{15a^4(a+bx)^{13/2}}{b^6} - \frac{20a^3(a+bx)^{15/2}}{b^6} + \frac{15\ldots}{}\right.\right.$$
$$= \frac{a^6\left(a+bx^2\right)^{11/2}}{11b^7} - \frac{6a^5\left(a+bx^2\right)^{13/2}}{13b^7} + \frac{a^4\left(a+bx^2\right)^{15/2}}{b^7} - \frac{20a^3\left(a+bx^2\right)^{17/2}}{17b^7} + \frac{15a^2\left(a+\ldots\right.}{19b\ldots}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.59

$$\frac{\left(a+bx^2\right)^{11/2}\left(1024a^6-5632a^5bx^2+18304a^4b^2x^4-45760a^3b^3x^6+97240a^2b^4x^8-184756ab^5x^{10}+323323b^6x^{12}\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(1024*a^6 - 5632*a^5*b*x^2 + 18304*a^4*b^2*x^4 - 45760*
a^3*b^3*x^6 + 97240*a^2*b^4*x^8 - 184756*a*b^5*x^10 + 323323*b^6*x^12))/(74
36429*b^7)
```

IntegrateAlgebraic [A] time = 0.05, size = 138, normalized size = 0.99

$$\frac{\sqrt{a+bx^2}\left(1024a^{11}-512a^{10}bx^2+384a^9b^2x^4-320a^8b^3x^6+280a^7b^4x^8-252a^6b^5x^{10}+231a^5b^6x^{12}+530959a^4b^7x^{14}+1826110a^3b^8x^{16}+2406690a^2b^9x^{18}+1431859ab^{10}x^{20}+323323b^{11}x^{22}\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^13*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(1024*a^11 - 512*a^10*b*x^2 + 384*a^9*b^2*x^4 - 320*a^8*b^
3*x^6 + 280*a^7*b^4*x^8 - 252*a^6*b^5*x^10 + 231*a^5*b^6*x^12 + 530959*a^4*
b^7*x^14 + 1826110*a^3*b^8*x^16 + 2406690*a^2*b^9*x^18 + 1431859*a*b^10*x^2
0 + 323323*b^11*x^22))/(7436429*b^7)
```

fricas [A] time = 1.28, size = 134, normalized size = 0.96

$$\frac{\left(323323\,b^{11}x^{22}+1431859\,ab^{10}x^{20}+2406690\,a^2b^9x^{18}+1826110\,a^3b^8x^{16}+530959\,a^4b^7x^{14}+231\,a^5b^6x^{12}-252\,a^6b^5x^{10}+280\,a^7b^4x^8-320\,a^8b^3x^6+384\,a^9b^2x^4-512\,a^{10}bx^2+1024\,a^{11}\right)\sqrt{bx^2+a}}{7436429\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7436429*(323323*b^11*x^22 + 1431859*a*b^10*x^20 + 2406690*a^2*b^9*x^18 +
1826110*a^3*b^8*x^16 + 530959*a^4*b^7*x^14 + 231*a^5*b^6*x^12 - 252*a^6*b^5
```

```
*x^10 + 280*a^7*b^4*x^8 - 320*a^8*b^3*x^6 + 384*a^9*b^2*x^4 - 512*a^10*b*x^
2 + 1024*a^11)*sqrt(b*x^2 + a)/b^7
```

giac [A] time = 1.10, size = 99, normalized size = 0.71

$$\frac{323323\left(bx^2+a\right)^{\frac{23}{2}}-2124694\left(bx^2+a\right)^{\frac{21}{2}}a+5870865\left(bx^2+a\right)^{\frac{19}{2}}a^2-8748740\left(bx^2+a\right)^{\frac{17}{2}}a^3+7436429\left(bx^2+a\right)^{\frac{15}{2}}a^4-3432198\left(bx^2+a\right)^{\frac{13}{2}}a^5+676039\left(bx^2+a\right)^{\frac{11}{2}}a^6}{7436429\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7436429*(323323*(b*x^2 + a)^(23/2) - 2124694*(b*x^2 + a)^(21/2)*a + 58708
65*(b*x^2 + a)^(19/2)*a^2 - 8748740*(b*x^2 + a)^(17/2)*a^3 + 7436429*(b*x^2
 + a)^(15/2)*a^4 - 3432198*(b*x^2 + a)^(13/2)*a^5 + 676039*(b*x^2 + a)^(11/
2)*a^6)/b^7
```

maple [A] time = 0.01, size = 80, normalized size = 0.57

$$\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(323323x^{12}b^6-184756a\,x^{10}b^5+97240a^2x^8b^4-45760a^3x^6b^3+18304a^4x^4b^2-5632a^5x^2b+1024a^6\right)}{7436429b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7436429*(b*x^2+a)^(11/2)*(323323*b^6*x^12-184756*a*b^5*x^10+97240*a^2*b^4
*x^8-45760*a^3*b^3*x^6+18304*a^4*b^2*x^4-5632*a^5*b*x^2+1024*a^6)/b^7
```

maxima [A] time = 1.45, size = 133, normalized size = 0.95

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^{12}}{23\,b}-\frac{4\left(bx^2+a\right)^{\frac{11}{2}}ax^{10}}{161\,b^2}+\frac{40\left(bx^2+a\right)^{\frac{11}{2}}a^2x^8}{3059\,b^3}-\frac{320\left(bx^2+a\right)^{\frac{11}{2}}a^3x^6}{52003\,b^4}+\frac{128\left(bx^2+a\right)^{\frac{11}{2}}a^4x^4}{52003\,b^5}-\frac{512\left(bx^2+a\right)^{\frac{11}{2}}a^5x^2}{676039\,b^6}+\frac{1024\left(bx^2+a\right)^{\frac{11}{2}}a^6}{7436429\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/23*(b*x^2 + a)^(11/2)*x^12/b - 4/161*(b*x^2 + a)^(11/2)*a*x^10/b^2 + 40/3
059*(b*x^2 + a)^(11/2)*a^2*x^8/b^3 - 320/52003*(b*x^2 + a)^(11/2)*a^3*x^6/b
^4 + 128/52003*(b*x^2 + a)^(11/2)*a^4*x^4/b^5 - 512/676039*(b*x^2 + a)^(11/
2)*a^5*x^2/b^6 + 1024/7436429*(b*x^2 + a)^(11/2)*a^6/b^7
```

mupad [B] time = 4.84, size = 130, normalized size = 0.93

$$\sqrt{b\,x^2+a}\,\left(\frac{1024\,a^{11}}{7436429\,b^7}+\frac{3713\,a^4\,x^{14}}{52003}+\frac{b^4\,x^{22}}{23}+\frac{12770\,a^3\,b\,x^{16}}{52003}+\frac{31\,a\,b^3\,x^{20}}{161}+\frac{3\,a^5\,x^{12}}{96577\,b}-\frac{36\,a^6\,x^{10}}{1062347\,b^2}+\frac{40\,a^7\,x^8}{1062347\,b^3}-\frac{320\,a^8\,x^6}{7436429\,b^4}+\frac{384\,a^9\,x^4}{7436429\,b^5}-\frac{512\,a^{10}\,x^2}{7436429\,b^6}+\frac{990\,a^2\,b^2\,x^{18}}{3059}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((1024*a^11)/(7436429*b^7) + (3713*a^4*x^14)/52003 + (b^4
*x^22)/23 + (12770*a^3*b*x^16)/52003 + (31*a*b^3*x^20)/161 + (3*a^5*x^12)/(
96577*b) - (36*a^6*x^10)/(1062347*b^2) + (40*a^7*x^8)/(1062347*b^3) - (320*
a^8*x^6)/(7436429*b^4) + (384*a^9*x^4)/(7436429*b^5) - (512*a^10*x^2)/(7436
429*b^6) + (990*a^2*b^2*x^18)/3059)
```

sympy [**A**] time = 79.37, size = 277, normalized size = 1.98

$$\begin{cases} \frac{1024a^{11}\sqrt{a+bx^2}}{7436429b^7} - \frac{512a^{10}x^2\sqrt{a+bx^2}}{7436429b^6} + \frac{384a^9x^4\sqrt{a+bx^2}}{7436429b^5} - \frac{320a^8x^6\sqrt{a+bx^2}}{7436429b^4} + \frac{40a^7x^8\sqrt{a+bx^2}}{1062347b^3} - \frac{36a^6x^{10}\sqrt{a+bx^2}}{1062347b^2} + \frac{3a^5x^{12}\sqrt{a+bx^2}}{96577b} + \frac{3713a^4x^{14}\sqrt{a+bx^2}}{52003} + \frac{12770a^3bx^{16}\sqrt{a+bx^2}}{52003} + \frac{990a^2b^2x^{18}\sqrt{a+bx^2}}{3059} + \frac{31ab^3x^{20}\sqrt{a+bx^2}}{161} + \frac{b^4x^{22}\sqrt{a+bx^2}}{23} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^{14}}{14} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((1024*a**11*sqrt(a + b*x**2)/(7436429*b**7) - 512*a**10*x**2*sqrt
(a + b*x**2)/(7436429*b**6) + 384*a**9*x**4*sqrt(a + b*x**2)/(7436429*b**5)
 - 320*a**8*x**6*sqrt(a + b*x**2)/(7436429*b**4) + 40*a**7*x**8*sqrt(a + b*
x**2)/(1062347*b**3) - 36*a**6*x**10*sqrt(a + b*x**2)/(1062347*b**2) + 3*a*
*5*x**12*sqrt(a + b*x**2)/(96577*b) + 3713*a**4*x**14*sqrt(a + b*x**2)/5200
3 + 12770*a**3*b*x**16*sqrt(a + b*x**2)/52003 + 990*a**2*b**2*x**18*sqrt(a
+ b*x**2)/3059 + 31*a*b**3*x**20*sqrt(a + b*x**2)/161 + b**4*x**22*sqrt(a +
 b*x**2)/23, Ne(b, 0)), (a**(9/2)*x**14/14, True))
```

3.402 $\int x^{11}\left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=122

$$-\frac{a^5\left(a+bx^2\right)^{11/2}}{11b^6}+\frac{5a^4\left(a+bx^2\right)^{13/2}}{13b^6}-\frac{2a^3\left(a+bx^2\right)^{15/2}}{3b^6}+\frac{10a^2\left(a+bx^2\right)^{17/2}}{17b^6}+\frac{\left(a+bx^2\right)^{21/2}}{21b^6}-\frac{5a\left(a+bx^2\right)^{19/2}}{19b^6}$$

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{10a^2\left(a+bx^2\right)^{17/2}}{17b^6}-\frac{2a^3\left(a+bx^2\right)^{15/2}}{3b^6}+\frac{5a^4\left(a+bx^2\right)^{13/2}}{13b^6}-\frac{a^5\left(a+bx^2\right)^{11/2}}{11b^6}+\frac{\left(a+bx^2\right)^{21/2}}{21b^6}-\frac{5a\left(a+bx^2\right)^{19/2}}{19b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a^5*(a + b*x^2)^(11/2))/(11*b^6) + (5*a^4*(a + b*x^2)^(13/2))/(13*b^6) -
(2*a^3*(a + b*x^2)^(15/2))/(3*b^6) + (10*a^2*(a + b*x^2)^(17/2))/(17*b^6) -
 (5*a*(a + b*x^2)^(19/2))/(19*b^6) + (a + b*x^2)^(21/2)/(21*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{11}\left(a+bx^2\right)^{9/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^5(a+bx)^{9/2}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^5(a+bx)^{9/2}}{b^5}+\frac{5a^4(a+bx)^{11/2}}{b^5}-\frac{10a^3(a+bx)^{13/2}}{b^5}+\frac{10a^2(a+bx)^{15/2}}{b^5}-\right.\right.$$

$$=-\frac{a^5\left(a+bx^2\right)^{11/2}}{11b^6}+\frac{5a^4\left(a+bx^2\right)^{13/2}}{13b^6}-\frac{2a^3\left(a+bx^2\right)^{15/2}}{3b^6}+\frac{10a^2\left(a+bx^2\right)^{17/2}}{17b^6}-\frac{5a\left(a\right.}{}$$

Mathematica [**A**] time = 0.04, size = 72, normalized size = 0.59

$$\frac{\left(a+bx^2\right)^{11/2}\left(-256a^5+1408a^4bx^2-4576a^3b^2x^4+11440a^2b^3x^6-24310ab^4x^8+46189b^5x^{10}\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-256*a^5 + 1408*a^4*b*x^2 - 4576*a^3*b^2*x^4 + 11440*a
^2*b^3*x^6 - 24310*a*b^4*x^8 + 46189*b^5*x^10))/(969969*b^6)
```

IntegrateAlgebraic [**A**] time = 0.04, size = 72, normalized size = 0.59

$$\frac{\left(a+bx^2\right)^{11/2}\left(-256a^5+1408a^4bx^2-4576a^3b^2x^4+11440a^2b^3x^6-24310ab^4x^8+46189b^5x^{10}\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-256*a^5 + 1408*a^4*b*x^2 - 4576*a^3*b^2*x^4 + 11440*a
^2*b^3*x^6 - 24310*a*b^4*x^8 + 46189*b^5*x^10))/(969969*b^6)
```

fricas [**A**] time = 0.84, size = 123, normalized size = 1.01

$$\frac{\left(46189\,b^{10}x^{20}+206635\,ab^{9}x^{18}+351780\,a^{2}b^{8}x^{16}+271414\,a^{3}b^{7}x^{14}+80773\,a^{4}b^{6}x^{12}+63\,a^{5}b^{5}x^{10}-70\,a^{6}b^{4}x^{8}+80\,a^{7}b^{3}x^{6}-96\,a^{8}b^{2}x^{4}+128\,a^{9}bx^{2}-256\,a^{10}\right)\sqrt{bx^2+a}}{969969\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/969969*(46189*b^10*x^20 + 206635*a*b^9*x^18 + 351780*a^2*b^8*x^16 + 27141
4*a^3*b^7*x^14 + 80773*a^4*b^6*x^12 + 63*a^5*b^5*x^10 - 70*a^6*b^4*x^8 + 80
*a^7*b^3*x^6 - 96*a^8*b^2*x^4 + 128*a^9*b*x^2 - 256*a^10)*sqrt(b*x^2 + a)/b
^6
```

giac [**A**] time = 1.01, size = 85, normalized size = 0.70

$$\frac{46189\left(bx^2+a\right)^{\frac{21}{2}}-255255\left(bx^2+a\right)^{\frac{19}{2}}a+570570\left(bx^2+a\right)^{\frac{17}{2}}a^2-646646\left(bx^2+a\right)^{\frac{15}{2}}a^3+373065\left(bx^2+a\right)^{\frac{13}{2}}a^4-88179\left(bx^2+a\right)^{\frac{11}{2}}a^5}{969969\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/969969*(46189*(b*x^2 + a)^(21/2) - 255255*(b*x^2 + a)^(19/2)*a + 570570*(
b*x^2 + a)^(17/2)*a^2 - 646646*(b*x^2 + a)^(15/2)*a^3 + 373065*(b*x^2 + a)^
(13/2)*a^4 - 88179*(b*x^2 + a)^(11/2)*a^5)/b^6
```

maple [A] time = 0.01, size = 69, normalized size = 0.57

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-46189b^{5}x^{10}+24310a\,b^{4}x^{8}-11440a^{2}b^{3}x^{6}+4576a^{3}b^{2}x^{4}-1408a^{4}b\,x^{2}+256a^{5}\right)}{969969b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/969969*(b*x^2+a)^(11/2)*(-46189*b^5*x^10+24310*a*b^4*x^8-11440*a^2*b^3*x
^6+4576*a^3*b^2*x^4-1408*a^4*b*x^2+256*a^5)/b^6
```

maxima [A] time = 1.48, size = 113, normalized size = 0.93

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}x^{10}}{21\,b}-\frac{10\left(bx^{2}+a\right)^{\frac{11}{2}}ax^{8}}{399\,b^{2}}+\frac{80\left(bx^{2}+a\right)^{\frac{11}{2}}a^{2}x^{6}}{6783\,b^{3}}-\frac{32\left(bx^{2}+a\right)^{\frac{11}{2}}a^{3}x^{4}}{6783\,b^{4}}+\frac{128\left(bx^{2}+a\right)^{\frac{11}{2}}a^{4}x^{2}}{88179\,b^{5}}-\frac{256\left(bx^{2}+a\right)^{\frac{11}{2}}a^{5}}{969969\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/21*(b*x^2 + a)^(11/2)*x^10/b - 10/399*(b*x^2 + a)^(11/2)*a*x^8/b^2 + 80/6
783*(b*x^2 + a)^(11/2)*a^2*x^6/b^3 - 32/6783*(b*x^2 + a)^(11/2)*a^3*x^4/b^4
 + 128/88179*(b*x^2 + a)^(11/2)*a^4*x^2/b^5 - 256/969969*(b*x^2 + a)^(11/2)
*a^5/b^6
```

mupad [B] time = 4.81, size = 119, normalized size = 0.98

$$\sqrt{b\,x^{2}+a}\,\left(\frac{1049\,a^{4}\,x^{12}}{12597}-\frac{256\,a^{10}}{969969\,b^{6}}+\frac{b^{4}\,x^{20}}{21}+\frac{1898\,a^{3}\,b\,x^{14}}{6783}+\frac{85\,a\,b^{3}\,x^{18}}{399}+\frac{3\,a^{5}\,x^{10}}{46189\,b}-\frac{10\,a^{6}\,x^{8}}{138567\,b^{2}}+\frac{80\,a^{7}\,x^{6}}{969969\,b^{3}}-\frac{32\,a^{8}\,x^{4}}{323323\,b^{4}}+\frac{128\,a^{9}\,x^{2}}{969969\,b^{5}}+\frac{820\,a^{2}\,b^{2}\,x^{16}}{2261}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((1049*a^4*x^12)/12597 - (256*a^10)/(969969*b^6) + (b^4*x
^20)/21 + (1898*a^3*b*x^14)/6783 + (85*a*b^3*x^18)/399 + (3*a^5*x^10)/(4618
9*b) - (10*a^6*x^8)/(138567*b^2) + (80*a^7*x^6)/(969969*b^3) - (32*a^8*x^4)
/(323323*b^4) + (128*a^9*x^2)/(969969*b^5) + (820*a^2*b^2*x^16)/2261)
```

sympy [A] time = 62.85, size = 253, normalized size = 2.07

$$\begin{cases} -\frac{256a^{10}\sqrt{a+bx^{2}}}{969969b^{6}}+\frac{128a^{9}x^{2}\sqrt{a+bx^{2}}}{969969b^{5}}-\frac{32a^{8}x^{4}\sqrt{a+bx^{2}}}{323323b^{4}}+\frac{80a^{7}x^{6}\sqrt{a+bx^{2}}}{969969b^{3}}-\frac{10a^{6}x^{8}\sqrt{a+bx^{2}}}{138567b^{2}}+\frac{3a^{5}x^{10}\sqrt{a+bx^{2}}}{46189b}+\frac{1049a^{4}x^{12}\sqrt{a+bx^{2}}}{12597}+\frac{1898a^{3}bx^{14}\sqrt{a+bx^{2}}}{6783}+\frac{820a^{2}b^{2}x^{16}\sqrt{a+bx^{2}}}{2261}+\frac{85ab^{3}x^{18}\sqrt{a+bx^{2}}}{399}+\frac{b^{4}x^{20}\sqrt{a+bx^{2}}}{21} & \text{for}\ b\neq 0\\ \frac{a^{\frac{9}{2}}x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-256*a**10*sqrt(a + b*x**2)/(969969*b**6) + 128*a**9*x**2*sqrt(a
 + b*x**2)/(969969*b**5) - 32*a**8*x**4*sqrt(a + b*x**2)/(323323*b**4) + 80
*a**7*x**6*sqrt(a + b*x**2)/(969969*b**3) - 10*a**6*x**8*sqrt(a + b*x**2)/(
138567*b**2) + 3*a**5*x**10*sqrt(a + b*x**2)/(46189*b) + 1049*a**4*x**12*sq
rt(a + b*x**2)/12597 + 1898*a**3*b*x**14*sqrt(a + b*x**2)/6783 + 820*a**2*b
**2*x**16*sqrt(a + b*x**2)/2261 + 85*a*b**3*x**18*sqrt(a + b*x**2)/399 + b*
*4*x**20*sqrt(a + b*x**2)/21, Ne(b, 0)), (a**(9/2)*x**12/12, True))
```

3.403 $\int x^9 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=101

$$\frac{a^4 \left(a+bx^2\right)^{11/2}}{11b^5}-\frac{4a^3 \left(a+bx^2\right)^{13/2}}{13b^5}+\frac{2a^2 \left(a+bx^2\right)^{15/2}}{5b^5}+\frac{\left(a+bx^2\right)^{19/2}}{19b^5}-\frac{4a \left(a+bx^2\right)^{17/2}}{17b^5}$$

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{2a^2 \left(a+bx^2\right)^{15/2}}{5b^5}-\frac{4a^3 \left(a+bx^2\right)^{13/2}}{13b^5}+\frac{a^4 \left(a+bx^2\right)^{11/2}}{11b^5}+\frac{\left(a+bx^2\right)^{19/2}}{19b^5}-\frac{4a \left(a+bx^2\right)^{17/2}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^4*(a + b*x^2)^(11/2))/(11*b^5) - (4*a^3*(a + b*x^2)^(13/2))/(13*b^5) + (
2*a^2*(a + b*x^2)^(15/2))/(5*b^5) - (4*a*(a + b*x^2)^(17/2))/(17*b^5) + (a
+ b*x^2)^(19/2)/(19*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^9 \left(a+bx^2\right)^{9/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^4 (a+bx)^{9/2} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4 (a+bx)^{9/2}}{b^4}-\frac{4a^3 (a+bx)^{11/2}}{b^4}+\frac{6a^2 (a+bx)^{13/2}}{b^4}-\frac{4a (a+bx)^{15/2}}{b^4}+\frac{(a+\right.\right.\\
&= \frac{a^4 \left(a+bx^2\right)^{11/2}}{11b^5}-\frac{4a^3 \left(a+bx^2\right)^{13/2}}{13b^5}+\frac{2a^2 \left(a+bx^2\right)^{15/2}}{5b^5}-\frac{4a \left(a+bx^2\right)^{17/2}}{17b^5}+\frac{\left(a+bx^2\right)^{1}}{19b^5}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.60

$$\frac{\left(a+bx^2\right)^{11/2}\left(128a^4-704a^3bx^2+2288a^2b^2x^4-5720ab^3x^6+12155b^4x^8\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^(9/2),x]
[Out] ((a + b*x^2)^(11/2)*(128*a^4 - 704*a^3*b*x^2 + 2288*a^2*b^2*x^4 - 5720*a*b^
3*x^6 + 12155*b^4*x^8))/(230945*b^5)
```

IntegrateAlgebraic [A] time = 0.04, size = 61, normalized size = 0.60

$$\frac{\left(a+bx^2\right)^{11/2}\left(128a^4-704a^3bx^2+2288a^2b^2x^4-5720ab^3x^6+12155b^4x^8\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9*(a + b*x^2)^(9/2),x]
[Out] ((a + b*x^2)^(11/2)*(128*a^4 - 704*a^3*b*x^2 + 2288*a^2*b^2*x^4 - 5720*a*b^
3*x^6 + 12155*b^4*x^8))/(230945*b^5)
```

fricas [A] time = 0.89, size = 112, normalized size = 1.11

$$\frac{\left(12155\,b^{9}x^{18}+55055\,ab^{8}x^{16}+95238\,a^{2}b^{7}x^{14}+75086\,a^{3}b^{6}x^{12}+23063\,a^{4}b^{5}x^{10}+35\,a^{5}b^{4}x^{8}-40\,a^{6}b^{3}x^{6}+48\,a^{7}b^{2}x^{4}-64\,a^{8}bx^{2}+128\,a^{9}\right)\sqrt{bx^{2}+a}}{230945\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="fricas")
[Out] 1/230945*(12155*b^9*x^18 + 55055*a*b^8*x^16 + 95238*a^2*b^7*x^14 + 75086*a^
3*b^6*x^12 + 23063*a^4*b^5*x^10 + 35*a^5*b^4*x^8 - 40*a^6*b^3*x^6 + 48*a^7*
b^2*x^4 - 64*a^8*b*x^2 + 128*a^9)*sqrt(b*x^2 + a)/b^5
```

giac [A] time = 1.08, size = 71, normalized size = 0.70

$$\frac{12155\left(bx^{2}+a\right)^{\frac{19}{2}}-54340\left(bx^{2}+a\right)^{\frac{17}{2}}a+92378\left(bx^{2}+a\right)^{\frac{15}{2}}a^{2}-71060\left(bx^{2}+a\right)^{\frac{13}{2}}a^{3}+20995\left(bx^{2}+a\right)^{\frac{11}{2}}a^{4}}{230945\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/230945*(12155*(b*x^2 + a)^(19/2) - 54340*(b*x^2 + a)^(17/2)*a + 92378*(b*
x^2 + a)^(15/2)*a^2 - 71060*(b*x^2 + a)^(13/2)*a^3 + 20995*(b*x^2 + a)^(11/
2)*a^4)/b^5
```

maple [A] time = 0.01, size = 58, normalized size = 0.57

$$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(12155b^{4}x^{8}-5720a\,b^{3}x^{6}+2288a^{2}b^{2}x^{4}-704a^{3}b\,x^{2}+128a^{4}\right)}{230945b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/230945*(b*x^2+a)^(11/2)*(12155*b^4*x^8-5720*a*b^3*x^6+2288*a^2*b^2*x^4-70
4*a^3*b*x^2+128*a^4)/b^5
```

maxima [A] time = 1.43, size = 93, normalized size = 0.92

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}x^{8}}{19\,b}-\frac{8\left(bx^{2}+a\right)^{\frac{11}{2}}ax^{6}}{323\,b^{2}}+\frac{16\left(bx^{2}+a\right)^{\frac{11}{2}}a^{2}x^{4}}{1615\,b^{3}}-\frac{64\left(bx^{2}+a\right)^{\frac{11}{2}}a^{3}x^{2}}{20995\,b^{4}}+\frac{128\left(bx^{2}+a\right)^{\frac{11}{2}}a^{4}}{230945\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/19*(b*x^2 + a)^(11/2)*x^8/b - 8/323*(b*x^2 + a)^(11/2)*a*x^6/b^2 + 16/161
5*(b*x^2 + a)^(11/2)*a^2*x^4/b^3 - 64/20995*(b*x^2 + a)^(11/2)*a^3*x^2/b^4
+ 128/230945*(b*x^2 + a)^(11/2)*a^4/b^5
```

mupad [B] time = 4.74, size = 108, normalized size = 1.07

$$\sqrt{b\,x^{2}+a}\,\left(\frac{128\,a^{9}}{230945\,b^{5}}+\frac{23063\,a^{4}\,x^{10}}{230945}+\frac{b^{4}\,x^{18}}{19}+\frac{6826\,a^{3}\,b\,x^{12}}{20995}+\frac{77\,a\,b^{3}\,x^{16}}{323}+\frac{7\,a^{5}\,x^{8}}{46189\,b}-\frac{8\,a^{6}\,x^{6}}{46189\,b^{2}}+\frac{48\,a^{7}\,x^{4}}{230945\,b^{3}}-\frac{64\,a^{8}\,x^{2}}{230945\,b^{4}}+\frac{666\,a^{2}\,b^{2}\,x^{14}}{1615}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^9)/(230945*b^5) + (23063*a^4*x^10)/230945 + (b^4*
x^18)/19 + (6826*a^3*b*x^12)/20995 + (77*a*b^3*x^16)/323 + (7*a^5*x^8)/(461
89*b) - (8*a^6*x^6)/(46189*b^2) + (48*a^7*x^4)/(230945*b^3) - (64*a^8*x^2)/
(230945*b^4) + (666*a^2*b^2*x^14)/1615)
```

sympy [A] time = 48.28, size = 230, normalized size = 2.28

$$\begin{cases}\frac{128a^{9}\sqrt{a+bx^{2}}}{230945b^{5}}-\frac{64a^{8}x^{2}\sqrt{a+bx^{2}}}{230945b^{4}}+\frac{48a^{7}x^{4}\sqrt{a+bx^{2}}}{230945b^{3}}-\frac{8a^{6}x^{6}\sqrt{a+bx^{2}}}{46189b^{2}}+\frac{7a^{5}x^{8}\sqrt{a+bx^{2}}}{46189b}+\frac{23063a^{4}x^{10}\sqrt{a+bx^{2}}}{230945}+\frac{6826a^{3}bx^{12}\sqrt{a+bx^{2}}}{20995}+\frac{666a^{2}b^{2}x^{14}\sqrt{a+bx^{2}}}{1615}+\frac{77ab^{3}x^{16}\sqrt{a+bx^{2}}}{323}+\frac{b^{4}x^{18}\sqrt{a+bx^{2}}}{19} & \text{for}\ b\neq 0\\ \frac{a^{\frac{9}{2}}x^{10}}{10} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((128*a**9*sqrt(a + b*x**2)/(230945*b**5) - 64*a**8*x**2*sqrt(a +
b*x**2)/(230945*b**4) + 48*a**7*x**4*sqrt(a + b*x**2)/(230945*b**3) - 8*a**
6*x**6*sqrt(a + b*x**2)/(46189*b**2) + 7*a**5*x**8*sqrt(a + b*x**2)/(46189*
b) + 23063*a**4*x**10*sqrt(a + b*x**2)/230945 + 6826*a**3*b*x**12*sqrt(a +
b*x**2)/20995 + 666*a**2*b**2*x**14*sqrt(a + b*x**2)/1615 + 77*a*b**3*x**16
*sqrt(a + b*x**2)/323 + b**4*x**18*sqrt(a + b*x**2)/19, Ne(b, 0)), (a**(9/2
)*x**10/10, True))
```

3.404 $\int x^7 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^2\right)^{11/2}}{11b^4}+\frac{3a^2\left(a+bx^2\right)^{13/2}}{13b^4}+\frac{\left(a+bx^2\right)^{17/2}}{17b^4}-\frac{a\left(a+bx^2\right)^{15/2}}{5b^4}$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{a^3\left(a+bx^2\right)^{11/2}}{11b^4}+\frac{\left(a+bx^2\right)^{17/2}}{17b^4}-\frac{a\left(a+bx^2\right)^{15/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^(11/2))/(11*b^4) + (3*a^2*(a + b*x^2)^(13/2))/(13*b^4) -
(a*(a + b*x^2)^(15/2))/(5*b^4) + (a + b*x^2)^(17/2)/(17*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7\left(a+bx^2\right)^{9/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{9/2}\, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^{9/2}}{b^3}+\frac{3a^2(a+bx)^{11/2}}{b^3}-\frac{3a(a+bx)^{13/2}}{b^3}+\frac{(a+bx)^{15/2}}{b^3}\right) dx, x, x^2\right)\\
&= -\frac{a^3\left(a+bx^2\right)^{11/2}}{11b^4}+\frac{3a^2\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{a\left(a+bx^2\right)^{15/2}}{5b^4}+\frac{\left(a+bx^2\right)^{17/2}}{17b^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{11/2}\left(-16a^3+88a^2bx^2-286ab^2x^4+715b^3x^6\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-16*a^3 + 88*a^2*b*x^2 - 286*a*b^2*x^4 + 715*b^3*x^6))
/(12155*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^{11/2}\left(-16a^3+88a^2bx^2-286ab^2x^4+715b^3x^6\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-16*a^3 + 88*a^2*b*x^2 - 286*a*b^2*x^4 + 715*b^3*x^6))
/(12155*b^4)
```

fricas [A] time = 0.73, size = 101, normalized size = 1.26

$$\frac{\left(715\,b^8x^{16}+3289\,ab^7x^{14}+5808\,a^2b^6x^{12}+4714\,a^3b^5x^{10}+1515\,a^4b^4x^8+5\,a^5b^3x^6-6\,a^6b^2x^4+8\,a^7bx^2-16\,a^8\right)\sqrt{bx^2+a}}{12155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/12155*(715*b^8*x^16 + 3289*a*b^7*x^14 + 5808*a^2*b^6*x^12 + 4714*a^3*b^5*
x^10 + 1515*a^4*b^4*x^8 + 5*a^5*b^3*x^6 - 6*a^6*b^2*x^4 + 8*a^7*b*x^2 - 16*
a^8)*sqrt(b*x^2 + a)/b^4
```

giac [A] time = 1.15, size = 57, normalized size = 0.71

$$\frac{715\left(bx^2+a\right)^{\frac{17}{2}}-2431\left(bx^2+a\right)^{\frac{15}{2}}a+2805\left(bx^2+a\right)^{\frac{13}{2}}a^2-1105\left(bx^2+a\right)^{\frac{11}{2}}a^3}{12155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/12155*(715*(b*x^2 + a)^(17/2) - 2431*(b*x^2 + a)^(15/2)*a + 2805*(b*x^2 +
 a)^(13/2)*a^2 - 1105*(b*x^2 + a)^(11/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-715b^{3}x^{6}+286a\,b^{2}x^{4}-88a^{2}b\,x^{2}+16a^{3}\right)}{12155b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(9/2),x)
```

```
[Out] -1/12155*(b*x^2+a)^(11/2)*(-715*b^3*x^6+286*a*b^2*x^4-88*a^2*b*x^2+16*a^3)/
b^4
```

maxima [A] time = 1.42, size = 73, normalized size = 0.91

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}x^{6}}{17\,b}-\frac{2\left(bx^{2}+a\right)^{\frac{11}{2}}ax^{4}}{85\,b^{2}}+\frac{8\left(bx^{2}+a\right)^{\frac{11}{2}}a^{2}x^{2}}{1105\,b^{3}}-\frac{16\left(bx^{2}+a\right)^{\frac{11}{2}}a^{3}}{12155\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/17*(b*x^2 + a)^(11/2)*x^6/b - 2/85*(b*x^2 + a)^(11/2)*a*x^4/b^2 + 8/1105*
(b*x^2 + a)^(11/2)*a^2*x^2/b^3 - 16/12155*(b*x^2 + a)^(11/2)*a^3/b^4
```

mupad [B] time = 4.73, size = 97, normalized size = 1.21

$$\sqrt{b\,x^2+a}\,\left(\frac{303\,a^4\,x^8}{2431}-\frac{16\,a^8}{12155\,b^4}+\frac{b^4\,x^{16}}{17}+\frac{4714\,a^3\,b\,x^{10}}{12155}+\frac{23\,a\,b^3\,x^{14}}{85}+\frac{a^5\,x^6}{2431\,b}-\frac{6\,a^6\,x^4}{12155\,b^2}+\frac{8\,a^7\,x^2}{12155\,b^3}+\frac{528\,a^2\,b^2\,x^{12}}{1105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((303*a^4*x^8)/2431 - (16*a^8)/(12155*b^4) + (b^4*x^16)/1
7 + (4714*a^3*b*x^10)/12155 + (23*a*b^3*x^14)/85 + (a^5*x^6)/(2431*b) - (6*
a^6*x^4)/(12155*b^2) + (8*a^7*x^2)/(12155*b^3) + (528*a^2*b^2*x^12)/1105)
```

sympy [A] time = 37.20, size = 204, normalized size = 2.55

$$\begin{cases} -\frac{16a^{8}\sqrt{a+bx^{2}}}{12155b^{4}}+\frac{8a^{7}x^{2}\sqrt{a+bx^{2}}}{12155b^{3}}-\frac{6a^{6}x^{4}\sqrt{a+bx^{2}}}{12155b^{2}}+\frac{a^{5}x^{6}\sqrt{a+bx^{2}}}{2431b}+\frac{303a^{4}x^{8}\sqrt{a+bx^{2}}}{2431}+\frac{4714a^{3}bx^{10}\sqrt{a+bx^{2}}}{12155}+\frac{528a^{2}b^{2}x^{12}\sqrt{a+bx^{2}}}{1105}+\frac{23ab^{3}x^{14}\sqrt{a+bx^{2}}}{85}+\frac{b^{4}x^{16}\sqrt{a+bx^{2}}}{17} & \text{for}\: b \neq 0 \\ \frac{a^{\frac{9}{2}}x^{8}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-16*a**8*sqrt(a + b*x**2)/(12155*b**4) + 8*a**7*x**2*sqrt(a + b*
x**2)/(12155*b**3) - 6*a**6*x**4*sqrt(a + b*x**2)/(12155*b**2) + a**5*x**6*
sqrt(a + b*x**2)/(2431*b) + 303*a**4*x**8*sqrt(a + b*x**2)/2431 + 4714*a**3
*b*x**10*sqrt(a + b*x**2)/12155 + 528*a**2*b**2*x**12*sqrt(a + b*x**2)/1105
 + 23*a*b**3*x**14*sqrt(a + b*x**2)/85 + b**4*x**16*sqrt(a + b*x**2)/17, Ne
(b, 0)), (a**(9/2)*x**8/8, True))
```

3.405 $\int x^5 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^2\right)^{11/2}}{11b^3}+\frac{\left(a+bx^2\right)^{15/2}}{15b^3}-\frac{2a\left(a+bx^2\right)^{13/2}}{13b^3}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{11/2}}{11b^3}+\frac{\left(a+bx^2\right)^{15/2}}{15b^3}-\frac{2a\left(a+bx^2\right)^{13/2}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(9/2),x]
```

```
[Out] (a^2*(a + b*x^2)^(11/2))/(11*b^3) - (2*a*(a + b*x^2)^(13/2))/(13*b^3) + (a
+ b*x^2)^(15/2)/(15*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{9/2} dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{9/2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{9/2}}{b^2}-\frac{2a(a+bx)^{11/2}}{b^2}+\frac{(a+bx)^{13/2}}{b^2}\right)dx, x, x^2\right)\\ &= \frac{a^2\left(a+bx^2\right)^{11/2}}{11b^3}-\frac{2a\left(a+bx^2\right)^{13/2}}{13b^3}+\frac{\left(a+bx^2\right)^{15/2}}{15b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{11/2}\left(8a^2-44abx^2+143b^2x^4\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(8*a^2 - 44*a*b*x^2 + 143*b^2*x^4))/(2145*b^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^2\right)^{11/2}\left(8a^2-44abx^2+143b^2x^4\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^(9/2),x]
```

```
[Out] ((a + b*x^2)^(11/2)*(8*a^2 - 44*a*b*x^2 + 143*b^2*x^4))/(2145*b^3)
```

fricas [A] time = 0.97, size = 90, normalized size = 1.53

$$\frac{\left(143\,b^7x^{14}+671\,ab^6x^{12}+1218\,a^2b^5x^{10}+1030\,a^3b^4x^8+355\,a^4b^3x^6+3\,a^5b^2x^4-4\,a^6bx^2+8\,a^7\right)\sqrt{bx^2+a}}{2145\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/2145*(143*b^7*x^14 + 671*a*b^6*x^12 + 1218*a^2*b^5*x^10 + 1030*a^3*b^4*x^
8 + 355*a^4*b^3*x^6 + 3*a^5*b^2*x^4 - 4*a^6*b*x^2 + 8*a^7)*sqrt(b*x^2 + a)/
b^3
```

giac [A] time = 0.96, size = 43, normalized size = 0.73

$$\frac{143\left(bx^2+a\right)^{\frac{15}{2}}-330\left(bx^2+a\right)^{\frac{13}{2}}a+195\left(bx^2+a\right)^{\frac{11}{2}}a^2}{2145\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/2145*(143*(b*x^2 + a)^(15/2) - 330*(b*x^2 + a)^(13/2)*a + 195*(b*x^2 + a)
^(11/2)*a^2)/b^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(143b^{2}x^{4}-44ab\,x^{2}+8a^{2}\right)}{2145b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/2145*(b*x^2+a)^(11/2)*(143*b^2*x^4-44*a*b*x^2+8*a^2)/b^3
```

maxima [A] time = 1.39, size = 53, normalized size = 0.90

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}x^{4}}{15\,b}-\frac{4\left(bx^{2}+a\right)^{\frac{11}{2}}ax^{2}}{195\,b^{2}}+\frac{8\left(bx^{2}+a\right)^{\frac{11}{2}}a^{2}}{2145\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/15*(b*x^2 + a)^(11/2)*x^4/b - 4/195*(b*x^2 + a)^(11/2)*a*x^2/b^2 + 8/2145
*(b*x^2 + a)^(11/2)*a^2/b^3
```

mupad [B] time = 4.60, size = 86, normalized size = 1.46

$$\sqrt{b\,x^{2}+a}\,\left(\frac{8\,a^{7}}{2145\,b^{3}}+\frac{71\,a^{4}\,x^{6}}{429}+\frac{b^{4}\,x^{14}}{15}+\frac{206\,a^{3}\,b\,x^{8}}{429}+\frac{61\,a\,b^{3}\,x^{12}}{195}+\frac{a^{5}\,x^{4}}{715\,b}-\frac{4\,a^{6}\,x^{2}}{2145\,b^{2}}+\frac{406\,a^{2}\,b^{2}\,x^{10}}{715}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^7)/(2145*b^3) + (71*a^4*x^6)/429 + (b^4*x^14)/15 +
(206*a^3*b*x^8)/429 + (61*a*b^3*x^12)/195 + (a^5*x^4)/(715*b) - (4*a^6*x^2)
/(2145*b^2) + (406*a^2*b^2*x^10)/715)
```

sympy [A] time = 27.02, size = 180, normalized size = 3.05

$$\begin{cases} \frac{8a^{7}\sqrt{a+bx^{2}}}{2145b^{3}}-\frac{4a^{6}x^{2}\sqrt{a+bx^{2}}}{2145b^{2}}+\frac{a^{5}x^{4}\sqrt{a+bx^{2}}}{715b}+\frac{71a^{4}x^{6}\sqrt{a+bx^{2}}}{429}+\frac{206a^{3}bx^{8}\sqrt{a+bx^{2}}}{429}+\frac{406a^{2}b^{2}x^{10}\sqrt{a+bx^{2}}}{715}+\frac{61ab^{3}x^{12}\sqrt{a+bx^{2}}}{195}+\frac{b^{4}x^{14}\sqrt{a+bx^{2}}}{15} & \text{for}\ b\neq 0\\ \frac{a^{\frac{9}{2}}x^{6}}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((8*a**7*sqrt(a + b*x**2)/(2145*b**3) - 4*a**6*x**2*sqrt(a + b*x**
2)/(2145*b**2) + a**5*x**4*sqrt(a + b*x**2)/(715*b) + 71*a**4*x**6*sqrt(a +
 b*x**2)/429 + 206*a**3*b*x**8*sqrt(a + b*x**2)/429 + 406*a**2*b**2*x**10*s
qrt(a + b*x**2)/715 + 61*a*b**3*x**12*sqrt(a + b*x**2)/195 + b**4*x**14*sqr
t(a + b*x**2)/15, Ne(b, 0)), (a**(9/2)*x**6/6, True))
```

3.406 $\int x^3 \left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^2\right)^{13/2}}{13b^2}-\frac{a\left(a+bx^2\right)^{11/2}}{11b^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{13/2}}{13b^2}-\frac{a\left(a+bx^2\right)^{11/2}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(9/2),x]
```

```
[Out] -(a*(a + b*x^2)^(11/2))/(11*b^2) + (a + b*x^2)^(13/2)/(13*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right)^{9/2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x(a+bx)^{9/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^{9/2}}{b}+\frac{(a+bx)^{11/2}}{b}\right)dx,x,x^2\right)\\ &= -\frac{a\left(a+bx^2\right)^{11/2}}{11b^2}+\frac{\left(a+bx^2\right)^{13/2}}{13b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{11/2}\left(11bx^2-2a\right)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(9/2),x]
[Out] ((a + b*x^2)^(11/2)*(-2*a + 11*b*x^2))/(143*b^2)
```

IntegrateAlgebraic [B] time = 0.03, size = 82, normalized size = 2.16

$$\frac{\sqrt{a+bx^2}\left(-2a^6+a^5bx^2+35a^4b^2x^4+90a^3b^3x^6+100a^2b^4x^8+53ab^5x^{10}+11b^6x^{12}\right)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^(9/2),x]
[Out] (Sqrt[a + b*x^2]*(-2*a^6 + a^5*b*x^2 + 35*a^4*b^2*x^4 + 90*a^3*b^3*x^6 + 10
0*a^2*b^4*x^8 + 53*a*b^5*x^10 + 11*b^6*x^12))/(143*b^2)
```

fricas [B] time = 1.31, size = 78, normalized size = 2.05

$$\frac{\left(11\,b^6x^{12}+53\,ab^5x^{10}+100\,a^2b^4x^8+90\,a^3b^3x^6+35\,a^4b^2x^4+a^5bx^2-2\,a^6\right)\sqrt{bx^2+a}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="fricas")
[Out] 1/143*(11*b^6*x^12 + 53*a*b^5*x^10 + 100*a^2*b^4*x^8 + 90*a^3*b^3*x^6 + 35*
a^4*b^2*x^4 + a^5*b*x^2 - 2*a^6)*sqrt(b*x^2 + a)/b^2
```

giac [A] time = 1.06, size = 29, normalized size = 0.76

$$\frac{11\left(bx^2+a\right)^{\frac{13}{2}}-13\left(bx^2+a\right)^{\frac{11}{2}}a}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="giac")
[Out] 1/143*(11*(b*x^2 + a)^(13/2) - 13*(b*x^2 + a)^(11/2)*a)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-11b\,x^{2}+2a\right)}{143b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(9/2),x)
[Out] -1/143*(b*x^2+a)^(11/2)*(-11*b*x^2+2*a)/b^2
```

maxima [A] time = 1.29, size = 33, normalized size = 0.87

$$\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}x^{2}}{13\,b}-\frac{2\left(bx^{2}+a\right)^{\frac{11}{2}}a}{143\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="maxima")
[Out] 1/13*(b*x^2 + a)^(11/2)*x^2/b - 2/143*(b*x^2 + a)^(11/2)*a/b^2
```

mupad [B] time = 4.74, size = 29, normalized size = 0.76

$$-\frac{13\,a\left(b\,x^{2}+a\right)^{11/2}-11\left(b\,x^{2}+a\right)^{13/2}}{143\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(9/2),x)
[Out] -(13*a*(a + b*x^2)^(11/2) - 11*(a + b*x^2)^(13/2))/(143*b^2)
```

sympy [A] time = 18.50, size = 156, normalized size = 4.11

$$\begin{cases} -\frac{2a^{6}\sqrt{a+bx^{2}}}{143b^{2}}+\frac{a^{5}x^{2}\sqrt{a+bx^{2}}}{143b}+\frac{35a^{4}x^{4}\sqrt{a+bx^{2}}}{143}+\frac{90a^{3}bx^{6}\sqrt{a+bx^{2}}}{143}+\frac{100a^{2}b^{2}x^{8}\sqrt{a+bx^{2}}}{143}+\frac{53ab^{3}x^{10}\sqrt{a+bx^{2}}}{143}+\frac{b^{4}x^{12}\sqrt{a+bx^{2}}}{13} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{9}{2}}x^{4}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(9/2),x)
[Out] Piecewise((-2*a**6*sqrt(a + b*x**2)/(143*b**2) + a**5*x**2*sqrt(a + b*x**2)
/(143*b) + 35*a**4*x**4*sqrt(a + b*x**2)/143 + 90*a**3*b*x**6*sqrt(a + b*x*
*2)/143 + 100*a**2*b**2*x**8*sqrt(a + b*x**2)/143 + 53*a*b**3*x**10*sqrt(a
+ b*x**2)/143 + b**4*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(9/2)*x**4/4
, True))
```

3.407 $\int x\left(a+bx^2\right)^{9/2}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(9/2),x]
```

```
[Out] (a + b*x^2)^(11/2)/(11*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{9/2}\,dx=\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(9/2),x]
```

```
[Out] (a + b*x^2)^(11/2)/(11*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^(9/2),x]
```

```
[Out] (a + b*x^2)^(11/2)/(11*b)
```

fricas [B] time = 0.94, size = 65, normalized size = 3.61

$$\frac{\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{bx^2+a}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/11*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^
2 + a^5)*sqrt(b*x^2 + a)/b
```

giac [A] time = 1.19, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/11*(b*x^2 + a)^(11/2)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/11*(b*x^2+a)^(11/2)/b
```

maxima [A] time = 1.32, size = 14, normalized size = 0.78

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(11/2)/b
```

mupad [B] time = 4.81, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^2+a\right)^{11/2}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(11/2)/(11*b)
```

sympy [A] time = 12.96, size = 133, normalized size = 7.39

$$\begin{cases} \frac{a^5\sqrt{a+bx^2}}{11b} + \frac{5a^4x^2\sqrt{a+bx^2}}{11} + \frac{10a^3bx^4\sqrt{a+bx^2}}{11} + \frac{10a^2b^2x^6\sqrt{a+bx^2}}{11} + \frac{5ab^3x^8\sqrt{a+bx^2}}{11} + \frac{b^4x^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((a**5*sqrt(a + b*x**2)/(11*b) + 5*a**4*x**2*sqrt(a + b*x**2)/11 +
 10*a**3*b*x**4*sqrt(a + b*x**2)/11 + 10*a**2*b**2*x**6*sqrt(a + b*x**2)/11
 + 5*a*b**3*x**8*sqrt(a + b*x**2)/11 + b**4*x**10*sqrt(a + b*x**2)/11, Ne(b
, 0)), (a**(9/2)*x**2/2, True))
```

3.408 $\int \frac{(a+bx^2)^{9/2}}{x} dx$

Optimal. Leaf size=108

$$a^{9/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+a^4\sqrt{a+bx^2}+\frac{1}{3}a^3\left(a+bx^2\right)^{3/2}+\frac{1}{5}a^2\left(a+bx^2\right)^{5/2}+\frac{1}{7}a\left(a+bx^2\right)^{7/2}+\frac{1}{9}\left(a+bx^2\right)^{9/2}$$

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 208}

$$a^4\sqrt{a+bx^2}+\frac{1}{3}a^3\left(a+bx^2\right)^{3/2}+\frac{1}{5}a^2\left(a+bx^2\right)^{5/2}+a^{9/2}\left(-\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{7}a\left(a+bx^2\right)^{7/2}+\frac{1}{9}\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x,x]
```

```
[Out] a^4*Sqrt[a + b*x^2] + (a^3*(a + b*x^2)^(3/2))/3 + (a^2*(a + b*x^2)^(5/2))/5
 + (a*(a + b*x^2)^(7/2))/7 + (a + b*x^2)^(9/2)/9 - a^(9/2)*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x}\,dx,x,x^2\right)$$

$$= \frac{1}{9}\left(a+bx^2\right)^{9/2} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx,x,x^2\right)$$

$$= \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx,x,x^2\right)$$

$$= \frac{1}{5}a^2\left(a+bx^2\right)^{5/2} + \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} + \frac{1}{2}a^3\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)$$

$$= \frac{1}{3}a^3\left(a+bx^2\right)^{3/2} + \frac{1}{5}a^2\left(a+bx^2\right)^{5/2} + \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} + \frac{1}{2}a^4\,\text{Subst}\left(\int \frac{\sqrt{a}}{}\right.$$

$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3\left(a+bx^2\right)^{3/2} + \frac{1}{5}a^2\left(a+bx^2\right)^{5/2} + \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} + \frac{1}{2}a^5$$

$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3\left(a+bx^2\right)^{3/2} + \frac{1}{5}a^2\left(a+bx^2\right)^{5/2} + \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} + \frac{a^5\,\text{S}}{}$$

$$= a^4\sqrt{a+bx^2} + \frac{1}{3}a^3\left(a+bx^2\right)^{3/2} + \frac{1}{5}a^2\left(a+bx^2\right)^{5/2} + \frac{1}{7}a\left(a+bx^2\right)^{7/2} + \frac{1}{9}\left(a+bx^2\right)^{9/2} - a^{9/2}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 0.78

$$\frac{1}{315}\sqrt{a+bx^2}\left(563a^4+506a^3bx^2+408a^2b^2x^4+185ab^3x^6+35b^4x^8\right) - a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^2*b^2*x^4 + 185*a*b^3*x^6
 + 35*b^4*x^8))/315 - a^(9/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.05, size = 84, normalized size = 0.78

$$\frac{1}{315}\sqrt{a+bx^2}\left(563a^4+506a^3bx^2+408a^2b^2x^4+185ab^3x^6+35b^4x^8\right)-a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^2*b^2*x^4 + 185*a*b^3*x^6
 + 35*b^4*x^8))/315 - a^(9/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

fricas [A] time = 1.08, size = 170, normalized size = 1.57

$$\left[\frac{1}{2}a^{\frac{9}{2}}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+\frac{1}{315}\left(35b^4x^8+185ab^3x^6+408a^2b^2x^4+506a^3bx^2+563a^4\right)\sqrt{bx^2+a},\ \sqrt{-a}a^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\frac{1}{315}\left(35b^4x^8+185ab^3x^6+408a^2b^2x^4+506a^3bx^2+563a^4\right)\sqrt{bx^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*a^(9/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/315*(3
5*b^4*x^8 + 185*a*b^3*x^6 + 408*a^2*b^2*x^4 + 506*a^3*b*x^2 + 563*a^4)*sqrt
(b*x^2 + a), sqrt(-a)*a^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 1/315*(35*b^4*
x^8 + 185*a*b^3*x^6 + 408*a^2*b^2*x^4 + 506*a^3*b*x^2 + 563*a^4)*sqrt(b*x^2
 + a)]
```

giac [A] time = 1.17, size = 90, normalized size = 0.83

$$\frac{a^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{9}\left(bx^2+a\right)^{\frac{9}{2}}+\frac{1}{7}\left(bx^2+a\right)^{\frac{7}{2}}a+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}a^2+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a^3+\sqrt{bx^2+a}\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="giac")
```

```
[Out] a^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/9*(b*x^2 + a)^(9/2) + 1/7
*(b*x^2 + a)^(7/2)*a + 1/5*(b*x^2 + a)^(5/2)*a^2 + 1/3*(b*x^2 + a)^(3/2)*a^
3 + sqrt(b*x^2 + a)*a^4
```

maple [A] time = 0.00, size = 94, normalized size = 0.87

$$-a^{\frac{9}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^2+a}\,a^4+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}a^3}{3}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}a^2}{5}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}a}{7}+\frac{\left(b\,x^2+a\right)^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x,x)
```

```
[Out] 1/9*(b*x^2+a)^(9/2)+1/7*a*(b*x^2+a)^(7/2)+1/5*a^2*(b*x^2+a)^(5/2)+1/3*a^3*(
b*x^2+a)^(3/2)-a^(9/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+a^4*(b*x^2+a)^
(1/2)
```

maxima [A] time = 1.40, size = 82, normalized size = 0.76

$$-a^{\frac{9}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{9}\left(bx^2+a\right)^{\frac{9}{2}}+\frac{1}{7}\left(bx^2+a\right)^{\frac{7}{2}}a+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}a^2+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a^3+\sqrt{bx^2+a}\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(9/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/9*(b*x^2 + a)^(9/2) + 1/7*(b*x^2
 + a)^(7/2)*a + 1/5*(b*x^2 + a)^(5/2)*a^2 + 1/3*(b*x^2 + a)^(3/2)*a^3 + sqr
t(b*x^2 + a)*a^4
```

mupad [B] time = 5.31, size = 87, normalized size = 0.81

$$\frac{a\left(b\,x^2+a\right)^{7/2}}{7}+\frac{\left(b\,x^2+a\right)^{9/2}}{9}+a^4\,\sqrt{b\,x^2+a}+\frac{a^3\left(b\,x^2+a\right)^{3/2}}{3}+\frac{a^2\left(b\,x^2+a\right)^{5/2}}{5}+a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x,x)
```

```
[Out] a^(9/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (a*(a + b*x^2)^(7/2))/7 +
 (a + b*x^2)^(9/2)/9 + a^4*(a + b*x^2)^(1/2) + (a^3*(a + b*x^2)^(3/2))/3 +
(a^2*(a + b*x^2)^(5/2))/5
```

sympy [A] time = 10.48, size = 160, normalized size = 1.48

$$\frac{563a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}}{315}+\frac{a^{\frac{9}{2}}\log\left(\frac{bx^2}{a}\right)}{2}-a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)+\frac{506a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}{315}+\frac{136a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{105}+\frac{37a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}{63}+\frac{\sqrt{a}\,b^4x^8\sqrt{1+\frac{bx^2}{a}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x,x)
```

```
[Out] 563*a**(9/2)*sqrt(1 + b*x**2/a)/315 + a**(9/2)*log(b*x**2/a)/2 - a**(9/2)*l
og(sqrt(1 + b*x**2/a) + 1) + 506*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)/315 + 1
36*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)/105 + 37*a**(3/2)*b**3*x**6*sqrt(1
 + b*x**2/a)/63 + sqrt(a)*b**4*x**8*sqrt(1 + b*x**2/a)/9
```

3.409 $\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx$

Optimal. Leaf size=118

$$-\frac{9}{2}a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{9}{2}a^3b\sqrt{a+bx^2}+\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}+\frac{9}{10}ab\left(a+bx^2\right.$$

Rubi [A] time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$\frac{3}{2}a^2b\left(a+bx^2\right)^{3/2}+\frac{9}{2}a^3b\sqrt{a+bx^2}-\frac{9}{2}a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{2x^2}+\frac{9}{14}b\left(a+bx^2\right)^{7/2}+\frac{9}{10}ab\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^3,x]
```

```
[Out] (9*a^3*b*Sqrt[a + b*x^2])/2 + (3*a^2*b*(a + b*x^2)^(3/2))/2 + (9*a*b*(a + b
*x^2)^(5/2))/10 + (9*b*(a + b*x^2)^(7/2))/14 - (a + b*x^2)^(9/2)/(2*x^2) -
(9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^2}\,dx,x,x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{9/2}}{2x^2} + \frac{1}{4}(9b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx,x,x^2\right)$$

$$= \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} + \frac{1}{4}(9ab)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx,x,x^2\right)$$

$$= \frac{9}{10}ab\left(a+bx^2\right)^{5/2} + \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} + \frac{1}{4}\left(9a^2b\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x\right.$$

$$= \frac{3}{2}a^2b\left(a+bx^2\right)^{3/2} + \frac{9}{10}ab\left(a+bx^2\right)^{5/2} + \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} + \frac{1}{4}\left(9a^3b\right)\text{Subst}\Bigg($$

$$= \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b\left(a+bx^2\right)^{3/2} + \frac{9}{10}ab\left(a+bx^2\right)^{5/2} + \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} +$$

$$= \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b\left(a+bx^2\right)^{3/2} + \frac{9}{10}ab\left(a+bx^2\right)^{5/2} + \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} +$$

$$= \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b\left(a+bx^2\right)^{3/2} + \frac{9}{10}ab\left(a+bx^2\right)^{5/2} + \frac{9}{14}b\left(a+bx^2\right)^{7/2} - \frac{\left(a+bx^2\right)^{9/2}}{2x^2} -$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.31

$$\frac{b\left(a+bx^2\right)^{11/2}\, {}_2F_1\left(2,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, 1 + (b*x^2)/a])/(11*
a^2)
```

IntegrateAlgebraic [A] time = 0.10, size = 90, normalized size = 0.76

$$\frac{\sqrt{a+bx^2}\left(-35a^4+388a^3bx^2+156a^2b^2x^4+58ab^3x^6+10b^4x^8\right)}{70x^2}-\frac{9}{2}a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-35*a^4 + 388*a^3*b*x^2 + 156*a^2*b^2*x^4 + 58*a*b^3*x^6
+ 10*b^4*x^8))/(70*x^2) - (9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

fricas [A] time = 1.22, size = 188, normalized size = 1.59

$$\left[\frac{315\,a^{\frac{7}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,\left(10\,b^4x^8+58\,ab^3x^6+156\,a^2b^2x^4+388\,a^3bx^2-35\,a^4\right)\sqrt{bx^2+a}}{140\,x^2},\ \frac{315\,\sqrt{-a}\,a^3bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(10\,b^4x^8+58\,ab^3x^6+156\,a^2b^2x^4+388\,a^3bx^2-35\,a^4\right)\sqrt{bx^2+a}}{70\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*a^(7/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2 - 35*a^
4)*sqrt(b*x^2 + a))/x^2, 1/70*(315*sqrt(-a)*a^3*b*x^2*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) + (10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2
- 35*a^4)*sqrt(b*x^2 + a))/x^2]
```

giac [A] time = 1.14, size = 116, normalized size = 0.98

$$\frac{\frac{315\,a^4b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+10\,\left(bx^2+a\right)^{\frac{7}{2}}b^2+28\,\left(bx^2+a\right)^{\frac{5}{2}}ab^2+70\,\left(bx^2+a\right)^{\frac{3}{2}}a^2b^2+280\,\sqrt{bx^2+a}\,a^3b^2-\frac{35\,\sqrt{bx^2+a}\,a^4b}{x^2}}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/70*(315*a^4*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 10*(b*x^2 + a
)^(7/2)*b^2 + 28*(b*x^2 + a)^(5/2)*a*b^2 + 70*(b*x^2 + a)^(3/2)*a^2*b^2 + 2
80*sqrt(b*x^2 + a)*a^3*b^2 - 35*sqrt(b*x^2 + a)*a^4*b/x^2)/b
```

maple [A] time = 0.01, size = 118, normalized size = 1.00

$$-\frac{9a^{\frac{7}{2}}b\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2}+\frac{9\sqrt{b\,x^{2}+a}\,a^{3}b}{2}+\frac{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}{2}+\frac{9\left(b\,x^{2}+a\right)^{\frac{5}{2}}ab}{10}+\frac{9\left(b\,x^{2}+a\right)^{\frac{7}{2}}b}{14}+\frac{\left(b\,x^{2}+a\right)^{\frac{9}{2}}b}{2a}-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(b*x^2+a)^(11/2)+1/2/a*b*(b*x^2+a)^(9/2)+9/14*b*(b*x^2+a)^(7/2)+
9/10*a*b*(b*x^2+a)^(5/2)+3/2*a^2*b*(b*x^2+a)^(3/2)-9/2*a^(7/2)*b*ln((2*a+2*
(b*x^2+a)^(1/2)*a^(1/2))/x)+9/2*a^3*b*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.43, size = 106, normalized size = 0.90

$$-\frac{9}{2}a^{\frac{7}{2}}b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{9}{14}\left(bx^{2}+a\right)^{\frac{7}{2}}b+\frac{\left(bx^{2}+a\right)^{\frac{9}{2}}b}{2a}+\frac{9}{10}\left(bx^{2}+a\right)^{\frac{5}{2}}ab+\frac{3}{2}\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}b+\frac{9}{2}\sqrt{bx^{2}+a}\,a^{3}b-\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}}{2ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="maxima")
```

```
[Out] -9/2*a^(7/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 9/14*(b*x^2 + a)^(7/2)*b + 1
/2*(b*x^2 + a)^(9/2)*b/a + 9/10*(b*x^2 + a)^(5/2)*a*b + 3/2*(b*x^2 + a)^(3/
2)*a^2*b + 9/2*sqrt(b*x^2 + a)*a^3*b - 1/2*(b*x^2 + a)^(11/2)/(a*x^2)
```

mupad [B] time = 5.45, size = 95, normalized size = 0.81

$$\frac{b\left(b\,x^{2}+a\right)^{7/2}}{7}+4\,a^{3}\,b\,\sqrt{b\,x^{2}+a}+a^{2}\,b\left(b\,x^{2}+a\right)^{3/2}-\frac{a^{4}\,\sqrt{b\,x^{2}+a}}{2\,x^{2}}+\frac{2\,a\,b\left(b\,x^{2}+a\right)^{5/2}}{5}+\frac{a^{7/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1\mathrm{i}}{\sqrt{a}}\right)9\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^3,x)
```

```
[Out] (b*(a + b*x^2)^(7/2))/7 + 4*a^3*b*(a + b*x^2)^(1/2) + a^2*b*(a + b*x^2)^(3/
2) - (a^4*(a + b*x^2)^(1/2))/(2*x^2) + (a^(7/2)*b*atan(((a + b*x^2)^(1/2)*1
i)/a^(1/2))*9i)/2 + (2*a*b*(a + b*x^2)^(5/2))/5
```

sympy [A] time = 9.27, size = 167, normalized size = 1.42

$$-\frac{a^{\frac{9}{2}}\sqrt{1+\frac{bx^{2}}{a}}}{2x^{2}}+\frac{194a^{\frac{7}{2}}b\sqrt{1+\frac{bx^{2}}{a}}}{35}+\frac{9a^{\frac{7}{2}}b\log\left(\frac{bx^{2}}{a}\right)}{4}-\frac{9a^{\frac{7}{2}}b\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{2}+\frac{78a^{\frac{5}{2}}b^{2}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}{35}+\frac{29a^{\frac{3}{2}}b^{3}x^{4}\sqrt{1+\frac{bx^{2}}{a}}}{35}+\frac{\sqrt{a}\,b^{4}x^{6}\sqrt{1+\frac{bx^{2}}{a}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**3,x)
```

```
[Out] -a**(9/2)*sqrt(1 + b*x**2/a)/(2*x**2) + 194*a**(7/2)*b*sqrt(1 + b*x**2/a)/3
5 + 9*a**(7/2)*b*log(b*x**2/a)/4 - 9*a**(7/2)*b*log(sqrt(1 + b*x**2/a) + 1)
/2 + 78*a**(5/2)*b**2*x**2*sqrt(1 + b*x**2/a)/35 + 29*a**(3/2)*b**3*x**4*sq
rt(1 + b*x**2/a)/35 + sqrt(a)*b**4*x**6*sqrt(1 + b*x**2/a)/7
```

3.410 $\int \frac{(a+bx^2)^{9/2}}{x^5}\,dx$

Optimal. Leaf size=126

$$-\frac{63}{8}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{63}{8}a^2b^2\sqrt{a+bx^2}+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}-\frac{\left(a+b\right.}{4x}$$

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$\frac{63}{8}a^2b^2\sqrt{a+bx^2}-\frac{63}{8}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{63}{40}b^2\left(a+bx^2\right)^{5/2}+\frac{21}{8}ab^2\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{4x^4}-\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^5,x]
```

```
[Out] (63*a^2*b^2*Sqrt[a + b*x^2])/8 + (21*a*b^2*(a + b*x^2)^(3/2))/8 + (63*b^2*(
a + b*x^2)^(5/2))/40 - (9*b*(a + b*x^2)^(7/2))/(8*x^2) - (a + b*x^2)^(9/2)/
(4*x^4) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^3}\,dx,x,x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{9/2}}{4x^4} + \frac{1}{8}(9b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^2}\,dx,x,x^2\right)$$

$$= -\frac{9b\left(a+bx^2\right)^{7/2}}{8x^2} - \frac{\left(a+bx^2\right)^{9/2}}{4x^4} + \frac{1}{16}\left(63b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx,x,x^2\right)$$

$$= \frac{63}{40}b^2\left(a+bx^2\right)^{5/2} - \frac{9b\left(a+bx^2\right)^{7/2}}{8x^2} - \frac{\left(a+bx^2\right)^{9/2}}{4x^4} + \frac{1}{16}\left(63ab^2\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,\right.$$

$$= \frac{21}{8}ab^2\left(a+bx^2\right)^{3/2} + \frac{63}{40}b^2\left(a+bx^2\right)^{5/2} - \frac{9b\left(a+bx^2\right)^{7/2}}{8x^2} - \frac{\left(a+bx^2\right)^{9/2}}{4x^4} + \frac{1}{16}\left(63a^2b^2\right)\text{Sub}$$

$$= \frac{63}{8}a^2b^2\sqrt{a+bx^2} + \frac{21}{8}ab^2\left(a+bx^2\right)^{3/2} + \frac{63}{40}b^2\left(a+bx^2\right)^{5/2} - \frac{9b\left(a+bx^2\right)^{7/2}}{8x^2} - \frac{\left(a+bx^2\right)^{9/}}{4x^4}$$

$$= \frac{63}{8}a^2b^2\sqrt{a+bx^2} + \frac{21}{8}ab^2\left(a+bx^2\right)^{3/2} + \frac{63}{40}b^2\left(a+bx^2\right)^{5/2} - \frac{9b\left(a+bx^2\right)^{7/2}}{8x^2} - \frac{\left(a+bx^2\right)^{9/}}{4x^4}$$

$$= \frac{63}{8}a^2b^2\sqrt{a+bx^2} + \frac{21}{8}ab^2\left(a+bx^2\right)^{3/2} + \frac{63}{40}b^2\left(a+bx^2\right)^{5/2} - \frac{9b\left(a+bx^2\right)^{7/2}}{8x^2} - \frac{\left(a+bx^2\right)^{9/}}{4x^4}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.31

$$-\frac{b^2\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(3,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^5,x]
```

```
[Out] -1/11*(b^2*(a + b*x^2)^(11/2)*Hypergeometric2F1[3, 11/2, 13/2, 1 + (b*x^2)/
a])/a^3
```

IntegrateAlgebraic [A] time = 0.11, size = 92, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(-10a^4-85a^3bx^2+288a^2b^2x^4+56ab^3x^6+8b^4x^8\right)}{40x^4}-\frac{63}{8}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^5,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-10*a^4 - 85*a^3*b*x^2 + 288*a^2*b^2*x^4 + 56*a*b^3*x^6 +
 8*b^4*x^8))/(40*x^4) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

fricas [A] time = 1.00, size = 192, normalized size = 1.52

$$\left[\frac{315\,a^{\frac{5}{2}}b^2x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(8\,b^4x^8+56\,ab^3x^6+288\,a^2b^2x^4-85\,a^3bx^2-10\,a^4\right)\sqrt{bx^2+a}}{80\,x^4},\ \frac{315\,\sqrt{-a}\,a^2b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(8\,b^4x^8+56\,ab^3x^6+288\,a^2b^2x^4-85\,a^3bx^2-10\,a^4\right)\sqrt{bx^2+a}}{40\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/80*(315*a^(5/2)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(8*b^4*x^8 + 56*a*b^3*x^6 + 288*a^2*b^2*x^4 - 85*a^3*b*x^2 - 10*a^4
)*sqrt(b*x^2 + a))/x^4, 1/40*(315*sqrt(-a)*a^2*b^2*x^4*arctan(sqrt(-a)/sqrt
(b*x^2 + a)) + (8*b^4*x^8 + 56*a*b^3*x^6 + 288*a^2*b^2*x^4 - 85*a^3*b*x^2 -
 10*a^4)*sqrt(b*x^2 + a))/x^4]
```

giac [A] time = 1.15, size = 124, normalized size = 0.98

$$\frac{\frac{315\,a^3b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\left(bx^2+a\right)^{\frac{5}{2}}b^3+40\left(bx^2+a\right)^{\frac{3}{2}}ab^3+240\,\sqrt{bx^2+a}\,a^2b^3-\frac{5\left(17\left(bx^2+a\right)^{\frac{3}{2}}a^3b^3-15\,\sqrt{bx^2+a}\,a^4b^3\right)}{b^2x^4}}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/40*(315*a^3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 8*(b*x^2 + a)
^(5/2)*b^3 + 40*(b*x^2 + a)^(3/2)*a*b^3 + 240*sqrt(b*x^2 + a)*a^2*b^3 - 5*(
17*(b*x^2 + a)^(3/2)*a^3*b^3 - 15*sqrt(b*x^2 + a)*a^4*b^3)/(b^2*x^4))/b
```

maple [A] time = 0.01, size = 148, normalized size = 1.17

$$-\frac{63 a^{\frac{5}{2}} b^{2} \ln\left(\frac{2a+2\sqrt{b x^{2}+a}\,\sqrt{a}}{x}\right)}{8}+\frac{63\sqrt{b x^{2}+a}\,a^{2} b^{2}}{8}+\frac{21\left(b x^{2}+a\right)^{\frac{3}{2}} a b^{2}}{8}+\frac{63\left(b x^{2}+a\right)^{\frac{5}{2}} b^{2}}{40}+\frac{9\left(b x^{2}+a\right)^{\frac{7}{2}} b^{2}}{8a}+\frac{7\left(b x^{2}+a\right)^{\frac{9}{2}} b^{2}}{8a^{2}}-\frac{7\left(b x^{2}+a\right)^{\frac{11}{2}} b}{8a^{2} x^{2}}-\frac{\left(b x^{2}+a\right)^{\frac{11}{2}}}{4a x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(b*x^2+a)^(11/2)-7/8/a^2*b/x^2*(b*x^2+a)^(11/2)+7/8/a^2*b^2*(b*x
^2+a)^(9/2)+9/8/a*b^2*(b*x^2+a)^(7/2)+63/40*b^2*(b*x^2+a)^(5/2)+21/8*a*b^2*
(b*x^2+a)^(3/2)-63/8*a^(5/2)*b^2*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+63/8
*a^2*b^2*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.46, size = 136, normalized size = 1.08

$$-\frac{63}{8} a^{\frac{5}{2}} b^{2} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{63}{40}\left(bx^{2}+a\right)^{\frac{5}{2}} b^{2}+\frac{7\left(bx^{2}+a\right)^{\frac{9}{2}} b^{2}}{8a^{2}}+\frac{9\left(bx^{2}+a\right)^{\frac{7}{2}} b^{2}}{8a}+\frac{21}{8}\left(bx^{2}+a\right)^{\frac{3}{2}} ab^{2}+\frac{63}{8}\sqrt{bx^{2}+a}\,a^{2} b^{2}-\frac{7\left(bx^{2}+a\right)^{\frac{11}{2}} b}{8a^{2} x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}}{4ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="maxima")
```

```
[Out] -63/8*a^(5/2)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) + 63/40*(b*x^2 + a)^(5/2)*b
^2 + 7/8*(b*x^2 + a)^(9/2)*b^2/a^2 + 9/8*(b*x^2 + a)^(7/2)*b^2/a + 21/8*(b*
x^2 + a)^(3/2)*a*b^2 + 63/8*sqrt(b*x^2 + a)*a^2*b^2 - 7/8*(b*x^2 + a)^(11/2
)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(11/2)/(a*x^4)
```

mupad [B] time = 5.65, size = 132, normalized size = 1.05

$$\frac{\frac{15\,a^{4}\,b^{2}\,\sqrt{b\,x^{2}+a}}{8}-\frac{17\,a^{3}\,b^{2}\,{\left(b\,x^{2}+a\right)}^{3/2}}{8}}{{\left(b\,x^{2}+a\right)}^{2}-2\,a\,\left(b\,x^{2}+a\right)+a^{2}}+\frac{b^{2}\,{\left(b\,x^{2}+a\right)}^{5/2}}{5}+a\,b^{2}\,{\left(b\,x^{2}+a\right)}^{3/2}+6\,a^{2}\,b^{2}\,\sqrt{b\,x^{2}+a}+\frac{a^{5/2}\,b^{2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,63{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^5,x)
```

```
[Out] ((15*a^4*b^2*(a + b*x^2)^(1/2))/8 - (17*a^3*b^2*(a + b*x^2)^(3/2))/8)/((a +
 b*x^2)^2 - 2*a*(a + b*x^2) + a^2) + (b^2*(a + b*x^2)^(5/2))/5 + (a^(5/2)*b
^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*63i)/8 + a*b^2*(a + b*x^2)^(3/2) +
6*a^2*b^2*(a + b*x^2)^(1/2)
```

sympy [A] time = 8.73, size = 175, normalized size = 1.39

$$-\frac{63a^{\frac{5}{2}}b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8}-\frac{a^{5}}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{19a^{4}\sqrt{b}}{8x^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{203a^{3}b^{\frac{3}{2}}}{40x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{43a^{2}b^{\frac{5}{2}}x}{5\sqrt{\frac{a}{bx^{2}}+1}}+\frac{8ab^{\frac{7}{2}}x^{3}}{5\sqrt{\frac{a}{bx^{2}}+1}}+\frac{b^{\frac{9}{2}}x^{5}}{5\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**5,x)
```

```
[Out] -63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**5/(4*sqrt(b)*x**5*sqrt(
a/(b*x**2) + 1)) - 19*a**4*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) + 203*a**3
*b**(3/2)/(40*x*sqrt(a/(b*x**2) + 1)) + 43*a**2*b**(5/2)*x/(5*sqrt(a/(b*x**
2) + 1)) + 8*a*b**(7/2)*x**3/(5*sqrt(a/(b*x**2) + 1)) + b**(9/2)*x**5/(5*sq
rt(a/(b*x**2) + 1))
```

3.411 $\int \frac{(a+bx^2)^{9/2}}{x^7}\,dx$

Optimal. Leaf size=126

$$-\frac{105}{16}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}+\frac{105}{16}ab^3\sqrt{a+bx^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}-\frac{3b\left(a+\right.}{8x}$$

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{105}{16}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}+\frac{105}{16}ab^3\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^7,x]
```

```
[Out] (105*a*b^3*Sqrt[a + b*x^2])/16 + (35*b^3*(a + b*x^2)^(3/2))/16 - (21*b^2*(a
 + b*x^2)^(5/2))/(16*x^2) - (3*b*(a + b*x^2)^(7/2))/(8*x^4) - (a + b*x^2)^(
9/2)/(6*x^6) - (105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/16
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{9/2}}{6x^6}+\frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}+\frac{1}{16}\left(21b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}+\frac{1}{32}\left(105b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x\right.\\
&= \frac{35}{16}b^3\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}+\frac{1}{32}\left(105ab^3\right)\text{Subst}\\
&= \frac{105}{16}ab^3\sqrt{a+bx^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}+\\
&= \frac{105}{16}ab^3\sqrt{a+bx^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}+\\
&= \frac{105}{16}ab^3\sqrt{a+bx^2}+\frac{35}{16}b^3\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{16x^2}-\frac{3b\left(a+bx^2\right)^{7/2}}{8x^4}-\frac{\left(a+bx^2\right)^{9/2}}{6x^6}-
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.31

$$\frac{b^3 \left(a+bx^2\right)^{11/2} {}_2F_1\left(4,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^7,x]
```

```
[Out] (b^3*(a + b*x^2)^(11/2)*Hypergeometric2F1[4, 11/2, 13/2, 1 + (b*x^2)/a])/(1
1*a^4)
```

IntegrateAlgebraic [A] time = 0.13, size = 92, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(-8a^4-50a^3bx^2-165a^2b^2x^4+208ab^3x^6+16b^4x^8\right)}{48x^6}-\frac{105}{16}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^4 - 50*a^3*b*x^2 - 165*a^2*b^2*x^4 + 208*a*b^3*x^6 +
 16*b^4*x^8))/(48*x^6) - (105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/16
```

fricas [A] time = 1.04, size = 190, normalized size = 1.51

$$\left[\frac{315\,a^{\frac{3}{2}}b^3x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(16\,b^4x^8+208\,ab^3x^6-165\,a^2b^2x^4-50\,a^3bx^2-8\,a^4\right)\sqrt{bx^2+a}}{96\,x^6},\ \frac{315\,\sqrt{-a}\,ab^3x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(16\,b^4x^8+208\,ab^3x^6-165\,a^2b^2x^4-50\,a^3bx^2-8\,a^4\right)\sqrt{bx^2+a}}{48\,x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(315*a^(3/2)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(16*b^4*x^8 + 208*a*b^3*x^6 - 165*a^2*b^2*x^4 - 50*a^3*b*x^2 - 8*a^
4)*sqrt(b*x^2 + a))/x^6, 1/48*(315*sqrt(-a)*a*b^3*x^6*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) + (16*b^4*x^8 + 208*a*b^3*x^6 - 165*a^2*b^2*x^4 - 50*a^3*b*x^2
- 8*a^4)*sqrt(b*x^2 + a))/x^6]
```

giac [A] time = 1.10, size = 124, normalized size = 0.98

$$\frac{\frac{315\,a^2b^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+16\left(bx^2+a\right)^{\frac{3}{2}}b^4+192\,\sqrt{bx^2+a}\,ab^4-\frac{165\left(bx^2+a\right)^{\frac{5}{2}}a^2b^4-280\left(bx^2+a\right)^{\frac{3}{2}}a^3b^4+123\,\sqrt{bx^2+a}\,a^4b^4}{b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(315*a^2*b^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 16*(b*x^2 + a
)^(3/2)*b^4 + 192*sqrt(b*x^2 + a)*a*b^4 - (165*(b*x^2 + a)^(5/2)*a^2*b^4 -
280*(b*x^2 + a)^(3/2)*a^3*b^4 + 123*sqrt(b*x^2 + a)*a^4*b^4)/(b^3*x^6))/b
```

maple [A] time = 0.01, size = 168, normalized size = 1.33

$$-\frac{105a^{\frac{3}{2}}b^3\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{16}+\frac{105\sqrt{b\,x^2+a}\,a\,b^3}{16}+\frac{35\left(b\,x^2+a\right)^{\frac{3}{2}}b^3}{16}+\frac{21\left(b\,x^2+a\right)^{\frac{5}{2}}b^3}{16a}+\frac{15\left(b\,x^2+a\right)^{\frac{7}{2}}b^3}{16a^2}+\frac{35\left(b\,x^2+a\right)^{\frac{9}{2}}b^3}{48a^3}-\frac{35\left(b\,x^2+a\right)^{\frac{11}{2}}b^2}{48a^3x^2}-\frac{5\left(b\,x^2+a\right)^{\frac{11}{2}}b}{24a^2x^4}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{6a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^7,x)
```

```
[Out] -1/6/a/x^6*(b*x^2+a)^(11/2)-5/24/a^2*b/x^4*(b*x^2+a)^(11/2)-35/48/a^3*b^2/x
^2*(b*x^2+a)^(11/2)+35/48/a^3*b^3*(b*x^2+a)^(9/2)+15/16/a^2*b^3*(b*x^2+a)^(
7/2)+21/16/a*b^3*(b*x^2+a)^(5/2)+35/16*b^3*(b*x^2+a)^(3/2)-105/16*a^(3/2)*b
^3*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+105/16*a*b^3*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.47, size = 156, normalized size = 1.24

$$-\frac{105}{16}\,a^{\frac{3}{2}}b^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{35}{16}\left(bx^2+a\right)^{\frac{3}{2}}b^3+\frac{35\left(bx^2+a\right)^{\frac{9}{2}}b^3}{48\,a^3}+\frac{15\left(bx^2+a\right)^{\frac{7}{2}}b^3}{16\,a^2}+\frac{21\left(bx^2+a\right)^{\frac{5}{2}}b^3}{16\,a}+\frac{105}{16}\,\sqrt{bx^2+a}\,ab^3-\frac{35\left(bx^2+a\right)^{\frac{11}{2}}b^2}{48\,a^3x^2}-\frac{5\left(bx^2+a\right)^{\frac{11}{2}}b}{24\,a^2x^4}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="maxima")
```

```
[Out] -105/16*a^(3/2)*b^3*arcsinh(a/(sqrt(a*b)*abs(x))) + 35/16*(b*x^2 + a)^(3/2)
*b^3 + 35/48*(b*x^2 + a)^(9/2)*b^3/a^3 + 15/16*(b*x^2 + a)^(7/2)*b^3/a^2 +
21/16*(b*x^2 + a)^(5/2)*b^3/a + 105/16*sqrt(b*x^2 + a)*a*b^3 - 35/48*(b*x^2
 + a)^(11/2)*b^2/(a^3*x^2) - 5/24*(b*x^2 + a)^(11/2)*b/(a^2*x^4) - 1/6*(b*x
^2 + a)^(11/2)/(a*x^6)
```

mupad [B] time = 5.99, size = 149, normalized size = 1.18

$$\frac{\frac{41\,a^4\,b^3\,\sqrt{b\,x^2+a}}{16}-\frac{35\,a^3\,b^3\,{\left(b\,x^2+a\right)}^{3/2}}{6}+\frac{55\,a^2\,b^3\,{\left(b\,x^2+a\right)}^{5/2}}{16}}{3\,a\,{\left(b\,x^2+a\right)}^2-3\,a^2\,\left(b\,x^2+a\right)-{\left(b\,x^2+a\right)}^3+a^3}+\frac{b^3\,{\left(b\,x^2+a\right)}^{3/2}}{3}+4\,a\,b^3\,\sqrt{b\,x^2+a}+\frac{a^{3/2}\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,105{}\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^7,x)
```

```
[Out] ((41*a^4*b^3*(a + b*x^2)^(1/2))/16 - (35*a^3*b^3*(a + b*x^2)^(3/2))/6 + (55
*a^2*b^3*(a + b*x^2)^(5/2))/16)/(3*a*(a + b*x^2)^2 - 3*a^2*(a + b*x^2) - (a
```

```
 + b*x^2)^3 + a^3) + (b^3*(a + b*x^2)^(3/2))/3 + (a^(3/2)*b^3*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*105i)/16 + 4*a*b^3*(a + b*x^2)^(1/2)
```

sympy [A] time = 7.12, size = 175, normalized size = 1.39

$$-\frac{105a^{\frac{3}{2}}b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16}-\frac{a^5}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{29a^4\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{215a^3b^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{43a^2b^{\frac{5}{2}}}{48x\sqrt{\frac{a}{bx^2}+1}}+\frac{14ab^{\frac{7}{2}}x}{3\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{9}{2}}x^3}{3\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**7,x)
```

```
[Out] -105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*x))/16 - a**5/(6*sqrt(b)*x**7*sqr
t(a/(b*x**2) + 1)) - 29*a**4*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1)) - 215*a
**3*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) + 43*a**2*b**(5/2)/(48*x*sqrt(a
/(b*x**2) + 1)) + 14*a*b**(7/2)*x/(3*sqrt(a/(b*x**2) + 1)) + b**(9/2)*x**3/
(3*sqrt(a/(b*x**2) + 1))
```

3.412 $\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx$

Optimal. Leaf size=128

$$\frac{315}{128}b^4\sqrt{a+bx^2}-\frac{315}{128}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{105b^3(a+bx^2)^{3/2}}{128x^2}-\frac{21b^2(a+bx^2)^{5/2}}{64x^4}-\frac{(a+bx^2)^{9/2}}{8x^8}-\frac{3b(a+bx^2}{16x^6}$$

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 50, 63, 208}

$$-\frac{21b^2(a+bx^2)^{5/2}}{64x^4}-\frac{105b^3(a+bx^2)^{3/2}}{128x^2}+\frac{315}{128}b^4\sqrt{a+bx^2}-\frac{315}{128}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{(a+bx^2)^{9/2}}{8x^8}-\frac{3b(a+bx^2)^{7/2}}{16x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^9,x]
```

```
[Out] (315*b^4*Sqrt[a + b*x^2])/128 - (105*b^3*(a + b*x^2)^(3/2))/(128*x^2) - (21
*b^2*(a + b*x^2)^(5/2))/(64*x^4) - (3*b*(a + b*x^2)^(7/2))/(16*x^6) - (a +
b*x^2)^(9/2)/(8*x^8) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/1
28
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{9/2}}{8x^8} + \frac{1}{16}(9b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{3b\left(a+bx^2\right)^{7/2}}{16x^6} - \frac{\left(a+bx^2\right)^{9/2}}{8x^8} + \frac{1}{32}\left(21b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4} - \frac{3b\left(a+bx^2\right)^{7/2}}{16x^6} - \frac{\left(a+bx^2\right)^{9/2}}{8x^8} + \frac{1}{128}\left(105b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx,\right.$$

$$= -\frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4} - \frac{3b\left(a+bx^2\right)^{7/2}}{16x^6} - \frac{\left(a+bx^2\right)^{9/2}}{8x^8} + \frac{1}{256}\left(315b^4\right)\text{Sub}$$

$$= \frac{315}{128}b^4\sqrt{a+bx^2} - \frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4} - \frac{3b\left(a+bx^2\right)^{7/2}}{16x^6} - \frac{\left(a+bx^2\right)^{9/2}}{8x^8} +$$

$$= \frac{315}{128}b^4\sqrt{a+bx^2} - \frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4} - \frac{3b\left(a+bx^2\right)^{7/2}}{16x^6} - \frac{\left(a+bx^2\right)^{9/2}}{8x^8} +$$

$$= \frac{315}{128}b^4\sqrt{a+bx^2} - \frac{105b^3\left(a+bx^2\right)^{3/2}}{128x^2} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{64x^4} - \frac{3b\left(a+bx^2\right)^{7/2}}{16x^6} - \frac{\left(a+bx^2\right)^{9/2}}{8x^8} -$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.30

$$-\frac{b^4\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(5,\frac{11}{2};\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^9,x]
```

```
[Out] -1/11*(b^4*(a + b*x^2)^(11/2)*Hypergeometric2F1[5, 11/2, 13/2, 1 + (b*x^2)/
a])/a^5
```

IntegrateAlgebraic [A] time = 0.15, size = 92, normalized size = 0.72

$$\frac{\sqrt{a+bx^2}\left(-16a^4-88a^3bx^2-210a^2b^2x^4-325ab^3x^6+128b^4x^8\right)}{128x^8} - \frac{315}{128}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^4 - 88*a^3*b*x^2 - 210*a^2*b^2*x^4 - 325*a*b^3*x^6
+ 128*b^4*x^8))/(128*x^8) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a
]])/128
```

fricas [A] time = 1.34, size = 189, normalized size = 1.48

$$\left[\frac{315\,\sqrt{a}\,b^{4}x^{8}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\left(128\,b^{4}x^{8}-325\,ab^{3}x^{6}-210\,a^{2}b^{2}x^{4}-88\,a^{3}bx^{2}-16\,a^{4}\right)\sqrt{bx^{2}+a}}{256\,x^{8}},\ \frac{315\,\sqrt{-a}\,b^{4}x^{8}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(128\,b^{4}x^{8}-325\,ab^{3}x^{6}-210\,a^{2}b^{2}x^{4}-88\,a^{3}bx^{2}-16\,a^{4}\right)\sqrt{bx^{2}+a}}{128\,x^{8}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/256*(315*sqrt(a)*b^4*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) + 2*(128*b^4*x^8 - 325*a*b^3*x^6 - 210*a^2*b^2*x^4 - 88*a^3*b*x^2 - 16
*a^4)*sqrt(b*x^2 + a))/x^8, 1/128*(315*sqrt(-a)*b^4*x^8*arctan(sqrt(-a)/sqr
t(b*x^2 + a)) + (128*b^4*x^8 - 325*a*b^3*x^6 - 210*a^2*b^2*x^4 - 88*a^3*b*x
^2 - 16*a^4)*sqrt(b*x^2 + a))/x^8]
```

giac [A] time = 1.13, size = 122, normalized size = 0.95

$$\frac{\frac{315\,ab^{5}\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+128\,\sqrt{bx^{2}+a}\,b^{5}-\frac{325\left(bx^{2}+a\right)^{\frac{7}{2}}ab^{5}-765\left(bx^{2}+a\right)^{\frac{5}{2}}a^{2}b^{5}+643\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}b^{5}-187\,\sqrt{bx^{2}+a}\,a^{4}b^{5}}{b^{4}x^{8}}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/128*(315*a*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x^2
 + a)*b^5 - (325*(b*x^2 + a)^(7/2)*a*b^5 - 765*(b*x^2 + a)^(5/2)*a^2*b^5 +
643*(b*x^2 + a)^(3/2)*a^3*b^5 - 187*sqrt(b*x^2 + a)*a^4*b^5)/(b^4*x^8))/b
```

maple [A] time = 0.02, size = 190, normalized size = 1.48

$$-\frac{315\sqrt{a}\,b^{4}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{128}+\frac{315\sqrt{b\,x^{2}+a}\,b^{4}}{128}+\frac{105\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{4}}{128a}+\frac{63\left(b\,x^{2}+a\right)^{\frac{5}{2}}b^{4}}{128a^{2}}+\frac{45\left(b\,x^{2}+a\right)^{\frac{7}{2}}b^{4}}{128a^{3}}+\frac{35\left(b\,x^{2}+a\right)^{\frac{9}{2}}b^{4}}{128a^{4}}-\frac{35\left(b\,x^{2}+a\right)^{\frac{11}{2}}b^{3}}{128a^{4}x^{2}}-\frac{5\left(b\,x^{2}+a\right)^{\frac{11}{2}}b^{2}}{64a^{3}x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}b}{16a^{2}x^{6}}-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{8a\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^9,x)
```

```
[Out] -1/8/a/x^8*(b*x^2+a)^(11/2)-1/16/a^2*b/x^6*(b*x^2+a)^(11/2)-5/64/a^3*b^2/x^
4*(b*x^2+a)^(11/2)-35/128/a^4*b^3/x^2*(b*x^2+a)^(11/2)+35/128/a^4*b^4*(b*x^
```

```
2+a)^(9/2)+45/128/a^3*b^4*(b*x^2+a)^(7/2)+63/128/a^2*b^4*(b*x^2+a)^(5/2)+10
5/128/a*b^4*(b*x^2+a)^(3/2)-315/128*a^(1/2)*b^4*ln((2*a+2*(b*x^2+a)^(1/2)*a
^(1/2))/x)+315/128*b^4*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.45, size = 178, normalized size = 1.39

$$-\frac{315}{128}\,\sqrt{a}\,b^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{315}{128}\,\sqrt{bx^2+a}\,b^4+\frac{35\,(bx^2+a)^{\frac{9}{2}}b^4}{128\,a^4}+\frac{45\,(bx^2+a)^{\frac{7}{2}}b^4}{128\,a^3}+\frac{63\,(bx^2+a)^{\frac{5}{2}}b^4}{128\,a^2}+\frac{105\,(bx^2+a)^{\frac{3}{2}}b^4}{128\,a}-\frac{35\,(bx^2+a)^{\frac{11}{2}}b^3}{128\,a^4x^2}-\frac{5\,(bx^2+a)^{\frac{11}{2}}b^2}{64\,a^3x^4}-\frac{(bx^2+a)^{\frac{11}{2}}b}{16\,a^2x^6}-\frac{(bx^2+a)^{\frac{11}{2}}}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="maxima")
```

```
[Out] -315/128*sqrt(a)*b^4*arcsinh(a/(sqrt(a*b)*abs(x))) + 315/128*sqrt(b*x^2 + a
)*b^4 + 35/128*(b*x^2 + a)^(9/2)*b^4/a^4 + 45/128*(b*x^2 + a)^(7/2)*b^4/a^3
 + 63/128*(b*x^2 + a)^(5/2)*b^4/a^2 + 105/128*(b*x^2 + a)^(3/2)*b^4/a - 35/
128*(b*x^2 + a)^(11/2)*b^3/(a^4*x^2) - 5/64*(b*x^2 + a)^(11/2)*b^2/(a^3*x^4
) - 1/16*(b*x^2 + a)^(11/2)*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(11/2)/(a*x^8)
```

mupad [B] time = 6.20, size = 105, normalized size = 0.82

$$b^4\,\sqrt{b\,x^2+a}-\frac{325\,a\,{\left(b\,x^2+a\right)}^{7/2}}{128\,x^8}+\frac{187\,a^4\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{643\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^8}+\frac{765\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}{128\,x^8}+\frac{\sqrt{a}\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,315\mathrm{i}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^9,x)
```

```
[Out] b^4*(a + b*x^2)^(1/2) + (a^(1/2)*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3
15i)/128 - (325*a*(a + b*x^2)^(7/2))/(128*x^8) + (187*a^4*(a + b*x^2)^(1/2)
)/(128*x^8) - (643*a^3*(a + b*x^2)^(3/2))/(128*x^8) + (765*a^2*(a + b*x^2)^
(5/2))/(128*x^8)
```

sympy [A] time = 7.74, size = 173, normalized size = 1.35

$$-\frac{315\sqrt{a}\,b^4\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128}-\frac{a^5}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{13a^4\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{149a^3b^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{535a^2b^{\frac{5}{2}}}{128x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{197ab^{\frac{7}{2}}}{128x\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{9}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**9,x)
```

```
[Out] -315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*x))/128 - a**5/(8*sqrt(b)*x**9*sqr
t(a/(b*x**2) + 1)) - 13*a**4*sqrt(b)/(16*x**7*sqrt(a/(b*x**2) + 1)) - 149*a
**3*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) - 535*a**2*b**(5/2)/(128*x**3*s
qrt(a/(b*x**2) + 1)) - 197*a*b**(7/2)/(128*x*sqrt(a/(b*x**2) + 1)) + b**(9/
2)*x/sqrt(a/(b*x**2) + 1)
```

3.413 $\int \frac{(a+bx^2)^{9/2}}{x^{11}} \, dx$

Optimal. Leaf size=131

$$-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}-\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}$$

Rubi [A] time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 208}

$$-\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^11,x]
```

```
[Out] (-63*b^4*Sqrt[a + b*x^2])/(256*x^2) - (21*b^3*(a + b*x^2)^(3/2))/(128*x^4)
- (21*b^2*(a + b*x^2)^(5/2))/(160*x^6) - (9*b*(a + b*x^2)^(7/2))/(80*x^8) -
 (a + b*x^2)^(9/2)/(10*x^10) - (63*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2
56*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^6}\,dx,x,x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}+\frac{1}{20}(9b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^5}\,dx,x,x^2\right)$$

$$= -\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}+\frac{1}{160}\left(63b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx,x,x^2\right)$$

$$= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}+\frac{1}{64}\left(21b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx,x,\right.$$

$$= -\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}+\frac{1}{256}\left(63b^4\right)\text{Subst}$$

$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}+$$

$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}+$$

$$= -\frac{63b^4\sqrt{a+bx^2}}{256x^2}-\frac{21b^3\left(a+bx^2\right)^{3/2}}{128x^4}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{160x^6}-\frac{9b\left(a+bx^2\right)^{7/2}}{80x^8}-\frac{\left(a+bx^2\right)^{9/2}}{10x^{10}}-$$

Mathematica [A] time = 0.05, size = 109, normalized size = 0.83

$$-\frac{128a^5+784a^4bx^2+2024a^3b^2x^4+2858a^2b^3x^6+315b^5x^{10}\sqrt{\frac{bx^2}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)+2455ab^4x^8+965b^5x^{10}}{1280x^{10}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^11,x]
```

```
[Out] -1/1280*(128*a^5 + 784*a^4*b*x^2 + 2024*a^3*b^2*x^4 + 2858*a^2*b^3*x^6 + 24
55*a*b^4*x^8 + 965*b^5*x^10 + 315*b^5*x^10*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt
[1 + (b*x^2)/a]])/(x^10*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.18, size = 92, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\left(-128a^4-656a^3bx^2-1368a^2b^2x^4-1490ab^3x^6-965b^4x^8\right)}{1280x^{10}}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-128*a^4 - 656*a^3*b*x^2 - 1368*a^2*b^2*x^4 - 1490*a*b^3*
x^6 - 965*b^4*x^8))/(1280*x^10) - (63*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/(256*Sqrt[a])
```

fricas [A] time = 1.13, size = 202, normalized size = 1.54

$$\left[\frac{315\sqrt{a}b^5x^{10}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(965ab^4x^8+1490a^2b^3x^6+1368a^3b^2x^4+656a^4bx^2+128a^5\right)\sqrt{bx^2+a}}{2560ax^{10}}, \frac{315\sqrt{-a}b^5x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(965ab^4x^8+1490a^2b^3x^6+1368a^3b^2x^4+656a^4bx^2+128a^5\right)\sqrt{bx^2+a}}{1280ax^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/2560*(315*sqrt(a)*b^5*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) - 2*(965*a*b^4*x^8 + 1490*a^2*b^3*x^6 + 1368*a^3*b^2*x^4 + 656*a^4*b
*x^2 + 128*a^5)*sqrt(b*x^2 + a))/(a*x^10), 1/1280*(315*sqrt(-a)*b^5*x^10*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) - (965*a*b^4*x^8 + 1490*a^2*b^3*x^6 + 1368*a
^3*b^2*x^4 + 656*a^4*b*x^2 + 128*a^5)*sqrt(b*x^2 + a))/(a*x^10)]
```

giac [A] time = 1.09, size = 121, normalized size = 0.92

$$\frac{\frac{315\,b^6\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{965\left(bx^2+a\right)^{\frac{9}{2}}b^6-2370\left(bx^2+a\right)^{\frac{7}{2}}ab^6+2688\left(bx^2+a\right)^{\frac{5}{2}}a^2b^6-1470\left(bx^2+a\right)^{\frac{3}{2}}a^3b^6+315\sqrt{bx^2+a}\,a^4b^6}{b^5x^{10}}}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*(315*b^6*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (965*(b*x^2 + a
)^(9/2)*b^6 - 2370*(b*x^2 + a)^(7/2)*a*b^6 + 2688*(b*x^2 + a)^(5/2)*a^2*b^6
 - 1470*(b*x^2 + a)^(3/2)*a^3*b^6 + 315*sqrt(b*x^2 + a)*a^4*b^6)/(b^5*x^10)
)/b
```

maple [B] time = 0.05, size = 213, normalized size = 1.63

$$-\frac{63b^5\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{256\sqrt{a}}+\frac{63\sqrt{b\,x^2+a}\,b^5}{256a}+\frac{21\left(b\,x^2+a\right)^{\frac{3}{2}}b^5}{256a^2}+\frac{63\left(b\,x^2+a\right)^{\frac{5}{2}}b^5}{1280a^3}+\frac{9\left(b\,x^2+a\right)^{\frac{7}{2}}b^5}{256a^4}+\frac{7\left(b\,x^2+a\right)^{\frac{9}{2}}b^5}{256a^5}-\frac{7\left(b\,x^2+a\right)^{\frac{11}{2}}b^4}{256a^5x^2}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}b^3}{128a^4x^4}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}b^2}{160a^3x^6}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}b}{80a^2x^8}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{10a\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^11,x)
```

```
[Out] -1/10/a/x^10*(b*x^2+a)^(11/2)-1/80/a^2*b/x^8*(b*x^2+a)^(11/2)-1/160/a^3*b^2
/x^6*(b*x^2+a)^(11/2)-1/128/a^4*b^3/x^4*(b*x^2+a)^(11/2)-7/256/a^5*b^4/x^2*
(b*x^2+a)^(11/2)+7/256/a^5*b^5*(b*x^2+a)^(9/2)+9/256/a^4*b^5*(b*x^2+a)^(7/2
)+63/1280/a^3*b^5*(b*x^2+a)^(5/2)+21/256/a^2*b^5*(b*x^2+a)^(3/2)-63/256/a^(
1/2)*b^5*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+63/256/a*b^5*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.48, size = 201, normalized size = 1.53

$$-\frac{63\,b^5\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\,\sqrt{a}}+\frac{7\left(bx^2+a\right)^{\frac{9}{2}}b^5}{256\,a^5}+\frac{9\left(bx^2+a\right)^{\frac{7}{2}}b^5}{256\,a^4}+\frac{63\left(bx^2+a\right)^{\frac{5}{2}}b^5}{1280\,a^3}+\frac{21\left(bx^2+a\right)^{\frac{3}{2}}b^5}{256\,a^2}+\frac{63\,\sqrt{bx^2+a}\,b^5}{256\,a}-\frac{7\left(bx^2+a\right)^{\frac{11}{2}}b^4}{256\,a^5x^2}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^3}{128\,a^4x^4}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^2}{160\,a^3x^6}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b}{80\,a^2x^8}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="maxima")
```

```
[Out] -63/256*b^5*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 7/256*(b*x^2 + a)^(9/2)
*b^5/a^5 + 9/256*(b*x^2 + a)^(7/2)*b^5/a^4 + 63/1280*(b*x^2 + a)^(5/2)*b^5/
a^3 + 21/256*(b*x^2 + a)^(3/2)*b^5/a^2 + 63/256*sqrt(b*x^2 + a)*b^5/a - 7/2
56*(b*x^2 + a)^(11/2)*b^4/(a^5*x^2) - 1/128*(b*x^2 + a)^(11/2)*b^3/(a^4*x^4
) - 1/160*(b*x^2 + a)^(11/2)*b^2/(a^3*x^6) - 1/80*(b*x^2 + a)^(11/2)*b/(a^2
*x^8) - 1/10*(b*x^2 + a)^(11/2)/(a*x^10)
```

mupad [B] time = 6.62, size = 106, normalized size = 0.81

$$\frac{237\,a\,{\left(b\,x^2+a\right)}^{7/2}}{128\,x^{10}}-\frac{193\,{\left(b\,x^2+a\right)}^{9/2}}{256\,x^{10}}-\frac{63\,a^4\,\sqrt{b\,x^2+a}}{256\,x^{10}}+\frac{147\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^{10}}-\frac{21\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}{10\,x^{10}}+\frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,63{}\mathrm{i}}{256\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^11,x)
```

```
[Out] (b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*63i)/(256*a^(1/2)) - (193*(a + b*
x^2)^(9/2))/(256*x^10) + (237*a*(a + b*x^2)^(7/2))/(128*x^10) - (63*a^4*(a
```

```
+ b*x^2)^(1/2))/(256*x^10) + (147*a^3*(a + b*x^2)^(3/2))/(128*x^10) - (21*a
^2*(a + b*x^2)^(5/2))/(10*x^10)
```

sympy [A] time = 9.31, size = 153, normalized size = 1.17

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{10x^9}-\frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{80x^7}-\frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{160x^5}-\frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{128x^3}-\frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{256x}-\frac{63b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{256\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**11,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(10*x**9) - 41*a**3*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(80*x**7) - 171*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(160*x**5) - 1
49*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(128*x**3) - 193*b**(9/2)*sqrt(a/(b*x**2
) + 1)/(256*x) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*x))/(256*sqrt(a))
```

3.414 $\int \frac{(a+bx^2)^{9/2}}{x^{13}} \, dx$

Optimal. Leaf size=155

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}-\frac{21b^5\sqrt{a+bx^2}}{1024ax^2}-\frac{21b^4\sqrt{a+bx^2}}{512x^4}-\frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8}-\frac{\left(a+bx^2\right)^{9/2}}{12x^{12}}-\frac{3b\left(a -}{40}$$

Rubi [A] time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}-\frac{21b^5\sqrt{a+bx^2}}{1024ax^2}-\frac{21b^4\sqrt{a+bx^2}}{512x^4}-\frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8}-\frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}}-\frac{\left(a+bx^2\right)^{9/2}}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^13,x]
```

```
[Out] (-21*b^4*Sqrt[a + b*x^2])/(512*x^4) - (21*b^5*Sqrt[a + b*x^2])/(1024*a*x^2)
 - (7*b^3*(a + b*x^2)^(3/2))/(128*x^6) - (21*b^2*(a + b*x^2)^(5/2))/(320*x^
8) - (3*b*(a + b*x^2)^(7/2))/(40*x^10) - (a + b*x^2)^(9/2)/(12*x^12) + (21*
b^6*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(1024*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{13}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^7}\,dx,x,x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{9/2}}{12x^{12}} + \frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^6}\,dx,x,x^2\right)$$
$$= -\frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{\left(a+bx^2\right)^{9/2}}{12x^{12}} + \frac{1}{80}\left(21b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx,x,x^2\right)$$
$$= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{\left(a+bx^2\right)^{9/2}}{12x^{12}} + \frac{1}{128}\left(21b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx,x\right.$$
$$= -\frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{\left(a+bx^2\right)^{9/2}}{12x^{12}} + \frac{1}{256}\left(21b^4\right)\text{Subst}$$
$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} - \frac{\left(a+bx^2\right)^{9/2}}{12x^{12}} +$$
$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} -$$
$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} -$$
$$= -\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{7b^3\left(a+bx^2\right)^{3/2}}{128x^6} - \frac{21b^2\left(a+bx^2\right)^{5/2}}{320x^8} - \frac{3b\left(a+bx^2\right)^{7/2}}{40x^{10}} -$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.25

$$-\frac{b^6\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(\frac{11}{2},7;\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^13,x]
```

```
[Out] -1/11*(b^6*(a + b*x^2)^(11/2)*Hypergeometric2F1[11/2, 7, 13/2, 1 + (b*x^2)/
a])/a^7
```

IntegrateAlgebraic [A] time = 0.18, size = 106, normalized size = 0.68

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}+\frac{\sqrt{a+bx^2}\,\left(-1280a^5-6272a^4bx^2-12144a^3b^2x^4-11432a^2b^3x^6-4910ab^4x^8-315b^5x^{10}\right)}{15360ax^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^13,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5 - 6272*a^4*b*x^2 - 12144*a^3*b^2*x^4 - 11432*a^
2*b^3*x^6 - 4910*a*b^4*x^8 - 315*b^5*x^10))/(15360*a*x^12) + (21*b^6*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/(1024*a^(3/2))
```

fricas [A] time = 1.25, size = 223, normalized size = 1.44

$$\left[\frac{315\,\sqrt{a}b^6x^{12}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,\left(315\,ab^5x^{10}+4910\,a^2b^4x^8+11432\,a^3b^3x^6+12144\,a^4b^2x^4+6272\,a^5bx^2+1280\,a^6\right)\sqrt{bx^2+a}}{30720\,a^2x^{12}},\ -\frac{315\,\sqrt{-a}\,b^6x^{12}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(315\,ab^5x^{10}+4910\,a^2b^4x^8+11432\,a^3b^3x^6+12144\,a^4b^2x^4+6272\,a^5bx^2+1280\,a^6\right)\sqrt{bx^2+a}}{15360\,a^2x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/30720*(315*sqrt(a)*b^6*x^12*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) - 2*(315*a*b^5*x^10 + 4910*a^2*b^4*x^8 + 11432*a^3*b^3*x^6 + 12144*
a^4*b^2*x^4 + 6272*a^5*b*x^2 + 1280*a^6)*sqrt(b*x^2 + a))/(a^2*x^12), -1/15
360*(315*sqrt(-a)*b^6*x^12*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (315*a*b^5*x^
10 + 4910*a^2*b^4*x^8 + 11432*a^3*b^3*x^6 + 12144*a^4*b^2*x^4 + 6272*a^5*b*
x^2 + 1280*a^6)*sqrt(b*x^2 + a))/(a^2*x^12)]
```

giac [A] time = 1.10, size = 143, normalized size = 0.92

$$-\frac{\frac{315\,b^7\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{315\,\left(bx^2+a\right)^{\frac{11}{2}}b^7+3335\,\left(bx^2+a\right)^{\frac{9}{2}}ab^7-5058\,\left(bx^2+a\right)^{\frac{7}{2}}a^2b^7+4158\,\left(bx^2+a\right)^{\frac{5}{2}}a^3b^7-1785\,\left(bx^2+a\right)^{\frac{3}{2}}a^4b^7+315\,\sqrt{bx^2+a}\,a^5b^7}{ab^6x^{12}}}{15360\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/15360*(315*b^7*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*x
^2 + a)^(11/2)*b^7 + 3335*(b*x^2 + a)^(9/2)*a*b^7 - 5058*(b*x^2 + a)^(7/2)*
a^2*b^7 + 4158*(b*x^2 + a)^(5/2)*a^3*b^7 - 1785*(b*x^2 + a)^(3/2)*a^4*b^7 +
 315*sqrt(b*x^2 + a)*a^5*b^7)/(a*b^6*x^12))/b
```

maple [A] time = 0.13, size = 233, normalized size = 1.50

$$\frac{21b^6\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{1024a^{\frac{3}{2}}}-\frac{21\sqrt{b\,x^2+a}\,b^6}{1024a^2}-\frac{7\left(b\,x^2+a\right)^{\frac{3}{2}}b^6}{1024a^3}-\frac{21\left(b\,x^2+a\right)^{\frac{5}{2}}b^6}{5120a^4}-\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}b^6}{1024a^5}-\frac{7\left(b\,x^2+a\right)^{\frac{9}{2}}b^6}{3072a^6}+\frac{7\left(b\,x^2+a\right)^{\frac{11}{2}}b^5}{3072a^6x^2}+\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}b^4}{1536a^5x^4}+\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}b^3}{1920a^4x^6}+\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}b^2}{960a^3x^8}+\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}b}{120a^2x^{10}}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{12a\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^13,x)
```

```
[Out] -1/12/a/x^12*(b*x^2+a)^(11/2)+1/120/a^2*b/x^10*(b*x^2+a)^(11/2)+1/960/a^3*b
^2/x^8*(b*x^2+a)^(11/2)+1/1920/a^4*b^3/x^6*(b*x^2+a)^(11/2)+1/1536/a^5*b^4/
x^4*(b*x^2+a)^(11/2)+7/3072/a^6*b^5/x^2*(b*x^2+a)^(11/2)-7/3072/a^6*b^6*(b*
x^2+a)^(9/2)-3/1024/a^5*b^6*(b*x^2+a)^(7/2)-21/5120/a^4*b^6*(b*x^2+a)^(5/2)
-7/1024/a^3*b^6*(b*x^2+a)^(3/2)+21/1024/a^(3/2)*b^6*ln((2*a+2*(b*x^2+a)^(1/
2)*a^(1/2))/x)-21/1024/a^2*b^6*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.51, size = 221, normalized size = 1.43

$$\frac{21\,b^{6}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{1024\,a^{\frac{3}{2}}} - \frac{7\,\left(bx^{2}+a\right)^{\frac{9}{2}}b^{6}}{3072\,a^{6}} - \frac{3\,\left(bx^{2}+a\right)^{\frac{7}{2}}b^{6}}{1024\,a^{5}} - \frac{21\,\left(bx^{2}+a\right)^{\frac{5}{2}}b^{6}}{5120\,a^{4}} - \frac{7\,\left(bx^{2}+a\right)^{\frac{3}{2}}b^{6}}{1024\,a^{3}} - \frac{21\,\sqrt{bx^{2}+a}\,b^{6}}{1024\,a^{2}} + \frac{7\,\left(bx^{2}+a\right)^{\frac{11}{2}}b^{5}}{3072\,a^{6}x^{2}} + \frac{\left(bx^{2}+a\right)^{\frac{11}{2}}b^{4}}{1536\,a^{5}x^{4}} + \frac{\left(bx^{2}+a\right)^{\frac{11}{2}}b^{3}}{1920\,a^{4}x^{6}} + \frac{\left(bx^{2}+a\right)^{\frac{11}{2}}b^{2}}{960\,a^{3}x^{8}} + \frac{\left(bx^{2}+a\right)^{\frac{11}{2}}b}{120\,a^{2}x^{10}} - \frac{\left(bx^{2}+a\right)^{\frac{11}{2}}}{12\,ax^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="maxima")
```

```
[Out] 21/1024*b^6*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 7/3072*(b*x^2 + a)^(9/2
)*b^6/a^6 - 3/1024*(b*x^2 + a)^(7/2)*b^6/a^5 - 21/5120*(b*x^2 + a)^(5/2)*b^
6/a^4 - 7/1024*(b*x^2 + a)^(3/2)*b^6/a^3 - 21/1024*sqrt(b*x^2 + a)*b^6/a^2
+ 7/3072*(b*x^2 + a)^(11/2)*b^5/(a^6*x^2) + 1/1536*(b*x^2 + a)^(11/2)*b^4/(
a^5*x^4) + 1/1920*(b*x^2 + a)^(11/2)*b^3/(a^4*x^6) + 1/960*(b*x^2 + a)^(11/
2)*b^2/(a^3*x^8) + 1/120*(b*x^2 + a)^(11/2)*b/(a^2*x^10) - 1/12*(b*x^2 + a)
^(11/2)/(a*x^12)
```

mupad [B] time = 6.68, size = 123, normalized size = 0.79

$$\frac{843\,a\,{\left(b\,x^2+a\right)}^{7/2}}{2560\,x^{12}}-\frac{667\,{\left(b\,x^2+a\right)}^{9/2}}{3072\,x^{12}}-\frac{21\,a^4\,\sqrt{b\,x^2+a}}{1024\,x^{12}}+\frac{119\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}{1024\,x^{12}}-\frac{693\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}{2560\,x^{12}}-\frac{21\,{\left(b\,x^2+a\right)}^{11/2}}{1024\,a\,x^{12}}-\frac{b^6\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,21{}\mathrm{i}}{1024\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^13,x)
```

```
[Out] (843*a*(a + b*x^2)^(7/2))/(2560*x^12) - (b^6*atan(((a + b*x^2)^(1/2)*1i)/a^
(1/2))*21i)/(1024*a^(3/2)) - (667*(a + b*x^2)^(9/2))/(3072*x^12) - (21*a^4*
(a + b*x^2)^(1/2))/(1024*x^12) + (119*a^3*(a + b*x^2)^(3/2))/(1024*x^12) -
(693*a^2*(a + b*x^2)^(5/2))/(2560*x^12) - (21*(a + b*x^2)^(11/2))/(1024*a*x
^12)
```

sympy [A] time = 14.98, size = 204, normalized size = 1.32

$$- \frac{a^{5}}{12 \sqrt{b} x^{13} \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{59 a^{4} \sqrt{b}}{120 x^{11} \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{1151 a^{3} b^{\frac{3}{2}}}{960 x^{9} \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{2947 a^{2} b^{\frac{5}{2}}}{1920 x^{7} \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{8171 a b^{\frac{7}{2}}}{7680 x^{5} \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{1045 b^{\frac{9}{2}}}{3072 x^{3} \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{21 b^{\frac{11}{2}}}{1024 a x \sqrt{\frac{a}{b x^{2}} + 1}} + \frac{21 b^{6} \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x} \right)}}{1024 a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**13,x)
```

```
[Out] -a**5/(12*sqrt(b)*x**13*sqrt(a/(b*x**2) + 1)) - 59*a**4*sqrt(b)/(120*x**11*
sqrt(a/(b*x**2) + 1)) - 1151*a**3*b**(3/2)/(960*x**9*sqrt(a/(b*x**2) + 1))
- 2947*a**2*b**(5/2)/(1920*x**7*sqrt(a/(b*x**2) + 1)) - 8171*a*b**(7/2)/(76
80*x**5*sqrt(a/(b*x**2) + 1)) - 1045*b**(9/2)/(3072*x**3*sqrt(a/(b*x**2) +
1)) - 21*b**(11/2)/(1024*a*x*sqrt(a/(b*x**2) + 1)) + 21*b**6*asinh(sqrt(a)/
(sqrt(b)*x))/(1024*a**(3/2))
```

3.415 $\int \frac{\left(a+bx^2\right)^{9/2}}{x^{15}}\,dx$

Optimal. Leaf size=179

$$-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}+\frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2}-\frac{3b^5\sqrt{a+bx^2}}{1024ax^4}-\frac{3b^4\sqrt{a+bx^2}}{256x^6}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}}-\frac{\left(a+bx\right.}{14x^{1}}$$

Rubi [A] time = 0.12, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 208}

$$\frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2}-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}-\frac{3b^5\sqrt{a+bx^2}}{1024ax^4}-\frac{3b^4\sqrt{a+bx^2}}{256x^6}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}}-\frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}}-\frac{\left(a+bx^2\right)^{9/2}}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^15,x]
```

```
[Out] (-3*b^4*Sqrt[a + b*x^2])/(256*x^6) - (3*b^5*Sqrt[a + b*x^2])/(1024*a*x^4) +
 (9*b^6*Sqrt[a + b*x^2])/(2048*a^2*x^2) - (3*b^3*(a + b*x^2)^(3/2))/(128*x^
8) - (3*b^2*(a + b*x^2)^(5/2))/(80*x^10) - (3*b*(a + b*x^2)^(7/2))/(56*x^12
) - (a + b*x^2)^(9/2)/(14*x^14) - (9*b^7*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(2048*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{15}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^8}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{28}(9b)\,\text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x^7}\,dx, x, x^2\right)$$
$$= -\frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{16}\left(3b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^6}\,dx, x, x^2\right)$$
$$= -\frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{32}\left(3b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx, x, x^2\right.$$
$$= -\frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{256}\left(9b^4\right)\text{Subst}\left(\int\right.$$
$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{\left(a+bx^2\right)^{9/2}}{14x^{14}} + \frac{1}{512}$$
$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\left(a+bx^2\right)^{7/2}}{56x^{12}} - \frac{(a}{}$$
$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\,(}{}$$
$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\,(}{}$$
$$= -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^3\left(a+bx^2\right)^{3/2}}{128x^8} - \frac{3b^2\left(a+bx^2\right)^{5/2}}{80x^{10}} - \frac{3b\,(}{}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.22

$$\frac{b^7\left(a+bx^2\right)^{11/2}\,{}_2F_1\left(\frac{11}{2},8;\frac{13}{2};\frac{bx^2}{a}+1\right)}{11a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^15,x]
```

```
[Out] (b^7*(a + b*x^2)^(11/2)*Hypergeometric2F1[11/2, 8, 13/2, 1 + (b*x^2)/a])/(1
1*a^8)
```

IntegrateAlgebraic [A] time = 0.20, size = 117, normalized size = 0.65

$$\frac{\sqrt{a+bx^2}\left(-5120a^6-24320a^5bx^2-44928a^4b^2x^4-39056a^3b^3x^6-14168a^2b^4x^8-210ab^5x^{10}+315b^6x^{12}\right)}{71680a^2x^{14}}-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^15,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-5120*a^6 - 24320*a^5*b*x^2 - 44928*a^4*b^2*x^4 - 39056*a
^3*b^3*x^6 - 14168*a^2*b^4*x^8 - 210*a*b^5*x^10 + 315*b^6*x^12))/(71680*a^2
*x^14) - (9*b^7*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2048*a^(5/2))
```

fricas [A] time = 1.16, size = 245, normalized size = 1.37

$$\left[\frac{315\sqrt{a}b^7x^{14}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(315ab^6x^{12}-210a^2b^5x^{10}-14168a^3b^4x^8-39056a^4b^3x^6-44928a^5b^2x^4-24320a^6bx^2-5120a^7\right)\sqrt{bx^2+a}}{143360a^3x^{14}}, \frac{315\sqrt{-a}b^7x^{14}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(315ab^6x^{12}-210a^2b^5x^{10}-14168a^3b^4x^8-39056a^4b^3x^6-44928a^5b^2x^4-24320a^6bx^2-5120a^7\right)\sqrt{bx^2+a}}{71680a^3x^{14}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="fricas")
```

```
[Out] [1/143360*(315*sqrt(a)*b^7*x^14*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(315*a*b^6*x^12 - 210*a^2*b^5*x^10 - 14168*a^3*b^4*x^8 - 39056
*a^4*b^3*x^6 - 44928*a^5*b^2*x^4 - 24320*a^6*b*x^2 - 5120*a^7)*sqrt(b*x^2 +
 a))/(a^3*x^14), 1/71680*(315*sqrt(-a)*b^7*x^14*arctan(sqrt(-a)/sqrt(b*x^2
+ a)) + (315*a*b^6*x^12 - 210*a^2*b^5*x^10 - 14168*a^3*b^4*x^8 - 39056*a^4*
b^3*x^6 - 44928*a^5*b^2*x^4 - 24320*a^6*b*x^2 - 5120*a^7)*sqrt(b*x^2 + a))/
(a^3*x^14)]
```

giac [A] time = 1.09, size = 160, normalized size = 0.89

$$\frac{\frac{315b^8\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{315\left(bx^2+a\right)^{\frac{13}{2}}b^8-2100\left(bx^2+a\right)^{\frac{11}{2}}ab^8-8393\left(bx^2+a\right)^{\frac{9}{2}}a^2b^8+9216\left(bx^2+a\right)^{\frac{7}{2}}a^3b^8-5943\left(bx^2+a\right)^{\frac{5}{2}}a^4b^8+2100\left(bx^2+a\right)^{\frac{3}{2}}a^5b^8-315\sqrt{bx^2+a}a^6b^8}{a^2b^7x^{14}}}{71680b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="giac")
```

```
[Out] 1/71680*(315*b^8*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b*
x^2 + a)^(13/2)*b^8 - 2100*(b*x^2 + a)^(11/2)*a*b^8 - 8393*(b*x^2 + a)^(9/2
)*a^2*b^8 + 9216*(b*x^2 + a)^(7/2)*a^3*b^8 - 5943*(b*x^2 + a)^(5/2)*a^4*b^8
 + 2100*(b*x^2 + a)^(3/2)*a^5*b^8 - 315*sqrt(b*x^2 + a)*a^6*b^8)/(a^2*b^7*x
^14))/b
```

maple [A] time = 0.34, size = 253, normalized size = 1.41

$$-\frac{9b^7\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{2048a^{\frac{5}{2}}}+\frac{9\sqrt{bx^2+a}\,b^7}{2048a^3}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}b^7}{2048a^4}+\frac{9\left(bx^2+a\right)^{\frac{5}{2}}b^7}{10240a^5}+\frac{9\left(bx^2+a\right)^{\frac{7}{2}}b^7}{14336a^6}+\frac{\left(bx^2+a\right)^{\frac{9}{2}}b^7}{2048a^7}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^6}{2048a^7x^2}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^5}{7168a^6x^4}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^4}{8960a^5x^6}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^3}{4480a^4x^8}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^2}{560a^3x^{10}}+\frac{\left(bx^2+a\right)^{\frac{11}{2}}b}{56a^2x^{12}}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{14a\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^15,x)
```

```
[Out] -1/14/a/x^14*(b*x^2+a)^(11/2)+1/56/a^2*b/x^12*(b*x^2+a)^(11/2)-1/560/a^3*b^
2/x^10*(b*x^2+a)^(11/2)-1/4480/a^4*b^3/x^8*(b*x^2+a)^(11/2)-1/8960/a^5*b^4/
x^6*(b*x^2+a)^(11/2)-1/7168/a^6*b^5/x^4*(b*x^2+a)^(11/2)-1/2048/a^7*b^6/x^2
*(b*x^2+a)^(11/2)+1/2048/a^7*b^7*(b*x^2+a)^(9/2)+9/14336/a^6*b^7*(b*x^2+a)^
(7/2)+9/10240/a^5*b^7*(b*x^2+a)^(5/2)+3/2048/a^4*b^7*(b*x^2+a)^(3/2)-9/2048
/a^(5/2)*b^7*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+9/2048/a^3*b^7*(b*x^2+a)
^(1/2)
```

maxima [A] time = 1.51, size = 241, normalized size = 1.35

$$-\frac{9\,b^7\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2048\,a^{\frac{5}{2}}}+\frac{\left(bx^2+a\right)^{\frac{9}{2}}b^7}{2048\,a^7}+\frac{9\left(bx^2+a\right)^{\frac{7}{2}}b^7}{14336\,a^6}+\frac{9\left(bx^2+a\right)^{\frac{5}{2}}b^7}{10240\,a^5}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}b^7}{2048\,a^4}+\frac{9\sqrt{bx^2+a}\,b^7}{2048\,a^3}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^6}{2048\,a^7x^2}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^5}{7168\,a^6x^4}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^4}{8960\,a^5x^6}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^3}{4480\,a^4x^8}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}b^2}{560\,a^3x^{10}}+\frac{\left(bx^2+a\right)^{\frac{11}{2}}b}{56\,a^2x^{12}}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{14\,ax^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="maxima")
```

```
[Out] -9/2048*b^7*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2048*(b*x^2 + a)^(9/2
)*b^7/a^7 + 9/14336*(b*x^2 + a)^(7/2)*b^7/a^6 + 9/10240*(b*x^2 + a)^(5/2)*b
^7/a^5 + 3/2048*(b*x^2 + a)^(3/2)*b^7/a^4 + 9/2048*sqrt(b*x^2 + a)*b^7/a^3
- 1/2048*(b*x^2 + a)^(11/2)*b^6/(a^7*x^2) - 1/7168*(b*x^2 + a)^(11/2)*b^5/(
a^6*x^4) - 1/8960*(b*x^2 + a)^(11/2)*b^4/(a^5*x^6) - 1/4480*(b*x^2 + a)^(11
/2)*b^3/(a^4*x^8) - 1/560*(b*x^2 + a)^(11/2)*b^2/(a^3*x^10) + 1/56*(b*x^2 +
 a)^(11/2)*b/(a^2*x^12) - 1/14*(b*x^2 + a)^(11/2)/(a*x^14)
```

mupad [B] time = 7.23, size = 140, normalized size = 0.78

$$\frac{9\,a\,{\left(b\,x^2+a\right)}^{7/2}}{70\,x^{14}}-\frac{1199\,{\left(b\,x^2+a\right)}^{9/2}}{10240\,x^{14}}-\frac{9\,a^4\,\sqrt{b\,x^2+a}}{2048\,x^{14}}+\frac{15\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}{512\,x^{14}}-\frac{849\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}{10240\,x^{14}}-\frac{15\,{\left(b\,x^2+a\right)}^{11/2}}{512\,a\,x^{14}}+\frac{9\,{\left(b\,x^2+a\right)}^{13/2}}{2048\,a^2\,x^{14}}+\frac{b^7\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,9{}\mathrm{i}}{2048\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^15,x)
```

```
[Out] (b^7*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*9i)/(2048*a^(5/2)) - (1199*(a + b
*x^2)^(9/2))/(10240*x^14) + (9*a*(a + b*x^2)^(7/2))/(70*x^14) - (9*a^4*(a +
 b*x^2)^(1/2))/(2048*x^14) + (15*a^3*(a + b*x^2)^(3/2))/(512*x^14) - (849*a
^2*(a + b*x^2)^(5/2))/(10240*x^14) - (15*(a + b*x^2)^(11/2))/(512*a*x^14) +
 (9*(a + b*x^2)^(13/2))/(2048*a^2*x^14)
```

sympy [A] time = 21.82, size = 231, normalized size = 1.29

$$-\frac{a^5}{14\sqrt{b}\,x^{15}\sqrt{\frac{a}{bx^2}+1}}-\frac{23a^4\sqrt{b}}{56x^{13}\sqrt{\frac{a}{bx^2}+1}}-\frac{541a^3b^{\frac{3}{2}}}{560x^{11}\sqrt{\frac{a}{bx^2}+1}}-\frac{5249a^2b^{\frac{5}{2}}}{4480x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{6653ab^{\frac{7}{2}}}{8960x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{1027b^{\frac{9}{2}}}{5120x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{11}{2}}}{2048ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{9b^{\frac{13}{2}}}{2048a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{9b^7\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2048a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**15,x)
```

```
[Out] -a**5/(14*sqrt(b)*x**15*sqrt(a/(b*x**2) + 1)) - 23*a**4*sqrt(b)/(56*x**13*s
qrt(a/(b*x**2) + 1)) - 541*a**3*b**(3/2)/(560*x**11*sqrt(a/(b*x**2) + 1)) -
 5249*a**2*b**(5/2)/(4480*x**9*sqrt(a/(b*x**2) + 1)) - 6653*a*b**(7/2)/(896
0*x**7*sqrt(a/(b*x**2) + 1)) - 1027*b**(9/2)/(5120*x**5*sqrt(a/(b*x**2) + 1
)) + 3*b**(11/2)/(2048*a*x**3*sqrt(a/(b*x**2) + 1)) + 9*b**(13/2)/(2048*a**
2*x*sqrt(a/(b*x**2) + 1)) - 9*b**7*asinh(sqrt(a)/(sqrt(b)*x))/(2048*a**(5/2
))
```

3.416 $\int x^6 \left(a+bx^2\right)^{9/2} \, dx$

Optimal. Leaf size=202

$$-\frac{45a^8 \tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}}+\frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx\right.$$

Rubi [A] time = 0.11, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {279, 321, 217, 206}

$$\frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}-\frac{45a^8\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^(9/2),x]
```

```
[Out] (45*a^7*x*Sqrt[a + b*x^2])/(32768*b^3) - (15*a^6*x^3*Sqrt[a + b*x^2])/(1638
4*b^2) + (3*a^5*x^5*Sqrt[a + b*x^2])/(4096*b) + (9*a^4*x^7*Sqrt[a + b*x^2])
/2048 + (3*a^3*x^7*(a + b*x^2)^(3/2))/256 + (3*a^2*x^7*(a + b*x^2)^(5/2))/1
28 + (9*a*x^7*(a + b*x^2)^(7/2))/224 + (x^7*(a + b*x^2)^(9/2))/16 - (45*a^8
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32768*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^6\left(a+bx^2\right)^{9/2}\,dx &= \frac{1}{16}x^7\left(a+bx^2\right)^{9/2}+\frac{1}{16}(9a)\int x^6\left(a+bx^2\right)^{7/2}\,dx\\
&= \frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}+\frac{1}{32}\left(9a^2\right)\int x^6\left(a+bx^2\right)^{5/2}\,dx\\
&= \frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}+\frac{1}{128}\left(15a^3\right)\int x^6\left(a+\right.\\
&= \frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}\\
&= \frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\\
&= \frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{22}\\
&= -\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a\\
&= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a\right.\\
&= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a\right.\\
&= \frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a\right.
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 138, normalized size = 0.68

$$
\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(315a^7-210a^6bx^2+168a^5b^2x^4+32624a^4b^3x^6+98432a^3b^4x^8+119040a^2b^5x^{10}+66560ab^6x^{12}+14336b^7x^{14}\right)-\frac{315a^{15/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{229376b^{7/2}}
$$

Antiderivative was successfully verified.

[In] Integrate[x^6*(a + b*x^2)^(9/2),x]

[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(315*a^7 - 210*a^6*b*x^2 + 168*a^5*b^2*x^4 + 32
624*a^4*b^3*x^6 + 98432*a^3*b^4*x^8 + 119040*a^2*b^5*x^10 + 66560*a*b^6*x^1
2 + 14336*b^7*x^14) - (315*a^(15/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 +
(b*x^2)/a]))/(229376*b^(7/2))

IntegrateAlgebraic [A] time = 0.19, size = 129, normalized size = 0.64

$$\frac{45a^8\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{32768b^{7/2}}+\frac{\sqrt{a+bx^2}\,\left(315a^7x-210a^6bx^3+168a^5b^2x^5+32624a^4b^3x^7+98432a^3b^4x^9+119040a^2b^5x^{11}+66560ab^6x^{13}+14336b^7x^{15}\right)}{229376b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6*(a + b*x^2)^(9/2),x]

[Out] (Sqrt[a + b*x^2]*(315*a^7*x - 210*a^6*b*x^3 + 168*a^5*b^2*x^5 + 32624*a^4*b
^3*x^7 + 98432*a^3*b^4*x^9 + 119040*a^2*b^5*x^11 + 66560*a*b^6*x^13 + 14336
*b^7*x^15))/(229376*b^3) + (45*a^8*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(32
768*b^(7/2))

fricas [A] time = 1.32, size = 255, normalized size = 1.26

$$\left[\frac{315\,a^8\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}\,x-a\right)+2\left(14336\,b^8x^{15}+66560\,ab^7x^{13}+119040\,a^2b^6x^{11}+98432\,a^3b^5x^9+32624\,a^4b^4x^7+168\,a^5b^3x^5-210\,a^6b^2x^3+315\,a^7bx\right)\sqrt{bx^2+a}}{458752\,b^4},\ \frac{315\,a^8\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(14336\,b^8x^{15}+66560\,ab^7x^{13}+119040\,a^2b^6x^{11}+98432\,a^3b^5x^9+32624\,a^4b^4x^7+168\,a^5b^3x^5-210\,a^6b^2x^3+315\,a^7bx\right)\sqrt{bx^2+a}}{229376\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="fricas")

[Out] [1/458752*(315*a^8*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(14336*b^8*x^15 + 66560*a*b^7*x^13 + 119040*a^2*b^6*x^11 + 98432*a^3*b^
5*x^9 + 32624*a^4*b^4*x^7 + 168*a^5*b^3*x^5 - 210*a^6*b^2*x^3 + 315*a^7*b*x
)*sqrt(b*x^2 + a))/b^4, 1/229376*(315*a^8*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) + (14336*b^8*x^15 + 66560*a*b^7*x^13 + 119040*a^2*b^6*x^11 + 984
32*a^3*b^5*x^9 + 32624*a^4*b^4*x^7 + 168*a^5*b^3*x^5 - 210*a^6*b^2*x^3 + 31
5*a^7*b*x)*sqrt(b*x^2 + a))/b^4]

giac [A] time = 1.21, size = 133, normalized size = 0.66

$$\frac{45\,a^8\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{32768\,b^{\frac{7}{2}}}+\frac{1}{229376}\left(\frac{315\,a^7}{b^3}-2\left(\frac{105\,a^6}{b^2}-4\left(\frac{21\,a^5}{b}+2\left(2039\,a^4+8\left(769\,a^3b+2\left(465\,a^2b^2+4\left(14\,b^4x^2+65\,ab^3\right)x^2\right)x^2\right)x^2\right)x^2\right)x^2\right)x^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="giac")

[Out] 45/32768*a^8*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2) + 1/229376*(315
a^7/b^3 - 2(105*a^6/b^2 - 4*(21*a^5/b + 2*(2039*a^4 + 8*(769*a^3*b + 2*(4

```
65*a^2*b^2 + 4*(14*b^4*x^2 + 65*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*x^2)*sqrt(b
*x^2 + a)*x
```

maple [A] time = 0.01, size = 169, normalized size = 0.84

$$-\frac{45a^8\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{32768b^{\frac{7}{2}}}-\frac{45\sqrt{b\,x^2+a}\,a^7x}{32768b^3}-\frac{15\left(b\,x^2+a\right)^{\frac{3}{2}}a^6x}{16384b^3}+\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}x^5}{16b}-\frac{3\left(b\,x^2+a\right)^{\frac{5}{2}}a^5x}{4096b^3}-\frac{9\left(b\,x^2+a\right)^{\frac{7}{2}}a^4x}{14336b^3}-\frac{5\left(b\,x^2+a\right)^{\frac{11}{2}}a\,x^3}{224b^2}-\frac{\left(b\,x^2+a\right)^{\frac{9}{2}}a^3x}{1792b^3}+\frac{5\left(b\,x^2+a\right)^{\frac{11}{2}}a^2x}{896b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/16*x^5*(b*x^2+a)^(11/2)/b-5/224*a/b^2*x^3*(b*x^2+a)^(11/2)+5/896*a^2/b^3*
x*(b*x^2+a)^(11/2)-1/1792*a^3/b^3*x*(b*x^2+a)^(9/2)-9/14336*a^4/b^3*x*(b*x^
2+a)^(7/2)-3/4096*a^5/b^3*x*(b*x^2+a)^(5/2)-15/16384*a^6/b^3*x*(b*x^2+a)^(3
/2)-45/32768*a^7*x*(b*x^2+a)^(1/2)/b^3-45/32768*a^8/b^(7/2)*ln(b^(1/2)*x+(b
*x^2+a)^(1/2))
```

maxima [A] time = 1.46, size = 161, normalized size = 0.80

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^5}{16\,b}-\frac{5\left(bx^2+a\right)^{\frac{11}{2}}ax^3}{224\,b^2}+\frac{5\left(bx^2+a\right)^{\frac{11}{2}}a^2x}{896\,b^3}-\frac{\left(bx^2+a\right)^{\frac{9}{2}}a^3x}{1792\,b^3}-\frac{9\left(bx^2+a\right)^{\frac{7}{2}}a^4x}{14336\,b^3}-\frac{3\left(bx^2+a\right)^{\frac{5}{2}}a^5x}{4096\,b^3}-\frac{15\left(bx^2+a\right)^{\frac{3}{2}}a^6x}{16384\,b^3}-\frac{45\sqrt{bx^2+a}\,a^7x}{32768\,b^3}-\frac{45\,a^8\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{32768\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/16*(b*x^2 + a)^(11/2)*x^5/b - 5/224*(b*x^2 + a)^(11/2)*a*x^3/b^2 + 5/896*
(b*x^2 + a)^(11/2)*a^2*x/b^3 - 1/1792*(b*x^2 + a)^(9/2)*a^3*x/b^3 - 9/14336
*(b*x^2 + a)^(7/2)*a^4*x/b^3 - 3/4096*(b*x^2 + a)^(5/2)*a^5*x/b^3 - 15/1638
4*(b*x^2 + a)^(3/2)*a^6*x/b^3 - 45/32768*sqrt(b*x^2 + a)*a^7*x/b^3 - 45/327
68*a^8*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\left(b\,x^2+a\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^6*(a + b*x^2)^(9/2), x)
```

sympy [A] time = 30.60, size = 258, normalized size = 1.28

$$\frac{45a^{\frac{15}{2}}x}{32768b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^{\frac{13}{2}}x^3}{32768b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{11}{2}}x^5}{16384b\sqrt{1+\frac{bx^2}{a}}}+\frac{4099a^{\frac{9}{2}}x^7}{28672\sqrt{1+\frac{bx^2}{a}}}+\frac{8191a^{\frac{7}{2}}bx^9}{14336\sqrt{1+\frac{bx^2}{a}}}+\frac{1699a^{\frac{5}{2}}b^2x^{11}}{1792\sqrt{1+\frac{bx^2}{a}}}+\frac{725a^{\frac{3}{2}}b^3x^{13}}{896\sqrt{1+\frac{bx^2}{a}}}+\frac{79\sqrt{a}\,b^4x^{15}}{224\sqrt{1+\frac{bx^2}{a}}}-\frac{45a^8\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{32768b^{\frac{7}{2}}}+\frac{b^5x^{17}}{16\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**(9/2),x)
```

```
[Out] 45*a**(15/2)*x/(32768*b**3*sqrt(1 + b*x**2/a)) + 15*a**(13/2)*x**3/(32768*b
**2*sqrt(1 + b*x**2/a)) - 3*a**(11/2)*x**5/(16384*b*sqrt(1 + b*x**2/a)) + 4
099*a**(9/2)*x**7/(28672*sqrt(1 + b*x**2/a)) + 8191*a**(7/2)*b*x**9/(14336*
sqrt(1 + b*x**2/a)) + 1699*a**(5/2)*b**2*x**11/(1792*sqrt(1 + b*x**2/a)) +
725*a**(3/2)*b**3*x**13/(896*sqrt(1 + b*x**2/a)) + 79*sqrt(a)*b**4*x**15/(2
24*sqrt(1 + b*x**2/a)) - 45*a**8*asinh(sqrt(b)*x/sqrt(a))/(32768*b**(7/2))
+ b**5*x**17/(16*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.417 $\int x^4 \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=178

$$\frac{9a^7\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}}-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+\right.$$

Rubi [A] time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{9a^7\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(9/2),x]
```

```
[Out] (-9*a^6*x*Sqrt[a + b*x^2])/(2048*b^2) + (3*a^5*x^3*Sqrt[a + b*x^2])/(1024*b
) + (3*a^4*x^5*Sqrt[a + b*x^2])/256 + (3*a^3*x^5*(a + b*x^2)^(3/2))/128 + (
3*a^2*x^5*(a + b*x^2)^(5/2))/80 + (3*a*x^5*(a + b*x^2)^(7/2))/56 + (x^5*(a
+ b*x^2)^(9/2))/14 + (9*a^7*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2048*b^(
5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\left(a+bx^2\right)^{9/2}\,dx=\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{1}{14}(9a)\int x^4\left(a+bx^2\right)^{7/2}\,dx$$
$$=\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{1}{8}\left(3a^2\right)\int x^4\left(a+bx^2\right)^{5/2}\,dx$$
$$=\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{1}{16}\left(3a^3\right)\int x^4\left(a+bx^2\right)^{3}$$
$$=\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{}{1}$$
$$=\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+$$
$$=\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}$$
$$=-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x$$
$$=-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x$$
$$=-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x$$

Mathematica [A] time = 0.19, size = 127, normalized size = 0.71

$$\frac{\sqrt{a+bx^2}\left(\frac{315a^{13/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}x\left(-315a^6+210a^5bx^2+14168a^4b^2x^4+39056a^3b^3x^6+44928a^2b^4x^8+24320ab^5x^{10}+5120b^6x^{12}\right)\right)}{71680b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-315*a^6 + 210*a^5*b*x^2 + 14168*a^4*b^2*x^4 +
 39056*a^3*b^3*x^6 + 44928*a^2*b^4*x^8 + 24320*a*b^5*x^10 + 5120*b^6*x^12)
```

```
+ (315*a^(13/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(71680*
b^(5/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 118, normalized size = 0.66

$$\frac{\sqrt{a+bx^2}\left(-315a^6x+210a^5bx^3+14168a^4b^2x^5+39056a^3b^3x^7+44928a^2b^4x^9+24320ab^5x^{11}+5120b^6x^{13}\right)}{71680b^2}-\frac{9a^7\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2048b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-315*a^6*x + 210*a^5*b*x^3 + 14168*a^4*b^2*x^5 + 39056*a^
3*b^3*x^7 + 44928*a^2*b^4*x^9 + 24320*a*b^5*x^11 + 5120*b^6*x^13))/(71680*b
^2) - (9*a^7*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2048*b^(5/2))
```

fricas [A] time = 1.53, size = 234, normalized size = 1.31

$$\left[\frac{315\,a^7\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(5120\,b^7x^{13}+24320\,ab^6x^{11}+44928\,a^2b^5x^9+39056\,a^3b^4x^7+14168\,a^4b^3x^5+210\,a^5b^2x^3-315\,a^6bx\right)\sqrt{bx^2+a}}{143360\,b^3},\,-\frac{315\,a^7\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(5120\,b^7x^{13}+24320\,ab^6x^{11}+44928\,a^2b^5x^9+39056\,a^3b^4x^7+14168\,a^4b^3x^5+210\,a^5b^2x^3-315\,a^6bx\right)\sqrt{bx^2+a}}{71680\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/143360*(315*a^7*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(5120*b^7*x^13 + 24320*a*b^6*x^11 + 44928*a^2*b^5*x^9 + 39056*a^3*b^4*x
^7 + 14168*a^4*b^3*x^5 + 210*a^5*b^2*x^3 - 315*a^6*b*x)*sqrt(b*x^2 + a))/b^
3, -1/71680*(315*a^7*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (5120*b^
7*x^13 + 24320*a*b^6*x^11 + 44928*a^2*b^5*x^9 + 39056*a^3*b^4*x^7 + 14168*a
^4*b^3*x^5 + 210*a^5*b^2*x^3 - 315*a^6*b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 1.17, size = 119, normalized size = 0.67

$$-\frac{9\,a^7\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2048\,b^{\frac{5}{2}}}-\frac{1}{71680}\left(\frac{315\,a^6}{b^2}-2\left(\frac{105\,a^5}{b}+4\left(1771\,a^4+2\left(2441\,a^3b+8\left(351\,a^2b^2+10\left(4\,b^4x^2+19\,ab^3\right)x^2\right)x^2\right)x^2\right)x^2\right)x^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -9/2048*a^7*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/71680*(315*a
^6/b^2 - 2*(105*a^5/b + 4*(1771*a^4 + 2*(2441*a^3*b + 8*(351*a^2*b^2 + 10*(
4*b^4*x^2 + 19*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.01, size = 149, normalized size = 0.84

$$\frac{9a^7\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2048b^{\frac{5}{2}}}+\frac{9\sqrt{b\,x^2+a}\,a^6x}{2048b^2}+\frac{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^5x}{1024b^2}+\frac{3\left(b\,x^2+a\right)^{\frac{5}{2}}a^4x}{1280b^2}+\frac{9\left(b\,x^2+a\right)^{\frac{7}{2}}a^3x}{4480b^2}+\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}x^3}{14b}+\frac{\left(b\,x^2+a\right)^{\frac{9}{2}}a^2x}{560b^2}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}ax}{56b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/14*x^3*(b*x^2+a)^(11/2)/b-1/56*a/b^2*x*(b*x^2+a)^(11/2)+1/560*a^2/b^2*x*(
b*x^2+a)^(9/2)+9/4480*a^3/b^2*x*(b*x^2+a)^(7/2)+3/1280*a^4/b^2*x*(b*x^2+a)^
(5/2)+3/1024*a^5/b^2*x*(b*x^2+a)^(3/2)+9/2048*a^6*x*(b*x^2+a)^(1/2)/b^2+9/2
048*a^7/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.40, size = 141, normalized size = 0.79

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^3}{14\,b}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}ax}{56\,b^2}+\frac{\left(bx^2+a\right)^{\frac{9}{2}}a^2x}{560\,b^2}+\frac{9\left(bx^2+a\right)^{\frac{7}{2}}a^3x}{4480\,b^2}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}a^4x}{1280\,b^2}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}a^5x}{1024\,b^2}+\frac{9\sqrt{bx^2+a}\,a^6x}{2048\,b^2}+\frac{9\,a^7\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2048\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/14*(b*x^2 + a)^(11/2)*x^3/b - 1/56*(b*x^2 + a)^(11/2)*a*x/b^2 + 1/560*(b*
x^2 + a)^(9/2)*a^2*x/b^2 + 9/4480*(b*x^2 + a)^(7/2)*a^3*x/b^2 + 3/1280*(b*x
^2 + a)^(5/2)*a^4*x/b^2 + 3/1024*(b*x^2 + a)^(3/2)*a^5*x/b^2 + 9/2048*sqrt(
b*x^2 + a)*a^6*x/b^2 + 9/2048*a^7*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(b\,x^2+a\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^(9/2), x)
```

sympy [A] time = 20.00, size = 231, normalized size = 1.30

$$-\frac{9a^{\frac{13}{2}}x}{2048b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{11}{2}}x^3}{2048b\sqrt{1+\frac{bx^2}{a}}}+\frac{1027a^{\frac{9}{2}}x^5}{5120\sqrt{1+\frac{bx^2}{a}}}+\frac{6653a^{\frac{7}{2}}bx^7}{8960\sqrt{1+\frac{bx^2}{a}}}+\frac{5249a^{\frac{5}{2}}b^2x^9}{4480\sqrt{1+\frac{bx^2}{a}}}+\frac{541a^{\frac{3}{2}}b^3x^{11}}{560\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}\,b^4x^{13}}{56\sqrt{1+\frac{bx^2}{a}}}+\frac{9a^7\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2048b^{\frac{5}{2}}}+\frac{b^5x^{15}}{14\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(9/2),x)
```

```
[Out] -9*a**(13/2)*x/(2048*b**2*sqrt(1 + b*x**2/a)) - 3*a**(11/2)*x**3/(2048*b*sq
rt(1 + b*x**2/a)) + 1027*a**(9/2)*x**5/(5120*sqrt(1 + b*x**2/a)) + 6653*a**
(7/2)*b*x**7/(8960*sqrt(1 + b*x**2/a)) + 5249*a**(5/2)*b**2*x**9/(4480*sqrt
(1 + b*x**2/a)) + 541*a**(3/2)*b**3*x**11/(560*sqrt(1 + b*x**2/a)) + 23*sqr
t(a)*b**4*x**13/(56*sqrt(1 + b*x**2/a)) + 9*a**7*asinh(sqrt(b)*x/sqrt(a))/(
2048*b**(5/2)) + b**5*x**15/(14*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.418 $\int x^2 \left(a+bx^2\right)^{9/2} \, dx$

Optimal. Leaf size=154

$$-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}}+\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}a$$

Rubi [A] time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}}+\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(9/2),x]
```

```
[Out] (21*a^5*x*Sqrt[a + b*x^2])/(1024*b) + (21*a^4*x^3*Sqrt[a + b*x^2])/512 + (7
*a^3*x^3*(a + b*x^2)^(3/2))/128 + (21*a^2*x^3*(a + b*x^2)^(5/2))/320 + (3*a
*x^3*(a + b*x^2)^(7/2))/40 + (x^3*(a + b*x^2)^(9/2))/12 - (21*a^6*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \left(a+bx^2\right)^{9/2}\,dx &= \frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\frac{1}{4}(3a)\int x^2\left(a+bx^2\right)^{7/2}\,dx\\
&= \frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\frac{1}{40}\left(21a^2\right)\int x^2\left(a+bx^2\right)^{5/2}\,dx\\
&= \frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\frac{1}{64}\left(21a^3\right)\int x^2\left(a+bx^2\cdots\right.\\
&= \frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}+\cdots\\
&= \frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\cdots\\
&= \frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}\cdots\\
&= \frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}\cdots\\
&= \frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}\cdots
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 116, normalized size = 0.75

$$
\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(315a^5+4910a^4bx^2+11432a^3b^2x^4+12144a^2b^3x^6+6272ab^4x^8+1280b^5x^{10}\right)-\frac{315a^{11/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{15360b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(315*a^5 + 4910*a^4*b*x^2 + 11432*a^3*b^2*x^4 +
 12144*a^2*b^3*x^6 + 6272*a*b^4*x^8 + 1280*b^5*x^10) - (315*a^(11/2)*ArcSin
h[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(15360*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 107, normalized size = 0.69

$$\frac{21a^6\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{1024b^{3/2}}+\frac{\sqrt{a+bx^2}\,\left(315a^5x+4910a^4bx^3+11432a^3b^2x^5+12144a^2b^3x^7+6272ab^4x^9+1280b^5x^{11}\right)}{15360b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(315*a^5*x + 4910*a^4*b*x^3 + 11432*a^3*b^2*x^5 + 12144*a^
2*b^3*x^7 + 6272*a*b^4*x^9 + 1280*b^5*x^11))/(15360*b) + (21*a^6*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/(1024*b^(3/2))
```

fricas [A] time = 1.49, size = 211, normalized size = 1.37

$$\left[\frac{315\,a^6\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\left(1280\,b^6x^{11}+6272\,ab^5x^9+12144\,a^2b^4x^7+11432\,a^3b^3x^5+4910\,a^4b^2x^3+315\,a^5bx\right)\sqrt{bx^2+a}}{30720\,b^2},\ \frac{315\,a^6\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(1280\,b^6x^{11}+6272\,ab^5x^9+12144\,a^2b^4x^7+11432\,a^3b^3x^5+4910\,a^4b^2x^3+315\,a^5bx\right)\sqrt{bx^2+a}}{15360\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(315*a^6*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(1280*b^6*x^11 + 6272*a*b^5*x^9 + 12144*a^2*b^4*x^7 + 11432*a^3*b^3*x^5
+ 4910*a^4*b^2*x^3 + 315*a^5*b*x)*sqrt(b*x^2 + a))/b^2, 1/15360*(315*a^6*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (1280*b^6*x^11 + 6272*a*b^5*x^9
 + 12144*a^2*b^4*x^7 + 11432*a^3*b^3*x^5 + 4910*a^4*b^2*x^3 + 315*a^5*b*x)*
sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 1.24, size = 105, normalized size = 0.68

$$\frac{21\,a^6\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{1024\,b^{\frac{3}{2}}}+\frac{1}{15360}\,\left(\frac{315\,a^5}{b}+2\,\left(2455\,a^4+4\,\left(1429\,a^3b+2\,\left(759\,a^2b^2+8\,\left(10\,b^4x^2+49\,ab^3\right)x^2\right)x^2\right)x^2\right)x^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 21/1024*a^6*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/15360*(315*a
^5/b + 2*(2455*a^4 + 4*(1429*a^3*b + 2*(759*a^2*b^2 + 8*(10*b^4*x^2 + 49*a*
b^3)*x^2)*x^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.01, size = 129, normalized size = 0.84

$$-\frac{21a^6\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{1024b^{\frac{3}{2}}}-\frac{21\sqrt{b\,x^2+a}\,a^5x}{1024b}-\frac{7\left(b\,x^2+a\right)^{\frac{3}{2}}a^4x}{512b}-\frac{7\left(b\,x^2+a\right)^{\frac{5}{2}}a^3x}{640b}-\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}a^2x}{320b}-\frac{\left(b\,x^2+a\right)^{\frac{9}{2}}ax}{120b}+\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}x}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(9/2),x)
```

```
[Out] 1/12*x*(b*x^2+a)^(11/2)/b-1/120*a/b*x*(b*x^2+a)^(9/2)-3/320*a^2/b*x*(b*x^2+
a)^(7/2)-7/640*a^3/b*x*(b*x^2+a)^(5/2)-7/512*a^4/b*x*(b*x^2+a)^(3/2)-21/102
4*a^5*x*(b*x^2+a)^(1/2)/b-21/1024*a^6/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.37, size = 121, normalized size = 0.79

$$\frac{\left(bx^2+a\right)^{\frac{11}{2}}x}{12\,b}-\frac{\left(bx^2+a\right)^{\frac{9}{2}}ax}{120\,b}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}a^2x}{320\,b}-\frac{7\left(bx^2+a\right)^{\frac{5}{2}}a^3x}{640\,b}-\frac{7\left(bx^2+a\right)^{\frac{3}{2}}a^4x}{512\,b}-\frac{21\sqrt{bx^2+a}\,a^5x}{1024\,b}-\frac{21\,a^6\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^(11/2)*x/b - 1/120*(b*x^2 + a)^(9/2)*a*x/b - 3/320*(b*x^2
+ a)^(7/2)*a^2*x/b - 7/640*(b*x^2 + a)^(5/2)*a^3*x/b - 7/512*(b*x^2 + a)^(3
/2)*a^4*x/b - 21/1024*sqrt(b*x^2 + a)*a^5*x/b - 21/1024*a^6*arcsinh(b*x/sqr
t(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^2+a\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^(9/2), x)
```

sympy [A] time = 13.57, size = 204, normalized size = 1.32

$$\frac{21a^{\frac{11}{2}}x}{1024b\sqrt{1+\frac{bx^2}{a}}}+\frac{1045a^{\frac{9}{2}}x^3}{3072\sqrt{1+\frac{bx^2}{a}}}+\frac{8171a^{\frac{7}{2}}bx^5}{7680\sqrt{1+\frac{bx^2}{a}}}+\frac{2947a^{\frac{5}{2}}b^2x^7}{1920\sqrt{1+\frac{bx^2}{a}}}+\frac{1151a^{\frac{3}{2}}b^3x^9}{960\sqrt{1+\frac{bx^2}{a}}}+\frac{59\sqrt{a}\,b^4x^{11}}{120\sqrt{1+\frac{bx^2}{a}}}-\frac{21a^6\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{1024b^{\frac{3}{2}}}+\frac{b^5x^{13}}{12\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(9/2),x)
```

```
[Out] 21*a**(11/2)*x/(1024*b*sqrt(1 + b*x**2/a)) + 1045*a**(9/2)*x**3/(3072*sqrt(
1 + b*x**2/a)) + 8171*a**(7/2)*b*x**5/(7680*sqrt(1 + b*x**2/a)) + 2947*a**(
5/2)*b**2*x**7/(1920*sqrt(1 + b*x**2/a)) + 1151*a**(3/2)*b**3*x**9/(960*sqr
t(1 + b*x**2/a)) + 59*sqrt(a)*b**4*x**11/(120*sqrt(1 + b*x**2/a)) - 21*a**6
*asinh(sqrt(b)*x/sqrt(a))/(1024*b**(3/2)) + b**5*x**13/(12*sqrt(a)*sqrt(1 +
 b*x**2/a))
```

3.419 $\int \left(a+bx^2\right)^{9/2} dx$

Optimal. Leaf size=122

$$\frac{63a^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}}+\frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x\left(a\right.$$

Rubi [A] time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x\left(a+bx^2\right)^{3/2}+\frac{21}{160}a^2x\left(a+bx^2\right)^{5/2}+\frac{63a^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}}+\frac{9}{80}ax\left(a+bx^2\right)^{7/2}+\frac{1}{10}x\left(a+bx^2\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2),x]
```

```
[Out] (63*a^4*x*Sqrt[a + b*x^2])/256 + (21*a^3*x*(a + b*x^2)^(3/2))/128 + (21*a^2
*x*(a + b*x^2)^(5/2))/160 + (9*a*x*(a + b*x^2)^(7/2))/80 + (x*(a + b*x^2)^(
9/2))/10 + (63*a^5*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{9/2} dx = \frac{1}{10}x\left(a+bx^2\right)^{9/2} + \frac{1}{10}(9a)\int \left(a+bx^2\right)^{7/2} dx$$
$$= \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(a+bx^2\right)^{9/2} + \frac{1}{80}\left(63a^2\right)\int \left(a+bx^2\right)^{5/2} dx$$
$$= \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2} + \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(a+bx^2\right)^{9/2} + \frac{1}{32}\left(21a^3\right)\int \left(a+bx^2\right)^{3/2} dx$$
$$= \frac{21}{128}a^3x\left(a+bx^2\right)^{3/2} + \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2} + \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(a+bx^2\right)^{9/2} + \frac{1}{128}(6$$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x\left(a+bx^2\right)^{3/2} + \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2} + \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(\right.$$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x\left(a+bx^2\right)^{3/2} + \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2} + \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(\right.$$
$$= \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x\left(a+bx^2\right)^{3/2} + \frac{21}{160}a^2x\left(a+bx^2\right)^{5/2} + \frac{9}{80}ax\left(a+bx^2\right)^{7/2} + \frac{1}{10}x\left(\right.$$

Mathematica [A] time = 0.14, size = 98, normalized size = 0.80

$$\frac{\sqrt{a+bx^2}\left(\frac{315a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}} + 965a^4x + 1490a^3bx^3 + 1368a^2b^2x^5 + 656ab^3x^7 + 128b^4x^9\right)}{1280}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(965*a^4*x + 1490*a^3*b*x^3 + 1368*a^2*b^2*x^5 + 656*a*b^3
*x^7 + 128*b^4*x^9 + (315*a^(9/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sq
rt[1 + (b*x^2)/a])))/1280
```

IntegrateAlgebraic [A] time = 0.15, size = 93, normalized size = 0.76

$$\frac{\sqrt{a+bx^2}\left(965a^4x + 1490a^3bx^3 + 1368a^2b^2x^5 + 656ab^3x^7 + 128b^4x^9\right)}{1280} - \frac{63a^5\log\left(\sqrt{a+bx^2} - \sqrt{b}x\right)}{256\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(965*a^4*x + 1490*a^3*b*x^3 + 1368*a^2*b^2*x^5 + 656*a*b^3
*x^7 + 128*b^4*x^9))/1280 - (63*a^5*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2
56*Sqrt[b])
```

fricas [A] time = 1.57, size = 190, normalized size = 1.56

$$\left[\frac{315\,a^{5}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\left(128\,b^{5}x^{9}+656\,ab^{4}x^{7}+1368\,a^{2}b^{3}x^{5}+1490\,a^{3}b^{2}x^{3}+965\,a^{4}bx\right)\sqrt{bx^{2}+a}}{2560\,b},-\frac{315\,a^{5}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-\left(128\,b^{5}x^{9}+656\,ab^{4}x^{7}+1368\,a^{2}b^{3}x^{5}+1490\,a^{3}b^{2}x^{3}+965\,a^{4}bx\right)\sqrt{bx^{2}+a}}{1280\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(315*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(128*b^5*x^9 + 656*a*b^4*x^7 + 1368*a^2*b^3*x^5 + 1490*a^3*b^2*x^3 + 965*
a^4*b*x)*sqrt(b*x^2 + a))/b, -1/1280*(315*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (128*b^5*x^9 + 656*a*b^4*x^7 + 1368*a^2*b^3*x^5 + 1490*a^3
*b^2*x^3 + 965*a^4*b*x)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 1.12, size = 91, normalized size = 0.75

$$-\frac{63\,a^{5}\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{256\,\sqrt{b}}+\frac{1}{1280}\left(965\,a^{4}+2\left(745\,a^{3}b+4\left(171\,a^{2}b^{2}+2\left(8\,b^{4}x^{2}+41\,ab^{3}\right)x^{2}\right)x^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -63/256*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/1280*(965*a^
4 + 2*(745*a^3*b + 4*(171*a^2*b^2 + 2*(8*b^4*x^2 + 41*a*b^3)*x^2)*x^2)*x^2)
*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.00, size = 96, normalized size = 0.79

$$\frac{63a^{5}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{256\sqrt{b}}+\frac{63\sqrt{b\,x^{2}+a}\;a^{4}x}{256}+\frac{21\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{3}x}{128}+\frac{21\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{2}x}{160}+\frac{9\left(b\,x^{2}+a\right)^{\frac{7}{2}}ax}{80}+\frac{\left(b\,x^{2}+a\right)^{\frac{9}{2}}x}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2),x)
```

```
[Out] 1/10*x*(b*x^2+a)^(9/2)+9/80*a*x*(b*x^2+a)^(7/2)+21/160*a^2*x*(b*x^2+a)^(5/2
)+21/128*a^3*x*(b*x^2+a)^(3/2)+63/256*a^4*x*(b*x^2+a)^(1/2)+63/256*a^5/b^(1
/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 88, normalized size = 0.72

$$\frac{1}{10}\left(bx^{2}+a\right)^{\frac{9}{2}}x+\frac{9}{80}\left(bx^{2}+a\right)^{\frac{7}{2}}ax+\frac{21}{160}\left(bx^{2}+a\right)^{\frac{5}{2}}a^{2}x+\frac{21}{128}\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}x+\frac{63}{256}\sqrt{bx^{2}+a}\,a^{4}x+\frac{63\,a^{5}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(9/2)*x + 9/80*(b*x^2 + a)^(7/2)*a*x + 21/160*(b*x^2 + a)^
(5/2)*a^2*x + 21/128*(b*x^2 + a)^(3/2)*a^3*x + 63/256*sqrt(b*x^2 + a)*a^4*x
 + 63/256*a^5*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 4.62, size = 37, normalized size = 0.30

$$\frac{x\left(b\,x^2+a\right)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{9}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2),x)
```

```
[Out] (x*(a + b*x^2)^(9/2)*hypergeom([-9/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(9/2)
```

sympy [A] time = 8.35, size = 151, normalized size = 1.24

$$\frac{193a^{\frac{9}{2}}x\sqrt{1+\frac{bx^2}{a}}}{256}+\frac{149a^{\frac{7}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{128}+\frac{171a^{\frac{5}{2}}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{160}+\frac{41a^{\frac{3}{2}}b^3x^7\sqrt{1+\frac{bx^2}{a}}}{80}+\frac{\sqrt{a}\,b^4x^9\sqrt{1+\frac{bx^2}{a}}}{10}+\frac{63a^5\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2),x)
```

```
[Out] 193*a**(9/2)*x*sqrt(1 + b*x**2/a)/256 + 149*a**(7/2)*b*x**3*sqrt(1 + b*x**2
/a)/128 + 171*a**(5/2)*b**2*x**5*sqrt(1 + b*x**2/a)/160 + 41*a**(3/2)*b**3*
x**7*sqrt(1 + b*x**2/a)/80 + sqrt(a)*b**4*x**9*sqrt(1 + b*x**2/a)/10 + 63*a
**5*asinh(sqrt(b)*x/sqrt(a))/(256*sqrt(b))
```

3.420 $\int \frac{\left(a+bx^2\right)^{9/2}}{x^2}\,dx$

Optimal. Leaf size=123

$$\frac{315}{128}a^4\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}+\frac{21}{16}ab$$

Rubi [A] time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}+\frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{315}{128}a^4\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}+\frac{21}{16}abx\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^2,x]
```

```
[Out] (315*a^3*b*x*Sqrt[a + b*x^2])/128 + (105*a^2*b*x*(a + b*x^2)^(3/2))/64 + (2
1*a*b*x*(a + b*x^2)^(5/2))/16 + (9*b*x*(a + b*x^2)^(7/2))/8 - (a + b*x^2)^(
9/2)/x + (315*a^4*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/128
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^2}\,dx &= -\frac{\left(a+bx^2\right)^{9/2}}{x}+(9b)\int \left(a+bx^2\right)^{7/2}\,dx\\
&= \frac{9}{8}bx\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{1}{8}(63ab)\int \left(a+bx^2\right)^{5/2}\,dx\\
&= \frac{21}{16}abx\left(a+bx^2\right)^{5/2}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{1}{16}\left(105a^2b\right)\int \left(a+bx^2\right)^{3/2}\,dx\\
&= \frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}+\frac{21}{16}abx\left(a+bx^2\right)^{5/2}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}-\frac{\left(a+bx^2\right)^{9/2}}{x}+\frac{1}{64}\left(315a^3b\right)\\
&= \frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}+\frac{21}{16}abx\left(a+bx^2\right)^{5/2}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}-\frac{(a+b}{x}\\
&= \frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}+\frac{21}{16}abx\left(a+bx^2\right)^{5/2}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}-\frac{(a+b}{x}\\
&= \frac{315}{128}a^3bx\sqrt{a+bx^2}+\frac{105}{64}a^2bx\left(a+bx^2\right)^{3/2}+\frac{21}{16}abx\left(a+bx^2\right)^{5/2}+\frac{9}{8}bx\left(a+bx^2\right)^{7/2}-\frac{(a+b}{x}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.42

$$
-\frac{a^4\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{9}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^2,x]
```

```
[Out] -((a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -1/2, 1/2, -((b*x^2)/a)])/(x
*Sqrt[1 + (b*x^2)/a]))
```

IntegrateAlgebraic [A] time = 0.16, size = 95, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(-128a^4+325a^3bx^2+210a^2b^2x^4+88ab^3x^6+16b^4x^8\right)}{128x}-\frac{315}{128}a^4\sqrt{b}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-128*a^4 + 325*a^3*b*x^2 + 210*a^2*b^2*x^4 + 88*a*b^3*x^6
 + 16*b^4*x^8))/(128*x) - (315*a^4*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/128
```

fricas [A] time = 1.20, size = 184, normalized size = 1.50

$$\left[\frac{315\,a^4\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(16\,b^4x^8+88\,ab^3x^6+210\,a^2b^2x^4+325\,a^3bx^2-128\,a^4\right)\sqrt{bx^2+a}}{256\,x},-\frac{315\,a^4\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(16\,b^4x^8+88\,ab^3x^6+210\,a^2b^2x^4+325\,a^3bx^2-128\,a^4\right)\sqrt{bx^2+a}}{128\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/256*(315*a^4*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(16*b^4*x^8 + 88*a*b^3*x^6 + 210*a^2*b^2*x^4 + 325*a^3*b*x^2 - 128*a^4)*
sqrt(b*x^2 + a))/x, -1/128*(315*a^4*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (16*b^4*x^8 + 88*a*b^3*x^6 + 210*a^2*b^2*x^4 + 325*a^3*b*x^2 - 128
*a^4)*sqrt(b*x^2 + a))/x]
```

giac [A] time = 1.22, size = 115, normalized size = 0.93

$$-\frac{315}{256}\,a^4\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a^5\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{128}\left(325\,a^3b+2\left(105\,a^2b^2+4\left(2\,b^4x^2+11\,ab^3\right)x^2\right)x^2\right)\sqrt{bx^2+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="giac")
```

```
[Out] -315/256*a^4*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^5*sqrt(b)/(
(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/128*(325*a^3*b + 2*(105*a^2*b^2 +
4*(2*b^4*x^2 + 11*a*b^3)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

maple [A] time = 0.00, size = 117, normalized size = 0.95

$$\frac{315a^4\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{128}+\frac{315\sqrt{b\,x^2+a}\,a^3bx}{128}+\frac{105\left(b\,x^2+a\right)^{\frac{3}{2}}a^2bx}{64}+\frac{21\left(b\,x^2+a\right)^{\frac{5}{2}}abx}{16}+\frac{9\left(b\,x^2+a\right)^{\frac{7}{2}}bx}{8}+\frac{\left(b\,x^2+a\right)^{\frac{9}{2}}bx}{a}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^2,x)
```

```
[Out] -1/a/x*(b*x^2+a)^(11/2)+1/a*b*x*(b*x^2+a)^(9/2)+9/8*b*x*(b*x^2+a)^(7/2)+21/
16*a*b*x*(b*x^2+a)^(5/2)+105/64*a^2*b*x*(b*x^2+a)^(3/2)+315/128*a^3*b*x*(b*
x^2+a)^(1/2)+315/128*a^4*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.46, size = 91, normalized size = 0.74

$$\frac{9}{8}\left(bx^2+a\right)^{\frac{7}{2}}bx+\frac{21}{16}\left(bx^2+a\right)^{\frac{5}{2}}abx+\frac{105}{64}\left(bx^2+a\right)^{\frac{3}{2}}a^2bx+\frac{315}{128}\sqrt{bx^2+a}\,a^3bx+\frac{315}{128}a^4\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+a\right)^{\frac{9}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="maxima")
```

```
[Out] 9/8*(b*x^2 + a)^(7/2)*b*x + 21/16*(b*x^2 + a)^(5/2)*a*b*x + 105/64*(b*x^2 +
 a)^(3/2)*a^2*b*x + 315/128*sqrt(b*x^2 + a)*a^3*b*x + 315/128*a^4*sqrt(b)*a
rcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(9/2)/x
```

mupad [B] time = 6.03, size = 40, normalized size = 0.33

$$-\frac{\left(b\,x^2+a\right)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{9}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^2,x)
```

```
[Out] -((a + b*x^2)^(9/2)*hypergeom([-9/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a
 + 1)^(9/2))
```

sympy [A] time = 7.47, size = 173, normalized size = 1.41

$$-\frac{a^{\frac{9}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{197a^{\frac{7}{2}}bx}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{535a^{\frac{5}{2}}b^2x^3}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{149a^{\frac{3}{2}}b^3x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{13\sqrt{a}\,b^4x^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{315a^4\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128}+\frac{b^5x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**2,x)
```

```
[Out] -a**(9/2)/(x*sqrt(1 + b*x**2/a)) + 197*a**(7/2)*b*x/(128*sqrt(1 + b*x**2/a)
) + 535*a**(5/2)*b**2*x**3/(128*sqrt(1 + b*x**2/a)) + 149*a**(3/2)*b**3*x**
5/(64*sqrt(1 + b*x**2/a)) + 13*sqrt(a)*b**4*x**7/(16*sqrt(1 + b*x**2/a)) +
315*a**4*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/128 + b**5*x**9/(8*sqrt(a)*sqrt(1
 + b*x**2/a))
```

3.421 $\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx$

Optimal. Leaf size=128

$$\frac{105}{16}a^3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}+\frac{35}{8}ab^2x\left(a+bx^2\right)^{3/2}-\frac{3b\left(a+bx^2\right)^{7/2}}{x}-\frac{(}{}$$

Rubi [A] time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{105}{16}a^3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}+\frac{35}{8}ab^2x\left(a+bx^2\right)^{3/2}-\frac{\left(a+bx^2\right)^{9/2}}{3x^3}-\frac{3b\left(a+bx^2\right)^{7/2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^4,x]
```

```
[Out] (105*a^2*b^2*x*Sqrt[a + b*x^2])/16 + (35*a*b^2*x*(a + b*x^2)^(3/2))/8 + (7*
b^2*x*(a + b*x^2)^(5/2))/2 - (3*b*(a + b*x^2)^(7/2))/x - (a + b*x^2)^(9/2)/
(3*x^3) + (105*a^3*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/16
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^4}\,dx &= -\frac{\left(a+bx^2\right)^{9/2}}{3x^3}+(3b)\int \frac{\left(a+bx^2\right)^{7/2}}{x^2}\,dx \\
&= -\frac{3b\left(a+bx^2\right)^{7/2}}{x}-\frac{\left(a+bx^2\right)^{9/2}}{3x^3}+\left(21b^2\right)\int \left(a+bx^2\right)^{5/2}\,dx \\
&= \frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}-\frac{3b\left(a+bx^2\right)^{7/2}}{x}-\frac{\left(a+bx^2\right)^{9/2}}{3x^3}+\frac{1}{2}\left(35ab^2\right)\int \left(a+bx^2\right)^{3/2}\,dx \\
&= \frac{35}{8}ab^2x\left(a+bx^2\right)^{3/2}+\frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}-\frac{3b\left(a+bx^2\right)^{7/2}}{x}-\frac{\left(a+bx^2\right)^{9/2}}{3x^3}+\frac{1}{8}\left(105a^2b^2\right)\int \\
&= \frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{35}{8}ab^2x\left(a+bx^2\right)^{3/2}+\frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}-\frac{3b\left(a+bx^2\right)^{7/2}}{x}-\frac{\left(a+bx^2\right.}{3x^3} \\
&= \frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{35}{8}ab^2x\left(a+bx^2\right)^{3/2}+\frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}-\frac{3b\left(a+bx^2\right)^{7/2}}{x}-\frac{\left(a+bx^2\right.}{3x^3} \\
&= \frac{105}{16}a^2b^2x\sqrt{a+bx^2}+\frac{35}{8}ab^2x\left(a+bx^2\right)^{3/2}+\frac{7}{2}b^2x\left(a+bx^2\right)^{5/2}-\frac{3b\left(a+bx^2\right)^{7/2}}{x}-\frac{\left(a+bx^2\right.}{3x^3}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.42

$$
-\frac{a^4\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^4,x]
```

```
[Out] -1/3*(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -3/2, -1/2, -((b*x^2)/a)]
)/(x^3*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.18, size = 95, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-16a^4-208a^3bx^2+165a^2b^2x^4+50ab^3x^6+8b^4x^8\right)}{48x^3}-\frac{105}{16}a^3b^{3/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^4 - 208*a^3*b*x^2 + 165*a^2*b^2*x^4 + 50*a*b^3*x^6
+ 8*b^4*x^8))/(48*x^3) - (105*a^3*b^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]])/16
```

fricas [A] time = 1.24, size = 189, normalized size = 1.48

$$\left[\frac{315\,a^3b^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8\,b^4x^8+50\,ab^3x^6+165\,a^2b^2x^4-208\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+a}}{96\,x^3},\,-\frac{315\,a^3\sqrt{-b}\,bx^3\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(8\,b^4x^8+50\,ab^3x^6+165\,a^2b^2x^4-208\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+a}}{48\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(315*a^3*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(8*b^4*x^8 + 50*a*b^3*x^6 + 165*a^2*b^2*x^4 - 208*a^3*b*x^2 - 16*a^4)*s
qrt(b*x^2 + a))/x^3, -1/48*(315*a^3*sqrt(-b)*b*x^3*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (8*b^4*x^8 + 50*a*b^3*x^6 + 165*a^2*b^2*x^4 - 208*a^3*b*x^2 -
16*a^4)*sqrt(b*x^2 + a))/x^3]
```

giac [A] time = 1.15, size = 160, normalized size = 1.25

$$-\frac{105}{32}\,a^3b^{\frac{3}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{1}{48}\left(165\,a^2b^2+2\left(4\,b^4x^2+25\,ab^3\right)x^2\right)\sqrt{bx^2+a}\,x+\frac{2\left(15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^4b^{\frac{3}{2}}-24\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^5b^{\frac{3}{2}}+13\,a^6b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="giac")
```

```
[Out] -105/32*a^3*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/48*(165*a^2*b^
2 + 2*(4*b^4*x^2 + 25*a*b^3)*x^2)*sqrt(b*x^2 + a)*x + 2/3*(15*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*a^4*b^(3/2) - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(
3/2) + 13*a^6*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.01, size = 146, normalized size = 1.14

$$\frac{105a^3b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16}+\frac{105\sqrt{b\,x^2+a}\,a^2b^2x}{16}+\frac{35\left(b\,x^2+a\right)^{\frac{3}{2}}a\,b^2x}{8}+\frac{7\left(b\,x^2+a\right)^{\frac{5}{2}}b^2x}{2}+\frac{3\left(b\,x^2+a\right)^{\frac{7}{2}}b^2x}{a}+\frac{8\left(b\,x^2+a\right)^{\frac{9}{2}}b^2x}{3a^2}-\frac{8\left(b\,x^2+a\right)^{\frac{11}{2}}b}{3a^2x}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^4,x)
```

```
[Out] -1/3/a/x^3*(b*x^2+a)^(11/2)-8/3/a^2*b/x*(b*x^2+a)^(11/2)+8/3/a^2*b^2*x*(b*x
^2+a)^(9/2)+3/a*b^2*x*(b*x^2+a)^(7/2)+7/2*b^2*x*(b*x^2+a)^(5/2)+35/8*a*b^2*
x*(b*x^2+a)^(3/2)+105/16*a^2*b^2*x*(b*x^2+a)^(1/2)+105/16*a^3*b^(3/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.42, size = 120, normalized size = 0.94

$$\frac{7}{2}\left(bx^2+a\right)^{\frac{5}{2}}b^2x+\frac{3\left(bx^2+a\right)^{\frac{7}{2}}b^2x}{a}+\frac{35}{8}\left(bx^2+a\right)^{\frac{3}{2}}ab^2x+\frac{105}{16}\sqrt{bx^2+a}\,a^2b^2x+\frac{105}{16}a^3b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{8\left(bx^2+a\right)^{\frac{9}{2}}b}{3\,ax}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="maxima")
```

```
[Out] 7/2*(b*x^2 + a)^(5/2)*b^2*x + 3*(b*x^2 + a)^(7/2)*b^2*x/a + 35/8*(b*x^2 + a
)^(3/2)*a*b^2*x + 105/16*sqrt(b*x^2 + a)*a^2*b^2*x + 105/16*a^3*b^(3/2)*arc
sinh(b*x/sqrt(a*b)) - 8/3*(b*x^2 + a)^(9/2)*b/(a*x) - 1/3*(b*x^2 + a)^(11/2
)/(a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{9/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^4,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^4, x)
```

sympy [A] time = 6.84, size = 175, normalized size = 1.37

$$-\frac{a^{\frac{9}{2}}}{3x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{14a^{\frac{7}{2}}b}{3x\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2x}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{215a^{\frac{3}{2}}b^3x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{29\sqrt{a}\,b^4x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{105a^3b^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16}+\frac{b^5x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**4,x)
```

```
[Out] -a**(9/2)/(3*x**3*sqrt(1 + b*x**2/a)) - 14*a**(7/2)*b/(3*x*sqrt(1 + b*x**2/
a)) - 43*a**(5/2)*b**2*x/(48*sqrt(1 + b*x**2/a)) + 215*a**(3/2)*b**3*x**3/(
48*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**4*x**5/(24*sqrt(1 + b*x**2/a)) + 105
*a**3*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))/16 + b**5*x**7/(6*sqrt(a)*sqrt(1 +
b*x**2/a))
```

3.422 $\int \frac{\left(a+bx^2\right)^{9/2}}{x^6}\,dx$

Optimal. Leaf size=129

$$\frac{63}{8}a^2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}+\frac{63}{8}ab^3x\sqrt{a+bx^2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}-\frac{3b\left(a+b\right.}{5x^3}$$

Rubi [A] time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$\frac{63}{8}a^2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}+\frac{63}{8}ab^3x\sqrt{a+bx^2}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}-\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^6,x]
```

```
[Out] (63*a*b^3*x*Sqrt[a + b*x^2])/8 + (21*b^3*x*(a + b*x^2)^(3/2))/4 - (21*b^2*(
a + b*x^2)^(5/2))/(5*x) - (3*b*(a + b*x^2)^(7/2))/(5*x^3) - (a + b*x^2)^(9/
2)/(5*x^5) + (63*a^2*b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^6}\,dx &= -\frac{\left(a+bx^2\right)^{9/2}}{5x^5}+\frac{1}{5}(9b)\int \frac{\left(a+bx^2\right)^{7/2}}{x^4}\,dx\\
&= -\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}+\frac{1}{5}\left(21b^2\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^2}\,dx\\
&= -\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}-\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}+\left(21b^3\right)\int \left(a+bx^2\right)^{3/2}\,dx\\
&= \frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}-\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}+\frac{1}{4}\left(63ab^3\right)\int \sqrt{a+\cdots}\\
&= \frac{63}{8}ab^3x\sqrt{a+bx^2}+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}-\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}\cdots\\
&= \frac{63}{8}ab^3x\sqrt{a+bx^2}+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}-\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}\cdots\\
&= \frac{63}{8}ab^3x\sqrt{a+bx^2}+\frac{21}{4}b^3x\left(a+bx^2\right)^{3/2}-\frac{21b^2\left(a+bx^2\right)^{5/2}}{5x}-\frac{3b\left(a+bx^2\right)^{7/2}}{5x^3}-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}\cdots
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.42

$$-\frac{a^4\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{9}{2},-\frac{5}{2};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^6,x]
```

```
[Out] -1/5*(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -5/2, -3/2, -((b*x^2)/a)]
)/(x^5*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.21, size = 95, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-8a^4-56a^3bx^2-288a^2b^2x^4+85ab^3x^6+10b^4x^8\right)}{40x^5}-\frac{63}{8}a^2b^{5/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^4 - 56*a^3*b*x^2 - 288*a^2*b^2*x^4 + 85*a*b^3*x^6 +
10*b^4*x^8))/(40*x^5) - (63*a^2*b^(5/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
)/8
```

fricas [A] time = 0.71, size = 191, normalized size = 1.48

$$\left[\frac{315\,a^2b^{\frac{5}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(10\,b^4x^8+85\,ab^3x^6-288\,a^2b^2x^4-56\,a^3bx^2-8\,a^4\right)\sqrt{bx^2+a}}{80\,x^5},-\frac{315\,a^2\sqrt{-b}\,b^2x^5\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(10\,b^4x^8+85\,ab^3x^6-288\,a^2b^2x^4-56\,a^3bx^2-8\,a^4\right)\sqrt{bx^2+a}}{40\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(315*a^2*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 - 8*a^4)*sq
rt(b*x^2 + a))/x^5, -1/40*(315*a^2*sqrt(-b)*b^2*x^5*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - (10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 -
 8*a^4)*sqrt(b*x^2 + a))/x^5]
```

giac [A] time = 1.08, size = 200, normalized size = 1.55

$$-\frac{63}{16}\,a^2b^{\frac{5}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{1}{8}\left(2\,b^4x^2+17\,ab^3\right)\sqrt{bx^2+a}\,x+\frac{4\left(25\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8a^3b^{\frac{5}{2}}-75\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6a^4b^{\frac{5}{2}}+105\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^5b^{\frac{5}{2}}-65\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^6b^{\frac{5}{2}}+18\,a^7b^{\frac{5}{2}}\right)}{5\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="giac")
```

```
[Out] -63/16*a^2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/8*(2*b^4*x^2 +
17*a*b^3)*sqrt(b*x^2 + a)*x + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b
^(5/2) - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(5/2) + 105*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*a^5*b^(5/2) - 65*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(
5/2) + 18*a^7*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.01, size = 166, normalized size = 1.29

$$\frac{63a^2b^{\frac{5}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8}+\frac{63\sqrt{b\,x^2+a}\,a\,b^3x}{8}+\frac{21\left(b\,x^2+a\right)^{\frac{3}{2}}b^3x}{4}+\frac{21\left(b\,x^2+a\right)^{\frac{5}{2}}b^3x}{5a}+\frac{18\left(b\,x^2+a\right)^{\frac{7}{2}}b^3x}{5a^2}+\frac{16\left(b\,x^2+a\right)^{\frac{9}{2}}b^3x}{5a^3}-\frac{16\left(b\,x^2+a\right)^{\frac{11}{2}}b^2}{5a^3x}-\frac{2\left(b\,x^2+a\right)^{\frac{11}{2}}b}{5a^2x^3}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^6,x)
```

```
[Out] -1/5/a/x^5*(b*x^2+a)^(11/2)-2/5/a^2*b/x^3*(b*x^2+a)^(11/2)-16/5/a^3*b^2/x*(
b*x^2+a)^(11/2)+16/5/a^3*b^3*x*(b*x^2+a)^(9/2)+18/5/a^2*b^3*x*(b*x^2+a)^(7/
2)+21/5/a*b^3*x*(b*x^2+a)^(5/2)+21/4*b^3*x*(b*x^2+a)^(3/2)+63/8*a*b^3*x*(b*
x^2+a)^(1/2)+63/8*a^2*b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.45, size = 140, normalized size = 1.09

$$\frac{21}{4}\left(bx^2+a\right)^{\frac{3}{2}}b^3x+\frac{18\left(bx^2+a\right)^{\frac{7}{2}}b^3x}{5\,a^2}+\frac{21\left(bx^2+a\right)^{\frac{5}{2}}b^3x}{5\,a}+\frac{63}{8}\sqrt{bx^2+a}\,ab^3x+\frac{63}{8}a^2b^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{16\left(bx^2+a\right)^{\frac{9}{2}}b^2}{5\,a^2x}-\frac{2\left(bx^2+a\right)^{\frac{11}{2}}b}{5\,a^2x^3}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="maxima")
```

```
[Out] 21/4*(b*x^2 + a)^(3/2)*b^3*x + 18/5*(b*x^2 + a)^(7/2)*b^3*x/a^2 + 21/5*(b*x
^2 + a)^(5/2)*b^3*x/a + 63/8*sqrt(b*x^2 + a)*a*b^3*x + 63/8*a^2*b^(5/2)*arc
sinh(b*x/sqrt(a*b)) - 16/5*(b*x^2 + a)^(9/2)*b^2/(a^2*x) - 2/5*(b*x^2 + a)^
(11/2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(11/2)/(a*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{9/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^6,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^6, x)
```

sympy [A] time = 7.82, size = 175, normalized size = 1.36

$$-\frac{a^{\frac{9}{2}}}{5x^5\sqrt{1+\frac{bx^2}{a}}}-\frac{8a^{\frac{7}{2}}b}{5x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2}{5x\sqrt{1+\frac{bx^2}{a}}}-\frac{203a^{\frac{3}{2}}b^3x}{40\sqrt{1+\frac{bx^2}{a}}}+\frac{19\sqrt{a}\,b^4x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{63a^2b^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{b^5x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**6,x)
```

```
[Out] -a**(9/2)/(5*x**5*sqrt(1 + b*x**2/a)) - 8*a**(7/2)*b/(5*x**3*sqrt(1 + b*x**
2/a)) - 43*a**(5/2)*b**2/(5*x*sqrt(1 + b*x**2/a)) - 203*a**(3/2)*b**3*x/(40
*sqrt(1 + b*x**2/a)) + 19*sqrt(a)*b**4*x**3/(8*sqrt(1 + b*x**2/a)) + 63*a**
2*b**(5/2)*asinh(sqrt(b)*x/sqrt(a))/8 + b**5*x**5/(4*sqrt(a)*sqrt(1 + b*x**
2/a))
```

3.423 $\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx$

Optimal. Leaf size=126

$$\frac{9}{2}ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{9}{2}b^4x\sqrt{a+bx^2}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{x}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}$$

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {277, 195, 217, 206}

$$-\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{x}+\frac{9}{2}b^4x\sqrt{a+bx^2}+\frac{9}{2}ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^8,x]
```

```
[Out] (9*b^4*x*Sqrt[a + b*x^2])/2 - (3*b^3*(a + b*x^2)^(3/2))/x - (3*b^2*(a + b*x
^2)^(5/2))/(5*x^3) - (9*b*(a + b*x^2)^(7/2))/(35*x^5) - (a + b*x^2)^(9/2)/(
7*x^7) + (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^8}\,dx &= -\frac{\left(a+bx^2\right)^{9/2}}{7x^7}+\frac{1}{7}(9b)\int \frac{\left(a+bx^2\right)^{7/2}}{x^6}\,dx \\
&= -\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}+\frac{1}{5}\left(9b^2\right)\int \frac{\left(a+bx^2\right)^{5/2}}{x^4}\,dx \\
&= -\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}+\left(3b^3\right)\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx \\
&= -\frac{3b^3\left(a+bx^2\right)^{3/2}}{x}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}+\left(9b^4\right)\int \sqrt{a+bx^2}\,d \\
&= \frac{9}{2}b^4x\sqrt{a+bx^2}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{x}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}+\frac{1}{2}(\\
&= \frac{9}{2}b^4x\sqrt{a+bx^2}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{x}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}+\frac{1}{2}(\\
&= \frac{9}{2}b^4x\sqrt{a+bx^2}-\frac{3b^3\left(a+bx^2\right)^{3/2}}{x}-\frac{3b^2\left(a+bx^2\right)^{5/2}}{5x^3}-\frac{9b\left(a+bx^2\right)^{7/2}}{35x^5}-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}+\frac{9}{2}ab
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.43

$$-\frac{a^4\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{9}{2},-\frac{7}{2};-\frac{5}{2};-\frac{bx^2}{a}\right)}{7x^7\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^8,x]
```

```
[Out] -1/7*(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -7/2, -5/2, -((b*x^2)/a)]
)/(x^7*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.22, size = 93, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-10a^4-58a^3bx^2-156a^2b^2x^4-388ab^3x^6+35b^4x^8\right)}{70x^7}-\frac{9}{2}ab^{7/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^8,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-10*a^4 - 58*a^3*b*x^2 - 156*a^2*b^2*x^4 - 388*a*b^3*x^6
+ 35*b^4*x^8))/(70*x^7) - (9*a*b^(7/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/2
```

fricas [A] time = 0.54, size = 187, normalized size = 1.48

$$\left[\frac{315\,ab^{\frac{7}{2}}x^7\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(35\,b^4x^8-388\,ab^3x^6-156\,a^2b^2x^4-58\,a^3bx^2-10\,a^4\right)\sqrt{bx^2+a}}{140\,x^7},-\frac{315\,a\sqrt{-b}\,b^3x^7\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(35\,b^4x^8-388\,ab^3x^6-156\,a^2b^2x^4-58\,a^3bx^2-10\,a^4\right)\sqrt{bx^2+a}}{70\,x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/140*(315*a*b^(7/2)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 - 10*a^4)*s
qrt(b*x^2 + a))/x^7, -1/70*(315*a*sqrt(-b)*b^3*x^7*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 -
 10*a^4)*sqrt(b*x^2 + a))/x^7]
```

giac [B] time = 1.20, size = 240, normalized size = 1.90

$$\frac{1}{2}\,\sqrt{bx^2+a}\,b^4x-\frac{9}{4}\,ab^{\frac{7}{2}}\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{4\left(175\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{12}a^2b^{\frac{7}{2}}-700\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{10}a^3b^{\frac{7}{2}}+1575\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{8}a^4b^{\frac{7}{2}}-1820\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{6}a^5b^{\frac{7}{2}}+1337\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{4}a^6b^{\frac{7}{2}}-504\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^{2}a^7b^{\frac{7}{2}}+97\,a^8b^{\frac{7}{2}}\right)}{35\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^4*x - 9/4*a*b^(7/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))
^2) + 4/35*(175*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/2) - 700*(sqrt(b)
*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^8
*a^4*b^(7/2) - 1820*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*b^(7/2) + 1337*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(7/2) - 504*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*a^7*b^(7/2) + 97*a^8*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.02, size = 186, normalized size = 1.48

$$\frac{9a\,b^{\frac{7}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2}+\frac{9\sqrt{b\,x^2+a}\,b^4x}{2}+\frac{3\left(b\,x^2+a\right)^{\frac{3}{2}}b^4x}{a}+\frac{12\left(b\,x^2+a\right)^{\frac{5}{2}}b^4x}{5a^2}+\frac{72\left(b\,x^2+a\right)^{\frac{7}{2}}b^4x}{35a^3}+\frac{64\left(b\,x^2+a\right)^{\frac{9}{2}}b^4x}{35a^4}-\frac{64\left(b\,x^2+a\right)^{\frac{11}{2}}b^3}{35a^4x}-\frac{8\left(b\,x^2+a\right)^{\frac{11}{2}}b^2}{35a^3x^3}-\frac{4\left(b\,x^2+a\right)^{\frac{11}{2}}b}{35a^2x^5}-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{7a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^8,x)
```

```
[Out] -1/7/a/x^7*(b*x^2+a)^(11/2)-4/35/a^2*b/x^5*(b*x^2+a)^(11/2)-8/35/a^3*b^2/x^
3*(b*x^2+a)^(11/2)-64/35/a^4*b^3/x*(b*x^2+a)^(11/2)+64/35/a^4*b^4*x*(b*x^2+
a)^(9/2)+72/35/a^3*b^4*x*(b*x^2+a)^(7/2)+12/5/a^2*b^4*x*(b*x^2+a)^(5/2)+3/a
*b^4*x*(b*x^2+a)^(3/2)+9/2*b^4*x*(b*x^2+a)^(1/2)+9/2*a*b^(7/2)*ln(b^(1/2)*x
+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.47, size = 160, normalized size = 1.27

$$\frac{9}{2}\sqrt{bx^2+a}\,b^4x+\frac{72\left(bx^2+a\right)^{\frac{7}{2}}b^4x}{35\,a^3}+\frac{12\left(bx^2+a\right)^{\frac{5}{2}}b^4x}{5\,a^2}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}b^4x}{a}+\frac{9}{2}\,ab^{\frac{7}{2}}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{64\left(bx^2+a\right)^{\frac{9}{2}}b^3}{35\,a^3x}-\frac{8\left(bx^2+a\right)^{\frac{11}{2}}b^2}{35\,a^3x^3}-\frac{4\left(bx^2+a\right)^{\frac{11}{2}}b}{35\,a^2x^5}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="maxima")
```

```
[Out] 9/2*sqrt(b*x^2 + a)*b^4*x + 72/35*(b*x^2 + a)^(7/2)*b^4*x/a^3 + 12/5*(b*x^2
 + a)^(5/2)*b^4*x/a^2 + 3*(b*x^2 + a)^(3/2)*b^4*x/a + 9/2*a*b^(7/2)*arcsinh
(b*x/sqrt(a*b)) - 64/35*(b*x^2 + a)^(9/2)*b^3/(a^3*x) - 8/35*(b*x^2 + a)^(1
1/2)*b^2/(a^3*x^3) - 4/35*(b*x^2 + a)^(11/2)*b/(a^2*x^5) - 1/7*(b*x^2 + a)^
(11/2)/(a*x^7)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{9/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^8,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^8, x)
```

sympy [A] time = 8.79, size = 167, normalized size = 1.33

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6}-\frac{29a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^4}-\frac{78a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^2}-\frac{194ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{35}-\frac{9ab^{\frac{7}{2}}\log\left(\frac{a}{bx^2}\right)}{4}+\frac{9ab^{\frac{7}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)}{2}+\frac{b^{\frac{9}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**8,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 29*a**3*b**(3/2)*sqrt(a/(b*x*
*2) + 1)/(35*x**4) - 78*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(35*x**2) - 194*
a*b**(7/2)*sqrt(a/(b*x**2) + 1)/35 - 9*a*b**(7/2)*log(a/(b*x**2))/4 + 9*a*b
**(7/2)*log(sqrt(a/(b*x**2) + 1) + 1)/2 + b**(9/2)*x**2*sqrt(a/(b*x**2) + 1
)/2
```

3.424 $\int \frac{\left(a+bx^2\right)^{9/2}}{x^{10}}\,dx$

Optimal. Leaf size=124

$$b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b^4\sqrt{a+bx^2}}{x}-\frac{b^3\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{b^2\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{\left(a+bx^2\right)^{9/2}}{9x^9}-\frac{b\left(a+bx^2\right)^{7/2}}{7x^7}$$

Rubi [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 206}

$$-\frac{b^4\sqrt{a+bx^2}}{x}-\frac{b^3\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{b^2\left(a+bx^2\right)^{5/2}}{5x^5}+b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b\left(a+bx^2\right)^{7/2}}{7x^7}-\frac{\left(a+bx^2\right)^{9/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^10,x]
```

```
[Out] -((b^4*Sqrt[a + b*x^2])/x) - (b^3*(a + b*x^2)^(3/2))/(3*x^3) - (b^2*(a + b*
x^2)^(5/2))/(5*x^5) - (b*(a + b*x^2)^(7/2))/(7*x^7) - (a + b*x^2)^(9/2)/(9*
x^9) + b^(9/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx = -\frac{(a+bx^2)^{9/2}}{9x^9} + b\int \frac{(a+bx^2)^{7/2}}{x^8}\,dx$$
$$= -\frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^2\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx$$
$$= -\frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^3\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx$$
$$= -\frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^4\int \frac{\sqrt{a+bx^2}}{x^2}\,dx$$
$$= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^5\int \frac{}{\sqrt{a}}$$
$$= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^5\,\text{Subst}$$
$$= -\frac{b^4\sqrt{a+bx^2}}{x} - \frac{b^3(a+bx^2)^{3/2}}{3x^3} - \frac{b^2(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9} + b^{9/2}\tanh$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.44

$$-\frac{a^4\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{9}{2},-\frac{9}{2};-\frac{7}{2};-\frac{bx^2}{a}\right)}{9x^9\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^10,x]
```

```
[Out] -1/9*(a^4*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/2, -9/2, -7/2, -((b*x^2)/a)]
)/(x^9*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.24, size = 90, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(-35a^4-185a^3bx^2-408a^2b^2x^4-506ab^3x^6-563b^4x^8\right)}{315x^9} - b^{9/2}\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^10,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-35*a^4 - 185*a^3*b*x^2 - 408*a^2*b^2*x^4 - 506*a*b^3*x^6
 - 563*b^4*x^8))/(315*x^9) - b^(9/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

fricas [A] time = 1.34, size = 184, normalized size = 1.48

$$\left[\frac{315\,b^{\frac{9}{2}}x^{9}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\left(563\,b^{4}x^{8}+506\,ab^{3}x^{6}+408\,a^{2}b^{2}x^{4}+185\,a^{3}bx^{2}+35\,a^{4}\right)\sqrt{bx^{2}+a}}{630\,x^{9}},\ -\frac{315\,\sqrt{-b}\,b^{4}x^{9}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(563\,b^{4}x^{8}+506\,ab^{3}x^{6}+408\,a^{2}b^{2}x^{4}+185\,a^{3}bx^{2}+35\,a^{4}\right)\sqrt{bx^{2}+a}}{315\,x^{9}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/630*(315*b^(9/2)*x^9*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2
*(563*b^4*x^8 + 506*a*b^3*x^6 + 408*a^2*b^2*x^4 + 185*a^3*b*x^2 + 35*a^4)*s
qrt(b*x^2 + a))/x^9, -1/315*(315*sqrt(-b)*b^4*x^9*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) + (563*b^4*x^8 + 506*a*b^3*x^6 + 408*a^2*b^2*x^4 + 185*a^3*b*x^2
+ 35*a^4)*sqrt(b*x^2 + a))/x^9]
```

giac [B] time = 1.12, size = 276, normalized size = 2.23

$$-\frac{1}{2}\,b^{\frac{9}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}\right)+\frac{2\left(1575\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{16}ab^{\frac{9}{2}}-6300\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{14}a^{2}b^{\frac{9}{2}}+21000\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}a^{3}b^{\frac{9}{2}}-31500\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{10}a^{4}b^{\frac{9}{2}}+39438\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}a^{5}b^{\frac{9}{2}}-26292\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}a^{6}b^{\frac{9}{2}}+13968\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}a^{7}b^{\frac{9}{2}}-3492\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}a^{8}b^{\frac{9}{2}}+563\,a^{9}b^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/2*b^(9/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/315*(1575*(sqrt(b)*x
- sqrt(b*x^2 + a))^16*a*b^(9/2) - 6300*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2
*b^(9/2) + 21000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(9/2) - 31500*(sqrt
(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(9/2) + 39438*(sqrt(b)*x - sqrt(b*x^2 + a
))^8*a^5*b^(9/2) - 26292*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(9/2) + 1396
8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(9/2) - 3492*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a^8*b^(9/2) + 563*a^9*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
 a)^9
```

maple [B] time = 0.03, size = 206, normalized size = 1.66

$$b^{\frac{9}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)+\frac{\sqrt{b\,x^{2}+a}\;b^{5}x}{a}+\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{5}x}{3a^{2}}+\frac{8\left(b\,x^{2}+a\right)^{\frac{5}{2}}b^{5}x}{15a^{3}}+\frac{16\left(b\,x^{2}+a\right)^{\frac{7}{2}}b^{5}x}{35a^{4}}+\frac{128\left(b\,x^{2}+a\right)^{\frac{9}{2}}b^{5}x}{315a^{5}}-\frac{128\left(b\,x^{2}+a\right)^{\frac{11}{2}}b^{4}}{315a^{5}x}-\frac{16\left(b\,x^{2}+a\right)^{\frac{11}{2}}b^{3}}{315a^{4}x^{3}}-\frac{8\left(b\,x^{2}+a\right)^{\frac{11}{2}}b^{2}}{315a^{3}x^{5}}-\frac{2\left(b\,x^{2}+a\right)^{\frac{11}{2}}b}{63a^{2}x^{7}}-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{9a\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^10,x)
```

```
[Out] -1/9/a/x^9*(b*x^2+a)^(11/2)-2/63/a^2*b/x^7*(b*x^2+a)^(11/2)-8/315/a^3*b^2/x
^5*(b*x^2+a)^(11/2)-16/315/a^4*b^3/x^3*(b*x^2+a)^(11/2)-128/315/a^5*b^4/x*(
b*x^2+a)^(11/2)+128/315/a^5*b^5*x*(b*x^2+a)^(9/2)+16/35/a^4*b^5*x*(b*x^2+a)
```

```
^(7/2)+8/15/a^3*b^5*x*(b*x^2+a)^(5/2)+2/3/a^2*b^5*x*(b*x^2+a)^(3/2)+1/a*b^5
*x*(b*x^2+a)^(1/2)+b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.51, size = 180, normalized size = 1.45

$$\frac{16\left(bx^2+a\right)^{\frac{7}{2}}b^5x}{35\,a^4}+\frac{8\left(bx^2+a\right)^{\frac{5}{2}}b^5x}{15\,a^3}+\frac{2\left(bx^2+a\right)^{\frac{3}{2}}b^5x}{3\,a^2}+\frac{\sqrt{bx^2+a}\,b^5x}{a}+b^{\frac{9}{2}}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{128\left(bx^2+a\right)^{\frac{9}{2}}b^4}{315\,a^4x}-\frac{16\left(bx^2+a\right)^{\frac{11}{2}}b^3}{315\,a^4x^3}-\frac{8\left(bx^2+a\right)^{\frac{11}{2}}b^2}{315\,a^3x^5}-\frac{2\left(bx^2+a\right)^{\frac{11}{2}}b}{63\,a^2x^7}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="maxima")
```

```
[Out] 16/35*(b*x^2 + a)^(7/2)*b^5*x/a^4 + 8/15*(b*x^2 + a)^(5/2)*b^5*x/a^3 + 2/3*
(b*x^2 + a)^(3/2)*b^5*x/a^2 + sqrt(b*x^2 + a)*b^5*x/a + b^(9/2)*arcsinh(b*x
/sqrt(a*b)) - 128/315*(b*x^2 + a)^(9/2)*b^4/(a^4*x) - 16/315*(b*x^2 + a)^(1
1/2)*b^3/(a^4*x^3) - 8/315*(b*x^2 + a)^(11/2)*b^2/(a^3*x^5) - 2/63*(b*x^2 +
 a)^(11/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(11/2)/(a*x^9)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{9/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^10,x)
```

```
[Out] int((a + b*x^2)^(9/2)/x^10, x)
```

sympy [A] time = 9.86, size = 160, normalized size = 1.29

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{9x^8}-\frac{37a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{63x^6}-\frac{136a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{105x^4}-\frac{506ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{315x^2}-\frac{563b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{315}-\frac{b^{\frac{9}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{9}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**10,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(9*x**8) - 37*a**3*b**(3/2)*sqrt(a/(b*x*
*2) + 1)/(63*x**6) - 136*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(105*x**4) - 50
6*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(315*x**2) - 563*b**(9/2)*sqrt(a/(b*x**2)
 + 1)/315 - b**(9/2)*log(a/(b*x**2))/2 + b**(9/2)*log(sqrt(a/(b*x**2) + 1)
+ 1)
```

3.425 $\int \frac{(a+bx^2)^{9/2}}{x^{12}} dx$

Optimal. Leaf size=21

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^12,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(11*a*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}} dx = -\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^12,x]
```

```
[Out] -1/11*(a + b*x^2)^(11/2)/(a*x^11)
```

IntegrateAlgebraic [B] time = 0.21, size = 75, normalized size = 3.57

$$\frac{\sqrt{a+bx^2}\left(-a^5-5a^4bx^2-10a^3b^2x^4-10a^2b^3x^6-5ab^4x^8-b^5x^{10}\right)}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^12,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-a^5 - 5*a^4*b*x^2 - 10*a^3*b^2*x^4 - 10*a^2*b^3*x^6 - 5*
a*b^4*x^8 - b^5*x^10))/(11*a*x^11)
```

fricas [B] time = 0.63, size = 68, normalized size = 3.24

$$-\frac{\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\sqrt{bx^2+a}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/11*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x
^2 + a^5)*sqrt(b*x^2 + a)/(a*x^11)
```

giac [B] time = 1.06, size = 167, normalized size = 7.95

$$\frac{2\left(11\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}b^{\frac{11}{2}}+165\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^2b^{\frac{11}{2}}+462\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^4b^{\frac{11}{2}}+330\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^6b^{\frac{11}{2}}+55\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^8b^{\frac{11}{2}}+a^{10}b^{\frac{11}{2}}\right)}{11\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="giac")
```

```
[Out] 2/11*(11*(sqrt(b)*x - sqrt(b*x^2 + a))^20*b^(11/2) + 165*(sqrt(b)*x - sqrt(
b*x^2 + a))^16*a^2*b^(11/2) + 462*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(1
1/2) + 330*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(11/2) + 55*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*a^8*b^(11/2) + a^10*b^(11/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^11
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{11a\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^12,x)
```

```
[Out] -1/11*(b*x^2+a)^(11/2)/a/x^11
```

maxima [A] time = 1.50, size = 17, normalized size = 0.81

$$-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{11\,ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/11*(b*x^2 + a)^(11/2)/(a*x^11)
```

mupad [B] time = 6.28, size = 111, normalized size = 5.29

$$-\frac{a^4\sqrt{b\,x^2+a}}{11\,x^{11}}-\frac{5\,b^4\sqrt{b\,x^2+a}}{11\,x^3}-\frac{10\,a\,b^3\sqrt{b\,x^2+a}}{11\,x^5}-\frac{5\,a^3\,b\sqrt{b\,x^2+a}}{11\,x^9}-\frac{b^5\sqrt{b\,x^2+a}}{11\,a\,x}-\frac{10\,a^2\,b^2\sqrt{b\,x^2+a}}{11\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^12,x)
```

```
[Out] - (a^4*(a + b*x^2)^(1/2))/(11*x^11) - (5*b^4*(a + b*x^2)^(1/2))/(11*x^3) -
(10*a*b^3*(a + b*x^2)^(1/2))/(11*x^5) - (5*a^3*b*(a + b*x^2)^(1/2))/(11*x^9
) - (b^5*(a + b*x^2)^(1/2))/(11*a*x) - (10*a^2*b^2*(a + b*x^2)^(1/2))/(11*x
^7)
```

sympy [B] time = 2.45, size = 150, normalized size = 7.14

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{11x^{10}}-\frac{5a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^8}-\frac{10a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^6}-\frac{10ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^4}-\frac{5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^2}-\frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{11a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**12,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(11*x**10) - 5*a**3*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(11*x**8) - 10*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(11*x**6) - 10*
a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(11*x**4) - 5*b**(9/2)*sqrt(a/(b*x**2) + 1)
/(11*x**2) - b**(11/2)*sqrt(a/(b*x**2) + 1)/(11*a)
```

3.426 $\int \frac{(a+bx^2)^{9/2}}{x^{14}} dx$

Optimal. Leaf size=44

$$\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^14,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(13*a*x^13) + (2*b*(a + b*x^2)^(11/2))/(143*a^2*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^{9/2}}{x^{14}} dx &= -\frac{(a+bx^2)^{11/2}}{13ax^{13}} - \frac{(2b)\int \frac{(a+bx^2)^{9/2}}{x^{12}} dx}{13a} \\
&= -\frac{(a+bx^2)^{11/2}}{13ax^{13}} + \frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^2\right)^{11/2}\left(2bx^2-11a\right)}{143a^2x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^14,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-11*a + 2*b*x^2))/(143*a^2*x^13)
```

IntegrateAlgebraic [A] time = 0.19, size = 86, normalized size = 1.95

$$\frac{\sqrt{a+bx^2}\left(-11a^6-53a^5bx^2-100a^4b^2x^4-90a^3b^3x^6-35a^2b^4x^8-ab^5x^{10}+2b^6x^{12}\right)}{143a^2x^{13}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^14,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-11*a^6 - 53*a^5*b*x^2 - 100*a^4*b^2*x^4 - 90*a^3*b^3*x^6
 - 35*a^2*b^4*x^8 - a*b^5*x^10 + 2*b^6*x^12))/(143*a^2*x^13)
```

fricas [B] time = 1.09, size = 82, normalized size = 1.86

$$\frac{\left(2\,b^6x^{12}-ab^5x^{10}-35\,a^2b^4x^8-90\,a^3b^3x^6-100\,a^4b^2x^4-53\,a^5bx^2-11\,a^6\right)\sqrt{bx^2+a}}{143\,a^2x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/143*(2*b^6*x^12 - a*b^5*x^10 - 35*a^2*b^4*x^8 - 90*a^3*b^3*x^6 - 100*a^4*
b^2*x^4 - 53*a^5*b*x^2 - 11*a^6)*sqrt(b*x^2 + a)/(a^2*x^13)
```

giac [B] time = 1.09, size = 328, normalized size = 7.45

$$\frac{4\left(143\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}b^{\frac{13}{2}}+429\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}ab^{\frac{13}{2}}+2145\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^2b^{\frac{13}{2}}+3003\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^3b^{\frac{13}{2}}+6006\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^4b^{\frac{13}{2}}+4290\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^5b^{\frac{13}{2}}+4290\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^6b^{\frac{13}{2}}+1430\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^7b^{\frac{13}{2}}+715\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^8b^{\frac{13}{2}}+65\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^9b^{\frac{13}{2}}+13\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{10}b^{\frac{13}{2}}-a^{11}b^{\frac{13}{2}}\right)}{143\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="giac")
```

```
[Out] 4/143*(143*(sqrt(b)*x - sqrt(b*x^2 + a))^22*b^(13/2) + 429*(sqrt(b)*x - sqr
t(b*x^2 + a))^20*a*b^(13/2) + 2145*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^2*b^(
13/2) + 3003*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^3*b^(13/2) + 6006*(sqrt(b)*
```

```
x - sqrt(b*x^2 + a))^14*a^4*b^(13/2) + 4290*(sqrt(b)*x - sqrt(b*x^2 + a))^1
2*a^5*b^(13/2) + 4290*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^6*b^(13/2) + 1430*
(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*b^(13/2) + 715*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*a^8*b^(13/2) + 65*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^9*b^(13/2) + 13
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*b^(13/2) - a^11*b^(13/2))/((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)^13
```

maple [A] time = 0.01, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-2b\,x^{2}+11a\right)}{143a^{2}x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^14,x)
```

```
[Out] -1/143*(b*x^2+a)^(11/2)*(-2*b*x^2+11*a)/x^13/a^2
```

maxima [A] time = 1.57, size = 36, normalized size = 0.82

$$\frac{2\left(bx^{2}+a\right)^{\frac{11}{2}}b}{143\,a^{2}x^{11}}-\frac{\left(bx^{2}+a\right)^{\frac{11}{2}}}{13\,ax^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="maxima")
```

```
[Out] 2/143*(b*x^2 + a)^(11/2)*b/(a^2*x^11) - 1/13*(b*x^2 + a)^(11/2)/(a*x^13)
```

mupad [B] time = 6.85, size = 131, normalized size = 2.98

$$\frac{2\,b^{6}\,\sqrt{b\,x^{2}+a}}{143\,a^{2}\,x}-\frac{35\,b^{4}\,\sqrt{b\,x^{2}+a}}{143\,x^{5}}-\frac{90\,a\,b^{3}\,\sqrt{b\,x^{2}+a}}{143\,x^{7}}-\frac{53\,a^{3}\,b\,\sqrt{b\,x^{2}+a}}{143\,x^{11}}-\frac{b^{5}\,\sqrt{b\,x^{2}+a}}{143\,a\,x^{3}}-\frac{a^{4}\,\sqrt{b\,x^{2}+a}}{13\,x^{13}}-\frac{100\,a^{2}\,b^{2}\,\sqrt{b\,x^{2}+a}}{143\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^14,x)
```

```
[Out] (2*b^6*(a + b*x^2)^(1/2))/(143*a^2*x) - (35*b^4*(a + b*x^2)^(1/2))/(143*x^5
) - (90*a*b^3*(a + b*x^2)^(1/2))/(143*x^7) - (53*a^3*b*(a + b*x^2)^(1/2))/(
143*x^11) - (b^5*(a + b*x^2)^(1/2))/(143*a*x^3) - (a^4*(a + b*x^2)^(1/2))/(
13*x^13) - (100*a^2*b^2*(a + b*x^2)^(1/2))/(143*x^9)
```

sympy [B] time = 2.98, size = 175, normalized size = 3.98

$$-\frac{a^{4}\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{13x^{12}}-\frac{53a^{3}b^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143x^{10}}-\frac{100a^{2}b^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143x^{8}}-\frac{90ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143x^{6}}-\frac{35b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143x^{4}}-\frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143ax^{2}}+\frac{2b^{\frac{13}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{143a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**14,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(13*x**12) - 53*a**3*b**(3/2)*sqrt(a/(b*
x**2) + 1)/(143*x**10) - 100*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(143*x**8)
- 90*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(143*x**6) - 35*b**(9/2)*sqrt(a/(b*x**
2) + 1)/(143*x**4) - b**(11/2)*sqrt(a/(b*x**2) + 1)/(143*a*x**2) + 2*b**(13
/2)*sqrt(a/(b*x**2) + 1)/(143*a**2)
```

3.427 $\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx$

Optimal. Leaf size=68

$$-\frac{8b^2\left(a+bx^2\right)^{11/2}}{2145a^3x^{11}}+\frac{4b\left(a+bx^2\right)^{11/2}}{195a^2x^{13}}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8b^2\left(a+bx^2\right)^{11/2}}{2145a^3x^{11}}+\frac{4b\left(a+bx^2\right)^{11/2}}{195a^2x^{13}}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^16,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(15*a*x^15) + (4*b*(a + b*x^2)^(11/2))/(195*a^2*x^13) -
 (8*b^2*(a + b*x^2)^(11/2))/(2145*a^3*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{16}}\,dx = -\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}} - \frac{(4b)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{14}}\,dx}{15a}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}} + \frac{4b\left(a+bx^2\right)^{11/2}}{195a^2x^{13}} + \frac{\left(8b^2\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{12}}\,dx}{195a^2}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}} + \frac{4b\left(a+bx^2\right)^{11/2}}{195a^2x^{13}} - \frac{8b^2\left(a+bx^2\right)^{11/2}}{2145a^3x^{11}}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^2\right)^{11/2}\left(143a^2-44abx^2+8b^2x^4\right)}{2145a^3x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^16,x]
[Out] -1/2145*((a + b*x^2)^(11/2)*(143*a^2 - 44*a*b*x^2 + 8*b^2*x^4))/(a^3*x^15)
```

IntegrateAlgebraic [A] time = 0.23, size = 97, normalized size = 1.43

$$\frac{\sqrt{a+bx^2}\left(-143a^7-671a^6bx^2-1218a^5b^2x^4-1030a^4b^3x^6-355a^3b^4x^8-3a^2b^5x^{10}+4ab^6x^{12}-8b^7x^{14}\right)}{2145a^3x^{15}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^16,x]
[Out] (Sqrt[a + b*x^2]*(-143*a^7 - 671*a^6*b*x^2 - 1218*a^5*b^2*x^4 - 1030*a^4*b^
3*x^6 - 355*a^3*b^4*x^8 - 3*a^2*b^5*x^10 + 4*a*b^6*x^12 - 8*b^7*x^14))/(214
5*a^3*x^15)
```

fricas [A] time = 1.28, size = 93, normalized size = 1.37

$$-\frac{\left(8\,b^7x^{14}-4\,ab^6x^{12}+3\,a^2b^5x^{10}+355\,a^3b^4x^8+1030\,a^4b^3x^6+1218\,a^5b^2x^4+671\,a^6bx^2+143\,a^7\right)\sqrt{bx^2+a}}{2145\,a^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/2145*(8*b^7*x^14 - 4*a*b^6*x^12 + 3*a^2*b^5*x^10 + 355*a^3*b^4*x^8 + 103
0*a^4*b^3*x^6 + 1218*a^5*b^2*x^4 + 671*a^6*b*x^2 + 143*a^7)*sqrt(b*x^2 + a)
/(a^3*x^15)
```

giac [B] time = 1.22, size = 354, normalized size = 5.21

$$\frac{16\left(1430\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{24}b^{\frac{15}{2}}+6435\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}ab^{\frac{15}{2}}+24453\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}a^2b^{\frac{15}{2}}+45045\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^3b^{\frac{15}{2}}+70785\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^4b^{\frac{15}{2}}+64350\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^5b^{\frac{15}{2}}+50050\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^6b^{\frac{15}{2}}+21450\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^7b^{\frac{15}{2}}+7800\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^8b^{\frac{15}{2}}+975\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^9b^{\frac{15}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^{10}b^{\frac{15}{2}}-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{11}b^{\frac{15}{2}}+a^{12}b^{\frac{15}{2}}\right)}{2145\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="giac")
```

```
[Out] 16/2145*(1430*(sqrt(b)*x - sqrt(b*x^2 + a))^24*b^(15/2) + 6435*(sqrt(b)*x -
 sqrt(b*x^2 + a))^22*a*b^(15/2) + 24453*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^
2*b^(15/2) + 45045*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^3*b^(15/2) + 70785*(s
qrt(b)*x - sqrt(b*x^2 + a))^16*a^4*b^(15/2) + 64350*(sqrt(b)*x - sqrt(b*x^2
 + a))^14*a^5*b^(15/2) + 50050*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^6*b^(15/2
) + 21450*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^7*b^(15/2) + 7800*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*a^8*b^(15/2) + 975*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^9*
b^(15/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^10*b^(15/2) - 15*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*a^11*b^(15/2) + a^12*b^(15/2))/((sqrt(b)*x - sqrt(b*
x^2 + a))^2 - a)^15
```

maple [A] time = 0.00, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(8b^2x^4-44ab\,x^2+143a^2\right)}{2145a^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^16,x)
```

```
[Out] -1/2145*(b*x^2+a)^(11/2)*(8*b^2*x^4-44*a*b*x^2+143*a^2)/x^15/a^3
```

maxima [A] time = 1.51, size = 56, normalized size = 0.82

$$-\frac{8\left(bx^2+a\right)^{\frac{11}{2}}b^2}{2145\,a^3x^{11}}+\frac{4\left(bx^2+a\right)^{\frac{11}{2}}b}{195\,a^2x^{13}}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{15\,ax^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="maxima")
```

```
[Out] -8/2145*(b*x^2 + a)^(11/2)*b^2/(a^3*x^11) + 4/195*(b*x^2 + a)^(11/2)*b/(a^2
*x^13) - 1/15*(b*x^2 + a)^(11/2)/(a*x^15)
```

mupad [B] time = 7.42, size = 151, normalized size = 2.22

$$\frac{4\,b^6\,\sqrt{b\,x^2+a}}{2145\,a^2\,x^3}-\frac{71\,b^4\,\sqrt{b\,x^2+a}}{429\,x^7}-\frac{206\,a\,b^3\,\sqrt{b\,x^2+a}}{429\,x^9}-\frac{61\,a^3\,b\,\sqrt{b\,x^2+a}}{195\,x^{13}}-\frac{b^5\,\sqrt{b\,x^2+a}}{715\,a\,x^5}-\frac{a^4\,\sqrt{b\,x^2+a}}{15\,x^{15}}-\frac{8\,b^7\,\sqrt{b\,x^2+a}}{2145\,a^3\,x}-\frac{406\,a^2\,b^2\,\sqrt{b\,x^2+a}}{715\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^16,x)
```

```
[Out] (4*b^6*(a + b*x^2)^(1/2))/(2145*a^2*x^3) - (71*b^4*(a + b*x^2)^(1/2))/(429*
x^7) - (206*a*b^3*(a + b*x^2)^(1/2))/(429*x^9) - (61*a^3*b*(a + b*x^2)^(1/2
))/(195*x^13) - (b^5*(a + b*x^2)^(1/2))/(715*a*x^5) - (a^4*(a + b*x^2)^(1/2
))/(15*x^15) - (8*b^7*(a + b*x^2)^(1/2))/(2145*a^3*x) - (406*a^2*b^2*(a + b
*x^2)^(1/2))/(715*x^11)
```

sympy [B] time = 3.93, size = 604, normalized size = 8.88

$$-\frac{143a^9b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^6\left(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}\right)}-\frac{957a^8b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^4\left(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}\right)}-\frac{2703a^7b^{\frac{13}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^2\left(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}\right)}-\frac{4137a^6b^{\frac{15}{2}}\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{3633a^5b^{\frac{17}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{1743a^4b^{\frac{19}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{357a^3b^{\frac{21}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{3a^2b^{\frac{23}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{12ab^{\frac{25}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}-\frac{8b^{\frac{27}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**16,x)
```

```
[Out] -143*a**9*b**(9/2)*sqrt(a/(b*x**2) + 1)/(x**6*(2145*a**5*b**4*x**8 + 4290*a
**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 957*a**8*b**(11/2)*sqrt(a/(b*x**2
) + 1)/(x**4*(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x
**12)) - 2703*a**7*b**(13/2)*sqrt(a/(b*x**2) + 1)/(x**2*(2145*a**5*b**4*x**
8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 4137*a**6*b**(15/2)*sqr
t(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b
**6*x**12) - 3633*a**5*b**(17/2)*x**2*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*
x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 1743*a**4*b**(19/2)*x
**4*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145
*a**3*b**6*x**12) - 357*a**3*b**(21/2)*x**6*sqrt(a/(b*x**2) + 1)/(2145*a**5
*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 3*a**2*b**(23/2
)*x**8*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2
145*a**3*b**6*x**12) - 12*a*b**(25/2)*x**10*sqrt(a/(b*x**2) + 1)/(2145*a**5
*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 8*b**(27/2)*x**
12*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*
a**3*b**6*x**12)
```

3.428 $\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx$

Optimal. Leaf size=92

$$\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}}-\frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}}+\frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}}-\frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}}+\frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^18,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(17*a*x^17) + (2*b*(a + b*x^2)^(11/2))/(85*a^2*x^15) -
(8*b^2*(a + b*x^2)^(11/2))/(1105*a^3*x^13) + (16*b^3*(a + b*x^2)^(11/2))/(1
2155*a^4*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{18}}\,dx = -\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}} - \frac{(6b)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{16}}\,dx}{17a}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{11/2}}{85a^2x^{15}} + \frac{\left(8b^2\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{14}}\,dx}{85a^2}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{11/2}}{85a^2x^{15}} - \frac{8b^2\left(a+bx^2\right)^{11/2}}{1105a^3x^{13}} - \frac{\left(16b^3\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{12}}\,dx}{1105a^3}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}} + \frac{2b\left(a+bx^2\right)^{11/2}}{85a^2x^{15}} - \frac{8b^2\left(a+bx^2\right)^{11/2}}{1105a^3x^{13}} + \frac{16b^3\left(a+bx^2\right)^{11/2}}{12155a^4x^{11}}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^2\right)^{11/2}\left(-715a^3+286a^2bx^2-88ab^2x^4+16b^3x^6\right)}{12155a^4x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^18,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-715*a^3 + 286*a^2*b*x^2 - 88*a*b^2*x^4 + 16*b^3*x^6))
/(12155*a^4*x^17)
```

IntegrateAlgebraic [A] time = 0.23, size = 108, normalized size = 1.17

$$\frac{\sqrt{a+bx^2}\left(-715a^8-3289a^7bx^2-5808a^6b^2x^4-4714a^5b^3x^6-1515a^4b^4x^8-5a^3b^5x^{10}+6a^2b^6x^{12}-8ab^7x^{14}+16b^8x^{16}\right)}{12155a^4x^{17}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^18,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-715*a^8 - 3289*a^7*b*x^2 - 5808*a^6*b^2*x^4 - 4714*a^5*b
^3*x^6 - 1515*a^4*b^4*x^8 - 5*a^3*b^5*x^10 + 6*a^2*b^6*x^12 - 8*a*b^7*x^14
+ 16*b^8*x^16))/(12155*a^4*x^17)
```

fricas [A] time = 1.88, size = 104, normalized size = 1.13

$$\frac{\left(16\,b^8x^{16}-8\,ab^7x^{14}+6\,a^2b^6x^{12}-5\,a^3b^5x^{10}-1515\,a^4b^4x^8-4714\,a^5b^3x^6-5808\,a^6b^2x^4-3289\,a^7bx^2-715\,a^8\right)\sqrt{bx^2+a}}{12155\,a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="fricas")
```

```
[Out] 1/12155*(16*b^8*x^16 - 8*a*b^7*x^14 + 6*a^2*b^6*x^12 - 5*a^3*b^5*x^10 - 151
5*a^4*b^4*x^8 - 4714*a^5*b^3*x^6 - 5808*a^6*b^2*x^4 - 3289*a^7*b*x^2 - 715*
a^8)*sqrt(b*x^2 + a)/(a^4*x^17)
```

giac [B] time = 1.30, size = 382, normalized size = 4.15

$$\frac{32\left(12155\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{26}b^{\frac{17}{2}}+65637\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{24}ab^{\frac{17}{2}}+233376\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}a^2b^{\frac{17}{2}}+466752\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}a^3b^{\frac{17}{2}}+692835\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}a^4b^{\frac{17}{2}}+668525\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^5b^{\frac{17}{2}}+486200\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^6b^{\frac{17}{2}}+221000\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^7b^{\frac{17}{2}}+71825\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^8b^{\frac{17}{2}}+9775\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^9b^{\frac{17}{2}}+680\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^{10}b^{\frac{17}{2}}-136\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^{11}b^{\frac{17}{2}}+17\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^{12}b^{\frac{17}{2}}-a^{13}b^{\frac{17}{2}}\right)}{12155\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="giac")
```

```
[Out] 32/12155*(12155*(sqrt(b)*x - sqrt(b*x^2 + a))^26*b^(17/2) + 65637*(sqrt(b)*
x - sqrt(b*x^2 + a))^24*a*b^(17/2) + 233376*(sqrt(b)*x - sqrt(b*x^2 + a))^2
2*a^2*b^(17/2) + 466752*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^3*b^(17/2) + 692
835*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^4*b^(17/2) + 668525*(sqrt(b)*x - sqr
t(b*x^2 + a))^16*a^5*b^(17/2) + 486200*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^6
*b^(17/2) + 221000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^7*b^(17/2) + 71825*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^8*b^(17/2) + 9775*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^9*b^(17/2) + 680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^10*b^(17/2) -
136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^11*b^(17/2) + 17*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a^12*b^(17/2) - a^13*b^(17/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^17
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}\left(-16b^3x^6+88a\,b^2x^4-286a^2b\,x^2+715a^3\right)}{12155a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^18,x)
```

```
[Out] -1/12155*(b*x^2+a)^(11/2)*(-16*b^3*x^6+88*a*b^2*x^4-286*a^2*b*x^2+715*a^3)/
x^17/a^4
```

maxima [A] time = 1.55, size = 76, normalized size = 0.83

$$\frac{16\left(bx^2+a\right)^{\frac{11}{2}}b^3}{12155\,a^4x^{11}}-\frac{8\left(bx^2+a\right)^{\frac{11}{2}}b^2}{1105\,a^3x^{13}}+\frac{2\left(bx^2+a\right)^{\frac{11}{2}}b}{85\,a^2x^{15}}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{17\,ax^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="maxima")
```

```
[Out] 16/12155*(b*x^2 + a)^(11/2)*b^3/(a^4*x^11) - 8/1105*(b*x^2 + a)^(11/2)*b^2/
(a^3*x^13) + 2/85*(b*x^2 + a)^(11/2)*b/(a^2*x^15) - 1/17*(b*x^2 + a)^(11/2)
/(a*x^17)
```

mupad [B] time = 8.17, size = 171, normalized size = 1.86

$$\frac{6\,b^6\,\sqrt{b\,x^2+a}}{12155\,a^2\,x^5}-\frac{303\,b^4\,\sqrt{b\,x^2+a}}{2431\,x^9}-\frac{4714\,a\,b^3\,\sqrt{b\,x^2+a}}{12155\,x^{11}}-\frac{23\,a^3\,b\,\sqrt{b\,x^2+a}}{85\,x^{15}}-\frac{b^5\,\sqrt{b\,x^2+a}}{2431\,a\,x^7}-\frac{a^4\,\sqrt{b\,x^2+a}}{17\,x^{17}}-\frac{8\,b^7\,\sqrt{b\,x^2+a}}{12155\,a^3\,x^3}+\frac{16\,b^8\,\sqrt{b\,x^2+a}}{12155\,a^4\,x}-\frac{528\,a^2\,b^2\,\sqrt{b\,x^2+a}}{1105\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^18,x)
```

```
[Out] (6*b^6*(a + b*x^2)^(1/2))/(12155*a^2*x^5) - (303*b^4*(a + b*x^2)^(1/2))/(24
31*x^9) - (4714*a*b^3*(a + b*x^2)^(1/2))/(12155*x^11) - (23*a^3*b*(a + b*x^
2)^(1/2))/(85*x^15) - (b^5*(a + b*x^2)^(1/2))/(2431*a*x^7) - (a^4*(a + b*x^
2)^(1/2))/(17*x^17) - (8*b^7*(a + b*x^2)^(1/2))/(12155*a^3*x^3) + (16*b^8*(
a + b*x^2)^(1/2))/(12155*a^4*x) - (528*a^2*b^2*(a + b*x^2)^(1/2))/(1105*x^1
3)
```

sympy [B] time = 4.97, size = 867, normalized size = 9.42

$$-\frac{715a^{11}b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}-\frac{5434a^{10}b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}-\frac{17820a^9b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}-\frac{32720a^8b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}-\frac{36370a^7b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}-\frac{24500a^6b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}-\frac{9268a^5b^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}-\frac{1520a^4b^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}+\frac{5a^3b^{\frac{35}{2}}x^{16}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}+\frac{30a^2b^{\frac{37}{2}}x^{18}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}+\frac{40ab^{\frac{39}{2}}x^{20}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}+\frac{16b^{\frac{41}{2}}x^{22}\sqrt{\frac{a}{bx^2}+1}}{12155a^7b^9x^{16}+36465a^6b^{10}x^{18}+36465a^5b^{11}x^{20}+12155a^4b^{12}x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**18,x)
```

```
[Out] -715*a**11*b**(19/2)*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a*
*6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 5434*a*
*10*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6
*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 17820*a**
9*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b
**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 32720*a**8*
b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**
10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 36370*a**7*b*
*(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10
*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 24500*a**6*b**(
29/2)*x**10*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*
x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 9268*a**5*b**(31
/2)*x**12*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x*
*18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 1520*a**4*b**(33/2
)*x**14*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**1
8 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 5*a**3*b**(35/2)*x**
16*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 3
6465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 30*a**2*b**(37/2)*x**18*s
qrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465
```

```
*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 40*a*b**(39/2)*x**20*sqrt(a/(
b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b
**11*x**20 + 12155*a**4*b**12*x**22) + 16*b**(41/2)*x**22*sqrt(a/(b*x**2) +
 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b**11*x**2
0 + 12155*a**4*b**12*x**22)
```

3.429 $\int \frac{(a+bx^2)^{9/2}}{x^{20}} dx$

Optimal. Leaf size=116

$$-\frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}}+\frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}}+\frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}}-\frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}}+\frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^20,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(19*a*x^19) + (8*b*(a + b*x^2)^(11/2))/(323*a^2*x^17) -
 (16*b^2*(a + b*x^2)^(11/2))/(1615*a^3*x^15) + (64*b^3*(a + b*x^2)^(11/2))/
(20995*a^4*x^13) - (128*b^4*(a + b*x^2)^(11/2))/(230945*a^5*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{20}}\,dx = -\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}} - \frac{(8b)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{18}}\,dx}{19a}$$

$$= -\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}} + \frac{8b\left(a+bx^2\right)^{11/2}}{323a^2x^{17}} + \frac{\left(48b^2\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{16}}\,dx}{323a^2}$$

$$= -\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}} + \frac{8b\left(a+bx^2\right)^{11/2}}{323a^2x^{17}} - \frac{16b^2\left(a+bx^2\right)^{11/2}}{1615a^3x^{15}} - \frac{\left(64b^3\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{14}}\,dx}{1615a^3}$$

$$= -\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}} + \frac{8b\left(a+bx^2\right)^{11/2}}{323a^2x^{17}} - \frac{16b^2\left(a+bx^2\right)^{11/2}}{1615a^3x^{15}} + \frac{64b^3\left(a+bx^2\right)^{11/2}}{20995a^4x^{13}} + \frac{\left(128b^4\right)\int \frac{(a+}{}}{20995}$$

$$= -\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}} + \frac{8b\left(a+bx^2\right)^{11/2}}{323a^2x^{17}} - \frac{16b^2\left(a+bx^2\right)^{11/2}}{1615a^3x^{15}} + \frac{64b^3\left(a+bx^2\right)^{11/2}}{20995a^4x^{13}} - \frac{128b^4\left(a+bx\right.}{230945a^5x}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.55

$$-\frac{\left(a+bx^2\right)^{11/2}\left(12155a^4-5720a^3bx^2+2288a^2b^2x^4-704ab^3x^6+128b^4x^8\right)}{230945a^5x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^20,x]
```

```
[Out] -1/230945*((a + b*x^2)^(11/2)*(12155*a^4 - 5720*a^3*b*x^2 + 2288*a^2*b^2*x^
4 - 704*a*b^3*x^6 + 128*b^4*x^8))/(a^5*x^19)
```

IntegrateAlgebraic [A] time = 0.25, size = 119, normalized size = 1.03

$$\frac{\sqrt{a+bx^2}\left(-12155a^9-55055a^8bx^2-95238a^7b^2x^4-75086a^6b^3x^6-23063a^5b^4x^8-35a^4b^5x^{10}+40a^3b^6x^{12}-48a^2b^7x^{14}+64ab^8x^{16}-128b^9x^{18}\right)}{230945a^5x^{19}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^20,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-12155*a^9 - 55055*a^8*b*x^2 - 95238*a^7*b^2*x^4 - 75086*
a^6*b^3*x^6 - 23063*a^5*b^4*x^8 - 35*a^4*b^5*x^10 + 40*a^3*b^6*x^12 - 48*a^
2*b^7*x^14 + 64*a*b^8*x^16 - 128*b^9*x^18))/(230945*a^5*x^19)
```

fricas [A] time = 1.60, size = 115, normalized size = 0.99

$$-\frac{\left(128\,b^9x^{18}-64\,ab^8x^{16}+48\,a^2b^7x^{14}-40\,a^3b^6x^{12}+35\,a^4b^5x^{10}+23063\,a^5b^4x^8+75086\,a^6b^3x^6+95238\,a^7b^2x^4+55055\,a^8bx^2+12155\,a^9\right)\sqrt{bx^2+a}}{230945\,a^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/230945*(128*b^9*x^18 - 64*a*b^8*x^16 + 48*a^2*b^7*x^14 - 40*a^3*b^6*x^12
 + 35*a^4*b^5*x^10 + 23063*a^5*b^4*x^8 + 75086*a^6*b^3*x^6 + 95238*a^7*b^2*
x^4 + 55055*a^8*b*x^2 + 12155*a^9)*sqrt(b*x^2 + a)/(a^5*x^19)
```

giac [B] time = 1.17, size = 408, normalized size = 3.52

$$\frac{256 \left(92378 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{28} b^{\frac{19}{2}} + 554268 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{26} a b^{\frac{19}{2}} + 1939938 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{24} a^2 b^{\frac{19}{2}} + 4018443 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{22} a^3 b^{\frac{19}{2}} + 5866003 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{20} a^4 b^{\frac{19}{2}} + 5773625 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{18} a^5 b^{\frac{19}{2}} + 4094025 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{16} a^6 b^{\frac{19}{2}} + 1889550 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{14} a^7 b^{\frac{19}{2}} + 581400 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{12} a^8 b^{\frac{19}{2}} + 80750 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{10} a^9 b^{\frac{19}{2}} + 3876 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{8} a^{10} b^{\frac{19}{2}} - 969 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{6} a^{11} b^{\frac{19}{2}} + 171 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{4} a^{12} b^{\frac{19}{2}} - 19 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^{2} a^{13} b^{\frac{19}{2}} + a^{14} b^{\frac{19}{2}}\right)}{230945 \left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 - a\right)^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="giac")
```

```
[Out] 256/230945*(92378*(sqrt(b)*x - sqrt(b*x^2 + a))^28*b^(19/2) + 554268*(sqrt(
b)*x - sqrt(b*x^2 + a))^26*a*b^(19/2) + 1939938*(sqrt(b)*x - sqrt(b*x^2 + a
))^24*a^2*b^(19/2) + 4018443*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^3*b^(19/2)
+ 5866003*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^4*b^(19/2) + 5773625*(sqrt(b)*
x - sqrt(b*x^2 + a))^18*a^5*b^(19/2) + 4094025*(sqrt(b)*x - sqrt(b*x^2 + a)
)^16*a^6*b^(19/2) + 1889550*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^7*b^(19/2) +
 581400*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^8*b^(19/2) + 80750*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a^9*b^(19/2) + 3876*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^1
0*b^(19/2) - 969*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^11*b^(19/2) + 171*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*a^12*b^(19/2) - 19*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^13*b^(19/2) + a^14*b^(19/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^19
```

maple [A] time = 0.01, size = 61, normalized size = 0.53

$$-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}} \left(128 b^4 x^8 - 704 a\, b^3 x^6 + 2288 a^2 b^2 x^4 - 5720 a^3 b\, x^2 + 12155 a^4\right)}{230945 a^5 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^20,x)
```

```
[Out] -1/230945*(b*x^2+a)^(11/2)*(128*b^4*x^8-704*a*b^3*x^6+2288*a^2*b^2*x^4-5720
*a^3*b*x^2+12155*a^4)/x^19/a^5
```

maxima [A] time = 1.57, size = 96, normalized size = 0.83

$$-\frac{128 \left(bx^2+a\right)^{\frac{11}{2}} b^4}{230945\, a^5 x^{11}} + \frac{64 \left(bx^2+a\right)^{\frac{11}{2}} b^3}{20995\, a^4 x^{13}} - \frac{16 \left(bx^2+a\right)^{\frac{11}{2}} b^2}{1615\, a^3 x^{15}} + \frac{8 \left(bx^2+a\right)^{\frac{11}{2}} b}{323\, a^2 x^{17}} - \frac{\left(bx^2+a\right)^{\frac{11}{2}}}{19\, a x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="maxima")
```

```
[Out] -128/230945*(b*x^2 + a)^(11/2)*b^4/(a^5*x^11) + 64/20995*(b*x^2 + a)^(11/2)
*b^3/(a^4*x^13) - 16/1615*(b*x^2 + a)^(11/2)*b^2/(a^3*x^15) + 8/323*(b*x^2
+ a)^(11/2)*b/(a^2*x^17) - 1/19*(b*x^2 + a)^(11/2)/(a*x^19)
```

mupad [B] time = 8.84, size = 191, normalized size = 1.65

$$\frac{8\,b^6\,\sqrt{b\,x^2+a}}{46189\,a^2\,x^7}-\frac{23063\,b^4\,\sqrt{b\,x^2+a}}{230945\,x^{11}}-\frac{6826\,a\,b^3\,\sqrt{b\,x^2+a}}{20995\,x^{13}}-\frac{77\,a^3\,b\,\sqrt{b\,x^2+a}}{323\,x^{17}}-\frac{7\,b^5\,\sqrt{b\,x^2+a}}{46189\,a\,x^9}-\frac{a^4\,\sqrt{b\,x^2+a}}{19\,x^{19}}-\frac{48\,b^7\,\sqrt{b\,x^2+a}}{230945\,a^3\,x^5}+\frac{64\,b^8\,\sqrt{b\,x^2+a}}{230945\,a^4\,x^3}-\frac{128\,b^9\,\sqrt{b\,x^2+a}}{230945\,a^5\,x}-\frac{666\,a^2\,b^2\,\sqrt{b\,x^2+a}}{1615\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^20,x)
```

```
[Out] (8*b^6*(a + b*x^2)^(1/2))/(46189*a^2*x^7) - (23063*b^4*(a + b*x^2)^(1/2))/(
230945*x^11) - (6826*a*b^3*(a + b*x^2)^(1/2))/(20995*x^13) - (77*a^3*b*(a +
 b*x^2)^(1/2))/(323*x^17) - (7*b^5*(a + b*x^2)^(1/2))/(46189*a*x^9) - (a^4*
(a + b*x^2)^(1/2))/(19*x^19) - (48*b^7*(a + b*x^2)^(1/2))/(230945*a^3*x^5)
+ (64*b^8*(a + b*x^2)^(1/2))/(230945*a^4*x^3) - (128*b^9*(a + b*x^2)^(1/2))
/(230945*a^5*x) - (666*a^2*b^2*(a + b*x^2)^(1/2))/(1615*x^15)
```

sympy [B] time = 6.26, size = 1182, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**20,x)
```

```
[Out] -12155*a**13*b**(33/2)*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 9237
80*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 +
230945*a**5*b**20*x**26) - 103675*a**12*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)
/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x*
*22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 388388*a**11*b**
(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**
17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5
*b**20*x**26) - 834988*a**10*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(230945*a*
*9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 92378
0*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 1127210*a**9*b**(41/2)*x**8
*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 +
1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**2
6) - 978810*a**8*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x*
*18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**1
9*x**24 + 230945*a**5*b**20*x**26) - 534060*a**7*b**(45/2)*x**12*sqrt(a/(b*
x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**
7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 167436
*a**6*b**(47/2)*x**14*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 92378
0*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 2
```

```
30945*a**5*b**20*x**26) - 23091*a**5*b**(49/2)*x**16*sqrt(a/(b*x**2) + 1)/(
230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**2
2 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 35*a**4*b**(51/2)*
x**18*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**
20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20
*x**26) - 280*a**3*b**(53/2)*x**20*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*
x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b*
*19*x**24 + 230945*a**5*b**20*x**26) - 560*a**2*b**(55/2)*x**22*sqrt(a/(b*x
**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7
*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 448*a*b
**(57/2)*x**24*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*
b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a
**5*b**20*x**26) - 128*b**(59/2)*x**26*sqrt(a/(b*x**2) + 1)/(230945*a**9*b*
*16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**
6*b**19*x**24 + 230945*a**5*b**20*x**26)
```

3.430 $\int \frac{(a+bx^2)^{9/2}}{x^{22}} dx$

Optimal. Leaf size=140

$$\frac{256b^5(a+bx^2)^{11/2}}{969969a^6x^{11}}-\frac{128b^4(a+bx^2)^{11/2}}{88179a^5x^{13}}+\frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}}-\frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}}+\frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}}-\frac{(a+bx^2)^{11/2}}{21ax^{21}}$$

Rubi [A] time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{256b^5(a+bx^2)^{11/2}}{969969a^6x^{11}}-\frac{128b^4(a+bx^2)^{11/2}}{88179a^5x^{13}}+\frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}}-\frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}}+\frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}}-\frac{(a+bx^2)^{11/2}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^22,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(21*a*x^21) + (10*b*(a + b*x^2)^(11/2))/(399*a^2*x^19)
- (80*b^2*(a + b*x^2)^(11/2))/(6783*a^3*x^17) + (32*b^3*(a + b*x^2)^(11/2))
/(6783*a^4*x^15) - (128*b^4*(a + b*x^2)^(11/2))/(88179*a^5*x^13) + (256*b^5
*(a + b*x^2)^(11/2))/(969969*a^6*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{22}}\,dx = -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} - \frac{(10b)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{20}}\,dx}{21a}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} + \frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}} + \frac{\left(80b^2\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{18}}\,dx}{399a^2}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} + \frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}} - \frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}} - \frac{\left(160b^3\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{16}}\,dx}{2261a^3}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} + \frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}} - \frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}} + \frac{32b^3\left(a+bx^2\right)^{11/2}}{6783a^4x^{15}} + \frac{\left(128b^4\right)\int}{678}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} + \frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}} - \frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}} + \frac{32b^3\left(a+bx^2\right)^{11/2}}{6783a^4x^{15}} - \frac{128b^4\left(a+\right.}{88179a}$$
$$= -\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}} + \frac{10b\left(a+bx^2\right)^{11/2}}{399a^2x^{19}} - \frac{80b^2\left(a+bx^2\right)^{11/2}}{6783a^3x^{17}} + \frac{32b^3\left(a+bx^2\right)^{11/2}}{6783a^4x^{15}} - \frac{128b^4\left(a+\right.}{88179a}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 0.54

$$\frac{\left(a+bx^2\right)^{11/2}\left(-46189a^5+24310a^4bx^2-11440a^3b^2x^4+4576a^2b^3x^6-1408ab^4x^8+256b^5x^{10}\right)}{969969a^6x^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^22,x]
```

```
[Out] ((a + b*x^2)^(11/2)*(-46189*a^5 + 24310*a^4*b*x^2 - 11440*a^3*b^2*x^4 + 457
6*a^2*b^3*x^6 - 1408*a*b^4*x^8 + 256*b^5*x^10))/(969969*a^6*x^21)
```

IntegrateAlgebraic [A] time = 0.26, size = 130, normalized size = 0.93

$$\frac{\sqrt{a+bx^2}\left(-46189a^{10}-206635a^9bx^2-351780a^8b^2x^4-271414a^7b^3x^6-80773a^6b^4x^8-63a^5b^5x^{10}+70a^4b^6x^{12}-80a^3b^7x^{14}+96a^2b^8x^{16}-128ab^9x^{18}+256b^{10}x^{20}\right)}{969969a^6x^{21}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^22,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-46189*a^10 - 206635*a^9*b*x^2 - 351780*a^8*b^2*x^4 - 271
414*a^7*b^3*x^6 - 80773*a^6*b^4*x^8 - 63*a^5*b^5*x^10 + 70*a^4*b^6*x^12 - 8
0*a^3*b^7*x^14 + 96*a^2*b^8*x^16 - 128*a*b^9*x^18 + 256*b^10*x^20))/(969969
*a^6*x^21)
```

fricas [A] time = 2.72, size = 126, normalized size = 0.90

$$\frac{\left(256\,b^{10}x^{20} - 128\,ab^{9}x^{18} + 96\,a^{2}b^{8}x^{16} - 80\,a^{3}b^{7}x^{14} + 70\,a^{4}b^{6}x^{12} - 63\,a^{5}b^{5}x^{10} - 80773\,a^{6}b^{4}x^{8} - 271414\,a^{7}b^{3}x^{6} - 351780\,a^{8}b^{2}x^{4} - 206635\,a^{9}bx^{2} - 46189\,a^{10}\right)\sqrt{bx^{2}+a}}{969969\,a^{6}x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="fricas")
```

```
[Out] 1/969969*(256*b^10*x^20 - 128*a*b^9*x^18 + 96*a^2*b^8*x^16 - 80*a^3*b^7*x^1
4 + 70*a^4*b^6*x^12 - 63*a^5*b^5*x^10 - 80773*a^6*b^4*x^8 - 271414*a^7*b^3*
x^6 - 351780*a^8*b^2*x^4 - 206635*a^9*b*x^2 - 46189*a^10)*sqrt(b*x^2 + a)/(
a^6*x^21)
```

giac [B] time = 1.25, size = 436, normalized size = 3.11

$$\frac{512\left(646646\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{30}b^{\frac{21}{2}} + 4157010\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{28}ab^{\frac{21}{2}} + 14549535\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{26}a^{2}b^{\frac{21}{2}} + 30715685\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{24}a^{3}b^{\frac{21}{2}} + 44618574\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{22}a^{4}b^{\frac{21}{2}} + 44265858\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{20}a^{5}b^{\frac{21}{2}} + 31009615\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{18}a^{6}b^{\frac{21}{2}} + 14346045\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{16}a^{7}b^{\frac{21}{2}} + 4273290\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{14}a^{8}b^{\frac{21}{2}} + 592382\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}a^{9}b^{\frac{21}{2}} + 20349\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{10}a^{10}b^{\frac{21}{2}} - 5985\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}a^{11}b^{\frac{21}{2}} + 1330\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}a^{12}b^{\frac{21}{2}} - 210\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}a^{13}b^{\frac{21}{2}} + 21\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}a^{14}b^{\frac{21}{2}} - a^{15}b^{\frac{21}{2}}\right)}{969969\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="giac")
```

```
[Out] 512/969969*(646646*(sqrt(b)*x - sqrt(b*x^2 + a))^30*b^(21/2) + 4157010*(sqr
t(b)*x - sqrt(b*x^2 + a))^28*a*b^(21/2) + 14549535*(sqrt(b)*x - sqrt(b*x^2
+ a))^26*a^2*b^(21/2) + 30715685*(sqrt(b)*x - sqrt(b*x^2 + a))^24*a^3*b^(21
/2) + 44618574*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^4*b^(21/2) + 44265858*(sq
rt(b)*x - sqrt(b*x^2 + a))^20*a^5*b^(21/2) + 31009615*(sqrt(b)*x - sqrt(b*x
^2 + a))^18*a^6*b^(21/2) + 14346045*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^7*b^
(21/2) + 4273290*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^8*b^(21/2) + 592382*(sq
rt(b)*x - sqrt(b*x^2 + a))^12*a^9*b^(21/2) + 20349*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^10*b^(21/2) - 5985*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^11*b^(21/2)
 + 1330*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^12*b^(21/2) - 210*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*a^13*b^(21/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^14*b^
(21/2) - a^15*b^(21/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^21
```

maple [A] time = 0.01, size = 72, normalized size = 0.51

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-256b^{5}x^{10}+1408a\,b^{4}x^{8}-4576a^{2}b^{3}x^{6}+11440a^{3}b^{2}x^{4}-24310a^{4}b\,x^{2}+46189a^{5}\right)}{969969a^{6}x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^22,x)
```

```
[Out] -1/969969*(b*x^2+a)^(11/2)*(-256*b^5*x^10+1408*a*b^4*x^8-4576*a^2*b^3*x^6+1
1440*a^3*b^2*x^4-24310*a^4*b*x^2+46189*a^5)/x^21/a^6
```

maxima [A] time = 1.53, size = 116, normalized size = 0.83

$$\frac{256\left(bx^2+a\right)^{\frac{11}{2}}b^5}{969969\,a^6x^{11}}-\frac{128\left(bx^2+a\right)^{\frac{11}{2}}b^4}{88179\,a^5x^{13}}+\frac{32\left(bx^2+a\right)^{\frac{11}{2}}b^3}{6783\,a^4x^{15}}-\frac{80\left(bx^2+a\right)^{\frac{11}{2}}b^2}{6783\,a^3x^{17}}+\frac{10\left(bx^2+a\right)^{\frac{11}{2}}b}{399\,a^2x^{19}}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{21\,ax^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="maxima")
```

```
[Out] 256/969969*(b*x^2 + a)^(11/2)*b^5/(a^6*x^11) - 128/88179*(b*x^2 + a)^(11/2)
*b^4/(a^5*x^13) + 32/6783*(b*x^2 + a)^(11/2)*b^3/(a^4*x^15) - 80/6783*(b*x^
2 + a)^(11/2)*b^2/(a^3*x^17) + 10/399*(b*x^2 + a)^(11/2)*b/(a^2*x^19) - 1/2
1*(b*x^2 + a)^(11/2)/(a*x^21)
```

mupad [B] time = 9.63, size = 211, normalized size = 1.51

$$\frac{10\,b^6\,\sqrt{b\,x^2+a}}{138567\,a^2\,x^9}-\frac{1049\,b^4\,\sqrt{b\,x^2+a}}{12597\,x^{13}}-\frac{1898\,a\,b^3\,\sqrt{b\,x^2+a}}{6783\,x^{15}}-\frac{85\,a^3\,b\,\sqrt{b\,x^2+a}}{399\,x^{19}}-\frac{3\,b^5\,\sqrt{b\,x^2+a}}{46189\,a\,x^{11}}-\frac{a^4\,\sqrt{b\,x^2+a}}{21\,x^{21}}-\frac{80\,b^7\,\sqrt{b\,x^2+a}}{969969\,a^3\,x^7}+\frac{32\,b^8\,\sqrt{b\,x^2+a}}{323323\,a^4\,x^5}-\frac{128\,b^9\,\sqrt{b\,x^2+a}}{969969\,a^5\,x^3}+\frac{256\,b^{10}\,\sqrt{b\,x^2+a}}{969969\,a^6\,x}-\frac{820\,a^2\,b^2\,\sqrt{b\,x^2+a}}{2261\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^22,x)
```

```
[Out] (10*b^6*(a + b*x^2)^(1/2))/(138567*a^2*x^9) - (1049*b^4*(a + b*x^2)^(1/2))/
(12597*x^13) - (1898*a*b^3*(a + b*x^2)^(1/2))/(6783*x^15) - (85*a^3*b*(a +
b*x^2)^(1/2))/(399*x^19) - (3*b^5*(a + b*x^2)^(1/2))/(46189*a*x^11) - (a^4*
(a + b*x^2)^(1/2))/(21*x^21) - (80*b^7*(a + b*x^2)^(1/2))/(969969*a^3*x^7)
+ (32*b^8*(a + b*x^2)^(1/2))/(323323*a^4*x^5) - (128*b^9*(a + b*x^2)^(1/2))
/(969969*a^5*x^3) + (256*b^10*(a + b*x^2)^(1/2))/(969969*a^6*x) - (820*a^2*
b^2*(a + b*x^2)^(1/2))/(2261*x^17)
```

sympy [B] time = 7.80, size = 1540, normalized size = 11.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**22,x)
```

```
[Out] -46189*a**15*b**(51/2)*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 484
9845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**2
6 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 437580*a**14*b**(
53/2)*x**2*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b
**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*
a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 1846845*a**13*b**(55/2)*x**4*
sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22
+ 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*
x**28 + 969969*a**6*b**30*x**30) - 4558554*a**12*b**(57/2)*x**6*sqrt(a/(b*x
```

```
**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a
**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969
969*a**6*b**30*x**30) - 7252938*a**11*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(
969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x
**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b*
*30*x**30) - 7715232*a**10*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(969969*a**
11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 969
9690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30)
 - 5487650*a**9*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x*
*20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b
**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 2516940*
a**8*b**(65/2)*x**14*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 48498
45*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26
+ 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 675513*a**7*b**(67/
2)*x**16*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**
26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a*
*7*b**29*x**28 + 969969*a**6*b**30*x**30) - 80836*a**6*b**(69/2)*x**18*sqrt
(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 96
99690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**2
8 + 969969*a**6*b**30*x**30) + 63*a**5*b**(71/2)*x**20*sqrt(a/(b*x**2) + 1)
/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27
*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*
b**30*x**30) + 630*a**4*b**(73/2)*x**22*sqrt(a/(b*x**2) + 1)/(969969*a**11*
b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 969969
0*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) +
1680*a**3*b**(75/2)*x**24*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 +
4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x
**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 2016*a**2*b**(
77/2)*x**26*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*
b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845
*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 1152*a*b**(79/2)*x**28*sqrt(
a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 969
9690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28
 + 969969*a**6*b**30*x**30) + 256*b**(81/2)*x**30*sqrt(a/(b*x**2) + 1)/(969
969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**2
4 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30
*x**30)
```

3.431 $\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx$

Optimal. Leaf size=164

$$-\frac{1024b^6\left(a+bx^2\right)^{11/2}}{7436429a^7x^{11}}+\frac{512b^5\left(a+bx^2\right)^{11/2}}{676039a^6x^{13}}-\frac{128b^4\left(a+bx^2\right)^{11/2}}{52003a^5x^{15}}+\frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}}-\frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}}+\frac{4b\,(}{1}$$

Rubi [A] time = 0.07, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{1024b^6\left(a+bx^2\right)^{11/2}}{7436429a^7x^{11}}+\frac{512b^5\left(a+bx^2\right)^{11/2}}{676039a^6x^{13}}-\frac{128b^4\left(a+bx^2\right)^{11/2}}{52003a^5x^{15}}+\frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}}-\frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}}+\frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}}-\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(9/2)/x^24,x]
```

```
[Out] -(a + b*x^2)^(11/2)/(23*a*x^23) + (4*b*(a + b*x^2)^(11/2))/(161*a^2*x^21) -
 (40*b^2*(a + b*x^2)^(11/2))/(3059*a^3*x^19) + (320*b^3*(a + b*x^2)^(11/2))
/(52003*a^4*x^17) - (128*b^4*(a + b*x^2)^(11/2))/(52003*a^5*x^15) + (512*b^
5*(a + b*x^2)^(11/2))/(676039*a^6*x^13) - (1024*b^6*(a + b*x^2)^(11/2))/(74
36429*a^7*x^11)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{9/2}}{x^{24}}\,dx &= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} - \frac{(12b)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{22}}\,dx}{23a} \\
&= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} + \frac{\left(40b^2\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{20}}\,dx}{161a^2} \\
&= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} - \frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}} - \frac{\left(320b^3\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^{18}}\,dx}{3059a^3} \\
&= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} - \frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}} + \frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}} + \frac{\left(1920b^4\right)\int}{5200} \\
&= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} - \frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}} + \frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}} - \frac{128b^4\left(a+b\right.}{52003a^5} \\
&= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} - \frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}} + \frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}} - \frac{128b^4\left(a+b\right.}{52003a^5} \\
&= -\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}} + \frac{4b\left(a+bx^2\right)^{11/2}}{161a^2x^{21}} - \frac{40b^2\left(a+bx^2\right)^{11/2}}{3059a^3x^{19}} + \frac{320b^3\left(a+bx^2\right)^{11/2}}{52003a^4x^{17}} - \frac{128b^4\left(a+b\right.}{52003a^5}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 86, normalized size = 0.52

$$
-\frac{\left(a+bx^2\right)^{11/2}\left(323323a^6-184756a^5bx^2+97240a^4b^2x^4-45760a^3b^3x^6+18304a^2b^4x^8-5632ab^5x^{10}+1024b^6x^{12}\right)}{7436429a^7x^{23}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(9/2)/x^24,x]
```

```
[Out] -1/7436429*((a + b*x^2)^(11/2)*(323323*a^6 - 184756*a^5*b*x^2 + 97240*a^4*b
^2*x^4 - 45760*a^3*b^3*x^6 + 18304*a^2*b^4*x^8 - 5632*a*b^5*x^10 + 1024*b^6
*x^12))/(a^7*x^23)
```

IntegrateAlgebraic [A] time = 0.27, size = 141, normalized size = 0.86

$$
\frac{\sqrt{a+bx^2}\left(-323323a^{11}-1431859a^{10}bx^2-2406690a^9b^2x^4-1826110a^8b^3x^6-530959a^7b^4x^8-231a^6b^5x^{10}+252a^5b^6x^{12}-280a^4b^7x^{14}+320a^3b^8x^{16}-384a^2b^9x^{18}+512ab^{10}x^{20}-1024b^{11}x^{22}\right)}{7436429a^7x^{23}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(9/2)/x^24,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-323323*a^11 - 1431859*a^10*b*x^2 - 2406690*a^9*b^2*x^4 -
 1826110*a^8*b^3*x^6 - 530959*a^7*b^4*x^8 - 231*a^6*b^5*x^10 + 252*a^5*b^6*
x^12 - 280*a^4*b^7*x^14 + 320*a^3*b^8*x^16 - 384*a^2*b^9*x^18 + 512*a*b^10*
x^20 - 1024*b^11*x^22))/(7436429*a^7*x^23)
```

fricas [A] time = 3.17, size = 137, normalized size = 0.84

$$-\frac{\left(1024\,b^{11}x^{22} - 512\,ab^{10}x^{20} + 384\,a^{2}b^{9}x^{18} - 320\,a^{3}b^{8}x^{16} + 280\,a^{4}b^{7}x^{14} - 252\,a^{5}b^{6}x^{12} + 231\,a^{6}b^{5}x^{10} + 530959\,a^{7}b^{4}x^{8} + 1826110\,a^{8}b^{3}x^{6} + 2406690\,a^{9}b^{2}x^{4} + 1431859\,a^{10}bx^{2} + 323323\,a^{11}\right)\sqrt{bx^{2} + a}}{7436429\,a^{7}x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/7436429*(1024*b^11*x^22 - 512*a*b^10*x^20 + 384*a^2*b^9*x^18 - 320*a^3*b
^8*x^16 + 280*a^4*b^7*x^14 - 252*a^5*b^6*x^12 + 231*a^6*b^5*x^10 + 530959*a
^7*b^4*x^8 + 1826110*a^8*b^3*x^6 + 2406690*a^9*b^2*x^4 + 1431859*a^10*b*x^2
 + 323323*a^11)*sqrt(b*x^2 + a)/(a^7*x^23)
```

giac [B] time = 1.23, size = 462, normalized size = 2.82

$$\frac{2048\left(4249388\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{32} b^{\frac{23}{2}} + 28683369\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{30} ab^{\frac{23}{2}} + 100922965\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{28} a^{2}b^{\frac{23}{2}} + 215656441\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{26} a^{3}b^{\frac{23}{2}} + 313006057\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{24} a^{4}b^{\frac{23}{2}} + 311653979\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{22} a^{5}b^{\frac{23}{2}} + 216800507\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{20} a^{6}b^{\frac{23}{2}} + 100105775\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{18} a^{7}b^{\frac{23}{2}} + 29173683\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{16} a^{8}b^{\frac{23}{2}} + 4004231\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{14} a^{9}b^{\frac{23}{2}} + 100947\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{12} a^{10}b^{\frac{23}{2}} - 33649\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{10} a^{11}b^{\frac{23}{2}} + 8855\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{8} a^{12}b^{\frac{23}{2}} - 1771\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{6} a^{13}b^{\frac{23}{2}} + 253\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{4} a^{14}b^{\frac{23}{2}} - 23\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{2} a^{15}b^{\frac{23}{2}} + a^{16}b^{\frac{23}{2}}\right)}{7436429\left(\left(\sqrt{b}x - \sqrt{bx^{2} + a}\right)^{2} - a\right)^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="giac")
```

```
[Out] 2048/7436429*(4249388*(sqrt(b)*x - sqrt(b*x^2 + a))^32*b^(23/2) + 28683369*
(sqrt(b)*x - sqrt(b*x^2 + a))^30*a*b^(23/2) + 100922965*(sqrt(b)*x - sqrt(b
*x^2 + a))^28*a^2*b^(23/2) + 215656441*(sqrt(b)*x - sqrt(b*x^2 + a))^26*a^3
*b^(23/2) + 313006057*(sqrt(b)*x - sqrt(b*x^2 + a))^24*a^4*b^(23/2) + 31165
3979*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^5*b^(23/2) + 216800507*(sqrt(b)*x -
 sqrt(b*x^2 + a))^20*a^6*b^(23/2) + 100105775*(sqrt(b)*x - sqrt(b*x^2 + a))
^18*a^7*b^(23/2) + 29173683*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^8*b^(23/2) +
 4004231*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^9*b^(23/2) + 100947*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*a^10*b^(23/2) - 33649*(sqrt(b)*x - sqrt(b*x^2 + a))^1
0*a^11*b^(23/2) + 8855*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^12*b^(23/2) - 1771
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^13*b^(23/2) + 253*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a^14*b^(23/2) - 23*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^15*b^(23/2)
+ a^16*b^(23/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^23
```

maple [A] time = 0.01, size = 83, normalized size = 0.51

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(1024x^{12}b^{6}-5632a\,x^{10}b^{5}+18304a^{2}x^{8}b^{4}-45760a^{3}x^{6}b^{3}+97240a^{4}x^{4}b^{2}-184756a^{5}x^{2}b+323323a^{6}\right)}{7436429a^{7}x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(9/2)/x^24,x)
```

```
[Out] -1/7436429*(b*x^2+a)^(11/2)*(1024*b^6*x^12-5632*a*b^5*x^10+18304*a^2*b^4*x^
8-45760*a^3*b^3*x^6+97240*a^4*b^2*x^4-184756*a^5*b*x^2+323323*a^6)/x^23/a^7
```

maxima [A] time = 1.59, size = 136, normalized size = 0.83

$$-\frac{1024\left(bx^2+a\right)^{\frac{11}{2}}b^6}{7436429\,a^7x^{11}}+\frac{512\left(bx^2+a\right)^{\frac{11}{2}}b^5}{676039\,a^6x^{13}}-\frac{128\left(bx^2+a\right)^{\frac{11}{2}}b^4}{52003\,a^5x^{15}}+\frac{320\left(bx^2+a\right)^{\frac{11}{2}}b^3}{52003\,a^4x^{17}}-\frac{40\left(bx^2+a\right)^{\frac{11}{2}}b^2}{3059\,a^3x^{19}}+\frac{4\left(bx^2+a\right)^{\frac{11}{2}}b}{161\,a^2x^{21}}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{23\,ax^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1024/7436429*(b*x^2 + a)^(11/2)*b^6/(a^7*x^11) + 512/676039*(b*x^2 + a)^(1
1/2)*b^5/(a^6*x^13) - 128/52003*(b*x^2 + a)^(11/2)*b^4/(a^5*x^15) + 320/520
03*(b*x^2 + a)^(11/2)*b^3/(a^4*x^17) - 40/3059*(b*x^2 + a)^(11/2)*b^2/(a^3*
x^19) + 4/161*(b*x^2 + a)^(11/2)*b/(a^2*x^21) - 1/23*(b*x^2 + a)^(11/2)/(a*
x^23)
```

mupad [B] time = 10.47, size = 231, normalized size = 1.41

$$\frac{36\,b^6\,\sqrt{b\,x^2+a}}{1062347\,a^2\,x^{11}}-\frac{3713\,b^4\,\sqrt{b\,x^2+a}}{52003\,x^{15}}-\frac{12770\,a\,b^3\,\sqrt{b\,x^2+a}}{52003\,x^{17}}-\frac{31\,a^3\,b\,\sqrt{b\,x^2+a}}{161\,x^{21}}-\frac{3\,b^5\,\sqrt{b\,x^2+a}}{96577\,a\,x^{13}}-\frac{a^4\,\sqrt{b\,x^2+a}}{23\,x^{23}}-\frac{40\,b^7\,\sqrt{b\,x^2+a}}{1062347\,a^3\,x^9}+\frac{320\,b^8\,\sqrt{b\,x^2+a}}{7436429\,a^4\,x^7}-\frac{384\,b^9\,\sqrt{b\,x^2+a}}{7436429\,a^5\,x^5}+\frac{512\,b^{10}\,\sqrt{b\,x^2+a}}{7436429\,a^6\,x^3}-\frac{1024\,b^{11}\,\sqrt{b\,x^2+a}}{7436429\,a^7\,x}-\frac{990\,a^2\,b^2\,\sqrt{b\,x^2+a}}{3059\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(9/2)/x^24,x)
```

```
[Out] (36*b^6*(a + b*x^2)^(1/2))/(1062347*a^2*x^11) - (3713*b^4*(a + b*x^2)^(1/2)
)/(52003*x^15) - (12770*a*b^3*(a + b*x^2)^(1/2))/(52003*x^17) - (31*a^3*b*(
a + b*x^2)^(1/2))/(161*x^21) - (3*b^5*(a + b*x^2)^(1/2))/(96577*a*x^13) - (
a^4*(a + b*x^2)^(1/2))/(23*x^23) - (40*b^7*(a + b*x^2)^(1/2))/(1062347*a^3*
x^9) + (320*b^8*(a + b*x^2)^(1/2))/(7436429*a^4*x^7) - (384*b^9*(a + b*x^2)
^(1/2))/(7436429*a^5*x^5) + (512*b^10*(a + b*x^2)^(1/2))/(7436429*a^6*x^3)
- (1024*b^11*(a + b*x^2)^(1/2))/(7436429*a^7*x) - (990*a^2*b^2*(a + b*x^2)^
(1/2))/(3059*x^19)
```

sympy [B] time = 9.07, size = 1950, normalized size = 11.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(9/2)/x**24,x)
```

```
[Out] -323323*a**17*b**(73/2)*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 4
4618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b
**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 74364
29*a**7*b**42*x**34) - 3371797*a**16*b**(75/2)*x**2*sqrt(a/(b*x**2) + 1)/(7
```

436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 15847689*a**15*b**(77/2)*x**4*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 44210595*a**14*b**(79/2)*x**6*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 81074994*a**13*b**(81/2)*x**8*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 102129258*a**12*b**(83/2)*x**10*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 89502546*a**11*b**(85/2)*x**12*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 53885062*a**10*b**(87/2)*x**14*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 21329935*a**9*b**(89/2)*x**16*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 5012953*a**8*b**(91/2)*x**18*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 531157*a**7*b**(93/2)*x**20*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 231*a**6*b**(95/2)*x**22*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 2772*a**5*b**(97/2)*x**24*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 9240*a**4*b**(99/2)*x**26*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 14784*a**3*b**(101/2)*x**28*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 1

```
48728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**
41*x**32 + 7436429*a**7*b**42*x**34) - 12672*a**2*b**(103/2)*x**30*sqrt(a/(
b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 1115
46435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**4
0*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 5632*a*b*
*(105/2)*x**32*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a
**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**2
8 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b
**42*x**34) - 1024*b**(107/2)*x**34*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**
36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 14872
8580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x
**32 + 7436429*a**7*b**42*x**34)
```

3.432 $\int x^5\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{448}\left(4x^2+9\right)^{7/2}-\frac{9}{160}\left(4x^2+9\right)^{5/2}+\frac{27}{64}\left(4x^2+9\right)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{448}\left(4x^2+9\right)^{7/2}-\frac{9}{160}\left(4x^2+9\right)^{5/2}+\frac{27}{64}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[9 + 4*x^2],x]
```

```
[Out] (27*(9 + 4*x^2)^(3/2))/64 - (9*(9 + 4*x^2)^(5/2))/160 + (9 + 4*x^2)^(7/2)/4
48
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{9+4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{9+4x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{81}{16}\sqrt{9+4x}-\frac{9}{8}(9+4x)^{3/2}+\frac{1}{16}(9+4x)^{5/2}\right)dx,x,x^2\right)\\ &= \frac{27}{64}\left(9+4x^2\right)^{3/2}-\frac{9}{160}\left(9+4x^2\right)^{5/2}+\frac{1}{448}\left(9+4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(4x^2+9\right)^{3/2}\left(10x^4-18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[9 + 4*x^2],x]
[Out] ((9 + 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))/280
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(4x^2+9\right)^{3/2}\left(10x^4-18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[9 + 4*x^2],x]
[Out] ((9 + 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))/280
```

fricas [A] time = 1.33, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^6+18\,x^4-54\,x^2+243\right)\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/280*(40*x^6 + 18*x^4 - 54*x^2 + 243)*sqrt(4*x^2 + 9)
```

giac [A] time = 1.03, size = 34, normalized size = 0.74

$$\frac{1}{448}\left(4\,x^2+9\right)^{\frac{7}{2}}-\frac{9}{160}\left(4\,x^2+9\right)^{\frac{5}{2}}+\frac{27}{64}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/448*(4*x^2 + 9)^(7/2) - 9/160*(4*x^2 + 9)^(5/2) + 27/64*(4*x^2 + 9)^(3/2)
```

maple [A] time = 0.00, size = 24, normalized size = 0.52

$$\frac{\left(4x^2+9\right)^{\frac{3}{2}}\left(10x^4-18x^2+27\right)}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2+9)^(1/2),x)
```

```
[Out] 1/280*(4*x^2+9)^(3/2)*(10*x^4-18*x^2+27)
```

maxima [A] time = 2.97, size = 40, normalized size = 0.87

$$\frac{1}{28}\left(4\,x^2+9\right)^{\frac{3}{2}}x^4-\frac{9}{140}\left(4\,x^2+9\right)^{\frac{3}{2}}x^2+\frac{27}{280}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/28*(4*x^2 + 9)^(3/2)*x^4 - 9/140*(4*x^2 + 9)^(3/2)*x^2 + 27/280*(4*x^2 +
9)^(3/2)
```

mupad [B] time = 4.55, size = 25, normalized size = 0.54

$$\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2\,x^6}{7}+\frac{9\,x^4}{70}-\frac{27\,x^2}{70}+\frac{243}{140}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x^2 + 9/4)^(1/2)*((9*x^4)/70 - (27*x^2)/70 + (2*x^6)/7 + 243/140)
```

sympy [A] time = 1.97, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{4x^2+9}}{7}+\frac{9x^4\sqrt{4x^2+9}}{140}-\frac{27x^2\sqrt{4x^2+9}}{140}+\frac{243\sqrt{4x^2+9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(4*x**2+9)**(1/2),x)
```

```
[Out] x**6*sqrt(4*x**2 + 9)/7 + 9*x**4*sqrt(4*x**2 + 9)/140 - 27*x**2*sqrt(4*x**2
 + 9)/140 + 243*sqrt(4*x**2 + 9)/280
```

3.433 $\int x^4\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{81}{256}\sqrt{4x^2+9}\,x+\frac{1}{6}\sqrt{4x^2+9}\,x^5+\frac{3}{32}\sqrt{4x^2+9}\,x^3+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 215}

$$\frac{1}{6}\sqrt{4x^2+9}\,x^5+\frac{3}{32}\sqrt{4x^2+9}\,x^3-\frac{81}{256}\sqrt{4x^2+9}\,x+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[9 + 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[9 + 4*x^2])/256 + (3*x^3*Sqrt[9 + 4*x^2])/32 + (x^5*Sqrt[9 + 4*
x^2])/6 + (729*ArcSinh[(2*x)/3])/512
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{9+4x^2}\,dx &= \frac{1}{6}x^5\sqrt{9+4x^2}+\frac{3}{2}\int\frac{x^4}{\sqrt{9+4x^2}}\,dx\\ &= \frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}-\frac{81}{32}\int\frac{x^2}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{9+4x^2}+\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}+\frac{729}{256}\int\frac{1}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{9+4x^2}+\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.62

$$\frac{1}{768}x\sqrt{4x^2+9}\left(128x^4+72x^2-243\right)+\frac{729}{512}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 + (729*ArcSinh[(2*x)/3])/
512
```

IntegrateAlgebraic [A] time = 0.05, size = 50, normalized size = 0.79

$$\frac{1}{768}\sqrt{4x^2+9}\left(128x^5+72x^3-243x\right)-\frac{729}{512}\log\left(\sqrt{4x^2+9}-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[9 + 4*x^2],x]
```

```
[Out] (Sqrt[9 + 4*x^2]*(-243*x + 72*x^3 + 128*x^5))/768 - (729*Log[-2*x + Sqrt[9
+ 4*x^2]])/512
```

fricas [A] time = 1.12, size = 42, normalized size = 0.67

$$\frac{1}{768}\left(128\,x^5+72\,x^3-243\,x\right)\sqrt{4\,x^2+9}-\frac{729}{512}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 + 72*x^3 - 243*x)*sqrt(4*x^2 + 9) - 729/512*log(-2*x + sqrt(
4*x^2 + 9))
```

giac [A] time = 1.17, size = 43, normalized size = 0.68

$$\frac{1}{768}\left(8\left(16x^2+9\right)x^2-243\right)\sqrt{4x^2+9}\,x-\frac{729}{512}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 + 9)*x^2 - 243)*sqrt(4*x^2 + 9)*x - 729/512*log(-2*x + sqr
t(4*x^2 + 9))
```

maple [A] time = 0.01, size = 46, normalized size = 0.73

$$\frac{\left(4x^2+9\right)^{\frac{3}{2}}x^3}{24}-\frac{9\left(4x^2+9\right)^{\frac{3}{2}}x}{128}+\frac{81\sqrt{4x^2+9}\,x}{256}+\frac{729\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2+9)^(1/2),x)
```

```
[Out] 1/24*x^3*(4*x^2+9)^(3/2)-9/128*x*(4*x^2+9)^(3/2)+81/256*x*(4*x^2+9)^(1/2)+7
29/512*arcsinh(2/3*x)
```

maxima [A] time = 2.94, size = 45, normalized size = 0.71

$$\frac{1}{24}\left(4x^2+9\right)^{\frac{3}{2}}x^3-\frac{9}{128}\left(4x^2+9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{4x^2+9}\,x+\frac{729}{512}\,\mathrm{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^2 + 9)^(3/2)*x^3 - 9/128*(4*x^2 + 9)^(3/2)*x + 81/256*sqrt(4*x^2
+ 9)*x + 729/512*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 30, normalized size = 0.48

$$\frac{729\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{512}+\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2x^5}{3}+\frac{3x^3}{8}-\frac{81x}{64}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2 + 9)^(1/2),x)
```

```
[Out] (729*asinh((2*x)/3))/512 + ((x^2 + 9/4)^(1/2)*((3*x^3)/8 - (81*x)/64 + (2*x
^5)/3))/2
```

sympy [A] time = 4.56, size = 75, normalized size = 1.19

$$\frac{2x^7}{3\sqrt{4x^2+9}}+\frac{15x^5}{8\sqrt{4x^2+9}}-\frac{27x^3}{64\sqrt{4x^2+9}}-\frac{729x}{256\sqrt{4x^2+9}}+\frac{729\operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**2+9)**(1/2),x)
```

```
[Out] 2*x**7/(3*sqrt(4*x**2 + 9)) + 15*x**5/(8*sqrt(4*x**2 + 9)) - 27*x**3/(64*sq
rt(4*x**2 + 9)) - 729*x/(256*sqrt(4*x**2 + 9)) + 729*asinh(2*x/3)/512
```

3.434 $\int x^3\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(4x^2+9\right)^{5/2}-\frac{3}{16}\left(4x^2+9\right)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(4x^2+9\right)^{5/2}-\frac{3}{16}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[9 + 4*x^2],x]
```

```
[Out] (-3*(9 + 4*x^2)^(3/2))/16 + (9 + 4*x^2)^(5/2)/80
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{9+4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x\sqrt{9+4x}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{9}{4}\sqrt{9+4x}+\frac{1}{4}(9+4x)^{3/2}\right)dx,x,x^2\right)\\&=-\frac{3}{16}\left(9+4x^2\right)^{3/2}+\frac{1}{80}\left(9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(2x^2-3\right)\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[9 + 4*x^2],x]
```

```
[Out] ((-3 + 2*x^2)*(9 + 4*x^2)^(3/2))/40
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(2x^2-3\right)\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[9 + 4*x^2],x]
```

```
[Out] ((-3 + 2*x^2)*(9 + 4*x^2)^(3/2))/40
```

fricas [A] time = 0.62, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8\,x^4+6\,x^2-27\right)\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 + 6*x^2 - 27)*sqrt(4*x^2 + 9)
```

giac [A] time = 1.01, size = 23, normalized size = 0.74

$$\frac{1}{80}\left(4\,x^2+9\right)^{\frac{5}{2}}-\frac{3}{16}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*(4*x^2 + 9)^(5/2) - 3/16*(4*x^2 + 9)^(3/2)
```

maple [A] time = 0.00, size = 19, normalized size = 0.61

$$\frac{\left(4x^2+9\right)^{\frac{3}{2}}\left(2x^2-3\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2+9)^(1/2),x)
```

```
[Out] 1/40*(4*x^2+9)^(3/2)*(2*x^2-3)
```

maxima [A] time = 2.96, size = 26, normalized size = 0.84

$$\frac{1}{20} \left(4 x^2 + 9\right)^{\frac{3}{2}} x^2 - \frac{3}{40} \left(4 x^2 + 9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/20*(4*x^2 + 9)^(3/2)*x^2 - 3/40*(4*x^2 + 9)^(3/2)
```

mupad [B] time = 0.02, size = 20, normalized size = 0.65

$$\sqrt{x^2 + \frac{9}{4}} \left(\frac{2 x^4}{5} + \frac{3 x^2}{10} - \frac{27}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2 + 9)^(1/2),x)
[Out] (x^2 + 9/4)^(1/2)*((3*x^2)/10 + (2*x^4)/5 - 27/20)
```

sympy [A] time = 0.63, size = 44, normalized size = 1.42

$$\frac{x^4 \sqrt{4x^2 + 9}}{5} + \frac{3x^2 \sqrt{4x^2 + 9}}{20} - \frac{27\sqrt{4x^2 + 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(4*x**2+9)**(1/2),x)
[Out] x**4*sqrt(4*x**2 + 9)/5 + 3*x**2*sqrt(4*x**2 + 9)/20 - 27*sqrt(4*x**2 + 9)/
40
```

3.435 $\int x^2\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=45

$$\frac{9}{32}\sqrt{4x^2+9}\,x+\frac{1}{4}\sqrt{4x^2+9}\,x^3-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 215}

$$\frac{1}{4}\sqrt{4x^2+9}\,x^3+\frac{9}{32}\sqrt{4x^2+9}\,x-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9*x*Sqrt[9 + 4*x^2])/32 + (x^3*Sqrt[9 + 4*x^2])/4 - (81*ArcSinh[(2*x)/3])/
64
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\sqrt{9+4x^2}\,dx = \frac{1}{4}x^3\sqrt{9+4x^2} + \frac{9}{4}\int \frac{x^2}{\sqrt{9+4x^2}}\,dx$$
$$= \frac{9}{32}x\sqrt{9+4x^2} + \frac{1}{4}x^3\sqrt{9+4x^2} - \frac{81}{32}\int \frac{1}{\sqrt{9+4x^2}}\,dx$$
$$= \frac{9}{32}x\sqrt{9+4x^2} + \frac{1}{4}x^3\sqrt{9+4x^2} - \frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.80

$$\sqrt{4x^2+9}\left(\frac{x^3}{4}+\frac{9x}{32}\right)-\frac{81}{64}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[9 + 4*x^2],x]
[Out] Sqrt[9 + 4*x^2]*((9*x)/32 + x^3/4) - (81*ArcSinh[(2*x)/3])/64
```

IntegrateAlgebraic [A] time = 0.03, size = 45, normalized size = 1.00

$$\frac{81}{64}\log\left(\sqrt{4x^2+9}-2x\right)+\frac{1}{32}\sqrt{4x^2+9}\left(8x^3+9x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[9 + 4*x^2],x]
[Out] (Sqrt[9 + 4*x^2]*(9*x + 8*x^3))/32 + (81*Log[-2*x + Sqrt[9 + 4*x^2]])/64
```

fricas [A] time = 1.15, size = 37, normalized size = 0.82

$$\frac{1}{32}\left(8\,x^3+9\,x\right)\sqrt{4\,x^2+9}+\frac{81}{64}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/32*(8*x^3 + 9*x)*sqrt(4*x^2 + 9) + 81/64*log(-2*x + sqrt(4*x^2 + 9))
```

giac [A] time = 1.05, size = 36, normalized size = 0.80

$$\frac{1}{32}\left(8\,x^2+9\right)\sqrt{4\,x^2+9}\,x+\frac{81}{64}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/32*(8*x^2 + 9)*sqrt(4*x^2 + 9)*x + 81/64*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.00, size = 32, normalized size = 0.71

$$\frac{\left(4x^{2}+9\right)^{\frac{3}{2}} x}{16}-\frac{9\sqrt{4x^{2}+9}\, x}{32}-\frac{81 \operatorname{arcsinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2+9)^(1/2),x)
[Out] 1/16*(4*x^2+9)^(3/2)*x-9/32*(4*x^2+9)^(1/2)*x-81/64*arcsinh(2/3*x)
```

maxima [A] time = 2.97, size = 31, normalized size = 0.69

$$\frac{1}{16}\left(4\,x^{2}+9\right)^{\frac{3}{2}} x-\frac{9}{32}\,\sqrt{4\,x^{2}+9}\,x-\frac{81}{64}\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/16*(4*x^2 + 9)^(3/2)*x - 9/32*sqrt(4*x^2 + 9)*x - 81/64*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.51

$$\frac{\left(x^{3}+\frac{9\,x}{8}\right)\sqrt{x^{2}+\frac{9}{4}}}{2}-\frac{81 \operatorname{asinh}\left(\frac{2\,x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2 + 9)^(1/2),x)
[Out] (((9*x)/8 + x^3)*(x^2 + 9/4)^(1/2))/2 - (81*asinh((2*x)/3))/64
```

sympy [A] time = 2.71, size = 54, normalized size = 1.20

$$\frac{x^{5}}{\sqrt{4x^{2}+9}}+\frac{27x^{3}}{8\sqrt{4x^{2}+9}}+\frac{81x}{32\sqrt{4x^{2}+9}}-\frac{81 \operatorname{asinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**2+9)**(1/2),x)
```

```
[Out] x**5/sqrt(4*x**2 + 9) + 27*x**3/(8*sqrt(4*x**2 + 9)) + 81*x/(32*sqrt(4*x**2
 + 9)) - 81*asinh(2*x/3)/64
```

3.436 $\int x\sqrt{9+4x^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9 + 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{9+4x^2}\,dx=\frac{1}{12}\left(9+4x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9 + 4*x^2)^(3/2)/12
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[9 + 4*x^2],x]
```

```
[Out] (9 + 4*x^2)^(3/2)/12
```

fricas [A] time = 1.28, size = 11, normalized size = 0.73

$$\frac{1}{12} \left(4 x^2 + 9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

giac [A] time = 1.04, size = 11, normalized size = 0.73

$$\frac{1}{12} \left(4 x^2 + 9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$\frac{\left(4x^2 + 9\right)^{\frac{3}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)*x,x)
```

```
[Out] 1/12*(4*x^2+9)^(3/2)
```

maxima [A] time = 1.31, size = 11, normalized size = 0.73

$$\frac{1}{12} \left(4 x^2 + 9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*x^2 + 9)^(3/2)
```

mupad [B] time = 0.02, size = 16, normalized size = 1.07

$$\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{4\,x^2}{3}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2 + 9)^(1/2),x)
[Out] ((x^2 + 9/4)^(1/2)*((4*x^2)/3 + 3))/2
```

sympy [B] time = 0.20, size = 27, normalized size = 1.80

$$\frac{x^2\sqrt{4x^2+9}}{3}+\frac{3\sqrt{4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2+9)**(1/2),x)
[Out] x**2*sqrt(4*x**2 + 9)/3 + 3*sqrt(4*x**2 + 9)/4
```

3.437 $\int \sqrt{9+4x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{4x^2+9}\,x+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{4x^2+9}\,x+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/2 + (9*ArcSinh[(2*x)/3])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{9+4x^2}\,dx &= \frac{1}{2}x\sqrt{9+4x^2}+\frac{9}{2}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{9+4x^2}+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{2}\sqrt{4x^2+9}\,x+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/2 + (9*ArcSinh[(2*x)/3])/4
```

IntegrateAlgebraic [A] time = 0.03, size = 37, normalized size = 1.37

$$\frac{1}{2}x\sqrt{4x^2+9} - \frac{9}{4}\log\left(\sqrt{4x^2+9} - 2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/2 - (9*Log[-2*x + Sqrt[9 + 4*x^2]])/4
```

fricas [A] time = 0.77, size = 29, normalized size = 1.07

$$\frac{1}{2}\sqrt{4x^2+9}\,x - \frac{9}{4}\log\left(-2x + \sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

giac [A] time = 0.99, size = 29, normalized size = 1.07

$$\frac{1}{2}\sqrt{4x^2+9}\,x - \frac{9}{4}\log\left(-2x + \sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.00, size = 20, normalized size = 0.74

$$\frac{\sqrt{4x^2+9}\,x}{2} + \frac{9\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2),x)
```

```
[Out] 9/4*arcsinh(2/3*x)+1/2*(4*x^2+9)^(1/2)*x
```

maxima [A] time = 2.98, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{4x^2+9}\,x+\frac{9}{4}\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(4*x^2 + 9)*x + 9/4*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.59

$$\frac{9\,\operatorname{asinh}\left(\frac{2\,x}{3}\right)}{4}+x\,\sqrt{x^2+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2),x)
```

```
[Out] (9*asinh((2*x)/3))/4 + x*(x^2 + 9/4)^(1/2)
```

sympy [A] time = 0.21, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\operatorname{asinh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2),x)
```

```
[Out] x*sqrt(4*x**2 + 9)/2 + 9*asinh(2*x/3)/4
```

3.438 $\int \frac{\sqrt{9+4x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{4x^2+9}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 207}

$$\sqrt{4x^2+9}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9+4x^2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{9+4x}}{x}\,dx, x, x^2\right)\\ &= \sqrt{9+4x^2}+\frac{9}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\ &= \sqrt{9+4x^2}+\frac{9}{4}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\ &= \sqrt{9+4x^2}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\sqrt{4x^2+9}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x,x]
[Out] Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 1.00

$$\sqrt{4x^2+9}-3\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 + 4*x^2]/x,x]
[Out] Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

fricas [A] time = 1.42, size = 44, normalized size = 1.47

$$\sqrt{4\,x^2+9}-3\,\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)+3\,\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(4*x^2 + 9) - 3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 3*log(-2*x + sqrt(4*x
^2 + 9) - 3)
```

giac [A] time = 1.03, size = 38, normalized size = 1.27

$$\sqrt{4\,x^2+9}-\frac{3}{2}\,\log\left(\sqrt{4\,x^2+9}+3\right)+\frac{3}{2}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(4*x^2 + 9) - 3/2*log(sqrt(4*x^2 + 9) + 3) + 3/2*log(sqrt(4*x^2 + 9) -
3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-3\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)+\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x,x)
```

```
[Out] (4*x^2+9)^(1/2)-3*arctanh(3/(4*x^2+9)^(1/2))
```

maxima [A] time = 2.94, size = 19, normalized size = 0.63

$$\sqrt{4\,x^2+9}-3\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(4*x^2 + 9) - 3*arcsinh(3/2/abs(x))
```

mupad [B] time = 0.03, size = 22, normalized size = 0.73

$$2\,\sqrt{x^2+\frac{9}{4}}-3\,\mathrm{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x,x)
[Out] 2*(x^2 + 9/4)^(1/2) - 3*atanh((2*(x^2 + 9/4)^(1/2))/3)
```

sympy **[A]** time = 1.25, size = 39, normalized size = 1.30

$$\frac{2x}{\sqrt{1+\frac{9}{4x^2}}}-3\operatorname{asinh}\left(\frac{3}{2x}\right)+\frac{9}{2x\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x,x)
[Out] 2*x/sqrt(1 + 9/(4*x**2)) - 3*asinh(3/(2*x)) + 9/(2*x*sqrt(1 + 9/(4*x**2)))
```

3.439 $\int \frac{\sqrt{9+4x^2}}{x^2}\,dx$

Optimal. Leaf size=25

$$2\sinh^{-1}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 215}

$$2\sinh^{-1}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 + 4*x^2]/x) + 2*ArcSinh[(2*x)/3]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9+4x^2}}{x^2}\,dx &= -\frac{\sqrt{9+4x^2}}{x}+4\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{\sqrt{9+4x^2}}{x}+2\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$2\sinh^{-1}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^2,x]
[Out] -(Sqrt[9 + 4*x^2]/x) + 2*ArcSinh[(2*x)/3]
```

IntegrateAlgebraic [A] time = 0.04, size = 35, normalized size = 1.40

$$-\frac{\sqrt{4x^2+9}}{x}-2\log\left(\sqrt{4x^2+9}-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 + 4*x^2]/x^2,x]
[Out] -(Sqrt[9 + 4*x^2]/x) - 2*Log[-2*x + Sqrt[9 + 4*x^2]]
```

fricas [A] time = 1.18, size = 35, normalized size = 1.40

$$-\frac{2\,x\log\left(-2\,x+\sqrt{4\,x^2+9}\right)+2\,x+\sqrt{4\,x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="fricas")
[Out] -(2*x*log(-2*x + sqrt(4*x^2 + 9)) + 2*x + sqrt(4*x^2 + 9))/x
```

giac [A] time = 1.15, size = 40, normalized size = 1.60

$$\frac{36}{\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9}-2\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="giac")
[Out] 36/((2*x - sqrt(4*x^2 + 9))^2 - 9) - 2*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.00, size = 34, normalized size = 1.36

$$\frac{4\sqrt{4x^2+9}\;x}{9}+2\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)-\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^2,x)
[Out] -1/9/x*(4*x^2+9)^(3/2)+4/9*(4*x^2+9)^(1/2)*x+2*arcsinh(2/3*x)
```

maxima [A] time = 2.94, size = 21, normalized size = 0.84

$$-\frac{\sqrt{4\,x^2+9}}{x}+2\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="maxima")
[Out] -sqrt(4*x^2 + 9)/x + 2*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$2\,\operatorname{asinh}\left(\frac{2\,x}{3}\right)-\frac{2\,\sqrt{x^2+\frac{9}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^2,x)
[Out] 2*asinh((2*x)/3) - (2*(x^2 + 9/4)^(1/2))/x
```

sympy [A] time = 0.25, size = 19, normalized size = 0.76

$$2\operatorname{asinh}\left(\frac{2x}{3}\right)-\frac{\sqrt{4x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**2,x)
[Out] 2*asinh(2*x/3) - sqrt(4*x**2 + 9)/x
```

3.440 $\int \frac{\sqrt{9+4x^2}}{x^3} \, dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 207}

$$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(2*x^2) - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9+4x^2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{9+4x}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{2x^2} + \text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\ &= -\frac{\sqrt{9+4x^2}}{2x^2} - \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.95

$$-\frac{\sqrt{4x^2+9}}{2x^2} - \frac{2}{3}\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^3,x]
[Out] -1/2*Sqrt[9 + 4*x^2]/x^2 - (2*ArcTanh[Sqrt[1 + (4*x^2)/9]])/3
```

IntegrateAlgebraic [A] time = 0.04, size = 39, normalized size = 1.00

$$-\frac{\sqrt{4x^2+9}}{2x^2} - \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 + 4*x^2]/x^3,x]
[Out] -1/2*Sqrt[9 + 4*x^2]/x^2 - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/3
```

fricas [A] time = 0.83, size = 57, normalized size = 1.46

$$-\frac{4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)+3\,\sqrt{4\,x^2+9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*x^2*log(-2*x + sqrt(4*x^2 + 9) + 3) - 4*x^2*log(-2*x + sqrt(4*x^2 +
 9) - 3) + 3*sqrt(4*x^2 + 9))/x^2
```

giac [A] time = 1.21, size = 43, normalized size = 1.10

$$-\frac{\sqrt{4\,x^2+9}}{2\,x^2}-\frac{1}{3}\,\log\left(\sqrt{4\,x^2+9}+3\right)+\frac{1}{3}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(4*x^2 + 9)/x^2 - 1/3*log(sqrt(4*x^2 + 9) + 3) + 1/3*log(sqrt(4*x^
2 + 9) - 3)
```

maple [A] time = 0.01, size = 41, normalized size = 1.05

$$-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{3}-\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{18x^2}+\frac{2\sqrt{4x^2+9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^3,x)
```

```
[Out] -1/18/x^2*(4*x^2+9)^(3/2)+2/9*(4*x^2+9)^(1/2)-2/3*arctanh(3/(4*x^2+9)^(1/2)
)
```

maxima [A] time = 2.81, size = 35, normalized size = 0.90

$$\frac{2}{9}\,\sqrt{4\,x^2+9}-\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}}{18\,x^2}-\frac{2}{3}\,\mathrm{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(4*x^2 + 9) - 1/18*(4*x^2 + 9)^(3/2)/x^2 - 2/3*arcsinh(3/2/abs(x))
```

mupad [B] time = 0.03, size = 25, normalized size = 0.64

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)}{3}-\frac{\sqrt{x^2+\frac{9}{4}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^3,x)
```

```
[Out] - (2*atanh((2*(x^2 + 9/4)^(1/2))/3))/3 - (x^2 + 9/4)^(1/2)/x^2
```

sympy [A] time = 1.67, size = 24, normalized size = 0.62

$$-\frac{2\,\text{asinh}\left(\frac{3}{2x}\right)}{3}-\frac{\sqrt{1+\frac{9}{4x^2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**3,x)
```

```
[Out] -2*asinh(3/(2*x))/3 - sqrt(1 + 9/(4*x**2))/x
```

3.441 $\int \frac{\sqrt{9+4x^2}}{x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{(4x^2+9)^{3/2}}{27x^3}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{(4x^2+9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^4,x]
```

```
[Out] -(9 + 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{9+4x^2}}{x^4}\,dx = -\frac{(9+4x^2)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{(4x^2+9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^4,x]
```

```
[Out] -1/27*(9 + 4*x^2)^(3/2)/x^3
```

IntegrateAlgebraic [A] time = 0.04, size = 25, normalized size = 1.39

$$\frac{(-4x^2-9)\sqrt{4x^2+9}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 + 4*x^2]/x^4,x]
[Out] ((-9 - 4*x^2)*Sqrt[9 + 4*x^2])/(27*x^3)
```

fricas [A] time = 1.09, size = 20, normalized size = 1.11

$$-\frac{8\,x^3+\left(4\,x^2+9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="fricas")
[Out] -1/27*(8*x^3 + (4*x^2 + 9)^(3/2))/x^3
```

giac [B] time = 1.01, size = 42, normalized size = 2.33

$$\frac{16\left(\left(2\,x-\sqrt{4\,x^2+9}\right)^4+27\right)}{\left(\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="giac")
[Out] 16*((2*x - sqrt(4*x^2 + 9))^4 + 27)/((2*x - sqrt(4*x^2 + 9))^2 - 9)^3
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^4,x)
[Out] -1/27*(4*x^2+9)^(3/2)/x^3
```

maxima [A] time = 2.94, size = 14, normalized size = 0.78

$$-\frac{\left(4\,x^{2}+9\right)^{\frac{3}{2}}}{27\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(4*x^2 + 9)^(3/2)/x^3
```

mupad [B] time = 0.03, size = 27, normalized size = 1.50

$$-\frac{18\,\sqrt{x^2+\frac{9}{4}}+8\,x^2\,\sqrt{x^2+\frac{9}{4}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^4,x)
```

```
[Out] -(18*(x^2 + 9/4)^(1/2) + 8*x^2*(x^2 + 9/4)^(1/2))/(27*x^3)
```

sympy [B] time = 0.93, size = 34, normalized size = 1.89

$$-\frac{8\sqrt{1+\frac{9}{4x^{2}}}}{27}-\frac{2\sqrt{1+\frac{9}{4x^{2}}}}{3x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**4,x)
```

```
[Out] -8*sqrt(1 + 9/(4*x**2))/27 - 2*sqrt(1 + 9/(4*x**2))/(3*x**2)
```

3.442 $\int \frac{\sqrt{9+4x^2}}{x^5} \, dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{4x^2+9}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{4x^4}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 207}

$$-\frac{\sqrt{4x^2+9}}{18x^2}-\frac{\sqrt{4x^2+9}}{4x^4}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 + 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(4*x^4) - Sqrt[9 + 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 + 4
*x^2]/3])/27
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{9+4x^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{9+4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4}-\frac{\sqrt{9+4x^2}}{18x^2}-\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4}-\frac{\sqrt{9+4x^2}}{18x^2}-\frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\
&= -\frac{\sqrt{9+4x^2}}{4x^4}-\frac{\sqrt{9+4x^2}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$-\frac{16\left(4x^2+9\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{4x^2}{9}+1\right)}{2187}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 + 4*x^2]/x^5,x]
```

```
[Out] (-16*(9 + 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (4*x^2)/9])/2187
```

IntegrateAlgebraic [A] time = 0.05, size = 46, normalized size = 0.81

$$\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)+\frac{\sqrt{4x^2+9}\,\left(-2x^2-9\right)}{36x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 + 4*x^2]/x^5,x]
```

```
[Out] ((-9 - 2*x^2)*Sqrt[9 + 4*x^2])/(36*x^4) + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/27
```

fricas [A] time = 1.74, size = 64, normalized size = 1.12

$$\frac{8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}\left(2\,x^2+9\right)}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/108*(8*x^4*log(-2*x + sqrt(4*x^2 + 9) + 3) - 8*x^4*log(-2*x + sqrt(4*x^2
+ 9) - 3) - 3*sqrt(4*x^2 + 9)*(2*x^2 + 9))/x^4
```

giac [A] time = 1.05, size = 55, normalized size = 0.96

$$-\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}+9\,\sqrt{4\,x^2+9}}{72\,x^4}+\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}+3\right)-\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*((4*x^2 + 9)^(3/2) + 9*sqrt(4*x^2 + 9))/x^4 + 1/27*log(sqrt(4*x^2 + 9
) + 3) - 1/27*log(sqrt(4*x^2 + 9) - 3)
```

maple [A] time = 0.01, size = 55, normalized size = 0.96

$$\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}+\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{162x^2}-\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{36x^4}-\frac{2\sqrt{4x^2+9}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+9)^(1/2)/x^5,x)
```

```
[Out] -1/36/x^4*(4*x^2+9)^(3/2)+1/162*(4*x^2+9)^(3/2)/x^2-2/81*(4*x^2+9)^(1/2)+2/
27*arctanh(3/(4*x^2+9)^(1/2))
```

maxima [A] time = 3.01, size = 49, normalized size = 0.86

$$-\frac{2}{81}\sqrt{4x^2+9}+\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{162x^2}-\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{36x^4}+\frac{2}{27}\operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(4*x^2 + 9) + 1/162*(4*x^2 + 9)^(3/2)/x^2 - 1/36*(4*x^2 + 9)^(3/2
)/x^4 + 2/27*arcsinh(3/2/abs(x))
```

mupad [B] time = 0.03, size = 45, normalized size = 0.79

$$\frac{2\operatorname{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{27}+\frac{\sqrt{x^2+\frac{9}{4}}\left(\frac{2}{3x^2}-\frac{1}{x^4}\right)}{2}-\frac{4\sqrt{x^2+\frac{9}{4}}}{9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + 9)^(1/2)/x^5,x)
```

```
[Out] (2*atanh((2*(x^2 + 9/4)^(1/2))/3))/27 + ((x^2 + 9/4)^(1/2)*(2/(3*x^2) - 1/x
^4))/2 - (4*(x^2 + 9/4)^(1/2))/(9*x^2)
```

sympy [A] time = 3.20, size = 63, normalized size = 1.11

$$\frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{27}-\frac{1}{9x\sqrt{1+\frac{9}{4x^2}}}-\frac{3}{4x^3\sqrt{1+\frac{9}{4x^2}}}-\frac{9}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+9)**(1/2)/x**5,x)
```

```
[Out] 2*asinh(3/(2*x))/27 - 1/(9*x*sqrt(1 + 9/(4*x**2))) - 3/(4*x**3*sqrt(1 + 9/(
4*x**2))) - 9/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

3.443 $\int x^5\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{448}\left(9-4x^2\right)^{7/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{27}{64}\left(9-4x^2\right)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{448}\left(9-4x^2\right)^{7/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{27}{64}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-27*(9 - 4*x^2)^(3/2))/64 + (9*(9 - 4*x^2)^(5/2))/160 - (9 - 4*x^2)^(7/2)/
448
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{9-4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int\sqrt{9-4x}\,x^2\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{81}{16}\sqrt{9-4x}-\frac{9}{8}(9-4x)^{3/2}+\frac{1}{16}(9-4x)^{5/2}\right)dx,x,x^2\right)\\&=-\frac{27}{64}\left(9-4x^2\right)^{3/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{1}{448}\left(9-4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{1}{280}\left(9-4x^2\right)^{3/2}\left(10x^4+18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[9 - 4*x^2],x]
[Out] -1/280*((9 - 4*x^2)^(3/2)*(27 + 18*x^2 + 10*x^4))
```

IntegrateAlgebraic [A] time = 0.02, size = 32, normalized size = 0.70

$$\frac{1}{280}\sqrt{9-4x^2}\left(40x^6-18x^4-54x^2-243\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[9 - 4*x^2],x]
[Out] (Sqrt[9 - 4*x^2]*(-243 - 54*x^2 - 18*x^4 + 40*x^6))/280
```

fricas [A] time = 0.97, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^6-18\,x^4-54\,x^2-243\right)\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/280*(40*x^6 - 18*x^4 - 54*x^2 - 243)*sqrt(-4*x^2 + 9)
```

giac [A] time = 1.02, size = 52, normalized size = 1.13

$$\frac{1}{448}\left(4\,x^2-9\right)^3\sqrt{-4\,x^2+9}+\frac{9}{160}\left(4\,x^2-9\right)^2\sqrt{-4\,x^2+9}-\frac{27}{64}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/448*(4*x^2 - 9)^3*sqrt(-4*x^2 + 9) + 9/160*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9)
 - 27/64*(-4*x^2 + 9)^(3/2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.74

$$\frac{(2x-3)\,(2x+3)\left(10x^4+18x^2+27\right)\sqrt{-4x^2+9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/280*(2*x-3)*(2*x+3)*(10*x^4+18*x^2+27)*(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.91, size = 40, normalized size = 0.87

$$-\frac{1}{28}\left(-4x^2+9\right)^{\frac{3}{2}}x^4-\frac{9}{140}\left(-4x^2+9\right)^{\frac{3}{2}}x^2-\frac{27}{280}\left(-4x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/28*(-4*x^2 + 9)^(3/2)*x^4 - 9/140*(-4*x^2 + 9)^(3/2)*x^2 - 27/280*(-4*x^
2 + 9)^(3/2)
```

mupad [B] time = 4.53, size = 28, normalized size = 0.61

$$-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(-\frac{4x^6}{7}+\frac{9x^4}{35}+\frac{27x^2}{35}+\frac{243}{70}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*((27*x^2)/35 + (9*x^4)/35 - (4*x^6)/7 + 243/70))/2
```

sympy [A] time = 2.01, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{9-4x^2}}{7}-\frac{9x^4\sqrt{9-4x^2}}{140}-\frac{27x^2\sqrt{9-4x^2}}{140}-\frac{243\sqrt{9-4x^2}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**6*sqrt(9 - 4*x**2)/7 - 9*x**4*sqrt(9 - 4*x**2)/140 - 27*x**2*sqrt(9 - 4*
x**2)/140 - 243*sqrt(9 - 4*x**2)/280
```

3.444 $\int x^4\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{81}{256}\sqrt{9-4x^2}\,x+\frac{1}{6}\sqrt{9-4x^2}\,x^5-\frac{3}{32}\sqrt{9-4x^2}\,x^3+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 216}

$$\frac{1}{6}\sqrt{9-4x^2}\,x^5-\frac{3}{32}\sqrt{9-4x^2}\,x^3-\frac{81}{256}\sqrt{9-4x^2}\,x+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[9 - 4*x^2])/256 - (3*x^3*Sqrt[9 - 4*x^2])/32 + (x^5*Sqrt[9 - 4*
x^2])/6 + (729*ArcSin[(2*x)/3])/512
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{9-4x^2}\,dx &= \frac{1}{6}x^5\sqrt{9-4x^2}+\frac{3}{2}\int\frac{x^4}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{81}{32}\int\frac{x^2}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{9-4x^2}-\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{729}{256}\int\frac{1}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{9-4x^2}-\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.62

$$\frac{1}{768}x\sqrt{9-4x^2}\left(128x^4-72x^2-243\right)+\frac{729}{512}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 + (729*ArcSin[(2*x)/3])/5
12
```

IntegrateAlgebraic [A] time = 0.15, size = 53, normalized size = 0.84

$$\frac{729}{256}\tan^{-1}\left(\frac{2x}{\sqrt{9-4x^2}-3}\right)+\frac{1}{768}\sqrt{9-4x^2}\left(128x^5-72x^3-243x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[9 - 4*x^2],x]
```

```
[Out] (Sqrt[9 - 4*x^2]*(-243*x - 72*x^3 + 128*x^5))/768 + (729*ArcTan[(2*x)/(-3 +
 Sqrt[9 - 4*x^2])])/256
```

fricas [A] time = 1.16, size = 45, normalized size = 0.71

$$\frac{1}{768}\left(128\,x^5-72\,x^3-243\,x\right)\sqrt{-4\,x^2+9}-\frac{729}{256}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 - 72*x^3 - 243*x)*sqrt(-4*x^2 + 9) - 729/256*arctan(1/2*(sqr
t(-4*x^2 + 9) - 3)/x)
```

giac [A] time = 1.08, size = 33, normalized size = 0.52

$$\frac{1}{768}\left(8\left(16x^2-9\right)x^2-243\right)\sqrt{-4x^2+9}\,x+\frac{729}{512}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(-4*x^2 + 9)*x + 729/512*arcsin(2/3*x)
```

maple [A] time = 0.01, size = 46, normalized size = 0.73

$$-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}x^3}{24}-\frac{9\left(-4x^2+9\right)^{\frac{3}{2}}x}{128}+\frac{81\sqrt{-4x^2+9}\,x}{256}+\frac{729\arcsin\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-4*x^2+9)^(1/2),x)
```

```
[Out] -1/24*x^3*(-4*x^2+9)^(3/2)-9/128*x*(-4*x^2+9)^(3/2)+81/256*x*(-4*x^2+9)^(1/
2)+729/512*arcsin(2/3*x)
```

maxima [A] time = 3.00, size = 45, normalized size = 0.71

$$-\frac{1}{24}\left(-4x^2+9\right)^{\frac{3}{2}}x^3-\frac{9}{128}\left(-4x^2+9\right)^{\frac{3}{2}}x+\frac{81}{256}\sqrt{-4x^2+9}\,x+\frac{729}{512}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(-4*x^2 + 9)^(3/2)*x^3 - 9/128*(-4*x^2 + 9)^(3/2)*x + 81/256*sqrt(-4*
x^2 + 9)*x + 729/512*arcsin(2/3*x)
```

mupad [B] time = 4.56, size = 32, normalized size = 0.51

$$\frac{729\,\mathrm{asin}\left(\frac{2x}{3}\right)}{512}-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(-\frac{2x^5}{3}+\frac{3x^3}{8}+\frac{81x}{64}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(9 - 4*x^2)^(1/2),x)
```

```
[Out] (729*asin((2*x)/3))/512 - ((9/4 - x^2)^(1/2)*((81*x)/64 + (3*x^3)/8 - (2*x^
5)/3))/2
```

sympy [A] time = 4.66, size = 167, normalized size = 2.65

$$\begin{cases} \frac{2ix^7}{3\sqrt{4x^2-9}} - \frac{15ix^5}{8\sqrt{4x^2-9}} - \frac{27ix^3}{64\sqrt{4x^2-9}} + \frac{729ix}{256\sqrt{4x^2-9}} - \frac{729i\operatorname{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{2x^7}{3\sqrt{9-4x^2}} + \frac{15x^5}{8\sqrt{9-4x^2}} + \frac{27x^3}{64\sqrt{9-4x^2}} - \frac{729x}{256\sqrt{9-4x^2}} + \frac{729\operatorname{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((2*I*x**7/(3*sqrt(4*x**2 - 9)) - 15*I*x**5/(8*sqrt(4*x**2 - 9)) -
 27*I*x**3/(64*sqrt(4*x**2 - 9)) + 729*I*x/(256*sqrt(4*x**2 - 9)) - 729*I*a
cosh(2*x/3)/512, 4*Abs(x**2)/9 > 1), (-2*x**7/(3*sqrt(9 - 4*x**2)) + 15*x**
5/(8*sqrt(9 - 4*x**2)) + 27*x**3/(64*sqrt(9 - 4*x**2)) - 729*x/(256*sqrt(9
- 4*x**2)) + 729*asin(2*x/3)/512, True))
```

3.445 $\int x^3\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(9-4x^2\right)^{5/2}-\frac{3}{16}\left(9-4x^2\right)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(9-4x^2\right)^{5/2}-\frac{3}{16}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-3*(9 - 4*x^2)^(3/2))/16 + (9 - 4*x^2)^(5/2)/80
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{9-4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int\sqrt{9-4x}\,x\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{9}{4}\sqrt{9-4x}-\frac{1}{4}(9-4x)^{3/2}\right)dx,x,x^2\right)\\ &= -\frac{3}{16}\left(9-4x^2\right)^{3/2}+\frac{1}{80}\left(9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$-\frac{1}{40}\left(9-4x^2\right)^{3/2}\left(2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[9 - 4*x^2],x]
[Out] -1/40*((9 - 4*x^2)^(3/2)*(3 + 2*x^2))
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(9-4x^2\right)^{3/2}\left(-2x^2-3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[9 - 4*x^2],x]
[Out] ((9 - 4*x^2)^(3/2)*(-3 - 2*x^2))/40
```

fricas [A] time = 1.33, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8\,x^4-6\,x^2-27\right)\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/40*(8*x^4 - 6*x^2 - 27)*sqrt(-4*x^2 + 9)
```

giac [A] time = 1.14, size = 32, normalized size = 1.03

$$\frac{1}{80}\left(4\,x^2-9\right)^2\sqrt{-4\,x^2+9}-\frac{3}{16}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/80*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9) - 3/16*(-4*x^2 + 9)^(3/2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$\frac{(2x-3)\,(2x+3)\left(2x^2+3\right)\sqrt{-4x^2+9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-4*x^2+9)^(1/2),x)
[Out] 1/40*(2*x-3)*(2*x+3)*(2*x^2+3)*(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.97, size = 26, normalized size = 0.84

$$-\frac{1}{20}\left(-4x^{2}+9\right)^{\frac{3}{2}}x^{2}-\frac{3}{40}\left(-4x^{2}+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*(-4*x^2 + 9)^(3/2)*x^2 - 3/40*(-4*x^2 + 9)^(3/2)
```

mupad [B] time = 0.02, size = 23, normalized size = 0.74

$$-\frac{\sqrt{\frac{9}{4}-x^{2}}\,\left(-\frac{4x^{4}}{5}+\frac{3x^{2}}{5}+\frac{27}{10}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*((3*x^2)/5 - (4*x^4)/5 + 27/10))/2
```

sympy [A] time = 0.67, size = 44, normalized size = 1.42

$$\frac{x^{4}\sqrt{9-4x^{2}}}{5}-\frac{3x^{2}\sqrt{9-4x^{2}}}{20}-\frac{27\sqrt{9-4x^{2}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-4*x**2+9)**(1/2),x)
```

```
[Out] x**4*sqrt(9 - 4*x**2)/5 - 3*x**2*sqrt(9 - 4*x**2)/20 - 27*sqrt(9 - 4*x**2)/
40
```

3.446 $\int x^2\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{9}{32}\sqrt{9-4x^2}\,x+\frac{1}{4}\sqrt{9-4x^2}\,x^3+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {279, 321, 216}

$$\frac{1}{4}\sqrt{9-4x^2}\,x^3-\frac{9}{32}\sqrt{9-4x^2}\,x+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[9 - 4*x^2],x]
```

```
[Out] (-9*x*Sqrt[9 - 4*x^2])/32 + (x^3*Sqrt[9 - 4*x^2])/4 + (81*ArcSin[(2*x)/3])/
64
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\sqrt{9-4x^2}\,dx = \frac{1}{4}x^3\sqrt{9-4x^2} + \frac{9}{4}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$$
$$= -\frac{9}{32}x\sqrt{9-4x^2} + \frac{1}{4}x^3\sqrt{9-4x^2} + \frac{81}{32}\int \frac{1}{\sqrt{9-4x^2}}\,dx$$
$$= -\frac{9}{32}x\sqrt{9-4x^2} + \frac{1}{4}x^3\sqrt{9-4x^2} + \frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.80

$$\sqrt{9-4x^2}\left(\frac{x^3}{4}-\frac{9x}{32}\right)+\frac{81}{64}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[9 - 4*x^2],x]
```

```
[Out] Sqrt[9 - 4*x^2]*((-9*x)/32 + x^3/4) + (81*ArcSin[(2*x)/3])/64
```

IntegrateAlgebraic [A] time = 0.09, size = 48, normalized size = 1.07

$$\frac{81}{32}\tan^{-1}\left(\frac{2x}{\sqrt{9-4x^2}-3}\right)+\frac{1}{32}\sqrt{9-4x^2}\left(8x^3-9x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[9 - 4*x^2],x]
```

```
[Out] (Sqrt[9 - 4*x^2]*(-9*x + 8*x^3))/32 + (81*ArcTan[(2*x)/(-3 + Sqrt[9 - 4*x^2
])])/32
```

fricas [A] time = 1.14, size = 40, normalized size = 0.89

$$\frac{1}{32}\left(8\,x^3-9\,x\right)\sqrt{-4\,x^2+9}-\frac{81}{32}\,\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 9*x)*sqrt(-4*x^2 + 9) - 81/32*arctan(1/2*(sqrt(-4*x^2 + 9) -
3)/x)
```

giac [A] time = 1.09, size = 26, normalized size = 0.58

$$\frac{1}{32}\left(8\,x^2-9\right)\sqrt{-4\,x^2+9}\,x+\frac{81}{64}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/32*(8*x^2 - 9)*sqrt(-4*x^2 + 9)*x + 81/64*arcsin(2/3*x)
```

maple [A] time = 0.01, size = 32, normalized size = 0.71

$$-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}x}{16}+\frac{9\sqrt{-4x^2+9}\;x}{32}+\frac{81\arcsin\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-4*x^2+9)^(1/2),x)
[Out] -1/16*(-4*x^2+9)^(3/2)*x+9/32*(-4*x^2+9)^(1/2)*x+81/64*arcsin(2/3*x)
```

maxima [A] time = 2.95, size = 31, normalized size = 0.69

$$-\frac{1}{16}\left(-4\,x^2+9\right)^{\frac{3}{2}}x+\frac{9}{32}\,\sqrt{-4\,x^2+9}\,x+\frac{81}{64}\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/16*(-4*x^2 + 9)^(3/2)*x + 9/32*sqrt(-4*x^2 + 9)*x + 81/64*arcsin(2/3*x)
```

mupad [B] time = 0.03, size = 27, normalized size = 0.60

$$\frac{81\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{64}-\frac{\sqrt{\frac{9}{4}-x^2}\;\left(\frac{9\,x}{8}-x^3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(9 - 4*x^2)^(1/2),x)
[Out] (81*asin((2*x)/3))/64 - ((9/4 - x^2)^(1/2)*((9*x)/8 - x^3))/2
```

sympy [A] time = 2.68, size = 124, normalized size = 2.76

$$\begin{cases} \frac{ix^5}{\sqrt{4x^2-9}} - \frac{27ix^3}{8\sqrt{4x^2-9}} + \frac{81ix}{32\sqrt{4x^2-9}} - \frac{81i\operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{x^5}{\sqrt{9-4x^2}} + \frac{27x^3}{8\sqrt{9-4x^2}} - \frac{81x}{32\sqrt{9-4x^2}} + \frac{81\operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((I*x**5/sqrt(4*x**2 - 9) - 27*I*x**3/(8*sqrt(4*x**2 - 9)) + 81*I*
x/(32*sqrt(4*x**2 - 9)) - 81*I*acosh(2*x/3)/64, 4*Abs(x**2)/9 > 1), (-x**5/
sqrt(9 - 4*x**2) + 27*x**3/(8*sqrt(9 - 4*x**2)) - 81*x/(32*sqrt(9 - 4*x**2)
) + 81*asin(2*x/3)/64, True))
```

3.447 $\int x\sqrt{9-4x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[9 - 4*x^2],x]
```

```
[Out] -(9 - 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{9-4x^2}\,dx=-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[9 - 4*x^2],x]
```

```
[Out] -1/12*(9 - 4*x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[9 - 4*x^2],x]
[Out] -1/12*(9 - 4*x^2)^(3/2)
```

fricas [A] time = 1.51, size = 18, normalized size = 1.20

$$\frac{1}{12}\left(4\,x^2-9\right)\sqrt{-4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/12*(4*x^2 - 9)*sqrt(-4*x^2 + 9)
```

giac [A] time = 1.08, size = 11, normalized size = 0.73

$$-\frac{1}{12}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/12*(-4*x^2 + 9)^(3/2)
```

maple [A] time = 0.00, size = 22, normalized size = 1.47

$$\frac{(2x-3)\,(2x+3)\,\sqrt{-4x^2+9}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)*x,x)
[Out] 1/12*(2*x-3)*(2*x+3)*(-4*x^2+9)^(1/2)
```

maxima [A] time = 1.28, size = 11, normalized size = 0.73

$$-\frac{1}{12}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/12*(-4*x^2 + 9)^(3/2)
```

mupad [B] time = 0.02, size = 18, normalized size = 1.20

$$\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{4\,x^2}{3}-3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(9 - 4*x^2)^(1/2),x)
[Out] ((9/4 - x^2)^(1/2)*((4*x^2)/3 - 3))/2
```

sympy [B] time = 0.20, size = 27, normalized size = 1.80

$$\frac{x^2\sqrt{9-4x^2}}{3}-\frac{3\sqrt{9-4x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x**2+9)**(1/2),x)
[Out] x**2*sqrt(9 - 4*x**2)/3 - 3*sqrt(9 - 4*x**2)/4
```

3.448 $\int \sqrt{9-4x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{9-4x^2}\,x+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{9-4x^2}\,x+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[9 - 4*x^2])/2 + (9*ArcSin[(2*x)/3])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{9-4x^2}\,dx &= \frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{2}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{2}\sqrt{9-4x^2}\,x+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2],x]
[Out] (x*Sqrt[9 - 4*x^2])/2 + (9*ArcSin[(2*x)/3])/4
```

IntegrateAlgebraic [A] time = 0.05, size = 41, normalized size = 1.52

$$\frac{1}{2}x\sqrt{9-4x^2}-\frac{9}{2}\tan^{-1}\left(\frac{\sqrt{9-4x^2}}{2x+3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 - 4*x^2],x]
[Out] (x*Sqrt[9 - 4*x^2])/2 - (9*ArcTan[Sqrt[9 - 4*x^2]/(3 + 2*x)])/2
```

fricas [A] time = 0.64, size = 32, normalized size = 1.19

$$\frac{1}{2}\sqrt{-4x^2+9}\,x-\frac{9}{2}\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-4*x^2 + 9)*x - 9/2*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

giac [A] time = 1.00, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-4x^2+9}\,x+\frac{9}{4}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.74

$$\frac{\sqrt{-4x^2+9}\,x}{2}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2),x)
```

```
[Out] 9/4*arcsin(2/3*x)+1/2*(-4*x^2+9)^(1/2)*x
```

maxima [A] time = 2.94, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-4x^2+9}\,x+\frac{9}{4}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

mupad [B] time = 0.02, size = 18, normalized size = 0.67

$$\frac{9\,\mathrm{asin}\left(\frac{2x}{3}\right)}{4}+x\sqrt{\frac{9}{4}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2),x)
```

```
[Out] (9*asin((2*x)/3))/4 + x*(9/4 - x^2)^(1/2)
```

sympy [A] time = 0.22, size = 22, normalized size = 0.81

$$\frac{x\sqrt{9-4x^2}}{2}+\frac{9\,\mathrm{asin}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2),x)
```

```
[Out] x*sqrt(9 - 4*x**2)/2 + 9*asin(2*x/3)/4
```

3.449 $\int \frac{\sqrt{9-4x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 206}

$$\sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x,x]
```

```
[Out] Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{9-4x^2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9-4x}}{x}\,dx, x, x^2\right)\\
&= \sqrt{9-4x^2} + \frac{9}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)\\
&= \sqrt{9-4x^2} - \frac{9}{4}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\\
&= \sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x,x]
[Out] Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 1.00

$$\sqrt{9-4x^2} - 3\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 - 4*x^2]/x,x]
[Out] Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

fricas [A] time = 1.34, size = 28, normalized size = 0.93

$$\sqrt{-4\,x^2+9} + 3\,\log\left(\frac{\sqrt{-4\,x^2+9} - 3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(-4*x^2 + 9) + 3*log((sqrt(-4*x^2 + 9) - 3)/x)
```

giac [A] time = 1.18, size = 40, normalized size = 1.33

$$\sqrt{-4\,x^2+9}-\frac{3}{2}\,\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{3}{2}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(-4*x^2 + 9) - 3/2*log(sqrt(-4*x^2 + 9) + 3) + 3/2*log(-sqrt(-4*x^2 + 9
) + 3)
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-3\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)+\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x,x)
```

```
[Out] (-4*x^2+9)^(1/2)-3*arctanh(3/(-4*x^2+9)^(1/2))
```

maxima [A] time = 2.86, size = 35, normalized size = 1.17

$$\sqrt{-4\,x^2+9}-3\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(-4*x^2 + 9) - 3*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.55, size = 32, normalized size = 1.07

$$3\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)+2\,\sqrt{\frac{9}{4}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x,x)
```

```
[Out] 3*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2) + 2*(9/4 - x^2)^(1/2)
```

sympy [A] time = 1.32, size = 76, normalized size = 2.53

$$\begin{cases} i\sqrt{4x^2-9} - 3\log(x) + \frac{3\log(x^2)}{2} + 3i\operatorname{asin}\left(\frac{3}{2x}\right) & \text{for } \frac{4|x^2|}{9} > 1 \\ \sqrt{9-4x^2} + \frac{3\log(x^2)}{2} - 3\log\left(\sqrt{1-\frac{4x^2}{9}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(4*x**2 - 9) - 3*log(x) + 3*log(x**2)/2 + 3*I*asin(3/(2*x)
), 4*Abs(x**2)/9 > 1), (sqrt(9 - 4*x**2) + 3*log(x**2)/2 - 3*log(sqrt(1 - 4
*x**2/9) + 1), True))
```

3.450 $\int \frac{\sqrt{9-4x^2}}{x^2} \, dx$

Optimal. Leaf size=25

$$-\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 216}

$$-\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - 4*x^2]/x) - 2*ArcSin[(2*x)/3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{9-4x^2}}{x^2} \, dx &= -\frac{\sqrt{9-4x^2}}{x}-4\int \frac{1}{\sqrt{9-4x^2}} \, dx\\
&= -\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\sqrt{9-4x^2}}{x}-2\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^2,x]
[Out] -(Sqrt[9 - 4*x^2]/x) - 2*ArcSin[(2*x)/3]
```

IntegrateAlgebraic [A] time = 0.05, size = 39, normalized size = 1.56

$$4\tan^{-1}\left(\frac{\sqrt{9-4x^2}}{2x+3}\right)-\frac{\sqrt{9-4x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 - 4*x^2]/x^2,x]
[Out] -(Sqrt[9 - 4*x^2]/x) + 4*ArcTan[Sqrt[9 - 4*x^2]/(3 + 2*x)]
```

fricas [A] time = 0.73, size = 36, normalized size = 1.44

$$\frac{4\,x\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)-\sqrt{-4\,x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="fricas")
[Out] (4*x*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x) - sqrt(-4*x^2 + 9))/x
```

giac [A] time = 1.14, size = 39, normalized size = 1.56

$$\frac{2\,x}{\sqrt{-4\,x^2+9}-3}-\frac{\sqrt{-4\,x^2+9}-3}{2\,x}-2\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="giac")
[Out] 2*x/(sqrt(-4*x^2 + 9) - 3) - 1/2*(sqrt(-4*x^2 + 9) - 3)/x - 2*arcsin(2/3*x)
```

maple [A] time = 0.00, size = 34, normalized size = 1.36

$$-\frac{4\sqrt{-4x^2+9}\,x}{9}-2\arcsin\left(\frac{2x}{3}\right)-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^2,x)
[Out] -1/9/x*(-4*x^2+9)^(3/2)-4/9*(-4*x^2+9)^(1/2)*x-2*arcsin(2/3*x)
```

maxima [A] time = 2.92, size = 21, normalized size = 0.84

$$-\frac{\sqrt{-4\,x^2+9}}{x}-2\,\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="maxima")
[Out] -sqrt(-4*x^2 + 9)/x - 2*arcsin(2/3*x)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$-2\,\mathrm{asin}\left(\frac{2\,x}{3}\right)-\frac{2\,\sqrt{\frac{9}{4}-x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^2,x)
[Out] - 2*asin((2*x)/3) - (2*(9/4 - x^2)^(1/2))/x
```

sympy [A] time = 0.24, size = 20, normalized size = 0.80

$$-2\,\mathrm{asin}\left(\frac{2x}{3}\right)-\frac{\sqrt{9-4x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**2,x)
[Out] -2*asin(2*x/3) - sqrt(9 - 4*x**2)/x
```

3.451 $\int \frac{\sqrt{9-4x^2}}{x^3} \, dx$

Optimal. Leaf size=39

$$\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{2x^2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(2*x^2) + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9-4x^2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9-4x}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{2x^2} - \text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9-4x^2}}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\\ &= -\frac{\sqrt{9-4x^2}}{2x^2} + \frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.95

$$\frac{2}{3}\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right) - \frac{\sqrt{9-4x^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^3,x]
[Out] -1/2*Sqrt[9 - 4*x^2]/x^2 + (2*ArcTanh[Sqrt[1 - (4*x^2)/9]])/3
```

IntegrateAlgebraic [A] time = 0.04, size = 39, normalized size = 1.00

$$\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right) - \frac{\sqrt{9-4x^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 - 4*x^2]/x^3,x]
[Out] -1/2*Sqrt[9 - 4*x^2]/x^2 + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/3
```

fricas [A] time = 1.15, size = 38, normalized size = 0.97

$$-\frac{4\,x^2\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right) + 3\,\sqrt{-4\,x^2+9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*x^2*log((sqrt(-4*x^2 + 9) - 3)/x) + 3*sqrt(-4*x^2 + 9))/x^2
```

giac [A] time = 1.07, size = 45, normalized size = 1.15

$$-\frac{\sqrt{-4\,x^2+9}}{2\,x^2}+\frac{1}{3}\,\log\left(\sqrt{-4\,x^2+9}+3\right)-\frac{1}{3}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-4*x^2 + 9)/x^2 + 1/3*log(sqrt(-4*x^2 + 9) + 3) - 1/3*log(-sqrt(-
4*x^2 + 9) + 3)
```

maple [A] time = 0.01, size = 41, normalized size = 1.05

$$\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{3}-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{18x^2}-\frac{2\sqrt{-4x^2+9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^3,x)
```

```
[Out] -1/18/x^2*(-4*x^2+9)^(3/2)-2/9*(-4*x^2+9)^(1/2)+2/3*arctanh(3/(-4*x^2+9)^(1
/2))
```

maxima [A] time = 2.99, size = 51, normalized size = 1.31

$$-\frac{2}{9}\,\sqrt{-4\,x^2+9}-\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}}{18\,x^2}+\frac{2}{3}\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(-4*x^2 + 9) - 1/18*(-4*x^2 + 9)^(3/2)/x^2 + 2/3*log(6*sqrt(-4*x^2
 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.66, size = 35, normalized size = 0.90

$$-\frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)}{3}-\frac{\sqrt{\frac{9}{4}-x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^3,x)
```

```
[Out] - (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2))/3 - (9/4 - x^2)^(1/2
)/x^2
```

sympy [A] time = 1.73, size = 97, normalized size = 2.49

$$\begin{cases} \frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} + \frac{1}{x\sqrt{-1+\frac{9}{4x^2}}} - \frac{9}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ -\frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{3} - \frac{i}{x\sqrt{1-\frac{9}{4x^2}}} + \frac{9i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**3,x)
```

```
[Out] Piecewise((2*acosh(3/(2*x))/3 + 1/(x*sqrt(-1 + 9/(4*x**2))) - 9/(4*x**3*sqr
t(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (-2*I*asin(3/(2*x))/3 - I/(x*sqr
t(1 - 9/(4*x**2))) + 9*I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

3.452 $\int \frac{\sqrt{9-4x^2}}{x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{\left(9-4x^2\right)^{3/2}}{27x^3}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\left(9-4x^2\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^4,x]
```

```
[Out] -(9 - 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{9-4x^2}}{x^4}\,dx=-\frac{\left(9-4x^2\right)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\left(9-4x^2\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^4,x]
```

```
[Out] -1/27*(9 - 4*x^2)^(3/2)/x^3
```

IntegrateAlgebraic [A] time = 0.03, size = 25, normalized size = 1.39

$$\frac{\sqrt{9-4x^2}\left(4x^2-9\right)}{27x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 - 4*x^2]/x^4,x]
```

```
[Out] (Sqrt[9 - 4*x^2]*(-9 + 4*x^2))/(27*x^3)
```

fricas [A] time = 1.31, size = 21, normalized size = 1.17

$$\frac{\left(4\,x^2-9\right)\sqrt{-4\,x^2+9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(4*x^2 - 9)*sqrt(-4*x^2 + 9)/x^3
```

giac [B] time = 1.05, size = 73, normalized size = 4.06

$$-\frac{2\,x^3\left(\frac{3\left(\sqrt{-4\,x^2+9}-3\right)^2}{x^2}-4\right)}{27\left(\sqrt{-4\,x^2+9}-3\right)^3}+\frac{\sqrt{-4\,x^2+9}-3}{18\,x}-\frac{\left(\sqrt{-4\,x^2+9}-3\right)^3}{216\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -2/27*x^3*(3*(sqrt(-4*x^2 + 9) - 3)^2/x^2 - 4)/(sqrt(-4*x^2 + 9) - 3)^3 + 1
/18*(sqrt(-4*x^2 + 9) - 3)/x - 1/216*(sqrt(-4*x^2 + 9) - 3)^3/x^3
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$\frac{(2x-3)\,(2x+3)\,\sqrt{-4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^4,x)
```

```
[Out] 1/27/x^3*(2*x-3)*(2*x+3)*(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.90, size = 14, normalized size = 0.78

$$-\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(-4*x^2 + 9)^(3/2)/x^3
```

mupad [B] time = 0.03, size = 31, normalized size = 1.72

$$\frac{8\,x^2\,\sqrt{\frac{9}{4}-x^2}-18\,\sqrt{\frac{9}{4}-x^2}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^4,x)
```

```
[Out] (8*x^2*(9/4 - x^2)^(1/2) - 18*(9/4 - x^2)^(1/2))/(27*x^3)
```

sympy [B] time = 0.97, size = 76, normalized size = 4.22

$$\begin{cases} \frac{8\sqrt{-1+\frac{9}{4x^2}}}{27}-\frac{2\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for}\ \frac{9}{4|x^2|}>1 \\ \frac{8i\sqrt{1-\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**4,x)
```

```
[Out] Piecewise((8*sqrt(-1 + 9/(4*x**2))/27 - 2*sqrt(-1 + 9/(4*x**2))/(3*x**2), 9
/(4*Abs(x**2)) > 1), (8*I*sqrt(1 - 9/(4*x**2))/27 - 2*I*sqrt(1 - 9/(4*x**2)
)/(3*x**2), True))
```

3.453 $\int \frac{\sqrt{9-4x^2}}{x^5} \, dx$

Optimal. Leaf size=57

$$\frac{\sqrt{9-4x^2}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{4x^4}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 206}

$$\frac{\sqrt{9-4x^2}}{18x^2}-\frac{\sqrt{9-4x^2}}{4x^4}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(4*x^4) + Sqrt[9 - 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 - 4
*x^2]/3])/27
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{9-4x^2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{9-4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} + \frac{\sqrt{9-4x^2}}{18x^2} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} + \frac{\sqrt{9-4x^2}}{18x^2} + \frac{1}{18}\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\\
&= -\frac{\sqrt{9-4x^2}}{4x^4} + \frac{\sqrt{9-4x^2}}{18x^2} + \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$-\frac{16\left(9-4x^2\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{4x^2}{9}\right)}{2187}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - 4*x^2]/x^5,x]
[Out] (-16*(9 - 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - (4*x^2)/9])/2187
```

IntegrateAlgebraic [A] time = 0.04, size = 46, normalized size = 0.81

$$\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)+\frac{\sqrt{9-4x^2}\,(2x^2-9)}{36x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[9 - 4*x^2]/x^5,x]
```

```
[Out] (Sqrt[9 - 4*x^2]*(-9 + 2*x^2))/(36*x^4) + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/27
```

fricas [A] time = 1.01, size = 45, normalized size = 0.79

$$-\frac{8\,x^4\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\,(2\,x^2-9)\sqrt{-4\,x^2+9}}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/108*(8*x^4*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 - 9)*sqrt(-4*x^2 + 9
))/x^4
```

giac [A] time = 1.06, size = 57, normalized size = 1.00

$$-\frac{(-4\,x^2+9)^{\frac{3}{2}}+9\,\sqrt{-4\,x^2+9}}{72\,x^4}+\frac{1}{27}\,\log\left(\sqrt{-4\,x^2+9}+3\right)-\frac{1}{27}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*((-4*x^2 + 9)^(3/2) + 9*sqrt(-4*x^2 + 9))/x^4 + 1/27*log(sqrt(-4*x^2
+ 9) + 3) - 1/27*log(-sqrt(-4*x^2 + 9) + 3)
```

maple [A] time = 0.00, size = 55, normalized size = 0.96

$$\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}-\frac{(-4x^2+9)^{\frac{3}{2}}}{162x^2}-\frac{(-4x^2+9)^{\frac{3}{2}}}{36x^4}-\frac{2\sqrt{-4x^2+9}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+9)^(1/2)/x^5,x)
```

```
[Out] -1/36/x^4*(-4*x^2+9)^(3/2)-1/162*(-4*x^2+9)^(3/2)/x^2-2/81*(-4*x^2+9)^(1/2)
+2/27*arctanh(3/(-4*x^2+9)^(1/2))
```

maxima [A] time = 3.02, size = 65, normalized size = 1.14

$$-\frac{2}{81}\sqrt{-4x^2+9}-\frac{(-4x^2+9)^{\frac{3}{2}}}{162x^2}-\frac{(-4x^2+9)^{\frac{3}{2}}}{36x^4}+\frac{2}{27}\log\left(\frac{6\sqrt{-4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(-4*x^2 + 9) - 1/162*(-4*x^2 + 9)^(3/2)/x^2 - 1/36*(-4*x^2 + 9)^(
3/2)/x^4 + 2/27*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 0.03, size = 49, normalized size = 0.86

$$\frac{\sqrt{\frac{9}{4}-x^2}}{9x^2}-\frac{2\ln\left(\sqrt{\frac{9}{4x^2}-1}-\frac{3\sqrt{\frac{1}{x^2}}}{2}\right)}{27}-\frac{\sqrt{\frac{9}{4}-x^2}}{2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9 - 4*x^2)^(1/2)/x^5,x)
```

```
[Out] (9/4 - x^2)^(1/2)/(9*x^2) - (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2)
)/2))/27 - (9/4 - x^2)^(1/2)/(2*x^4)
```

sympy [A] time = 3.27, size = 139, normalized size = 2.44

$$\begin{cases} \frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{27}-\frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}}+\frac{3}{4x^3\sqrt{-1+\frac{9}{4x^2}}}-\frac{9}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{27}+\frac{i}{9x\sqrt{1-\frac{9}{4x^2}}}-\frac{3i}{4x^3\sqrt{1-\frac{9}{4x^2}}}+\frac{9i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+9)**(1/2)/x**5,x)
```

```
[Out] Piecewise((2*acosh(3/(2*x))/27 - 1/(9*x*sqrt(-1 + 9/(4*x**2))) + 3/(4*x**3*
sqrt(-1 + 9/(4*x**2))) - 9/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2))
> 1), (-2*I*asin(3/(2*x))/27 + I/(9*x*sqrt(1 - 9/(4*x**2))) - 3*I/(4*x**3*s
qrt(1 - 9/(4*x**2))) + 9*I/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

3.454 $\int x^5\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{448}\left(4x^2-9\right)^{7/2}+\frac{9}{160}\left(4x^2-9\right)^{5/2}+\frac{27}{64}\left(4x^2-9\right)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{448}\left(4x^2-9\right)^{7/2}+\frac{9}{160}\left(4x^2-9\right)^{5/2}+\frac{27}{64}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (27*(-9 + 4*x^2)^(3/2))/64 + (9*(-9 + 4*x^2)^(5/2))/160 + (-9 + 4*x^2)^(7/2
)/448
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5\sqrt{-9+4x^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2\sqrt{-9+4x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{81}{16}\sqrt{-9+4x}+\frac{9}{8}(-9+4x)^{3/2}+\frac{1}{16}(-9+4x)^{5/2}\right)dx,x,x^2\right)\\
&= \frac{27}{64}\left(-9+4x^2\right)^{3/2}+\frac{9}{160}\left(-9+4x^2\right)^{5/2}+\frac{1}{448}\left(-9+4x^2\right)^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(4x^2-9\right)^{3/2}\left(10x^4+18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-9 + 4*x^2],x]
```

```
[Out] ((-9 + 4*x^2)^(3/2)*(27 + 18*x^2 + 10*x^4))/280
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(4x^2-9\right)^{3/2}\left(10x^4+18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[-9 + 4*x^2],x]
```

```
[Out] ((-9 + 4*x^2)^(3/2)*(27 + 18*x^2 + 10*x^4))/280
```

fricas [A] time = 1.29, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^6-18\,x^4-54\,x^2-243\right)\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/280*(40*x^6 - 18*x^4 - 54*x^2 - 243)*sqrt(4*x^2 - 9)
```

giac [A] time = 0.97, size = 34, normalized size = 0.74

$$\frac{1}{448}\left(4\,x^2-9\right)^{\frac{7}{2}}+\frac{9}{160}\left(4\,x^2-9\right)^{\frac{5}{2}}+\frac{27}{64}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/448*(4*x^2 - 9)^(7/2) + 9/160*(4*x^2 - 9)^(5/2) + 27/64*(4*x^2 - 9)^(3/2)
```

maple [A] time = 0.01, size = 34, normalized size = 0.74

$$\frac{(2x-3)\,(2x+3)\left(10x^4+18x^2+27\right)\sqrt{4x^2-9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/280*(2*x-3)*(2*x+3)*(10*x^4+18*x^2+27)*(4*x^2-9)^(1/2)
```

maxima [A] time = 2.94, size = 40, normalized size = 0.87

$$\frac{1}{28}\left(4x^2-9\right)^{\frac{3}{2}}x^4+\frac{9}{140}\left(4x^2-9\right)^{\frac{3}{2}}x^2+\frac{27}{280}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/28*(4*x^2 - 9)^(3/2)*x^4 + 9/140*(4*x^2 - 9)^(3/2)*x^2 + 27/280*(4*x^2 -
9)^(3/2)
```

mupad [B] time = 5.31, size = 28, normalized size = 0.61

$$-\sqrt{4x^2-9}\left(-\frac{x^6}{7}+\frac{9x^4}{140}+\frac{27x^2}{140}+\frac{243}{280}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(4*x^2 - 9)^(1/2),x)
```

```
[Out] -(4*x^2 - 9)^(1/2)*((27*x^2)/140 + (9*x^4)/140 - x^6/7 + 243/280)
```

sympy [A] time = 1.92, size = 61, normalized size = 1.33

$$\frac{x^6\sqrt{4x^2-9}}{7}-\frac{9x^4\sqrt{4x^2-9}}{140}-\frac{27x^2\sqrt{4x^2-9}}{140}-\frac{243\sqrt{4x^2-9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(4*x**2-9)**(1/2),x)
```

```
[Out] x**6*sqrt(4*x**2 - 9)/7 - 9*x**4*sqrt(4*x**2 - 9)/140 - 27*x**2*sqrt(4*x**2
 - 9)/140 - 243*sqrt(4*x**2 - 9)/280
```

3.455 $\int x^4\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{81}{256}\sqrt{4x^2-9}\,x-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{6}\sqrt{4x^2-9}\,x^5-\frac{3}{32}\sqrt{4x^2-9}\,x^3$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$\frac{1}{6}\sqrt{4x^2-9}\,x^5-\frac{3}{32}\sqrt{4x^2-9}\,x^3-\frac{81}{256}\sqrt{4x^2-9}\,x-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[-9 + 4*x^2])/256 - (3*x^3*Sqrt[-9 + 4*x^2])/32 + (x^5*Sqrt[-9 +
 4*x^2])/6 - (729*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/512
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^4\sqrt{-9+4x^2}\,dx &= \frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{3}{2}\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx\\
&= -\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{81}{32}\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{256}\int \frac{1}{\sqrt{-9+4x^2}}\,dx\\
&= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{256}\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,-\right.\\
&= -\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{512}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.68

$$\frac{1}{768}x\sqrt{4x^2-9}\left(128x^4-72x^2-243\right)-\frac{729}{512}\log\left(\sqrt{4x^2-9}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 - (729*Log[2*x + Sqrt[-9
 + 4*x^2]])/512
```

IntegrateAlgebraic [A] time = 0.05, size = 50, normalized size = 0.69

$$\frac{729}{512}\log\left(\sqrt{4x^2-9}-2x\right)+\frac{1}{768}\sqrt{4x^2-9}\left(128x^5-72x^3-243x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (Sqrt[-9 + 4*x^2]*(-243*x - 72*x^3 + 128*x^5))/768 + (729*Log[-2*x + Sqrt[-
9 + 4*x^2]])/512
```

fricas [A] time = 1.22, size = 42, normalized size = 0.58

$$\frac{1}{768}\left(128\,x^5-72\,x^3-243\,x\right)\sqrt{4\,x^2-9}+\frac{729}{512}\,\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 - 72*x^3 - 243*x)*sqrt(4*x^2 - 9) + 729/512*log(-2*x + sqrt(
4*x^2 - 9))
```

giac [A] time = 1.19, size = 44, normalized size = 0.61

$$\frac{1}{768}\left(8\left(16\,x^2-9\right)x^2-243\right)\sqrt{4\,x^2-9}\,x+\frac{729}{512}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(4*x^2 - 9)*x + 729/512*log(abs(-2*x +
 sqrt(4*x^2 - 9)))
```

maple [A] time = 0.01, size = 61, normalized size = 0.85

$$\frac{\left(4x^2-9\right)^{\frac{3}{2}}x^3}{24}+\frac{9\left(4x^2-9\right)^{\frac{3}{2}}x}{128}+\frac{81\sqrt{4x^2-9}\;x}{256}-\frac{729\sqrt{4}\;\ln\left(\sqrt{4}\;x+\sqrt{4x^2-9}\right)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/24*x^3*(4*x^2-9)^(3/2)+9/128*x*(4*x^2-9)^(3/2)+81/256*x*(4*x^2-9)^(1/2)-7
29/1024*ln(x*4^(1/2)+(4*x^2-9)^(1/2))*4^(1/2)
```

maxima [A] time = 2.92, size = 57, normalized size = 0.79

$$\frac{1}{24}\left(4\,x^2-9\right)^{\frac{3}{2}}x^3+\frac{9}{128}\left(4\,x^2-9\right)^{\frac{3}{2}}x+\frac{81}{256}\,\sqrt{4\,x^2-9}\,x-\frac{729}{512}\,\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^2 - 9)^(3/2)*x^3 + 9/128*(4*x^2 - 9)^(3/2)*x + 81/256*sqrt(4*x^2
- 9)*x - 729/512*log(8*x + 4*sqrt(4*x^2 - 9))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{4\,x^2-9}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^4*(4*x^2 - 9)^(1/2), x)
```

sympy [A] time = 4.56, size = 167, normalized size = 2.32

$$\begin{cases} \frac{2x^7}{3\sqrt{4x^2-9}} - \frac{15x^5}{8\sqrt{4x^2-9}} - \frac{27x^3}{64\sqrt{4x^2-9}} + \frac{729x}{256\sqrt{4x^2-9}} - \frac{729\operatorname{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{2ix^7}{3\sqrt{9-4x^2}} + \frac{15ix^5}{8\sqrt{9-4x^2}} + \frac{27ix^3}{64\sqrt{9-4x^2}} - \frac{729ix}{256\sqrt{9-4x^2}} + \frac{729i\operatorname{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*x**7/(3*sqrt(4*x**2 - 9)) - 15*x**5/(8*sqrt(4*x**2 - 9)) - 27*
x**3/(64*sqrt(4*x**2 - 9)) + 729*x/(256*sqrt(4*x**2 - 9)) - 729*acosh(2*x/3
)/512, 4*Abs(x**2)/9 > 1), (-2*I*x**7/(3*sqrt(9 - 4*x**2)) + 15*I*x**5/(8*s
qrt(9 - 4*x**2)) + 27*I*x**3/(64*sqrt(9 - 4*x**2)) - 729*I*x/(256*sqrt(9 -
4*x**2)) + 729*I*asin(2*x/3)/512, True))
```

3.456 $\int x^3\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(4x^2-9\right)^{5/2}+\frac{3}{16}\left(4x^2-9\right)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(4x^2-9\right)^{5/2}+\frac{3}{16}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (3*(-9 + 4*x^2)^(3/2))/16 + (-9 + 4*x^2)^(5/2)/80
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{-9+4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int x\sqrt{-9+4x}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{9}{4}\sqrt{-9+4x}+\frac{1}{4}(-9+4x)^{3/2}\right)dx,x,x^2\right)\\&=\frac{3}{16}\left(-9+4x^2\right)^{3/2}+\frac{1}{80}\left(-9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(2x^2+3\right)\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-9 + 4*x^2],x]
```

```
[Out] ((3 + 2*x^2)*(-9 + 4*x^2)^(3/2))/40
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(2x^2+3\right)\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[-9 + 4*x^2],x]
```

```
[Out] ((3 + 2*x^2)*(-9 + 4*x^2)^(3/2))/40
```

fricas [A] time = 1.04, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8\,x^4-6\,x^2-27\right)\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 6*x^2 - 27)*sqrt(4*x^2 - 9)
```

giac [A] time = 1.07, size = 23, normalized size = 0.74

$$\frac{1}{80}\left(4\,x^2-9\right)^{\frac{5}{2}}+\frac{3}{16}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/80*(4*x^2 - 9)^(5/2) + 3/16*(4*x^2 - 9)^(3/2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$\frac{(2x-3)\,(2x+3)\left(2x^2+3\right)\sqrt{4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/40*(2*x-3)*(2*x+3)*(2*x^2+3)*(4*x^2-9)^(1/2)
```

maxima [A] time = 2.91, size = 26, normalized size = 0.84

$$\frac{1}{20}\left(4\,x^2-9\right)^{\frac{3}{2}}x^2+\frac{3}{40}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/20*(4*x^2 - 9)^(3/2)*x^2 + 3/40*(4*x^2 - 9)^(3/2)
```

mupad [B] time = 5.51, size = 23, normalized size = 0.74

$$-\sqrt{4\,x^2-9}\,\left(-\frac{x^4}{5}+\frac{3\,x^2}{20}+\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*x^2 - 9)^(1/2),x)
[Out] -(4*x^2 - 9)^(1/2)*((3*x^2)/20 - x^4/5 + 27/40)
```

sympy [A] time = 0.66, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4x^2-9}}{5}-\frac{3x^2\sqrt{4x^2-9}}{20}-\frac{27\sqrt{4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(4*x**2-9)**(1/2),x)
[Out] x**4*sqrt(4*x**2 - 9)/5 - 3*x**2*sqrt(4*x**2 - 9)/20 - 27*sqrt(4*x**2 - 9)/
40
```

3.457 $\int x^2\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{9}{32}\sqrt{4x^2-9}\,x-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{4}\sqrt{4x^2-9}\,x^3$$

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 206}

$$\frac{1}{4}\sqrt{4x^2-9}\,x^3-\frac{9}{32}\sqrt{4x^2-9}\,x-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (-9*x*Sqrt[-9 + 4*x^2])/32 + (x^3*Sqrt[-9 + 4*x^2])/4 - (81*ArcTanh[(2*x)/S
qrt[-9 + 4*x^2]])/64
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{-9+4x^2}\,dx &= \frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{9}{4}\int\frac{x^2}{\sqrt{-9+4x^2}}\,dx\\
&= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{32}\int\frac{1}{\sqrt{-9+4x^2}}\,dx\\
&= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{32}\,\mathrm{Subst}\left(\int\frac{1}{1-4x^2}\,dx,x,\frac{x}{\sqrt{-9+4x^2}}\right)\\
&= -\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{64}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.85

$$\sqrt{4x^2-9}\left(\frac{x^3}{4}-\frac{9x}{32}\right)-\frac{81}{64}\log\left(\sqrt{4x^2-9}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-9 + 4*x^2],x]
[Out] Sqrt[-9 + 4*x^2]*((-9*x)/32 + x^3/4) - (81*Log[2*x + Sqrt[-9 + 4*x^2]])/64
```

IntegrateAlgebraic [A] time = 0.03, size = 45, normalized size = 0.83

$$\frac{81}{64}\log\left(\sqrt{4x^2-9}-2x\right)+\frac{1}{32}\sqrt{4x^2-9}\left(8x^3-9x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[-9 + 4*x^2],x]
[Out] (Sqrt[-9 + 4*x^2]*(-9*x + 8*x^3))/32 + (81*Log[-2*x + Sqrt[-9 + 4*x^2]])/64
```

fricas [A] time = 0.95, size = 37, normalized size = 0.69

$$\frac{1}{32}\left(8x^3-9x\right)\sqrt{4x^2-9}+\frac{81}{64}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 9*x)*sqrt(4*x^2 - 9) + 81/64*log(-2*x + sqrt(4*x^2 - 9))
```

giac [A] time = 1.16, size = 37, normalized size = 0.69

$$\frac{1}{32}\left(8\,x^2-9\right)\sqrt{4\,x^2-9}\,x+\frac{81}{64}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*(8*x^2 - 9)*sqrt(4*x^2 - 9)*x + 81/64*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

maple [A] time = 0.01, size = 47, normalized size = 0.87

$$\frac{\left(4x^2-9\right)^{\frac{3}{2}}x}{16}+\frac{9\sqrt{4x^2-9}\,x}{32}-\frac{81\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2-9)^(1/2),x)
```

```
[Out] 1/16*(4*x^2-9)^(3/2)*x+9/32*(4*x^2-9)^(1/2)*x-81/128*4^(1/2)*ln(4^(1/2)*x+(
4*x^2-9)^(1/2))
```

maxima [A] time = 2.89, size = 43, normalized size = 0.80

$$\frac{1}{16}\left(4\,x^2-9\right)^{\frac{3}{2}}x+\frac{9}{32}\,\sqrt{4\,x^2-9}\,x-\frac{81}{64}\,\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*x^2 - 9)^(3/2)*x + 9/32*sqrt(4*x^2 - 9)*x - 81/64*log(8*x + 4*sqrt(
4*x^2 - 9))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\sqrt{4\,x^2-9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^2*(4*x^2 - 9)^(1/2), x)
```

sympy [A] time = 2.80, size = 124, normalized size = 2.30

$$\begin{cases} \frac{x^5}{\sqrt{4x^2-9}} - \frac{27x^3}{8\sqrt{4x^2-9}} + \frac{81x}{32\sqrt{4x^2-9}} - \frac{81\operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{ix^5}{\sqrt{9-4x^2}} + \frac{27ix^3}{8\sqrt{9-4x^2}} - \frac{81ix}{32\sqrt{9-4x^2}} + \frac{81i\operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((x**5/sqrt(4*x**2 - 9) - 27*x**3/(8*sqrt(4*x**2 - 9)) + 81*x/(32*
sqrt(4*x**2 - 9)) - 81*acosh(2*x/3)/64, 4*Abs(x**2)/9 > 1), (-I*x**5/sqrt(9
 - 4*x**2) + 27*I*x**3/(8*sqrt(9 - 4*x**2)) - 81*I*x/(32*sqrt(9 - 4*x**2))
+ 81*I*asin(2*x/3)/64, True))
```

3.458 $\int x\sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (-9 + 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{-9+4x^2}\,dx=\frac{1}{12}\left(-9+4x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-9 + 4*x^2],x]
```

```
[Out] (-9 + 4*x^2)^(3/2)/12
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[-9 + 4*x^2],x]
[Out] (-9 + 4*x^2)^(3/2)/12
```

fricas [A] time = 0.77, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

giac [A] time = 1.03, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

maple [A] time = 0.00, size = 22, normalized size = 1.47

$$\frac{(2x-3)(2x+3)\sqrt{4x^2-9}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)*x,x)
[Out] 1/12*(2*x-3)*(2*x+3)*(4*x^2-9)^(1/2)
```

maxima [A] time = 1.30, size = 11, normalized size = 0.73

$$\frac{1}{12}\left(4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/12*(4*x^2 - 9)^(3/2)
```

mupad [B]   time = 0.06, size = 11, normalized size = 0.73

$$\frac{\left(4\,x^2-9\right)^{3/2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2 - 9)^(1/2),x)
```

```
[Out] (4*x^2 - 9)^(3/2)/12
```

sympy [B]   time = 0.20, size = 27, normalized size = 1.80

$$\frac{x^2\sqrt{4x^2-9}}{3}-\frac{3\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2-9)**(1/2),x)
```

```
[Out] x**2*sqrt(4*x**2 - 9)/3 - 3*sqrt(4*x**2 - 9)/4
```

3.459 $\int \sqrt{-9+4x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2])/2 - (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{-9+4x^2}\,dx = \frac{1}{2}x\sqrt{-9+4x^2} - \frac{9}{2}\int \frac{1}{\sqrt{-9+4x^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{-9+4x^2} - \frac{9}{2}\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)$$
$$= \frac{1}{2}x\sqrt{-9+4x^2} - \frac{9}{4}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.03

$$\frac{1}{2}x\sqrt{4x^2-9} - \frac{9}{4}\log\left(\sqrt{4x^2-9}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2],x]
[Out] (x*Sqrt[-9 + 4*x^2])/2 - (9*Log[2*x + Sqrt[-9 + 4*x^2]])/4
```

IntegrateAlgebraic [A] time = 0.03, size = 37, normalized size = 1.03

$$\frac{1}{2}\sqrt{4x^2-9}\,x + \frac{9}{4}\log\left(\sqrt{4x^2-9}-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 + 4*x^2],x]
[Out] (x*Sqrt[-9 + 4*x^2])/2 + (9*Log[-2*x + Sqrt[-9 + 4*x^2]])/4
```

fricas [A] time = 0.78, size = 29, normalized size = 0.81

$$\frac{1}{2}\sqrt{4x^2-9}\,x + \frac{9}{4}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(4*x^2 - 9)*x + 9/4*log(-2*x + sqrt(4*x^2 - 9))
```

giac [A] time = 1.11, size = 30, normalized size = 0.83

$$\frac{1}{2}\sqrt{4x^2-9}\,x + \frac{9}{4}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(4*x^2 - 9)*x + 9/4*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

maple [A] time = 0.00, size = 35, normalized size = 0.97

$$\frac{\sqrt{4x^2-9}\,x}{2}-\frac{9\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2),x)
[Out] 1/2*(4*x^2-9)^(1/2)*x-9/8*4^(1/2)*ln(4^(1/2)*x+(4*x^2-9)^(1/2))
```

maxima [A] time = 2.97, size = 31, normalized size = 0.86

$$\frac{1}{2}\sqrt{4x^2-9}\,x-\frac{9}{4}\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(4*x^2 - 9)*x - 9/4*log(8*x + 4*sqrt(4*x^2 - 9))
```

mupad [B] time = 5.03, size = 29, normalized size = 0.81

$$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(2x+\sqrt{4x^2-9}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2),x)
[Out] (x*(4*x^2 - 9)^(1/2))/2 - (9*log(2*x + (4*x^2 - 9)^(1/2)))/4
```

sympy [A] time = 0.22, size = 22, normalized size = 0.61

$$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\operatorname{acosh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2),x)
[Out] x*sqrt(4*x**2 - 9)/2 - 9*acosh(2*x/3)/4
```

3.460 $\int \frac{\sqrt{-9+4x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 203}

$$\sqrt{4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x,x]
```

```
[Out] Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-9+4x^2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{-9+4x}}{x}\,dx, x, x^2\right)\\ &= \sqrt{-9+4x^2} - \frac{9}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\ &= \sqrt{-9+4x^2} - \frac{9}{4}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\ &= \sqrt{-9+4x^2} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\sqrt{4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x,x]
[Out] Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 1.00

$$\sqrt{4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 + 4*x^2]/x,x]
[Out] Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

fricas [A] time = 0.90, size = 28, normalized size = 0.93

$$\sqrt{4\,x^2-9} - 6\,\arctan\left(-\frac{2}{3}\,x + \frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="fricas")
[Out] sqrt(4*x^2 - 9) - 6*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9))
```

giac [A] time = 1.01, size = 24, normalized size = 0.80

$$\sqrt{4\,x^2-9}-3\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="giac")
[Out] sqrt(4*x^2 - 9) - 3*arctan(1/3*sqrt(4*x^2 - 9))
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$3\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)+\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x,x)
[Out] (4*x^2-9)^(1/2)+3*arctan(3/(4*x^2-9)^(1/2))
```

maxima [A] time = 3.01, size = 19, normalized size = 0.63

$$\sqrt{4\,x^2-9}+3\,\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x,x, algorithm="maxima")
[Out] sqrt(4*x^2 - 9) + 3*arcsin(3/2/abs(x))
```

mupad [B] time = 5.34, size = 24, normalized size = 0.80

$$\sqrt{4\,x^2-9}-3\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x,x)
```

```
[Out] (4*x^2 - 9)^(1/2) - 3*atan((4*x^2 - 9)^(1/2)/3)
```

sympy **[C]** time = 1.37, size = 82, normalized size = 2.73

$$\begin{cases} \sqrt{4x^2 - 9} - 3i\log(x) + \frac{3i\log(x^2)}{2} + 3\,\mathrm{asin}\left(\frac{3}{2x}\right) & \text{for } \frac{4|x^2|}{9} > 1 \\ i\sqrt{9 - 4x^2} + \frac{3i\log(x^2)}{2} - 3i\log\left(\sqrt{1 - \frac{4x^2}{9}} + 1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x,x)
```

```
[Out] Piecewise((sqrt(4*x**2 - 9) - 3*I*log(x) + 3*I*log(x**2)/2 + 3*asin(3/(2*x)
), 4*Abs(x**2)/9 > 1), (I*sqrt(9 - 4*x**2) + 3*I*log(x**2)/2 - 3*I*log(sqrt
(1 - 4*x**2/9) + 1), True))
```

3.461 $\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx$

Optimal. Leaf size=34

$$2\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)-\frac{\sqrt{4x^2-9}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 206}

$$2\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)-\frac{\sqrt{4x^2-9}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 + 4*x^2]/x) + 2*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx = -\frac{\sqrt{-9+4x^2}}{x} + 4\int \frac{1}{\sqrt{-9+4x^2}}\,dx$$
$$= -\frac{\sqrt{-9+4x^2}}{x} + 4\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)$$
$$= -\frac{\sqrt{-9+4x^2}}{x} + 2\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.41

$$-\frac{\sqrt{4x^2-9} + \frac{2x\sqrt{4x^2-9}\,\sin^{-1}\left(\frac{2x}{3}\right)}{\sqrt{9-4x^2}}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^2,x]
```

```
[Out] -((Sqrt[-9 + 4*x^2] + (2*x*Sqrt[-9 + 4*x^2]*ArcSin[(2*x)/3])/Sqrt[9 - 4*x^2
])/x)
```

IntegrateAlgebraic [A] time = 0.04, size = 35, normalized size = 1.03

$$-\frac{\sqrt{4x^2-9}}{x} - 2\log\left(\sqrt{4x^2-9} - 2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 + 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 + 4*x^2]/x) - 2*Log[-2*x + Sqrt[-9 + 4*x^2]]
```

fricas [A] time = 1.27, size = 35, normalized size = 1.03

$$-\frac{2\,x\log\left(-2\,x + \sqrt{4\,x^2-9}\right) + 2\,x + \sqrt{4\,x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*x*log(-2*x + sqrt(4*x^2 - 9)) + 2*x + sqrt(4*x^2 - 9))/x
```

giac [A] time = 1.23, size = 44, normalized size = 1.29

$$-\frac{36}{\left(2x-\sqrt{4x^2-9}\right)^2+9}-\log\left(\left(2x-\sqrt{4x^2-9}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="giac")
[Out] -36/((2*x - sqrt(4*x^2 - 9))^2 + 9) - log((2*x - sqrt(4*x^2 - 9))^2)
```

maple [A] time = 0.00, size = 48, normalized size = 1.41

$$-\frac{4\sqrt{4x^2-9}\,x}{9}+\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^2,x)
[Out] 1/9/x*(4*x^2-9)^(3/2)-4/9*(4*x^2-9)^(1/2)*x+4^(1/2)*ln(4^(1/2)*x+(4*x^2-9)^
(1/2))
```

maxima [A] time = 2.93, size = 33, normalized size = 0.97

$$-\frac{\sqrt{4x^2-9}}{x}+2\log\left(8x+4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="maxima")
[Out] -sqrt(4*x^2 - 9)/x + 2*log(8*x + 4*sqrt(4*x^2 - 9))
```

mupad [B] time = 5.55, size = 39, normalized size = 1.15

$$-\frac{\sqrt{4x^2-9}}{x}-\frac{2\,\mathrm{asin}\left(\frac{2x}{3}\right)\sqrt{4x^2-9}}{3\sqrt{1-\frac{4x^2}{9}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^2,x)
[Out] - (4*x^2 - 9)^(1/2)/x - (2*asin((2*x)/3)*(4*x^2 - 9)^(1/2))/(3*(1 - (4*x^2)
/9)^(1/2))
```

sympy [A] time = 0.25, size = 19, normalized size = 0.56

$$2\operatorname{acosh}\left(\frac{2x}{3}\right) - \frac{\sqrt{4x^2 - 9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**2,x)
```

```
[Out] 2*acosh(2*x/3) - sqrt(4*x**2 - 9)/x
```

3.462 $\int \frac{\sqrt{-9+4x^2}}{x^3} dx$

Optimal. Leaf size=39

$$\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{2x^2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 203}

$$\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[-9 + 4*x^2]/(2*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9+4x}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{2x^2} + \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{2x^2} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\
&= -\frac{\sqrt{-9+4x^2}}{2x^2} + \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.41

$$-\frac{12x^2 + 4\sqrt{9-4x^2}\,x^2\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right) - 27}{6x^2\sqrt{4x^2-9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^3,x]
```

```
[Out] -1/6*(-27 + 12*x^2 + 4*x^2*Sqrt[9 - 4*x^2]*ArcTanh[Sqrt[1 - (4*x^2)/9]])/(x
^2*Sqrt[-9 + 4*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right) - \frac{\sqrt{4x^2-9}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 + 4*x^2]/x^3,x]
```

```
[Out] -1/2*Sqrt[-9 + 4*x^2]/x^2 + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/3
```

fricas [A] time = 0.82, size = 38, normalized size = 0.97

$$\frac{8\,x^2\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)-3\,\sqrt{4\,x^2-9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/6*(8*x^2*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) - 3*sqrt(4*x^2 - 9))/x^2
```

giac [A] time = 1.07, size = 29, normalized size = 0.74

$$-\frac{\sqrt{4\,x^2-9}}{2\,x^2}+\frac{2}{3}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="giac")
[Out] -1/2*sqrt(4*x^2 - 9)/x^2 + 2/3*arctan(1/3*sqrt(4*x^2 - 9))
```

maple [A] time = 0.00, size = 41, normalized size = 1.05

$$-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{3}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{18x^2}-\frac{2\sqrt{4x^2-9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^3,x)
[Out] 1/18/x^2*(4*x^2-9)^(3/2)-2/9*(4*x^2-9)^(1/2)-2/3*arctan(3/(4*x^2-9)^(1/2))
```

maxima [A] time = 2.96, size = 35, normalized size = 0.90

$$-\frac{2}{9}\,\sqrt{4\,x^2-9}+\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}}{18\,x^2}-\frac{2}{3}\,\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="maxima")
[Out] -2/9*sqrt(4*x^2 - 9) + 1/18*(4*x^2 - 9)^(3/2)/x^2 - 2/3*arcsin(3/2/abs(x))
```

mupad [B] time = 5.33, size = 29, normalized size = 0.74

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{3}-\frac{\sqrt{4\,x^2-9}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^3,x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/3 - (4*x^2 - 9)^(1/2)/(2*x^2)
```

sympy [A] time = 1.74, size = 97, normalized size = 2.49

$$\begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{3}+\frac{i}{x\sqrt{-1+\frac{9}{4x^2}}}-\frac{9i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2\,\mathrm{asin}\left(\frac{3}{2x}\right)}{3}-\frac{1}{x\sqrt{1-\frac{9}{4x^2}}}+\frac{9}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**3,x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/3 + I/(x*sqrt(-1 + 9/(4*x**2))) - 9*I/(4*x**3
*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (-2*asin(3/(2*x))/3 - 1/(x*s
qrt(1 - 9/(4*x**2))) + 9/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

3.463 $\int \frac{\sqrt{-9+4x^2}}{x^4} \, dx$

Optimal. Leaf size=18

$$\frac{\left(4x^2-9\right)^{3/2}}{27x^3}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\left(4x^2-9\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^4,x]
```

```
[Out] (-9 + 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-9+4x^2}}{x^4} \, dx = \frac{\left(-9+4x^2\right)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(4x^2-9\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^4,x]
```

```
[Out] (-9 + 4*x^2)^(3/2)/(27*x^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{\left(4x^2-9\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 + 4*x^2]/x^4,x]
[Out] (-9 + 4*x^2)^(3/2)/(27*x^3)
```

fricas [A] time = 0.92, size = 20, normalized size = 1.11

$$\frac{8\,x^3+\left(4\,x^2-9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/27*(8*x^3 + (4*x^2 - 9)^(3/2))/x^3
```

giac [B] time = 1.10, size = 42, normalized size = 2.33

$$\frac{16\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^4+27\right)}{\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="giac")
[Out] 16*((2*x - sqrt(4*x^2 - 9))^4 + 27)/((2*x - sqrt(4*x^2 - 9))^2 + 9)^3
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$\frac{(2x-3)\,(2x+3)\,\sqrt{4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^4,x)
[Out] 1/27/x^3*(2*x-3)*(2*x+3)*(4*x^2-9)^(1/2)
```

maxima [A] time = 3.01, size = 14, normalized size = 0.78

$$\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/27*(4*x^2 - 9)^(3/2)/x^3
```

mupad [B] time = 5.10, size = 31, normalized size = 1.72

$$\frac{4\,x^2\,\sqrt{4\,x^2-9}-9\,\sqrt{4\,x^2-9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^4,x)
```

```
[Out] (4*x^2*(4*x^2 - 9)^(1/2) - 9*(4*x^2 - 9)^(1/2))/(27*x^3)
```

sympy [B] time = 0.98, size = 76, normalized size = 4.22

$$\begin{cases} \frac{8i\sqrt{-1+\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for } \frac{9}{4|x^2|}>1 \\ \frac{8\sqrt{1-\frac{9}{4x^2}}}{27}-\frac{2\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**4,x)
```

```
[Out] Piecewise((8*I*sqrt(-1 + 9/(4*x**2))/27 - 2*I*sqrt(-1 + 9/(4*x**2))/(3*x**2
), 9/(4*Abs(x**2)) > 1), (8*sqrt(1 - 9/(4*x**2))/27 - 2*sqrt(1 - 9/(4*x**2)
)/(3*x**2), True))
```

3.464 $\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{4x^4}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 203}

$$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{\sqrt{4x^2-9}}{4x^4}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[-9 + 4*x^2]/(4*x^4) + Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 +
 4*x^2]/3])/27
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9+4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4} + \frac{\sqrt{-9+4x^2}}{18x^2} + \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4} + \frac{\sqrt{-9+4x^2}}{18x^2} + \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\
&= -\frac{\sqrt{-9+4x^2}}{4x^4} + \frac{\sqrt{-9+4x^2}}{18x^2} + \frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$\frac{16\left(4x^2-9\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{4x^2}{9}\right)}{2187}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + 4*x^2]/x^5,x]
```

```
[Out] (16*(-9 + 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - (4*x^2)/9])/2187
```

IntegrateAlgebraic [A] time = 0.04, size = 46, normalized size = 0.81

$$\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)+\frac{\sqrt{4x^2-9}\,\left(2x^2-9\right)}{36x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 + 4*x^2]/x^5,x]
```

```
[Out] ((-9 + 2*x^2)*Sqrt[-9 + 4*x^2])/(36*x^4) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/2
7
```

fricas [A] time = 1.06, size = 45, normalized size = 0.79

$$\frac{16\,x^4\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}\left(2\,x^2-9\right)}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/108*(16*x^4*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9)*(2*x
^2 - 9))/x^4
```

giac [A] time = 0.98, size = 41, normalized size = 0.72

$$\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}-9\,\sqrt{4\,x^2-9}}{72\,x^4}+\frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/72*((4*x^2 - 9)^(3/2) - 9*sqrt(4*x^2 - 9))/x^4 + 2/27*arctan(1/3*sqrt(4*x
^2 - 9))
```

maple [A] time = 0.00, size = 55, normalized size = 0.96

$$-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{162x^2}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{36x^4}-\frac{2\sqrt{4x^2-9}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-9)^(1/2)/x^5,x)
```

```
[Out] 1/36/x^4*(4*x^2-9)^(3/2)+1/162*(4*x^2-9)^(3/2)/x^2-2/81*(4*x^2-9)^(1/2)-2/2
7*arctan(3/(4*x^2-9)^(1/2))
```

maxima [A] time = 2.96, size = 49, normalized size = 0.86

$$-\frac{2}{81}\sqrt{4x^2-9}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{162x^2}+\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{36x^4}-\frac{2}{27}\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/81*sqrt(4*x^2 - 9) + 1/162*(4*x^2 - 9)^(3/2)/x^2 + 1/36*(4*x^2 - 9)^(3/2
)/x^4 - 2/27*arcsin(3/2/abs(x))
```

mupad [B] time = 5.19, size = 43, normalized size = 0.75

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4x^2-9}}{3}\right)}{27}-\frac{\frac{\sqrt{4x^2-9}}{8}-\frac{\left(4x^2-9\right)^{3/2}}{72}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 9)^(1/2)/x^5,x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/27 - ((4*x^2 - 9)^(1/2)/8 - (4*x^2 - 9)^(3/2)
/72)/x^4
```

sympy [A] time = 3.36, size = 139, normalized size = 2.44

$$\begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}}+\frac{3i}{4x^3\sqrt{-1+\frac{9}{4x^2}}}-\frac{9i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2\,\mathrm{asin}\left(\frac{3}{2x}\right)}{27}+\frac{1}{9x\sqrt{1-\frac{9}{4x^2}}}-\frac{3}{4x^3\sqrt{1-\frac{9}{4x^2}}}+\frac{9}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-9)**(1/2)/x**5,x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + 3*I/(4*x
**3*sqrt(-1 + 9/(4*x**2))) - 9*I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x
**2)) > 1), (-2*asin(3/(2*x))/27 + 1/(9*x*sqrt(1 - 9/(4*x**2))) - 3/(4*x**3
*sqrt(1 - 9/(4*x**2))) + 9/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

3.465 $\int x^5\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{448}\left(-4x^2-9\right)^{7/2}-\frac{9}{160}\left(-4x^2-9\right)^{5/2}-\frac{27}{64}\left(-4x^2-9\right)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{448}\left(-4x^2-9\right)^{7/2}-\frac{9}{160}\left(-4x^2-9\right)^{5/2}-\frac{27}{64}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-27*(-9 - 4*x^2)^(3/2))/64 - (9*(-9 - 4*x^2)^(5/2))/160 - (-9 - 4*x^2)^(7/
2)/448
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{-9-4x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int\sqrt{-9-4x}\,x^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{81}{16}\sqrt{-9-4x}+\frac{9}{8}(-9-4x)^{3/2}+\frac{1}{16}(-9-4x)^{5/2}\right)dx,x,x^2\right)\\ &= -\frac{27}{64}\left(-9-4x^2\right)^{3/2}-\frac{9}{160}\left(-9-4x^2\right)^{5/2}-\frac{1}{448}\left(-9-4x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{1}{280}\left(-4x^2-9\right)^{3/2}\left(10x^4-18x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-9 - 4*x^2],x]
[Out] -1/280*((-9 - 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.59

$$\frac{1}{280}\left(-4x^2-9\right)^{3/2}\left(-10x^4+18x^2-27\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[-9 - 4*x^2],x]
[Out] ((-9 - 4*x^2)^(3/2)*(-27 + 18*x^2 - 10*x^4))/280
```

fricas [A] time = 0.99, size = 28, normalized size = 0.61

$$\frac{1}{280}\left(40\,x^6+18\,x^4-54\,x^2+243\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/280*(40*x^6 + 18*x^4 - 54*x^2 + 243)*sqrt(-4*x^2 - 9)
```

giac [A] time = 1.06, size = 37, normalized size = 0.80

$$\frac{1}{448}\left(4\,x^2+9\right)^{\frac{7}{2}}i-\frac{9}{160}\left(4\,x^2+9\right)^{\frac{5}{2}}i+\frac{27}{64}\left(4\,x^2+9\right)^{\frac{3}{2}}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/448*(4*x^2 + 9)^(7/2)*i - 9/160*(4*x^2 + 9)^(5/2)*i + 27/64*(4*x^2 + 9)^(
3/2)*i
```

maple [A] time = 0.00, size = 24, normalized size = 0.52

$$-\frac{\left(10x^4-18x^2+27\right)\left(-4x^2-9\right)^{\frac{3}{2}}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/280*(10*x^4-18*x^2+27)*(-4*x^2-9)^(3/2)
```

maxima [A] time = 2.95, size = 40, normalized size = 0.87

$$-\frac{1}{28}\left(-4\,x^2-9\right)^{\frac{3}{2}}x^4+\frac{9}{140}\left(-4\,x^2-9\right)^{\frac{3}{2}}x^2-\frac{27}{280}\left(-4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/28*(-4*x^2 - 9)^(3/2)*x^4 + 9/140*(-4*x^2 - 9)^(3/2)*x^2 - 27/280*(-4*x^
2 - 9)^(3/2)
```

mupad [B] time = 5.13, size = 27, normalized size = 0.59

$$\sqrt{-4\,x^2-9}\,\left(\frac{x^6}{7}+\frac{9\,x^4}{140}-\frac{27\,x^2}{140}+\frac{243}{280}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] (- 4*x^2 - 9)^(1/2)*((9*x^4)/140 - (27*x^2)/140 + x^6/7 + 243/280)
```

sympy [A] time = 2.08, size = 68, normalized size = 1.48

$$\frac{x^6\sqrt{-4x^2-9}}{7}+\frac{9x^4\sqrt{-4x^2-9}}{140}-\frac{27x^2\sqrt{-4x^2-9}}{140}+\frac{243\sqrt{-4x^2-9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-4*x**2-9)**(1/2),x)
```

```
[Out] x**6*sqrt(-4*x**2 - 9)/7 + 9*x**4*sqrt(-4*x**2 - 9)/140 - 27*x**2*sqrt(-4*x
**2 - 9)/140 + 243*sqrt(-4*x**2 - 9)/280
```

3.466 $\int x^4\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{81}{256}\sqrt{-4x^2-9}\,x-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)+\frac{1}{6}\sqrt{-4x^2-9}\,x^5+\frac{3}{32}\sqrt{-4x^2-9}\,x^3$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 203}

$$\frac{1}{6}\sqrt{-4x^2-9}\,x^5+\frac{3}{32}\sqrt{-4x^2-9}\,x^3-\frac{81}{256}\sqrt{-4x^2-9}\,x-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-81*x*Sqrt[-9 - 4*x^2])/256 + (3*x^3*Sqrt[-9 - 4*x^2])/32 + (x^5*Sqrt[-9 -
 4*x^2])/6 - (729*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/512
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{-9-4x^2}\,dx &= \frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{3}{2}\int\frac{x^4}{\sqrt{-9-4x^2}}\,dx\\ &= \frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}+\frac{81}{32}\int\frac{x^2}{\sqrt{-9-4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{256}\int\frac{1}{\sqrt{-9-4x^2}}\,dx\\ &= -\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{256}\text{Subst}\left(\int\frac{1}{1+4x^2}\,dx,x,\frac{}{\sqrt{-}}\right.\\ &= -\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.67

$$\frac{1}{768}x\sqrt{-4x^2-9}\left(128x^4+72x^2-243\right)-\frac{729}{512}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 - 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 - (729*ArcTan[(2*x)/Sqrt
[-9 - 4*x^2]])/512
```

IntegrateAlgebraic [C] time = 0.05, size = 54, normalized size = 0.75

$$\frac{1}{768}\sqrt{-4x^2-9}\left(128x^5+72x^3-243x\right)-\frac{729}{512}i\log\left(\sqrt{-4x^2-9}-2ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (Sqrt[-9 - 4*x^2]*(-243*x + 72*x^3 + 128*x^5))/768 - ((729*I)/512)*Log[(-2*
I)*x + Sqrt[-9 - 4*x^2]]
```

fricas [C] time = 0.99, size = 72, normalized size = 1.00

$$\frac{1}{768}\left(128\,x^{5}+72\,x^{3}-243\,x\right)\sqrt{-4\,x^{2}-9}-\frac{729}{1024}i\,\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^{2}-9}}{x}\right)+\frac{729}{1024}i\,\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^{2}-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(128*x^5 + 72*x^3 - 243*x)*sqrt(-4*x^2 - 9) - 729/1024*I*log(-(8*x +
4*I*sqrt(-4*x^2 - 9))/x) + 729/1024*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-4\,x^{2}-9}\,x^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9)*x^4, x)
```

maple [A] time = 0.01, size = 55, normalized size = 0.76

$$-\frac{\left(-4x^{2}-9\right)^{\frac{3}{2}}x^{3}}{24}+\frac{9\left(-4x^{2}-9\right)^{\frac{3}{2}}x}{128}+\frac{81\sqrt{-4x^{2}-9}\,x}{256}-\frac{729\arctan\left(\frac{2x}{\sqrt{-4x^{2}-9}}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/24*x^3*(-4*x^2-9)^(3/2)+9/128*x*(-4*x^2-9)^(3/2)+81/256*x*(-4*x^2-9)^(1/
2)-729/512*arctan(2*x/(-4*x^2-9)^(1/2))
```

maxima [C] time = 2.94, size = 45, normalized size = 0.62

$$-\frac{1}{24}\left(-4\,x^{2}-9\right)^{\frac{3}{2}}x^{3}+\frac{9}{128}\left(-4\,x^{2}-9\right)^{\frac{3}{2}}x+\frac{81}{256}\,\sqrt{-4\,x^{2}-9}\,x+\frac{729}{512}i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(-4*x^2 - 9)^(3/2)*x^3 + 9/128*(-4*x^2 - 9)^(3/2)*x + 81/256*sqrt(-4*
x^2 - 9)*x + 729/512*I*arcsinh(2/3*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{-4\,x^2-9}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^4*(- 4*x^2 - 9)^(1/2), x)
```

sympy [C] time = 4.67, size = 83, normalized size = 1.15

$$\frac{2ix^7}{3\sqrt{4x^2+9}}+\frac{15ix^5}{8\sqrt{4x^2+9}}-\frac{27ix^3}{64\sqrt{4x^2+9}}-\frac{729ix}{256\sqrt{4x^2+9}}+\frac{729i\operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*x**7/(3*sqrt(4*x**2 + 9)) + 15*I*x**5/(8*sqrt(4*x**2 + 9)) - 27*I*x**3/
(64*sqrt(4*x**2 + 9)) - 729*I*x/(256*sqrt(4*x**2 + 9)) + 729*I*asinh(2*x/3)
/512
```

3.467 $\int x^3\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{80}\left(-4x^2-9\right)^{5/2}+\frac{3}{16}\left(-4x^2-9\right)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{80}\left(-4x^2-9\right)^{5/2}+\frac{3}{16}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (3*(-9 - 4*x^2)^(3/2))/16 + (-9 - 4*x^2)^(5/2)/80
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{-9-4x^2}\,dx&=\frac{1}{2}\text{Subst}\left(\int\sqrt{-9-4x}\,x\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{9}{4}\sqrt{-9-4x}-\frac{1}{4}(-9-4x)^{3/2}\right)dx,x,x^2\right)\\&=\frac{3}{16}\left(-9-4x^2\right)^{3/2}+\frac{1}{80}\left(-9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(-4x^2-9\right)^{3/2}\left(3-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-9 - 4*x^2],x]
[Out] ((-9 - 4*x^2)^(3/2)*(3 - 2*x^2))/40
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{40}\left(-4x^2-9\right)^{3/2}\left(3-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[-9 - 4*x^2],x]
[Out] ((-9 - 4*x^2)^(3/2)*(3 - 2*x^2))/40
```

fricas [A] time = 1.24, size = 23, normalized size = 0.74

$$\frac{1}{40}\left(8\,x^4+6\,x^2-27\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/40*(8*x^4 + 6*x^2 - 27)*sqrt(-4*x^2 - 9)
```

giac [A] time = 0.98, size = 25, normalized size = 0.81

$$\frac{1}{80}\left(4\,x^2+9\right)^{\frac{5}{2}}i-\frac{3}{16}\left(4\,x^2+9\right)^{\frac{3}{2}}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/80*(4*x^2 + 9)^(5/2)*i - 3/16*(4*x^2 + 9)^(3/2)*i
```

maple [A] time = 0.00, size = 19, normalized size = 0.61

$$-\frac{\left(2x^2-3\right)\left(-4x^2-9\right)^{\frac{3}{2}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-4*x^2-9)^(1/2),x)
```

[Out] -1/40*(2*x^2-3)*(-4*x^2-9)^(3/2)

maxima [A] time = 3.03, size = 26, normalized size = 0.84

$$-\frac{1}{20}\left(-4x^2-9\right)^{\frac{3}{2}}x^2+\frac{3}{40}\left(-4x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*(-4*x^2 - 9)^(3/2)*x^2 + 3/40*(-4*x^2 - 9)^(3/2)
```

mupad [B] time = 5.15, size = 22, normalized size = 0.71

$$\sqrt{-4x^2-9}\left(\frac{x^4}{5}+\frac{3x^2}{20}-\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] (- 4*x^2 - 9)^(1/2)*((3*x^2)/20 + x^4/5 - 27/40)
```

sympy [A] time = 0.69, size = 49, normalized size = 1.58

$$\frac{x^4\sqrt{-4x^2-9}}{5}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-4*x**2-9)**(1/2),x)
```

```
[Out] x**4*sqrt(-4*x**2 - 9)/5 + 3*x**2*sqrt(-4*x**2 - 9)/20 - 27*sqrt(-4*x**2 -
9)/40
```

3.468 $\int x^2\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=54

$$\frac{9}{32}\sqrt{-4x^2-9}\,x+\frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)+\frac{1}{4}\sqrt{-4x^2-9}\,x^3$$

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {279, 321, 217, 203}

$$\frac{1}{4}\sqrt{-4x^2-9}\,x^3+\frac{9}{32}\sqrt{-4x^2-9}\,x+\frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-9 - 4*x^2],x]
```

```
[Out] (9*x*Sqrt[-9 - 4*x^2])/32 + (x^3*Sqrt[-9 - 4*x^2])/4 + (81*ArcTan[(2*x)/Sqr
t[-9 - 4*x^2]])/64
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{-9-4x^2}\,dx &= \frac{1}{4}x^3\sqrt{-9-4x^2} - \frac{9}{4}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx \\
&= \frac{9}{32}x\sqrt{-9-4x^2} + \frac{1}{4}x^3\sqrt{-9-4x^2} + \frac{81}{32}\int \frac{1}{\sqrt{-9-4x^2}}\,dx \\
&= \frac{9}{32}x\sqrt{-9-4x^2} + \frac{1}{4}x^3\sqrt{-9-4x^2} + \frac{81}{32}\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right) \\
&= \frac{9}{32}x\sqrt{-9-4x^2} + \frac{1}{4}x^3\sqrt{-9-4x^2} + \frac{81}{64}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.80

$$\frac{1}{64}\left(2x\sqrt{-4x^2-9}\,\left(8x^2+9\right)+81\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-9 - 4*x^2],x]
[Out] (2*x*Sqrt[-9 - 4*x^2]*(9 + 8*x^2) + 81*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/64
```

IntegrateAlgebraic [C] time = 0.04, size = 49, normalized size = 0.91

$$\frac{1}{32}\sqrt{-4x^2-9}\,\left(8x^3+9x\right)+\frac{81}{64}i\log\left(\sqrt{-4x^2-9}\,-2ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[-9 - 4*x^2],x]
[Out] (Sqrt[-9 - 4*x^2]*(9*x + 8*x^3))/32 + ((81*I)/64)*Log[(-2*I)*x + Sqrt[-9 -
4*x^2]]
```

fricas [C] time = 0.90, size = 67, normalized size = 1.24

$$\frac{1}{32}\left(8\,x^3+9\,x\right)\sqrt{-4\,x^2-9}\,+\frac{81}{128}i\,\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-9}}{x}\right)-\frac{81}{128}i\,\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 + 9*x)*sqrt(-4*x^2 - 9) + 81/128*I*log(-(8*x + 4*I*sqrt(-4*x^2
- 9))/x) - 81/128*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4\,x^2-9}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9)*x^2, x)
```

maple [A] time = 0.00, size = 41, normalized size = 0.76

$$-\frac{\left(-4x^2-9\right)^{\frac{3}{2}}x}{16}-\frac{9\sqrt{-4x^2-9}\;x}{32}+\frac{81\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/16*(-4*x^2-9)^(3/2)*x-9/32*(-4*x^2-9)^(1/2)*x+81/64*arctan(2/(-4*x^2-9)^
(1/2)*x)
```

maxima [C] time = 2.98, size = 31, normalized size = 0.57

$$-\frac{1}{16}\left(-4\,x^2-9\right)^{\frac{3}{2}}x-\frac{9}{32}\sqrt{-4\,x^2-9}\,x-\frac{81}{64}i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(-4*x^2 - 9)^(3/2)*x - 9/32*sqrt(-4*x^2 - 9)*x - 81/64*I*arcsinh(2/3*
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\sqrt{-4\,x^2-9}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^2*(- 4*x^2 - 9)^(1/2), x)
```

sympy [C] time = 2.76, size = 61, normalized size = 1.13

$$\frac{ix^5}{\sqrt{4x^2+9}}+\frac{27ix^3}{8\sqrt{4x^2+9}}+\frac{81ix}{32\sqrt{4x^2+9}}-\frac{81i\operatorname{asinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-4*x**2-9)**(1/2),x)
```

```
[Out] I*x**5/sqrt(4*x**2 + 9) + 27*I*x**3/(8*sqrt(4*x**2 + 9)) + 81*I*x/(32*sqrt(
4*x**2 + 9)) - 81*I*asinh(2*x/3)/64
```

3.469 $\int x\sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-9 - 4*x^2],x]
```

```
[Out] -(-9 - 4*x^2)^(3/2)/12
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{-9-4x^2}\,dx=-\frac{1}{12}\left(-9-4x^2\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/12*(-9 - 4*x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/12*(-9 - 4*x^2)^(3/2)
```

fricas [A] time = 0.76, size = 18, normalized size = 1.20

$$\frac{1}{12}\left(4\,x^2+9\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^2 + 9)*sqrt(-4*x^2 - 9)
```

giac [A] time = 1.08, size = 12, normalized size = 0.80

$$\frac{1}{12}\left(4\,x^2+9\right)^{\frac{3}{2}} i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*x^2 + 9)^(3/2)*i
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$-\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)*x,x)
```

```
[Out] -1/12*(-4*x^2-9)^(3/2)
```

maxima [A] time = 1.31, size = 11, normalized size = 0.73

$$-\frac{1}{12}\left(-4\,x^2-9\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-4*x^2 - 9)^(3/2)
```

mupad [B] time = 0.07, size = 11, normalized size = 0.73

$$-\frac{\left(-4\,x^2-9\right)^{3/2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(- 4*x^2 - 9)^(1/2),x)
[Out] -(- 4*x^2 - 9)^(3/2)/12
```

sympy [B] time = 0.22, size = 31, normalized size = 2.07

$$\frac{x^2\sqrt{-4x^2-9}}{3}+\frac{3\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*x**2-9)**(1/2),x)
[Out] x**2*sqrt(-4*x**2 - 9)/3 + 3*sqrt(-4*x**2 - 9)/4
```

3.470 $\int \sqrt{-9-4x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {195, 217, 203}

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 - 4*x^2])/2 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-9-4x^2}\,dx &= \frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{2}\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{2}\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)\\ &= \frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{4}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{1}{4}\left(2x\sqrt{-4x^2-9}-9\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2],x]
[Out] (2*x*Sqrt[-9 - 4*x^2] - 9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

IntegrateAlgebraic [C] time = 0.03, size = 41, normalized size = 1.14

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{4}i\log\left(\sqrt{-4x^2-9}-2ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 - 4*x^2],x]
[Out] (x*Sqrt[-9 - 4*x^2])/2 - ((9*I)/4)*Log[(-2*I)*x + Sqrt[-9 - 4*x^2]]
```

fricas [C] time = 0.74, size = 59, normalized size = 1.64

$$\frac{1}{2}\sqrt{-4x^2-9}\,x-\frac{9}{8}i\log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right)+\frac{9}{8}i\log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-4*x^2 - 9)*x - 9/8*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) + 9/8*I
*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4x^2-9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9), x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.81

$$\frac{\sqrt{-4x^2-9}\,x}{2}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2),x)
```

```
[Out] -9/4*arctan(2/(-4*x^2-9)^(1/2)*x)+1/2*(-4*x^2-9)^(1/2)*x
```

maxima [C] time = 2.99, size = 19, normalized size = 0.53

$$\frac{1}{2}\sqrt{-4\,x^2-9}\,x+\frac{9}{4}i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 - 9)*x + 9/4*I*arcsinh(2/3*x)
```

mupad [B] time = 5.03, size = 28, normalized size = 0.78

$$\frac{x\,\sqrt{-4\,x^2-9}}{2}-\frac{9\,\mathrm{atan}\left(\frac{2\,x}{\sqrt{-4\,x^2-9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2),x)
```

```
[Out] (x*(- 4*x^2 - 9)^(1/2))/2 - (9*atan((2*x)/(- 4*x^2 - 9)^(1/2)))/4
```

sympy [A] time = 0.39, size = 34, normalized size = 0.94

$$\frac{x\sqrt{-4x^2-9}}{2}-\frac{9\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2),x)
```

```
[Out] x*sqrt(-4*x**2 - 9)/2 - 9*atan(2*x/sqrt(-4*x**2 - 9))/4
```

3.471 $\int \frac{\sqrt{-9-4x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{-4x^2-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 204}

$$\sqrt{-4x^2-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x,x]
```

```
[Out] Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-9-4x^2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9-4x}}{x}\,dx, x, x^2\right)\\ &= \sqrt{-9-4x^2} - \frac{9}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)\\ &= \sqrt{-9-4x^2} + \frac{9}{4}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\ &= \sqrt{-9-4x^2} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\sqrt{-4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x,x]
[Out] Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 1.00

$$\sqrt{-4x^2-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 - 4*x^2]/x,x]
[Out] Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]
```

fricas [C] time = 1.10, size = 52, normalized size = 1.73

$$\sqrt{-4\,x^2-9} - \frac{3}{2}i\,\log\left(-\frac{6\left(i\sqrt{-4\,x^2-9}-3\right)}{x}\right) + \frac{3}{2}i\,\log\left(-\frac{6\left(-i\sqrt{-4\,x^2-9}-3\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-4*x^2 - 9) - 3/2*I*log(-6*(I*sqrt(-4*x^2 - 9) - 3)/x) + 3/2*I*log(-6*
(-I*sqrt(-4*x^2 - 9) - 3)/x)
```

giac [A] time = 1.17, size = 24, normalized size = 0.80

$$\sqrt{-4\,x^2-9}\,-3\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-4*x^2 - 9) - 3*arctan(1/3*sqrt(-4*x^2 - 9))
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$3\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)+\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x*(-4*x^2-9)^(1/2),x)
```

```
[Out] (-4*x^2-9)^(1/2)+3*arctan(3/(-4*x^2-9)^(1/2))
```

maxima [C] time = 2.93, size = 35, normalized size = 1.17

$$\sqrt{-4\,x^2-9}\,+3i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-4*x^2 - 9) + 3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.73, size = 24, normalized size = 0.80

$$\sqrt{-4\,x^2-9}\,-3\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x,x)
```

```
[Out] (- 4*x^2 - 9)^(1/2) - 3*atan((- 4*x^2 - 9)^(1/2)/3)
```

sympy [C] time = 1.26, size = 44, normalized size = 1.47

$$\frac{2ix}{\sqrt{1+\frac{9}{4x^2}}} - 3i\operatorname{asinh}\left(\frac{3}{2x}\right) + \frac{9i}{2x\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x*(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*x/sqrt(1 + 9/(4*x**2)) - 3*I*asinh(3/(2*x)) + 9*I/(2*x*sqrt(1 + 9/(4*x*
*2)))
```

3.472 $\int \frac{\sqrt{-9-4x^2}}{x^2} \, dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{-4x^2-9}}{x}-2\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {277, 217, 203}

$$-\frac{\sqrt{-4x^2-9}}{x}-2\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 - 4*x^2]/x) - 2*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx = -\frac{\sqrt{-9-4x^2}}{x} - 4\int \frac{1}{\sqrt{-9-4x^2}}\,dx$$
$$= -\frac{\sqrt{-9-4x^2}}{x} - 4\,\mathrm{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)$$
$$= -\frac{\sqrt{-9-4x^2}}{x} - 2\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.44

$$\frac{\sqrt{-4x^2-9}\left(2x\sinh^{-1}\left(\frac{2x}{3}\right)-\sqrt{4x^2+9}\right)}{x\sqrt{4x^2+9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^2,x]
```

```
[Out] (Sqrt[-9 - 4*x^2]*(-Sqrt[9 + 4*x^2] + 2*x*ArcSinh[(2*x)/3]))/(x*Sqrt[9 + 4*
x^2])
```

IntegrateAlgebraic [C] time = 0.04, size = 39, normalized size = 1.15

$$-\frac{\sqrt{-4x^2-9}}{x} - 2i\log\left(\sqrt{-4x^2-9}-2ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 - 4*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-9 - 4*x^2]/x) - (2*I)*Log[(-2*I)*x + Sqrt[-9 - 4*x^2]]
```

fricas [C] time = 0.81, size = 64, normalized size = 1.88

$$\frac{-i\,x\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-9}}{x}\right)+i\,x\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-9}}{x}\right)-\sqrt{-4\,x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (-I*x*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) + I*x*log(-(8*x - 4*I*sqrt(-4*x^
2 - 9))/x) - sqrt(-4*x^2 - 9))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-4\,x^2-9}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 - 9)/x^2, x)
```

maple [A] time = 0.00, size = 43, normalized size = 1.26

$$\frac{4\sqrt{-4x^2-9}\,x}{9}-2\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)+\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^2,x)
```

```
[Out] 1/9/x*(-4*x^2-9)^(3/2)+4/9*(-4*x^2-9)^(1/2)*x-2*arctan(2/(-4*x^2-9)^(1/2)*x
)
```

maxima [C] time = 2.92, size = 21, normalized size = 0.62

$$-\frac{\sqrt{-4\,x^2-9}}{x}+2i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(-4*x^2 - 9)/x + 2*I*arcsinh(2/3*x)
```

mupad [B] time = 4.77, size = 41, normalized size = 1.21

$$-\frac{\sqrt{-4\,x^2-9}}{x}-\frac{\mathrm{asin}\left(\frac{x\,2\mathrm{i}}{3}\right)\,\sqrt{-4\,x^2-9}\,2\mathrm{i}}{3\,\sqrt{\frac{4\,x^2}{9}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^2,x)
```

```
[Out] - (- 4*x^2 - 9)^(1/2)/x - (asin((x*2i)/3)*(- 4*x^2 - 9)^(1/2)*2i)/(3*((4*x^
2)/9 + 1)^(1/2))
```

sympy [A] time = 0.42, size = 32, normalized size = 0.94

$$-2\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{\sqrt{-4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**2,x)
```

```
[Out] -2*atan(2*x/sqrt(-4*x**2 - 9)) - sqrt(-4*x**2 - 9)/x
```

3.473 $\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{-4x^2-9}}{2x^2}-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 47, 63, 204}

$$-\frac{\sqrt{-4x^2-9}}{2x^2}-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^3,x]
```

```
[Out] -Sqrt[-9 - 4*x^2]/(2*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-9-4x}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{-9-4x^2}}{2x^2} - \operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{-9-4x^2}}{2x^2} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\ &= -\frac{\sqrt{-9-4x^2}}{2x^2} - \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.41

$$\frac{12x^2 + 4\sqrt{4x^2+9}\,x^2\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right) + 27}{6x^2\sqrt{-4x^2-9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^3,x]
```

```
[Out] (27 + 12*x^2 + 4*x^2*Sqrt[9 + 4*x^2]*ArcTanh[Sqrt[1 + (4*x^2)/9]])/(6*x^2*S
qrt[-9 - 4*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 1.00

$$-\frac{\sqrt{-4x^2-9}}{2x^2} - \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 - 4*x^2]/x^3,x]
```

```
[Out] -1/2*Sqrt[-9 - 4*x^2]/x^2 - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/3
```

fricas [C] time = 1.01, size = 65, normalized size = 1.67

$$\frac{-2i\,x^2\log\left(-\frac{4\left(i\,\sqrt{-4\,x^2-9}-3\right)}{3\,x}\right)+2i\,x^2\log\left(-\frac{4\left(-i\,\sqrt{-4\,x^2-9}-3\right)}{3\,x}\right)-3\,\sqrt{-4\,x^2-9}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(-2*I*x^2*log(-4/3*(I*sqrt(-4*x^2 - 9) - 3)/x) + 2*I*x^2*log(-4/3*(-I*s
qrt(-4*x^2 - 9) - 3)/x) - 3*sqrt(-4*x^2 - 9))/x^2
```

giac [A] time = 1.21, size = 29, normalized size = 0.74

$$-\frac{\sqrt{-4\,x^2-9}}{2\,x^2}-\frac{2}{3}\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-4*x^2 - 9)/x^2 - 2/3*arctan(1/3*sqrt(-4*x^2 - 9))
```

maple [A] time = 0.00, size = 41, normalized size = 1.05

$$\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{3}+\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{18x^2}+\frac{2\sqrt{-4x^2-9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^3,x)
```

```
[Out] 1/18/x^2*(-4*x^2-9)^(3/2)+2/9*(-4*x^2-9)^(1/2)+2/3*arctan(3/(-4*x^2-9)^(1/2
))
```

maxima [C] time = 2.90, size = 51, normalized size = 1.31

$$\frac{2}{9}\,\sqrt{-4\,x^2-9}\,+\frac{\left(-4\,x^2-9\right)^{\frac{3}{2}}}{18\,x^2}+\frac{2}{3}i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(-4*x^2 - 9) + 1/18*(-4*x^2 - 9)^(3/2)/x^2 + 2/3*I*log(6*sqrt(4*x^2
 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.79, size = 29, normalized size = 0.74

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{3}-\frac{\sqrt{-4\,x^2-9}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^3,x)
[Out] - (2*atan((- 4*x^2 - 9)^(1/2)/3))/3 - (- 4*x^2 - 9)^(1/2)/(2*x^2)
```

sympy [C] time = 1.70, size = 27, normalized size = 0.69

$$-\frac{2i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{3}-\frac{i\sqrt{1+\frac{9}{4x^2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**3,x)
[Out] -2*I*asinh(3/(2*x))/3 - I*sqrt(1 + 9/(4*x**2))/x
```

3.474 $\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx$

Optimal. Leaf size=18

$$\frac{\left(-4x^2-9\right)^{3/2}}{27x^3}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\left(-4x^2-9\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^4,x]
```

```
[Out] (-9 - 4*x^2)^(3/2)/(27*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx=\frac{\left(-9-4x^2\right)^{3/2}}{27x^3}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(-4x^2-9\right)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^4,x]
```

```
[Out] (-9 - 4*x^2)^(3/2)/(27*x^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{(-4x^2-9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 - 4*x^2]/x^4,x]
[Out] (-9 - 4*x^2)^(3/2)/(27*x^3)
```

fricas [A] time = 0.69, size = 14, normalized size = 0.78

$$\frac{(-4\,x^2-9)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/27*(-4*x^2 - 9)^(3/2)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-4\,x^2-9}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(-4*x^2 - 9)/x^4, x)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{(-4x^2-9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^4,x)
[Out] 1/27*(-4*x^2-9)^(3/2)/x^3
```

maxima [A] time = 2.89, size = 14, normalized size = 0.78

$$\frac{\left(-4\,x^2-9\right)^{\frac{3}{2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/27*(-4*x^2 - 9)^(3/2)/x^3
```

mupad [B] time = 4.74, size = 31, normalized size = 1.72

$$-\frac{4\,x^2\,\sqrt{-4\,x^2-9}\,+9\,\sqrt{-4\,x^2-9}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x^2 - 9)^(1/2)/x^4,x)
```

```
[Out] -(4*x^2*(- 4*x^2 - 9)^(1/2) + 9*(- 4*x^2 - 9)^(1/2))/(27*x^3)
```

sympy [C] time = 1.00, size = 37, normalized size = 2.06

$$-\frac{8i\sqrt{1+\frac{9}{4x^2}}}{27}-\frac{2i\sqrt{1+\frac{9}{4x^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-9)**(1/2)/x**4,x)
```

```
[Out] -8*I*sqrt(1 + 9/(4*x**2))/27 - 2*I*sqrt(1 + 9/(4*x**2))/(3*x**2)
```

3.475 $\int \frac{\sqrt{-9-4x^2}}{x^5} dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)-\frac{\sqrt{-4x^2-9}}{4x^4}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 47, 51, 63, 204}

$$-\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{\sqrt{-4x^2-9}}{4x^4}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 - 4*x^2]/x^5,x]
```

```
[Out] -Sqrt[-9 - 4*x^2]/(4*x^4) - Sqrt[-9 - 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 -
 4*x^2]/3])/27
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-9-4x}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\
&= -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} + \frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$
\frac{16\left(-4x^2-9\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{4x^2}{9}+1\right)}{2187}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 - 4*x^2]/x^5,x]
```

```
[Out] (16*(-9 - 4*x^2)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (4*x^2)/9])/2187
```

IntegrateAlgebraic [A] time = 0.03, size = 46, normalized size = 0.81

$$\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)+\frac{\sqrt{-4x^2-9}\left(-2x^2-9\right)}{36x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-9 - 4*x^2]/x^5,x]
```

```
[Out] (Sqrt[-9 - 4*x^2]*(-9 - 2*x^2))/(36*x^4) + (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/2
7
```

fricas [C] time = 1.06, size = 72, normalized size = 1.26

$$\frac{-4i\,x^4\log\left(-\frac{4\left(i\sqrt{-4\,x^2-9}+3\right)}{27\,x}\right)+4i\,x^4\log\left(-\frac{4\left(-i\sqrt{-4\,x^2-9}+3\right)}{27\,x}\right)-3\left(2\,x^2+9\right)\sqrt{-4\,x^2-9}}{108\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/108*(-4*I*x^4*log(-4/27*(I*sqrt(-4*x^2 - 9) + 3)/x) + 4*I*x^4*log(-4/27*(
-I*sqrt(-4*x^2 - 9) + 3)/x) - 3*(2*x^2 + 9)*sqrt(-4*x^2 - 9))/x^4
```

giac [A] time = 1.11, size = 43, normalized size = 0.75

$$-\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}i+9\sqrt{-4\,x^2-9}}{72\,x^4}+\frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*((4*x^2 + 9)^(3/2)*i + 9*sqrt(-4*x^2 - 9))/x^4 + 2/27*arctan(1/3*sqrt
(-4*x^2 - 9))
```

maple [A] time = 0.01, size = 55, normalized size = 0.96

$$-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}-\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{162x^2}+\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{36x^4}-\frac{2\sqrt{-4x^2-9}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-9)^(1/2)/x^5,x)
```

[Out] 1/36/x^4*(-4*x^2-9)^(3/2)-1/162*(-4*x^2-9)^(3/2)/x^2-2/81*(-4*x^2-9)^(1/2)-2/27*arctan(3/(-4*x^2-9)^(1/2))

maxima [C] time = 2.99, size = 65, normalized size = 1.14

$$-\frac{2}{81}\sqrt{-4x^2-9}-\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{162x^2}+\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{36x^4}-\frac{2}{27}i\log\left(\frac{6\sqrt{4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="maxima")

[Out] -2/81*sqrt(-4*x^2 - 9) - 1/162*(-4*x^2 - 9)^(3/2)/x^2 + 1/36*(-4*x^2 - 9)^(3/2)/x^4 - 2/27*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))

mupad [B] time = 4.69, size = 43, normalized size = 0.75

$$\frac{2\operatorname{atan}\left(\frac{\sqrt{-4x^2-9}}{3}\right)}{27}-\frac{\frac{\sqrt{-4x^2-9}}{8}-\frac{\left(-4x^2-9\right)^{3/2}}{72}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((- 4*x^2 - 9)^(1/2)/x^5,x)

[Out] (2*atan((- 4*x^2 - 9)^(1/2)/3))/27 - ((- 4*x^2 - 9)^(1/2)/8 - (- 4*x^2 - 9)^(3/2)/72)/x^4

sympy [C] time = 3.37, size = 68, normalized size = 1.19

$$\frac{2i\operatorname{asinh}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{1+\frac{9}{4x^2}}}-\frac{3i}{4x^3\sqrt{1+\frac{9}{4x^2}}}-\frac{9i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x**2-9)**(1/2)/x**5,x)

[Out] 2*I*asinh(3/(2*x))/27 - I/(9*x*sqrt(1 + 9/(4*x**2))) - 3*I/(4*x**3*sqrt(1 + 9/(4*x**2))) - 9*I/(8*x**5*sqrt(1 + 9/(4*x**2)))

3.476 $\int \frac{x^5}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{2a\left(a+bx^2\right)^{3/2}}{3b^3}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^2\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{2a\left(a+bx^2\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*Sqrt[a + b*x^2])/b^3 - (2*a*(a + b*x^2)^(3/2))/(3*b^3) + (a + b*x^2)^(
5/2)/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt{a+bx}} - \frac{2a\sqrt{a+bx}}{b^2} + \frac{(a+bx)^{3/2}}{b^2}\right) dx, x, x^2\right)$$
$$= \frac{a^2\sqrt{a+bx^2}}{b^3} - \frac{2a\left(a+bx^2\right)^{3/2}}{3b^3} + \frac{\left(a+bx^2\right)^{5/2}}{5b^3}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\left(8a^2-4abx^2+3b^2x^4\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(8*a^2 - 4*a*b*x^2 + 3*b^2*x^4))/(15*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\left(8a^2-4abx^2+3b^2x^4\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(8*a^2 - 4*a*b*x^2 + 3*b^2*x^4))/(15*b^3)
```

fricas [A] time = 0.80, size = 35, normalized size = 0.62

$$\frac{\left(3\,b^2x^4-4\,abx^2+8\,a^2\right)\sqrt{bx^2+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*b^2*x^4 - 4*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/b^3
```

giac [A] time = 1.15, size = 46, normalized size = 0.82

$$\frac{\sqrt{bx^2+a}\,a^2}{b^3}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}-10\left(bx^2+a\right)^{\frac{3}{2}}a}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)*a^2/b^3 + 1/15*(3*(b*x^2 + a)^(5/2) - 10*(b*x^2 + a)^(3/2)*
a)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.64

$$\frac{\sqrt{b\,x^2+a}\,\left(3b^2x^4-4ab\,x^2+8a^2\right)}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(3*b^2*x^4-4*a*b*x^2+8*a^2)/b^3
```

maxima [A] time = 1.32, size = 53, normalized size = 0.95

$$\frac{\sqrt{bx^2+a}\,x^4}{5\,b}-\frac{4\,\sqrt{bx^2+a}\,ax^2}{15\,b^2}+\frac{8\,\sqrt{bx^2+a}\,a^2}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(b*x^2 + a)*x^4/b - 4/15*sqrt(b*x^2 + a)*a*x^2/b^2 + 8/15*sqrt(b*x^
2 + a)*a^2/b^3
```

mupad [B] time = 4.67, size = 36, normalized size = 0.64

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,a^2}{15\,b^3}+\frac{x^4}{5\,b}-\frac{4\,a\,x^2}{15\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^2)/(15*b^3) + x^4/(5*b) - (4*a*x^2)/(15*b^2))
```

sympy [A] time = 0.83, size = 68, normalized size = 1.21

$$\begin{cases} \frac{8a^2\sqrt{a+bx^2}}{15b^3} - \frac{4ax^2\sqrt{a+bx^2}}{15b^2} + \frac{x^4\sqrt{a+bx^2}}{5b} & \text{for } b \neq 0 \\ \frac{x^6}{6\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*a*x**2*sqrt(a + b*x**2)/(1
5*b**2) + x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)), (x**6/(6*sqrt(a)), True))
```

3.477 $\int \frac{x^4}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=73

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2],x]
```

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) + (3*a^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2}}\,dx = \frac{x^3\sqrt{a+bx^2}}{4b} - \frac{(3a)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{\left(3a^2\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^2}$$
$$= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.85

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right) + \sqrt{b}\,x\sqrt{a+bx^2}\,\left(2bx^2-3a\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a + 2*b*x^2) + 3*a^2*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 63, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\,\left(2bx^3-3ax\right)}{8b^2} - \frac{3a^2\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a*x + 2*b*x^3))/(8*b^2) - (3*a^2*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/(8*b^(5/2))
```

fricas [A] time = 0.68, size = 124, normalized size = 1.70

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(2\,b^2x^3-3\,abx\right)\sqrt{bx^2+a}}{16\,b^3}, -\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(2\,b^2x^3-3\,abx\right)\sqrt{bx^2+a}}{8\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 1.04, size = 54, normalized size = 0.74

$$\frac{1}{8}\sqrt{bx^2+a}\,x\left(\frac{2x^2}{b}-\frac{3a}{b^2}\right)-\frac{3a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 59, normalized size = 0.81

$$\frac{\sqrt{b\,x^2+a}\;x^3}{4b}+\frac{3a^2\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}-\frac{3\sqrt{b\,x^2+a}\;ax}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^3*(b*x^2+a)^(1/2)/b-3/8*a*x*(b*x^2+a)^(1/2)/b^2+3/8*a^2/b^(5/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.35, size = 51, normalized size = 0.70

$$\frac{\sqrt{bx^2+a}\,x^3}{4\,b}-\frac{3\,\sqrt{bx^2+a}\,ax}{8\,b^2}+\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*x^3/b - 3/8*sqrt(b*x^2 + a)*a*x/b^2 + 3/8*a^2*arcsinh(b
*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 4.03, size = 95, normalized size = 1.30

$$-\frac{3a^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqrt(1 + b*x*
*2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + x**5/(4*sqrt(a)*sqr
t(1 + b*x**2/a))
```

3.478 $\int \frac{x^3}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, x^2\right)\\ &= -\frac{a\sqrt{a+bx^2}}{b^2}+\frac{\left(a+bx^2\right)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{(bx^2-2a)\sqrt{a+bx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2],x]
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.75

$$\frac{(bx^2-2a)\sqrt{a+bx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + b*x^2],x]
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

fricas [A] time = 1.10, size = 23, normalized size = 0.64

$$\frac{\sqrt{bx^2+a}\,(bx^2-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

giac [A] time = 1.09, size = 30, normalized size = 0.83

$$\frac{(bx^2+a)^{\frac{3}{2}}}{3\,b^2}-\frac{\sqrt{bx^2+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/3*(b*x^2 + a)^(3/2)/b^2 - sqrt(b*x^2 + a)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$-\frac{\sqrt{b\,x^2+a}\,\left(-b\,x^2+2a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-b*x^2+2*a)/b^2
```

maxima [A] time = 1.34, size = 33, normalized size = 0.92

$$\frac{\sqrt{bx^2+a}\,x^2}{3\,b}-\frac{2\,\sqrt{bx^2+a}\,a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*x^2/b - 2/3*sqrt(b*x^2 + a)*a/b^2
```

mupad [B] time = 4.73, size = 24, normalized size = 0.67

$$-\frac{\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(1/2),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

sympy [A] time = 0.49, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne
(b, 0)), (x**4/(4*sqrt(a)), True))
```

3.479 $\int \frac{x^2}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a+bx^2}}{2b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 206}

$$\frac{x\sqrt{a+bx^2}}{2b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx = \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$
$$= \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$
$$= \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{x\sqrt{a+bx^2}}{2b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2],x]
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 51, normalized size = 1.04

$$\frac{a\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{3/2}} + \frac{x\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a + b*x^2],x]
[Out] (x*Sqrt[a + b*x^2])/(2*b) + (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3
/2))
```

fricas [A] time = 1.25, size = 93, normalized size = 1.90

$$\left[\frac{2\sqrt{bx^2+a}\,bx + a\sqrt{b}\,\log\left(-2\,bx^2 + 2\sqrt{bx^2+a}\,\sqrt{b}\,x - a\right)}{4\,b^2}, \frac{\sqrt{bx^2+a}\,bx + a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*x + a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b^2]
```

giac [A] time = 1.18, size = 40, normalized size = 0.82

$$\frac{\sqrt{bx^2+a}\,x}{2\,b}+\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3
/2)
```

maple [A] time = 0.00, size = 39, normalized size = 0.80

$$-\frac{a\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{b\,x^2+a}\;x}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.26, size = 31, normalized size = 0.63

$$\frac{\sqrt{bx^2+a}\,x}{2\,b}-\frac{a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b - 1/2*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

mupad [B] time = 4.82, size = 56, normalized size = 1.14

$$\begin{cases} \frac{x^3}{3\,\sqrt{a}} & \text{if } b=0 \\ \frac{x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, x^3/(3*a^(1/2)), b ~= 0, (x*(a + b*x^2)^(1/2))/(2*b) - (a
*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)))
```

sympy [A] time = 2.25, size = 42, normalized size = 0.86

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{a\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)
)
```

3.480 $\int \frac{x}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{a+bx^2}}{b}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.077, Rules used = {261}

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]/b
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}}{b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2],x]
```

```
[Out] Sqrt[a + b*x^2]/b
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a + b*x^2],x]
[Out] Sqrt[a + b*x^2]/b
```

fricas [A] time = 1.04, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] sqrt(b*x^2 + a)/b
```

giac [A] time = 1.00, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] sqrt(b*x^2 + a)/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\sqrt{b\,x^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2),x)
[Out] (b*x^2+a)^(1/2)/b
```

maxima [A] time = 1.40, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)/b
```

mupad [B] time = 4.58, size = 13, normalized size = 0.87

$$\frac{\sqrt{b\,x^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)/b
```

sympy [A] time = 0.38, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

3.481 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 1.12

$$-\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a + b*x^2],x]
```

```
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]/Sqrt[b])
```

fricas [A] time = 1.34, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

giac [A] time = 1.24, size = 23, normalized size = 0.92

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\, x + \sqrt{b\, x^2 + a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a)^(1/2),x)

[Out] ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)

maxima [A] time = 1.36, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)

mupad [B] time = 0.12, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\, x + \sqrt{b\, x^2 + a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*x^2)^(1/2),x)

[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)

sympy [A] time = 1.02, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a)**(1/2),x)

[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)

3.482 $\int \frac{1}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2]),x]
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^2]),x]
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

fricas [A] time = 0.90, size = 60, normalized size = 2.40

$$\left[\frac{\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)}{2\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)/sqrt(a), sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a))/a]
```

giac [A] time = 0.99, size = 22, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.38, size = 17, normalized size = 0.68

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

mupad [B] time = 4.87, size = 19, normalized size = 0.76

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] -atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(1/2)
```

sympy [A] time = 1.06, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

3.483 $\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}}{ax}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

IntegrateAlgebraic [A] time = 0.04, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + b*x^2]),x]
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

fricas [A] time = 1.16, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(b*x^2 + a)/(a*x)
```

giac [A] time = 1.16, size = 30, normalized size = 1.58

$$\frac{2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$-\frac{\sqrt{b\,x^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/2),x)
[Out] -(b*x^2+a)^(1/2)/a/x
```

maxima [A] time = 1.35, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x^2 + a)/(a*x)
```

mupad [B] time = 4.60, size = 17, normalized size = 0.89

$$-\frac{\sqrt{b\,x^2+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] -(a + b*x^2)^(1/2)/(a*x)
```

sympy [A] time = 0.69, size = 19, normalized size = 1.00

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

3.484 $\int \frac{1}{x^3 \sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=50

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{\sqrt{a+bx^2}}{2ax^2}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2
))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{a+bx^2}}{2ax^2}-\frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a}\\ &= -\frac{\sqrt{a+bx^2}}{2ax^2}-\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a}\\ &= -\frac{\sqrt{a+bx^2}}{2ax^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.22

$$\frac{b\sqrt{a+bx^2}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\sqrt{\frac{bx^2}{a}+1}}-\frac{a}{2bx^2}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*(-1/2*a/(b*x^2) + ArcTanh[Sqrt[1 + (b*x^2)/a]]/(2*Sqrt[1
 + (b*x^2)/a])))/a^2
```

IntegrateAlgebraic [A] time = 0.07, size = 50, normalized size = 1.00

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2]/(a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3
/2))
```

fricas [A] time = 1.38, size = 105, normalized size = 2.10

$$\left[\frac{\sqrt{a}bx^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}\,a}{4a^2x^2}, -\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\,a}{2a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*x^2)]
```

giac [A] time = 1.09, size = 51, normalized size = 1.02

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{bx^2+a}\,b}{ax^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^2 + a)*b
/(a*x^2))/b
```

maple [A] time = 0.00, size = 48, normalized size = 0.96

$$\frac{b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))
/x)
```

maxima [A] time = 1.35, size = 36, normalized size = 0.72

$$\frac{b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)/(a*x^2)
```

mupad [B] time = 4.75, size = 38, normalized size = 0.76

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (a + b*x^2)^(1/2)/(2*a*x
^2)
```

sympy [A] time = 2.25, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**
(3/2))
```

3.485 $\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=44

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2]),x]

[Out] -Sqrt[a + b*x^2]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx &= -\frac{\sqrt{a+bx^2}}{3ax^3}-\frac{(2b)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}\\ &= -\frac{\sqrt{a+bx^2}}{3ax^3}+\frac{2b\sqrt{a+bx^2}}{3a^2x}\end{aligned}$$

Mathematica [A]   time = 0.01, size = 29, normalized size = 0.66

$$-\frac{(a-2bx^2)\sqrt{a+bx^2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2]),x]
[Out] -1/3*((a - 2*b*x^2)*Sqrt[a + b*x^2])/(a^2*x^3)
```

IntegrateAlgebraic [A]   time = 0.06, size = 31, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\,(2bx^2-a)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[a + b*x^2]),x]
[Out] (Sqrt[a + b*x^2]*(-a + 2*b*x^2))/(3*a^2*x^3)
```

fricas [A]   time = 1.66, size = 27, normalized size = 0.61

$$\frac{(2\,bx^2-a)\sqrt{bx^2+a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

giac [A]   time = 1.15, size = 55, normalized size = 1.25

$$\frac{4\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3
```

maple [A] time = 0.00, size = 26, normalized size = 0.59

$$-\frac{\sqrt{b\,x^2+a}\,\left(-2b\,x^2+a\right)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3
```

maxima [A] time = 1.27, size = 36, normalized size = 0.82

$$\frac{2\sqrt{bx^2+a}\,b}{3\,a^2x}-\frac{\sqrt{bx^2+a}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(b*x^2 + a)*b/(a^2*x) - 1/3*sqrt(b*x^2 + a)/(a*x^3)
```

mupad [B] time = 4.62, size = 25, normalized size = 0.57

$$-\frac{\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3)
```

sympy [A] time = 0.90, size = 46, normalized size = 1.05

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a**2)
```

3.486 $\int \frac{1}{x^5 \sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=74

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}-\frac{\sqrt{a+bx^2}}{4ax^4}$$

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}-\frac{\sqrt{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(4*a*x^4) + (3*b*Sqrt[a + b*x^2])/(8*a^2*x^2) - (3*b^2*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{8a}\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} + \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{16a^2}\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{8a^2}\\
&= -\frac{\sqrt{a+bx^2}}{4ax^4} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.50

$$
-\frac{b^2\sqrt{a+bx^2}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)}{a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2]),x]
[Out] -((b^2*Sqrt[a + b*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^2)/a])/a^3)
```

IntegrateAlgebraic [A] time = 0.07, size = 62, normalized size = 0.84

$$
\frac{\sqrt{a+bx^2}\left(3bx^2-2a\right)}{8a^2x^4} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a + 3*b*x^2))/(8*a^2*x^4) - (3*b^2*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/(8*a^(5/2))
```

fricas [A] time = 0.90, size = 135, normalized size = 1.82

$$\left[\frac{3\sqrt{a}\,b^2x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,abx^2-2\,a^2\right)\sqrt{bx^2+a}}{16\,a^3x^4},\ \frac{3\sqrt{-a}\,b^2x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3\,abx^2-2\,a^2\right)\sqrt{bx^2+a}}{8\,a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(3*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/(a^3*x^4), 1/8*(3*sqrt(-a)*b^2*x
^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/
(a^3*x^4)]
```

giac [A] time = 1.11, size = 75, normalized size = 1.01

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\left(bx^2+a\right)^{\frac{3}{2}}b^3-5\sqrt{bx^2+a}\,ab^3}{a^2b^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)
^(3/2)*b^3 - 5*sqrt(b*x^2 + a)*a*b^3)/(a^2*b^2*x^4))/b
```

maple [A] time = 0.00, size = 68, normalized size = 0.92

$$-\frac{3b^2\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}+\frac{3\sqrt{b\,x^2+a}\,b}{8a^2x^2}-\frac{\sqrt{b\,x^2+a}}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)^(1/2)/a/x^4+3/8*b*(b*x^2+a)^(1/2)/a^2/x^2-3/8/a^(5/2)*b^2*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.29, size = 56, normalized size = 0.76

$$-\frac{3\,b^{2}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{3\,\sqrt{bx^{2}+a}\,b}{8\,a^{2}x^{2}}-\frac{\sqrt{bx^{2}+a}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/8*sqrt(b*x^2 + a)*b/(a^2
*x^2) - 1/4*sqrt(b*x^2 + a)/(a*x^4)
```

mupad [B] time = 4.73, size = 57, normalized size = 0.77

$$\frac{3\,\left(b\,x^{2}+a\right)^{3/2}}{8\,a^{2}\,x^{4}}-\frac{5\,\sqrt{b\,x^{2}+a}}{8\,a\,x^{4}}-\frac{3\,b^{2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(1/2)),x)
```

```
[Out] (3*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (5*(a + b*x^2)^(1/2))/(8*a*x^4) - (3*b^
2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

sympy [A] time = 4.25, size = 97, normalized size = 1.31

$$-\frac{1}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{\sqrt{b}}{8ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b^{\frac{3}{2}}}{8a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3b^{2}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] -1/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)/(8*a*x**3*sqrt(a/(b*x**2
) + 1)) + 3*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**2*asinh(sqrt(a)
/(sqrt(b)*x))/(8*a**(5/2))
```

3.487 $\int \frac{x^5}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{3/2}}{3b^3}$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^2}{b^3\sqrt{a+bx^2}}-\frac{2a\sqrt{a+bx^2}}{b^3}+\frac{\left(a+bx^2\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(3/2),x]
```

```
[Out] -(a^2/(b^3*Sqrt[a + b*x^2])) - (2*a*Sqrt[a + b*x^2])/b^3 + (a + b*x^2)^(3/2
)/(3*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{3/2}} - \frac{2a}{b^2\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b^2}\right)dx, x, x^2\right)$$
$$= -\frac{a^2}{b^3\sqrt{a+bx^2}} - \frac{2a\sqrt{a+bx^2}}{b^3} + \frac{\left(a+bx^2\right)^{3/2}}{3b^3}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.69

$$\frac{-8a^2 - 4abx^2 + b^2x^4}{3b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(3/2),x]
[Out] (-8*a^2 - 4*a*b*x^2 + b^2*x^4)/(3*b^3*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 38, normalized size = 0.69

$$\frac{-8a^2 - 4abx^2 + b^2x^4}{3b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2)^(3/2),x]
[Out] (-8*a^2 - 4*a*b*x^2 + b^2*x^4)/(3*b^3*Sqrt[a + b*x^2])
```

fricas [A] time = 1.00, size = 46, normalized size = 0.84

$$\frac{\left(b^2x^4 - 4\,abx^2 - 8\,a^2\right)\sqrt{bx^2+a}}{3\left(b^4x^2 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] 1/3*(b^2*x^4 - 4*a*b*x^2 - 8*a^2)*sqrt(b*x^2 + a)/(b^4*x^2 + a*b^3)
```

giac [A] time = 0.98, size = 52, normalized size = 0.95

$$-\frac{a^2}{\sqrt{bx^2+a}\,b^3}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}b^6-6\,\sqrt{bx^2+a}\,ab^6}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -a^2/(sqrt(b*x^2 + a)*b^3) + 1/3*((b*x^2 + a)^(3/2)*b^6 - 6*sqrt(b*x^2 + a)
*a*b^6)/b^9
```

maple [A] time = 0.01, size = 36, normalized size = 0.65

$$-\frac{-b^2x^4+4ab\,x^2+8a^2}{3\sqrt{b\,x^2+a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/3*(-b^2*x^4+4*a*b*x^2+8*a^2)/(b*x^2+a)^(1/2)/b^3
```

maxima [A] time = 1.31, size = 53, normalized size = 0.96

$$\frac{x^4}{3\,\sqrt{bx^2+a}\,b}-\frac{4\,ax^2}{3\,\sqrt{bx^2+a}\,b^2}-\frac{8\,a^2}{3\,\sqrt{bx^2+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/(sqrt(b*x^2 + a)*b) - 4/3*a*x^2/(sqrt(b*x^2 + a)*b^2) - 8/3*a^2/(sq
rt(b*x^2 + a)*b^3)
```

mupad [B] time = 4.72, size = 41, normalized size = 0.75

$$-\frac{6\,a\,\left(b\,x^2+a\right)-{\left(b\,x^2+a\right)}^2+3\,a^2}{3\,b^3\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(3/2),x)
```

```
[Out] -(6*a*(a + b*x^2) - (a + b*x^2)^2 + 3*a^2)/(3*b^3*(a + b*x^2)^(1/2))
```

sympy [A] time = 0.94, size = 68, normalized size = 1.24

$$\begin{cases} -\frac{8a^2}{3b^3\sqrt{a+bx^2}} - \frac{4ax^2}{3b^2\sqrt{a+bx^2}} + \frac{x^4}{3b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-8*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*a*x**2/(3*b**2*sqrt(a + b*
x**2)) + x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(3/2)), True))
```

3.488 $\int \frac{x^4}{(a+bx^2)^{3/2}} dx$

Optimal. Leaf size=68

$$-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{x^3}{b\sqrt{a+bx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {288, 321, 217, 206}

$$\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}-\frac{x^3}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(3/2),x]
```

```
[Out] -(x^3/(b*Sqrt[a + b*x^2])) + (3*x*Sqrt[a + b*x^2])/(2*b^2) - (3*a*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b}\\
&= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{(3a)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}\\
&= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}\\
&= -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.04

$$\frac{\sqrt{b}\,x\left(3a+bx^2\right)-3a^{3/2}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(3/2),x]
[Out] (Sqrt[b]*x*(3*a + b*x^2) - 3*a^(3/2)*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x
)/Sqrt[a]])/(2*b^(5/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.09, size = 60, normalized size = 0.88

$$\frac{3a\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{5/2}}+\frac{3ax+bx^3}{2b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*x + b*x^3)/(2*b^2*Sqrt[a + b*x^2]) + (3*a*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(2*b^(5/2))
```

fricas [A] time = 1.28, size = 159, normalized size = 2.34

$$\left[\frac{3\left(abx^2+a^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(b^2x^3+3\,abx\right)\sqrt{bx^2+a}}{4\left(b^4x^2+ab^3\right)},\frac{3\left(abx^2+a^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(b^2x^3+3\,abx\right)\sqrt{bx^2+a}}{2\left(b^4x^2+ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*b*x^2 + a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/2*(3*(a*
b*x^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (b^2*x^3 + 3*a*b
*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 1.15, size = 51, normalized size = 0.75

$$\frac{x\left(\frac{x^2}{b}+\frac{3\,a}{b^2}\right)}{2\,\sqrt{bx^2+a}}+\frac{3\,a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*(x^2/b + 3*a/b^2)/sqrt(b*x^2 + a) + 3/2*a*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 57, normalized size = 0.84

$$\frac{x^3}{2\sqrt{b\,x^2+a}\;b}+\frac{3ax}{2\sqrt{b\,x^2+a}\;b^2}-\frac{3a\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*x^3/b/(b*x^2+a)^(1/2)+3/2*a/b^2*x/(b*x^2+a)^(1/2)-3/2*a/b^(5/2)*ln(b^(1
/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.32, size = 49, normalized size = 0.72

$$\frac{x^3}{2\,\sqrt{bx^2+a}\,b}+\frac{3\,ax}{2\,\sqrt{bx^2+a}\,b^2}-\frac{3\,a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(sqrt(b*x^2 + a)*b) + 3/2*a*x/(sqrt(b*x^2 + a)*b^2) - 3/2*a*arcsinh
(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(3/2), x)
```

sympy [A] time = 3.26, size = 71, normalized size = 1.04

$$\frac{3\sqrt{a}\,x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] 3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b
**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))
```

3.489 $\int \frac{x^3}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(3/2),x]
```

```
[Out] a/(b^2*Sqrt[a + b*x^2]) + Sqrt[a + b*x^2]/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx,x,x^2\right)\\ &= \frac{a}{b^2\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.75

$$\frac{2a + bx^2}{b^2\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(3/2),x]
[Out] (2*a + b*x^2)/(b^2*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 24, normalized size = 0.75

$$\frac{2a + bx^2}{b^2\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2)^(3/2),x]
[Out] (2*a + b*x^2)/(b^2*Sqrt[a + b*x^2])
```

fricas [A] time = 0.92, size = 34, normalized size = 1.06

$$\frac{\left(bx^2 + 2\,a\right)\sqrt{bx^2 + a}}{b^3x^2 + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] (b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^3*x^2 + a*b^2)
```

giac [A] time = 1.06, size = 32, normalized size = 1.00

$$\frac{\frac{\sqrt{bx^2+a}}{b} + \frac{a}{\sqrt{bx^2+a}\,b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] (sqrt(b*x^2 + a)/b + a/(sqrt(b*x^2 + a)*b))/b
```

maple [A] time = 0.00, size = 23, normalized size = 0.72

$$\frac{b\,x^2 + 2a}{\sqrt{b\,x^2 + a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] (b*x^2+2*a)/(b*x^2+a)^(1/2)/b^2
```

maxima [A] time = 1.38, size = 32, normalized size = 1.00

$$\frac{x^2}{\sqrt{bx^2+a}\,b}+\frac{2\,a}{\sqrt{bx^2+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(b*x^2 + a)*b) + 2*a/(sqrt(b*x^2 + a)*b^2)
```

mupad [B] time = 4.70, size = 22, normalized size = 0.69

$$\frac{b\,x^2+2\,a}{b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(3/2),x)
```

```
[Out] (2*a + b*x^2)/(b^2*(a + b*x^2)^(1/2))
```

sympy [A] time = 0.58, size = 41, normalized size = 1.28

$$\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0
)), (x**4/(4*a**(3/2)), True))
```

3.490 $\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {288, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(3/2),x]
```

```
[Out] -(x/(b*Sqrt[a + b*x^2])) + ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{x}{b\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}$$
$$= -\frac{x}{b\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b}$$
$$= -\frac{x}{b\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 1.37

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{b}\,x}{b^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[b]*x) + Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(
b^(3/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.06, size = 46, normalized size = 1.07

$$-\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^2)^(3/2),x]
```

```
[Out] -(x/(b*Sqrt[a + b*x^2])) - Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]/b^(3/2)
```

fricas [A] time = 0.76, size = 130, normalized size = 3.02

$$\left[-\frac{2\sqrt{bx^2+a}\,bx-\left(bx^2+a\right)\sqrt{b}\,\log\left(-2\,bx^2-2\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\left(b^3x^2+ab^2\right)},-\frac{\sqrt{bx^2+a}\,bx+\left(bx^2+a\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b^3x^2+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(b*x^2 + a)*b*x - (b*x^2 + a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a))/(b^3*x^2 + a*b^2), -(sqrt(b*x^2 + a)*b*x + (b*x^2
+ a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(b^3*x^2 + a*b^2)]
```

giac [A] time = 1.23, size = 39, normalized size = 0.91

$$-\frac{x}{\sqrt{bx^2+a}\,b}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -x/(sqrt(b*x^2 + a)*b) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.01, size = 37, normalized size = 0.86

$$-\frac{x}{\sqrt{b\,x^2+a}\;b}+\frac{\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.39, size = 29, normalized size = 0.67

$$-\frac{x}{\sqrt{bx^2+a}\,b}+\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/(sqrt(b*x^2 + a)*b) + arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

mupad [B] time = 0.09, size = 36, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}-\frac{x}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(3/2),x)
```

```
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(3/2) - x/(b*(a + b*x^2)^(1/2))
```

sympy [A] time = 1.71, size = 37, normalized size = 0.86

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}\,b\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))
```

3.491 $\int \frac{x}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(3/2),x]
[Out] -(1/(b*Sqrt[a + b*x^2]))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{3/2}}\,dx = -\frac{1}{b\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(3/2),x]
[Out] -(1/(b*Sqrt[a + b*x^2]))
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x^2)^(3/2),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2]))
```

fricas [A] time = 0.97, size = 24, normalized size = 1.50

$$-\frac{\sqrt{bx^2+a}}{b^2x^2+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)/(b^2*x^2 + a*b)
```

giac [A] time = 1.05, size = 14, normalized size = 0.88

$$-\frac{1}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(b*x^2 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{\sqrt{b\,x^2+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/b/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.32, size = 14, normalized size = 0.88

$$-\frac{1}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(b*x^2 + a)*b)
```

mupad [**B**] time = 0.04, size = 14, normalized size = 0.88

$$-\frac{1}{b\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(3/2),x)
```

```
[Out] -1/(b*(a + b*x^2)^(1/2))
```

sympy [**A**] time = 0.54, size = 24, normalized size = 1.50

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
```

3.492 $\int \frac{1}{\left(a+bx^2\right)^{3/2}} dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+bx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {191}

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}} dx = \frac{x}{a\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

fricas [A] time = 1.04, size = 23, normalized size = 1.44

$$\frac{\sqrt{bx^2 + a}\, x}{abx^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

giac [A] time = 1.12, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{bx^2 + a}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x}{\sqrt{b\,x^2 + a}\; a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2),x)
```

```
[Out] x/a/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.33, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{bx^2 + a}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.88

$$\frac{x}{a\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(3/2),x)
```

```
[Out] x/(a*(a + b*x^2)^(1/2))
```

sympy [A] time = 0.63, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.493 $\int \frac{1}{x(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{1}{a\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{1}{a\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{1}{a\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}\\
&= \frac{1}{a\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}\\
&= \frac{1}{a\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.80

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(3/2)),x]
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^2)/a]/(a*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 41, normalized size = 1.00

$$\frac{1}{a\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^(3/2)),x]
```

[Out] 1/(a*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(3/2)

fricas [A] time = 0.61, size = 126, normalized size = 3.07

$$\left[\frac{(bx^2+a)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\sqrt{bx^2+a}\,a}{2\left(a^2bx^2+a^3\right)}, \frac{(bx^2+a)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\,a}{a^2bx^2+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] [1/2*((b*x^2 + a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(b*x^2 + a)*a)/(a^2*b*x^2 + a^3), ((b*x^2 + a)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*b*x^2 + a^3)]

giac [A] time = 1.12, size = 39, normalized size = 0.95

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{1}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="giac")

[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/(sqrt(b*x^2 + a)*a)

maple [A] time = 0.00, size = 43, normalized size = 1.05

$$-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{a^{\frac{3}{2}}}+\frac{1}{\sqrt{b\,x^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^2+a)^(3/2),x)

[Out] 1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)

maxima [A] time = 1.36, size = 31, normalized size = 0.76

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{1}{\sqrt{bx^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 1/(sqrt(b*x^2 + a)*a)
```

mupad [B] time = 4.78, size = 33, normalized size = 0.80

$$\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(3/2)),x)
```

```
[Out] 1/(a*(a + b*x^2)^(1/2)) - atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(3/2)
```

sympy [B] time = 1.73, size = 184, normalized size = 4.49

$$\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**
2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/
(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) +
2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2)
+ 2*a**(7/2)*b*x**2)
```

3.494 $\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 191}

$$-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(1/(a*x*Sqrt[a + b*x^2])) - (2*b*x)/(a^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{1}{ax\sqrt{a+bx^2}}-\frac{(2b)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{a}\\ &= -\frac{1}{ax\sqrt{a+bx^2}}-\frac{2bx}{a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+2bx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(3/2)),x]
[Out] -((a + 2*b*x^2)/(a^2*x*Sqrt[a + b*x^2]))
```

IntegrateAlgebraic [A] time = 0.05, size = 28, normalized size = 0.74

$$\frac{-a-2bx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^(3/2)),x]
[Out] (-a - 2*b*x^2)/(a^2*x*Sqrt[a + b*x^2])
```

fricas [A] time = 1.04, size = 35, normalized size = 0.92

$$-\frac{\left(2\,bx^2+a\right)\sqrt{bx^2+a}}{a^2bx^3+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] -(2*b*x^2 + a)*sqrt(b*x^2 + a)/(a^2*b*x^3 + a^3*x)
```

giac [A] time = 1.15, size = 50, normalized size = 1.32

$$-\frac{bx}{\sqrt{bx^2+a}\,a^2}+\frac{2\,\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] -b*x/(sqrt(b*x^2 + a)*a^2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a)*a)
```

maple [A] time = 0.00, size = 26, normalized size = 0.68

$$-\frac{2b\,x^{2}+a}{\sqrt{b\,x^{2}+a}\;a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -(2*b*x^2+a)/x/(b*x^2+a)^(1/2)/a^2
```

maxima [A] time = 1.29, size = 34, normalized size = 0.89

$$-\frac{2\,bx}{\sqrt{bx^{2}+a}\,a^{2}}-\frac{1}{\sqrt{bx^{2}+a}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b*x/(sqrt(b*x^2 + a)*a^2) - 1/(sqrt(b*x^2 + a)*a*x)
```

mupad [B] time = 4.64, size = 35, normalized size = 0.92

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(\frac{1}{a}+\frac{2\,b\,x^{2}}{a^{2}}\right)}{b\,x^{3}+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(3/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(1/a + (2*b*x^2)/a^2))/(a*x + b*x^3)
```

sympy [A] time = 0.86, size = 46, normalized size = 1.21

$$-\frac{1}{a\sqrt{b}\,x^{2}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{2\sqrt{b}}{a^{2}\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] -1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2)
+ 1))
```

3.495 $\int \frac{1}{x^3(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3b}{2a^2\sqrt{a+bx^2}}-\frac{1}{2ax^2\sqrt{a+bx^2}}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{3\sqrt{a+bx^2}}{2a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{1}{ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] 1/(a*x^2*Sqrt[a + b*x^2]) - (3*Sqrt[a + b*x^2])/(2*a^2*x^2) + (3*b*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{1}{ax^2\sqrt{a+bx^2}}+\frac{3\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}\\
&= \frac{1}{ax^2\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{2a^2x^2}-\frac{(3b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^2}\\
&= \frac{1}{ax^2\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{2a^2x^2}-\frac{3\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^2}\\
&= \frac{1}{ax^2\sqrt{a+bx^2}}-\frac{3\sqrt{a+bx^2}}{2a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.51

$$-\frac{b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^2}{a}+1\right)}{a^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] -((b*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^2)/a])/(a^2*Sqrt[a + b*x^2]))
```

IntegrateAlgebraic [A] time = 0.07, size = 60, normalized size = 0.87

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{-a-3bx^2}{2a^2x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-a - 3*b*x^2)/(2*a^2*x^2*Sqrt[a + b*x^2]) + (3*b*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(2*a^(5/2))
```

fricas [A] time = 1.12, size = 171, normalized size = 2.48

$$\left[\frac{3\left(b^{2}x^{4}+abx^{2}\right)\sqrt{a}\log\left(-\frac{bx^{2}+2\sqrt{bx^{2}+a}\sqrt{a}+2a}{x^{2}}\right)-2\left(3abx^{2}+a^{2}\right)\sqrt{bx^{2}+a}}{4\left(a^{3}bx^{4}+a^{4}x^{2}\right)},-\frac{3\left(b^{2}x^{4}+abx^{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(3abx^{2}+a^{2}\right)\sqrt{bx^{2}+a}}{2\left(a^{3}bx^{4}+a^{4}x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b^2*x^4 + a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a)
 + 2*a)/x^2) - 2*(3*a*b*x^2 + a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2),
-1/2*(3*(b^2*x^4 + a*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*
a*b*x^2 + a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

giac [A] time = 1.12, size = 72, normalized size = 1.04

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^{2}}-\frac{3\left(bx^{2}+a\right)b-2\,ab}{2\left(\left(bx^{2}+a\right)^{\frac{3}{2}}-\sqrt{bx^{2}+a}\,a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/2*(3*(b*x^2 + a)
*b - 2*a*b)/(((b*x^2 + a)^(3/2) - sqrt(b*x^2 + a)*a)*a^2)
```

maple [A] time = 0.01, size = 63, normalized size = 0.91

$$\frac{3b\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{5}{2}}}-\frac{3b}{2\sqrt{b\,x^{2}+a}\,a^{2}}-\frac{1}{2\sqrt{b\,x^{2}+a}\,a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a^2/(b*x^2+a)^(1/2)+3/2/a^(5/2)*b*ln((2*a+
2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.31, size = 51, normalized size = 0.74

$$\frac{3\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{3\,b}{2\,\sqrt{bx^2+a}\,a^2}-\frac{1}{2\,\sqrt{bx^2+a}\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 3/2*b/(sqrt(b*x^2 + a)*a^2) -
 1/2/(sqrt(b*x^2 + a)*a*x^2)
```

mupad [B] time = 4.94, size = 53, normalized size = 0.77

$$\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{1}{2\,a\,x^2\,\sqrt{b\,x^2+a}}-\frac{3\,b}{2\,a^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(3/2)),x)
```

```
[Out] (3*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) - 1/(2*a*x^2*(a + b*x^2)
^(1/2)) - (3*b)/(2*a^2*(a + b*x^2)^(1/2))
```

sympy [A] time = 3.39, size = 73, normalized size = 1.06

$$-\frac{1}{2a\sqrt{b}\,x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] -1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*
x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))
```

3.496 $\int \frac{1}{x^4(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{8b^2x}{3a^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}-\frac{1}{3ax^3\sqrt{a+bx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 191}

$$\frac{8b^2x}{3a^3\sqrt{a+bx^2}}+\frac{4b}{3a^2x\sqrt{a+bx^2}}-\frac{1}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -1/(3*a*x^3*Sqrt[a + b*x^2]) + (4*b)/(3*a^2*x*Sqrt[a + b*x^2]) + (8*b^2*x)/
(3*a^3*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{3/2}}\,dx = -\frac{1}{3ax^3\sqrt{a+bx^2}} - \frac{(4b)\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a}$$

$$= -\frac{1}{3ax^3\sqrt{a+bx^2}} + \frac{4b}{3a^2x\sqrt{a+bx^2}} + \frac{\left(8b^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}$$

$$= -\frac{1}{3ax^3\sqrt{a+bx^2}} + \frac{4b}{3a^2x\sqrt{a+bx^2}} + \frac{8b^2x}{3a^3\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.61

$$-\frac{a^2-4abx^2-8b^2x^4}{3a^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(3/2)),x]
[Out] -1/3*(a^2 - 4*a*b*x^2 - 8*b^2*x^4)/(a^3*x^3*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.08, size = 42, normalized size = 0.64

$$\frac{-a^2+4abx^2+8b^2x^4}{3a^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^(3/2)),x]
[Out] (-a^2 + 4*a*b*x^2 + 8*b^2*x^4)/(3*a^3*x^3*Sqrt[a + b*x^2])
```

fricas [A] time = 0.74, size = 50, normalized size = 0.76

$$\frac{\left(8\,b^2x^4+4\,abx^2-a^2\right)\sqrt{bx^2+a}}{3\left(a^3bx^5+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] 1/3*(8*b^2*x^4 + 4*a*b*x^2 - a^2)*sqrt(b*x^2 + a)/(a^3*b*x^5 + a^4*x^3)
```

giac [A]   time = 1.16, size = 106, normalized size = 1.61

$$\frac{b^2x}{\sqrt{bx^2+a}\,a^3}-\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4 b^{\frac{3}{2}}-12\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2 ab^{\frac{3}{2}}+5\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] b^2*x/(sqrt(b*x^2 + a)*a^3) - 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2
) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 5*a^2*b^(3/2))/(((sqrt(b
)*x - sqrt(b*x^2 + a))^2 - a)^3*a^2)
```

maple [A]   time = 0.01, size = 37, normalized size = 0.56

$$-\frac{-8b^2x^4-4ab\,x^2+a^2}{3\sqrt{b\,x^2+a}\;a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/3*(-8*b^2*x^4-4*a*b*x^2+a^2)/x^3/(b*x^2+a)^(1/2)/a^3
```

maxima [A]   time = 1.31, size = 54, normalized size = 0.82

$$\frac{8\,b^2x}{3\,\sqrt{bx^2+a}\,a^3}+\frac{4\,b}{3\,\sqrt{bx^2+a}\,a^2x}-\frac{1}{3\,\sqrt{bx^2+a}\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*b^2*x/(sqrt(b*x^2 + a)*a^3) + 4/3*b/(sqrt(b*x^2 + a)*a^2*x) - 1/3/(sqrt
(b*x^2 + a)*a*x^3)
```

mupad [B]   time = 5.12, size = 38, normalized size = 0.58

$$\frac{-a^2+4\,a\,b\,x^2+8\,b^2\,x^4}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(3/2)),x)
```

[Out] (8*b^2*x^4 - a^2 + 4*a*b*x^2)/(3*a^3*x^3*(a + b*x^2)^(1/2))

sympy [B] time = 1.23, size = 233, normalized size = 3.53

$$-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(b*x**2+a)**(3/2),x)

[Out] -a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6)

3.497 $\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {288, 321, 217, 206}

$$-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(5/2),x]
```

```
[Out] -x^5/(3*b*(a + b*x^2)^(3/2)) - (5*x^3)/(3*b^2*Sqrt[a + b*x^2]) + (5*x*Sqrt[
a + b*x^2])/(2*b^3) - (5*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2)
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}+\frac{5\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{3b}\\ &= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b^2}\\ &= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{(5a)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^3}\\ &= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{(5a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^3}\\ &= -\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 90, normalized size = 0.99

$$\frac{\sqrt{b}\,x\left(15a^2+20abx^2+3b^2x^4\right)-15a^{3/2}\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6b^{7/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*x*(15*a^2 + 20*a*b*x^2 + 3*b^2*x^4) - 15*a^(3/2)*(a + b*x^2)*Sqrt[
1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(6*b^(7/2)*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 72, normalized size = 0.79

$$\frac{15a^2x+20abx^3+3b^2x^5}{6b^3\left(a+bx^2\right)^{3/2}}+\frac{5a\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^2)^(5/2),x]
```

```
[Out] (15*a^2*x + 20*a*b*x^3 + 3*b^2*x^5)/(6*b^3*(a + b*x^2)^(3/2)) + (5*a*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(7/2))
```

fricas [A] time = 1.17, size = 227, normalized size = 2.49

$$\left[\frac{15\left(ab^2x^4+2\,a^2bx^2+a^3\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(3\,b^3x^5+20\,ab^2x^3+15\,a^2bx\right)\sqrt{bx^2+a}}{12\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)},\ \frac{15\left(ab^2x^4+2\,a^2bx^2+a^3\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(3\,b^3x^5+20\,ab^2x^3+15\,a^2bx\right)\sqrt{bx^2+a}}{6\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) + 2*(3*b^3*x^5 + 20*a*b^2*x^3 + 15*a^2*b*x)*sqrt(b*x
^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/6*(15*(a*b^2*x^4 + 2*a^2*b*x^
2 + a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*b^3*x^5 + 20*a*b^
2*x^3 + 15*a^2*b*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

giac [A] time = 1.21, size = 65, normalized size = 0.71

$$\frac{\left(x^2\left(\frac{3\,x^2}{b}+\frac{20\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)x}{6\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{5\,a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(x^2*(3*x^2/b + 20*a/b^2) + 15*a^2/b^3)*x/(b*x^2 + a)^(3/2) + 5/2*a*log
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.01, size = 75, normalized size = 0.82

$$\frac{x^5}{2\left(b\,x^2+a\right)^{\frac{3}{2}}b}+\frac{5a\,x^3}{6\left(b\,x^2+a\right)^{\frac{3}{2}}b^2}+\frac{5ax}{2\sqrt{b\,x^2+a}\;b^3}-\frac{5a\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/2*x^5/b/(b*x^2+a)^(3/2)+5/6*a/b^2*x^3/(b*x^2+a)^(3/2)+5/2*a/b^3*x/(b*x^2+
a)^(1/2)-5/2*a/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.41, size = 89, normalized size = 0.98

$$\frac{x^{5}}{2\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{5\,ax\left(\frac{3\,x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}\right)}{6\,b}+\frac{5\,ax}{6\,\sqrt{bx^{2}+a}\,b^{3}}-\frac{5\,a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^5/((b*x^2 + a)^(3/2)*b) + 5/6*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/
((b*x^2 + a)^(3/2)*b^2))/b + 5/6*a*x/(sqrt(b*x^2 + a)*b^3) - 5/2*a*arcsinh(
b*x/sqrt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^6/(a + b*x^2)^(5/2), x)
```

sympy [B] time = 5.12, size = 367, normalized size = 4.03

$$-\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^{2}}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^{2}\sqrt{1+\frac{bx^{2}}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{20a^{39}b^{\frac{47}{2}}x^{3}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{38}b^{\frac{49}{2}}x^{5}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^{2}}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(5/2),x)
```

```
[Out] -15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2
)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2
/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/
(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt
```

```
(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x*
*2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47/2)
*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**
2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(51/2)*sqrt(
1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a))
```

3.498 $\int \frac{x^5}{(a+bx^2)^{5/2}} \, dx$

Optimal. Leaf size=54

$$-\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{2a}{b^3\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(5/2),x]
```

```
[Out] -a^2/(3*b^3*(a + b*x^2)^(3/2)) + (2*a)/(b^3*Sqrt[a + b*x^2]) + Sqrt[a + b*x
^2]/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{5/2}} - \frac{2a}{b^2(a+bx)^{3/2}} + \frac{1}{b^2\sqrt{a+bx}}\right) dx, x, x^2\right)$$
$$= -\frac{a^2}{3b^3\left(a+bx^2\right)^{3/2}} + \frac{2a}{b^3\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^3}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.72

$$\frac{8a^2+12abx^2+3b^2x^4}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(5/2),x]
[Out] (8*a^2 + 12*a*b*x^2 + 3*b^2*x^4)/(3*b^3*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.72

$$\frac{8a^2+12abx^2+3b^2x^4}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2)^(5/2),x]
[Out] (8*a^2 + 12*a*b*x^2 + 3*b^2*x^4)/(3*b^3*(a + b*x^2)^(3/2))
```

fricas [A] time = 1.14, size = 58, normalized size = 1.07

$$\frac{\left(3\,b^2x^4+12\,abx^2+8\,a^2\right)\sqrt{bx^2+a}}{3\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*(3*b^2*x^4 + 12*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/(b^5*x^4 + 2*a*b^4*x^2
 + a^2*b^3)
```

giac [A] time = 1.02, size = 44, normalized size = 0.81

$$\frac{\sqrt{bx^2+a}}{b^3} + \frac{6\left(bx^2+a\right)a - a^2}{3\left(bx^2+a\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b^3 + 1/3*(6*(b*x^2 + a)*a - a^2)/((b*x^2 + a)^(3/2)*b^3)
```

maple [A] time = 0.00, size = 36, normalized size = 0.67

$$\frac{3b^2x^4 + 12ab\,x^2 + 8a^2}{3\left(b\,x^2+a\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(3*b^2*x^4+12*a*b*x^2+8*a^2)/(b*x^2+a)^(3/2)/b^3
```

maxima [A] time = 1.35, size = 52, normalized size = 0.96

$$\frac{x^4}{\left(bx^2+a\right)^{\frac{3}{2}}b} + \frac{4\,ax^2}{\left(bx^2+a\right)^{\frac{3}{2}}b^2} + \frac{8\,a^2}{3\left(bx^2+a\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] x^4/((b*x^2 + a)^(3/2)*b) + 4*a*x^2/((b*x^2 + a)^(3/2)*b^2) + 8/3*a^2/((b*x
^2 + a)^(3/2)*b^3)
```

mupad [B] time = 5.20, size = 38, normalized size = 0.70

$$\frac{2\,a\,\left(b\,x^2+a\right)+\left(b\,x^2+a\right)^2-\frac{a^2}{3}}{b^3\,\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(5/2),x)
```

```
[Out] (2*a*(a + b*x^2) + (a + b*x^2)^2 - a^2/3)/(b^3*(a + b*x^2)^(3/2))
```

sympy [A] time = 1.09, size = 138, normalized size = 2.56

$$\begin{cases} \frac{8a^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{12abx^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{3b^2x^4}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)
) + 12*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2))
+ 3*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)), N
e(b, 0)), (x**6/(6*a**(5/2)), True))
```

3.499 $\int \frac{x^4}{\left(a+bx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{x}{b^2\sqrt{a+bx^2}}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {288, 217, 206}

$$-\frac{x}{b^2\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(5/2),x]
```

```
[Out] -x^3/(3*b*(a + b*x^2)^(3/2)) - x/(b^2*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]]/b^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{x^3}{3b\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx}{b}$$

$$= -\frac{x^3}{3b\left(a+bx^2\right)^{3/2}} - \frac{x}{b^2\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}$$

$$= -\frac{x^3}{3b\left(a+bx^2\right)^{3/2}} - \frac{x}{b^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^2}$$

$$= -\frac{x^3}{3b\left(a+bx^2\right)^{3/2}} - \frac{x}{b^2\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.12, size = 80, normalized size = 1.25

$$\frac{3\sqrt{a}\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) - \sqrt{b}\,x\left(3a+4bx^2\right)}{3b^{5/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(Sqrt[b]*x*(3*a + 4*b*x^2)) + 3*Sqrt[a]*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*A
rcSinh[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(5/2)*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 58, normalized size = 0.91

$$\frac{-3ax-4bx^3}{3b^2\left(a+bx^2\right)^{3/2}} - \frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (-3*a*x - 4*b*x^3)/(3*b^2*(a + b*x^2)^(3/2)) - Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]]/b^(5/2)
```

fricas [A] time = 0.72, size = 199, normalized size = 3.11

$$\left[\frac{3\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}x-a\right)-2\left(4\,b^{2}x^{3}+3\,abx\right)\sqrt{bx^{2}+a}}{6\left(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3}\right)},-\frac{3\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)+\left(4\,b^{2}x^{3}+3\,abx\right)\sqrt{bx^{2}+a}}{3\left(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) - 2*(4*b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*
b^4*x^2 + a^2*b^3), -1/3*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) + (4*b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^5*x^4
+ 2*a*b^4*x^2 + a^2*b^3)]
```

giac [A] time = 1.05, size = 51, normalized size = 0.80

$$-\frac{x\left(\frac{4\,x^{2}}{b}+\frac{3\,a}{b^{2}}\right)}{3\left(bx^{2}+a\right)^{\frac{3}{2}}}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(4*x^2/b + 3*a/b^2)/(b*x^2 + a)^(3/2) - log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 54, normalized size = 0.84

$$-\frac{x^{3}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b}-\frac{x}{\sqrt{b\,x^{2}+a}\;b^{2}}+\frac{\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*x^3/b/(b*x^2+a)^(3/2)-x/b^2/(b*x^2+a)^(1/2)+1/b^(5/2)*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))
```

maxima [A] time = 1.37, size = 65, normalized size = 1.02

$$-\frac{1}{3}\,x\left(\frac{3\,x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}\right)-\frac{x}{3\,\sqrt{bx^{2}+a}\,b^{2}}+\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - 1/3*x/
(sqrt(b*x^2 + a)*b^2) + arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(5/2), x)
```

sympy [B] time = 2.99, size = 303, normalized size = 4.73

$$\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*
b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a
)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*
a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1
+ b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a
) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3
/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqr
t(1 + b*x**2/a))
```

3.500 $\int \frac{x^3}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(5/2),x]
```

```
[Out] a/(3*b^2*(a + b*x^2)^(3/2)) - 1/(b^2*Sqrt[a + b*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)dx,x,x^2\right)\\ &= \frac{a}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{1}{b^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{-2a-3bx^2}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(5/2),x]
[Out] (-2*a - 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 28, normalized size = 0.78

$$\frac{-2a-3bx^2}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2)^(5/2),x]
[Out] (-2*a - 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.88, size = 47, normalized size = 1.31

$$-\frac{\left(3\,bx^2+2\,a\right)\sqrt{bx^2+a}}{3\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] -1/3*(3*b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 1.10, size = 24, normalized size = 0.67

$$-\frac{3\,bx^2+2\,a}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] -1/3*(3*b*x^2 + 2*a)/((b*x^2 + a)^(3/2)*b^2)
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$-\frac{3b\,x^{2}+2a}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(5/2),x)
[Out] -1/3*(3*b*x^2+2*a)/(b*x^2+a)^(3/2)/b^2
```

maxima [A] time = 1.27, size = 33, normalized size = 0.92

$$-\frac{x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}-\frac{2\,a}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -x^2/((b*x^2 + a)^(3/2)*b) - 2/3*a/((b*x^2 + a)^(3/2)*b^2)
```

mupad [B] time = 5.17, size = 24, normalized size = 0.67

$$-\frac{3\,b\,x^{2}+2\,a}{3\,b^{2}\left(b\,x^{2}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(5/2),x)
[Out] -(2*a + 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

sympy [A] time = 1.07, size = 92, normalized size = 2.56

$$\begin{cases} -\frac{2a}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}}-\frac{3bx^{2}}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}} & \text{for}\ b\neq 0 \\ \frac{x^{4}}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(5/2),x)
[Out] Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2))
- 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b
, 0)), (x**4/(4*a**(5/2)), True))
```

3.501 $\int \frac{x^2}{\left(a+bx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=21

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(5/2),x]
```

```
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{5/2}} \, dx = \frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(5/2),x]
```

```
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 21, normalized size = 1.00

$$\frac{x^3}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^2)^(5/2),x]
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

fricas [B] time = 1.11, size = 37, normalized size = 1.76

$$\frac{\sqrt{bx^2+a}\,x^3}{3\left(ab^2x^4+2\,a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x^2 + a)*x^3/(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)
```

giac [A] time = 1.08, size = 17, normalized size = 0.81

$$\frac{x^3}{3\left(bx^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*x^3/((b*x^2 + a)^(3/2)*a)
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{x^3}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(5/2),x)
[Out] 1/3*x^3/a/(b*x^2+a)^(3/2)
```

maxima [A] time = 1.31, size = 34, normalized size = 1.62

$$-\frac{x}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{x}{3\sqrt{bx^2+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -1/3*x/((b*x^2 + a)^(3/2)*b) + 1/3*x/(sqrt(b*x^2 + a)*a*b)
```

mupad [B] time = 5.13, size = 17, normalized size = 0.81

$$\frac{x^3}{3\,a\,\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(5/2),x)
[Out] x^3/(3*a*(a + b*x^2)^(3/2))
```

sympy [B] time = 0.76, size = 44, normalized size = 2.10

$$\frac{x^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(5/2),x)
[Out] x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

3.502 $\int \frac{x}{(a+bx^2)^{5/2}} \, dx$

Optimal. Leaf size=18

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{5/2}} \, dx = -\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*1/(b*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x^2)^(5/2),x]
[Out] -1/3*1/(b*(a + b*x^2)^(3/2))
```

fricas [B] time = 0.49, size = 35, normalized size = 1.94

$$-\frac{\sqrt{bx^2+a}}{3\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] -1/3*sqrt(b*x^2 + a)/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

giac [A] time = 1.02, size = 14, normalized size = 0.78

$$-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] -1/3/((b*x^2 + a)^(3/2)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{1}{3\left(b\,x^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2),x)
[Out] -1/3/b/(b*x^2+a)^(3/2)
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3/((b*x^2 + a)^(3/2)*b)
```

mupad [B] time = 4.94, size = 14, normalized size = 0.78

$$-\frac{1}{3\,b\,\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(5/2),x)
```

```
[Out] -1/(3*b*(a + b*x^2)^(3/2))
```

sympy [A] time = 1.00, size = 46, normalized size = 2.56

$$\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(b
, 0)), (x**2/(2*a**(5/2)), True))
```

3.503 $\int \frac{1}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\ &= \frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{x(3a+2bx^2)}{3a^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-5/2),x]
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 29, normalized size = 0.74

$$\frac{x(3a+2bx^2)}{3a^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-5/2),x]
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

fricas [A] time = 1.06, size = 47, normalized size = 1.21

$$\frac{(2\,bx^3+3\,ax)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

giac [A] time = 1.04, size = 27, normalized size = 0.69

$$\frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

maple [A] time = 0.00, size = 26, normalized size = 0.67

$$\frac{(2b\,x^2+3a)\,x}{3\,(b\,x^2+a)^{\frac{3}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(2*b*x^2+3*a)/(b*x^2+a)^(3/2)/a^2
```

maxima [A] time = 1.34, size = 31, normalized size = 0.79

$$\frac{2\,x}{3\,\sqrt{bx^2+a}\,a^2}+\frac{x}{3\,(bx^2+a)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

mupad [B] time = 4.89, size = 28, normalized size = 0.72

$$\frac{2\,x\,(b\,x^2+a)+a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(5/2),x)
```

```
[Out] (2*x*(a + b*x^2) + a*x)/(3*a^2*(a + b*x^2)^(3/2))
```

sympy [B] time = 0.83, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)
) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*
x**2/a))
```

3.504 $\int \frac{1}{x(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{1}{a^2\sqrt{a+bx^2}}+\frac{1}{3a\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{1}{a^2\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{1}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*(a + b*x^2)^(3/2)) + 1/(a^2*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]]/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, x^2\right)\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)}{2a}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{1}{a^2\sqrt{a+bx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a^2}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{1}{a^2\sqrt{a+bx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{a^2b}\\
&= \frac{1}{3a\left(a+bx^2\right)^{3/2}}+\frac{1}{a^2\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.61

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx^2}{a}+1\right)}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(5/2)),x]
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x^2)/a]/(3*a*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 54, normalized size = 0.92

$$\frac{4a+3bx^2}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (4*a + 3*b*x^2)/(3*a^2*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]/a^(5/2)
```

fricas [A] time = 1.02, size = 197, normalized size = 3.34

$$\left[\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,abx^2+4\,a^2\right)\sqrt{bx^2+a}}{6\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)},\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3\,abx^2+4\,a^2\right)\sqrt{bx^2+a}}{3\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(3*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4
+ 2*a^4*b*x^2 + a^5), 1/3*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a)*arctan(sq
rt(-a)/sqrt(b*x^2 + a)) + (3*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4
 + 2*a^4*b*x^2 + a^5)]
```

giac [A] time = 1.15, size = 50, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,bx^2+4\,a}{3\left(bx^2+a\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(3*b*x^2 + 4*a)/((b*x
^2 + a)^(3/2)*a^2)
```

maple [A] time = 0.00, size = 57, normalized size = 0.97

$$\frac{1}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a}-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{a^{\frac{5}{2}}}+\frac{1}{\sqrt{b\,x^2+a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3/a/(b*x^2+a)^(3/2)+1/a^2/(b*x^2+a)^(1/2)-1/a^(5/2)*ln((2*a+2*(b*x^2+a)^(
1/2)*a^(1/2))/x)
```

maxima [A] time = 1.34, size = 45, normalized size = 0.76

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{1}{\sqrt{bx^2+a}\,a^2}+\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/(sqrt(b*x^2 + a)*a^2) + 1/3/((b*
x^2 + a)^(3/2)*a)
```

mupad [B] time = 5.20, size = 47, normalized size = 0.80

$$\frac{\frac{b\,x^2+a}{a^2}+\frac{1}{3\,a}}{{\left(b\,x^2+a\right)}^{3/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(5/2)),x)
```

```
[Out] ((a + b*x^2)/a^2 + 1/(3*a))/(a + b*x^2)^(3/2) - atanh((a + b*x^2)^(1/2)/a^(
1/2))/a^(5/2)
```

sympy [B] time = 2.89, size = 740, normalized size = 12.54

$$\frac{8a^7\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{3a^7\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{6a^7\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{14a^6bx^2\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{9a^6bx^2\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{18a^6bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{6a^5b^2x^4\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{9a^5b^2x^4\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{18a^5b^2x^4\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{3a^4b^3x^6\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{6a^4b^3x^6\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(5/2),x)
```

```
[Out] 8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)
*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 1
8*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a*
*7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(
15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a
```

```
)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)
*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**
2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sq
rt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**
2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a*
*(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x
**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(sqr
t(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2
*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2
) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) -
 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b
*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)
```

3.505 $\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{8bx}{3a^3\sqrt{a+bx^2}}-\frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{ax\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {271, 192, 191}

$$-\frac{8bx}{3a^3\sqrt{a+bx^2}}-\frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(3/2))) - (4*b*x)/(3*a^2*(a + b*x^2)^(3/2)) - (8*b*x)/
(3*a^3*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx = -\frac{1}{ax\left(a+bx^2\right)^{3/2}} - \frac{(4b)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{a}$$
$$= -\frac{1}{ax\left(a+bx^2\right)^{3/2}} - \frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}} - \frac{(8b)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}$$
$$= -\frac{1}{ax\left(a+bx^2\right)^{3/2}} - \frac{4bx}{3a^2\left(a+bx^2\right)^{3/2}} - \frac{8bx}{3a^3\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.70

$$\frac{-3a^2-12abx^2-8b^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(5/2)),x]
[Out] (-3*a^2 - 12*a*b*x^2 - 8*b^2*x^4)/(3*a^3*x*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 42, normalized size = 0.70

$$\frac{-3a^2-12abx^2-8b^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^(5/2)),x]
[Out] (-3*a^2 - 12*a*b*x^2 - 8*b^2*x^4)/(3*a^3*x*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.63, size = 59, normalized size = 0.98

$$-\frac{\left(8\,b^2x^4+12\,abx^2+3\,a^2\right)\sqrt{bx^2+a}}{3\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*b^2*x^4 + 12*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a)/(a^3*b^2*x^5 + 2*a^4*
b*x^3 + a^5*x)
```

giac [A] time = 1.20, size = 64, normalized size = 1.07

$$-\frac{x\left(\frac{5\,b^2x^2}{a^3}+\frac{6\,b}{a^2}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\,\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(5*b^2*x^2/a^3 + 6*b/a^2)/(b*x^2 + a)^(3/2) + 2*sqrt(b)/(((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)*a^2)
```

maple [A] time = 0.00, size = 39, normalized size = 0.65

$$-\frac{8b^2x^4+12ab\,x^2+3a^2}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\,a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(8*b^2*x^4+12*a*b*x^2+3*a^2)/x/(b*x^2+a)^(3/2)/a^3
```

maxima [A] time = 1.34, size = 50, normalized size = 0.83

$$-\frac{8\,bx}{3\,\sqrt{bx^2+a}\,a^3}-\frac{4\,bx}{3\left(bx^2+a\right)^{\frac{3}{2}}a^2}-\frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*b*x/((b*x^2 + a)^(3/2)*a^2) - 1/((b*x^
2 + a)^(3/2)*a*x)
```

mupad [B] time = 5.18, size = 42, normalized size = 0.70

$$\frac{4\,a\,\left(b\,x^2+a\right)-8\left(b\,x^2+a\right)^2+a^2}{3\,a^3\,x\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(5/2)),x)
```

```
[Out] (4*a*(a + b*x^2) - 8*(a + b*x^2)^2 + a^2)/(3*a^3*x*(a + b*x^2)^(3/2))
```

sympy [B] time = 1.29, size = 165, normalized size = 2.75

$$-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] -3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a
**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*
a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/
(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)
```

3.506 $\int \frac{1}{x^3(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5b}{2a^3\sqrt{a+bx^2}}-\frac{5b}{6a^2\left(a+bx^2\right)^{3/2}}-\frac{1}{2ax^2\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{5\sqrt{a+bx^2}}{2a^3x^2}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] 1/(3*a*x^2*(a + b*x^2)^(3/2)) + 5/(3*a^2*x^2*Sqrt[a + b*x^2]) - (5*Sqrt[a +
 b*x^2])/(2*a^3*x^2) + (5*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^2\right)}{6a}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}+\frac{5\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3x^2}-\frac{(5b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a^3}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3x^2}-\frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a^3}\\
&= \frac{1}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5}{3a^2x^2\sqrt{a+bx^2}}-\frac{5\sqrt{a+bx^2}}{2a^3x^2}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.42

$$-\frac{b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx^2}{a}+1\right)}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/3*(b*Hypergeometric2F1[-3/2, 2, -1/2, 1 + (b*x^2)/a])/(a^2*(a + b*x^2)^(
3/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 71, normalized size = 0.81

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{-3a^2-20abx^2-15b^2x^4}{6a^3x^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2 - 20*a*b*x^2 - 15*b^2*x^4)/(6*a^3*x^2*(a + b*x^2)^(3/2)) + (5*b*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

fricas [A] time = 0.68, size = 241, normalized size = 2.74

$$\left[\frac{15\left(b^3x^6+2ab^2x^4+a^2bx^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(15ab^2x^4+20a^2bx^2+3a^3\right)\sqrt{bx^2+a}}{12\left(a^4b^2x^6+2a^5bx^4+a^6x^2\right)}, -\frac{15\left(b^3x^6+2ab^2x^4+a^2bx^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(15ab^2x^4+20a^2bx^2+3a^3\right)\sqrt{bx^2+a}}{6\left(a^4b^2x^6+2a^5bx^4+a^6x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(15*a*b^2*x^4 + 20*a^2*b*x^2 + 3*a^3)*sq
rt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(b^3*x^6 + 2
*a*b^2*x^4 + a^2*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*a*b
^2*x^4 + 20*a^2*b*x^2 + 3*a^3)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4
+ a^6*x^2)]
```

giac [A] time = 1.08, size = 73, normalized size = 0.83

$$-\frac{5\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^3}-\frac{6\left(bx^2+a\right)b+ab}{3\left(bx^2+a\right)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx^2+a}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) - 1/3*(6*(b*x^2 + a)
*b + a*b)/((b*x^2 + a)^(3/2)*a^3) - 1/2*sqrt(b*x^2 + a)/(a^3*x^2)
```

maple [A] time = 0.01, size = 78, normalized size = 0.89

$$-\frac{5b}{6\left(b\,x^{2}+a\right)^{\frac{3}{2}} a^{2}}+\frac{5b\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{7}{2}}}-\frac{5b}{2\sqrt{b\,x^{2}+a}\,a^{3}}-\frac{1}{2\left(b\,x^{2}+a\right)^{\frac{3}{2}} a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(3/2)-5/6*b/a^2/(b*x^2+a)^(3/2)-5/2*b/a^3/(b*x^2+a)^(1
/2)+5/2/a^(7/2)*b*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.35, size = 66, normalized size = 0.75

$$\frac{5\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{7}{2}}}-\frac{5\,b}{2\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{5\,b}{6\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{1}{2\left(bx^{2}+a\right)^{\frac{3}{2}}ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 5/2*b/(sqrt(b*x^2 + a)*a^3) -
 5/6*b/((b*x^2 + a)^(3/2)*a^2) - 1/2/((b*x^2 + a)^(3/2)*a*x^2)
```

mupad [B] time = 5.25, size = 73, normalized size = 0.83

$$\frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}-\frac{1}{2\,a\,x^{2}\left(b\,x^{2}+a\right)^{3/2}}-\frac{10\,b}{3\,a^{2}\left(b\,x^{2}+a\right)^{3/2}}-\frac{5\,b^{2}\,x^{2}}{2\,a^{3}\left(b\,x^{2}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(5/2)),x)
```

```
[Out] (5*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - 1/(2*a*x^2*(a + b*x^2)
^(3/2)) - (10*b)/(3*a^2*(a + b*x^2)^(3/2)) - (5*b^2*x^2)/(2*a^3*(a + b*x^2)
^(3/2))
```

sympy [B] time = 5.17, size = 864, normalized size = 9.82

$$-\frac{6a^{17}\sqrt{1+\frac{bx^{2}}{a}}}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{46a^{16}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{15a^{16}bx^{2}\log\left(\frac{bx^{2}}{a}\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}+\frac{30a^{16}bx^{2}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{70a^{15}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{45a^{15}b^{2}x^{4}\log\left(\frac{bx^{2}}{a}\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}+\frac{90a^{15}b^{2}x^{4}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{30a^{14}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{45a^{14}b^{3}x^{6}\log\left(\frac{bx^{2}}{a}\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}+\frac{90a^{14}b^{3}x^{6}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}-\frac{15a^{13}b^{4}x^{8}\log\left(\frac{bx^{2}}{a}\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}+\frac{30a^{13}b^{4}x^{8}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{12a^{\frac{39}{2}}x^{2}+36a^{\frac{37}{2}}bx^{4}+36a^{\frac{35}{2}}b^{2}x^{6}+12a^{\frac{33}{2}}b^{3}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(5/2),x)

[Out] -6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 + b*x
**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 +
12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2
+ 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) +
30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/
2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*b**
2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**
(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x**2/
a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a
**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12*a*
*(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*
b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*
a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a*
*14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sqrt(
1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*
b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(12*
a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2
)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)
*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x*
*8)
```

3.507 $\int \frac{1}{x^4 (a+bx^2)^{5/2}} \, dx$

Optimal. Leaf size=86

$$\frac{16b^2x}{3a^4\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2b}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {271, 192, 191}

$$\frac{16b^2x}{3a^4\sqrt{a+bx^2}}+\frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2b}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(3/2)) + (2*b)/(a^2*x*(a + b*x^2)^(3/2)) + (8*b^2*x
)/(3*a^3*(a + b*x^2)^(3/2)) + (16*b^2*x)/(3*a^4*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^{5/2}}\,dx = -\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}} - \frac{(2b)\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx}{a}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}} + \frac{2b}{a^2x\left(a+bx^2\right)^{3/2}} + \frac{\left(8b^2\right)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{a^2}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}} + \frac{2b}{a^2x\left(a+bx^2\right)^{3/2}} + \frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{\left(16b^2\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^3}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{3/2}} + \frac{2b}{a^2x\left(a+bx^2\right)^{3/2}} + \frac{8b^2x}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{16b^2x}{3a^4\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.62

$$\frac{-a^3+6a^2bx^2+24ab^2x^4+16b^3x^6}{3a^4x^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-a^3 + 6*a^2*b*x^2 + 24*a*b^2*x^4 + 16*b^3*x^6)/(3*a^4*x^3*(a + b*x^2)^(3/
2))
```

IntegrateAlgebraic [A] time = 0.09, size = 53, normalized size = 0.62

$$\frac{-a^3+6a^2bx^2+24ab^2x^4+16b^3x^6}{3a^4x^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-a^3 + 6*a^2*b*x^2 + 24*a*b^2*x^4 + 16*b^3*x^6)/(3*a^4*x^3*(a + b*x^2)^(3/
2))
```

fricas [A] time = 1.16, size = 72, normalized size = 0.84

$$\frac{\left(16\,b^3x^6+24\,ab^2x^4+6\,a^2bx^2-a^3\right)\sqrt{bx^2+a}}{3\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(16*b^3*x^6 + 24*a*b^2*x^4 + 6*a^2*b*x^2 - a^3)*sqrt(b*x^2 + a)/(a^4*b^
2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

giac [A] time = 1.26, size = 121, normalized size = 1.41

$$\frac{x\left(\frac{8\,b^3x^2}{a^4}+\frac{9\,b^2}{a^3}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{4\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-9\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+4\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(8*b^3*x^2/a^4 + 9*b^2/a^3)/(b*x^2 + a)^(3/2) - 4/3*(3*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*b^(3/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 4
*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^3)
```

maple [A] time = 0.01, size = 48, normalized size = 0.56

$$-\frac{-16b^3x^6-24a\,b^2x^4-6a^2b\,x^2+a^3}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-16*b^3*x^6-24*a*b^2*x^4-6*a^2*b*x^2+a^3)/x^3/(b*x^2+a)^(3/2)/a^4
```

maxima [A] time = 1.40, size = 72, normalized size = 0.84

$$\frac{16\,b^2x}{3\,\sqrt{bx^2+a}\,a^4}+\frac{8\,b^2x}{3\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{2\,b}{\left(bx^2+a\right)^{\frac{3}{2}}a^2x}-\frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/3*b^2*x/(sqrt(b*x^2 + a)*a^4) + 8/3*b^2*x/((b*x^2 + a)^(3/2)*a^3) + 2*b/
((b*x^2 + a)^(3/2)*a^2*x) - 1/3/((b*x^2 + a)^(3/2)*a*x^3)
```

mupad [B] time = 5.00, size = 78, normalized size = 0.91

$$-\frac{6\,a^2\,\left(b\,x^2+a\right)-24\,a\,{\left(b\,x^2+a\right)}^2+16\,{\left(b\,x^2+a\right)}^3+a^3}{{\left(b\,x^2+a\right)}^{3/2}\,\left(\frac{3\,a^5\,x}{b}-\frac{3\,a^4\,x\,\left(b\,x^2+a\right)}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(5/2)),x)
```

```
[Out] -(6*a^2*(a + b*x^2) - 24*a*(a + b*x^2)^2 + 16*(a + b*x^2)^3 + a^3)/((a + b*
x^2)^(3/2)*((3*a^5*x)/b - (3*a^4*x*(a + b*x^2))/b))
```

sympy [B] time = 1.73, size = 354, normalized size = 4.12

$$-\frac{a^4 b^{\frac{19}{2}} \sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{5a^3 b^{\frac{21}{2}} x^2 \sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{30a^2 b^{\frac{23}{2}} x^4 \sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{40ab^{\frac{25}{2}} x^6 \sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{16b^{\frac{27}{2}} x^8 \sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] -a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4
+ 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(21/2)*x**2*sqrt(a/(b*
x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a*
*4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x
**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 40*a*b**
(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*
a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**8*sqrt(a/(b*x**2) +
1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12
*x**8)
```

3.508 $\int \frac{x^{10}}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{3x^3}{b^4\sqrt{a+bx^2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {288, 321, 217, 206}

$$-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{3x^3}{b^4\sqrt{a+bx^2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2)^(9/2),x]
```

```
[Out] -x^9/(7*b*(a + b*x^2)^(7/2)) - (9*x^7)/(35*b^2*(a + b*x^2)^(5/2)) - (3*x^5)
/(5*b^3*(a + b*x^2)^(3/2)) - (3*x^3)/(b^4*Sqrt[a + b*x^2]) + (9*x*Sqrt[a +
b*x^2])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}+\frac{9\int \frac{x^8}{\left(a+bx^2\right)^{7/2}}\,dx}{7b}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}+\frac{9\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx}{5b^2}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}+\frac{3\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{3x^3}{b^4\sqrt{a+bx^2}}+\frac{9\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{b^4}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{3x^3}{b^4\sqrt{a+bx^2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{(9a}{}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{3x^3}{b^4\sqrt{a+bx^2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{(9a}{}$$

$$= -\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}-\frac{9x^7}{35b^2\left(a+bx^2\right)^{5/2}}-\frac{3x^5}{5b^3\left(a+bx^2\right)^{3/2}}-\frac{3x^3}{b^4\sqrt{a+bx^2}}+\frac{9x\sqrt{a+bx^2}}{2b^5}-\frac{9a}{}$$

Mathematica [A] time = 0.20, size = 114, normalized size = 0.87

$$\frac{\sqrt{b}\,x\left(315a^4+1050a^3bx^2+1218a^2b^2x^4+528ab^3x^6+35b^4x^8\right)-315a^{3/2}\left(a+bx^2\right)^3\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{70b^{11/2}\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2)^(9/2),x]
```

```
[Out] (Sqrt[b]*x*(315*a^4 + 1050*a^3*b*x^2 + 1218*a^2*b^2*x^4 + 528*a*b^3*x^6 + 3
5*b^4*x^8) - 315*a^(3/2)*(a + b*x^2)^3*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]
*x)/Sqrt[a]])/(70*b^(11/2)*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.23, size = 94, normalized size = 0.72

$$\frac{315a^4x + 1050a^3bx^3 + 1218a^2b^2x^5 + 528ab^3x^7 + 35b^4x^9}{70b^5\left(a + bx^2\right)^{7/2}} + \frac{9a\log\left(\sqrt{a + bx^2} - \sqrt{b}\,x\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^10/(a + b*x^2)^(9/2),x]
```

```
[Out] (315*a^4*x + 1050*a^3*b*x^3 + 1218*a^2*b^2*x^5 + 528*a*b^3*x^7 + 35*b^4*x^9
)/(70*b^5*(a + b*x^2)^(7/2)) + (9*a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2
*b^(11/2))
```

fricas [A] time = 2.11, size = 359, normalized size = 2.74

$$\left[\frac{315\left(ab^4x^8 + 4a^2b^3x^6 + 6a^3b^2x^4 + 4a^4bx^2 + a^5\right)\sqrt{b}\log\left(-2bx^2 + 2\sqrt{bx^2 + a}\sqrt{b}\,x - a\right) + 2\left(35b^5x^9 + 528ab^4x^7 + 1218a^2b^3x^5 + 1050a^3b^2x^3 + 315a^4bx\right)\sqrt{bx^2 + a}}{140\left(b^{10}x^8 + 4ab^9x^6 + 6a^2b^8x^4 + 4a^3b^7x^2 + a^4b^6\right)}, \frac{315\left(ab^4x^8 + 4a^2b^3x^6 + 6a^3b^2x^4 + 4a^4bx^2 + a^5\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right) + \left(35b^5x^9 + 528ab^4x^7 + 1218a^2b^3x^5 + 1050a^3b^2x^3 + 315a^4bx\right)\sqrt{bx^2 + a}}{70\left(b^{10}x^8 + 4ab^9x^6 + 6a^2b^8x^4 + 4a^3b^7x^2 + a^4b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(35*b^5*x^9 +
528*a*b^4*x^7 + 1218*a^2*b^3*x^5 + 1050*a^3*b^2*x^3 + 315*a^4*b*x)*sqrt(b*x
^2 + a))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)
, 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (35*b^5*x^9 + 528*a*b^4*x^7
+ 1218*a^2*b^3*x^5 + 1050*a^3*b^2*x^3 + 315*a^4*b*x)*sqrt(b*x^2 + a))/(b^10
*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)]
```

giac [A] time = 1.19, size = 91, normalized size = 0.69

$$\frac{\left(\left(\left(x^2\left(\frac{35\,x^2}{b} + \frac{528\,a}{b^2}\right) + \frac{1218\,a^2}{b^3}\right)x^2 + \frac{1050\,a^3}{b^4}\right)x^2 + \frac{315\,a^4}{b^5}\right)x}{70\left(bx^2 + a\right)^{\frac{7}{2}}} + \frac{9\,a\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2 + a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
+ 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(11/2)
```

maple [A] time = 0.03, size = 111, normalized size = 0.85

$$\frac{x^{9}}{2\left(b\,x^{2}+a\right)^{\frac{7}{2}} b}+\frac{9 a\,x^{7}}{14\left(b\,x^{2}+a\right)^{\frac{7}{2}} b^{2}}+\frac{9 a\,x^{5}}{10\left(b\,x^{2}+a\right)^{\frac{5}{2}} b^{3}}+\frac{3 a\,x^{3}}{2\left(b\,x^{2}+a\right)^{\frac{3}{2}} b^{4}}+\frac{9 a x}{2\sqrt{b\,x^{2}+a}\;b^{5}}-\frac{9 a \ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{2 b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/2*x^9/b/(b*x^2+a)^(7/2)+9/14*a/b^2*x^7/(b*x^2+a)^(7/2)+9/10*a/b^3*x^5/(b*
x^2+a)^(5/2)+3/2*a/b^4*x^3/(b*x^2+a)^(3/2)+9/2*a/b^5*x/(b*x^2+a)^(1/2)-9/2*
a/b^(11/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [B] time = 1.69, size = 285, normalized size = 2.18

$$\frac{x^{9}}{2\left(bx^{2}+a\right)^{\frac{7}{2}} b}+\frac{9\left(\frac{35 x^{6}}{\left(bx^{2}+a\right)^{\frac{7}{2}} b}+\frac{70 a x^{4}}{\left(bx^{2}+a\right)^{\frac{7}{2}} b^{2}}+\frac{56 a^{2} x^{2}}{\left(bx^{2}+a\right)^{\frac{7}{2}} b^{3}}+\frac{16 a^{3}}{\left(bx^{2}+a\right)^{\frac{7}{2}} b^{4}}\right) a x}{70 b}+\frac{3 a x\left(\frac{15 x^{4}}{\left(bx^{2}+a\right)^{\frac{5}{2}} b}+\frac{20 a x^{2}}{\left(bx^{2}+a\right)^{\frac{5}{2}} b^{2}}+\frac{8 a^{2}}{\left(bx^{2}+a\right)^{\frac{5}{2}} b^{3}}\right)}{10 b^{2}}+\frac{3 a x\left(\frac{3 x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}} b}+\frac{2 a}{\left(bx^{2}+a\right)^{\frac{3}{2}} b^{2}}\right)}{2 b^{3}}+\frac{9 a^{2} x^{3}}{2\left(bx^{2}+a\right)^{\frac{5}{2}} b^{4}}-\frac{417 a x}{70 \sqrt{bx^{2}+a}\, b^{5}}-\frac{51 a^{2} x}{70\left(bx^{2}+a\right)^{\frac{3}{2}} b^{5}}+\frac{261 a^{3} x}{70\left(bx^{2}+a\right)^{\frac{5}{2}} b^{5}}-\frac{9 a \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2 b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^9/((b*x^2 + a)^(7/2)*b) + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x
^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((
b*x^2 + a)^(7/2)*b^4))*a*x/b + 3/10*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*
a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 + 3/2*a*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 + 9/2*a^2
*x^3/((b*x^2 + a)^(5/2)*b^4) - 417/70*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*a^2
*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*a^3*x/((b*x^2 + a)^(5/2)*b^5) - 9/2*a*a
rcsinh(b*x/sqrt(a*b))/b^(11/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{\left(b\,x^{2}+a\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^10/(a + b*x^2)^(9/2), x)
```

sympy [B] time = 13.42, size = 3181, normalized size = 24.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**2+a)**(9/2),x)
```

```
[Out] -315*a**(311/2)*b**66*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(3
09/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1
 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a*
*(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x*
*8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a)
+ 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*a**(309/2)*b**6
7*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2
)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) +
1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(14
9/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x
**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)
*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 4725*a**(307/2)*b**68*x**4*sqrt(1 +
 b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x*
*2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)
*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(
1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a*
*(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**
12*sqrt(1 + b*x**2/a)) - 6300*a**(305/2)*b**69*x**6*sqrt(1 + b*x**2/a)*asin
h(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**
(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**
4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) +
 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(15
3/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x
**2/a)) - 4725*a**(303/2)*b**70*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqr
t(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145
/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x*
*2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2
)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt
(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*
a**(301/2)*b**71*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(
309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(
1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a
**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x
**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a)
 + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 315*a**(299/2)*b**7
2*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/
2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) +
 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(1
```

49/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*
x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2
)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 315*a**155*b**(133/2)*x/(70*a**(30
9/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1
+ b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**
(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**
8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) +
70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 1995*a**154*b**(135/2
)*x**3/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(14
5/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x
2/a) + 1400*a(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/
2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqr
t(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 5313
*a**153*b**(137/2)*x**5/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*
a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*
x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a
) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**
(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 +
b*x**2/a)) + 7647*a**152*b**(139/2)*x**7/(70*a**(309/2)*b**(143/2)*sqrt(1 +
b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(3
05/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*
sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 4
20*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2
)*x**12*sqrt(1 + b*x**2/a)) + 6323*a**151*b**(141/2)*x**9/(70*a**(309/2)*b*
*(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**
2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)
*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(
1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**
(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 2907*a**150*b**(143/2)*x**11
/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x
2*sqrt(1 + b*x2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a)
+ 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**
(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 +
b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 633*a**149
*b**(145/2)*x**13/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(30
7/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*s
qrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 10
50*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2
)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2
/a)) + 35*a**148*b**(147/2)*x**15/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2
/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b
(147/2)*x4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1
+ b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(
299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12
*sqrt(1 + b*x**2/a))

3.509 $\int \frac{x^9}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}}+\frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}}-\frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}}+\frac{4a}{b^5\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^5}$$

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}}+\frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}}-\frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}}+\frac{4a}{b^5\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2)^(9/2),x]
```

```
[Out] -a^4/(7*b^5*(a + b*x^2)^(7/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - (2*a^2
)/(b^5*(a + b*x^2)^(3/2)) + (4*a)/(b^5*Sqrt[a + b*x^2]) + Sqrt[a + b*x^2]/b
^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{(a+bx)^{9/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^4}{b^4(a+bx)^{9/2}} - \frac{4a^3}{b^4(a+bx)^{7/2}} + \frac{6a^2}{b^4(a+bx)^{5/2}} - \frac{4a}{b^4(a+bx)^{3/2}} + \frac{1}{b^4\sqrt{a+bx}}\right) d$$
$$= -\frac{a^4}{7b^5\left(a+bx^2\right)^{7/2}} + \frac{4a^3}{5b^5\left(a+bx^2\right)^{5/2}} - \frac{2a^2}{b^5\left(a+bx^2\right)^{3/2}} + \frac{4a}{b^5\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^5}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.65

$$\frac{128a^4 + 448a^3bx^2 + 560a^2b^2x^4 + 280ab^3x^6 + 35b^4x^8}{35b^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2)^(9/2),x]
```

```
[Out] (128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^4*x^8)/(3
5*b^5*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 61, normalized size = 0.65

$$\frac{128a^4 + 448a^3bx^2 + 560a^2b^2x^4 + 280ab^3x^6 + 35b^4x^8}{35b^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9/(a + b*x^2)^(9/2),x]
```

```
[Out] (128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^4*x^8)/(3
5*b^5*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.29, size = 102, normalized size = 1.09

$$\frac{\left(35\,b^4x^8 + 280\,ab^3x^6 + 560\,a^2b^2x^4 + 448\,a^3bx^2 + 128\,a^4\right)\sqrt{bx^2+a}}{35\left(b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a^
4)*sqrt(b*x^2 + a)/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 +
 a^4*b^5)
```

giac [A] time = 1.13, size = 72, normalized size = 0.77

$$\frac{\sqrt{bx^2+a}}{b^5}+\frac{140\left(bx^2+a\right)^3a-70\left(bx^2+a\right)^2a^2+28\left(bx^2+a\right)a^3-5\,a^4}{35\left(bx^2+a\right)^{\frac{7}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b^5 + 1/35*(140*(b*x^2 + a)^3*a - 70*(b*x^2 + a)^2*a^2 + 28
*(b*x^2 + a)*a^3 - 5*a^4)/((b*x^2 + a)^(7/2)*b^5)
```

maple [A] time = 0.01, size = 58, normalized size = 0.62

$$\frac{35b^4x^8+280a\,b^3x^6+560a^2b^2x^4+448a^3b\,x^2+128a^4}{35\left(b\,x^2+a\right)^{\frac{7}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/35*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128*a^4)/(b*x^
2+a)^(7/2)/b^5
```

maxima [A] time = 1.64, size = 92, normalized size = 0.98

$$\frac{x^8}{\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,ax^6}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{16\,a^2x^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{64\,a^3x^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b^4}+\frac{128\,a^4}{35\left(bx^2+a\right)^{\frac{7}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] x^8/((b*x^2 + a)^(7/2)*b) + 8*a*x^6/((b*x^2 + a)^(7/2)*b^2) + 16*a^2*x^4/((
b*x^2 + a)^(7/2)*b^3) + 64/5*a^3*x^2/((b*x^2 + a)^(7/2)*b^4) + 128/35*a^4/(
(b*x^2 + a)^(7/2)*b^5)
```

mupad [B] time = 4.94, size = 80, normalized size = 0.85

$$\frac{\sqrt{b\,x^2+a}}{b^5}+\frac{4\,a}{b^5\,\sqrt{b\,x^2+a}}-\frac{2\,a^2}{b^5\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{4\,a^3}{5\,b^5\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{a^4}{7\,b^5\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2)^(9/2),x)
```

```
[Out] (a + b*x^2)^(1/2)/b^5 + (4*a)/(b^5*(a + b*x^2)^(1/2)) - (2*a^2)/(b^5*(a + b
*x^2)^(3/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - a^4/(7*b^5*(a + b*x^2)^(
7/2))
```

sympy [A] time = 6.39, size = 454, normalized size = 4.83

$$\begin{cases} \frac{128a^4}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} + \frac{448a^3bx^2}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} + \frac{560a^2b^2x^4}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} + \frac{280ab^3x^6}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} + \frac{35b^4x^8}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^{10}}{10a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((128*a**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sqr
t(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a + b*
x**2)) + 448*a**3*b*x**2/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**
2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a
 + b*x**2)) + 560*a**2*b**2*x**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*
b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**
6*sqrt(a + b*x**2)) + 280*a*b**3*x**6/(35*a**3*b**5*sqrt(a + b*x**2) + 105*
a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**
8*x**6*sqrt(a + b*x**2)) + 35*b**4*x**8/(35*a**3*b**5*sqrt(a + b*x**2) + 10
5*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b
**8*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**10/(10*a**(9/2)), True))
```

3.510 $\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=106

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{x}{b^4\sqrt{a+bx^2}}-\frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}}-\frac{x^7}{7b\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {288, 217, 206}

$$-\frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}}-\frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{x}{b^4\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{x^7}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2)^(9/2),x]
```

```
[Out] -x^7/(7*b*(a + b*x^2)^(7/2)) - x^5/(5*b^2*(a + b*x^2)^(5/2)) - x^3/(3*b^3*(
a + b*x^2)^(3/2)) - x/(b^4*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]]/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^2\right)^{9/2}}\,dx = -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{x^6}{\left(a+bx^2\right)^{7/2}}\,dx}{b}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^4}{\left(a+bx^2\right)^{5/2}}\,dx}{b^2}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^4}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x\right.}{b^4}$$

$$= -\frac{x^7}{7b\left(a+bx^2\right)^{7/2}} - \frac{x^5}{5b^2\left(a+bx^2\right)^{5/2}} - \frac{x^3}{3b^3\left(a+bx^2\right)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Mathematica [A] time = 0.13, size = 101, normalized size = 0.95

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}\sqrt{a+bx^2}} - \frac{x\left(105a^3+350a^2bx^2+406ab^2x^4+176b^3x^6\right)}{105b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/105*(x*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4 + 176*b^3*x^6))/(b^4*(a
+ b*x^2)^(7/2)) + (Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]
)/(b^(9/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.20, size = 80, normalized size = 0.75

$$\frac{-105a^3x-350a^2bx^3-406ab^2x^5-176b^3x^7}{105b^4\left(a+bx^2\right)^{7/2}}-\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/(a + b*x^2)^(9/2),x]
```

```
[Out] (-105*a^3*x - 350*a^2*b*x^3 - 406*a*b^2*x^5 - 176*b^3*x^7)/(105*b^4*(a + b*
x^2)^(7/2)) - Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]/b^(9/2)
```

fricas [A] time = 2.09, size = 331, normalized size = 3.12

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(176\,b^4x^7+406\,ab^3x^5+350\,a^2b^2x^3+105\,a^3bx\right)\sqrt{bx^2+a}}{210\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)},-\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(176\,b^4x^7+406\,ab^3x^5+350\,a^2b^2x^3+105\,a^3bx\right)\sqrt{bx^2+a}}{105\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(176*b^4*x^7 + 406
*a*b^3*x^5 + 350*a^2*b^2*x^3 + 105*a^3*b*x)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a
*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(b^4*x^8 +
 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)) + (176*b^4*x^7 + 406*a*b^3*x^5 + 350*a^2*b^2*x^3 + 105*a
^3*b*x)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6
*x^2 + a^4*b^5)]
```

giac [A] time = 1.20, size = 78, normalized size = 0.74

$$-\frac{\left(2\left(x^2\left(\frac{88\,x^2}{b}+\frac{203\,a}{b^2}\right)+\frac{175\,a^2}{b^3}\right)x^2+\frac{105\,a^3}{b^4}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/(
b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple [A] time = 0.02, size = 88, normalized size = 0.83

$$-\frac{x^7}{7\left(b\,x^2+a\right)^{\frac{7}{2}}b}-\frac{x^5}{5\left(b\,x^2+a\right)^{\frac{5}{2}}b^2}-\frac{x^3}{3\left(b\,x^2+a\right)^{\frac{3}{2}}b^3}-\frac{x}{\sqrt{b\,x^2+a}\;b^4}+\frac{\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7*x^7/b/(b*x^2+a)^(7/2)-1/5*x^5/b^2/(b*x^2+a)^(5/2)-1/3*x^3/b^3/(b*x^2+a
)^(3/2)-x/b^4/(b*x^2+a)^(1/2)+1/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [B] time = 1.67, size = 255, normalized size = 2.41

$$-\frac{1}{35}\left(\frac{35\,x^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{70\,ax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{\left(bx^2+a\right)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{\left(bx^2+a\right)^{\frac{7}{2}}b^4}\right)x-\frac{x\left(\frac{15\,x^4}{\left(bx^2+a\right)^{\frac{5}{2}}b}+\frac{20\,ax^2}{\left(bx^2+a\right)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{\left(bx^2+a\right)^{\frac{5}{2}}b^3}\right)}{15\,b}-\frac{x\left(\frac{3\,x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)}{3\,b^2}-\frac{ax^3}{\left(bx^2+a\right)^{\frac{5}{2}}b^3}+\frac{139\,x}{105\,\sqrt{bx^2+a}\,b^4}+\frac{17\,ax}{105\left(bx^2+a\right)^{\frac{3}{2}}b^4}-\frac{29\,a^2x}{35\left(bx^2+a\right)^{\frac{5}{2}}b^4}+\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*x - 1/15
*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2
/((b*x^2 + a)^(5/2)*b^3))/b - 1/3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*
x^2 + a)^(3/2)*b^2))/b^2 - a*x^3/((b*x^2 + a)^(5/2)*b^3) + 139/105*x/(sqrt(
b*x^2 + a)*b^4) + 17/105*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*a^2*x/((b*x^2
+ a)^(5/2)*b^4) + arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(b\,x^2+a\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2)^(9/2),x)
```

```
[Out] int(x^8/(a + b*x^2)^(9/2), x)
```

sympy [B] time = 9.04, size = 2980, normalized size = 28.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**2+a)**(9/2),x)
```

```
[Out] 105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(2
05/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1
+ b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**
(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**
8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) +
```

```
105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b**46
*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)
*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1
575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105
/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x*
*2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)
*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x**4*sqrt(1 +
 b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x*
*2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)
*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(
1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a*
*(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x*
*12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(1 + b*x**2/a)*asi
nh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a*
*(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x*
*4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a)
+ 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(1
09/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b
*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/s
qrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(1
01/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*
x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197
/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sq
rt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 63
0*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a
**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqr
t(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100
*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)
*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/
a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 105*a**(193/2)*b
**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(
99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a
) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b*
*(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 +
 b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(1
93/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b**(91/2)*x/(105*a*
*(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt
(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*
a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*
x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a
) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 665*a**101*b**(93
/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(
101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b
*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(19
7/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*s
```

```
qrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 1
771*a**100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2
)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2
/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b
**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
 + b*x**2/a)) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2)*b**(99/2)*sqrt(1
+ b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(
201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6
*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) +
630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111
/2)*x**12*sqrt(1 + b*x**2/a)) - 2096*a**98*b**(99/2)*x**9/(105*a**(205/2)*b
**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**
2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)
*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(
1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a*
*(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 934*a**97*b**(101/2)*x**11/
(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x*
*2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a)
+ 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(
107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b
*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 176*a**96*
b**(103/2)*x**13/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203
/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sq
rt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 157
5*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)
*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2
/a))
```

3.511 $\int \frac{x^7}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=75

$$\frac{a^3}{7b^4\left(a+bx^2\right)^{7/2}}-\frac{3a^2}{5b^4\left(a+bx^2\right)^{5/2}}+\frac{a}{b^4\left(a+bx^2\right)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}$$

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a^3}{7b^4\left(a+bx^2\right)^{7/2}}-\frac{3a^2}{5b^4\left(a+bx^2\right)^{5/2}}+\frac{a}{b^4\left(a+bx^2\right)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(9/2),x]
```

```
[Out] a^3/(7*b^4*(a + b*x^2)^(7/2)) - (3*a^2)/(5*b^4*(a + b*x^2)^(5/2)) + a/(b^4*
(a + b*x^2)^(3/2)) - 1/(b^4*Sqrt[a + b*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{9/2}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{9/2}}+\frac{3a^2}{b^3(a+bx)^{7/2}}-\frac{3a}{b^3(a+bx)^{5/2}}+\frac{1}{b^3(a+bx)^{3/2}}\right)dx,x,x^2\right)$$
$$= \frac{a^3}{7b^4\left(a+bx^2\right)^{7/2}}-\frac{3a^2}{5b^4\left(a+bx^2\right)^{5/2}}+\frac{a}{b^4\left(a+bx^2\right)^{3/2}}-\frac{1}{b^4\sqrt{a+bx^2}}$$

Mathematica [**A**] time = 0.02, size = 50, normalized size = 0.67

$$\frac{-16a^3-56a^2bx^2-70ab^2x^4-35b^3x^6}{35b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(9/2),x]
```

```
[Out] (-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6)/(35*b^4*(a + b*x^2)^(7
/2))
```

IntegrateAlgebraic [**A**] time = 0.03, size = 50, normalized size = 0.67

$$\frac{-16a^3-56a^2bx^2-70ab^2x^4-35b^3x^6}{35b^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2)^(9/2),x]
```

```
[Out] (-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6)/(35*b^4*(a + b*x^2)^(7
/2))
```

fricas [**A**] time = 0.90, size = 91, normalized size = 1.21

$$-\frac{\left(35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3\right)\sqrt{bx^2+a}}{35\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^2 + a)/(
b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

giac [A] time = 0.98, size = 55, normalized size = 0.73

$$-\frac{35\left(bx^2+a\right)^3-35\left(bx^2+a\right)^2a+21\left(bx^2+a\right)a^2-5\,a^3}{35\left(bx^2+a\right)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 - 5*a^3)/
((b*x^2 + a)^(7/2)*b^4)
```

maple [A] time = 0.01, size = 47, normalized size = 0.63

$$-\frac{35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3}{35\left(b\,x^2+a\right)^{\frac{7}{2}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/35*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)/(b*x^2+a)^(7/2)/b^4
```

maxima [A] time = 1.36, size = 73, normalized size = 0.97

$$-\frac{x^6}{\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{2\,ax^4}{\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,a^2x^2}{5\left(bx^2+a\right)^{\frac{7}{2}}b^3}-\frac{16\,a^3}{35\left(bx^2+a\right)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -x^6/((b*x^2 + a)^(7/2)*b) - 2*a*x^4/((b*x^2 + a)^(7/2)*b^2) - 8/5*a^2*x^2/
((b*x^2 + a)^(7/2)*b^3) - 16/35*a^3/((b*x^2 + a)^(7/2)*b^4)
```

mupad [B] time = 4.88, size = 63, normalized size = 0.84

$$\frac{a}{b^4\left(b\,x^2+a\right)^{3/2}}-\frac{1}{b^4\,\sqrt{b\,x^2+a}}-\frac{3\,a^2}{5\,b^4\left(b\,x^2+a\right)^{5/2}}+\frac{a^3}{7\,b^4\left(b\,x^2+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(9/2),x)
```

```
[Out] a/(b^4*(a + b*x^2)^(3/2)) - 1/(b^4*(a + b*x^2)^(1/2)) - (3*a^2)/(5*b^4*(a +
 b*x^2)^(5/2)) + a^3/(7*b^4*(a + b*x^2)^(7/2))
```

sympy [A] time = 6.08, size = 364, normalized size = 4.85

$$\begin{cases} -\frac{16a^3}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} - \frac{56a^2bx^2}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} - \frac{70ab^2x^4}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} - \frac{35b^3x^6}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^8}{8a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sqr
t(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a + b*
x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2
*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a
+ b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*
x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqr
t(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**
5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*s
qrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True))
```

3.512 $\int \frac{x^6}{(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=21

$$\frac{x^7}{7a(a+bx^2)^{7/2}}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{x^7}{7a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2)^(9/2),x]
```

```
[Out] x^7/(7*a*(a + b*x^2)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^2)^{9/2}} dx = \frac{x^7}{7a(a+bx^2)^{7/2}}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{x^7}{7a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2)^(9/2),x]
```

```
[Out] x^7/(7*a*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 21, normalized size = 1.00

$$\frac{x^7}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(a + b*x^2)^(9/2),x]
```

```
[Out] x^7/(7*a*(a + b*x^2)^(7/2))
```

fricas [B] time = 1.52, size = 59, normalized size = 2.81

$$\frac{\sqrt{bx^2+a}\,x^7}{7\left(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*x^7/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*
b*x^2 + a^5)
```

giac [A] time = 1.22, size = 17, normalized size = 0.81

$$\frac{x^7}{7\left(bx^2+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{x^7}{7\left(b\,x^2+a\right)^{\frac{7}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7*x^7/a/(b*x^2+a)^(7/2)
```

maxima [B] time = 1.33, size = 103, normalized size = 4.90

$$-\frac{x^5}{2\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{5\,ax^3}{8\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{x}{14\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{x}{7\sqrt{bx^2+a}\,ab^3}+\frac{3\,ax}{56\left(bx^2+a\right)^{\frac{5}{2}}b^3}-\frac{15\,a^2x}{56\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 1/14*x
/((b*x^2 + a)^(3/2)*b^3) + 1/7*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*a*x/((b*x^2
 + a)^(5/2)*b^3) - 15/56*a^2*x/((b*x^2 + a)^(7/2)*b^3)
```

mupad [B] time = 4.76, size = 68, normalized size = 3.24

$$\frac{x}{7\,a\,b^3\,\sqrt{b\,x^2+a}}-\frac{3\,x}{7\,b^3\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{a^2\,x}{7\,b^3\,{\left(b\,x^2+a\right)}^{7/2}}+\frac{3\,a\,x}{7\,b^3\,{\left(b\,x^2+a\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2)^(9/2),x)
```

```
[Out] x/(7*a*b^3*(a + b*x^2)^(1/2)) - (3*x)/(7*b^3*(a + b*x^2)^(3/2)) - (a^2*x)/(
7*b^3*(a + b*x^2)^(7/2)) + (3*a*x)/(7*b^3*(a + b*x^2)^(5/2))
```

sympy [B] time = 1.47, size = 95, normalized size = 4.52

$$\frac{x^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 +
 b*x**2/a))
```

3.513 $\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{2a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{1}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(9/2),x]
```

```
[Out] -a^2/(7*b^3*(a + b*x^2)^(7/2)) + (2*a)/(5*b^3*(a + b*x^2)^(5/2)) - 1/(3*b^3
*(a + b*x^2)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^{9/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{9/2}} - \frac{2a}{b^2(a+bx)^{7/2}} + \frac{1}{b^2(a+bx)^{5/2}}\right) dx, x, x^2\right)$$
$$= -\frac{a^2}{7b^3\left(a+bx^2\right)^{7/2}} + \frac{2a}{5b^3\left(a+bx^2\right)^{5/2}} - \frac{1}{3b^3\left(a+bx^2\right)^{3/2}}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{-8a^2-28abx^2-35b^2x^4}{105b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(9/2),x]
[Out] (-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4)/(105*b^3*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{-8a^2-28abx^2-35b^2x^4}{105b^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2)^(9/2),x]
[Out] (-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4)/(105*b^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.75, size = 80, normalized size = 1.36

$$-\frac{\left(35\,b^2x^4+28\,abx^2+8\,a^2\right)\sqrt{bx^2+a}}{105\left(b^7x^8+4\,ab^6x^6+6\,a^2b^5x^4+4\,a^3b^4x^2+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="fricas")
[Out] -1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/(b^7*x^8 + 4*a*b^6
*x^6 + 6*a^2*b^5*x^4 + 4*a^3*b^4*x^2 + a^4*b^3)
```

giac [A] time = 0.95, size = 41, normalized size = 0.69

$$-\frac{35\left(bx^2+a\right)^2-42\left(bx^2+a\right)a+15\,a^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/((b*x^2 + a)^(7/2)*b^
3)
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$-\frac{35b^2x^4+28ab\,x^2+8a^2}{105\left(b\,x^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/105*(35*b^2*x^4+28*a*b*x^2+8*a^2)/(b*x^2+a)^(7/2)/b^3
```

maxima [A] time = 1.41, size = 53, normalized size = 0.90

$$-\frac{x^4}{3\left(bx^2+a\right)^{\frac{7}{2}}b}-\frac{4\,ax^2}{15\left(bx^2+a\right)^{\frac{7}{2}}b^2}-\frac{8\,a^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*x^4/((b*x^2 + a)^(7/2)*b) - 4/15*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 8/105
*a^2/((b*x^2 + a)^(7/2)*b^3)
```

mupad [B] time = 4.81, size = 41, normalized size = 0.69

$$-\frac{35\left(b\,x^2+a\right)^2-42\,a\,\left(b\,x^2+a\right)+15\,a^2}{105\,b^3\left(b\,x^2+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(9/2),x)
```

```
[Out] -(35*(a + b*x^2)^2 - 42*a*(a + b*x^2) + 15*a^2)/(105*b^3*(a + b*x^2)^(7/2))
```

sympy [A]   time = 5.91, size = 272, normalized size = 4.61

$$\begin{cases} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{35b^2x^4}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqr
t(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b
*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*
sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a
+ b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x
**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqr
t(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True))
```

3.514 $\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=44

$$\frac{2bx^7}{35a^2(a+bx^2)^{7/2}}+\frac{x^5}{5a(a+bx^2)^{7/2}}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{2bx^7}{35a^2(a+bx^2)^{7/2}}+\frac{x^5}{5a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2)^(9/2),x]
```

```
[Out] x^5/(5*a*(a + b*x^2)^(7/2)) + (2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx &= \frac{x^5}{5a(a+bx^2)^{7/2}}+\frac{(2b)\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx}{5a}\\ &= \frac{x^5}{5a(a+bx^2)^{7/2}}+\frac{2bx^7}{35a^2(a+bx^2)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{7ax^5 + 2bx^7}{35a^2 (a + bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2)^(9/2),x]
[Out] (7*a*x^5 + 2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 31, normalized size = 0.70

$$\frac{x^5 (7a + 2bx^2)}{35a^2 (a + bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a + b*x^2)^(9/2),x]
[Out] (x^5*(7*a + 2*b*x^2))/(35*a^2*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.83, size = 71, normalized size = 1.61

$$\frac{(2\,bx^7 + 7\,ax^5)\sqrt{bx^2 + a}}{35\,(a^2b^4x^8 + 4\,a^3b^3x^6 + 6\,a^4b^2x^4 + 4\,a^5bx^2 + a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
[Out] 1/35*(2*b*x^7 + 7*a*x^5)*sqrt(b*x^2 + a)/(a^2*b^4*x^8 + 4*a^3*b^3*x^6 + 6*a
^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

giac [A] time = 0.99, size = 29, normalized size = 0.66

$$\frac{x^5\left(\frac{2\,bx^2}{a^2} + \frac{7}{a}\right)}{35\,(bx^2 + a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$\frac{\left(2b\,x^{2}+7a\right)x^{5}}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/35*x^5*(2*b*x^2+7*a)/(b*x^2+a)^(7/2)/a^2
```

maxima [B] time = 1.38, size = 85, normalized size = 1.93

$$-\frac{x^{3}}{4\left(bx^{2}+a\right)^{\frac{7}{2}}b}+\frac{3\,x}{140\left(bx^{2}+a\right)^{\frac{5}{2}}b^{2}}+\frac{2\,x}{35\sqrt{bx^{2}+a}\,a^{2}b^{2}}+\frac{x}{35\left(bx^{2}+a\right)^{\frac{3}{2}}ab^{2}}-\frac{3\,ax}{28\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/((b*x^2 + a)^(7/2)*b) + 3/140*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*x/(
sqrt(b*x^2 + a)*a^2*b^2) + 1/35*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*a*x/((b*
x^2 + a)^(7/2)*b^2)
```

mupad [B] time = 4.88, size = 68, normalized size = 1.55

$$\frac{2\,x}{35\,a^{2}\,b^{2}\,\sqrt{b\,x^{2}+a}}-\frac{8\,x}{35\,b^{2}\,\left(b\,x^{2}+a\right)^{5/2}}+\frac{x}{35\,a\,b^{2}\,\left(b\,x^{2}+a\right)^{3/2}}+\frac{a\,x}{7\,b^{2}\,\left(b\,x^{2}+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(9/2),x)
```

```
[Out] (2*x)/(35*a^2*b^2*(a + b*x^2)^(1/2)) - (8*x)/(35*b^2*(a + b*x^2)^(5/2)) + x
/(35*a*b^2*(a + b*x^2)^(3/2)) + (a*x)/(7*b^2*(a + b*x^2)^(7/2))
```

sympy [B] time = 1.60, size = 199, normalized size = 4.52

$$\frac{7ax^{5}}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{9}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{7}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}+35a^{\frac{5}{2}}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{2bx^{7}}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{9}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{7}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}+35a^{\frac{5}{2}}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] 7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*
x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(
9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a)
+ 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))
```

3.515 $\int \frac{x^3}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=38

$$\frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(9/2),x]
```

```
[Out] a/(7*b^2*(a + b*x^2)^(7/2)) - 1/(5*b^2*(a + b*x^2)^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^{9/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{9/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{9/2}}+\frac{1}{b(a+bx)^{7/2}}\right)dx,x,x^2\right)\\ &= \frac{a}{7b^2\left(a+bx^2\right)^{7/2}}-\frac{1}{5b^2\left(a+bx^2\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{-2a-7bx^2}{35b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(9/2),x]
[Out] (-2*a - 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 28, normalized size = 0.74

$$\frac{-2a-7bx^2}{35b^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2)^(9/2),x]
[Out] (-2*a - 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))
```

fricas [B] time = 0.94, size = 69, normalized size = 1.82

$$-\frac{\left(7\,bx^2+2\,a\right)\sqrt{bx^2+a}}{35\left(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
[Out] -1/35*(7*b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^
4 + 4*a^3*b^3*x^2 + a^4*b^2)
```

giac [A] time = 0.83, size = 24, normalized size = 0.63

$$-\frac{7\,bx^2+2\,a}{35\left(bx^2+a\right)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
[Out] -1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2)
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{7b\,x^{2}+2a}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(9/2),x)
[Out] -1/35*(7*b*x^2+2*a)/(b*x^2+a)^(7/2)/b^2
```

maxima [A] time = 1.30, size = 33, normalized size = 0.87

$$-\frac{x^{2}}{5\left(bx^{2}+a\right)^{\frac{7}{2}} b}-\frac{2\,a}{35\left(bx^{2}+a\right)^{\frac{7}{2}} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
[Out] -1/5*x^2/((b*x^2 + a)^(7/2)*b) - 2/35*a/((b*x^2 + a)^(7/2)*b^2)
```

mupad [B] time = 4.82, size = 24, normalized size = 0.63

$$-\frac{7\,b\,x^{2}+2\,a}{35\,b^{2}\left(b\,x^{2}+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(9/2),x)
[Out] -(2*a + 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))
```

sympy [A] time = 5.86, size = 180, normalized size = 4.74

$$\begin{cases} -\frac{2a}{35a^{3}b^{2}\sqrt{a+bx^{2}}+105a^{2}b^{3}x^{2}\sqrt{a+bx^{2}}+105ab^{4}x^{4}\sqrt{a+bx^{2}}+35b^{5}x^{6}\sqrt{a+bx^{2}}}-\frac{7bx^{2}}{35a^{3}b^{2}\sqrt{a+bx^{2}}+105a^{2}b^{3}x^{2}\sqrt{a+bx^{2}}+105ab^{4}x^{4}\sqrt{a+bx^{2}}+35b^{5}x^{6}\sqrt{a+bx^{2}}} & \text{for}\ b\neq 0\\ \frac{x^{4}}{4a^{\frac{9}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(9/2),x)
[Out] Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a
+ b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2
)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a +
b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2))
, Ne(b, 0)), (x**4/(4*a**(9/2)), True))
```

3.516 $\int \frac{x^2}{(a+bx^2)^{9/2}} \, dx$

Optimal. Leaf size=68

$$\frac{8b^2x^7}{105a^3\left(a+bx^2\right)^{7/2}}+\frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}}+\frac{x^3}{3a\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{8b^2x^7}{105a^3\left(a+bx^2\right)^{7/2}}+\frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}}+\frac{x^3}{3a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2)^(9/2),x]
```

```
[Out] x^3/(3*a*(a + b*x^2)^(7/2)) + (4*b*x^5)/(15*a^2*(a + b*x^2)^(7/2)) + (8*b^2
*x^7)/(105*a^3*(a + b*x^2)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{x^3}{3a\left(a+bx^2\right)^{7/2}} + \frac{(4b)\int \frac{x^4}{\left(a+bx^2\right)^{9/2}}\,dx}{3a}$$
$$= \frac{x^3}{3a\left(a+bx^2\right)^{7/2}} + \frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(8b^2\right)\int \frac{x^6}{\left(a+bx^2\right)^{9/2}}\,dx}{15a^2}$$
$$= \frac{x^3}{3a\left(a+bx^2\right)^{7/2}} + \frac{4bx^5}{15a^2\left(a+bx^2\right)^{7/2}} + \frac{8b^2x^7}{105a^3\left(a+bx^2\right)^{7/2}}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$\frac{x^3\left(35a^2+28abx^2+8b^2x^4\right)}{105a^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2)^(9/2),x]
[Out] (x^3*(35*a^2 + 28*a*b*x^2 + 8*b^2*x^4))/(105*a^3*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 42, normalized size = 0.62

$$\frac{x^3\left(35a^2+28abx^2+8b^2x^4\right)}{105a^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x^2)^(9/2),x]
[Out] (x^3*(35*a^2 + 28*a*b*x^2 + 8*b^2*x^4))/(105*a^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 0.78, size = 82, normalized size = 1.21

$$\frac{\left(8\,b^2x^7+28\,abx^5+35\,a^2x^3\right)\sqrt{bx^2+a}}{105\left(a^3b^4x^8+4\,a^4b^3x^6+6\,a^5b^2x^4+4\,a^6bx^2+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*b^2*x^7 + 28*a*b*x^5 + 35*a^2*x^3)*sqrt(b*x^2 + a)/(a^3*b^4*x^8 +
4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

giac [A] time = 1.08, size = 43, normalized size = 0.63

$$\frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{7\,b}{a^2}\right)+\frac{35}{a}\right)x^3}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$\frac{\left(8b^2x^4+28ab\,x^2+35a^2\right)x^3}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/105*x^3*(8*b^2*x^4+28*a*b*x^2+35*a^2)/(b*x^2+a)^(7/2)/a^3
```

maxima [A] time = 1.39, size = 70, normalized size = 1.03

$$-\frac{x}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{8\,x}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,x}{105\left(bx^2+a\right)^{\frac{3}{2}}a^2b}+\frac{x}{35\left(bx^2+a\right)^{\frac{5}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*x/((b*x^2 + a)^(7/2)*b) + 8/105*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*x/((
b*x^2 + a)^(3/2)*a^2*b) + 1/35*x/((b*x^2 + a)^(5/2)*a*b)
```

mupad [B] time = 4.76, size = 70, normalized size = 1.03

$$\frac{8\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}}-\frac{x}{7\,b\left(b\,x^2+a\right)^{7/2}}+\frac{4\,x}{105\,a^2\,b\left(b\,x^2+a\right)^{3/2}}+\frac{x}{35\,a\,b\left(b\,x^2+a\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(9/2),x)
```

```
[Out] (8*x)/(105*a^3*b*(a + b*x^2)^(1/2)) - x/(7*b*(a + b*x^2)^(7/2)) + (4*x)/(10
5*a^2*b*(a + b*x^2)^(3/2)) + x/(35*a*b*(a + b*x^2)^(5/2))
```

sympy [B] time = 1.87, size = 517, normalized size = 7.60

$$\frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}+\frac{63a^4bx^5}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}+\frac{36a^3b^2x^7}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}+\frac{8a^2b^3x^9}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] 35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(
1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))
+ 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*s
qrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13
/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/
a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b
*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420
*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 +
b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(1
7/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a)
 + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqr
t(1 + b*x**2/a))
```

3.517 $\int \frac{x}{\left(a+bx^2\right)^{9/2}} \, dx$

Optimal. Leaf size=18

$$-\frac{1}{7b\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/(7*b*(a + b*x^2)^(7/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{9/2}} \, dx = -\frac{1}{7b\left(a+bx^2\right)^{7/2}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*1/(b*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{7b\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*1/(b*(a + b*x^2)^(7/2))
```

fricas [B] time = 0.94, size = 57, normalized size = 3.17

$$-\frac{\sqrt{bx^2+a}}{7\left(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/7*sqrt(b*x^2 + a)/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2
 + a^4*b)
```

giac [A] time = 0.97, size = 14, normalized size = 0.78

$$-\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/7/((b*x^2 + a)^(7/2)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{1}{7\left(b\,x^2+a\right)^{\frac{7}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/7/b/(b*x^2+a)^(7/2)
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$-\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7/((b*x^2 + a)^(7/2)*b)
```

mupad [B] time = 4.60, size = 14, normalized size = 0.78

$$-\frac{1}{7\,b\,\left(b\,x^2+a\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(9/2),x)
```

```
[Out] -1/(7*b*(a + b*x^2)^(7/2))
```

sympy [A] time = 5.73, size = 90, normalized size = 5.00

$$\begin{cases} -\frac{1}{7a^3b\sqrt{a+bx^2}+21a^2b^2x^2\sqrt{a+bx^2}+21ab^3x^4\sqrt{a+bx^2}+7b^4x^6\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(9/2),x)
```

```
[Out] Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x**
2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(b,
 0)), (x**2/(2*a**(9/2)), True))
```

3.518 $\int \frac{1}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=77

$$\frac{16x}{35a^4\sqrt{a+bx^2}}+\frac{8x}{35a^3\left(a+bx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{16x}{35a^4\sqrt{a+bx^2}}+\frac{8x}{35a^3\left(a+bx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-9/2),x]
```

```
[Out] x/(7*a*(a + b*x^2)^(7/2)) + (6*x)/(35*a^2*(a + b*x^2)^(5/2)) + (8*x)/(35*a^
3*(a + b*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{x}{7a\left(a+bx^2\right)^{7/2}} + \frac{6\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= \frac{x}{7a\left(a+bx^2\right)^{7/2}} + \frac{6x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{24\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2}$$

$$= \frac{x}{7a\left(a+bx^2\right)^{7/2}} + \frac{6x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{8x}{35a^3\left(a+bx^2\right)^{3/2}} + \frac{16\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{35a^3}$$

$$= \frac{x}{7a\left(a+bx^2\right)^{7/2}} + \frac{6x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{8x}{35a^3\left(a+bx^2\right)^{3/2}} + \frac{16x}{35a^4\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.66

$$\frac{x\left(35a^3+70a^2bx^2+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-9/2),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(a + b*x^2)
^(7/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 51, normalized size = 0.66

$$\frac{x\left(35a^3+70a^2bx^2+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-9/2),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(a + b*x^2)
^(7/2))
```

fricas [A] time = 1.08, size = 91, normalized size = 1.18

$$\frac{\left(16\,b^3x^7+56\,ab^2x^5+70\,a^2bx^3+35\,a^3x\right)\sqrt{bx^2+a}}{35\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*sqrt(b*x^2 + a)/
(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

giac [A] time = 1.12, size = 55, normalized size = 0.71

$$\frac{\left(2\left(4x^2\left(\frac{2b^3x^2}{a^4}+\frac{7b^2}{a^3}\right)+\frac{35b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^2
 + a)^(7/2)
```

maple [A] time = 0.00, size = 48, normalized size = 0.62

$$\frac{\left(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3\right)x}{35\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/35*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/(b*x^2+a)^(7/2)/a^4
```

maxima [A] time = 1.37, size = 61, normalized size = 0.79

$$\frac{16x}{35\sqrt{bx^2+a}\,a^4}+\frac{8x}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3}+\frac{6x}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2}+\frac{x}{7\left(bx^2+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*x/(sqrt(b*x^2 + a)*a^4) + 8/35*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*x/((b
*x^2 + a)^(5/2)*a^2) + 1/7*x/((b*x^2 + a)^(7/2)*a)
```

mupad [B] time = 4.60, size = 61, normalized size = 0.79

$$\frac{16\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{6\,x}{35\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{x}{7\,a\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(9/2),x)
```

```
[Out] (16*x)/(35*a^4*(a + b*x^2)^(1/2)) + (8*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*x
)/(35*a^2*(a + b*x^2)^(5/2)) + x/(7*a*(a + b*x^2)^(7/2))
```

sympy [B] time = 2.19, size = 1265, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(9/2),x)
```

```
[Out] 35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 +
 b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**
3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 21
0*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1
+ b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*
x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**
2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(
27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x*
*2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/
2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(
1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*
b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 +
 b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x
**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a
**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b
*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13
/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqr
t(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2
)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a
))
```

3.519 $\int \frac{1}{x(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{1}{a^4\sqrt{a+bx^2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}+\frac{1}{7a\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$\frac{1}{a^4\sqrt{a+bx^2}}+\frac{1}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{1}{5a^2\left(a+bx^2\right)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{1}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] 1/(7*a*(a + b*x^2)^(7/2)) + 1/(5*a^2*(a + b*x^2)^(5/2)) + 1/(3*a^3*(a + b*x
^2)^(3/2)) + 1/(a^4*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(
9/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{9/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{7/2}}\,dx, x, x^2\right)}{2a}$$

$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{1}{5a^2\left(a+bx^2\right)^{5/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, x^2\right)}{2a^2}$$

$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{1}{5a^2\left(a+bx^2\right)^{5/2}} + \frac{1}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)}{2a^3}$$

$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{1}{5a^2\left(a+bx^2\right)^{5/2}} + \frac{1}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{1}{a^4\sqrt{a+bx^2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{2a^4}$$

$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{1}{5a^2\left(a+bx^2\right)^{5/2}} + \frac{1}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{1}{a^4\sqrt{a+bx^2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,d\right.}{a^4}$$

$$= \frac{1}{7a\left(a+bx^2\right)^{7/2}} + \frac{1}{5a^2\left(a+bx^2\right)^{5/2}} + \frac{1}{3a^3\left(a+bx^2\right)^{3/2}} + \frac{1}{a^4\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.38

$$\frac{{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] Hypergeometric2F1[-7/2, 1, -5/2, 1 + (b*x^2)/a]/(7*a*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 76, normalized size = 0.80

$$\frac{176a^3+406a^2bx^2+350ab^2x^4+105b^3x^6}{105a^4\left(a+bx^2\right)^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (176*a^3 + 406*a^2*b*x^2 + 350*a*b^2*x^4 + 105*b^3*x^6)/(105*a^4*(a + b*x^2
)^(7/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(9/2)
```

fricas [B] time = 1.01, size = 329, normalized size = 3.46

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(105\,ab^3x^6+350\,a^2b^2x^4+406\,a^3bx^2+176\,a^4\right)\sqrt{bx^2+a}}{210\left(a^5b^4x^8+4\,a^6b^3x^6+6\,a^7b^2x^4+4\,a^8bx^2+a^9\right)},\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(105\,ab^3x^6+350\,a^2b^2x^4+406\,a^3bx^2+176\,a^4\right)\sqrt{bx^2+a}}{105\left(a^5b^4x^8+4\,a^6b^3x^6+6\,a^7b^2x^4+4\,a^8bx^2+a^9\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(105*a*b^3*x^6
 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^2 + a))/(a^5*b^4*x^8
 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*b*x^2 + a^9), 1/105*(105*(b^4*x^8
+ 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a)*arctan(sqrt(-a)
/sqrt(b*x^2 + a)) + (105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*
a^4)*sqrt(b*x^2 + a))/(a^5*b^4*x^8 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*
b*x^2 + a^9)]
```

giac [A] time = 1.08, size = 81, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}+\frac{105\left(bx^2+a\right)^3+35\left(bx^2+a\right)^2a+21\left(bx^2+a\right)a^2+15\,a^3}{105\left(bx^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + 1/105*(105*(b*x^2 + a)^3
+ 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2)*a^4)
```

maple [A] time = 0.01, size = 85, normalized size = 0.89

$$\frac{1}{7\left(b\,x^{2}+a\right)^{\frac{7}{2}} a}+\frac{1}{5\left(b\,x^{2}+a\right)^{\frac{5}{2}} a^{2}}+\frac{1}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} a^{3}}-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{a^{\frac{9}{2}}}+\frac{1}{\sqrt{b\,x^{2}+a}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(9/2),x)
```

```
[Out] 1/7/a/(b*x^2+a)^(7/2)+1/5/a^2/(b*x^2+a)^(5/2)+1/3/a^3/(b*x^2+a)^(3/2)+1/a^4
/(b*x^2+a)^(1/2)-1/a^(9/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.36, size = 73, normalized size = 0.77

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{9}{2}}}+\frac{1}{\sqrt{bx^{2}+a}\,a^{4}}+\frac{1}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{1}{5\left(bx^{2}+a\right)^{\frac{5}{2}}a^{2}}+\frac{1}{7\left(bx^{2}+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 1/(sqrt(b*x^2 + a)*a^4) + 1/3/((b*
x^2 + a)^(3/2)*a^3) + 1/5/((b*x^2 + a)^(5/2)*a^2) + 1/7/((b*x^2 + a)^(7/2)*
a)
```

mupad [B] time = 4.85, size = 75, normalized size = 0.79

$$\frac{\frac{b\,x^{2}+a}{5\,a^{2}}+\frac{1}{7\,a}+\frac{\left(b\,x^{2}+a\right)^{2}}{3\,a^{3}}+\frac{\left(b\,x^{2}+a\right)^{3}}{a^{4}}}{\left(b\,x^{2}+a\right)^{7/2}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((a + b*x^2)/(5*a^2) + 1/(7*a) + (a + b*x^2)^2/(3*a^3) + (a + b*x^2)^3/a^4)
/(a + b*x^2)^(7/2) - atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(9/2)
```

sympy [B] time = 7.90, size = 5250, normalized size = 55.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(9/2),x)
```

```
[Out] 352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*
a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8
 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2
)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*
a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(
71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100
*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x*
*12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/
2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x**2/
a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 +
 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b*
*5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a
**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20
) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*
x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/
2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25
200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*
x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(210*a
**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/
2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 441
00*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*
x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a**31
*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
+ 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)/(2
10*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**
(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b
**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 4725*a
**30*b**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*
a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8
 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2
)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*
a**(53/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(
210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a*
*(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10
```

```
+ 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*
b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23630
*a**29*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
+ 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210*a*
*(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2
)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4410
0*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x
**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a**29
*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
*18 + 210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2/a)
/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*
a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**1
0 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2
)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 220
50*a**28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9
450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*
x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(
59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 +
210*a**(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a) +
 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) +
31442*a**27*b**5*x**10*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b
*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65
/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2
5200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9
*x**18 + 210*a**(53/2)*b**10*x**20) + 26460*a**27*b**5*x**10*log(b*x**2/a)/
(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a
**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10
 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)
*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 5292
0*a**27*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71
/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a
**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**1
2 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)
*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 19924*a**26*b**6*x**12*sqrt(1 +
b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*
```

b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450
*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**
20) + 22050*a**26*b**6*x**12*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*
b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(6
5/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**
9*x**18 + 210*a**(53/2)*b**10*x**20) - 44100*a**26*b**6*x**12*log(sqrt(1 +
b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2
*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(6
3/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 +
9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**1
0*x**20) + 8162*a**25*b**7*x**14*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a
(71/2)*b*x2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**25*b**7*x**14*log(
b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
+ 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*
b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450
*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**
20) - 25200*a**25*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2
100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6
+ 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)
*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100
*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1960*a**24*b**8*x**16*
sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*
b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a
(63/2)*b5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**
14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*
b**10*x**20) + 4725*a**24*b**8*x**16*log(b*x**2/a)/(210*a**(73/2) + 2100*a*
*(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 441
00*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*
x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(5
5/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 9450*a**24*b**8*x**16*log(sq
rt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/
2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5292
0*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*
x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/
2)*b**10*x**20) + 210*a**23*b**9*x**18*sqrt(1 + b*x**2/a)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**
6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2
)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 210
0*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**23*b**9*x**18
*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2
*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(6

```
3/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 +
 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**1
0*x**20) - 2100*a**23*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2)
 + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*
x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(6
1/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 105*a**22*b**10*x*
*20*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b
**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a*
*(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**1
4 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b
**10*x**20) - 210*a**22*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73
/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b*
*3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a*
*(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16
 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)
```

3.520 $\int \frac{1}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{128bx}{35a^5\sqrt{a+bx^2}}-\frac{64bx}{35a^4\left(a+bx^2\right)^{3/2}}-\frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {271, 192, 191}

$$-\frac{128bx}{35a^5\sqrt{a+bx^2}}-\frac{64bx}{35a^4\left(a+bx^2\right)^{3/2}}-\frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(1/(a*x*(a + b*x^2)^(7/2))) - (8*b*x)/(7*a^2*(a + b*x^2)^(7/2)) - (48*b*x)
/(35*a^3*(a + b*x^2)^(5/2)) - (64*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (128*b*
x)/(35*a^5*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^{9/2}}\,dx = -\frac{1}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8b)\int \frac{1}{\left(a+bx^2\right)^{9/2}}\,dx}{a}$$

$$= -\frac{1}{ax\left(a+bx^2\right)^{7/2}} - \frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{(48b)\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7a^2}$$

$$= -\frac{1}{ax\left(a+bx^2\right)^{7/2}} - \frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{(192b)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^3}$$

$$= -\frac{1}{ax\left(a+bx^2\right)^{7/2}} - \frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{64bx}{35a^4\left(a+bx^2\right)^{3/2}} - \frac{(128b)\int}{3}$$

$$= -\frac{1}{ax\left(a+bx^2\right)^{7/2}} - \frac{8bx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{48bx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{64bx}{35a^4\left(a+bx^2\right)^{3/2}} - \frac{128b}{35a^5\sqrt{a-}}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 0.64

$$\frac{-35a^4-280a^3bx^2-560a^2b^2x^4-448ab^3x^6-128b^4x^8}{35a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-35*a^4 - 280*a^3*b*x^2 - 560*a^2*b^2*x^4 - 448*a*b^3*x^6 - 128*b^4*x^8)/(
35*a^5*x*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 64, normalized size = 0.64

$$\frac{-35a^4-280a^3bx^2-560a^2b^2x^4-448ab^3x^6-128b^4x^8}{35a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-35*a^4 - 280*a^3*b*x^2 - 560*a^2*b^2*x^4 - 448*a*b^3*x^6 - 128*b^4*x^8)/(
35*a^5*x*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.61, size = 103, normalized size = 1.03

$$-\frac{\left(128\,b^4x^8+448\,ab^3x^6+560\,a^2b^2x^4+280\,a^3bx^2+35\,a^4\right)\sqrt{bx^2+a}}{35\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*a
^4)*sqrt(b*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*
x^3 + a^9*x)
```

giac [A] time = 1.12, size = 90, normalized size = 0.90

$$-\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\,\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)*
x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

maple [A] time = 0.01, size = 61, normalized size = 0.61

$$-\frac{128b^4x^8+448a\,b^3x^6+560a^2b^2x^4+280a^3b\,x^2+35a^4}{35\left(b\,x^2+a\right)^{\frac{7}{2}}\,a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/35*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^4+280*a^3*b*x^2+35*a^4)/x/(b
*x^2+a)^(7/2)/a^5
```

maxima [A] time = 1.43, size = 82, normalized size = 0.82

$$-\frac{128\,bx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,bx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{48\,bx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{8\,bx}{7\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{1}{\left(bx^2+a\right)^{\frac{7}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/
35*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*b*x/((b*x^2 + a)^(7/2)*a^2) - 1/((b*x^
2 + a)^(7/2)*a*x)
```

mupad [B] time = 4.71, size = 76, normalized size = 0.76

$$-\frac{\frac{1}{a^4}+\frac{128\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{29\,b\,x}{35\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{13\,b\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{b\,x}{7\,a^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(9/2)),x)
```

```
[Out] - (1/a^4 + (128*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (29*b*x)/(35*a^4*(
a + b*x^2)^(3/2)) - (13*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (b*x)/(7*a^2*(a +
 b*x^2)^(7/2))
```

sympy [B] time = 2.93, size = 400, normalized size = 4.00

$$-\frac{35a^4b^{\frac{33}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}-\frac{280a^3b^{\frac{35}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}-\frac{560a^2b^{\frac{37}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}-\frac{448ab^{\frac{39}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}-\frac{128b^{\frac{41}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}+140a^8b^{17}x^2+210a^7b^{18}x^4+140a^6b^{19}x^6+35a^5b^{20}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] -35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 280*a
**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 560*a
**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 448*a
*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 +
 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b**(
41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*
a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
```

3.521 $\int \frac{1}{x^3(a+bx^2)^{9/2}} dx$

Optimal. Leaf size=126

$$\frac{9b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{9b}{2a^5\sqrt{a+bx^2}}-\frac{3b}{2a^4\left(a+bx^2\right)^{3/2}}-\frac{9b}{10a^3\left(a+bx^2\right)^{5/2}}-\frac{9b}{14a^2\left(a+bx^2\right)^{7/2}}-\frac{1}{2ax^2\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 208}

$$-\frac{9\sqrt{a+bx^2}}{2a^5x^2}+\frac{3}{a^4x^2\sqrt{a+bx^2}}+\frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}}+\frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}}+\frac{9b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{1}{7ax^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] 1/(7*a*x^2*(a + b*x^2)^(7/2)) + 9/(35*a^2*x^2*(a + b*x^2)^(5/2)) + 3/(5*a^3
*x^2*(a + b*x^2)^(3/2)) + 3/(a^4*x^2*Sqrt[a + b*x^2]) - (9*Sqrt[a + b*x^2])
/(2*a^5*x^2) + (9*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{9/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{7/2}}\,dx, x, x^2\right)}{14a}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{9\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, x^2\right)}{10a^2}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx\right.}{2a^3}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} + \frac{9\operatorname{Sub}}{}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9\sqrt{a+}}{2a^5}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9\sqrt{a+}}{2a^5}$$

$$= \frac{1}{7ax^2\left(a+bx^2\right)^{7/2}} + \frac{9}{35a^2x^2\left(a+bx^2\right)^{5/2}} + \frac{3}{5a^3x^2\left(a+bx^2\right)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9\sqrt{a+}}{2a^5}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.29

$$-\frac{b\,{}_2F_1\left(-\frac{7}{2},2;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/7*(b*Hypergeometric2F1[-7/2, 2, -5/2, 1 + (b*x^2)/a])/(a^2*(a + b*x^2)^(
7/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 93, normalized size = 0.74

$$\frac{9b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}+\frac{-35a^4-528a^3bx^2-1218a^2b^2x^4-1050ab^3x^6-315b^4x^8}{70a^5x^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-35*a^4 - 528*a^3*b*x^2 - 1218*a^2*b^2*x^4 - 1050*a*b^3*x^6 - 315*b^4*x^8)
/(70*a^5*x^2*(a + b*x^2)^(7/2)) + (9*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2
*a^(11/2))
```

fricas [A] time = 1.00, size = 373, normalized size = 2.96

$$\left[\frac{315\left(b^5x^{10}+4ab^4x^8+6a^2b^3x^6+4a^3b^2x^4+a^4bx^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(315ab^4x^8+1050a^2b^3x^6+1218a^3b^2x^4+528a^4bx^2+35a^5\right)\sqrt{bx^2+a}}{140\left(a^6b^4x^{10}+4a^7b^3x^8+6a^8b^2x^6+4a^9bx^4+a^{10}x^2\right)},-\frac{315\left(b^5x^{10}+4ab^4x^8+6a^2b^3x^6+4a^3b^2x^4+a^4bx^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(315ab^4x^8+1050a^2b^3x^6+1218a^3b^2x^4+528a^4bx^2+35a^5\right)\sqrt{bx^2+a}}{70\left(a^6b^4x^{10}+4a^7b^3x^8+6a^8b^2x^6+4a^9bx^4+a^{10}x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*b
*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(315*
a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*s
qrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4
 + a^10*x^2), -1/70*(315*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^
2*x^4 + a^4*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (315*a*b^4*x
^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x
^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10
*x^2)]
```

giac [A] time = 1.13, size = 104, normalized size = 0.83

$$-\frac{9\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^5}-\frac{\sqrt{bx^2+a}}{2\,a^5x^2}-\frac{140\,\left(bx^2+a\right)^3b+35\,\left(bx^2+a\right)^2ab+14\,\left(bx^2+a\right)a^2b+5\,a^3b}{35\,\left(bx^2+a\right)^{\frac{7}{2}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -9/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/2*sqrt(b*x^2 + a
)/(a^5*x^2) - 1/35*(140*(b*x^2 + a)^3*b + 35*(b*x^2 + a)^2*a*b + 14*(b*x^2
+ a)*a^2*b + 5*a^3*b)/((b*x^2 + a)^(7/2)*a^5)
```

maple [A] time = 0.01, size = 108, normalized size = 0.86

$$-\frac{9b}{14\left(b\,x^2+a\right)^{\frac{7}{2}}a^2}-\frac{9b}{10\left(b\,x^2+a\right)^{\frac{5}{2}}a^3}-\frac{1}{2\left(b\,x^2+a\right)^{\frac{7}{2}}a\,x^2}-\frac{3b}{2\left(b\,x^2+a\right)^{\frac{3}{2}}a^4}+\frac{9b\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{11}{2}}}-\frac{9b}{2\sqrt{b\,x^2+a}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/2/a/x^2/(b*x^2+a)^(7/2)-9/14*b/a^2/(b*x^2+a)^(7/2)-9/10*b/a^3/(b*x^2+a)^
(5/2)-3/2*b/a^4/(b*x^2+a)^(3/2)-9/2*b/a^5/(b*x^2+a)^(1/2)+9/2/a^(11/2)*b*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.39, size = 96, normalized size = 0.76

$$\frac{9\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{11}{2}}}-\frac{9\,b}{2\,\sqrt{bx^2+a}\,a^5}-\frac{3\,b}{2\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{9\,b}{10\left(bx^2+a\right)^{\frac{5}{2}}a^3}-\frac{9\,b}{14\left(bx^2+a\right)^{\frac{7}{2}}a^2}-\frac{1}{2\left(bx^2+a\right)^{\frac{7}{2}}ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 9/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(11/2) - 9/2*b/(sqrt(b*x^2 + a)*a^5)
- 3/2*b/((b*x^2 + a)^(3/2)*a^4) - 9/10*b/((b*x^2 + a)^(5/2)*a^3) - 9/14*b/(
(b*x^2 + a)^(7/2)*a^2) - 1/2/((b*x^2 + a)^(7/2)*a*x^2)
```

mupad [B] time = 4.96, size = 113, normalized size = 0.90

$$\frac{9\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{11/2}}-\frac{\frac{b}{7\,a}+\frac{3\,b\,\left(b\,x^2+a\right)^2}{5\,a^3}+\frac{3\,b\,\left(b\,x^2+a\right)^3}{a^4}-\frac{9\,b\,\left(b\,x^2+a\right)^4}{2\,a^5}+\frac{9\,b\,\left(b\,x^2+a\right)}{35\,a^2}}{a\,\left(b\,x^2+a\right)^{7/2}-\left(b\,x^2+a\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(9/2)),x)
```

```
[Out] (9*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(11/2)) - (b/(7*a) + (3*b*(a +
b*x^2)^2)/(5*a^3) + (3*b*(a + b*x^2)^3)/a^4 - (9*b*(a + b*x^2)^4)/(2*a^5) +
 (9*b*(a + b*x^2))/(35*a^2))/(a*(a + b*x^2)^(7/2) - (a + b*x^2)^(9/2))
```

sympy [B] time = 12.43, size = 5540, normalized size = 43.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] -70*a**49*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4
+ 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*
b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 1680
0*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x*
*20 + 140*a**(87/2)*b**10*x**22) - 1476*a**48*b*x**2*sqrt(1 + b*x**2/a)/(14
0*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16
800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**
5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a*
*(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22)
 - 315*a**48*b*x**2*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*
x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(9
9/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 +
 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b*
*9*x**20 + 140*a**(87/2)*b**10*x**22) + 630*a**48*b*x**2*log(sqrt(1 + b*x**
2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b
**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*
a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x*
*16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)
*b**10*x**22) - 9822*a**47*b**2*x**4*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**
2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b
**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*
a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**
18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 3150*a**47*b*
*2*x**4*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*
a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x*
*10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(9
3/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 1
40*a**(87/2)*b**10*x**22) + 6300*a**47*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1
)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6
 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2
)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 63
```

00*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x **22) - 33956*a**46*b**3*x**6*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 140 0*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x** 8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/ 2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 14 00*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 14175*a**46*b**3*x** 6*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(10 3/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b **7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a** (87/2)*b**10*x**22) + 28350*a**46*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(14 0*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16 800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b** 5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a* *(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 71940*a**45*b**4*x**8*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a** (105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 2 9400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b* *6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a* *(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 37800*a**45*b**4*x**8*log (b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)* b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280 *a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x **16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 75600*a**45*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(140*a** (107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a **(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x** 12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/ 2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 10 0260*a**44*b**5*x**10*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(10 5/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 2940 0*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6* x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(8 9/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 66150*a**44*b**5*x**10*log(b *x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b* *2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a **(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x** 16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)* b**10*x**22) + 132300*a**44*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(140*a** (107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a **(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x** 12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/ 2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 94 396*a**43*b**6*x**12*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105 /2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400

*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x
14 + 16800*a(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89
/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 79380*a**43*b**6*x**12*log(b*
x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**
2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a*
*(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**1
6 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b
10*x22) + 158760*a**43*b**6*x**12*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(
107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a*
*(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**1
2 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2
)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 597
72*a**42*b**7*x**14*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/
2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*
a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x*
*14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/
2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 66150*a**42*b**7*x**14*log(b*x
2/a)/(140*a(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2
*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**
(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16
 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b*
*10*x**22) + 132300*a**42*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(1
07/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**
(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12
 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)
*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 2448
6*a**41*b**8*x**16*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2
)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a
(99/2)*b4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**
14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2
)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 37800*a**41*b**8*x**16*log(b*x*
*2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*
x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(
97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16
+ 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**
10*x**22) + 75600*a**41*b**8*x**16*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107
/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(1
01/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 +
 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b
8*x18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 5880*a
40*b9*x**18*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b
*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(
99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14
+ 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b
9*x20 + 140*a**(87/2)*b**10*x**22) - 14175*a**40*b**9*x**18*log(b*x**2/

a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**
6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/
2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6
300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*
x**22) + 28350*a**40*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)
*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/
2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29
400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8
*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 630*a**39
*b**10*x**20*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x*
*4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/
2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 1
6800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9
*x**20 + 140*a**(87/2)*b**10*x**22) - 3150*a**39*b**10*x**20*log(b*x**2/a)/
(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 +
 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*
b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300
*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**
22) + 6300*a**39*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x*
*2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*
b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400
*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x*
*18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 315*a**38*b*
*11*x**22*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 630
0*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*
x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**
(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 +
 140*a**(87/2)*b**10*x**22) + 630*a**38*b**11*x**22*log(sqrt(1 + b*x**2/a)
+ 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x
6 + 16800*a(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(9
7/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 +
 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**1
0*x**22)

3.522 $\int \frac{1}{x^4\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=132

$$\frac{256b^2x}{21a^6\sqrt{a+bx^2}}+\frac{128b^2x}{21a^5\left(a+bx^2\right)^{3/2}}+\frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}}+\frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}}+\frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}$$

Rubi [A] time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {271, 192, 191}

$$\frac{256b^2x}{21a^6\sqrt{a+bx^2}}+\frac{128b^2x}{21a^5\left(a+bx^2\right)^{3/2}}+\frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}}+\frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}}+\frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/(3*a*x^3*(a + b*x^2)^(7/2)) + (10*b)/(3*a^2*x*(a + b*x^2)^(7/2)) + (80*b
^2*x)/(21*a^3*(a + b*x^2)^(7/2)) + (32*b^2*x)/(7*a^4*(a + b*x^2)^(5/2)) + (
128*b^2*x)/(21*a^5*(a + b*x^2)^(3/2)) + (256*b^2*x)/(21*a^6*Sqrt[a + b*x^2]
)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{9/2}}\,dx = -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} - \frac{(10b)\int \frac{1}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{3a}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80b^2\right)\int \frac{1}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^2}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(160b^2\right)\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7a^3}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}} + \frac{\left(128b^2 \right.}{}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}} + \frac{1}{21a^5\left(\right.}$$

$$= -\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10b}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{80b^2x}{21a^3\left(a+bx^2\right)^{7/2}} + \frac{32b^2x}{7a^4\left(a+bx^2\right)^{5/2}} + \frac{1}{21a^5\left(\right.}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 0.57

$$\frac{-7a^5 + 70a^4bx^2 + 560a^3b^2x^4 + 1120a^2b^3x^6 + 896ab^4x^8 + 256b^5x^{10}}{21a^6x^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*a*b^4*x^8
 + 256*b^5*x^10)/(21*a^6*x^3*(a + b*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 75, normalized size = 0.57

$$\frac{-7a^5 + 70a^4bx^2 + 560a^3b^2x^4 + 1120a^2b^3x^6 + 896ab^4x^8 + 256b^5x^{10}}{21a^6x^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*a*b^4*x^8
 + 256*b^5*x^10)/(21*a^6*x^3*(a + b*x^2)^(7/2))
```

fricas [A] time = 1.16, size = 116, normalized size = 0.88

$$\frac{\left(256\,b^5x^{10}+896\,ab^4x^8+1120\,a^2b^3x^6+560\,a^3b^2x^4+70\,a^4bx^2-7\,a^5\right)\sqrt{bx^2+a}}{21\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 + 7
0*a^4*b*x^2 - 7*a^5)*sqrt(b*x^2 + a)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*
b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

giac [A] time = 1.16, size = 147, normalized size = 1.11

$$\frac{\left(\left(x^2\left(\frac{158\,b^5x^2}{a^6}+\frac{511\,b^4}{a^5}\right)+\frac{560\,b^3}{a^4}\right)x^2+\frac{210\,b^2}{a^3}\right)x}{21\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{4\left(6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+7\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^3
)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) - 15
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

maple [A] time = 0.01, size = 72, normalized size = 0.55

$$-\frac{-256b^5x^{10}-896a\,b^4x^8-1120a^2b^3x^6-560a^3b^2x^4-70a^4b\,x^2+7a^5}{21\left(b\,x^2+a\right)^{\frac{7}{2}}a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(9/2),x)
```

```
[Out] -1/21*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3*x^6-560*a^3*b^2*x^4-70*a^4*
b*x^2+7*a^5)/x^3/(b*x^2+a)^(7/2)/a^6
```

maxima [A] time = 1.42, size = 108, normalized size = 0.82

$$\frac{256\,b^2x}{21\,\sqrt{bx^2+a}\,a^6}+\frac{128\,b^2x}{21\,\left(bx^2+a\right)^{\frac{3}{2}}a^5}+\frac{32\,b^2x}{7\,\left(bx^2+a\right)^{\frac{5}{2}}a^4}+\frac{80\,b^2x}{21\,\left(bx^2+a\right)^{\frac{7}{2}}a^3}+\frac{10\,b}{3\,\left(bx^2+a\right)^{\frac{7}{2}}a^2x}-\frac{1}{3\,\left(bx^2+a\right)^{\frac{7}{2}}ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 256/21*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*b^2*x/((b*x^2 + a)^(3/2)*a^5) +
 32/7*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*b^2*x/((b*x^2 + a)^(7/2)*a^3) +
 10/3*b/((b*x^2 + a)^(7/2)*a^2*x) - 1/3/((b*x^2 + a)^(7/2)*a*x^3)
```

mupad [B] time = 4.80, size = 97, normalized size = 0.73

$$\frac{\frac{128\,b}{21\,a^5}+\frac{256\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{1}{3\,a^2}+\frac{19\,b\,x^2}{21\,a^3}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{32\,b}{21\,a^4\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{b^2\,x}{7\,a^3\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((128*b)/(21*a^5) + (256*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) - (1/(3*a
^2) + (19*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5/2)) - (32*b)/(21*a^4*x*(a +
b*x^2)^(3/2)) + (b^2*x)/(7*a^3*(a + b*x^2)^(7/2))
```

sympy [B] time = 3.84, size = 668, normalized size = 5.06

$$-\frac{7a^6b^{\frac{51}{2}}\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{63a^5b^{\frac{53}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{630a^4b^{\frac{55}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{1680a^3b^{\frac{57}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{2016a^2b^{\frac{59}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{1152ab^{\frac{61}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}+\frac{256b^{\frac{63}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] -7*a**6*b**(51/2)*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**
26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10
+ 21*a**6*b**30*x**12) + 63*a**5*b**(53/2)*x**2*sqrt(a/(b*x**2) + 1)/(21*a*
*11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**2
8*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 630*a**4*b**(55/2)*x
**4*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*
a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**3
0*x**12) + 1680*a**3*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x*
*2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105
*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 2016*a**2*b**(59/2)*x**8*sqrt(a/
(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*
```

```
x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) +
1152*a*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**1
0*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x
**10 + 21*a**6*b**30*x**12) + 256*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(21*
a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b*
*28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)
```

3.523 $\int \frac{x^5}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{1}{320}\left(4x^2+9\right)^{5/2}-\frac{3}{32}\left(4x^2+9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2+9}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{320}\left(4x^2+9\right)^{5/2}-\frac{3}{32}\left(4x^2+9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[9 + 4*x^2],x]
[Out] (81*Sqrt[9 + 4*x^2])/64 - (3*(9 + 4*x^2)^(3/2))/32 + (9 + 4*x^2)^(5/2)/320
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{9+4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{9+4x}}-\frac{9}{8}\sqrt{9+4x}+\frac{1}{16}(9+4x)^{3/2}\right)dx,x,x^2\right)\\ &= \frac{81}{64}\sqrt{9+4x^2}-\frac{3}{32}\left(9+4x^2\right)^{3/2}+\frac{1}{320}\left(9+4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{4x^2+9}\left(2x^4-6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[9 + 4*x^2],x]
[Out] (Sqrt[9 + 4*x^2]*(27 - 6*x^2 + 2*x^4))/40
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{4x^2+9}\left(2x^4-6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[9 + 4*x^2],x]
[Out] (Sqrt[9 + 4*x^2]*(27 - 6*x^2 + 2*x^4))/40
```

fricas [A] time = 0.97, size = 23, normalized size = 0.50

$$\frac{1}{40}\left(2x^4-6x^2+27\right)\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/40*(2*x^4 - 6*x^2 + 27)*sqrt(4*x^2 + 9)
```

giac [A] time = 1.13, size = 34, normalized size = 0.74

$$\frac{1}{320}\left(4x^2+9\right)^{\frac{5}{2}}-\frac{3}{32}\left(4x^2+9\right)^{\frac{3}{2}}+\frac{81}{64}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/320*(4*x^2 + 9)^(5/2) - 3/32*(4*x^2 + 9)^(3/2) + 81/64*sqrt(4*x^2 + 9)
```

maple [A] time = 0.00, size = 24, normalized size = 0.52

$$\frac{\sqrt{4x^2+9}\,\left(2x^4-6x^2+27\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/40*(4*x^2+9)^(1/2)*(2*x^4-6*x^2+27)
```

maxima [A] time = 2.89, size = 40, normalized size = 0.87

$$\frac{1}{20}\sqrt{4x^2+9}\,x^4 - \frac{3}{20}\sqrt{4x^2+9}\,x^2 + \frac{27}{40}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(4*x^2 + 9)*x^4 - 3/20*sqrt(4*x^2 + 9)*x^2 + 27/40*sqrt(4*x^2 + 9)
```

mupad [B] time = 0.02, size = 21, normalized size = 0.46

$$\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{x^4}{5}-\frac{3x^2}{5}+\frac{27}{10}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2 + 9)^(1/2),x)
```

```
[Out] ((x^2 + 9/4)^(1/2)*(x^4/5 - (3*x^2)/5 + 27/10))/2
```

sympy [A] time = 1.29, size = 44, normalized size = 0.96

$$\frac{x^4\sqrt{4x^2+9}}{20} - \frac{3x^2\sqrt{4x^2+9}}{20} + \frac{27\sqrt{4x^2+9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(4*x**2+9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 + 9)/20 - 3*x**2*sqrt(4*x**2 + 9)/20 + 27*sqrt(4*x**2 + 9)
/40
```

3.524 $\int \frac{x^4}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=45

$$-\frac{27}{128}\sqrt{4x^2+9}\,x+\frac{1}{16}\sqrt{4x^2+9}\,x^3+\frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 215}

$$\frac{1}{16}\sqrt{4x^2+9}\,x^3-\frac{27}{128}\sqrt{4x^2+9}\,x+\frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[9 + 4*x^2],x]
```

```
[Out] (-27*x*Sqrt[9 + 4*x^2])/128 + (x^3*Sqrt[9 + 4*x^2])/16 + (243*ArcSinh[(2*x)
/3])/256
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{9+4x^2}}\,dx &= \frac{1}{16}x^3\sqrt{9+4x^2}-\frac{27}{16}\int \frac{x^2}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9+4x^2}+\frac{1}{16}x^3\sqrt{9+4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9+4x^2}+\frac{1}{16}x^3\sqrt{9+4x^2}+\frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.76

$$\frac{1}{256}\left(2x\sqrt{4x^2+9}\left(8x^2-27\right)+243\sinh^{-1}\left(\frac{2x}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[9 + 4*x^2],x]
[Out] (2*x*Sqrt[9 + 4*x^2]*(-27 + 8*x^2) + 243*ArcSinh[(2*x)/3])/256
```

IntegrateAlgebraic [A] time = 0.06, size = 45, normalized size = 1.00

$$\frac{1}{128}\sqrt{4x^2+9}\left(8x^3-27x\right)-\frac{243}{256}\log\left(\sqrt{4x^2+9}-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[9 + 4*x^2],x]
[Out] (Sqrt[9 + 4*x^2]*(-27*x + 8*x^3))/128 - (243*Log[-2*x + Sqrt[9 + 4*x^2]])/2
56
```

fricas [A] time = 0.82, size = 37, normalized size = 0.82

$$\frac{1}{128}\left(8x^3-27x\right)\sqrt{4x^2+9}-\frac{243}{256}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/128*(8*x^3 - 27*x)*sqrt(4*x^2 + 9) - 243/256*log(-2*x + sqrt(4*x^2 + 9))
```

giac [A] time = 1.09, size = 36, normalized size = 0.80

$$\frac{1}{128}\left(8x^2-27\right)\sqrt{4x^2+9}\,x-\frac{243}{256}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/128*(8*x^2 - 27)*sqrt(4*x^2 + 9)*x - 243/256*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.01, size = 34, normalized size = 0.76

$$\frac{\sqrt{4x^2+9}\,x^3}{16}-\frac{27\sqrt{4x^2+9}\,x}{128}+\frac{243\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2+9)^(1/2),x)
[Out] 243/256*arcsinh(2/3*x)-27/128*(4*x^2+9)^(1/2)*x+1/16*x^3*(4*x^2+9)^(1/2)
```

maxima [A] time = 2.91, size = 33, normalized size = 0.73

$$\frac{1}{16}\sqrt{4x^2+9}\,x^3-\frac{27}{128}\sqrt{4x^2+9}\,x+\frac{243}{256}\,\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] 1/16*sqrt(4*x^2 + 9)*x^3 - 27/128*sqrt(4*x^2 + 9)*x + 243/256*arcsinh(2/3*x
)
```

mupad [B] time = 0.03, size = 25, normalized size = 0.56

$$\frac{243\,\operatorname{asinh}\left(\frac{2x}{3}\right)}{256}-\frac{\sqrt{x^2+\frac{9}{4}}\left(\frac{27x}{32}-\frac{x^3}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2 + 9)^(1/2),x)
[Out] (243*asinh((2*x)/3))/256 - ((x^2 + 9/4)^(1/2)*((27*x)/32 - x^3/4))/2
```

sympy [A] time = 0.73, size = 39, normalized size = 0.87

$$\frac{x^3\sqrt{4x^2+9}}{16}-\frac{27x\sqrt{4x^2+9}}{128}+\frac{243\,\operatorname{asinh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(4*x**2+9)**(1/2),x)
[Out] x**3*sqrt(4*x**2 + 9)/16 - 27*x*sqrt(4*x**2 + 9)/128 + 243*asinh(2*x/3)/256
```

3.525 $\int \frac{x^3}{\sqrt{9+4x^2}} dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(4x^2+9\right)^{3/2}-\frac{9}{16}\sqrt{4x^2+9}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(4x^2+9\right)^{3/2}-\frac{9}{16}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[9 + 4*x^2],x]
```

```
[Out] (-9*Sqrt[9 + 4*x^2])/16 + (9 + 4*x^2)^(3/2)/48
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{9+4x^2}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{9+4x}} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{9}{4\sqrt{9+4x}}+\frac{1}{4}\sqrt{9+4x}\right) dx, x, x^2\right)\\
&= -\frac{9}{16}\sqrt{9+4x^2}+\frac{1}{48}\left(9+4x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\left(2x^2-9\right)\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[9 + 4*x^2],x]
[Out] ((-9 + 2*x^2)*Sqrt[9 + 4*x^2])/24
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\left(2x^2-9\right)\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[9 + 4*x^2],x]
[Out] ((-9 + 2*x^2)*Sqrt[9 + 4*x^2])/24
```

fricas [A] time = 0.91, size = 18, normalized size = 0.58

$$\frac{1}{24}\sqrt{4x^2+9}\left(2x^2-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/24*sqrt(4*x^2 + 9)*(2*x^2 - 9)
```

giac [A] time = 1.13, size = 23, normalized size = 0.74

$$\frac{1}{48}\left(4x^2+9\right)^{\frac{3}{2}}-\frac{9}{16}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/48*(4*x^2 + 9)^(3/2) - 9/16*sqrt(4*x^2 + 9)
```

maple [A] time = 0.00, size = 19, normalized size = 0.61

$$\frac{\sqrt{4x^2+9}\,\left(2x^2-9\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/24*(4*x^2+9)^(1/2)*(2*x^2-9)
```

maxima [A] time = 2.91, size = 26, normalized size = 0.84

$$\frac{1}{12}\,\sqrt{4\,x^2+9}\,x^2-\frac{3}{8}\,\sqrt{4\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(4*x^2 + 9)*x^2 - 3/8*sqrt(4*x^2 + 9)
```

mupad [B] time = 0.02, size = 15, normalized size = 0.48

$$\sqrt{x^2+\frac{9}{4}}\,\left(\frac{x^2}{6}-\frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x^2 + 9/4)^(1/2)*(x^2/6 - 3/4)
```

sympy [A] time = 0.38, size = 27, normalized size = 0.87

$$\frac{x^2\sqrt{4x^2+9}}{12}-\frac{3\sqrt{4x^2+9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**2+9)**(1/2),x)
```

```
[Out] x**2*sqrt(4*x**2 + 9)/12 - 3*sqrt(4*x**2 + 9)/8
```

3.526 $\int \frac{x^2}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 215}

$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3])/16
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{9+4x^2}}\,dx &= \frac{1}{8}x\sqrt{9+4x^2}-\frac{9}{8}\int \frac{1}{\sqrt{9+4x^2}}\,dx\\ &= \frac{1}{8}x\sqrt{9+4x^2}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3])/16
```

IntegrateAlgebraic [A] time = 0.04, size = 37, normalized size = 1.37

$$\frac{1}{8}\sqrt{4x^2+9}\,x+\frac{9}{16}\log\left(\sqrt{4x^2+9}-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[9 + 4*x^2])/8 + (9*Log[-2*x + Sqrt[9 + 4*x^2]])/16
```

fricas [A] time = 0.91, size = 29, normalized size = 1.07

$$\frac{1}{8}\sqrt{4\,x^2+9}\,x+\frac{9}{16}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(4*x^2 + 9)*x + 9/16*log(-2*x + sqrt(4*x^2 + 9))
```

giac [A] time = 1.07, size = 29, normalized size = 1.07

$$\frac{1}{8}\sqrt{4\,x^2+9}\,x+\frac{9}{16}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^2 + 9)*x + 9/16*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.01, size = 20, normalized size = 0.74

$$\frac{\sqrt{4x^2+9}\,x}{8}-\frac{9\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2+9)^(1/2),x)
```

```
[Out] -9/16*arcsinh(2/3*x)+1/8*(4*x^2+9)^(1/2)*x
```

maxima [A] time = 2.92, size = 19, normalized size = 0.70

$$\frac{1}{8}\sqrt{4x^2+9}\,x - \frac{9}{16}\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(4*x^2 + 9)*x - 9/16*arcsinh(2/3*x)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.63

$$\frac{x\sqrt{x^2+\frac{9}{4}}}{4} - \frac{9\,\operatorname{asinh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2 + 9)^(1/2),x)
```

```
[Out] (x*(x^2 + 9/4)^(1/2))/4 - (9*asinh((2*x)/3))/16
```

sympy [A] time = 0.23, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2+9}}{8} - \frac{9\,\operatorname{asinh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*x**2+9)**(1/2),x)
```

```
[Out] x*sqrt(4*x**2 + 9)/8 - 9*asinh(2*x/3)/16
```

3.527 $\int \frac{x}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{1}{4}\sqrt{4x^2+9}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{4}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[9 + 4*x^2],x]
```

```
[Out] Sqrt[9 + 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{9+4x^2}}\,dx = \frac{1}{4}\sqrt{9+4x^2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{4}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[9 + 4*x^2],x]
```

```
[Out] Sqrt[9 + 4*x^2]/4
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{4}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[9 + 4*x^2],x]
```

```
[Out] Sqrt[9 + 4*x^2]/4
```

fricas [A] time = 0.91, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

giac [A] time = 1.12, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$\frac{\sqrt{4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/4*(4*x^2+9)^(1/2)
```

maxima [A] time = 1.32, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(4*x^2 + 9)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.60

$$\frac{\sqrt{x^2 + \frac{9}{4}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2 + 9)^(1/2),x)
[Out] (x^2 + 9/4)^(1/2)/2
```

sympy [A] time = 0.15, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2 + 9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2+9)**(1/2),x)
[Out] sqrt(4*x**2 + 9)/4
```

3.528 $\int \frac{1}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] ArcSinh[(2*x)/3]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] ArcSinh[(2*x)/3]/2
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 2.00

$$-\frac{1}{2}\log\left(\sqrt{4x^2+9}-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] -1/2*Log[-2*x + Sqrt[9 + 4*x^2]]
```

fricas [B] time = 0.88, size = 16, normalized size = 1.60

$$-\frac{1}{2} \log\left(-2\,x + \sqrt{4\,x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

giac [B] time = 1.17, size = 16, normalized size = 1.60

$$-\frac{1}{2} \log\left(-2\,x + \sqrt{4\,x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

maple [A] time = 0.00, size = 7, normalized size = 0.70

$$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/2*arcsinh(2/3*x)
```

maxima [A] time = 2.96, size = 6, normalized size = 0.60

$$\frac{1}{2}\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

[Out] 1/2*arcsinh(2/3*x)

mupad [B] time = 0.03, size = 6, normalized size = 0.60

$$\frac{\operatorname{asinh}\left(\frac{2\,x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^2 + 9)^(1/2),x)

[Out] asinh((2*x)/3)/2

sympy [A] time = 0.15, size = 7, normalized size = 0.70

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x**2+9)**(1/2),x)

[Out] asinh(2*x/3)/2

3.529 $\int \frac{1}{x\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 207}

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[9 + 4*x^2]/3]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{9+4x^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)$$
$$= -\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[9 + 4*x^2]),x]
[Out] -1/3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[9 + 4*x^2]),x]
[Out] -1/3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

fricas [B] time = 1.04, size = 35, normalized size = 1.75

$$-\frac{1}{3}\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)+\frac{1}{3}\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 1/3*log(-2*x + sqrt(4*x^2 + 9) - 3)
```

giac [B] time = 1.06, size = 29, normalized size = 1.45

$$-\frac{1}{6}\log\left(\sqrt{4\,x^2+9}+3\right)+\frac{1}{6}\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*log(sqrt(4*x^2 + 9) + 3) + 1/6*log(sqrt(4*x^2 + 9) - 3)
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$-\frac{\operatorname{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4*x^2+9)^(1/2),x)
```

```
[Out] -1/3*arctanh(3/(4*x^2+9)^(1/2))
```

maxima [A] time = 2.96, size = 9, normalized size = 0.45

$$-\frac{1}{3} \operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*arcsinh(3/2/abs(x))
```

mupad [B] time = 0.04, size = 12, normalized size = 0.60

$$-\frac{\operatorname{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] -atanh((2*(x^2 + 9/4)^(1/2))/3)/3
```

sympy [A] time = 1.02, size = 8, normalized size = 0.40

$$-\frac{\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x**2+9)**(1/2),x)
```

```
[Out] -asinh(3/(2*x))/3
```

3.530 $\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{9+4x^2}}{9x}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -1/9*Sqrt[9 + 4*x^2]/x
```

IntegrateAlgebraic [A] time = 0.03, size = 18, normalized size = 1.00

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[9 + 4*x^2]),x]
[Out] -1/9*Sqrt[9 + 4*x^2]/x
```

fricas [A] time = 0.90, size = 18, normalized size = 1.00

$$-\frac{2x+\sqrt{4x^2+9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/9*(2*x + sqrt(4*x^2 + 9))/x
```

giac [A] time = 1.12, size = 23, normalized size = 1.28

$$\frac{4}{\left(2x-\sqrt{4x^2+9}\right)^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 4/((2*x - sqrt(4*x^2 + 9))^2 - 9)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4*x^2+9)^(1/2),x)
[Out] -1/9*(4*x^2+9)^(1/2)/x
```

maxima [A] time = 2.98, size = 14, normalized size = 0.78

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(4*x^2 + 9)/x
```

mupad [B] time = 0.02, size = 12, normalized size = 0.67

$$-\frac{2\sqrt{x^2+\frac{9}{4}}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] -(2*(x^2 + 9/4)^(1/2))/(9*x)
```

sympy [A] time = 0.76, size = 15, normalized size = 0.83

$$-\frac{2\sqrt{1+\frac{9}{4x^2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4*x**2+9)**(1/2),x)
```

```
[Out] -2*sqrt(1 + 9/(4*x**2))/9
```

3.531 $\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{18x^2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 207}

$$\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{18x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(18*x^2) + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/27
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{18x^2}-\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{9+4x^2}}{18x^2}-\frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\ &= -\frac{\sqrt{9+4x^2}}{18x^2}+\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.95

$$\frac{1}{54}\left(4\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right)-\frac{3\sqrt{4x^2+9}}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[9 + 4*x^2]),x]
[Out] ((-3*Sqrt[9 + 4*x^2])/x^2 + 4*ArcTanh[Sqrt[1 + (4*x^2)/9]])/54
```

IntegrateAlgebraic [A] time = 0.04, size = 39, normalized size = 1.00

$$\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{18x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[9 + 4*x^2]),x]
[Out] -1/18*Sqrt[9 + 4*x^2]/x^2 + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/27
```

fricas [A] time = 0.97, size = 57, normalized size = 1.46

$$\frac{4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(4*x^2*log(-2*x + sqrt(4*x^2 + 9) + 3) - 4*x^2*log(-2*x + sqrt(4*x^2 +
 9) - 3) - 3*sqrt(4*x^2 + 9))/x^2
```

giac [A] time = 1.10, size = 43, normalized size = 1.10

$$-\frac{\sqrt{4\,x^2+9}}{18\,x^2}+\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}+3\right)-\frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(4*x^2 + 9)/x^2 + 1/27*log(sqrt(4*x^2 + 9) + 3) - 1/27*log(sqrt(4
*x^2 + 9) - 3)
```

maple [A] time = 0.00, size = 30, normalized size = 0.77

$$\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}-\frac{\sqrt{4x^2+9}}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4*x^2+9)^(1/2),x)
```

```
[Out] -1/18*(4*x^2+9)^(1/2)/x^2+2/27*arctanh(3/(4*x^2+9)^(1/2))
```

maxima [A] time = 2.94, size = 24, normalized size = 0.62

$$-\frac{\sqrt{4\,x^2+9}}{18\,x^2}+\frac{2}{27}\,\mathrm{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(4*x^2 + 9)/x^2 + 2/27*arcsinh(3/2/abs(x))
```

mupad [B] time = 0.03, size = 25, normalized size = 0.64

$$\frac{2\,\mathrm{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)}{27}-\frac{\sqrt{x^2+\frac{9}{4}}}{9\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] (2*atanh((2*(x^2 + 9/4)^(1/2))/3))/27 - (x^2 + 9/4)^(1/2)/(9*x^2)
```

sympy [A] time = 2.10, size = 44, normalized size = 1.13

$$\frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{27} - \frac{1}{9x\sqrt{1+\frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4*x**2+9)**(1/2),x)
```

```
[Out] 2*asinh(3/(2*x))/27 - 1/(9*x*sqrt(1 + 9/(4*x**2))) - 1/(4*x**3*sqrt(1 + 9/(
4*x**2)))
```

3.532 $\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{8\sqrt{4x^2+9}}{243x}-\frac{\sqrt{4x^2+9}}{27x^3}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{8\sqrt{4x^2+9}}{243x}-\frac{\sqrt{4x^2+9}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(27*x^3) + (8*Sqrt[9 + 4*x^2])/(243*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx &= -\frac{\sqrt{9+4x^2}}{27x^3}-\frac{8}{27}\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx\\ &= -\frac{\sqrt{9+4x^2}}{27x^3}+\frac{8\sqrt{9+4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 0.73

$$-\frac{\left(9-8x^2\right)\sqrt{\frac{4x^2}{9}+1}}{81x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[9 + 4*x^2]),x]
[Out] -1/81*((9 - 8*x^2)*Sqrt[1 + (4*x^2)/9])/x^3
```

IntegrateAlgebraic [A] time = 0.04, size = 25, normalized size = 0.68

$$\frac{\sqrt{4x^2+9}\left(8x^2-9\right)}{243x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[9 + 4*x^2]),x]
[Out] (Sqrt[9 + 4*x^2]*(-9 + 8*x^2))/(243*x^3)
```

fricas [A] time = 1.08, size = 28, normalized size = 0.76

$$\frac{16\,x^3+\left(8\,x^2-9\right)\sqrt{4\,x^2+9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/243*(16*x^3 + (8*x^2 - 9)*sqrt(4*x^2 + 9))/x^3
```

giac [A] time = 1.22, size = 42, normalized size = 1.14

$$\frac{32\left(\left(2\,x-\sqrt{4\,x^2+9}\right)^2-3\right)}{\left(\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 32*((2*x - sqrt(4*x^2 + 9))^2 - 3)/((2*x - sqrt(4*x^2 + 9))^2 - 9)^3
```

maple [A] time = 0.00, size = 22, normalized size = 0.59

$$\frac{\sqrt{4x^2+9}\;\left(8x^2-9\right)}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/243*(4*x^2+9)^(1/2)*(8*x^2-9)/x^3
```

maxima [A] time = 2.86, size = 29, normalized size = 0.78

$$\frac{8\sqrt{4x^2+9}}{243x} - \frac{\sqrt{4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/243*sqrt(4*x^2 + 9)/x - 1/27*sqrt(4*x^2 + 9)/x^3
```

mupad [B] time = 0.02, size = 19, normalized size = 0.51

$$\sqrt{x^2+\frac{9}{4}}\left(\frac{16}{243x} - \frac{2}{27x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] (x^2 + 9/4)^(1/2)*(16/(243*x) - 2/(27*x^3))
```

sympy [A] time = 1.29, size = 32, normalized size = 0.86

$$\frac{16\sqrt{1+\frac{9}{4x^2}}}{243} - \frac{2\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4*x**2+9)**(1/2),x)
```

```
[Out] 16*sqrt(1 + 9/(4*x**2))/243 - 2*sqrt(1 + 9/(4*x**2))/(27*x**2)
```

3.533 $\int \frac{1}{x^5 \sqrt{9+4x^2}} \, dx$

Optimal. Leaf size=57

$$\frac{\sqrt{4x^2+9}}{54x^2} - \frac{2}{81} \tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right) - \frac{\sqrt{4x^2+9}}{36x^4}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 207}

$$\frac{\sqrt{4x^2+9}}{54x^2} - \frac{\sqrt{4x^2+9}}{36x^4} - \frac{2}{81} \tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[9 + 4*x^2]),x]
```

```
[Out] -Sqrt[9 + 4*x^2]/(36*x^4) + Sqrt[9 + 4*x^2]/(54*x^2) - (2*ArcTanh[Sqrt[9 +
4*x^2]/3])/81
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{36x^4} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{36x^4} + \frac{\sqrt{9+4x^2}}{54x^2} + \frac{1}{27}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{9+4x}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9+4x^2}}{36x^4} + \frac{\sqrt{9+4x^2}}{54x^2} + \frac{1}{54}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{9+4x^2}\right)\\
&= -\frac{\sqrt{9+4x^2}}{36x^4} + \frac{\sqrt{9+4x^2}}{54x^2} - \frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9+4x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$-\frac{16}{729}\sqrt{4x^2+9}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{4x^2}{9}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[9 + 4*x^2]),x]
[Out] (-16*Sqrt[9 + 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (4*x^2)/9])/729
```

IntegrateAlgebraic [A] time = 0.04, size = 46, normalized size = 0.81

$$\frac{\left(2x^2-3\right)\sqrt{4x^2+9}}{108x^4} - \frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[9 + 4*x^2]),x]
```

```
[Out] ((-3 + 2*x^2)*Sqrt[9 + 4*x^2])/(108*x^4) - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/8
1
```

fricas [A] time = 0.91, size = 64, normalized size = 1.12

$$-\frac{8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}\left(2\,x^2-3\right)}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/324*(8*x^4*log(-2*x + sqrt(4*x^2 + 9) + 3) - 8*x^4*log(-2*x + sqrt(4*x^2
 + 9) - 3) - 3*sqrt(4*x^2 + 9)*(2*x^2 - 3))/x^4
```

giac [A] time = 1.18, size = 55, normalized size = 0.96

$$\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}-15\,\sqrt{4\,x^2+9}}{216\,x^4}-\frac{1}{81}\,\log\left(\sqrt{4\,x^2+9}+3\right)+\frac{1}{81}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((4*x^2 + 9)^(3/2) - 15*sqrt(4*x^2 + 9))/x^4 - 1/81*log(sqrt(4*x^2 +
9) + 3) + 1/81*log(sqrt(4*x^2 + 9) - 3)
```

maple [A] time = 0.00, size = 44, normalized size = 0.77

$$-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{81}+\frac{\sqrt{4x^2+9}}{54x^2}-\frac{\sqrt{4x^2+9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4*x^2+9)^(1/2),x)
```

```
[Out] -1/36*(4*x^2+9)^(1/2)/x^4+1/54*(4*x^2+9)^(1/2)/x^2-2/81*arctanh(3/(4*x^2+9)
^(1/2))
```

maxima [A] time = 2.95, size = 38, normalized size = 0.67

$$\frac{\sqrt{4\,x^2+9}}{54\,x^2}-\frac{\sqrt{4\,x^2+9}}{36\,x^4}-\frac{2}{81}\,\mathrm{arsinh}\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(4*x^2 + 9)/x^2 - 1/36*sqrt(4*x^2 + 9)/x^4 - 2/81*arcsinh(3/2/abs(
x))
```

mupad [B] time = 0.03, size = 33, normalized size = 0.58

$$\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2}{27\,x^2}-\frac{1}{9\,x^4}\right)}{2}-\frac{2\,\mathrm{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(4*x^2 + 9)^(1/2)),x)
```

```
[Out] ((x^2 + 9/4)^(1/2)*(2/(27*x^2) - 1/(9*x^4)))/2 - (2*atanh((2*(x^2 + 9/4)^(1
/2))/3))/81
```

sympy [A] time = 3.89, size = 63, normalized size = 1.11

$$-\frac{2\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{81}+\frac{1}{27x\sqrt{1+\frac{9}{4x^2}}}+\frac{1}{36x^3\sqrt{1+\frac{9}{4x^2}}}-\frac{1}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4*x**2+9)**(1/2),x)
```

```
[Out] -2*asinh(3/(2*x))/81 + 1/(27*x*sqrt(1 + 9/(4*x**2))) + 1/(36*x**3*sqrt(1 +
9/(4*x**2))) - 1/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

3.534 $\int \frac{x^5}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{320}\left(9-4x^2\right)^{5/2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{81}{64}\sqrt{9-4x^2}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{320}\left(9-4x^2\right)^{5/2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{81}{64}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[9 - 4*x^2],x]
```

```
[Out] (-81*Sqrt[9 - 4*x^2])/64 + (3*(9 - 4*x^2)^(3/2))/32 - (9 - 4*x^2)^(5/2)/320
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{9-4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{9-4x}}-\frac{9}{8}\sqrt{9-4x}+\frac{1}{16}(9-4x)^{3/2}\right)dx,x,x^2\right)\\ &= -\frac{81}{64}\sqrt{9-4x^2}+\frac{3}{32}\left(9-4x^2\right)^{3/2}-\frac{1}{320}\left(9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{1}{40}\sqrt{9-4x^2}\left(2x^4+6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[9 - 4*x^2],x]
[Out] -1/40*(Sqrt[9 - 4*x^2]*(27 + 6*x^2 + 2*x^4))
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{9-4x^2}\left(-2x^4-6x^2-27\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[9 - 4*x^2],x]
[Out] (Sqrt[9 - 4*x^2]*(-27 - 6*x^2 - 2*x^4))/40
```

fricas [A] time = 1.10, size = 23, normalized size = 0.50

$$-\frac{1}{40}\left(2x^4+6x^2+27\right)\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/40*(2*x^4 + 6*x^2 + 27)*sqrt(-4*x^2 + 9)
```

giac [A] time = 1.24, size = 43, normalized size = 0.93

$$-\frac{1}{320}\left(4x^2-9\right)^2\sqrt{-4x^2+9}+\frac{3}{32}\left(-4x^2+9\right)^{\frac{3}{2}}-\frac{81}{64}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/320*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9) + 3/32*(-4*x^2 + 9)^(3/2) - 81/64*sqr
t(-4*x^2 + 9)
```

maple [A] time = 0.00, size = 34, normalized size = 0.74

$$\frac{(2x-3)(2x+3)\left(2x^4+6x^2+27\right)}{40\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/40*(2*x-3)*(2*x+3)*(2*x^4+6*x^2+27)/(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.95, size = 40, normalized size = 0.87

$$-\frac{1}{20}\sqrt{-4x^2+9}\,x^4-\frac{3}{20}\sqrt{-4x^2+9}\,x^2-\frac{27}{40}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(-4*x^2 + 9)*x^4 - 3/20*sqrt(-4*x^2 + 9)*x^2 - 27/40*sqrt(-4*x^2
+ 9)
```

mupad [B] time = 0.04, size = 23, normalized size = 0.50

$$-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{x^4}{5}+\frac{3x^2}{5}+\frac{27}{10}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*((3*x^2)/5 + x^4/5 + 27/10))/2
```

sympy [A] time = 1.29, size = 46, normalized size = 1.00

$$-\frac{x^4\sqrt{9-4x^2}}{20}-\frac{3x^2\sqrt{9-4x^2}}{20}-\frac{27\sqrt{9-4x^2}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x**4*sqrt(9 - 4*x**2)/20 - 3*x**2*sqrt(9 - 4*x**2)/20 - 27*sqrt(9 - 4*x**2
)/40
```

3.535 $\int \frac{x^4}{\sqrt{9-4x^2}} \, dx$

Optimal. Leaf size=45

$$-\frac{27}{128}\sqrt{9-4x^2}\,x-\frac{1}{16}\sqrt{9-4x^2}\,x^3+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 216}

$$-\frac{1}{16}\sqrt{9-4x^2}\,x^3-\frac{27}{128}\sqrt{9-4x^2}\,x+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[9 - 4*x^2],x]
```

```
[Out] (-27*x*Sqrt[9 - 4*x^2])/128 - (x^3*Sqrt[9 - 4*x^2])/16 + (243*ArcSin[(2*x)/
3])/256
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{9-4x^2}}\,dx &= -\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{27}{16}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9-4x^2}-\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{243}{128}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{27}{128}x\sqrt{9-4x^2}-\frac{1}{16}x^3\sqrt{9-4x^2}+\frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.76

$$\frac{1}{256}\left(243\sin^{-1}\left(\frac{2x}{3}\right)-2x\sqrt{9-4x^2}\left(8x^2+27\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[9 - 4*x^2],x]
[Out] (-2*x*Sqrt[9 - 4*x^2]*(27 + 8*x^2) + 243*ArcSin[(2*x)/3])/256
```

IntegrateAlgebraic [A] time = 0.09, size = 48, normalized size = 1.07

$$\frac{243}{128}\tan^{-1}\left(\frac{2x}{\sqrt{9-4x^2}-3}\right)+\frac{1}{128}\sqrt{9-4x^2}\left(-8x^3-27x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[9 - 4*x^2],x]
[Out] (Sqrt[9 - 4*x^2]*(-27*x - 8*x^3))/128 + (243*ArcTan[(2*x)/(-3 + Sqrt[9 - 4*
x^2])])/128
```

fricas [A] time = 0.76, size = 40, normalized size = 0.89

$$-\frac{1}{128}\left(8x^3+27x\right)\sqrt{-4x^2+9}-\frac{243}{128}\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/128*(8*x^3 + 27*x)*sqrt(-4*x^2 + 9) - 243/128*arctan(1/2*(sqrt(-4*x^2 +
9) - 3)/x)
```

giac [A] time = 1.14, size = 26, normalized size = 0.58

$$-\frac{1}{128}\left(8x^2+27\right)\sqrt{-4x^2+9}\,x+\frac{243}{256}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/128*(8*x^2 + 27)*sqrt(-4*x^2 + 9)*x + 243/256*arcsin(2/3*x)
```

maple [A] time = 0.01, size = 34, normalized size = 0.76

$$-\frac{\sqrt{-4x^2+9}\,x^3}{16}-\frac{27\sqrt{-4x^2+9}\,x}{128}+\frac{243\arcsin\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-4*x^2+9)^(1/2),x)
```

```
[Out] 243/256*arcsin(2/3*x)-27/128*(-4*x^2+9)^(1/2)*x-1/16*x^3*(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.93, size = 33, normalized size = 0.73

$$-\frac{1}{16}\sqrt{-4\,x^2+9}\,x^3-\frac{27}{128}\sqrt{-4\,x^2+9}\,x+\frac{243}{256}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(-4*x^2 + 9)*x^3 - 27/128*sqrt(-4*x^2 + 9)*x + 243/256*arcsin(2/3
*x)
```

mupad [B] time = 0.03, size = 27, normalized size = 0.60

$$\frac{243\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{256}-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{x^3}{4}+\frac{27\,x}{32}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(9 - 4*x^2)^(1/2),x)
```

```
[Out] (243*asin((2*x)/3))/256 - ((9/4 - x^2)^(1/2)*((27*x)/32 + x^3/4))/2
```

sympy [A] time = 0.73, size = 39, normalized size = 0.87

$$-\frac{x^3\sqrt{9-4x^2}}{16}-\frac{27x\sqrt{9-4x^2}}{128}+\frac{243\,\mathrm{asin}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x**3*sqrt(9 - 4*x**2)/16 - 27*x*sqrt(9 - 4*x**2)/128 + 243*asin(2*x/3)/256
```

3.536 $\int \frac{x^3}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(9-4x^2\right)^{3/2}-\frac{9}{16}\sqrt{9-4x^2}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(9-4x^2\right)^{3/2}-\frac{9}{16}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[9 - 4*x^2],x]
```

```
[Out] (-9*Sqrt[9 - 4*x^2])/16 + (9 - 4*x^2)^(3/2)/48
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{9-4x}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{9}{4\sqrt{9-4x}}-\frac{1}{4}\sqrt{9-4x}\right)dx, x, x^2\right)\\ &= -\frac{9}{16}\sqrt{9-4x^2}+\frac{1}{48}\left(9-4x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$-\frac{1}{24}\sqrt{9-4x^2}\left(2x^2+9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[9 - 4*x^2],x]
[Out] -1/24*(Sqrt[9 - 4*x^2]*(9 + 2*x^2))
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\sqrt{9-4x^2}\left(-2x^2-9\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[9 - 4*x^2],x]
[Out] (Sqrt[9 - 4*x^2]*(-9 - 2*x^2))/24
```

fricas [A] time = 1.17, size = 18, normalized size = 0.58

$$-\frac{1}{24}\left(2x^2+9\right)\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/24*(2*x^2 + 9)*sqrt(-4*x^2 + 9)
```

giac [A] time = 0.98, size = 23, normalized size = 0.74

$$\frac{1}{48}\left(-4x^2+9\right)^{\frac{3}{2}}-\frac{9}{16}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/48*(-4*x^2 + 9)^(3/2) - 9/16*sqrt(-4*x^2 + 9)
```

maple [A] time = 0.01, size = 29, normalized size = 0.94

$$\frac{(2x-3)(2x+3)\left(2x^2+9\right)}{24\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/24*(2*x-3)*(2*x+3)*(2*x^2+9)/(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.90, size = 26, normalized size = 0.84

$$-\frac{1}{12}\sqrt{-4x^2+9}\,x^2-\frac{3}{8}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(-4*x^2 + 9)*x^2 - 3/8*sqrt(-4*x^2 + 9)
```

mupad [B] time = 0.02, size = 18, normalized size = 0.58

$$-\frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{x^2}{3}+\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(9 - 4*x^2)^(1/2),x)
```

```
[Out] -((9/4 - x^2)^(1/2)*(x^2/3 + 3/2))/2
```

sympy [A] time = 0.38, size = 29, normalized size = 0.94

$$-\frac{x^2\sqrt{9-4x^2}}{12}-\frac{3\sqrt{9-4x^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**2+9)**(1/2),x)
```

```
[Out] -x**2*sqrt(9 - 4*x**2)/12 - 3*sqrt(9 - 4*x**2)/8
```

3.537 $\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=27

$$\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 216}

$$\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[9 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[9 - 4*x^2])/8 + (9*ArcSin[(2*x)/3])/16
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{9-4x^2}}\,dx &= -\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{8}\int \frac{1}{\sqrt{9-4x^2}}\,dx\\ &= -\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{9}{16}\sin^{-1}\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[9 - 4*x^2],x]
```

```
[Out] -1/8*(x*Sqrt[9 - 4*x^2]) + (9*ArcSin[(2*x)/3])/16
```

IntegrateAlgebraic [A] time = 0.06, size = 40, normalized size = 1.48

$$\frac{9}{8}\tan^{-1}\left(\frac{2x}{\sqrt{9-4x^2}-3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[9 - 4*x^2],x]
```

```
[Out] -1/8*(x*Sqrt[9 - 4*x^2]) + (9*ArcTan[(2*x)/(-3 + Sqrt[9 - 4*x^2])])/8
```

fricas [A] time = 0.95, size = 32, normalized size = 1.19

$$-\frac{1}{8}\sqrt{-4x^2+9}\,x-\frac{9}{8}\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 + 9)*x - 9/8*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

giac [A] time = 1.18, size = 19, normalized size = 0.70

$$-\frac{1}{8}\sqrt{-4x^2+9}\,x+\frac{9}{16}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^2 + 9)*x + 9/16*arcsin(2/3*x)
```

maple [A] time = 0.01, size = 20, normalized size = 0.74

$$-\frac{\sqrt{-4x^2+9}\,x}{8}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2+9)^(1/2),x)
```

```
[Out] 9/16*arcsin(2/3*x)-1/8*(-4*x^2+9)^(1/2)*x
```

maxima [A] time = 2.88, size = 19, normalized size = 0.70

$$-\frac{1}{8}\sqrt{-4x^2+9}\,x+\frac{9}{16}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/8*sqrt(-4*x^2 + 9)*x + 9/16*arcsin(2/3*x)
```

mupad [B] time = 0.02, size = 19, normalized size = 0.70

$$\frac{9\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{16}-\frac{x\,\sqrt{\frac{9}{4}-x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(9 - 4*x^2)^(1/2),x)
[Out] (9*asin((2*x)/3))/16 - (x*(9/4 - x^2)^(1/2))/4
```

sympy [A] time = 0.23, size = 22, normalized size = 0.81

$$-\frac{x\sqrt{9-4x^2}}{8}+\frac{9\,\mathrm{asin}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2+9)**(1/2),x)
[Out] -x*sqrt(9 - 4*x**2)/8 + 9*asin(2*x/3)/16
```

3.538 $\int \frac{x}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[9 - 4*x^2],x]
```

```
[Out] -Sqrt[9 - 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{1}{4}\sqrt{9-4x^2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[9 - 4*x^2],x]
```

```
[Out] -1/4*Sqrt[9 - 4*x^2]
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[9 - 4*x^2],x]
```

```
[Out] -1/4*Sqrt[9 - 4*x^2]
```

fricas [A] time = 1.05, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-4*x^2 + 9)
```

giac [A] time = 1.13, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-4*x^2 + 9)
```

maple [A] time = 0.00, size = 22, normalized size = 1.47

$$\frac{(2x-3)(2x+3)}{4\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/4*(2*x-3)*(2*x+3)/(-4*x^2+9)^(1/2)
```

maxima [A] time = 1.33, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-4*x^2 + 9)
```

mupad [B] time = 4.56, size = 11, normalized size = 0.73

$$-\frac{\sqrt{\frac{9}{4}-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(9 - 4*x^2)^(1/2),x)
[Out] -(9/4 - x^2)^(1/2)/2
```

sympy [A] time = 0.15, size = 12, normalized size = 0.80

$$-\frac{\sqrt{9-4x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x**2+9)**(1/2),x)
[Out] -sqrt(9 - 4*x**2)/4
```

3.539 $\int \frac{1}{\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 - 4*x^2],x]
```

```
[Out] ArcSin[(2*x)/3]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 - 4*x^2],x]
```

```
[Out] ArcSin[(2*x)/3]/2
```

IntegrateAlgebraic [C] time = 0.03, size = 24, normalized size = 2.40

$$\frac{1}{2}i\log\left(\sqrt{9-4x^2}-2ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[9 - 4*x^2],x]
[Out] (I/2)*Log[(-2*I)*x + Sqrt[9 - 4*x^2]]
```

fricas [B] time = 0.72, size = 19, normalized size = 1.90

$$-\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

giac [A] time = 1.12, size = 6, normalized size = 0.60

$$\frac{1}{2}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 1/2*arcsin(2/3*x)
```

maple [A] time = 0.00, size = 7, normalized size = 0.70

$$\frac{\arcsin\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+9)^(1/2),x)
[Out] 1/2*arcsin(2/3*x)
```

maxima [A] time = 2.98, size = 6, normalized size = 0.60

$$\frac{1}{2}\arcsin\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsin(2/3*x)
```

mupad [B] time = 0.01, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(\frac{2\,x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9 - 4*x^2)^(1/2),x)
[Out] asin((2*x)/3)/2
```

sympy [A] time = 0.15, size = 7, normalized size = 0.70

$$\frac{\operatorname{asin}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+9)**(1/2),x)
[Out] asin(2*x/3)/2
```

3.540 $\int \frac{1}{x\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 206}

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[9 - 4*x^2]/3]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{9-4x^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\right)$$
$$= -\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[9 - 4*x^2]),x]
[Out] -1/3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[9 - 4*x^2]),x]
[Out] -1/3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

fricas [A] time = 0.85, size = 18, normalized size = 0.90

$$\frac{1}{3}\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] 1/3*log((sqrt(-4*x^2 + 9) - 3)/x)
```

giac [**B**] time = 0.94, size = 31, normalized size = 1.55

$$-\frac{1}{6}\,\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{1}{6}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] -1/6*log(sqrt(-4*x^2 + 9) + 3) + 1/6*log(-sqrt(-4*x^2 + 9) + 3)
```

maple [**A**] time = 0.00, size = 15, normalized size = 0.75

$$-\frac{\operatorname{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-4*x^2+9)^(1/2),x)
[Out] -1/3*arctanh(3/(-4*x^2+9)^(1/2))
```

maxima [**A**] time = 2.99, size = 25, normalized size = 1.25

$$-\frac{1}{3}\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="maxima")
[Out] -1/3*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [**B**] time = 0.12, size = 20, normalized size = 1.00

$$\frac{\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(9 - 4*x^2)^(1/2)),x)
[Out] log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2)/3
```

sympy [A] time = 1.06, size = 26, normalized size = 1.30

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-acosh(3/(2*x))/3, 9/(4*Abs(x**2)) > 1), (I*asin(3/(2*x))/3, Tru
e))
```

3.541 $\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{9-4x^2}}{9x}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -1/9*Sqrt[9 - 4*x^2]/x
```

IntegrateAlgebraic [A] time = 0.04, size = 18, normalized size = 1.00

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[9 - 4*x^2]),x]
[Out] -1/9*Sqrt[9 - 4*x^2]/x
```

fricas [A] time = 1.09, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="fricas")
[Out] -1/9*sqrt(-4*x^2 + 9)/x
```

giac [B] time = 1.12, size = 33, normalized size = 1.83

$$\frac{2\,x}{9\left(\sqrt{-4\,x^2+9}-3\right)}-\frac{\sqrt{-4\,x^2+9}-3}{18\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="giac")
[Out] 2/9*x/(sqrt(-4*x^2 + 9) - 3) - 1/18*(sqrt(-4*x^2 + 9) - 3)/x
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$\frac{(2x-3)\,(2x+3)}{9\sqrt{-4x^2+9}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-4*x^2+9)^(1/2),x)
[Out] 1/9/x*(2*x-3)*(2*x+3)/(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.90, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-4\,x^2+9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-4*x^2 + 9)/x
```

mupad [B] time = 0.02, size = 14, normalized size = 0.78

$$-\frac{2\sqrt{\frac{9}{4}-x^2}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] -(2*(9/4 - x^2)^(1/2))/(9*x)
```

sympy [A] time = 0.80, size = 41, normalized size = 2.28

$$\begin{cases} -\frac{2\sqrt{-1+\frac{9}{4x^2}}}{9} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ -\frac{2i\sqrt{1-\frac{9}{4x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 9/(4*x**2))/9, 9/(4*Abs(x**2)) > 1), (-2*I*sqrt(1 -
 9/(4*x**2))/9, True))
```

3.542 $\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 206}

$$-\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(18*x^2) - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/27
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{18x^2} + \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{18x^2} - \frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\\
&= -\frac{\sqrt{9-4x^2}}{18x^2} - \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.95

$$\frac{1}{54}\left(-\frac{3\sqrt{9-4x^2}}{x^2} - 4\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[9 - 4*x^2]),x]
[Out] ((-3*Sqrt[9 - 4*x^2])/x^2 - 4*ArcTanh[Sqrt[1 - (4*x^2)/9]])/54
```

IntegrateAlgebraic [A] time = 0.04, size = 39, normalized size = 1.00

$$-\frac{\sqrt{9-4x^2}}{18x^2} - \frac{2}{27}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[9 - 4*x^2]),x]
[Out] -1/18*Sqrt[9 - 4*x^2]/x^2 - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/27
```

fricas [A] time = 1.36, size = 38, normalized size = 0.97

$$\frac{4\,x^2\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right) - 3\,\sqrt{-4\,x^2+9}}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(4*x^2*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*sqrt(-4*x^2 + 9))/x^2
```

giac [A] time = 1.21, size = 45, normalized size = 1.15

$$-\frac{\sqrt{-4\,x^2+9}}{18\,x^2}-\frac{1}{27}\,\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{1}{27}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(-4*x^2 + 9)/x^2 - 1/27*log(sqrt(-4*x^2 + 9) + 3) + 1/27*log(-sqr
t(-4*x^2 + 9) + 3)
```

maple [A] time = 0.01, size = 30, normalized size = 0.77

$$-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}-\frac{\sqrt{-4x^2+9}}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-4*x^2+9)^(1/2),x)
```

```
[Out] -1/18*(-4*x^2+9)^(1/2)/x^2-2/27*arctanh(3/(-4*x^2+9)^(1/2))
```

maxima [A] time = 2.93, size = 40, normalized size = 1.03

$$-\frac{\sqrt{-4\,x^2+9}}{18\,x^2}-\frac{2}{27}\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(-4*x^2 + 9)/x^2 - 2/27*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x)
)
```

mupad [B] time = 0.03, size = 35, normalized size = 0.90

$$\frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)}{27}-\frac{\sqrt{\frac{9}{4}-x^2}}{9\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2))/27 - (9/4 - x^2)^(1/2)
/(9*x^2)
```

sympy [A] time = 2.17, size = 99, normalized size = 2.54

$$\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{27} + \frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ \frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{1-\frac{9}{4x^2}}} + \frac{i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(3/(2*x))/27 + 1/(9*x*sqrt(-1 + 9/(4*x**2))) - 1/(4*x**3
*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (2*I*asin(3/(2*x))/27 - I/(9
*x*sqrt(1 - 9/(4*x**2))) + I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

3.543 $\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=37

$$-\frac{8\sqrt{9-4x^2}}{243x}-\frac{\sqrt{9-4x^2}}{27x^3}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$-\frac{8\sqrt{9-4x^2}}{243x}-\frac{\sqrt{9-4x^2}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(27*x^3) - (8*Sqrt[9 - 4*x^2])/(243*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx&=-\frac{\sqrt{9-4x^2}}{27x^3}+\frac{8}{27}\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx\\&=-\frac{\sqrt{9-4x^2}}{27x^3}-\frac{8\sqrt{9-4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 0.73

$$-\frac{\sqrt{1-\frac{4x^2}{9}}\,(8x^2+9)}{81x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -1/81*(Sqrt[1 - (4*x^2)/9]*(9 + 8*x^2))/x^3
```

IntegrateAlgebraic [A] time = 0.06, size = 25, normalized size = 0.68

$$\frac{\left(-8x^2-9\right)\sqrt{9-4x^2}}{243x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[9 - 4*x^2]),x]
```

```
[Out] ((-9 - 8*x^2)*Sqrt[9 - 4*x^2])/(243*x^3)
```

fricas [A] time = 1.20, size = 21, normalized size = 0.57

$$-\frac{\left(8\,x^2+9\right)\sqrt{-4\,x^2+9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/243*(8*x^2 + 9)*sqrt(-4*x^2 + 9)/x^3
```

giac [B] time = 1.13, size = 73, normalized size = 1.97

$$\frac{2\,x^3\left(\frac{9\left(\sqrt{-4\,x^2+9}-3\right)^2}{x^2}+4\right)}{243\left(\sqrt{-4\,x^2+9}-3\right)^3}-\frac{\sqrt{-4\,x^2+9}-3}{54\,x}-\frac{\left(\sqrt{-4\,x^2+9}-3\right)^3}{1944\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 2/243*x^3*(9*(sqrt(-4*x^2 + 9) - 3)^2/x^2 + 4)/(sqrt(-4*x^2 + 9) - 3)^3 - 1
/54*(sqrt(-4*x^2 + 9) - 3)/x - 1/1944*(sqrt(-4*x^2 + 9) - 3)^3/x^3
```

maple [A] time = 0.00, size = 32, normalized size = 0.86

$$\frac{(2x-3)\,(2x+3)\left(8x^2+9\right)}{243\sqrt{-4x^2+9}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-4*x^2+9)^(1/2),x)
```

```
[Out] 1/243*(2*x-3)*(2*x+3)*(8*x^2+9)/x^3/(-4*x^2+9)^(1/2)
```

maxima [A] time = 2.97, size = 29, normalized size = 0.78

$$-\frac{8\sqrt{-4x^2+9}}{243x}-\frac{\sqrt{-4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/243*sqrt(-4*x^2 + 9)/x - 1/27*sqrt(-4*x^2 + 9)/x^3
```

mupad [B] time = 0.02, size = 22, normalized size = 0.59

$$-\sqrt{\frac{9}{4}-x^2}\left(\frac{16}{243x}+\frac{2}{27x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] -(9/4 - x^2)^(1/2)*(16/(243*x) + 2/(27*x^3))
```

sympy [A] time = 1.33, size = 80, normalized size = 2.16

$$\begin{cases} -\frac{16\sqrt{-1+\frac{9}{4x^2}}}{243}-\frac{2\sqrt{-1+\frac{9}{4x^2}}}{27x^2} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{16i\sqrt{1-\frac{9}{4x^2}}}{243}-\frac{2i\sqrt{1-\frac{9}{4x^2}}}{27x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-16*sqrt(-1 + 9/(4*x**2))/243 - 2*sqrt(-1 + 9/(4*x**2))/(27*x**2
), 9/(4*Abs(x**2)) > 1), (-16*I*sqrt(1 - 9/(4*x**2))/243 - 2*I*sqrt(1 - 9/(
4*x**2))/(27*x**2), True))
```

3.544 $\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{36x^4}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 206}

$$-\frac{\sqrt{9-4x^2}}{54x^2}-\frac{\sqrt{9-4x^2}}{36x^4}-\frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[9 - 4*x^2]),x]
```

```
[Out] -Sqrt[9 - 4*x^2]/(36*x^4) - Sqrt[9 - 4*x^2]/(54*x^2) - (2*ArcTanh[Sqrt[9 -
4*x^2]/3])/81
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^3}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{36x^4} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{36x^4} - \frac{\sqrt{9-4x^2}}{54x^2} + \frac{1}{27}\operatorname{Subst}\left(\int \frac{1}{\sqrt{9-4x}\,x}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{9-4x^2}}{36x^4} - \frac{\sqrt{9-4x^2}}{54x^2} - \frac{1}{54}\operatorname{Subst}\left(\int \frac{1}{\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{9-4x^2}\right)\\
&= -\frac{\sqrt{9-4x^2}}{36x^4} - \frac{\sqrt{9-4x^2}}{54x^2} - \frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$-\frac{16}{729}\sqrt{9-4x^2}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{4x^2}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[9 - 4*x^2]),x]
[Out] (-16*Sqrt[9 - 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (4*x^2)/9])/729
```

IntegrateAlgebraic [A] time = 0.04, size = 46, normalized size = 0.81

$$\frac{\sqrt{9-4x^2}\left(-2x^2-3\right)}{108x^4} - \frac{2}{81}\tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[9 - 4*x^2]),x]
```

[Out] (Sqrt[9 - 4*x^2]*(-3 - 2*x^2))/(108*x^4) - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/81

fricas [A] time = 0.92, size = 45, normalized size = 0.79

$$\frac{8\,x^4\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\left(2\,x^2+3\right)\sqrt{-4\,x^2+9}}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="fricas")

[Out] 1/324*(8*x^4*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 + 3)*sqrt(-4*x^2 + 9))/x^4

giac [A] time = 0.98, size = 57, normalized size = 1.00

$$\frac{\left(-4\,x^2+9\right)^{\frac{3}{2}}-15\,\sqrt{-4\,x^2+9}}{216\,x^4}-\frac{1}{81}\,\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{1}{81}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="giac")

[Out] 1/216*((-4*x^2 + 9)^(3/2) - 15*sqrt(-4*x^2 + 9))/x^4 - 1/81*log(sqrt(-4*x^2 + 9) + 3) + 1/81*log(-sqrt(-4*x^2 + 9) + 3)

maple [A] time = 0.00, size = 44, normalized size = 0.77

$$-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{81}-\frac{\sqrt{-4x^2+9}}{54x^2}-\frac{\sqrt{-4x^2+9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(-4*x^2+9)^(1/2),x)

[Out] -1/36*(-4*x^2+9)^(1/2)/x^4-1/54*(-4*x^2+9)^(1/2)/x^2-2/81*arctanh(3/(-4*x^2+9)^(1/2))

maxima [A] time = 2.93, size = 54, normalized size = 0.95

$$-\frac{\sqrt{-4\,x^2+9}}{54\,x^2}-\frac{\sqrt{-4\,x^2+9}}{36\,x^4}-\frac{2}{81}\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(-4*x^2 + 9)/x^2 - 1/36*sqrt(-4*x^2 + 9)/x^4 - 2/81*log(6*sqrt(-4
*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 4.51, size = 49, normalized size = 0.86

$$\frac{2\ln\left(\sqrt{\frac{9}{4x^2}-1}-\sqrt{\frac{9}{4x^2}}\right)}{81}-\frac{\sqrt{\frac{9}{4}-x^2}\left(\frac{2}{27x^2}+\frac{1}{9x^4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(9 - 4*x^2)^(1/2)),x)
```

```
[Out] (2*log((9/(4*x^2) - 1)^(1/2) - (9/(4*x^2))^(1/2)))/81 - ((9/4 - x^2)^(1/2)*
(2/(27*x^2) + 1/(9*x^4)))/2
```

sympy [A] time = 3.96, size = 136, normalized size = 2.39

$$\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{81}+\frac{1}{27x\sqrt{-1+\frac{9}{4x^2}}}-\frac{1}{36x^3\sqrt{-1+\frac{9}{4x^2}}}-\frac{1}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{9}{4|x^2|}>1 \\ \frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{1-\frac{9}{4x^2}}}+\frac{i}{36x^3\sqrt{1-\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-4*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(3/(2*x))/81 + 1/(27*x*sqrt(-1 + 9/(4*x**2))) - 1/(36*x*
*3*sqrt(-1 + 9/(4*x**2))) - 1/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2
)) > 1), (2*I*asin(3/(2*x))/81 - I/(27*x*sqrt(1 - 9/(4*x**2))) + I/(36*x**3
*sqrt(1 - 9/(4*x**2))) + I/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

3.545 $\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{1}{320}\left(4x^2-9\right)^{5/2}+\frac{3}{32}\left(4x^2-9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2-9}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{320}\left(4x^2-9\right)^{5/2}+\frac{3}{32}\left(4x^2-9\right)^{3/2}+\frac{81}{64}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (81*Sqrt[-9 + 4*x^2])/64 + (3*(-9 + 4*x^2)^(3/2))/32 + (-9 + 4*x^2)^(5/2)/3
20
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{81}{16\sqrt{-9+4x}}+\frac{9}{8}\sqrt{-9+4x}+\frac{1}{16}(-9+4x)^{3/2}\right)dx, x, x^2\right)\\
&= \frac{81}{64}\sqrt{-9+4x^2}+\frac{3}{32}\left(-9+4x^2\right)^{3/2}+\frac{1}{320}\left(-9+4x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{4x^2-9}\left(2x^4+6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-9 + 4*x^2],x]
[Out] (Sqrt[-9 + 4*x^2]*(27 + 6*x^2 + 2*x^4))/40
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{4x^2-9}\left(2x^4+6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[-9 + 4*x^2],x]
[Out] (Sqrt[-9 + 4*x^2]*(27 + 6*x^2 + 2*x^4))/40
```

fricas [A] time = 1.07, size = 23, normalized size = 0.50

$$\frac{1}{40}\left(2x^4+6x^2+27\right)\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/40*(2*x^4 + 6*x^2 + 27)*sqrt(4*x^2 - 9)
```

giac [A] time = 1.12, size = 34, normalized size = 0.74

$$\frac{1}{320}\left(4x^2-9\right)^{\frac{5}{2}}+\frac{3}{32}\left(4x^2-9\right)^{\frac{3}{2}}+\frac{81}{64}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/320*(4*x^2 - 9)^(5/2) + 3/32*(4*x^2 - 9)^(3/2) + 81/64*sqrt(4*x^2 - 9)
```

maple [A] time = 0.00, size = 34, normalized size = 0.74

$$\frac{(2x-3)(2x+3)\left(2x^4+6x^2+27\right)}{40\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/40*(2*x-3)*(2*x+3)*(2*x^4+6*x^2+27)/(4*x^2-9)^(1/2)
```

maxima [A] time = 2.93, size = 40, normalized size = 0.87

$$\frac{1}{20}\sqrt{4x^2-9}\,x^4+\frac{3}{20}\sqrt{4x^2-9}\,x^2+\frac{27}{40}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(4*x^2 - 9)*x^4 + 3/20*sqrt(4*x^2 - 9)*x^2 + 27/40*sqrt(4*x^2 - 9)
```

mupad [B] time = 4.74, size = 22, normalized size = 0.48

$$\sqrt{4x^2-9}\,\left(\frac{x^4}{20}+\frac{3x^2}{20}+\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^2 - 9)^(1/2),x)
```

```
[Out] (4*x^2 - 9)^(1/2)*((3*x^2)/20 + x^4/20 + 27/40)
```

sympy [A] time = 1.25, size = 44, normalized size = 0.96

$$\frac{x^4\sqrt{4x^2-9}}{20}+\frac{3x^2\sqrt{4x^2-9}}{20}+\frac{27\sqrt{4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(4*x**2-9)**(1/2),x)
```

```
[Out] x**4*sqrt(4*x**2 - 9)/20 + 3*x**2*sqrt(4*x**2 - 9)/20 + 27*sqrt(4*x**2 - 9)
/40
```

3.546 $\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{27}{128}\sqrt{4x^2-9}\,x+\frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{16}\sqrt{4x^2-9}\,x^3$$

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 206}

$$\frac{1}{16}\sqrt{4x^2-9}\,x^3+\frac{27}{128}\sqrt{4x^2-9}\,x+\frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (27*x*Sqrt[-9 + 4*x^2])/128 + (x^3*Sqrt[-9 + 4*x^2])/16 + (243*ArcTanh[(2*x
)/Sqrt[-9 + 4*x^2]])/256
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx = \frac{1}{16}x^3\sqrt{-9+4x^2} + \frac{27}{16}\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx$$
$$= \frac{27}{128}x\sqrt{-9+4x^2} + \frac{1}{16}x^3\sqrt{-9+4x^2} + \frac{243}{128}\int \frac{1}{\sqrt{-9+4x^2}}\,dx$$
$$= \frac{27}{128}x\sqrt{-9+4x^2} + \frac{1}{16}x^3\sqrt{-9+4x^2} + \frac{243}{128}\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)$$
$$= \frac{27}{128}x\sqrt{-9+4x^2} + \frac{1}{16}x^3\sqrt{-9+4x^2} + \frac{243}{256}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.80

$$\frac{1}{256}\left(2x\sqrt{4x^2-9}\,\left(8x^2+27\right)+243\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (2*x*Sqrt[-9 + 4*x^2]*(27 + 8*x^2) + 243*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/2
56
```

IntegrateAlgebraic [A] time = 0.06, size = 45, normalized size = 0.83

$$\frac{1}{128}\sqrt{4x^2-9}\,\left(8x^3+27x\right)-\frac{243}{256}\log\left(\sqrt{4x^2-9}\,-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (Sqrt[-9 + 4*x^2]*(27*x + 8*x^3))/128 - (243*Log[-2*x + Sqrt[-9 + 4*x^2]])/
256
```

fricas [A] time = 1.09, size = 37, normalized size = 0.69

$$\frac{1}{128}\left(8\,x^3+27\,x\right)\sqrt{4\,x^2-9}-\frac{243}{256}\,\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*x^3 + 27*x)*sqrt(4*x^2 - 9) - 243/256*log(-2*x + sqrt(4*x^2 - 9))
```

giac [A] time = 1.24, size = 37, normalized size = 0.69

$$\frac{1}{128}\left(8\,x^2+27\right)\sqrt{4\,x^2-9}\,x-\frac{243}{256}\,\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*(8*x^2 + 27)*sqrt(4*x^2 - 9)*x - 243/256*log(abs(-2*x + sqrt(4*x^2 -
9)))
```

maple [A] time = 0.01, size = 49, normalized size = 0.91

$$\frac{\sqrt{4x^2-9}\;x^3}{16}+\frac{27\sqrt{4x^2-9}\;x}{128}+\frac{243\sqrt{4}\;\ln\left(\sqrt{4}\;x+\sqrt{4x^2-9}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/16*x^3*(4*x^2-9)^(1/2)+27/128*(4*x^2-9)^(1/2)*x+243/512*4^(1/2)*ln(4^(1/2
)*x+(4*x^2-9)^(1/2))
```

maxima [A] time = 2.81, size = 45, normalized size = 0.83

$$\frac{1}{16}\,\sqrt{4\,x^2-9}\,x^3+\frac{27}{128}\,\sqrt{4\,x^2-9}\,x+\frac{243}{256}\,\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(4*x^2 - 9)*x^3 + 27/128*sqrt(4*x^2 - 9)*x + 243/256*log(8*x + 4*s
qrt(4*x^2 - 9))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\sqrt{4\,x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^2 - 9)^(1/2),x)
```

```
[Out] int(x^4/(4*x^2 - 9)^(1/2), x)
```

sympy [A] time = 0.72, size = 39, normalized size = 0.72

$$\frac{x^3\sqrt{4x^2-9}}{16}+\frac{27x\sqrt{4x^2-9}}{128}+\frac{243\operatorname{acosh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(4*x**2-9)**(1/2),x)
```

```
[Out] x**3*sqrt(4*x**2 - 9)/16 + 27*x*sqrt(4*x**2 - 9)/128 + 243*acosh(2*x/3)/256
```

3.547 $\int \frac{x^3}{\sqrt{-9+4x^2}} \, dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{4x^2-9}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (9*Sqrt[-9 + 4*x^2])/16 + (-9 + 4*x^2)^(3/2)/48
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{-9+4x^2}} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\sqrt{-9+4x}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{9}{4\sqrt{-9+4x}}+\frac{1}{4}\sqrt{-9+4x}\right) dx, x, x^2\right)\\
&= \frac{9}{16}\sqrt{-9+4x^2}+\frac{1}{48}\left(-9+4x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\left(2x^2+9\right)\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-9 + 4*x^2],x]
[Out] ((9 + 2*x^2)*Sqrt[-9 + 4*x^2])/24
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\left(2x^2+9\right)\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[-9 + 4*x^2],x]
[Out] ((9 + 2*x^2)*Sqrt[-9 + 4*x^2])/24
```

fricas [A] time = 0.90, size = 18, normalized size = 0.58

$$\frac{1}{24}\sqrt{4\,x^2-9}\left(2\,x^2+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/24*sqrt(4*x^2 - 9)*(2*x^2 + 9)
```

giac [A] time = 0.97, size = 23, normalized size = 0.74

$$\frac{1}{48}\left(4\,x^2-9\right)^{\frac{3}{2}}+\frac{9}{16}\sqrt{4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/48*(4*x^2 - 9)^(3/2) + 9/16*sqrt(4*x^2 - 9)
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$\frac{(2x-3)\,(2x+3)\left(2x^2+9\right)}{24\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2-9)^(1/2),x)
[Out] 1/24*(2*x-3)*(2*x+3)*(2*x^2+9)/(4*x^2-9)^(1/2)
```

maxima [A] time = 2.92, size = 26, normalized size = 0.84

$$\frac{1}{12}\sqrt{4x^2-9}\,x^2+\frac{3}{8}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/12*sqrt(4*x^2 - 9)*x^2 + 3/8*sqrt(4*x^2 - 9)
```

mupad [B] time = 4.86, size = 18, normalized size = 0.58

$$\frac{\left(2x^2+9\right)\sqrt{4x^2-9}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2 - 9)^(1/2),x)
[Out] ((2*x^2 + 9)*(4*x^2 - 9)^(1/2))/24
```

sympy [A] time = 0.37, size = 27, normalized size = 0.87

$$\frac{x^2\sqrt{4x^2-9}}{12}+\frac{3\sqrt{4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**2-9)**(1/2),x)
[Out] x**2*sqrt(4*x**2 - 9)/12 + 3*sqrt(4*x**2 - 9)/8
```

3.548 $\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=36

$$\frac{1}{8}\sqrt{4x^2-9}\,x+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 206}

$$\frac{1}{8}\sqrt{4x^2-9}\,x+\frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-9 + 4*x^2],x]
```

```
[Out] (x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/16
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx &= \frac{1}{8}x\sqrt{-9+4x^2} + \frac{9}{8}\int \frac{1}{\sqrt{-9+4x^2}}\,dx \\ &= \frac{1}{8}x\sqrt{-9+4x^2} + \frac{9}{8}\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right) \\ &= \frac{1}{8}x\sqrt{-9+4x^2} + \frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{1}{8}\sqrt{4x^2-9}\,x + \frac{9}{16}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-9 + 4*x^2],x]
[Out] (x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/16
```

IntegrateAlgebraic [A] time = 0.04, size = 37, normalized size = 1.03

$$\frac{1}{8}x\sqrt{4x^2-9} - \frac{9}{16}\log\left(\sqrt{4x^2-9} - 2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[-9 + 4*x^2],x]
[Out] (x*Sqrt[-9 + 4*x^2])/8 - (9*Log[-2*x + Sqrt[-9 + 4*x^2]])/16
```

fricas [A] time = 0.96, size = 29, normalized size = 0.81

$$\frac{1}{8}\sqrt{4x^2-9}\,x - \frac{9}{16}\log\left(-2x + \sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(4*x^2 - 9)*x - 9/16*log(-2*x + sqrt(4*x^2 - 9))
```

giac [A] time = 1.18, size = 30, normalized size = 0.83

$$\frac{1}{8}\sqrt{4x^2-9}\,x - \frac{9}{16}\log\left(\left|-2x + \sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/8*sqrt(4*x^2 - 9)*x - 9/16*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

maple [A] time = 0.01, size = 35, normalized size = 0.97

$$\frac{\sqrt{4x^2-9}\;x}{8}+\frac{9\sqrt{4}\;\ln\left(\sqrt{4}\;x+\sqrt{4x^2-9}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2-9)^(1/2),x)
[Out] 1/8*(4*x^2-9)^(1/2)*x+9/32*4^(1/2)*ln(4^(1/2)*x+(4*x^2-9)^(1/2))
```

maxima [A] time = 2.91, size = 31, normalized size = 0.86

$$\frac{1}{8}\sqrt{4\,x^2-9}\,x+\frac{9}{16}\,\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/8*sqrt(4*x^2 - 9)*x + 9/16*log(8*x + 4*sqrt(4*x^2 - 9))
```

mupad [B] time = 0.10, size = 29, normalized size = 0.81

$$\frac{9\;\ln\left(x+\frac{\sqrt{4\,x^2-9}}{2}\right)}{16}+\frac{x\,\sqrt{4\,x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^2 - 9)^(1/2),x)
[Out] (9*log(x + (4*x^2 - 9)^(1/2)/2))/16 + (x*(4*x^2 - 9)^(1/2))/8
```

sympy [A] time = 0.22, size = 22, normalized size = 0.61

$$\frac{x\sqrt{4x^2-9}}{8}+\frac{9\,\operatorname{acosh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*x**2-9)**(1/2),x)
[Out] x*sqrt(4*x**2 - 9)/8 + 9*acosh(2*x/3)/16
```

3.549 $\int \frac{x}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{1}{4}\sqrt{4x^2-9}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{4}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-9 + 4*x^2],x]
```

```
[Out] Sqrt[-9 + 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{-9+4x^2}}\,dx=\frac{1}{4}\sqrt{-9+4x^2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{4}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-9 + 4*x^2],x]
```

```
[Out] Sqrt[-9 + 4*x^2]/4
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{4}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[-9 + 4*x^2],x]
```

```
[Out] Sqrt[-9 + 4*x^2]/4
```

fricas [A] time = 1.04, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(4*x^2 - 9)
```

giac [A] time = 1.07, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(4*x^2 - 9)
```

maple [A] time = 0.00, size = 22, normalized size = 1.47

$$\frac{(2x-3)(2x+3)}{4\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/4*(2*x-3)*(2*x+3)/(4*x^2-9)^(1/2)
```

maxima [A] time = 1.27, size = 11, normalized size = 0.73

$$\frac{1}{4}\sqrt{4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(4*x^2 - 9)
```

mupad [B] time = 0.14, size = 11, normalized size = 0.73

$$\frac{\sqrt{4\,x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^2 - 9)^(1/2),x)
[Out] (4*x^2 - 9)^(1/2)/4
```

sympy [A] time = 0.15, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*x**2-9)**(1/2),x)
[Out] sqrt(4*x**2 - 9)/4
```

3.550 $\int \frac{1}{\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 + 4*x^2],x]
```

```
[Out] ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9+4x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x}{\sqrt{-9+4x^2}}\right)\\ &= \frac{1}{2}\tanh^{-1}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.00, size = 43, normalized size = 2.26

$$\frac{1}{4}\log\left(\frac{2x}{\sqrt{4x^2-9}}+1\right)-\frac{1}{4}\log\left(1-\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 + 4*x^2],x]
[Out] -1/4*Log[1 - (2*x)/Sqrt[-9 + 4*x^2]] + Log[1 + (2*x)/Sqrt[-9 + 4*x^2]]/4
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.05

$$-\frac{1}{2}\log\left(\sqrt{4x^2-9}-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-9 + 4*x^2],x]
[Out] -1/2*Log[-2*x + Sqrt[-9 + 4*x^2]]
```

fricas [A] time = 0.71, size = 16, normalized size = 0.84

$$-\frac{1}{2}\log\left(-2x+\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(-2*x + sqrt(4*x^2 - 9))
```

giac [A] time = 0.98, size = 17, normalized size = 0.89

$$-\frac{1}{2}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] -1/2*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

maple [A] time = 0.00, size = 22, normalized size = 1.16

$$\frac{\sqrt{4}\,\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2-9)^(1/2),x)
[Out] 1/4*4^(1/2)*ln(4^(1/2)*x+(4*x^2-9)^(1/2))
```

maxima [A] time = 2.96, size = 18, normalized size = 0.95

$$\frac{1}{2} \log\left(8\,x + 4\,\sqrt{4\,x^2 - 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(8*x + 4*sqrt(4*x^2 - 9))
```

mupad [B] time = 4.75, size = 16, normalized size = 0.84

$$\frac{\ln\left(2\,x + \sqrt{4\,x^2 - 9}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 9)^(1/2),x)
```

```
[Out] log(2*x + (4*x^2 - 9)^(1/2))/2
```

sympy [A] time = 0.16, size = 7, normalized size = 0.37

$$\frac{\operatorname{acosh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2-9)**(1/2),x)
```

```
[Out] acosh(2*x/3)/2
```

3.551 $\int \frac{1}{x\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 203}

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-9+4x^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)$$
$$= \frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-9 + 4*x^2]),x]
[Out] ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-9 + 4*x^2]),x]
[Out] ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

fricas [A] time = 0.93, size = 18, normalized size = 0.90

$$\frac{2}{3}\arctan\left(-\frac{2}{3}x+\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 2/3*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9))
```

giac [A] time = 1.06, size = 14, normalized size = 0.70

$$\frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/3*arctan(1/3*sqrt(4*x^2 - 9))
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$-\frac{\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4*x^2-9)^(1/2),x)
[Out] -1/3*arctan(3/(4*x^2-9)^(1/2))
```

maxima [A] time = 2.91, size = 9, normalized size = 0.45

$$-\frac{1}{3}\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/3*arcsin(3/2/abs(x))
```

mupad [B] time = 0.12, size = 20, normalized size = 1.00

$$\frac{\ln\left(\frac{\sqrt{4x^2-9}+3i}{x}\right)1i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4*x^2 - 9)^(1/2)),x)
[Out] (log(((4*x^2 - 9)^(1/2) + 3i)/x)*1i)/3
```

sympy [A] time = 1.07, size = 26, normalized size = 1.30

$$\begin{cases} \frac{i\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ -\frac{\mathrm{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(3/(2*x))/3, 9/(4*Abs(x**2)) > 1), (-asin(3/(2*x))/3, Tru
e))
```

3.552 $\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{4x^2-9}}{9x}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\sqrt{4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{-9+4x^2}}{9x}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\sqrt{4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(9*x)
```

IntegrateAlgebraic [A] time = 0.03, size = 18, normalized size = 1.00

$$\frac{\sqrt{4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[-9 + 4*x^2]),x]
[Out] Sqrt[-9 + 4*x^2]/(9*x)
```

fricas [A] time = 0.89, size = 18, normalized size = 1.00

$$\frac{2x+\sqrt{4x^2-9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/9*(2*x + sqrt(4*x^2 - 9))/x
```

giac [A] time = 1.06, size = 23, normalized size = 1.28

$$\frac{4}{\left(2x-\sqrt{4x^2-9}\right)^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 4/((2*x - sqrt(4*x^2 - 9))^2 + 9)
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$\frac{(2x-3)(2x+3)}{9\sqrt{4x^2-9}\ x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4*x^2-9)^(1/2),x)
[Out] 1/9/x*(2*x-3)*(2*x+3)/(4*x^2-9)^(1/2)
```

maxima [A] time = 2.84, size = 14, normalized size = 0.78

$$\frac{\sqrt{4x^2-9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(4*x^2 - 9)/x
```

mupad [B] time = 4.75, size = 14, normalized size = 0.78

$$\frac{\sqrt{4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(4*x^2 - 9)^(1/2)),x)
```

```
[Out] (4*x^2 - 9)^(1/2)/(9*x)
```

sympy [A] time = 0.81, size = 37, normalized size = 2.06

$$\begin{cases} \frac{2i\sqrt{-1+\frac{9}{4x^2}}}{9} & \text{for}\ \frac{9}{4|x^2|} > 1 \\ \frac{2\sqrt{1-\frac{9}{4x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*sqrt(-1 + 9/(4*x**2))/9, 9/(4*Abs(x**2)) > 1), (2*sqrt(1 - 9
/(4*x**2))/9, True))
```

3.553 $\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/27
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{9}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{18x^2}+\frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\
&= \frac{\sqrt{-9+4x^2}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.38

$$\frac{4}{81}\sqrt{4x^2-9}\left(\frac{9}{8x^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{4x^2}{9}}\right)}{2\sqrt{1-\frac{4x^2}{9}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-9 + 4*x^2]),x]
[Out] (4*Sqrt[-9 + 4*x^2]*(9/(8*x^2) + ArcTanh[Sqrt[1 - (4*x^2)/9]]/(2*Sqrt[1 - (
4*x^2)/9])))/81
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 1.00

$$\frac{\sqrt{4x^2-9}}{18x^2}+\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[-9 + 4*x^2]),x]
[Out] Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/27
```

fricas [A] time = 1.39, size = 38, normalized size = 0.97

$$\frac{8\,x^2\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/54*(8*x^2*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9))/x^2
```

giac [A] time = 1.10, size = 29, normalized size = 0.74

$$\frac{\sqrt{4\,x^2-9}}{18\,x^2}+\frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/18*sqrt(4*x^2 - 9)/x^2 + 2/27*arctan(1/3*sqrt(4*x^2 - 9))
```

maple [A] time = 0.01, size = 30, normalized size = 0.77

$$-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}+\frac{\sqrt{4x^2-9}}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4*x^2-9)^(1/2),x)
[Out] 1/18*(4*x^2-9)^(1/2)/x^2-2/27*arctan(3/(4*x^2-9)^(1/2))
```

maxima [A] time = 2.93, size = 24, normalized size = 0.62

$$\frac{\sqrt{4\,x^2-9}}{18\,x^2}-\frac{2}{27}\,\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] 1/18*sqrt(4*x^2 - 9)/x^2 - 2/27*arcsin(3/2/abs(x))
```

mupad [B] time = 4.88, size = 29, normalized size = 0.74

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{27}+\frac{\sqrt{4\,x^2-9}}{18\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(4*x^2 - 9)^(1/2)),x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/27 + (4*x^2 - 9)^(1/2)/(18*x^2)
```

sympy [A] time = 2.13, size = 99, normalized size = 2.54

$$\begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{27}-\frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}}+\frac{i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4|x^2|}>1 \\ -\frac{2\,\mathrm{asin}\left(\frac{3}{2x}\right)}{27}+\frac{1}{9x\sqrt{1-\frac{9}{4x^2}}}-\frac{1}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + I/(4*x**
3*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**2)) > 1), (-2*asin(3/(2*x))/27 + 1/(9
*x*sqrt(1 - 9/(4*x**2))) - 1/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

3.554 $\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(27*x^3) + (8*Sqrt[-9 + 4*x^2])/(243*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx &= \frac{\sqrt{-9+4x^2}}{27x^3}+\frac{8}{27}\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx\\ &= \frac{\sqrt{-9+4x^2}}{27x^3}+\frac{8\sqrt{-9+4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.68

$$\frac{\sqrt{4x^2-9}\left(8x^2+9\right)}{243x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] (Sqrt[-9 + 4*x^2]*(9 + 8*x^2))/(243*x^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 25, normalized size = 0.68

$$\frac{\sqrt{4x^2-9}\left(8x^2+9\right)}{243x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] (Sqrt[-9 + 4*x^2]*(9 + 8*x^2))/(243*x^3)
```

fricas [A] time = 0.71, size = 28, normalized size = 0.76

$$\frac{16\,x^3+\left(8\,x^2+9\right)\sqrt{4\,x^2-9}}{243\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/243*(16*x^3 + (8*x^2 + 9)*sqrt(4*x^2 - 9))/x^3
```

giac [A] time = 1.13, size = 42, normalized size = 1.14

$$\frac{32\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^2+3\right)}{\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 32*((2*x - sqrt(4*x^2 - 9))^2 + 3)/((2*x - sqrt(4*x^2 - 9))^2 + 9)^3
```

maple [A] time = 0.00, size = 32, normalized size = 0.86

$$\frac{(2x-3)\,(2x+3)\left(8x^2+9\right)}{243\sqrt{4x^2-9}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/243*(2*x-3)*(2*x+3)*(8*x^2+9)/x^3/(4*x^2-9)^(1/2)
```

maxima [A] time = 2.96, size = 29, normalized size = 0.78

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/243*sqrt(4*x^2 - 9)/x + 1/27*sqrt(4*x^2 - 9)/x^3
```

mupad [B] time = 4.87, size = 31, normalized size = 0.84

$$\frac{8x^2\sqrt{4x^2-9}+9\sqrt{4x^2-9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(4*x^2 - 9)^(1/2)),x)
```

```
[Out] (8*x^2*(4*x^2 - 9)^(1/2) + 9*(4*x^2 - 9)^(1/2))/(243*x^3)
```

sympy [A] time = 1.35, size = 76, normalized size = 2.05

$$\begin{cases} \frac{16i\sqrt{-1+\frac{9}{4x^2}}}{243}+\frac{2i\sqrt{-1+\frac{9}{4x^2}}}{27x^2} & \text{for } \frac{9}{4|x^2|}>1 \\ \frac{16\sqrt{1-\frac{9}{4x^2}}}{243}+\frac{2\sqrt{1-\frac{9}{4x^2}}}{27x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((16*I*sqrt(-1 + 9/(4*x**2))/243 + 2*I*sqrt(-1 + 9/(4*x**2))/(27*x
**2), 9/(4*Abs(x**2)) > 1), (16*sqrt(1 - 9/(4*x**2))/243 + 2*sqrt(1 - 9/(4*
x**2))/(27*x**2), True))
```

3.555 $\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{4x^2-9}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)+\frac{\sqrt{4x^2-9}}{36x^4}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{4x^2-9}}{54x^2}+\frac{\sqrt{4x^2-9}}{36x^4}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] Sqrt[-9 + 4*x^2]/(36*x^4) + Sqrt[-9 + 4*x^2]/(54*x^2) + (2*ArcTan[Sqrt[-9 +
 4*x^2]/3])/81
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4} + \frac{\sqrt{-9+4x^2}}{54x^2} + \frac{1}{27}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-9+4x}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4} + \frac{\sqrt{-9+4x^2}}{54x^2} + \frac{1}{54}\,\text{Subst}\left(\int \frac{1}{\frac{9}{4}+\frac{x^2}{4}}\,dx, x, \sqrt{-9+4x^2}\right)\\
&= \frac{\sqrt{-9+4x^2}}{36x^4} + \frac{\sqrt{-9+4x^2}}{54x^2} + \frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9+4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$\frac{16}{729}\sqrt{4x^2-9}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{4x^2}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-9 + 4*x^2]),x]
[Out] (16*Sqrt[-9 + 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (4*x^2)/9])/729
```

IntegrateAlgebraic [A] time = 0.04, size = 46, normalized size = 0.81

$$\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)+\frac{\sqrt{4x^2-9}\,\left(2x^2+3\right)}{108x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[-9 + 4*x^2]),x]
```

```
[Out] ((3 + 2*x^2)*Sqrt[-9 + 4*x^2])/(108*x^4) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/8
1
```

fricas [A] time = 1.11, size = 45, normalized size = 0.79

$$\frac{16\,x^4 \arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}\,\left(2\,x^2+3\right)}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(16*x^4*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9)*(2*x
^2 + 3))/x^4
```

giac [A] time = 1.04, size = 41, normalized size = 0.72

$$\frac{\left(4\,x^2-9\right)^{\frac{3}{2}}+15\,\sqrt{4\,x^2-9}}{216\,x^4}+\frac{2}{81}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*((4*x^2 - 9)^(3/2) + 15*sqrt(4*x^2 - 9))/x^4 + 2/81*arctan(1/3*sqrt(4
*x^2 - 9))
```

maple [A] time = 0.01, size = 44, normalized size = 0.77

$$-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{81}+\frac{\sqrt{4x^2-9}}{54x^2}+\frac{\sqrt{4x^2-9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4*x^2-9)^(1/2),x)
```

```
[Out] 1/36*(4*x^2-9)^(1/2)/x^4+1/54*(4*x^2-9)^(1/2)/x^2-2/81*arctan(3/(4*x^2-9)^(
1/2))
```

maxima [A] time = 2.92, size = 38, normalized size = 0.67

$$\frac{\sqrt{4\,x^2-9}}{54\,x^2}+\frac{\sqrt{4\,x^2-9}}{36\,x^4}-\frac{2}{81}\,\arcsin\left(\frac{3}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/54*sqrt(4*x^2 - 9)/x^2 + 1/36*sqrt(4*x^2 - 9)/x^4 - 2/81*arcsin(3/2/abs(x
))
```

mupad [B] time = 5.32, size = 57, normalized size = 1.00

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{81}+\frac{\frac{10\,\sqrt{4\,x^2-9}}{9}+\frac{2\,{\left(4\,x^2-9\right)}^{3/2}}{27}}{72\,x^2+{\left(4\,x^2-9\right)}^2-81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(4*x^2 - 9)^(1/2)),x)
```

```
[Out] (2*atan((4*x^2 - 9)^(1/2)/3))/81 + ((10*(4*x^2 - 9)^(1/2))/9 + (2*(4*x^2 -
9)^(3/2))/27)/(72*x^2 + (4*x^2 - 9)^2 - 81)
```

sympy [A] time = 3.82, size = 136, normalized size = 2.39

$$\begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{-1+\frac{9}{4x^2}}}+\frac{i}{36x^3\sqrt{-1+\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for}\ \frac{9}{4\left|x^2\right|}>1 \\ -\frac{2\,\mathrm{asin}\left(\frac{3}{2x}\right)}{81}+\frac{1}{27x\sqrt{1-\frac{9}{4x^2}}}-\frac{1}{36x^3\sqrt{1-\frac{9}{4x^2}}}-\frac{1}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(3/(2*x))/81 - I/(27*x*sqrt(-1 + 9/(4*x**2))) + I/(36*x
**3*sqrt(-1 + 9/(4*x**2))) + I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9/(4*Abs(x**
2)) > 1), (-2*asin(3/(2*x))/81 + 1/(27*x*sqrt(1 - 9/(4*x**2))) - 1/(36*x**3
*sqrt(1 - 9/(4*x**2))) - 1/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

3.556 $\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{320}\left(-4x^2-9\right)^{5/2}-\frac{3}{32}\left(-4x^2-9\right)^{3/2}-\frac{81}{64}\sqrt{-4x^2-9}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{1}{320}\left(-4x^2-9\right)^{5/2}-\frac{3}{32}\left(-4x^2-9\right)^{3/2}-\frac{81}{64}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (-81*Sqrt[-9 - 4*x^2])/64 - (3*(-9 - 4*x^2)^(3/2))/32 - (-9 - 4*x^2)^(5/2)/
320
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{-9-4x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{81}{16\sqrt{-9-4x}}+\frac{9}{8}\sqrt{-9-4x}+\frac{1}{16}(-9-4x)^{3/2}\right)dx,x,x^2\right)\\ &= -\frac{81}{64}\sqrt{-9-4x^2}-\frac{3}{32}\left(-9-4x^2\right)^{3/2}-\frac{1}{320}\left(-9-4x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{1}{40}\sqrt{-4x^2-9}\left(2x^4-6x^2+27\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-9 - 4*x^2],x]
[Out] -1/40*(Sqrt[-9 - 4*x^2]*(27 - 6*x^2 + 2*x^4))
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.59

$$\frac{1}{40}\sqrt{-4x^2-9}\left(-2x^4+6x^2-27\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[-9 - 4*x^2],x]
[Out] (Sqrt[-9 - 4*x^2]*(-27 + 6*x^2 - 2*x^4))/40
```

fricas [A] time = 0.63, size = 23, normalized size = 0.50

$$-\frac{1}{40}\left(2\,x^4-6\,x^2+27\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] -1/40*(2*x^4 - 6*x^2 + 27)*sqrt(-4*x^2 - 9)
```

giac [A] time = 1.00, size = 36, normalized size = 0.78

$$-\frac{1}{320}\left(4\,x^2+9\right)^{\frac{5}{2}}i+\frac{3}{32}\left(4\,x^2+9\right)^{\frac{3}{2}}i-\frac{81}{64}\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] -1/320*(4*x^2 + 9)^(5/2)*i + 3/32*(4*x^2 + 9)^(3/2)*i - 81/64*sqrt(-4*x^2 -
 9)
```

maple [A] time = 0.00, size = 24, normalized size = 0.52

$$-\frac{\left(2x^4-6x^2+27\right)\sqrt{-4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/40*(2*x^4-6*x^2+27)*(-4*x^2-9)^(1/2)
```

maxima [A] time = 2.91, size = 40, normalized size = 0.87

$$-\frac{1}{20}\sqrt{-4x^2-9}\,x^4+\frac{3}{20}\sqrt{-4x^2-9}\,x^2-\frac{27}{40}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(-4*x^2 - 9)*x^4 + 3/20*sqrt(-4*x^2 - 9)*x^2 - 27/40*sqrt(-4*x^2
- 9)
```

mupad [B] time = 5.29, size = 23, normalized size = 0.50

$$-\sqrt{-4x^2-9}\,\left(\frac{x^4}{20}-\frac{3x^2}{20}+\frac{27}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] -(- 4*x^2 - 9)^(1/2)*(x^4/20 - (3*x^2)/20 + 27/40)
```

sympy [A] time = 1.21, size = 49, normalized size = 1.07

$$-\frac{x^4\sqrt{-4x^2-9}}{20}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**4*sqrt(-4*x**2 - 9)/20 + 3*x**2*sqrt(-4*x**2 - 9)/20 - 27*sqrt(-4*x**2
- 9)/40
```

3.557 $\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{27}{128}\sqrt{-4x^2-9}\,x+\frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{1}{16}\sqrt{-4x^2-9}\,x^3$$

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 203}

$$-\frac{1}{16}\sqrt{-4x^2-9}\,x^3+\frac{27}{128}\sqrt{-4x^2-9}\,x+\frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (27*x*Sqrt[-9 - 4*x^2])/128 - (x^3*Sqrt[-9 - 4*x^2])/16 + (243*ArcTan[(2*x)
/Sqrt[-9 - 4*x^2]])/256
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx &= -\frac{1}{16}x^3\sqrt{-9-4x^2} - \frac{27}{16}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx\\ &= \frac{27}{128}x\sqrt{-9-4x^2} - \frac{1}{16}x^3\sqrt{-9-4x^2} + \frac{243}{128}\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\ &= \frac{27}{128}x\sqrt{-9-4x^2} - \frac{1}{16}x^3\sqrt{-9-4x^2} + \frac{243}{128}\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)\\ &= \frac{27}{128}x\sqrt{-9-4x^2} - \frac{1}{16}x^3\sqrt{-9-4x^2} + \frac{243}{256}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.80

$$\frac{1}{256}\left(2x\sqrt{-4x^2-9}\,\left(27-8x^2\right)+243\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (2*x*(27 - 8*x^2)*Sqrt[-9 - 4*x^2] + 243*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/25
6
```

IntegrateAlgebraic [C] time = 0.04, size = 49, normalized size = 0.91

$$\frac{1}{128}\sqrt{-4x^2-9}\,\left(27x-8x^3\right)+\frac{243}{256}i\log\left(\sqrt{-4x^2-9}\,-2ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (Sqrt[-9 - 4*x^2]*(27*x - 8*x^3))/128 + ((243*I)/256)*Log[(-2*I)*x + Sqrt[-
9 - 4*x^2]]
```

fricas [C] time = 0.64, size = 67, normalized size = 1.24

$$-\frac{1}{128}\left(8\,x^3-27\,x\right)\sqrt{-4\,x^2-9}\,+\frac{243}{512}i\,\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-9}}{x}\right)-\frac{243}{512}i\,\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

[Out] -1/128*(8*x^3 - 27*x)*sqrt(-4*x^2 - 9) + 243/512*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) - 243/512*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-4\,x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="giac")

[Out] integrate(x^4/sqrt(-4*x^2 - 9), x)

maple [A] time = 0.00, size = 43, normalized size = 0.80

$$-\frac{\sqrt{-4x^2-9}\;x^3}{16}+\frac{27\sqrt{-4x^2-9}\;x}{128}+\frac{243\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(-4*x^2-9)^(1/2),x)

[Out] 243/256*arctan(2/(-4*x^2-9)^(1/2)*x)+27/128*(-4*x^2-9)^(1/2)*x-1/16*x^3*(-4*x^2-9)^(1/2)

maxima [C] time = 2.95, size = 33, normalized size = 0.61

$$-\frac{1}{16}\sqrt{-4\,x^2-9}\,x^3+\frac{27}{128}\sqrt{-4\,x^2-9}\,x-\frac{243}{256}i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="maxima")

[Out] -1/16*sqrt(-4*x^2 - 9)*x^3 + 27/128*sqrt(-4*x^2 - 9)*x - 243/256*I*arcsinh(2/3*x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\sqrt{-4\,x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(- 4*x^2 - 9)^(1/2),x)

```
[Out] int(x^4/(- 4*x^2 - 9)^(1/2), x)
```

sympy [A] time = 0.89, size = 53, normalized size = 0.98

$$-\frac{x^3\sqrt{-4x^2-9}}{16}+\frac{27x\sqrt{-4x^2-9}}{128}+\frac{243\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x**3*sqrt(-4*x**2 - 9)/16 + 27*x*sqrt(-4*x**2 - 9)/128 + 243*atan(2*x/sqrt
(-4*x**2 - 9))/256
```

3.558 $\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{48}\left(-4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{-4x^2-9}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{48}\left(-4x^2-9\right)^{3/2}+\frac{9}{16}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (9*Sqrt[-9 - 4*x^2])/16 + (-9 - 4*x^2)^(3/2)/48
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{-9-4x}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{9}{4\sqrt{-9-4x}}-\frac{1}{4}\sqrt{-9-4x}\right)dx, x, x^2\right)\\
&= \frac{9}{16}\sqrt{-9-4x^2}+\frac{1}{48}\left(-9-4x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\sqrt{-4x^2-9}\left(9-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-9 - 4*x^2],x]
[Out] (Sqrt[-9 - 4*x^2]*(9 - 2*x^2))/24
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{24}\sqrt{-4x^2-9}\left(9-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[-9 - 4*x^2],x]
[Out] (Sqrt[-9 - 4*x^2]*(9 - 2*x^2))/24
```

fricas [A] time = 0.95, size = 18, normalized size = 0.58

$$-\frac{1}{24}\left(2\,x^2-9\right)\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] -1/24*(2*x^2 - 9)*sqrt(-4*x^2 - 9)
```

giac [A] time = 1.08, size = 24, normalized size = 0.77

$$-\frac{1}{48}\left(4\,x^2+9\right)^{\frac{3}{2}}i+\frac{9}{16}\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] -1/48*(4*x^2 + 9)^(3/2)*i + 9/16*sqrt(-4*x^2 - 9)
```

maple [A] time = 0.00, size = 19, normalized size = 0.61

$$-\frac{\left(2x^2-9\right)\sqrt{-4x^2-9}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^2-9)^(1/2),x)
[Out] -1/24*(2*x^2-9)*(-4*x^2-9)^(1/2)
```

maxima [A] time = 2.96, size = 26, normalized size = 0.84

$$-\frac{1}{12}\,\sqrt{-4\,x^2-9}\,x^2+\frac{3}{8}\,\sqrt{-4\,x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/12*sqrt(-4*x^2 - 9)*x^2 + 3/8*sqrt(-4*x^2 - 9)
```

mupad [B] time = 5.07, size = 18, normalized size = 0.58

$$-\frac{\left(2\,x^2-9\right)\,\sqrt{-4\,x^2-9}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(- 4*x^2 - 9)^(1/2),x)
[Out] -((2*x^2 - 9)*(- 4*x^2 - 9)^(1/2))/24
```

sympy [A] time = 0.37, size = 31, normalized size = 1.00

$$-\frac{x^2\sqrt{-4x^2-9}}{12}+\frac{3\sqrt{-4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**2-9)**(1/2),x)
[Out] -x**2*sqrt(-4*x**2 - 9)/12 + 3*sqrt(-4*x**2 - 9)/8
```

3.559 $\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{8}\sqrt{-4x^2-9}\,x-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {321, 217, 203}

$$-\frac{1}{8}\sqrt{-4x^2-9}\,x-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[-9 - 4*x^2])/8 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx &= -\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{8}\int \frac{1}{\sqrt{-9-4x^2}}\,dx\\ &= -\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{8}\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx,x,\frac{x}{\sqrt{-9-4x^2}}\right)\\ &= -\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$-\frac{1}{8}\sqrt{-4x^2-9}\,x-\frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/8*(x*Sqrt[-9 - 4*x^2]) - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16
```

IntegrateAlgebraic [C] time = 0.03, size = 41, normalized size = 1.14

$$-\frac{1}{8}\sqrt{-4x^2-9}\,x-\frac{9}{16}i\log\left(\sqrt{-4x^2-9}-2ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/8*(x*Sqrt[-9 - 4*x^2]) - ((9*I)/16)*Log[(-2*I)*x + Sqrt[-9 - 4*x^2]]
```

fricas [C] time = 0.68, size = 59, normalized size = 1.64

$$-\frac{1}{8}\sqrt{-4\,x^2-9}\,x-\frac{9}{32}i\,\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-9}}{x}\right)+\frac{9}{32}i\,\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 - 9)*x - 9/32*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) + 9/3
2*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-4\,x^2 - 9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(-4*x^2 - 9), x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.81

$$-\frac{\sqrt{-4x^2-9}\;x}{8}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2-9)^(1/2),x)
[Out] -9/16*arctan(2/(-4*x^2-9)^(1/2)*x)-1/8*(-4*x^2-9)^(1/2)*x
```

maxima [C] time = 2.93, size = 19, normalized size = 0.53

$$-\frac{1}{8}\sqrt{-4\,x^2-9}\,x+\frac{9}{16}i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/8*sqrt(-4*x^2 - 9)*x + 9/16*I*arcsinh(2/3*x)
```

mupad [B] time = 0.12, size = 31, normalized size = 0.86

$$-\frac{x\sqrt{-4\,x^2-9}}{8}+\frac{\ln\left(x-\frac{\sqrt{-4\,x^2-9}\;1\mathrm{i}}{2}\right)\,9\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(- 4*x^2 - 9)^(1/2),x)
[Out] (log(x - ((- 4*x^2 - 9)^(1/2)*1i)/2)*9i)/16 - (x*(- 4*x^2 - 9)^(1/2))/8
```

sympy [A] time = 0.39, size = 36, normalized size = 1.00

$$-\frac{x\sqrt{-4x^2-9}}{8}-\frac{9\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2-9)**(1/2),x)
```

```
[Out] -x*sqrt(-4*x**2 - 9)/8 - 9*atan(2*x/sqrt(-4*x**2 - 9))/16
```

3.560 $\int \frac{x}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -Sqrt[-9 - 4*x^2]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{-9-4x^2}}\,dx=-\frac{1}{4}\sqrt{-9-4x^2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/4*Sqrt[-9 - 4*x^2]
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[-9 - 4*x^2],x]
```

```
[Out] -1/4*Sqrt[-9 - 4*x^2]
```

fricas [A] time = 1.04, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-4*x^2 - 9)
```

giac [A] time = 1.13, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-4*x^2 - 9)
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$-\frac{\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2-9)^(1/2)*x,x)
```

```
[Out] -1/4*(-4*x^2-9)^(1/2)
```

maxima [A] time = 1.32, size = 11, normalized size = 0.73

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-4*x^2 - 9)
```

mupad [**B**] time = 4.99, size = 11, normalized size = 0.73

$$-\frac{\sqrt{-4\,x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(- 4*x^2 - 9)^(1/2),x)
```

```
[Out] -(- 4*x^2 - 9)^(1/2)/4
```

sympy [**A**] time = 0.16, size = 14, normalized size = 0.93

$$-\frac{\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*x**2-9)**(1/2),x)
```

```
[Out] -sqrt(-4*x**2 - 9)/4
```

3.561 $\int \frac{1}{\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 - 4*x^2],x]
```

```
[Out] ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9-4x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{x}{\sqrt{-9-4x^2}}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 - 4*x^2],x]
```

```
[Out] ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2
```

IntegrateAlgebraic [C] time = 0.02, size = 24, normalized size = 1.26

$$\frac{1}{2}i\log\left(\sqrt{-4x^2-9}-2ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-9 - 4*x^2],x]
```

```
[Out] (I/2)*Log[(-2*I)*x + Sqrt[-9 - 4*x^2]]
```

fricas [C] time = 0.85, size = 47, normalized size = 2.47

$$\frac{1}{4}i\,\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-9}}{x}\right)-\frac{1}{4}i\,\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 9))/x) - 1/4*I*log(-(8*x - 4*I*sqrt(-4*
x^2 - 9))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-4\,x^2-9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-4*x^2 - 9), x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-4*x^2-9)^(1/2),x)

[Out] 1/2*arctan(2/(-4*x^2-9)^(1/2)*x)

maxima [C] time = 2.98, size = 6, normalized size = 0.32

$$-\frac{1}{2} i \operatorname{arsinh}\left(\frac{2}{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2-9)^(1/2),x, algorithm="maxima")

[Out] -1/2*I*arcsinh(2/3*x)

mupad [B] time = 0.11, size = 15, normalized size = 0.79

$$\frac{\operatorname{atan}\left(\frac{2 x}{\sqrt{-4 x^2-9}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(- 4*x^2 - 9)^(1/2),x)

[Out] atan((2*x)/(- 4*x^2 - 9)^(1/2))/2

sympy [A] time = 0.32, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x**2-9)**(1/2),x)

[Out] atan(2*x/sqrt(-4*x**2 - 9))/2

3.562 $\int \frac{1}{x\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 204}

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-9-4x^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\right)$$
$$= \frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-9 - 4*x^2]),x]
[Out] ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-9 - 4*x^2]),x]
[Out] ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

fricas [C] time = 0.74, size = 43, normalized size = 2.15

$$-\frac{1}{6}i\log\left(-\frac{2\left(i\sqrt{-4x^2-9}+3\right)}{3x}\right)+\frac{1}{6}i\log\left(-\frac{2\left(-i\sqrt{-4x^2-9}+3\right)}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] -1/6*I*log(-2/3*(I*sqrt(-4*x^2 - 9) + 3)/x) + 1/6*I*log(-2/3*(-I*sqrt(-4*x^
2 - 9) + 3)/x)
```

giac [A] time = 1.13, size = 14, normalized size = 0.70

$$\frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] 1/3*arctan(1/3*sqrt(-4*x^2 - 9))
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$-\frac{\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-4*x^2-9)^(1/2),x)
[Out] -1/3*arctan(3/(-4*x^2-9)^(1/2))
```

maxima [C] time = 2.94, size = 25, normalized size = 1.25

$$-\frac{1}{3}i\log\left(\frac{6\sqrt{4x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -1/3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 5.27, size = 14, normalized size = 0.70

$$\frac{\mathrm{atan}\left(\frac{\sqrt{-4x^2-9}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- 4*x^2 - 9)^(1/2)),x)
[Out] atan((- 4*x^2 - 9)^(1/2)/3)/3
```

sympy [C] time = 1.02, size = 8, normalized size = 0.40

$$\frac{i \operatorname{asinh}\left(\frac{3}{2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-4*x**2-9)**(1/2),x)
```

```
[Out] I*asinh(3/(2*x))/3
```

3.563 $\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx=\frac{\sqrt{-9-4x^2}}{9x}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(9*x)
```

IntegrateAlgebraic [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[-9 - 4*x^2]),x]
[Out] Sqrt[-9 - 4*x^2]/(9*x)
```

fricas [A] time = 0.77, size = 14, normalized size = 0.78

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="fricas")
[Out] 1/9*sqrt(-4*x^2 - 9)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4\,x^2-9}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-4*x^2 - 9)*x^2), x)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\sqrt{-4x^2-9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-4*x^2-9)^(1/2),x)
[Out] 1/9/x*(-4*x^2-9)^(1/2)
```

maxima [A] time = 2.95, size = 14, normalized size = 0.78

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(-4*x^2 - 9)/x
```

mupad [B] time = 5.04, size = 14, normalized size = 0.78

$$\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(- 4*x^2 - 9)^(1/2)),x)
```

```
[Out] (- 4*x^2 - 9)^(1/2)/(9*x)
```

sympy [C] time = 0.77, size = 15, normalized size = 0.83

$$\frac{2i\sqrt{1+\frac{9}{4x^2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*sqrt(1 + 9/(4*x**2))/9
```

3.564 $\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 204}

$$\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(18*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/27
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^2}\,dx, x, x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{18x^2}-\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)\\ &= \frac{\sqrt{-9-4x^2}}{18x^2}+\frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\ &= \frac{\sqrt{-9-4x^2}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.38

$$-\frac{4}{81}\sqrt{-4x^2-9}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{4x^2}{9}+1}\right)}{2\sqrt{\frac{4x^2}{9}+1}}-\frac{9}{8x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-9 - 4*x^2]),x]
[Out] (-4*Sqrt[-9 - 4*x^2]*(-9/(8*x^2) + ArcTanh[Sqrt[1 + (4*x^2)/9]]/(2*Sqrt[1 +
 (4*x^2)/9])))/81
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{\sqrt{-4x^2-9}}{18x^2}-\frac{2}{27}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[-9 - 4*x^2]),x]
[Out] Sqrt[-9 - 4*x^2]/(18*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/27
```

fricas [C] time = 0.73, size = 65, normalized size = 1.67

$$\frac{-2i\,x^2\log\left(-\frac{4\left(i\sqrt{-4\,x^2-9}-3\right)}{27\,x}\right)+2i\,x^2\log\left(-\frac{4\left(-i\sqrt{-4\,x^2-9}-3\right)}{27\,x}\right)+3\sqrt{-4\,x^2-9}}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(-2*I*x^2*log(-4/27*(I*sqrt(-4*x^2 - 9) - 3)/x) + 2*I*x^2*log(-4/27*(-
I*sqrt(-4*x^2 - 9) - 3)/x) + 3*sqrt(-4*x^2 - 9))/x^2
```

giac [A] time = 1.07, size = 29, normalized size = 0.74

$$\frac{\sqrt{-4\,x^2-9}}{18\,x^2}-\frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(-4*x^2 - 9)/x^2 - 2/27*arctan(1/3*sqrt(-4*x^2 - 9))
```

maple [A] time = 0.00, size = 30, normalized size = 0.77

$$\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}+\frac{\sqrt{-4x^2-9}}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-4*x^2-9)^(1/2),x)
```

```
[Out] 1/18*(-4*x^2-9)^(1/2)/x^2+2/27*arctan(3/(-4*x^2-9)^(1/2))
```

maxima [C] time = 2.92, size = 40, normalized size = 1.03

$$\frac{\sqrt{-4\,x^2-9}}{18\,x^2}+\frac{2}{27}i\log\left(\frac{6\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(-4*x^2 - 9)/x^2 + 2/27*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x)
)
```

mupad [B] time = 5.18, size = 29, normalized size = 0.74

$$\frac{\sqrt{-4\,x^2-9}}{18\,x^2}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(- 4*x^2 - 9)^(1/2)),x)
```

```
[Out] (- 4*x^2 - 9)^(1/2)/(18*x^2) - (2*atan((- 4*x^2 - 9)^(1/2)/3))/27
```

sympy [C] time = 2.06, size = 46, normalized size = 1.18

$$-\frac{2i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{27}+\frac{i}{9x\sqrt{1+\frac{9}{4x^2}}}+\frac{i}{4x^3\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-4*x**2-9)**(1/2),x)
```

```
[Out] -2*I*asinh(3/(2*x))/27 + I/(9*x*sqrt(1 + 9/(4*x**2))) + I/(4*x**3*sqrt(1 +
9/(4*x**2)))
```

3.565 $\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8\sqrt{-4x^2-9}}{243x}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {271, 264}

$$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8\sqrt{-4x^2-9}}{243x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(27*x^3) - (8*Sqrt[-9 - 4*x^2])/(243*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx&=\frac{\sqrt{-9-4x^2}}{27x^3}-\frac{8}{27}\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx\\&=\frac{\sqrt{-9-4x^2}}{27x^3}-\frac{8\sqrt{-9-4x^2}}{243x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.68

$$\frac{\left(9-8x^2\right)\sqrt{-4x^2-9}}{243x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] ((9 - 8*x^2)*Sqrt[-9 - 4*x^2])/(243*x^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 25, normalized size = 0.68

$$\frac{(9-8x^2)\sqrt{-4x^2-9}}{243x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] ((9 - 8*x^2)*Sqrt[-9 - 4*x^2])/(243*x^3)
```

fricas [A] time = 0.89, size = 21, normalized size = 0.57

$$-\frac{(8x^2-9)\sqrt{-4x^2-9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/243*(8*x^2 - 9)*sqrt(-4*x^2 - 9)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4x^2-9}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-4*x^2 - 9)*x^4), x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.59

$$-\frac{(8x^2-9)\sqrt{-4x^2-9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-4*x^2-9)^(1/2),x)
```

```
[Out] -1/243*(8*x^2-9)/x^3*(-4*x^2-9)^(1/2)
```

maxima [A] time = 2.96, size = 29, normalized size = 0.78

$$-\frac{8\sqrt{-4x^2-9}}{243x}+\frac{\sqrt{-4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="maxima")
[Out] -8/243*sqrt(-4*x^2 - 9)/x + 1/27*sqrt(-4*x^2 - 9)/x^3
```

mupad [B] time = 5.08, size = 31, normalized size = 0.84

$$-\frac{8x^2\sqrt{-4x^2-9}-9\sqrt{-4x^2-9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(- 4*x^2 - 9)^(1/2)),x)
[Out] -(8*x^2*(- 4*x^2 - 9)^(1/2) - 9*(- 4*x^2 - 9)^(1/2))/(243*x^3)
```

sympy [C] time = 1.29, size = 36, normalized size = 0.97

$$-\frac{16i\sqrt{1+\frac{9}{4x^2}}}{243}+\frac{2i\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-4*x**2-9)**(1/2),x)
[Out] -16*I*sqrt(1 + 9/(4*x**2))/243 + 2*I*sqrt(1 + 9/(4*x**2))/(27*x**2)
```

3.566 $\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-4x^2-9}}{54x^2}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)+\frac{\sqrt{-4x^2-9}}{36x^4}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 51, 63, 204}

$$-\frac{\sqrt{-4x^2-9}}{54x^2}+\frac{\sqrt{-4x^2-9}}{36x^4}+\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] Sqrt[-9 - 4*x^2]/(36*x^4) - Sqrt[-9 - 4*x^2]/(54*x^2) + (2*ArcTan[Sqrt[-9 -
 4*x^2]/3])/81
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^3}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9-4x^2}}{36x^4} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x^2}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9-4x^2}}{36x^4} - \frac{\sqrt{-9-4x^2}}{54x^2} + \frac{1}{27}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-9-4x}\,x}\,dx, x, x^2\right)\\
&= \frac{\sqrt{-9-4x^2}}{36x^4} - \frac{\sqrt{-9-4x^2}}{54x^2} - \frac{1}{54}\operatorname{Subst}\left(\int \frac{1}{-\frac{9}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{-9-4x^2}\right)\\
&= \frac{\sqrt{-9-4x^2}}{36x^4} - \frac{\sqrt{-9-4x^2}}{54x^2} + \frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-9-4x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.56

$$\frac{16}{729}\sqrt{-4x^2-9}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{4x^2}{9}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-9 - 4*x^2]),x]
[Out] (16*Sqrt[-9 - 4*x^2]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (4*x^2)/9])/729
```

IntegrateAlgebraic [A] time = 0.03, size = 46, normalized size = 0.81

$$\frac{2}{81}\tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)+\frac{\sqrt{-4x^2-9}\,\left(3-2x^2\right)}{108x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[-9 - 4*x^2]),x]
```

```
[Out] (Sqrt[-9 - 4*x^2]*(3 - 2*x^2))/(108*x^4) + (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/8
1
```

fricas [C] time = 0.87, size = 72, normalized size = 1.26

$$\frac{-4i\,x^4\log\left(-\frac{4\left(i\sqrt{-4\,x^2-9}+3\right)}{81\,x}\right)+4i\,x^4\log\left(-\frac{4\left(-i\sqrt{-4\,x^2-9}+3\right)}{81\,x}\right)-3\left(2\,x^2-3\right)\sqrt{-4\,x^2-9}}{324\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(-4*I*x^4*log(-4/81*(I*sqrt(-4*x^2 - 9) + 3)/x) + 4*I*x^4*log(-4/81*(
-I*sqrt(-4*x^2 - 9) + 3)/x) - 3*(2*x^2 - 3)*sqrt(-4*x^2 - 9))/x^4
```

giac [A] time = 1.11, size = 43, normalized size = 0.75

$$-\frac{\left(4\,x^2+9\right)^{\frac{3}{2}}i-15\sqrt{-4\,x^2-9}}{216\,x^4}+\frac{2}{81}\arctan\left(\frac{1}{3}\sqrt{-4\,x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/216*((4*x^2 + 9)^(3/2)*i - 15*sqrt(-4*x^2 - 9))/x^4 + 2/81*arctan(1/3*sq
rt(-4*x^2 - 9))
```

maple [A] time = 0.01, size = 44, normalized size = 0.77

$$-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{81}-\frac{\sqrt{-4x^2-9}}{54x^2}+\frac{\sqrt{-4x^2-9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-4*x^2-9)^(1/2),x)
```

```
[Out] 1/36*(-4*x^2-9)^(1/2)/x^4-1/54*(-4*x^2-9)^(1/2)/x^2-2/81*arctan(3/(-4*x^2-9
)^(1/2))
```

maxima [C] time = 2.89, size = 54, normalized size = 0.95

$$-\frac{\sqrt{-4\,x^2-9}}{54\,x^2}+\frac{\sqrt{-4\,x^2-9}}{36\,x^4}-\frac{2}{81}i\log\left(\frac{6\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(-4*x^2 - 9)/x^2 + 1/36*sqrt(-4*x^2 - 9)/x^4 - 2/81*I*log(6*sqrt(
4*x^2 + 9)/abs(x) + 18/abs(x))
```

mupad [B] time = 5.07, size = 60, normalized size = 1.05

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{81}-\frac{\frac{10\,\sqrt{-4\,x^2-9}}{9}+\frac{2\,{\left(-4\,x^2-9\right)}^{3/2}}{27}}{72\,x^2-{\left(4\,x^2+9\right)}^2+81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(- 4*x^2 - 9)^(1/2)),x)
```

```
[Out] (2*atan((- 4*x^2 - 9)^(1/2)/3))/81 - ((10*(- 4*x^2 - 9)^(1/2))/9 + (2*(- 4*
x^2 - 9)^(3/2))/27)/(72*x^2 - (4*x^2 + 9)^2 + 81)
```

sympy [C] time = 3.88, size = 65, normalized size = 1.14

$$\frac{2i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{81}-\frac{i}{27x\sqrt{1+\frac{9}{4x^2}}}-\frac{i}{36x^3\sqrt{1+\frac{9}{4x^2}}}+\frac{i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-4*x**2-9)**(1/2),x)
```

```
[Out] 2*I*asinh(3/(2*x))/81 - I/(27*x*sqrt(1 + 9/(4*x**2))) - I/(36*x**3*sqrt(1 +
 9/(4*x**2))) + I/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

3.567 $\int \frac{1}{\sqrt{9+bx^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/3]/Sqrt[b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx = \frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/3]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 1.65

$$-\frac{\log\left(\sqrt{bx^2+9}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[9 + b*x^2],x]
```

```
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[9 + b*x^2]]/Sqrt[b])
```

fricas [B] time = 0.70, size = 65, normalized size = 3.82

$$\left[\frac{\log\left(-\sqrt{b}\,x-\sqrt{bx^2+9}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+9}\sqrt{-b}-3\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(-sqrt(b)*x - sqrt(b*x^2 + 9))/sqrt(b), -2*sqrt(-b)*arctan((sqrt(b*x^2
+ 9)*sqrt(-b) - 3*sqrt(-b))/(b*x))/b]
```

giac [A] time = 1.14, size = 22, normalized size = 1.29

$$-\frac{\log\left(-\sqrt{b}\,x+\sqrt{bx^2+9}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(b)*x + sqrt(b*x^2 + 9))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 1.24

$$\frac{\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+9}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+9)^(1/2),x)
```

```
[Out] ln(b^(1/2)*x+(b*x^2+9)^(1/2))/b^(1/2)
```

maxima [A] time = 1.30, size = 11, normalized size = 0.65

$$\frac{\operatorname{arsinh}\left(\frac{1}{3}\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*sqrt(b)*x)/sqrt(b)
```

mupad [B] time = 0.04, size = 11, normalized size = 0.65

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + 9)^(1/2),x)
```

```
[Out] asinh((b^(1/2)*x)/3)/b^(1/2)
```

sympy [A] time = 0.95, size = 14, normalized size = 0.82

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+9)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/3)/sqrt(b)
```

3.568 $\int \frac{1}{\sqrt{9-bx^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 - b*x^2],x]
```

```
[Out] ArcSin[(Sqrt[b]*x)/3]/Sqrt[b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx = \frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 - b*x^2],x]
```

```
[Out] ArcSin[(Sqrt[b]*x)/3]/Sqrt[b]
```

IntegrateAlgebraic [B] time = 0.04, size = 35, normalized size = 2.06

$$\frac{\sqrt{-b}\,\log\left(\sqrt{9-bx^2}-\sqrt{-b}\,x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[9 - b*x^2],x]
[Out] (Sqrt[-b]*Log[-(Sqrt[-b]*x) + Sqrt[9 - b*x^2]])/b
```

fricas [B] time = 0.90, size = 58, normalized size = 3.41

$$\left[-\frac{\sqrt{-b}\,\log\left(-\sqrt{-b}\,x-\sqrt{-bx^2+9}\right)}{b}, -\frac{2\,\arctan\left(\frac{\sqrt{-bx^2+9}-3}{\sqrt{b}\,x}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-b)*log(-sqrt(-b)*x - sqrt(-b*x^2 + 9))/b, -2*arctan((sqrt(-b*x^2 +
9) - 3)/(sqrt(b)*x))/sqrt(b)]
```

giac [B] time = 1.11, size = 27, normalized size = 1.59

$$-\frac{\log\left(-\sqrt{-b}\,x+\sqrt{-bx^2+9}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="giac")
[Out] -log(-sqrt(-b)*x + sqrt(-b*x^2 + 9))/sqrt(-b)
```

maple [A] time = 0.00, size = 21, normalized size = 1.24

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+9}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+9)^(1/2),x)
[Out] 1/b^(1/2)*arctan(b^(1/2)*x/(-b*x^2+9)^(1/2))
```

maxima [A] time = 2.83, size = 11, normalized size = 0.65

$$\frac{\arcsin\left(\frac{1}{3}\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/3*sqrt(b)*x)/sqrt(b)
```

mupad [B] time = 0.04, size = 15, normalized size = 0.88

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{-b}\,x}{3}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9 - b*x^2)^(1/2),x)
```

```
[Out] asinh(((-b)^(1/2)*x)/3)/(-b)^(1/2)
```

sympy [A] time = 1.01, size = 39, normalized size = 2.29

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{9} > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+9)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2)/9 > 1), (asin(sqrt(b)
*x/3)/sqrt(b), True))
```

3.569 $\int \frac{1}{\sqrt{-9+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-9+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{-9+bx^2}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-9+bx^2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 + b*x^2],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 1.12

$$-\frac{\log\left(\sqrt{bx^2-9}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-9 + b*x^2],x]
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[-9 + b*x^2]]/Sqrt[b])
```

fricas [A] time = 1.13, size = 57, normalized size = 2.28

$$\left[\frac{\log\left(2\,bx^2+2\,\sqrt{bx^2-9}\,\sqrt{b}\,x-9\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2-9}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(2*b*x^2 + 2*sqrt(b*x^2 - 9)*sqrt(b)*x - 9)/sqrt(b), -sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 - 9))/b]
```

giac [A] time = 1.14, size = 23, normalized size = 0.92

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2-9}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="giac")
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 - 9)))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\, x + \sqrt{b\, x^2 - 9}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-9)^(1/2),x)
```

```
[Out] ln(b^(1/2)*x+(b*x^2-9)^(1/2))/b^(1/2)
```

maxima [A] time = 1.33, size = 24, normalized size = 0.96

$$\frac{\log\left(2\, bx + 2\, \sqrt{bx^2 - 9}\, \sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + 2*sqrt(b*x^2 - 9)*sqrt(b))/sqrt(b)
```

mupad [B] time = 0.08, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b\, x^2 - 9} + \sqrt{b}\, x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 - 9)^(1/2),x)
```

```
[Out] log((b*x^2 - 9)^(1/2) + b^(1/2)*x)/b^(1/2)
```

sympy [A] time = 1.03, size = 39, normalized size = 1.56

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}} & \text{for}\: \frac{\left|bx^2\right|}{9} > 1 \\ -\frac{i \operatorname{asin}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2)/9 > 1), (-I*asin(sqrt(b)
*x/3)/sqrt(b), True))
```

3.570 $\int \frac{1}{\sqrt{-9-bx^2}} \, dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-9 - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-9-bx^2}} \, dx &= \text{Subst}\left(\int \frac{1}{1+bx^2} \, dx, x, \frac{x}{\sqrt{-9-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-9-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-9 - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.04, size = 35, normalized size = 1.35

$$\frac{\sqrt{-b}\,\log\left(\sqrt{-bx^2-9}-\sqrt{-b}\,x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-9 - b*x^2],x]
```

```
[Out] (Sqrt[-b]*Log[-(Sqrt[-b]*x) + Sqrt[-9 - b*x^2]])/b
```

fricas [A] time = 0.92, size = 68, normalized size = 2.62

$$\left[-\frac{\sqrt{-b}\,\log\left(-2\,bx^2+2\,\sqrt{-bx^2-9}\,\sqrt{-b}\,x-9\right)}{2\,b},-\frac{\arctan\left(\frac{\sqrt{-bx^2-9}\,\sqrt{b}\,x}{bx^2+9}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-2*b*x^2 + 2*sqrt(-b*x^2 - 9)*sqrt(-b)*x - 9)/b, -arctan
(sqrt(-b*x^2 - 9)*sqrt(b)*x/(b*x^2 + 9))/sqrt(b)]
```

giac [A] time = 1.20, size = 28, normalized size = 1.08

$$-\frac{\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2-9}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-b*x^2 - 9)))/sqrt(-b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2-9}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-9)^(1/2),x)
```

```
[Out] arctan(x*b^(1/2)/(-b*x^2-9)^(1/2))/b^(1/2)
```

maxima [C] time = 1.38, size = 12, normalized size = 0.46

$$-\frac{i\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*arcsinh(1/3*sqrt(b)*x)/sqrt(b)
```

mupad [B] time = 0.09, size = 25, normalized size = 0.96

$$\frac{\ln\left(\sqrt{-b\,x^2-9}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- b*x^2 - 9)^(1/2),x)
```

```
[Out] log((- b*x^2 - 9)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

sympy [C] time = 0.97, size = 17, normalized size = 0.65

$$-\frac{i\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-9)**(1/2),x)
```

```
[Out] -I*asinh(sqrt(b)*x/3)/sqrt(b)
```

3.571 $\int \frac{1}{\sqrt{\pi+bx^2}}\,dx$

Optimal. Leaf size=19

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Pi + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\pi+bx^2}}\,dx=\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Pi + b*x^2],x]
```

```
[Out] ArcSinh[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 1.47

$$-\frac{\log\left(\sqrt{bx^2+\pi}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[Pi + b*x^2],x]
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[Pi + b*x^2]]/Sqrt[b])
```

fricas [B] time = 0.99, size = 59, normalized size = 3.11

$$\left[\frac{\log\left(-\pi-2\,bx^2-2\,\sqrt{\pi+bx^2}\,\sqrt{b}\,x\right)}{2\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{\pi+bx^2}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-pi - 2*b*x^2 - 2*sqrt(pi + b*x^2)*sqrt(b)*x)/sqrt(b), -sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(pi + b*x^2))/b]
```

giac [A] time = 1.12, size = 22, normalized size = 1.16

$$-\frac{\log\left(-\sqrt{b}\,x+\sqrt{\pi+bx^2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="giac")
[Out] -log(-sqrt(b)*x + sqrt(pi + b*x^2))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 1.11

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+\pi}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+Pi)^(1/2),x)
[Out] ln(b^(1/2)*x+(b*x^2+Pi)^(1/2))/b^(1/2)
```

maximo [A] time = 1.33, size = 13, normalized size = 0.68

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*x/sqrt(pi*b))/sqrt(b)
```

mupad [B] time = 5.12, size = 20, normalized size = 1.05

$$\frac{\ln\left(\sqrt{b}\ x + \sqrt{b\,x^2 + \Pi}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi + b*x^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*x + (Pi + b*x^2)^(1/2))/b^(1/2)
```

sympy [A] time = 0.98, size = 17, normalized size = 0.89

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+pi)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(pi))/sqrt(b)
```

3.572 $\int \frac{1}{\sqrt{\pi-bx^2}}\,dx$

Optimal. Leaf size=19

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Pi - b*x^2],x]
```

```
[Out] ArcSin[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\pi - bx^2}}\,dx = \frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Pi - b*x^2],x]
```

```
[Out] ArcSin[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 1.84

$$\frac{\sqrt{-b}\,\log\left(\sqrt{\pi-bx^2}-\sqrt{-b}\,x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[Pi - b*x^2],x]
```

```
[Out] (Sqrt[-b]*Log[-(Sqrt[-b]*x) + Sqrt[Pi - b*x^2]])/b
```

fricas [A] time = 0.93, size = 62, normalized size = 3.26

$$\left[-\frac{\sqrt{-b}\,\log\left(-\pi+2\,bx^2-2\,\sqrt{\pi-bx^2}\,\sqrt{-b}\,x\right)}{2\,b},-\frac{\arctan\left(-\frac{\sqrt{b}\,x}{\sqrt{\pi-bx^2}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-pi + 2*b*x^2 - 2*sqrt(pi - b*x^2)*sqrt(-b)*x)/b, -arcta
n(-sqrt(b)*x/sqrt(pi - b*x^2))/sqrt(b)]
```

giac [B] time = 1.05, size = 28, normalized size = 1.47

$$-\frac{\log\left(\left|-\sqrt{-b}\,x+\sqrt{\pi-bx^2}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(pi - b*x^2)))/sqrt(-b)
```

maple [A] time = 0.01, size = 21, normalized size = 1.11

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+Pi)^(1/2),x)
```

```
[Out] 1/b^(1/2)*arctan(b^(1/2)*x/(-b*x^2+Pi)^(1/2))
```

maxima [A] time = 2.86, size = 13, normalized size = 0.68

$$\frac{\arcsin\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(b*x/sqrt(pi*b))/sqrt(b)
```

mupad [B] time = 0.10, size = 25, normalized size = 1.32

$$\frac{\ln\left(\sqrt{\Pi - b\,x^2} + \sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(Pi - b*x^2)^(1/2),x)
```

```
[Out] log((Pi - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

sympy [A] time = 1.04, size = 46, normalized size = 2.42

$$\begin{cases} -\frac{i\,\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{\pi} > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+pi)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (asin
(sqrt(b)*x/sqrt(pi))/sqrt(b), True))
```

3.573 $\int \frac{1}{\sqrt{-\pi+bx^2}}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Pi + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\pi+bx^2}}\,dx = \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{-\pi+bx^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-\pi+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Pi + b*x^2],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.04, size = 30, normalized size = 1.11

$$-\frac{\log\left(\sqrt{bx^2-\pi}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-Pi + b*x^2],x]
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[-Pi + b*x^2]]/Sqrt[b])
```

fricas [A] time = 0.92, size = 74, normalized size = 2.74

$$\left[\frac{\log\left(-\pi+2\,bx^2+2\,\sqrt{-\pi+bx^2}\,\sqrt{b}\,x\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(-\frac{\sqrt{-\pi+bx^2}\,\sqrt{-b}\,x}{\pi-bx^2}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-pi + 2*b*x^2 + 2*sqrt(-pi + b*x^2)*sqrt(b)*x)/sqrt(b), -sqrt(-b)*
arctan(-sqrt(-pi + b*x^2)*sqrt(-b)*x/(pi - b*x^2))/b]
```

giac [A] time = 1.14, size = 25, normalized size = 0.93

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{-\pi+bx^2}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(-pi + b*x^2)))/sqrt(b)
```

maple [A] time = 0.00, size = 23, normalized size = 0.85

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}-\pi}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-Pi)^(1/2),x)
[Out] ln(b^(1/2)*x+(b*x^2-Pi)^(1/2))/b^(1/2)
```

maxima [A] time = 1.39, size = 26, normalized size = 0.96

$$\frac{\log\left(2\,bx+2\,\sqrt{-\pi+bx^{2}}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-pi)^(1/2),x, algorithm="maxima")
[Out] log(2*b*x + 2*sqrt(-pi + b*x^2)*sqrt(b))/sqrt(b)
```

mupad [B] time = 0.13, size = 22, normalized size = 0.81

$$\frac{\ln\left(\sqrt{b\,x^{2}-\Pi}+\sqrt{b}\,x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 - Pi)^(1/2),x)
[Out] log((b*x^2 - Pi)^(1/2) + b^(1/2)*x)/b^(1/2)
```

sympy [A] time = 1.05, size = 46, normalized size = 1.70

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for}\ \frac{\left|bx^{2}\right|}{\pi}>1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-pi)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (-I*asin
(sqrt(b)*x/sqrt(pi))/sqrt(b), True))
```

3.574 $\int \frac{1}{\sqrt{-\pi-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Pi - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\pi-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-\pi-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-\pi-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Pi - b*x^2],x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.04, size = 37, normalized size = 1.32

$$\frac{\sqrt{-b}\,\log\left(\sqrt{-bx^2-\pi}-\sqrt{-b}\,x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-Pi - b*x^2],x]
[Out] (Sqrt[-b]*Log[-(Sqrt[-b]*x) + Sqrt[-Pi - b*x^2]])/b
```

fricas [A] time = 0.86, size = 74, normalized size = 2.64

$$\left[-\frac{\sqrt{-b}\,\log\left(-\pi-2\,bx^2+2\,\sqrt{-\pi-bx^2}\,\sqrt{-b}\,x\right)}{2\,b},-\frac{\arctan\left(\frac{\sqrt{-\pi-bx^2}\,\sqrt{b}\,x}{\pi+bx^2}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-b)*log(-pi - 2*b*x^2 + 2*sqrt(-pi - b*x^2)*sqrt(-b)*x)/b, -arct
an(sqrt(-pi - b*x^2)*sqrt(b)*x/(pi + b*x^2))/sqrt(b)]
```

giac [A] time = 1.16, size = 30, normalized size = 1.07

$$-\frac{\log\left(\left|-\sqrt{-b}\,x+\sqrt{-\pi-bx^2}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="giac")
[Out] -log(abs(-sqrt(-b)*x + sqrt(-pi - b*x^2)))/sqrt(-b)
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2-\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-Pi)^(1/2),x)
[Out] arctan(x*b^(1/2)/(-b*x^2-Pi)^(1/2))/b^(1/2)
```

maxima [C] time = 1.35, size = 14, normalized size = 0.50

$$-\frac{i\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="maxima")
[Out] -I*arcsinh(b*x/sqrt(pi*b))/sqrt(b)
```

mupad [B] time = 5.09, size = 27, normalized size = 0.96

$$\frac{\ln\left(\sqrt{-b\,x^2-\Pi}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- Pi - b*x^2)^(1/2),x)
[Out] log((- Pi - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

sympy [C] time = 1.00, size = 20, normalized size = 0.71

$$-\frac{i\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-pi)**(1/2),x)
[Out] -I*asinh(sqrt(b)*x/sqrt(pi))/sqrt(b)
```

3.575 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.00, size = 28, normalized size = 1.12

$$-\frac{\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a + b*x^2],x]
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]/Sqrt[b])
```

fricas [A] time = 0.90, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

giac [A] time = 1.08, size = 23, normalized size = 0.92

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x)
[Out] 1/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.35, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/2),x)
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

sympy [A] time = 1.01, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

3.576 $\int \frac{1}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - b*x^2],x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.04, size = 35, normalized size = 1.35

$$\frac{\sqrt{-b}\,\log\left(\sqrt{a-bx^2}-\sqrt{-b}\,x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a - b*x^2],x]
[Out] (Sqrt[-b]*Log[-(Sqrt[-b]*x) + Sqrt[a - b*x^2]])/b
```

fricas [A] time = 0.69, size = 72, normalized size = 2.77

$$\left[-\frac{\sqrt{-b}\,\log\left(2\,bx^2-2\,\sqrt{-bx^2+a}\,\sqrt{-b}\,x-a\right)}{2\,b},\,-\frac{\arctan\left(\frac{\sqrt{-bx^2+a}\,\sqrt{b}\,x}{bx^2-a}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-b)*log(2*b*x^2 - 2*sqrt(-b*x^2 + a)*sqrt(-b)*x - a)/b, -arctan(
sqrt(-b*x^2 + a)*sqrt(b)*x/(b*x^2 - a))/sqrt(b)]
```

giac [A] time = 1.15, size = 28, normalized size = 1.08

$$-\frac{\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2+a}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-b*x^2 + a)))/sqrt(-b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^{2}+a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2),x)
[Out] arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 2.96, size = 13, normalized size = 0.50

$$\frac{\arcsin\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] arcsin(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 0.11, size = 25, normalized size = 0.96

$$\frac{\ln\left(\sqrt{a-b\,x^{2}}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2)^(1/2),x)
[Out] log((a - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

sympy [A] time = 1.07, size = 46, normalized size = 1.77

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\ \left|\frac{bx^{2}}{a}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2/a) > 1), (asin(s
qrt(b)*x/sqrt(a))/sqrt(b), True))
```

3.577 $\int \frac{1}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a+bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{-a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.03, size = 30, normalized size = 1.11

$$-\frac{\log\left(\sqrt{bx^2-a}-\sqrt{b}\,x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-a + b*x^2],x]
```

```
[Out] -(Log[-(Sqrt[b]*x) + Sqrt[-a + b*x^2]]/Sqrt[b])
```

fricas [A] time = 0.97, size = 63, normalized size = 2.33

$$\left[\frac{\log\left(2\,bx^2+2\,\sqrt{bx^2-a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2-a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b*x^2 + 2*sqrt(b*x^2 - a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 - a))/b]
```

giac [A] time = 1.05, size = 25, normalized size = 0.93

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2-a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2 - a)))/sqrt(b)
```

maple [A] time = 0.00, size = 23, normalized size = 0.85

$$\frac{\ln\left(\sqrt{b}\, x+\sqrt{b\, x^{2}-a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2-a)^(1/2),x)
[Out] ln(b^(1/2)*x+(b*x^2-a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.32, size = 26, normalized size = 0.96

$$\frac{\log\left(2\, bx+2\, \sqrt{bx^{2}-a}\, \sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] log(2*b*x + 2*sqrt(b*x^2 - a)*sqrt(b))/sqrt(b)
```

mupad [B] time = 0.12, size = 22, normalized size = 0.81

$$\frac{\ln\left(\sqrt{b\, x^{2}-a}+\sqrt{b}\, x\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 - a)^(1/2),x)
[Out] log((b*x^2 - a)^(1/2) + b^(1/2)*x)/b^(1/2)
```

sympy [A] time = 1.09, size = 46, normalized size = 1.70

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\: \left|\frac{bx^{2}}{a}\right| > 1 \\ - \frac{i \operatorname{asin}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-a)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2/a) > 1), (-I*asin(s
qrt(b)*x/sqrt(a))/sqrt(b), True))
```

3.578 $\int \frac{1}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-a - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a-bx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-a-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-a - b*x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.04, size = 37, normalized size = 1.32

$$\frac{\sqrt{-b}\,\log\left(\sqrt{-a-bx^2}-\sqrt{-b}\,x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[-b]*Log[-(Sqrt[-b]*x) + Sqrt[-a - b*x^2]])/b
```

fricas [A] time = 0.92, size = 74, normalized size = 2.64

$$\left[-\frac{\sqrt{-b}\,\log\left(-2\,bx^2+2\,\sqrt{-bx^2-a}\,\sqrt{-b}\,x-a\right)}{2\,b},\ -\frac{\arctan\left(\frac{\sqrt{-bx^2-a}\,\sqrt{b}\,x}{bx^2+a}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-2*b*x^2 + 2*sqrt(-b*x^2 - a)*sqrt(-b)*x - a)/b, -arctan
(sqrt(-b*x^2 - a)*sqrt(b)*x/(b*x^2 + a))/sqrt(b)]
```

giac [A] time = 1.27, size = 30, normalized size = 1.07

$$-\frac{\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2-a}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(-b)*x + sqrt(-b*x^2 - a)))/sqrt(-b)
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^{2}-a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2-a)^(1/2),x)
[Out] arctan(x*b^(1/2)/(-b*x^2-a)^(1/2))/b^(1/2)
```

maxima [C] time = 1.32, size = 14, normalized size = 0.50

$$-\frac{i\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] -I*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 5.13, size = 27, normalized size = 0.96

$$\frac{\ln\left(\sqrt{-b\,x^{2}-a}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- a - b*x^2)^(1/2),x)
[Out] log((- a - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

sympy [C] time = 1.02, size = 20, normalized size = 0.71

$$-\frac{i\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-a)**(1/2),x)
[Out] -I*asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

3.579 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=16

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2-x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{a^2-x^2}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

IntegrateAlgebraic [C] time = 0.03, size = 24, normalized size = 1.50

$$i\log\left(\sqrt{a^2-x^2}-ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a^2 - x^2],x]
```

```
[Out] I*Log[(-I)*x + Sqrt[a^2 - x^2]]
```

fricas [A] time = 0.83, size = 23, normalized size = 1.44

$$-2\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

giac [A] time = 1.07, size = 9, normalized size = 0.56

$$\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x/a)*sgn(a)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2-x^2)^(1/2),x)
```

```
[Out] arctan(x/(a^2-x^2)^(1/2))
```

maxima [A] time = 2.89, size = 6, normalized size = 0.38

$$\arcsin\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x/a)
```

mupad [B] time = 4.74, size = 14, normalized size = 0.88

$$\operatorname{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 - x^2)^(1/2),x)
```

```
[Out] atan(x/(a^2 - x^2)^(1/2))
```

sympy [A] time = 1.02, size = 19, normalized size = 1.19

$$\begin{cases} -i \operatorname{acosh}\left(\frac{x}{a}\right) & \text{for } \left|\frac{x^2}{a^2}\right| > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2-x**2)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x/a), Abs(x**2/a**2) > 1), (asin(x/a), True))
```

3.580 $\int \frac{x^{1+m}\left(a(2+m)+b(3+m)x^2\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=17

$$x^{m+2}\sqrt{a+bx^2}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {449}

$$x^{m+2}\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] x^(2 + m)*Sqrt[a + b*x^2]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{1+m}\left(a(2+m)+b(3+m)x^2\right)}{\sqrt{a+bx^2}}\,dx = x^{2+m}\sqrt{a+bx^2}$$

Mathematica [C] time = 0.11, size = 104, normalized size = 6.12

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\left(b(m+3)x^2\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};-\frac{bx^2}{a}\right)+a(m+4)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*(a*(4 + m)*Hypergeometric2F1[1/2, (2 + m)/2,
 (4 + m)/2, -((b*x^2)/a)] + b*(3 + m)*x^2*Hypergeometric2F1[1/2, (4 + m)/2,
 (6 + m)/2, -((b*x^2)/a)]))/((4 + m)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{x^{1+m}\left(a(2+m)+b(3+m)x^2\right)}{\sqrt{a+bx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],
x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a +
b*x^2], x]
```

fricas [A] time = 0.89, size = 16, normalized size = 0.94

$$\sqrt{bx^2+a}\,xx^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(b*x^2 + a)*x*x^(m + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b(m+3)x^2+a(m+2)\right)x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*(m + 3)*x^2 + a*(m + 2))*x^(m + 1)/sqrt(b*x^2 + a), x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\sqrt{b\,x^2+a}\;x^{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m+1)*(a*(m+2)+b*(m+3)*x^2)/(b*x^2+a)^(1/2),x)
```

```
[Out] x^(m+2)*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.88, size = 16, normalized size = 0.94

$$\sqrt{bx^2+a}\,x^2x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] sqrt(b*x^2 + a)*x^2*x^m
```

mupad [B] time = 5.19, size = 24, normalized size = 1.41

$$\frac{x^{m+1}\,\left(b\,x^3+a\,x\right)}{\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m + 1)*(a*(m + 2) + b*x^2*(m + 3)))/(a + b*x^2)^(1/2),x)
```

```
[Out] (x^(m + 1)*(a*x + b*x^3))/(a + b*x^2)^(1/2)
```

sympy [C] time = 10.06, size = 202, normalized size = 11.88

$$\frac{\sqrt{a}mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{\sqrt{a}\,x^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\Gamma\left(\frac{m}{2}+2\right)}+\frac{bmx^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+3\right)}+\frac{3bx^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*(a*(2+m)+b*(3+m)*x**2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*m*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x**2
*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + sqrt(a)*x**2*x**m*gamma(m/2 + 1)*h
yper((1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/gamma(m/2 + 2) +
 b*m*x**4*x**m*gamma(m/2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_
polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3)) + 3*b*x**4*x**m*gamma(m/2 + 2)*hy
per((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(
m/2 + 3))
```

3.581 $\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right) dx$

Optimal. Leaf size=17

$$x^{m+2}\sqrt{a+bx^2}$$

Rubi [C] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 7.47, number of steps used = 5, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {365, 364}

$$\frac{ax^{m+2}\sqrt{\frac{bx^2}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}+\frac{b(m+3)x^{m+4}\sqrt{\frac{bx^2}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};-\frac{bx^2}{a}\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3 + m))/Sqrt[a +
b*x^2],x]
```

```
[Out] (a*x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/
2, -((b*x^2)/a)])/Sqrt[a + b*x^2] + (b*(3 + m)*x^(4 + m)*Sqrt[1 + (b*x^2)/a
]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, -((b*x^2)/a)])/((4 + m)*Sqrt
[a + b*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)dx = (a(2+m))\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx+(b(3+m))\int \frac{x^{3+m}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{\left(a(2+m)\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}} + \frac{\left(b(3+m)\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{3+m}}{\sqrt{1+\frac{bx^2}{a}}}\, d}{\sqrt{a+bx^2}}$$

$$= \frac{ax^{2+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}} + \frac{b(3+m)x^{4+m}\sqrt{1+\frac{bx^2}{a}}\;{}_2F}{(4+m)\sqrt{a}}$$

Mathematica [C] time = 0.07, size = 104, normalized size = 6.12

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\left(b(m+3)x^2\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};-\frac{bx^2}{a}\right)+a(m+4)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3 + m))/Sqr
t[a + b*x^2],x]
```

```
[Out] (x^(2 + m)*Sqrt[1 + (b*x^2)/a]*(a*(4 + m)*Hypergeometric2F1[1/2, (2 + m)/2,
 (4 + m)/2, -((b*x^2)/a)] + b*(3 + m)*x^2*Hypergeometric2F1[1/2, (4 + m)/2,
 (6 + m)/2, -((b*x^2)/a)]))/((4 + m)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [F] time = 1.89, size = 0, normalized size = 0.00

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3
+ m))/Sqrt[a + b*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m
)*x^(3 + m))/Sqrt[a + b*x^2], x]
```

fricas [A] time = 0.86, size = 18, normalized size = 1.06

$$\frac{\sqrt{bx^2+a}\,x^{m+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x^(m + 3)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b(m+3)x^{m+3}}{\sqrt{bx^2+a}} + \frac{a(m+2)x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(b*(m + 3)*x^(m + 3)/sqrt(b*x^2 + a) + a*(m + 2)*x^(m + 1)/sqrt(b*
x^2 + a), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(m+2)\,a\,x^{m+1}}{\sqrt{b\,x^2+a}} + \frac{(m+3)\,b\,x^{m+3}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*(m+2)*x^(m+1)/(b*x^2+a)^(1/2)+b*(m+3)*x^(m+3)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(a*(m+2)*x^(m+1)/(b*x^2+a)^(1/2)+b*(m+3)*x^(m+3)/(b*x^2+a)^(1/2),x)
```

maxima [A] time = 1.89, size = 16, normalized size = 0.94

$$\sqrt{bx^2+a}\,x^2x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x
, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*x^2*x^m
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{a\,x^{m+1}\,(m+2)}{\sqrt{b\,x^2+a}} + \frac{b\,x^{m+3}\,(m+3)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(m + 1)*(m + 2))/(a + b*x^2)^(1/2) + (b*x^(m + 3)*(m + 3))/(a + b*
x^2)^(1/2),x)
```

```
[Out] int((a*x^(m + 1)*(m + 2))/(a + b*x^2)^(1/2) + (b*x^(m + 3)*(m + 3))/(a + b*
x^2)^(1/2), x)
```

sympy [C] time = 5.29, size = 105, normalized size = 6.18

$$\frac{\sqrt{a}\,x^{2}x^{m}\,(m+2)\,\Gamma\left(\frac{m}{2}+1\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{bx^{4}x^{m}\,(m+3)\,\Gamma\left(\frac{m}{2}+2\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\\frac{m}{2}+3\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*(2+m)*x**(1+m)/(b*x**2+a)**(1/2)+b*(3+m)*x**(3+m)/(b*x**2+a)**(
1/2),x)
```

```
[Out] sqrt(a)*x**2*x**m*(m + 2)*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,),
b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + b*x**4*x**m*(m + 3)*gamma(m/
2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*gamma(m/2 + 3))
```

3.582 $\int \frac{x^{-1+m}\left(am+b(-1+m)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {449}

$$\frac{x^m}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] x^m/Sqrt[a + b*x^2]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-1+m}\left(am+b(-1+m)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{x^m}{\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.11, size = 103, normalized size = 6.87

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\left(b(m-1)x^2\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)+a(m+2)\,{}_2F_1\left(\frac{3}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*(a*(2 + m)*Hypergeometric2F1[3/2, m/2, (2 + m)/2,
-((b*x^2)/a)] + b*(-1 + m)*x^2*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)]))/(a*(2 + m)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+m}\left(am+b(-1+m)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(
3/2), x]
```

fricas [A] time = 1.19, size = 16, normalized size = 1.07

$$\frac{xx^{m-1}}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] x*x^(m - 1)/sqrt(b*x^2 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b(m-1)x^2+am\right)x^{m-1}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*(m - 1)*x^2 + a*m)*x^(m - 1)/(b*x^2 + a)^(3/2), x)
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$\frac{x^m}{\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-1)*(a*m+b*(m-1)*x^2)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/(b*x^2+a)^(1/2)*x^m
```

maxima [A] time = 1.91, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] x^m/sqrt(b*x^2 + a)
```

mupad [B] time = 5.43, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(a*m + b*x^2*(m - 1)))/(a + b*x^2)^(3/2),x)
```

```
[Out] x^m/(a + b*x^2)^(1/2)
```

sympy [C] time = 52.96, size = 97, normalized size = 6.47

$$\frac{mx^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+1\right)}+\frac{bx^2x^m\left(m-1\right)\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*(a*m+b*(-1+m)*x**2)/(b*x**2+a)**(3/2),x)
```

```
[Out] m*x**m*gamma(m/2)*hyper((3/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(
2*sqrt(a)*gamma(m/2 + 1)) + b*x**2*x**m*(m - 1)*gamma(m/2 + 1)*hyper((3/2,
m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 2))
```

3.583 $$\int \left(-\frac{bx^{1+m}}{\left(a+bx^2\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right) dx$$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^2}}$$

Rubi [C] time = 0.07, antiderivative size = 123, normalized size of antiderivative = 8.20, number of steps used = 5, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {365, 364}

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}-\frac{bx^{m+2}\sqrt{\frac{bx^2}{a}+1}\ {}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x^2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -((b*x^2)/a
)])/Sqrt[a + b*x^2] - (b*x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/
2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*(2 + m)*Sqrt[a + b*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(-\frac{bx^{1+m}}{\left(a+bx^2\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right)dx = -\left(b\int \frac{x^{1+m}}{\left(a+bx^2\right)^{3/2}}\,dx\right)+m\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx$$

$$= -\frac{\left(b\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{1+m}}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2}}+\frac{\left(m\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{x^{-1+m}}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{x^m\sqrt{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}-\frac{bx^{2+m}\sqrt{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{a(2+m)\sqrt{a+bx^2}}$$

Mathematica [C] time = 0.06, size = 103, normalized size = 6.87

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\left(b(m-1)x^2\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)+a(m+2)\,{}_2F_1\left(\frac{3}{2},\frac{m}{2};\frac{m+2}{2};-\frac{bx^2}{a}\right)\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x^
2],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^2)/a]*(a*(2 + m)*Hypergeometric2F1[3/2, m/2, (2 + m)/2,
-((b*x^2)/a)] + b*(-1 + m)*x^2*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)]))/(a*(2 + m)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \left(-\frac{bx^{1+m}}{\left(a+bx^2\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt
[a + b*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m
))/Sqrt[a + b*x^2], x]
```

fricas [A] time = 0.94, size = 26, normalized size = 1.73

$$\frac{\sqrt{bx^2+a}\,x^{m+1}}{bx^3+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="fricas")
```

```
[Out] sqrt(b*x^2 + a)*x^(m + 1)/(b*x^3 + a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{mx^{m-1}}{\sqrt{bx^2+a}} - \frac{bx^{m+1}}{\left(bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(m*x^(m - 1)/sqrt(b*x^2 + a) - b*x^(m + 1)/(b*x^2 + a)^(3/2), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int -\frac{b\,x^{m+1}}{\left(b\,x^2+a\right)^{\frac{3}{2}}} + \frac{m\,x^{m-1}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-b*x^(m+1)/(b*x^2+a)^(3/2)+m*x^(m-1)/(b*x^2+a)^(1/2),x)
```

```
[Out] int(-b*x^(m+1)/(b*x^2+a)^(3/2)+m*x^(m-1)/(b*x^2+a)^(1/2),x)
```

maxima [A] time = 1.92, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorith
m="maxima")
```

```
[Out] x^m/sqrt(b*x^2 + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$-\int \frac{b\,x^{m+1}}{\left(b\,x^2+a\right)^{3/2}} - \frac{m\,x^{m-1}}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^(m - 1))/(a + b*x^2)^(1/2) - (b*x^(m + 1))/(a + b*x^2)^(3/2),x)
```

```
[Out] -int((b*x^(m + 1))/(a + b*x^2)^(3/2) - (m*x^(m - 1))/(a + b*x^2)^(1/2), x)
```

sympy [C] time = 5.59, size = 94, normalized size = 6.27

$$\frac{mx^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{m}{2}+1\right)}-\frac{bx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*x**(1+m)/(b*x**2+a)**(3/2)+m*x**(-1+m)/(b*x**2+a)**(1/2),x)
```

```
[Out] m*x**m*gamma(m/2)*hyper((1/2, m/2), (m/2 + 1,), b*x**2*exp_polar(I*pi)/a)/(
2*sqrt(a)*gamma(m/2 + 1)) - b*x**2*x**m*gamma(m/2 + 1)*hyper((3/2, m/2 + 1)
, (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 2))
```

3.584 $\int x^7 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{9a^2\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^4}-\frac{9a\left(a+bx^2\right)^{10/3}}{20b^4}$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{7/3}}{14b^4}-\frac{3a^3\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^4}-\frac{9a\left(a+bx^2\right)^{10/3}}{20b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(4/3))/(8*b^4) + (9*a^2*(a + b*x^2)^(7/3))/(14*b^4) - (
9*a*(a + b*x^2)^(10/3))/(20*b^4) + (3*(a + b*x^2)^(13/3))/(26*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7 \sqrt[3]{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3 \sqrt[3]{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3 \sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}\right)dx, x, x^2\right)\\ &= -\frac{3a^3\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{9a^2\left(a+bx^2\right)^{7/3}}{14b^4}-\frac{9a\left(a+bx^2\right)^{10/3}}{20b^4}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{4/3}\left(-81a^3+108a^2bx^2-126ab^2x^4+140b^3x^6\right)}{3640b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(-81*a^3 + 108*a^2*b*x^2 - 126*a*b^2*x^4 + 140*b^3*x^6
))/(3640*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 50, normalized size = 0.62

$$-\frac{3\left(a+bx^2\right)^{4/3}\left(81a^3-108a^2bx^2+126ab^2x^4-140b^3x^6\right)}{3640b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3)*(81*a^3 - 108*a^2*b*x^2 + 126*a*b^2*x^4 - 140*b^3*x^6
))/(3640*b^4)
```

fricas [A] time = 0.80, size = 57, normalized size = 0.71

$$\frac{3\left(140\,b^4x^8+14\,ab^3x^6-18\,a^2b^2x^4+27\,a^3bx^2-81\,a^4\right)\left(bx^2+a\right)^{\frac{1}{3}}}{3640\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/3640*(140*b^4*x^8 + 14*a*b^3*x^6 - 18*a^2*b^2*x^4 + 27*a^3*b*x^2 - 81*a^4
)*(b*x^2 + a)^(1/3)/b^4
```

giac [A] time = 0.92, size = 57, normalized size = 0.71

$$\frac{3\left(140\left(bx^2+a\right)^{\frac{13}{3}}-546\left(bx^2+a\right)^{\frac{10}{3}}a+780\left(bx^2+a\right)^{\frac{7}{3}}a^2-455\left(bx^2+a\right)^{\frac{4}{3}}a^3\right)}{3640\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/3640*(140*(b*x^2 + a)^(13/3) - 546*(b*x^2 + a)^(10/3)*a + 780*(b*x^2 + a)
^(7/3)*a^2 - 455*(b*x^2 + a)^(4/3)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-140b^{3}x^{6}+126a\,b^{2}x^{4}-108a^{2}b\,x^{2}+81a^{3}\right)}{3640b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(1/3),x)
[Out] -3/3640*(b*x^2+a)^(4/3)*(-140*b^3*x^6+126*a*b^2*x^4-108*a^2*b*x^2+81*a^3)/b
^4
```

maxima [A] time = 1.37, size = 64, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{13}{3}}}{26\,b^{4}}-\frac{9\left(bx^{2}+a\right)^{\frac{10}{3}}a}{20\,b^{4}}+\frac{9\left(bx^{2}+a\right)^{\frac{7}{3}}a^{2}}{14\,b^{4}}-\frac{3\left(bx^{2}+a\right)^{\frac{4}{3}}a^{3}}{8\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 3/26*(b*x^2 + a)^(13/3)/b^4 - 9/20*(b*x^2 + a)^(10/3)*a/b^4 + 9/14*(b*x^2 +
 a)^(7/3)*a^2/b^4 - 3/8*(b*x^2 + a)^(4/3)*a^3/b^4
```

mupad [B] time = 5.25, size = 55, normalized size = 0.69

$$\left(b\,x^{2}+a\right)^{1/3}\left(\frac{3\,x^{8}}{26}-\frac{243\,a^{4}}{3640\,b^{4}}+\frac{3\,a\,x^{6}}{260\,b}-\frac{27\,a^{2}\,x^{4}}{1820\,b^{2}}+\frac{81\,a^{3}\,x^{2}}{3640\,b^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(1/3),x)
[Out] (a + b*x^2)^(1/3)*((3*x^8)/26 - (243*a^4)/(3640*b^4) + (3*a*x^6)/(260*b) -
(27*a^2*x^4)/(1820*b^2) + (81*a^3*x^2)/(3640*b^3))
```

sympy [B] time = 2.85, size = 1795, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(1/3),x)
```

```
[Out] -243*a**(73/3)*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x*
*2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8
+ 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 243*a**(73/3)/(3640*a*
*20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7
*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x
**12) - 1377*a**(70/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 2184
0*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a
**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 1458*a*
*(70/3)*b*x**2/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*
x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**
10 + 3640*a**14*b**10*x**12) - 3213*a**(67/3)*b**2*x**4*(1 + b*x**2/a)**(1/
3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800
*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a*
*14*b**10*x**12) + 3645*a**(67/3)*b**2*x**4/(3640*a**20*b**4 + 21840*a**19*
b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**
8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) - 3927*a**(64/3)*
b**3*x**6*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 +
54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 218
40*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 4860*a**(64/3)*b**3*x**6/(3
640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**1
7*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b
**10*x**12) - 2163*a**(61/3)*b**4*x**8*(1 + b*x**2/a)**(1/3)/(3640*a**20*b*
*4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6
+ 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12)
+ 3645*a**(61/3)*b**4*x**8/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600
*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a*
*15*b**9*x**10 + 3640*a**14*b**10*x**12) + 1827*a**(58/3)*b**5*x**10*(1 + b
*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6
*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x*
*10 + 3640*a**14*b**10*x**12) + 1458*a**(58/3)*b**5*x**10/(3640*a**20*b**4
+ 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 5
4600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 6
573*a**(55/3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**
19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*
b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 243*a**(55/3
)*b**6*x**12/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x*
*4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10
 + 3640*a**14*b**10*x**12) + 8787*a**(52/3)*b**7*x**14*(1 + b*x**2/a)**(1/3
)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*
a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**
14*b**10*x**12) + 6498*a**(49/3)*b**8*x**16*(1 + b*x**2/a)**(1/3)/(3640*a**
20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*
x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x*
*12) + 2562*a**(46/3)*b**9*x**18*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 2
1840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 5460
```

```
0*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 420*
a**(43/3)*b**10*x**20*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*
b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**
8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12)
```

3.585 $\int x^5 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{4/3}}{8b^3}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^3}-\frac{3a\left(a+bx^2\right)^{7/3}}{7b^3}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{4/3}}{8b^3}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^3}-\frac{3a\left(a+bx^2\right)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(4/3))/(8*b^3) - (3*a*(a + b*x^2)^(7/3))/(7*b^3) + (3*(a
 + b*x^2)^(10/3))/(20*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt[3]{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt[3]{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2\sqrt[3]{a+bx}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{3a^2\left(a+bx^2\right)^{4/3}}{8b^3}-\frac{3a\left(a+bx^2\right)^{7/3}}{7b^3}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{4/3}\left(9a^2-12abx^2+14b^2x^4\right)}{280b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(4/3)*(9*a^2 - 12*a*b*x^2 + 14*b^2*x^4))/(280*b^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{4/3}\left(9a^2-12abx^2+14b^2x^4\right)}{280b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(4/3)*(9*a^2 - 12*a*b*x^2 + 14*b^2*x^4))/(280*b^3)
```

fricas [A] time = 0.74, size = 46, normalized size = 0.78

$$\frac{3\left(14\,b^3x^6+2\,ab^2x^4-3\,a^2bx^2+9\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}}{280\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/280*(14*b^3*x^6 + 2*a*b^2*x^4 - 3*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(1/3)/b^
3
```

giac [A] time = 1.03, size = 43, normalized size = 0.73

$$\frac{3\left(14\left(bx^2+a\right)^{\frac{10}{3}}-40\left(bx^2+a\right)^{\frac{7}{3}}a+35\left(bx^2+a\right)^{\frac{4}{3}}a^2\right)}{280\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] 3/280*(14*(b*x^2 + a)^(10/3) - 40*(b*x^2 + a)^(7/3)*a + 35*(b*x^2 + a)^(4/3
)*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{3 \left(b\,x^{2}+a \right)^{\frac{4}{3}} \left(14 b^{2} x^{4}-12 a b\,x^{2}+9 a^{2}\right)}{280 b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/3),x)
```

```
[Out] 3/280*(b*x^2+a)^(4/3)*(14*b^2*x^4-12*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.34, size = 47, normalized size = 0.80

$$\frac{3 \left(bx^{2}+a\right)^{\frac{10}{3}}}{20 \, b^{3}} - \frac{3 \left(bx^{2}+a\right)^{\frac{7}{3}} a}{7 \, b^{3}} + \frac{3 \left(bx^{2}+a\right)^{\frac{4}{3}} a^{2}}{8 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^2 + a)^(10/3)/b^3 - 3/7*(b*x^2 + a)^(7/3)*a/b^3 + 3/8*(b*x^2 + a)
^(4/3)*a^2/b^3
```

mupad [B] time = 4.80, size = 44, normalized size = 0.75

$$\left(b\,x^2+a\right)^{1/3}\,\left(\frac{3\,x^6}{20}+\frac{27\,a^3}{280\,b^3}+\frac{3\,a\,x^4}{140\,b}-\frac{9\,a^2\,x^2}{280\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(1/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((3*x^6)/20 + (27*a^3)/(280*b^3) + (3*a*x^4)/(140*b) - (9
*a^2*x^2)/(280*b^2))
```

sympy [B] time = 1.88, size = 700, normalized size = 11.86

$$\frac{27 a^{\frac{34}{3}} \sqrt[3]{1 + \frac{b x^{2}}{a}}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}} - \frac{27 a^{\frac{34}{3}}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}} + \frac{72 a^{\frac{31}{3}} b x^{2} \sqrt[3]{1 + \frac{b x^{2}}{a}}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}} - \frac{81 a^{\frac{31}{3}} b x^{2}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}} + \frac{60 a^{\frac{28}{3}} b^{2} x^{4} \sqrt[3]{1 + \frac{b x^{2}}{a}}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}} - \frac{81 a^{\frac{28}{3}} b^{2} x^{4}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}} + \frac{60 a^{\frac{25}{3}} b^{3} x^{6} \sqrt[3]{1 + \frac{b x^{2}}{a}}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}} - \frac{27 a^{\frac{25}{3}} b^{3} x^{6}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}} + \frac{135 a^{\frac{22}{3}} b^{4} x^{8} \sqrt[3]{1 + \frac{b x^{2}}{a}}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}} + \frac{132 a^{\frac{19}{3}} b^{5} x^{10} \sqrt[3]{1 + \frac{b x^{2}}{a}}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}} + \frac{42 a^{\frac{16}{3}} b^{6} x^{12} \sqrt[3]{1 + \frac{b x^{2}}{a}}}{280 a^{8} b^{3} + 840 a^{7} b^{4} x^{2} + 840 a^{6} b^{5} x^{4} + 280 a^{5} b^{6} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(34/3)*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 84
0*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 27*a**(34/3)/(280*a**8*b**3 + 840*
a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 72*a**(31/3)*b*
```

```
x**2*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b
**5*x**4 + 280*a**5*b**6*x**6) - 81*a**(31/3)*b*x**2/(280*a**8*b**3 + 840*a
**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 60*a**(28/3)*b**
2*x**4*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6
*b**5*x**4 + 280*a**5*b**6*x**6) - 81*a**(28/3)*b**2*x**4/(280*a**8*b**3 +
840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 60*a**(25/3
)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840
*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 27*a**(25/3)*b**3*x**6/(280*a**8*b*
*3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 135*a*
*(22/3)*b**4*x**8*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2
 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 132*a**(19/3)*b**5*x**10*(1 +
 b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4
+ 280*a**5*b**6*x**6) + 42*a**(16/3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(280*
a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6)
```

3.586 $\int x^3 \sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b^2}-\frac{3a\left(a+bx^2\right)^{4/3}}{8b^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b^2}-\frac{3a\left(a+bx^2\right)^{4/3}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(4/3))/(8*b^2) + (3*(a + b*x^2)^(7/3))/(14*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt[3]{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt[3]{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a\sqrt[3]{a+bx}}{b}+\frac{(a+bx)^{4/3}}{b}\right)dx,x,x^2\right)\\
&= -\frac{3a\left(a+bx^2\right)^{4/3}}{8b^2}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{4/3}\left(4bx^2-3a\right)}{56b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(4/3)*(-3*a + 4*b*x^2))/(56*b^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 1.03

$$-\frac{3\sqrt[3]{a+bx^2}\left(3a^2-abx^2-4b^2x^4\right)}{56b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^(1/3),x]
[Out] (-3*(a + b*x^2)^(1/3)*(3*a^2 - a*b*x^2 - 4*b^2*x^4))/(56*b^2)
```

fricas [A] time = 0.70, size = 34, normalized size = 0.89

$$\frac{3\left(4\,b^2x^4+abx^2-3\,a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}}{56\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/56*(4*b^2*x^4 + a*b*x^2 - 3*a^2)*(b*x^2 + a)^(1/3)/b^2
```

giac [A] time = 1.20, size = 29, normalized size = 0.76

$$\frac{3\left(4\left(bx^2+a\right)^{\frac{7}{3}}-7\left(bx^2+a\right)^{\frac{4}{3}}a\right)}{56\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] 3/56*(4*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^(4/3)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-4b\,x^{2}+3a\right)}{56b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/3),x)
```

```
[Out] -3/56*(b*x^2+a)^(4/3)*(-4*b*x^2+3*a)/b^2
```

maxima [A] time = 1.32, size = 30, normalized size = 0.79

$$\frac{3\left(bx^{2}+a\right)^{\frac{7}{3}}}{14\,b^{2}}-\frac{3\left(bx^{2}+a\right)^{\frac{4}{3}}a}{8\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b^2 - 3/8*(b*x^2 + a)^(4/3)*a/b^2
```

mupad [B] time = 4.72, size = 33, normalized size = 0.87

$$\left(b\,x^{2}+a\right)^{1/3}\left(\frac{3\,x^{4}}{14}-\frac{9\,a^{2}}{56\,b^{2}}+\frac{3\,a\,x^{2}}{56\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(1/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((3*x^4)/14 - (9*a^2)/(56*b^2) + (3*a*x^2)/(56*b))
```

sympy [B] time = 1.22, size = 223, normalized size = 5.87

$$-\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^{2}}{a}}}{56a^{2}b^{2}+56ab^{3}x^{2}}+\frac{9a^{\frac{13}{3}}}{56a^{2}b^{2}+56ab^{3}x^{2}}-\frac{6a^{\frac{10}{3}}bx^{2}\sqrt[3]{1+\frac{bx^{2}}{a}}}{56a^{2}b^{2}+56ab^{3}x^{2}}+\frac{9a^{\frac{10}{3}}bx^{2}}{56a^{2}b^{2}+56ab^{3}x^{2}}+\frac{15a^{\frac{7}{3}}b^{2}x^{4}\sqrt[3]{1+\frac{bx^{2}}{a}}}{56a^{2}b^{2}+56ab^{3}x^{2}}+\frac{12a^{\frac{4}{3}}b^{3}x^{6}\sqrt[3]{1+\frac{bx^{2}}{a}}}{56a^{2}b^{2}+56ab^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(1/3),x)
```

```
[Out] -9*a**(13/3)*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*a**(
13/3)/(56*a**2*b**2 + 56*a*b**3*x**2) - 6*a**(10/3)*b*x**2*(1 + b*x**2/a)**
(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*a**(10/3)*b*x**2/(56*a**2*b**2 +
56*a*b**3*x**2) + 15*a**(7/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2
 + 56*a*b**3*x**2) + 12*a**(4/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(56*a**2*b
**2 + 56*a*b**3*x**2)
```

3.587 $\int x\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt[3]{a+bx^2}\,dx=\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

fricas [A] time = 0.63, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

giac [A] time = 1.12, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\left(b\,x^2+a\right)^{\frac{4}{3}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/3),x)
```

```
[Out] 3/8*(b*x^2+a)^(4/3)/b
```

maxima [A] time = 1.37, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b
```

mupad [B] time = 4.66, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x^2+a\right)^{4/3}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(1/3),x)
```

```
[Out] (3*(a + b*x^2)^(4/3))/(8*b)
```

sympy [A] time = 0.20, size = 42, normalized size = 2.33

$$\begin{cases} \frac{3a\sqrt[3]{a+bx^2}}{8b} + \frac{3x^2\sqrt[3]{a+bx^2}}{8} & \text{for}\ b \neq 0 \\ \frac{\sqrt[3]{a}\,x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/3),x)
```

```
[Out] Piecewise((3*a*(a + b*x**2)**(1/3)/(8*b) + 3*x**2*(a + b*x**2)**(1/3)/8, Ne
(b, 0)), (a**(1/3)*x**2/2, True))
```

3.588 $\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx$

Optimal. Leaf size=101

$$\frac{3}{2}\sqrt[3]{a+bx^2}+\frac{3}{4}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\,\log(x)$$

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 57, 617, 204, 31}

$$\frac{3}{2}\sqrt[3]{a+bx^2}+\frac{3}{4}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\,\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/2 - (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^2)^
(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3)
- (a + b*x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} + \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} - \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{1}{4}\left(3\sqrt[3]{a}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right) - \frac{1}{4}\left(3a^{2/3}\right)\operatorname{Subst}\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) + \frac{1}{2}\left(3\sqrt[3]{a}\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.\\
&= \frac{3}{2}\sqrt[3]{a+bx^2} - \frac{1}{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 126, normalized size = 1.25

$$\frac{1}{4}\left(-\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)+6\sqrt[3]{a+bx^2}+2\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x,x]
```

```
[Out] (6*(a + b*x^2)^(1/3) - 2*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/
a^(1/3))/Sqrt[3]] + 2*a^(1/3)*Log[a^(1/3) - (a + b*x^2)^(1/3)] - a^(1/3)*Lo
g[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/4
```

IntegrateAlgebraic [A] time = 0.10, size = 133, normalized size = 1.32

$$-\frac{1}{4}\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)+\frac{3}{2}\sqrt[3]{a+bx^2}+\frac{1}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)-\frac{1}{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/2 - (Sqrt[3]*a^(1/3)*ArcTan[1/Sqrt[3] + (2*(a + b*x^2
)^(1/3))/(Sqrt[3]*a^(1/3))])/2 + (a^(1/3)*Log[-a^(1/3) + (a + b*x^2)^(1/3)]
)/2 - (a^(1/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]
)/4
```

fricas [A] time = 1.42, size = 102, normalized size = 1.01

$$-\frac{1}{2}\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)-\frac{1}{4}a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\left(bx^2+a\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqrt
(3)*a)/a) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 1/2*a^(1/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a)^
(1/3)
```

giac [A] time = 2.41, size = 98, normalized size = 0.97

$$-\frac{1}{2}\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{1}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{2}\left(bx^2+a\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
```

```
2/3)) + 1/2*a^(1/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/2*(b*x^2 + a)
^(1/3)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x,x)
[Out] int((b*x^2+a)^(1/3)/x,x)
```

maxima [A] time = 3.03, size = 97, normalized size = 0.96

$$-\frac{1}{2}\,\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}\,a^{\frac{1}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{1}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\left(bx^{2}+a\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x,x, algorithm="maxima")
[Out] -1/2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(1/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a)^(1/3
)
```

mupad [B] time = 4.74, size = 115, normalized size = 1.14

$$\frac{a^{1/3}\,\ln\left(\frac{9\,a\,\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{4/3}}{4}\right)}{2}+\frac{3\,\left(b\,x^2+a\right)^{1/3}}{2}-\frac{a^{1/3}\,\ln\left(\frac{9\,a^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+\frac{9\,a\,\left(b\,x^2+a\right)^{1/3}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+a^{1/3}\,\ln\left(\frac{9\,a\,\left(b\,x^2+a\right)^{1/3}}{2}-9\,a^{4/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x,x)
[Out] (a^(1/3)*log((9*a*(a + b*x^2)^(1/3))/4 - (9*a^(4/3))/4))/2 + (3*(a + b*x^2)
^(1/3))/2 - (a^(1/3)*log((9*a^(4/3)*((3^(1/2)*1i)/2 + 1/2))/2 + (9*a*(a + b
*x^2)^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/2 + a^(1/3)*log((9*a*(a + b*x^2)^(1
/3))/2 - 9*a^(4/3)*((3^(1/2)*1i)/4 - 1/4))*((3^(1/2)*1i)/4 - 1/4)
```

sympy [C] time = 1.07, size = 46, normalized size = 0.46

$$-\frac{\sqrt[3]{b}\,x^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x,x)
```

```
[Out] -b**(1/3)*x**(2/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(2/3))
```

3.589 $\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx$

Optimal. Leaf size=107

$$\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}-\frac{\sqrt[3]{a+bx^2}}{2x^2}$$

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 47, 57, 617, 204, 31}

$$\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}-\frac{\sqrt[3]{a+bx^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^3,x]
```

```
[Out] -(a + b*x^2)^(1/3)/(2*x^2) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqr
t[3]*a^(1/3))])/(2*Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/
3) - (a + b*x^2)^(1/3)])/(4*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2}+\frac{1}{6}b\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2}-\frac{b\log(x)}{6a^{2/3}}-\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{a}\ldots\right.}{4\sqrt[3]{a}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{2/3}}\\
&= -\frac{\sqrt[3]{a+bx^2}}{2x^2}-\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.35

$$\frac{3b\left(a+bx^2\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{bx^2}{a}+1\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^3,x]
```

```
[Out] (3*b*(a + b*x^2)^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, 1 + (b*x^2)/a])/(8*a^
2)
```

IntegrateAlgebraic [A] time = 0.19, size = 139, normalized size = 1.30

$$\frac{b\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{6a^{2/3}}-\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{12a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,a^{2/3}}-\frac{\sqrt[3]{a+bx^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/3)/x^3,x]
```

```
[Out] -1/2*(a + b*x^2)^(1/3)/x^2 - (b*ArcTan[1/Sqrt[3] + (2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(2*Sqrt[3]*a^(2/3)) + (b*Log[-a^(1/3) + (a + b*x^2)^(1/3)
])/(6*a^(2/3)) - (b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(
2/3)])/(12*a^(2/3))
```

fricas [A] time = 0.83, size = 155, normalized size = 1.45

$$-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}}abx^2\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-2\left(a^2\right)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+6\left(bx^2+a\right)^{\frac{1}{3}}a^2}{12a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(3)*(a^2)^(1/6)*a*b*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^
(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*b*x^2
*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) -
 2*(a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^2 + a)
^(1/3)*a^2)/(a^2*x^2)
```

giac [A] time = 2.48, size = 115, normalized size = 1.07

$$-\frac{\frac{2\sqrt{3}b^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{b^{2}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2b^{2}\log\left(\left|\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{6\left(bx^{2}+a\right)^{\frac{1}{3}}b}{x^{2}}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/12*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(2/3) + b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(2/3) - 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*
x^2 + a)^(1/3)*b/x^2)/b
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^3,x)
```

maxima [A] time = 2.91, size = 103, normalized size = 0.96

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{6a^{\frac{2}{3}}}-\frac{b\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}+\frac{b\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\left(bx^{2}+a\right)^{\frac{1}{3}}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(2/3) - 1/12*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(2/3) + 1/6*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3) - 1/2*(b*x^2 +
a)^(1/3)/x^2
```

mupad [B] time = 4.88, size = 125, normalized size = 1.17

$$\frac{b\,\ln\left(\frac{3\,b\,\left(b\,x^2+a\right)^{1/3}}{2}-\frac{3\,a^{1/3}\,b}{2}\right)}{6\,a^{2/3}}-\frac{\left(b\,x^2+a\right)^{1/3}}{2\,x^2}-\frac{\ln\left(\frac{3\,a^{1/3}\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{4}+\frac{3\,b\,\left(b\,x^2+a\right)^{1/3}}{2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{12\,a^{2/3}}-\frac{\ln\left(\frac{3\,a^{1/3}\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{4}+\frac{3\,b\,\left(b\,x^2+a\right)^{1/3}}{2}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{12\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x^3,x)
```

```
[Out] (b*log((3*b*(a + b*x^2)^(1/3))/2 - (3*a^(1/3)*b)/2))/(6*a^(2/3)) - (a + b*x
^2)^(1/3)/(2*x^2) - (log((3*a^(1/3)*(b - 3^(1/2)*b*1i))/4 + (3*b*(a + b*x^2
)^(1/3))/2)*(b - 3^(1/2)*b*1i))/(12*a^(2/3)) - (log((3*a^(1/3)*(b + 3^(1/2)
*b*1i))/4 + (3*b*(a + b*x^2)^(1/3))/2)*(b + 3^(1/2)*b*1i))/(12*a^(2/3))
```

sympy [C] time = 1.20, size = 42, normalized size = 0.39

$$-\frac{\sqrt[3]{b}\,\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**3,x)
```

```
[Out] -b**(1/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(4/3)*gamma(5/3))
```

3.590 $\int \frac{\sqrt[3]{a+bx^2}}{x^5} dx$

Optimal. Leaf size=135

$$-\frac{b^2 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}+\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{5/3}}+\frac{b^2 \log(x)}{18a^{5/3}}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}-\frac{\sqrt[3]{a+bx^2}}{4x^4}$$

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {266, 47, 51, 57, 617, 204, 31}

$$-\frac{b^2 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}+\frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{5/3}}+\frac{b^2 \log(x)}{18a^{5/3}}-\frac{b\sqrt[3]{a+bx^2}}{12ax^2}-\frac{\sqrt[3]{a+bx^2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/x^5,x]
```

```
[Out] -(a + b*x^2)^(1/3)/(4*x^4) - (b*(a + b*x^2)^(1/3))/(12*a*x^2) + (b^2*ArcTan
[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(5/3)) +
(b^2*Log[x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(12*a^(5
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[3]{a+bx^2}}{4x^4} + \frac{1}{12}b\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)}{18a}$$
$$= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2\log(x)}{18a^{5/3}} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{12a^{5/3}} + \frac{b^2\,\text{Subst}\left(\int \frac{}{a^{2/}}\right.}{}$$
$$= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2\log(x)}{18a^{5/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{6a^{5/3}}$$
$$= -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{5/3}} + \frac{b^2\log(x)}{18a^{5/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.29

$$-\frac{3b^2\left(a+bx^2\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};\frac{bx^2}{a}+1\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/x^5,x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, 1 + (b*x^2)/a])/(8
*a^3)
```

IntegrateAlgebraic [A] time = 0.20, size = 158, normalized size = 1.17

$$-\frac{b^2\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{18a^{5/3}} + \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{36a^{5/3}} + \frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{5/3}} + \frac{\left(-3a-bx^2\right)\sqrt[3]{a+bx^2}}{12ax^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/3)/x^5,x]
```

```
[Out] ((-3*a - b*x^2)*(a + b*x^2)^(1/3))/(12*a*x^4) + (b^2*ArcTan[1/Sqrt[3] + (2*
(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(5/3)) - (b^2*Log[-a^(1
/3) + (a + b*x^2)^(1/3)])/(18*a^(5/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a + b*
x^2)^(1/3) + (a + b*x^2)^(2/3)])/(36*a^(5/3))
```

fricas [A] time = 0.82, size = 199, normalized size = 1.47

$$\frac{2\,\sqrt{3}\,ab^2x^4\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\,\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\,\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3\,a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)-2\,\left(-a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)-3\left(a^2bx^2+3\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}}{36\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="fricas")
```

```
[Out] 1/36*(2*sqrt(3)*a*b^2*x^4*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(
1/3)*a - 2*sqrt(3)*(b*x^2 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2)
 + (-a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^2 +
 a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a -
(-a^2)^(2/3)) - 3*(a^2*b*x^2 + 3*a^3)*(b*x^2 + a)^(1/3))/(a^3*x^4)
```

giac [A] time = 2.37, size = 140, normalized size = 1.04

$$\frac{\frac{2\,\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\,b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left(\left(bx^2+a\right)^{\frac{4}{3}}b^3+2\left(bx^2+a\right)^{\frac{1}{3}}ab^3\right)}{ab^2x^4}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="giac")
```

```
[Out] 1/36*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(5/3) + b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3))/a^(5/3) - 2*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*
x^2 + a)^(4/3)*b^3 + 2*(b*x^2 + a)^(1/3)*a*b^3)/(a*b^2*x^4))/b
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/x^5,x)
```

```
[Out] int((b*x^2+a)^(1/3)/x^5,x)
```

maxima [A] time = 2.91, size = 155, normalized size = 1.15

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{18\,a^{\frac{5}{3}}}+\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{36\,a^{\frac{5}{3}}}-\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18\,a^{\frac{5}{3}}}-\frac{\left(bx^2+a\right)^{\frac{4}{3}}b^2+2\left(bx^2+a\right)^{\frac{1}{3}}ab^2}{12\left(\left(bx^2+a\right)^2a-2\left(bx^2+a\right)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(5/3) + 1/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(5/3) - 1/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(5/3) - 1/12*(
(b*x^2 + a)^(4/3)*b^2 + 2*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2*a - 2*(b*
x^2 + a)*a^2 + a^3)
```

mupad [B] time = 5.11, size = 217, normalized size = 1.61

$$\frac{b^2\ln\left(\frac{b^2}{2\,(-a)^{2/3}}-\frac{b^2\,\left(b\,x^2+a\right)^{1/3}}{2\,a}\right)}{18\,(-a)^{5/3}}-\frac{\ln\left(\frac{b^2+\sqrt{3}\,b^2\,1\mathrm{i}}{4\,(-a)^{2/3}}+\frac{b^2\,\left(b\,x^2+a\right)^{1/3}}{2\,a}\right)\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{36\,(-a)^{5/3}}-\frac{\frac{b^2\,\left(b\,x^2+a\right)^{1/3}}{3}+\frac{b^2\,\left(b\,x^2+a\right)^{4/3}}{6\,a}}{2\left(b\,x^2+a\right)^2-4\,a\,\left(b\,x^2+a\right)+2\,a^2}+\frac{b^2\ln\left(\frac{b^2\,\left(b\,x^2+a\right)^{1/3}}{2\,a}-\frac{b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,(-a)^{2/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18\,(-a)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/x^5,x)
```

```
[Out] (b^2*log(b^2/(2*(-a)^(2/3)) - (b^2*(a + b*x^2)^(1/3))/(2*a)))/(18*(-a)^(5/3
)) - (log((3^(1/2)*b^2*1i + b^2)/(4*(-a)^(2/3)) + (b^2*(a + b*x^2)^(1/3))/(
2*a))*(3^(1/2)*b^2*1i + b^2))/(36*(-a)^(5/3)) - ((b^2*(a + b*x^2)^(1/3))/3
+ (b^2*(a + b*x^2)^(4/3))/(6*a))/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2
) + (b^2*log((b^2*(a + b*x^2)^(1/3))/(2*a) - (b^2*((3^(1/2)*1i)/2 - 1/2))/(
2*(-a)^(2/3)))*((3^(1/2)*1i)/2 - 1/2))/(18*(-a)^(5/3))
```

sympy [C] time = 1.39, size = 42, normalized size = 0.31

$$-\frac{\sqrt[3]{b}\,\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{5}{3}\\\frac{8}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{10}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/x**5,x)
```

```
[Out] -b**(1/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(10/3)*gamma(8/3))
```

3.591 $\int x^7 \left(a+bx^2\right)^{2/3} dx$

Optimal. Leaf size=80

$$-\frac{3a^3\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{9a^2\left(a+bx^2\right)^{8/3}}{16b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{28b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{22b^4}$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{8/3}}{16b^4}-\frac{3a^3\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{28b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{22b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(5/3))/(10*b^4) + (9*a^2*(a + b*x^2)^(8/3))/(16*b^4) -
(9*a*(a + b*x^2)^(11/3))/(22*b^4) + (3*(a + b*x^2)^(14/3))/(28*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7\left(a+bx^2\right)^{2/3} dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{2/3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}\right)dx,x,x^2\right)\\
&= -\frac{3a^3\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{9a^2\left(a+bx^2\right)^{8/3}}{16b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{22b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{28b^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{5/3}\left(-81a^3+135a^2bx^2-180ab^2x^4+220b^3x^6\right)}{6160b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(5/3)*(-81*a^3 + 135*a^2*b*x^2 - 180*a*b^2*x^4 + 220*b^3*x^6
))/(6160*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 50, normalized size = 0.62

$$-\frac{3\left(a+bx^2\right)^{5/3}\left(81a^3-135a^2bx^2+180ab^2x^4-220b^3x^6\right)}{6160b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*(a + b*x^2)^(5/3)*(81*a^3 - 135*a^2*b*x^2 + 180*a*b^2*x^4 - 220*b^3*x^6
))/(6160*b^4)
```

fricas [A] time = 0.79, size = 57, normalized size = 0.71

$$\frac{3\left(220\,b^4x^8+40\,ab^3x^6-45\,a^2b^2x^4+54\,a^3bx^2-81\,a^4\right)\left(bx^2+a\right)^{\frac{2}{3}}}{6160\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/6160*(220*b^4*x^8 + 40*a*b^3*x^6 - 45*a^2*b^2*x^4 + 54*a^3*b*x^2 - 81*a^4
)*(b*x^2 + a)^(2/3)/b^4
```

giac [A] time = 0.98, size = 57, normalized size = 0.71

$$\frac{3\left(220\left(bx^2+a\right)^{\frac{14}{3}}-840\left(bx^2+a\right)^{\frac{11}{3}}a+1155\left(bx^2+a\right)^{\frac{8}{3}}a^2-616\left(bx^2+a\right)^{\frac{5}{3}}a^3\right)}{6160\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/6160*(220*(b*x^2 + a)^(14/3) - 840*(b*x^2 + a)^(11/3)*a + 1155*(b*x^2 + a
)^(8/3)*a^2 - 616*(b*x^2 + a)^(5/3)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(-220b^{3}x^{6}+180a\,b^{2}x^{4}-135a^{2}b\,x^{2}+81a^{3}\right)}{6160b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^(2/3),x)
```

```
[Out] -3/6160*(b*x^2+a)^(5/3)*(-220*b^3*x^6+180*a*b^2*x^4-135*a^2*b*x^2+81*a^3)/b
^4
```

maxima [A] time = 1.30, size = 64, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{14}{3}}}{28\,b^{4}}-\frac{9\left(bx^{2}+a\right)^{\frac{11}{3}}a}{22\,b^{4}}+\frac{9\left(bx^{2}+a\right)^{\frac{8}{3}}a^{2}}{16\,b^{4}}-\frac{3\left(bx^{2}+a\right)^{\frac{5}{3}}a^{3}}{10\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/28*(b*x^2 + a)^(14/3)/b^4 - 9/22*(b*x^2 + a)^(11/3)*a/b^4 + 9/16*(b*x^2 +
 a)^(8/3)*a^2/b^4 - 3/10*(b*x^2 + a)^(5/3)*a^3/b^4
```

mupad [B] time = 4.63, size = 55, normalized size = 0.69

$$\left(b\,x^2+a\right)^{2/3}\,\left(\frac{3\,x^8}{28}-\frac{243\,a^4}{6160\,b^4}+\frac{3\,a\,x^6}{154\,b}-\frac{27\,a^2\,x^4}{1232\,b^2}+\frac{81\,a^3\,x^2}{3080\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^(2/3),x)
```

```
[Out] (a + b*x^2)^(2/3)*((3*x^8)/28 - (243*a^4)/(6160*b^4) + (3*a*x^6)/(154*b) -
(27*a^2*x^4)/(1232*b^2) + (81*a^3*x^2)/(3080*b^3))
```

sympy [B] time = 2.94, size = 1795, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(2/3),x)
```

```
[Out] -243*a**(74/3)*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x*
*2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8
 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 243*a**(74/3)/(6160*a
**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b*
*7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10
*x**12) - 1296*a**(71/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36
960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 9240
0*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 1458
*a**(71/3)*b*x**2/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b*
*6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9
*x**10 + 6160*a**14*b**10*x**12) - 2808*a**(68/3)*b**2*x**4*(1 + b*x**2/a)*
*(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 1
23200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 61
60*a**14*b**10*x**12) + 3645*a**(68/3)*b**2*x**4/(6160*a**20*b**4 + 36960*a
**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**
16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 3120*a**(
65/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x
**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**
8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 4860*a**(65/3)*b**3*
x**6/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123
200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160
*a**14*b**10*x**12) - 1050*a**(62/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(6160*
a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b
**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**1
0*x**12) + 3645*a**(62/3)*b**4*x**8/(6160*a**20*b**4 + 36960*a**19*b**5*x**
2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8
+ 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 4032*a**(59/3)*b**5*x*
*10*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*
a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a*
*15*b**9*x**10 + 6160*a**14*b**10*x**12) + 1458*a**(59/3)*b**5*x**10/(6160*
a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b
**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**1
0*x**12) + 11004*a**(56/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**
4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6
+ 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12)
+ 243*a**(56/3)*b**6*x**12/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400
*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a
**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 14352*a**(53/3)*b**7*x**14*(1 +
 b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b*
*6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9
*x**10 + 6160*a**14*b**10*x**12) + 10485*a**(50/3)*b**8*x**16*(1 + b*x**2/a
)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 +
 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 +
6160*a**14*b**10*x**12) + 4080*a**(47/3)*b**9*x**18*(1 + b*x**2/a)**(2/3)/(
6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a*
```

```
*17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14
*b**10*x**12) + 660*a**(44/3)*b**10*x**20*(1 + b*x**2/a)**(2/3)/(6160*a**20
*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x
**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**
12)
```

3.592 $\int x^5 \left(a+bx^2\right)^{2/3} dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{5/3}}{10b^3}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^3}-\frac{3a\left(a+bx^2\right)^{8/3}}{8b^3}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{5/3}}{10b^3}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^3}-\frac{3a\left(a+bx^2\right)^{8/3}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(5/3))/(10*b^3) - (3*a*(a + b*x^2)^(8/3))/(8*b^3) + (3*(
a + b*x^2)^(11/3))/(22*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^2\right)^{2/3} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^{2/3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2(a+bx)^{2/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}\right) dx, x, x^2\right)\\
&= \frac{3a^2\left(a+bx^2\right)^{5/3}}{10b^3}-\frac{3a\left(a+bx^2\right)^{8/3}}{8b^3}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{5/3}\left(9a^2-15abx^2+20b^2x^4\right)}{440b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(2/3),x]
[Out] (3*(a + b*x^2)^(5/3)*(9*a^2 - 15*a*b*x^2 + 20*b^2*x^4))/(440*b^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{5/3}\left(9a^2-15abx^2+20b^2x^4\right)}{440b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^(2/3),x]
[Out] (3*(a + b*x^2)^(5/3)*(9*a^2 - 15*a*b*x^2 + 20*b^2*x^4))/(440*b^3)
```

fricas [A] time = 0.77, size = 46, normalized size = 0.78

$$\frac{3\left(20\,b^3x^6+5\,ab^2x^4-6\,a^2bx^2+9\,a^3\right)\left(bx^2+a\right)^{\frac{2}{3}}}{440\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/440*(20*b^3*x^6 + 5*a*b^2*x^4 - 6*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(2/3)/b^
3
```

giac [A] time = 0.97, size = 43, normalized size = 0.73

$$\frac{3\left(20\left(bx^2+a\right)^{\frac{11}{3}}-55\left(bx^2+a\right)^{\frac{8}{3}}a+44\left(bx^2+a\right)^{\frac{5}{3}}a^2\right)}{440\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] 3/440*(20*(b*x^2 + a)^(11/3) - 55*(b*x^2 + a)^(8/3)*a + 44*(b*x^2 + a)^(5/3
)*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{3 \left(b \,x^{2}+a \right)^{\frac{5}{3}} \left(20 b^{2} x^{4}-15 a b \,x^{2}+9 a^{2}\right)}{440 b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(2/3),x)
[Out] 3/440*(b*x^2+a)^(5/3)*(20*b^2*x^4-15*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.35, size = 47, normalized size = 0.80

$$\frac{3 \left(bx^{2}+a\right)^{\frac{11}{3}}}{22 \, b^{3}}-\frac{3 \left(bx^{2}+a\right)^{\frac{8}{3}} a}{8 \, b^{3}}+\frac{3 \left(bx^{2}+a\right)^{\frac{5}{3}} a^{2}}{10 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] 3/22*(b*x^2 + a)^(11/3)/b^3 - 3/8*(b*x^2 + a)^(8/3)*a/b^3 + 3/10*(b*x^2 + a
)^(5/3)*a^2/b^3
```

mupad [B] time = 4.66, size = 44, normalized size = 0.75

$${\left(b\,x^2+a\right)}^{2/3}\,\left(\frac{3\,x^6}{22}+\frac{27\,a^3}{440\,b^3}+\frac{3\,a\,x^4}{88\,b}-\frac{9\,a^2\,x^2}{220\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(2/3),x)
[Out] (a + b*x^2)^(2/3)*((3*x^6)/22 + (27*a^3)/(440*b^3) + (3*a*x^4)/(88*b) - (9*
a^2*x^2)/(220*b^2))
```

sympy [B] time = 2.00, size = 700, normalized size = 11.86

$$\frac{27 a^{\frac{35}{3}} \left(1 + \frac{b x^{2}}{a}\right)^{\frac{2}{3}}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}} - \frac{27 a^{\frac{35}{3}}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}} + \frac{63 a^{\frac{32}{3}} b x^{2} \left(1 + \frac{b x^{2}}{a}\right)^{\frac{2}{3}}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}} - \frac{81 a^{\frac{32}{3}} b x^{2}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}} + \frac{42 a^{\frac{29}{3}} b^{2} x^{4} \left(1 + \frac{b x^{2}}{a}\right)^{\frac{2}{3}}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}} - \frac{81 a^{\frac{29}{3}} b^{2} x^{4}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}} + \frac{78 a^{\frac{26}{3}} b^{3} x^{6} \left(1 + \frac{b x^{2}}{a}\right)^{\frac{2}{3}}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}} - \frac{27 a^{\frac{26}{3}} b^{3} x^{6}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}} + \frac{207 a^{\frac{23}{3}} b^{4} x^{8} \left(1 + \frac{b x^{2}}{a}\right)^{\frac{2}{3}}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}} + \frac{195 a^{\frac{20}{3}} b^{5} x^{10} \left(1 + \frac{b x^{2}}{a}\right)^{\frac{2}{3}}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}} + \frac{60 a^{\frac{17}{3}} b^{6} x^{12} \left(1 + \frac{b x^{2}}{a}\right)^{\frac{2}{3}}}{440 a^{8} b^{3} + 1320 a^{7} b^{4} x^{2} + 1320 a^{6} b^{5} x^{4} + 440 a^{5} b^{6} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(2/3),x)
[Out] 27*a**(35/3)*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1
320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 27*a**(35/3)/(440*a**8*b**3 + 13
20*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 63*a**(32/3
```

```
)*b*x**2*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*
a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 81*a**(32/3)*b*x**2/(440*a**8*b**3 +
 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 42*a**(2
9/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 +
 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 81*a**(29/3)*b**2*x**4/(440*a*
*8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) +
 78*a**(26/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b*
*4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 27*a**(26/3)*b**3*x**
6/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**
6*x**6) + 207*a**(23/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 13
20*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 195*a**(20/
3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 +
1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 60*a**(17/3)*b**6*x**12*(1 + b*
x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 +
 440*a**5*b**6*x**6)
```

3.593 $\int x^3 \left(a+bx^2\right)^{2/3} dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{8/3}}{16b^2}-\frac{3a\left(a+bx^2\right)^{5/3}}{10b^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{8/3}}{16b^2}-\frac{3a\left(a+bx^2\right)^{5/3}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(5/3))/(10*b^2) + (3*(a + b*x^2)^(8/3))/(16*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{2/3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^{2/3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a(a+bx)^{2/3}}{b}+\frac{(a+bx)^{5/3}}{b}\right)dx,x,x^2\right)\\ &= -\frac{3a\left(a+bx^2\right)^{5/3}}{10b^2}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{5/3}\left(5bx^2-3a\right)}{80b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(2/3),x]
[Out] (3*(a + b*x^2)^(5/3)*(-3*a + 5*b*x^2))/(80*b^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 1.03

$$-\frac{3\left(a+bx^2\right)^{2/3}\left(3a^2-2abx^2-5b^2x^4\right)}{80b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^(2/3),x]
[Out] (-3*(a + b*x^2)^(2/3)*(3*a^2 - 2*a*b*x^2 - 5*b^2*x^4))/(80*b^2)
```

fricas [A] time = 0.83, size = 35, normalized size = 0.92

$$\frac{3\left(5\,b^2x^4+2\,abx^2-3\,a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{80\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/80*(5*b^2*x^4 + 2*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3)/b^2
```

giac [A] time = 1.08, size = 29, normalized size = 0.76

$$\frac{3\left(5\left(bx^2+a\right)^{\frac{8}{3}}-8\left(bx^2+a\right)^{\frac{5}{3}}a\right)}{80\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] 3/80*(5*(b*x^2 + a)^(8/3) - 8*(b*x^2 + a)^(5/3)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(-5b\,x^{2}+3a\right)}{80b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(2/3),x)
```

```
[Out] -3/80*(b*x^2+a)^(5/3)*(-5*b*x^2+3*a)/b^2
```

maxima [A] time = 1.35, size = 30, normalized size = 0.79

$$\frac{3\left(bx^{2}+a\right)^{\frac{8}{3}}}{16\,b^{2}}-\frac{3\left(bx^{2}+a\right)^{\frac{5}{3}}a}{10\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x^2 + a)^(8/3)/b^2 - 3/10*(b*x^2 + a)^(5/3)*a/b^2
```

mupad [B] time = 4.71, size = 33, normalized size = 0.87

$$\left(b\,x^{2}+a\right)^{2/3}\left(\frac{3\,x^{4}}{16}-\frac{9\,a^{2}}{80\,b^{2}}+\frac{3\,a\,x^{2}}{40\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(2/3),x)
```

```
[Out] (a + b*x^2)^(2/3)*((3*x^4)/16 - (9*a^2)/(80*b^2) + (3*a*x^2)/(40*b))
```

sympy [A] time = 0.75, size = 66, normalized size = 1.74

$$\begin{cases}-\frac{9a^{2}\left(a+bx^{2}\right)^{\frac{2}{3}}}{80b^{2}}+\frac{3ax^{2}\left(a+bx^{2}\right)^{\frac{2}{3}}}{40b}+\frac{3x^{4}\left(a+bx^{2}\right)^{\frac{2}{3}}}{16} & \text{for}\ b\neq 0\\ \frac{a^{\frac{2}{3}}x^{4}}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((-9*a**2*(a + b*x**2)**(2/3)/(80*b**2) + 3*a*x**2*(a + b*x**2)**(
2/3)/(40*b) + 3*x**4*(a + b*x**2)**(2/3)/16, Ne(b, 0)), (a**(2/3)*x**4/4, T
rue))
```

3.594 $\int x\left(a+bx^2\right)^{2/3}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(2/3),x]
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{2/3}\,dx=\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(2/3),x]
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^(2/3),x]
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

fricas [A] time = 0.77, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

giac [A] time = 1.06, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\left(b\,x^2+a\right)^{\frac{5}{3}}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(2/3),x)
[Out] 3/10*(b*x^2+a)^(5/3)/b
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b
```

mupad [B] time = 4.58, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x^2+a\right)^{5/3}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(2/3),x)
```

```
[Out] (3*(a + b*x^2)^(5/3))/(10*b)
```

sympy [A] time = 0.37, size = 42, normalized size = 2.33

$$\begin{cases} \frac{3a\left(a+bx^2\right)^{\frac{2}{3}}}{10b}+\frac{3x^2\left(a+bx^2\right)^{\frac{2}{3}}}{10} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{2}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((3*a*(a + b*x**2)**(2/3)/(10*b) + 3*x**2*(a + b*x**2)**(2/3)/10,
Ne(b, 0)), (a**(2/3)*x**2/2, True))
```

3.595 $\int \frac{(a+bx^2)^{2/3}}{x} dx$

Optimal. Leaf size=101

$$\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}\left(a+bx^2\right)^{2/3}$$

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 55, 617, 204, 31}

$$\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}\left(a+bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(2/3))/4 + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^2)^
(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3)
- (a + b*x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{2/3}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx,x,x^2\right)\\ &= \frac{3}{4}\left(a+bx^2\right)^{2/3}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\ &= \frac{3}{4}\left(a+bx^2\right)^{2/3}-\frac{1}{2}a^{2/3}\log(x)-\frac{1}{4}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)+\frac{1}{4}(3a)\,\text{Subst}\\ &= \frac{3}{4}\left(a+bx^2\right)^{2/3}-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.\\ &= \frac{3}{4}\left(a+bx^2\right)^{2/3}+\frac{1}{2}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 93, normalized size = 0.92

$$\frac{1}{4}\left(3\left(a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\left(a+bx^2\right)^{2/3}\right)+2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2a^{2/3}\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x,x]
```

```
[Out] (2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*
a^(2/3)*Log[x] + 3*((a + b*x^2)^(2/3) + a^(2/3)*Log[a^(1/3) - (a + b*x^2)^(
1/3)]))/4
```

IntegrateAlgebraic [A] time = 0.09, size = 133, normalized size = 1.32

$$\frac{1}{2}a^{2/3}\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)-\frac{1}{4}a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)+\frac{1}{2}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{4}\left(a+bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(2/3))/4 + (Sqrt[3]*a^(2/3)*ArcTan[1/Sqrt[3] + (2*(a + b*x^2
)^(1/3))/(Sqrt[3]*a^(1/3))])/2 + (a^(2/3)*Log[-a^(1/3) + (a + b*x^2)^(1/3)]
)/2 - (a^(2/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]
)/4
```

fricas [A] time = 0.94, size = 122, normalized size = 1.21

$$\frac{1}{2}\sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{1}{3}}}{3a}\right)-\frac{1}{4}\left(a^2\right)^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)+\frac{1}{2}\left(a^2\right)^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+\frac{3}{4}\left(bx^2+a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)
*(a^2)^(1/3))/a) - 1/4*(a^2)^(1/3)*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a
+ (b*x^2 + a)^(1/3)*(a^2)^(2/3)) + 1/2*(a^2)^(1/3)*log((b*x^2 + a)^(1/3)*a
- (a^2)^(2/3)) + 3/4*(b*x^2 + a)^(2/3)
```

giac [A] time = 2.36, size = 98, normalized size = 0.97

$$\frac{1}{2}\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}a^{\frac{2}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{4}\left(bx^2+a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - 1/4*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
```

```
/3)) + 1/2*a^(2/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/4*(b*x^2 + a)^
(2/3)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x,x)
```

maxima [A] time = 2.97, size = 97, normalized size = 0.96

$$\frac{1}{2}\,\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}\,a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\left(bx^2+a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - 1/4*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3)) + 1/2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/4*(b*x^2 + a)^(2/3)
```

mupad [B] time = 4.67, size = 125, normalized size = 1.24

$$\frac{3\left(b\,x^2+a\right)^{2/3}}{4}+\frac{a^{2/3}\,\ln\left(\frac{9\,a^2\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{7/3}}{4}\right)}{2}-\frac{a^{2/3}\,\ln\left(\frac{9\,a^2\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^2}{4}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+a^{2/3}\,\ln\left(\frac{9\,a^2\left(b\,x^2+a\right)^{1/3}}{4}-9\,a^{7/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)^2\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x,x)
```

```
[Out] (3*(a + b*x^2)^(2/3))/4 + (a^(2/3)*log((9*a^2*(a + b*x^2)^(1/3))/4 - (9*a^(
7/3))/4))/2 - (a^(2/3)*log((9*a^2*(a + b*x^2)^(1/3))/4 - (9*a^(7/3)*((3^(1/
2)*1i)/2 + 1/2)^2)/4)*((3^(1/2)*1i)/2 + 1/2))/2 + a^(2/3)*log((9*a^2*(a + b
*x^2)^(1/3))/4 - 9*a^(7/3)*((3^(1/2)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4)
```

sympy [C] time = 1.12, size = 46, normalized size = 0.46

$$-\frac{b^{\frac{2}{3}}x^{\frac{4}{3}}\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x,x)
```

```
[Out] -b**(2/3)*x**(4/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(1/3))
```

3.596 $\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=104

$$-\frac{(a+bx^2)^{2/3}}{2x^2}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}$$

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 47, 55, 617, 204, 31}

$$-\frac{(a+bx^2)^{2/3}}{2x^2}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^3,x]
```

```
[Out] -(a + b*x^2)^(2/3)/(2*x^2) + (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3)
 - (a + b*x^2)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{2/3}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2} + \frac{1}{3}b\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right) - \frac{b\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}} - \frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= -\frac{\left(a+bx^2\right)^{2/3}}{2x^2} + \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.36

$$\frac{3b(a+bx^2)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{bx^2}{a}+1\right)}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^3,x]
```

```
[Out] (3*b*(a + b*x^2)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + (b*x^2)/a])/(10*a
^2)
```

IntegrateAlgebraic [A] time = 0.18, size = 136, normalized size = 1.31

$$-\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{6\sqrt[3]{a}}-\frac{\left(a+bx^2\right)^{2/3}}{2x^2}+\frac{b\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{3\sqrt[3]{a}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(2/3)/x^3,x]
```

```
[Out] -1/2*(a + b*x^2)^(2/3)/x^2 + (b*ArcTan[1/Sqrt[3] + (2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)) + (b*Log[-a^(1/3) + (a + b*x^2)^(1/3)])
/(3*a^(1/3)) - (b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/
3)])/(6*a^(1/3))
```

fricas [A] time = 0.67, size = 290, normalized size = 2.79

$$\left[\frac{3\sqrt{\frac{1}{3}}abx^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^2+3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^2}\right)-a^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2a^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\left(bx^2+a\right)^{\frac{2}{3}}a}{6ax^2},\ \frac{6\sqrt{\frac{1}{3}}a^{\frac{2}{3}}bx^2\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2a^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\left(bx^2+a\right)^{\frac{2}{3}}a}{6ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x^2*sqrt(-1/a^(2/3))*log((2*b*x^2 + 3*sqrt(1/3)*(2*(b
*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - a^(2/3)*b*x^2*log((b*x^2 + a)^(2
/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x^2*log((b*x^2 + a
)^(1/3) - a^(1/3)) - 3*(b*x^2 + a)^(2/3)*a)/(a*x^2), 1/6*(6*sqrt(1/3)*a^(2/
3)*b*x^2*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3
)*b*x^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^
(2/3)*b*x^2*log((b*x^2 + a)^(1/3) - a^(1/3)) - 3*(b*x^2 + a)^(2/3)*a)/(a*x^
2)]
```

giac [A] time = 2.37, size = 116, normalized size = 1.12

$$\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\,b^2\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}-\frac{3\,(bx^2+a)^{\frac{2}{3}}b}{x^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(1/3) - b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(1/3) + 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x^
2 + a)^(2/3)*b/x^2)/b
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^3,x)
```

maxima [A] time = 2.97, size = 103, normalized size = 0.99

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}}-\frac{b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}}+\frac{b\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}-\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(1/3) - 1/6*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(1/3) + 1/3*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(1/3) - 1/2*(b*x^2 + a)
^(2/3)/x^2
```

mupad [B] time = 4.89, size = 136, normalized size = 1.31

$$\frac{b\ln\left(a^{1/3}b^2-b^2\left(bx^2+a\right)^{1/3}\right)}{3a^{1/3}}-\frac{\left(bx^2+a\right)^{2/3}}{2x^2}-\frac{\ln\left(\frac{a^{1/3}\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4}-b^2\left(bx^2+a\right)^{1/3}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6a^{1/3}}-\frac{\ln\left(\frac{a^{1/3}\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4}-b^2\left(bx^2+a\right)^{1/3}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x^3,x)
```

```
[Out] (b*log(a^(1/3)*b^2 - b^2*(a + b*x^2)^(1/3)))/(3*a^(1/3)) - (a + b*x^2)^(2/3
)/(2*x^2) - (log((a^(1/3)*(b - 3^(1/2)*b*1i)^2)/4 - b^2*(a + b*x^2)^(1/3))*
(b - 3^(1/2)*b*1i))/(6*a^(1/3)) - (log((a^(1/3)*(b + 3^(1/2)*b*1i)^2)/4 - b
^2*(a + b*x^2)^(1/3))*(b + 3^(1/2)*b*1i))/(6*a^(1/3))
```

sympy [C] time = 1.22, size = 42, normalized size = 0.40

$$-\frac{b^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**3,x)
```

```
[Out] -b**(2/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(2/3)*gamma(4/3))
```

3.597 $\int \frac{(a+bx^2)^{2/3}}{x^5} \, dx$

Optimal. Leaf size=135

$$-\frac{b^2 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}-\frac{b^2 \tan^{-1}\left(\frac{2 \sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{6\sqrt{3}\, a^{4/3}}+\frac{b^2 \log(x)}{18a^{4/3}}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}-\frac{\left(a+bx^2\right)^{2/3}}{4x^4}$$

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {266, 47, 51, 55, 617, 204, 31}

$$-\frac{b^2 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}-\frac{b^2 \tan^{-1}\left(\frac{2 \sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3} \sqrt[3]{a}}\right)}{6\sqrt{3}\, a^{4/3}}+\frac{b^2 \log(x)}{18a^{4/3}}-\frac{b\left(a+bx^2\right)^{2/3}}{6ax^2}-\frac{\left(a+bx^2\right)^{2/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(2/3)/x^5,x]
```

```
[Out] -(a + b*x^2)^(2/3)/(4*x^4) - (b*(a + b*x^2)^(2/3))/(6*a*x^2) - (b^2*ArcTan[
(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(4/3)) + (
b^2*Log[x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(12*a^(4/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{2/3}}{x^5}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4} + \frac{1}{6}b\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4} - \frac{b\left(a+bx^2\right)^{2/3}}{6ax^2} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{18a}$$
$$= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4} - \frac{b\left(a+bx^2\right)^{2/3}}{6ax^2} + \frac{b^2\log(x)}{18a^{4/3}} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{12a^{4/3}} - \frac{b^2\operatorname{Sub}}{}$$
$$= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4} - \frac{b\left(a+bx^2\right)^{2/3}}{6ax^2} + \frac{b^2\log(x)}{18a^{4/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\right.}{6}$$
$$= -\frac{\left(a+bx^2\right)^{2/3}}{4x^4} - \frac{b\left(a+bx^2\right)^{2/3}}{6ax^2} - \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{4/3}} + \frac{b^2\log(x)}{18a^{4/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+b}\right.}{12a^{4/3}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.29

$$-\frac{3b^2\left(a+bx^2\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};\frac{bx^2}{a}+1\right)}{10a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(2/3)/x^5,x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(5/3)*Hypergeometric2F1[5/3, 3, 8/3, 1 + (b*x^2)/a])/(1
0*a^3)
```

IntegrateAlgebraic [A] time = 0.22, size = 158, normalized size = 1.17

$$-\frac{b^2\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{18a^{4/3}} + \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{36a^{4/3}} - \frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{4/3}} + \frac{\left(-3a-2bx^2\right)\left(a+bx^2\right)^{2/3}}{12ax^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(2/3)/x^5,x]
```

```
[Out] ((-3*a - 2*b*x^2)*(a + b*x^2)^(2/3))/(12*a*x^4) - (b^2*ArcTan[1/Sqrt[3] + (
2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(4/3)) - (b^2*Log[-a^
(1/3) + (a + b*x^2)^(1/3)])/(18*a^(4/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a +
b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(36*a^(4/3))
```

fricas [**A**] time = 0.79, size = 380, normalized size = 2.81

$$\left[\frac{3\sqrt{\frac{1}{3}}ab^2x^4\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx^2-3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x^2}\right)+(-a)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3\left(2abx^2+3a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{36a^2x^4}, -\frac{6\sqrt{\frac{1}{3}}ab^2x^4\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2(-a)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3\left(2abx^2+3a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{36a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="fricas")
```

```
[Out] [1/36*(3*sqrt(1/3)*a*b^2*x^4*sqrt((-a)^(1/3)/a)*log((2*b*x^2 - 3*sqrt(1/3)*
(2*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt(
(-a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + (-a)^(2/3)*b^2
*x^4*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2
*(-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) - 3*(2*a*b*x^2 + 3*
a^2)*(b*x^2 + a)^(2/3))/(a^2*x^4), -1/36*(6*sqrt(1/3)*a*b^2*x^4*sqrt(-(-a)^
(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/
3)/a)) - (-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^
(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) + (-a)^(1/
3)) + 3*(2*a*b*x^2 + 3*a^2)*(b*x^2 + a)^(2/3))/(a^2*x^4)]
```

giac [**A**] time = 2.55, size = 141, normalized size = 1.04

$$-\frac{\frac{2\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3\left(2\left(bx^2+a\right)^{\frac{5}{3}}b^3+\left(bx^2+a\right)^{\frac{2}{3}}ab^3\right)}{ab^2x^4}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="giac")
```

```
[Out] -1/36*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(4/3) - b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(4/3) + 2*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*(2*
(b*x^2 + a)^(5/3)*b^3 + (b*x^2 + a)^(2/3)*a*b^3)/(a*b^2*x^4))/b
```

maple [**F**] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{2}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(2/3)/x^5,x)
```

```
[Out] int((b*x^2+a)^(2/3)/x^5,x)
```

maxima [A] time = 3.02, size = 155, normalized size = 1.15

$$-\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{18\,a^{\frac{4}{3}}}+\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{36\,a^{\frac{4}{3}}}-\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18\,a^{\frac{4}{3}}}-\frac{2\left(bx^2+a\right)^{\frac{5}{3}}b^2+\left(bx^2+a\right)^{\frac{2}{3}}ab^2}{12\left(\left(bx^2+a\right)^2a-2\left(bx^2+a\right)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) + 1/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(4/3) - 1/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(4/3) - 1/12*
(2*(b*x^2 + a)^(5/3)*b^2 + (b*x^2 + a)^(2/3)*a*b^2)/((b*x^2 + a)^2*a - 2*(b
*x^2 + a)*a^2 + a^3)
```

mupad [B] time = 5.16, size = 212, normalized size = 1.57

$$\frac{(-1)^{1/3}\,b^2\,\ln\left(\left(b\,x^2+a\right)^{1/3}-(-1)^{2/3}\,a^{1/3}\right)}{18\,a^{4/3}}-\frac{\frac{b^2\left(b\,x^2+a\right)^{2/3}}{6}+\frac{b^2\left(b\,x^2+a\right)^{5/3}}{3\,a}}{2\left(b\,x^2+a\right)^2-4\,a\,\left(b\,x^2+a\right)+2\,a^2}+\frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\left(b\,x^2+a\right)^{1/3}}{36\,a^2}-\frac{(-1)^{2/3}\,b^4\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{36\,a^{5/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18\,a^{4/3}}-\frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\left(b\,x^2+a\right)^{1/3}}{36\,a^2}-\frac{(-1)^{2/3}\,b^4\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{36\,a^{5/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(2/3)/x^5,x)
```

```
[Out] ((-1)^(1/3)*b^2*log((a + b*x^2)^(1/3) - (-1)^(2/3)*a^(1/3)))/(18*a^(4/3)) -
 ((b^2*(a + b*x^2)^(2/3))/6 + (b^2*(a + b*x^2)^(5/3))/(3*a))/(2*(a + b*x^2)
^2 - 4*a*(a + b*x^2) + 2*a^2) + ((-1)^(1/3)*b^2*log((b^4*(a + b*x^2)^(1/3))
/(36*a^2) - ((-1)^(2/3)*b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(36*a^(5/3)))*((3^(1/
2)*1i)/2 - 1/2))/(18*a^(4/3)) - ((-1)^(1/3)*b^2*log((b^4*(a + b*x^2)^(1/3))
/(36*a^2) - ((-1)^(2/3)*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/(36*a^(5/3)))*((3^(1/
2)*1i)/2 + 1/2))/(18*a^(4/3))
```

sympy [C] time = 1.41, size = 42, normalized size = 0.31

$$-\frac{b^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(2/3)/x**5,x)
```

```
[Out] -b**(2/3)*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(8/3)*gamma(7/3))
```

3.598 $\int x^7 \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=80

$$-\frac{3a^3\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{9a^2\left(a+bx^2\right)^{10/3}}{20b^4}+\frac{3\left(a+bx^2\right)^{16/3}}{32b^4}-\frac{9a\left(a+bx^2\right)^{13/3}}{26b^4}$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{3a^3\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{3\left(a+bx^2\right)^{16/3}}{32b^4}-\frac{9a\left(a+bx^2\right)^{13/3}}{26b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(7/3))/(14*b^4) + (9*a^2*(a + b*x^2)^(10/3))/(20*b^4) -
 (9*a*(a + b*x^2)^(13/3))/(26*b^4) + (3*(a + b*x^2)^(16/3))/(32*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^{4/3}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^{4/3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{4/3}}{b^3}+\frac{3a^2(a+bx)^{7/3}}{b^3}-\frac{3a(a+bx)^{10/3}}{b^3}+\frac{(a+bx)^{13/3}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{3a^3\left(a+bx^2\right)^{7/3}}{14b^4}+\frac{9a^2\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{9a\left(a+bx^2\right)^{13/3}}{26b^4}+\frac{3\left(a+bx^2\right)^{16/3}}{32b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{7/3}\left(-81a^3+189a^2bx^2-315ab^2x^4+455b^3x^6\right)}{14560b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3)*(-81*a^3 + 189*a^2*b*x^2 - 315*a*b^2*x^4 + 455*b^3*x^6
))/(14560*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 50, normalized size = 0.62

$$-\frac{3\left(a+bx^2\right)^{7/3}\left(81a^3-189a^2bx^2+315ab^2x^4-455b^3x^6\right)}{14560b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3)*(81*a^3 - 189*a^2*b*x^2 + 315*a*b^2*x^4 - 455*b^3*x^6
))/(14560*b^4)
```

fricas [A] time = 0.79, size = 68, normalized size = 0.85

$$\frac{3\left(455\,b^5x^{10}+595\,ab^4x^8+14\,a^2b^3x^6-18\,a^3b^2x^4+27\,a^4bx^2-81\,a^5\right)\left(bx^2+a\right)^{\frac{1}{3}}}{14560\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/14560*(455*b^5*x^10 + 595*a*b^4*x^8 + 14*a^2*b^3*x^6 - 18*a^3*b^2*x^4 + 2
7*a^4*b*x^2 - 81*a^5)*(b*x^2 + a)^(1/3)/b^4
```

giac [A] time = 0.65, size = 57, normalized size = 0.71

$$\frac{3\left(455\left(bx^2+a\right)^{\frac{16}{3}}-1680\left(bx^2+a\right)^{\frac{13}{3}}a+2184\left(bx^2+a\right)^{\frac{10}{3}}a^2-1040\left(bx^2+a\right)^{\frac{7}{3}}a^3\right)}{14560\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="giac")
```

[Out] 3/14560*(455*(b*x^2 + a)^(16/3) - 1680*(b*x^2 + a)^(13/3)*a + 2184*(b*x^2 + a)^(10/3)*a^2 - 1040*(b*x^2 + a)^(7/3)*a^3)/b^4

maple [A] time = 0.00, size = 47, normalized size = 0.59

$$-\frac{3\left(b\,x^2+a\right)^{\frac{7}{3}}\left(-455b^3x^6+315a\,b^2x^4-189a^2b\,x^2+81a^3\right)}{14560b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(b*x^2+a)^(4/3),x)

[Out] -3/14560*(b*x^2+a)^(7/3)*(-455*b^3*x^6+315*a*b^2*x^4-189*a^2*b*x^2+81*a^3)/b^4

maxima [A] time = 1.38, size = 64, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{16}{3}}}{32\,b^4}-\frac{9\left(bx^2+a\right)^{\frac{13}{3}}a}{26\,b^4}+\frac{9\left(bx^2+a\right)^{\frac{10}{3}}a^2}{20\,b^4}-\frac{3\left(bx^2+a\right)^{\frac{7}{3}}a^3}{14\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="maxima")

[Out] 3/32*(b*x^2 + a)^(16/3)/b^4 - 9/26*(b*x^2 + a)^(13/3)*a/b^4 + 9/20*(b*x^2 + a)^(10/3)*a^2/b^4 - 3/14*(b*x^2 + a)^(7/3)*a^3/b^4

mupad [B] time = 5.19, size = 64, normalized size = 0.80

$$\left(b\,x^2+a\right)^{1/3}\left(\frac{51\,a\,x^8}{416}+\frac{3\,b\,x^{10}}{32}-\frac{243\,a^5}{14560\,b^4}+\frac{3\,a^2\,x^6}{1040\,b}-\frac{27\,a^3\,x^4}{7280\,b^2}+\frac{81\,a^4\,x^2}{14560\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(a + b*x^2)^(4/3),x)

[Out] (a + b*x^2)^(1/3)*((51*a*x^8)/416 + (3*b*x^10)/32 - (243*a^5)/(14560*b^4) + (3*a^2*x^6)/(1040*b) - (27*a^3*x^4)/(7280*b^2) + (81*a^4*x^2)/(14560*b^3))

sympy [A] time = 6.58, size = 136, normalized size = 1.70

$$\begin{cases} -\frac{243a^5\sqrt[3]{a+bx^2}}{14560b^4}+\frac{81a^4x^2\sqrt[3]{a+bx^2}}{14560b^3}-\frac{27a^3x^4\sqrt[3]{a+bx^2}}{7280b^2}+\frac{3a^2x^6\sqrt[3]{a+bx^2}}{1040b}+\frac{51ax^8\sqrt[3]{a+bx^2}}{416}+\frac{3bx^{10}\sqrt[3]{a+bx^2}}{32} & \text{for } b\neq 0 \\ \frac{a^{\frac{4}{3}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-243*a**5*(a + b*x**2)**(1/3)/(14560*b**4) + 81*a**4*x**2*(a + b
*x**2)**(1/3)/(14560*b**3) - 27*a**3*x**4*(a + b*x**2)**(1/3)/(7280*b**2) +
 3*a**2*x**6*(a + b*x**2)**(1/3)/(1040*b) + 51*a*x**8*(a + b*x**2)**(1/3)/4
16 + 3*b*x**10*(a + b*x**2)**(1/3)/32, Ne(b, 0)), (a**(4/3)*x**8/8, True))
```

3.599 $\int x^5 \left(a+bx^2\right)^{4/3}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+bx^2\right)^{7/3}}{14b^3}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^3}-\frac{3a\left(a+bx^2\right)^{10/3}}{10b^3}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^2\right)^{7/3}}{14b^3}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^3}-\frac{3a\left(a+bx^2\right)^{10/3}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(7/3))/(14*b^3) - (3*a*(a + b*x^2)^(10/3))/(10*b^3) + (3
*(a + b*x^2)^(13/3))/(26*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{4/3}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^{4/3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{4/3}}{b^2}-\frac{2a(a+bx)^{7/3}}{b^2}+\frac{(a+bx)^{10/3}}{b^2}\right)dx,x,x^2\right)\\ &= \frac{3a^2\left(a+bx^2\right)^{7/3}}{14b^3}-\frac{3a\left(a+bx^2\right)^{10/3}}{10b^3}+\frac{3\left(a+bx^2\right)^{13/3}}{26b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{7/3}\left(9a^2-21abx^2+35b^2x^4\right)}{910b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(4/3),x]
[Out] (3*(a + b*x^2)^(7/3)*(9*a^2 - 21*a*b*x^2 + 35*b^2*x^4))/(910*b^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{7/3}\left(9a^2-21abx^2+35b^2x^4\right)}{910b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^(4/3),x]
[Out] (3*(a + b*x^2)^(7/3)*(9*a^2 - 21*a*b*x^2 + 35*b^2*x^4))/(910*b^3)
```

fricas [A] time = 0.79, size = 57, normalized size = 0.97

$$\frac{3\left(35\,b^4x^8+49\,ab^3x^6+2\,a^2b^2x^4-3\,a^3bx^2+9\,a^4\right)\left(bx^2+a\right)^{\frac{1}{3}}}{910\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] 3/910*(35*b^4*x^8 + 49*a*b^3*x^6 + 2*a^2*b^2*x^4 - 3*a^3*b*x^2 + 9*a^4)*(b*
x^2 + a)^(1/3)/b^3
```

giac [A] time = 0.65, size = 43, normalized size = 0.73

$$\frac{3\left(35\left(bx^2+a\right)^{\frac{13}{3}}-91\left(bx^2+a\right)^{\frac{10}{3}}a+65\left(bx^2+a\right)^{\frac{7}{3}}a^2\right)}{910\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] 3/910*(35*(b*x^2 + a)^(13/3) - 91*(b*x^2 + a)^(10/3)*a + 65*(b*x^2 + a)^(7/
3)*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(35b^{2}x^{4}-21ab\,x^{2}+9a^{2}\right)}{910b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(4/3),x)
```

```
[Out] 3/910*(b*x^2+a)^(7/3)*(35*b^2*x^4-21*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.33, size = 47, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{13}{3}}}{26\,b^{3}}-\frac{3\left(bx^{2}+a\right)^{\frac{10}{3}}a}{10\,b^{3}}+\frac{3\left(bx^{2}+a\right)^{\frac{7}{3}}a^{2}}{14\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/26*(b*x^2 + a)^(13/3)/b^3 - 3/10*(b*x^2 + a)^(10/3)*a/b^3 + 3/14*(b*x^2 +
 a)^(7/3)*a^2/b^3
```

mupad [B] time = 5.10, size = 53, normalized size = 0.90

$$\left(b\,x^{2}+a\right)^{1/3}\left(\frac{21\,a\,x^{6}}{130}+\frac{3\,b\,x^{8}}{26}+\frac{27\,a^{4}}{910\,b^{3}}+\frac{3\,a^{2}\,x^{4}}{455\,b}-\frac{9\,a^{3}\,x^{2}}{910\,b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^(4/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((21*a*x^6)/130 + (3*b*x^8)/26 + (27*a^4)/(910*b^3) + (3*
a^2*x^4)/(455*b) - (9*a^3*x^2)/(910*b^2))
```

sympy [A] time = 3.98, size = 112, normalized size = 1.90

$$\begin{cases}\frac{27a^{4}\sqrt[3]{a+bx^{2}}}{910b^{3}}-\frac{9a^{3}x^{2}\sqrt[3]{a+bx^{2}}}{910b^{2}}+\frac{3a^{2}x^{4}\sqrt[3]{a+bx^{2}}}{455b}+\frac{21ax^{6}\sqrt[3]{a+bx^{2}}}{130}+\frac{3bx^{8}\sqrt[3]{a+bx^{2}}}{26} & \text{for } b\neq 0\\ \frac{a^{\frac{4}{3}}x^{6}}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((27*a**4*(a + b*x**2)**(1/3)/(910*b**3) - 9*a**3*x**2*(a + b*x**2
)**(1/3)/(910*b**2) + 3*a**2*x**4*(a + b*x**2)**(1/3)/(455*b) + 21*a*x**6*(
a + b*x**2)**(1/3)/130 + 3*b*x**8*(a + b*x**2)**(1/3)/26, Ne(b, 0)), (a**(4
/3)*x**6/6, True))
```

3.600 $\int x^3 \left(a+bx^2\right)^{4/3} dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{10/3}}{20b^2}-\frac{3a\left(a+bx^2\right)^{7/3}}{14b^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{10/3}}{20b^2}-\frac{3a\left(a+bx^2\right)^{7/3}}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(7/3))/(14*b^2) + (3*(a + b*x^2)^(10/3))/(20*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^{4/3} dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^{4/3} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^{4/3}}{b}+\frac{(a+bx)^{7/3}}{b}\right) dx, x, x^2\right)\\
&= -\frac{3a\left(a+bx^2\right)^{7/3}}{14b^2}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{7/3}\left(7bx^2-3a\right)}{140b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(4/3),x]
[Out] (3*(a + b*x^2)^(7/3)*(-3*a + 7*b*x^2))/(140*b^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 28, normalized size = 0.74

$$-\frac{3\left(3a-7bx^2\right)\left(a+bx^2\right)^{7/3}}{140b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^(4/3),x]
[Out] (-3*(3*a - 7*b*x^2)*(a + b*x^2)^(7/3))/(140*b^2)
```

fricas [A] time = 0.98, size = 45, normalized size = 1.18

$$\frac{3\left(7\,b^3x^6+11\,ab^2x^4+a^2bx^2-3\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}}{140\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] 3/140*(7*b^3*x^6 + 11*a*b^2*x^4 + a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(1/3)/b^2
```

giac [A] time = 0.69, size = 29, normalized size = 0.76

$$\frac{3\left(7\left(bx^2+a\right)^{\frac{10}{3}}-10\left(bx^2+a\right)^{\frac{7}{3}}a\right)}{140\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] 3/140*(7*(b*x^2 + a)^(10/3) - 10*(b*x^2 + a)^(7/3)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(-7b\,x^{2}+3a\right)}{140b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(4/3),x)
```

```
[Out] -3/140*(b*x^2+a)^(7/3)*(-7*b*x^2+3*a)/b^2
```

maxima [A] time = 1.32, size = 30, normalized size = 0.79

$$\frac{3\left(bx^{2}+a\right)^{\frac{10}{3}}}{20\,b^{2}}-\frac{3\left(bx^{2}+a\right)^{\frac{7}{3}}a}{14\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^2 + a)^(10/3)/b^2 - 3/14*(b*x^2 + a)^(7/3)*a/b^2
```

mupad [B] time = 5.05, size = 42, normalized size = 1.11

$$\left(b\,x^{2}+a\right)^{1/3}\left(\frac{33\,a\,x^{4}}{140}+\frac{3\,b\,x^{6}}{20}-\frac{9\,a^{3}}{140\,b^{2}}+\frac{3\,a^{2}\,x^{2}}{140\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(4/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((33*a*x^4)/140 + (3*b*x^6)/20 - (9*a^3)/(140*b^2) + (3*a
^2*x^2)/(140*b))
```

sympy [A] time = 2.54, size = 88, normalized size = 2.32

$$\begin{cases} -\frac{9a^{3}\sqrt[3]{a+bx^{2}}}{140b^{2}}+\frac{3a^{2}x^{2}\sqrt[3]{a+bx^{2}}}{140b}+\frac{33ax^{4}\sqrt[3]{a+bx^{2}}}{140}+\frac{3bx^{6}\sqrt[3]{a+bx^{2}}}{20} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{4}{3}}x^{4}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-9*a**3*(a + b*x**2)**(1/3)/(140*b**2) + 3*a**2*x**2*(a + b*x**2
)**(1/3)/(140*b) + 33*a*x**4*(a + b*x**2)**(1/3)/140 + 3*b*x**6*(a + b*x**2
)**(1/3)/20, Ne(b, 0)), (a**(4/3)*x**4/4, True))
```

3.601 $\int x\left(a+bx^2\right)^{4/3}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^{4/3}\,dx=\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^(4/3),x]
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

fricas [B] time = 1.14, size = 32, normalized size = 1.78

$$\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] 3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)/b
```

giac [A] time = 0.63, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{7}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] 3/14*(b*x^2 + a)^(7/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\left(b\,x^2+a\right)^{\frac{7}{3}}}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(4/3),x)
[Out] 3/14*(b*x^2+a)^(7/3)/b
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{7}{3}}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b
```

mupad [B] time = 4.99, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x^2+a\right)^{7/3}}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(4/3),x)
```

```
[Out] (3*(a + b*x^2)^(7/3))/(14*b)
```

sympy [A] time = 1.37, size = 65, normalized size = 3.61

$$\begin{cases} \frac{3a^2\sqrt[3]{a+bx^2}}{14b}+\frac{3ax^2\sqrt[3]{a+bx^2}}{7}+\frac{3bx^4\sqrt[3]{a+bx^2}}{14} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{4}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((3*a**2*(a + b*x**2)**(1/3)/(14*b) + 3*a*x**2*(a + b*x**2)**(1/3)
/7 + 3*b*x**4*(a + b*x**2)**(1/3)/14, Ne(b, 0)), (a**(4/3)*x**2/2, True))
```

3.602 $\int \frac{\left(a+bx^2\right)^{4/3}}{x}\,dx$

Optimal. Leaf size=117

$$\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}$$

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 57, 617, 204, 31}

$$\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{1}{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x,x]
```

```
[Out] (3*a*(a + b*x^2)^(1/3))/2 + (3*(a + b*x^2)^(4/3))/8 - (Sqrt[3]*a^(4/3)*ArcT
an[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/2 - (a^(4/3)*Log[x])
/2 + (3*a^(4/3)*Log[a^(1/3) - (a + b*x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{4/3}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{8}\left(a+bx^2\right)^{4/3}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx,x,x^2\right)\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^2\right)\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}-\frac{1}{2}a^{4/3}\log(x)-\frac{1}{4}\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^2}\right)\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+\frac{1}{2}\left(3a^{4/3}\right)\text{Subs}\\
&= \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}\left(a+bx^2\right)^{4/3}-\frac{1}{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{\right.}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 144, normalized size = 1.23

$$\frac{1}{8}\left(4a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-2a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)-4\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+3bx^2\sqrt[3]{a+bx^2}+15a\sqrt[3]{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x,x]
```

```
[Out] (15*a*(a + b*x^2)^(1/3) + 3*b*x^2*(a + b*x^2)^(1/3) - 4*Sqrt[3]*a^(4/3)*Arc
Tan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) -
(a + b*x^2)^(1/3)] - 2*a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
 + b*x^2)^(2/3)])/8
```

IntegrateAlgebraic [A] time = 0.10, size = 142, normalized size = 1.21

$$\frac{1}{2}a^{4/3}\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)-\frac{1}{4}a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)-\frac{1}{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{8}\sqrt[3]{a+bx^2}\left(5a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(4/3)/x,x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(5*a + b*x^2))/8 - (Sqrt[3]*a^(4/3)*ArcTan[1/Sqrt[3] +
 (2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/2 + (a^(4/3)*Log[-a^(1/3) + (a +
 b*x^2)^(1/3)])/2 - (a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a +
 b*x^2)^(2/3)])/4
```

fricas [A] time = 1.40, size = 111, normalized size = 0.95

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{4}\,a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{8}\left(bx^2+5\,a\right)\left(bx^2+a\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqrt
(3)*a)/a) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 1/2*a^(4/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/8*(b*x^2 + 5*a
)*(b*x^2 + a)^(1/3)
```

giac [A] time = 1.46, size = 110, normalized size = 0.94

$$-\frac{1}{2}\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}\,a^{\frac{4}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{4}{3}}\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{8}\left(bx^2+a\right)^{\frac{4}{3}}+\frac{3}{2}\left(bx^2+a\right)^{\frac{1}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(4/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/8*(b*x^2 + a)
^(4/3) + 3/2*(b*x^2 + a)^(1/3)*a
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x,x)
```

maxima [A] time = 3.06, size = 109, normalized size = 0.93

$$-\frac{1}{2}\,\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{4}\,a^{\frac{4}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{2}\,a^{\frac{4}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{8}\left(bx^{2}+a\right)^{\frac{4}{3}}+\frac{3}{2}\left(bx^{2}+a\right)^{\frac{1}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/2*a^(4/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/8*(b*x^2 + a)^(4/3
) + 3/2*(b*x^2 + a)^(1/3)*a
```

mupad [B] time = 5.00, size = 133, normalized size = 1.14

$$\frac{3\,a\,{\left(b\,x^2+a\right)}^{1/3}}{2}+\frac{3\,{\left(b\,x^2+a\right)}^{4/3}}{8}+\frac{a^{4/3}\,\ln\left(\frac{9\,a^2\,{\left(b\,x^2+a\right)}^{1/3}}{2}-\frac{9\,a^{7/3}}{2}\right)}{2}-\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+\frac{9\,a^2\,{\left(b\,x^2+a\right)}^{1/3}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+a^{4/3}\,\ln\left(9\,a^{7/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)-\frac{9\,a^2\,{\left(b\,x^2+a\right)}^{1/3}}{2}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x,x)
```

```
[Out] (3*a*(a + b*x^2)^(1/3))/2 + (3*(a + b*x^2)^(4/3))/8 + (a^(4/3)*log((9*a^2*(
a + b*x^2)^(1/3))/2 - (9*a^(7/3))/2))/2 - (a^(4/3)*log((9*a^(7/3)*((3^(1/2)
*1i)/2 + 1/2))/2 + (9*a^2*(a + b*x^2)^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/2 +
```

```
 a^(4/3)*log(9*a^(7/3)*((3^(1/2)*1i)/4 - 1/4) - (9*a^2*(a + b*x^2)^(1/3))/2
)*((3^(1/2)*1i)/4 - 1/4)
```

sympy [C] time = 1.30, size = 49, normalized size = 0.42

$$-\frac{b^{\frac{4}{3}}x^{\frac{8}{3}}\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{4}{3}\\-\frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x,x)
```

```
[Out] -b**(4/3)*x**(8/3)*gamma(-4/3)*hyper((-4/3, -4/3), (-1/3,), a*exp_polar(I*p
i)/(b*x**2))/(2*gamma(-1/3))
```

3.603 $\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx$

Optimal. Leaf size=116

$$-\frac{(a+bx^2)^{4/3}}{2x^2}+2b\sqrt[3]{a+bx^2}+\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{2\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)$$

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {266, 47, 50, 57, 617, 204, 31}

$$-\frac{(a+bx^2)^{4/3}}{2x^2}+2b\sqrt[3]{a+bx^2}+\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)-\frac{2\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^3,x]
```

```
[Out] 2*b*(a + b*x^2)^(1/3) - (a + b*x^2)^(4/3)/(2*x^2) - (2*a^(1/3)*b*ArcTan[(a^
(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log
[x])/3 + a^(1/3)*b*Log[a^(1/3) - (a + b*x^2)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{4/3}}{2x^2} + \frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^2\right)$$
$$= 2b\sqrt[3]{a+bx^2} - \frac{\left(a+bx^2\right)^{4/3}}{2x^2} + \frac{1}{3}(2ab)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)$$
$$= 2b\sqrt[3]{a+bx^2} - \frac{\left(a+bx^2\right)^{4/3}}{2x^2} - \frac{2}{3}\sqrt[3]{a}\,b\log(x) - \left(\sqrt[3]{a}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right) - \Big($$
$$= 2b\sqrt[3]{a+bx^2} - \frac{\left(a+bx^2\right)^{4/3}}{2x^2} - \frac{2}{3}\sqrt[3]{a}\,b\log(x) + \sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) + \left(2\sqrt[3]{a}\,b\right)\text{Subst}\Big($$
$$= 2b\sqrt[3]{a+bx^2} - \frac{\left(a+bx^2\right)^{4/3}}{2x^2} - \frac{2\sqrt[3]{a}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\sqrt[3]{a}\,b\log(x) + \sqrt[3]{a}\,b\log\Big(\sqrt[3]{a} - \sqrt[3]{a}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.32

$$\frac{3b\left(a+bx^2\right)^{7/3}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{bx^2}{a}+1\right)}{14a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^3,x]
```

```
[Out] (3*b*(a + b*x^2)^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, 1 + (b*x^2)/a])/(14*
a^2)
```

IntegrateAlgebraic [A] time = 0.13, size = 147, normalized size = 1.27

$$-\frac{1}{3}\sqrt[3]{a}\,b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)+\frac{\sqrt[3]{a+bx^2}\left(3bx^2-a\right)}{2x^2}+\frac{2}{3}\sqrt[3]{a}\,b\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)-\frac{2\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(4/3)/x^3,x]
```

```
[Out] ((a + b*x^2)^(1/3)*(-a + 3*b*x^2))/(2*x^2) - (2*a^(1/3)*b*ArcTan[1/Sqrt[3]
+ (2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] + (2*a^(1/3)*b*Log[-a^(
1/3) + (a + b*x^2)^(1/3)])/3 - (a^(1/3)*b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)
^(1/3) + (a + b*x^2)^(2/3)])/3
```

fricas [A] time = 1.57, size = 129, normalized size = 1.11

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}bx^2\arctan\left(\frac{2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+2\,a^{\frac{1}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\left(3\,bx^2-a\right)\left(bx^2+a\right)^{\frac{1}{3}}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*a^(1/3)*b*x^2*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/
3) + sqrt(3)*a)/a) + 2*a^(1/3)*b*x^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1
/3)*a^(1/3) + a^(2/3)) - 4*a^(1/3)*b*x^2*log((b*x^2 + a)^(1/3) - a^(1/3)) -
 3*(3*b*x^2 - a)*(b*x^2 + a)^(1/3))/x^2
```

giac [A] time = 1.42, size = 131, normalized size = 1.13

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+2\,a^{\frac{1}{3}}b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}b^2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)-9\left(bx^2+a\right)^{\frac{1}{3}}b^2+\frac{3\left(bx^2+a\right)^{\frac{1}{3}}ab}{x^2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/6*(4*sqrt(3)*a^(1/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/
3))/a^(1/3)) + 2*a^(1/3)*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1
/3) + a^(2/3)) - 4*a^(1/3)*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) - 9*(b
*x^2 + a)^(1/3)*b^2 + 3*(b*x^2 + a)^(1/3)*a*b/x^2)/b
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{4}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^3,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^3,x)
```

maxima [A] time = 2.90, size = 116, normalized size = 1.00

$$-\frac{2}{3}\sqrt{3}a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{3}a^{\frac{1}{3}}b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{2}{3}a^{\frac{1}{3}}b\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\left(bx^2+a\right)^{\frac{1}{3}}b-\frac{\left(bx^2+a\right)^{\frac{1}{3}}a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*a^(1/3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a
^(1/3)) - 1/3*a^(1/3)*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 2/3*a^(1/3)*b*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a
)^(1/3)*b - 1/2*(b*x^2 + a)^(1/3)*a/x^2
```

mupad [B] time = 5.42, size = 141, normalized size = 1.22

$$\frac{3b\left(bx^2+a\right)^{1/3}}{2}-\frac{a\left(bx^2+a\right)^{1/3}}{2x^2}+\frac{2a^{1/3}b\ln\left(6a^{4/3}b-6ab\left(bx^2+a\right)^{1/3}\right)}{3}+\frac{a^{1/3}b\ln\left(6ab\left(bx^2+a\right)^{1/3}-3a^{4/3}b\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}-\frac{a^{1/3}b\ln\left(3a^{4/3}b\left(1+\sqrt{3}\,1\mathrm{i}\right)+6ab\left(bx^2+a\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x^3,x)
```

```
[Out] (3*b*(a + b*x^2)^(1/3))/2 - (a*(a + b*x^2)^(1/3))/(2*x^2) + (2*a^(1/3)*b*lo
g(6*a^(4/3)*b - 6*a*b*(a + b*x^2)^(1/3)))/3 + (a^(1/3)*b*log(6*a*b*(a + b*x
^2)^(1/3) - 3*a^(4/3)*b*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/3 - (a^(1/3)*b*
log(3*a^(4/3)*b*(3^(1/2)*1i + 1) + 6*a*b*(a + b*x^2)^(1/3))*(3^(1/2)*1i + 1
))/3
```

sympy [C] time = 1.41, size = 46, normalized size = 0.40

$$-\frac{b^{\frac{4}{3}}x^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**3,x)
```

```
[Out] -b**(4/3)*x**(2/3)*gamma(-1/3)*hyper((-4/3, -1/3), (2/3,), a*exp_polar(I*pi
)/(b*x**2))/(2*gamma(2/3))
```

3.604 $\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx$

Optimal. Leaf size=132

$$\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}-\frac{b\sqrt[3]{a+bx^2}}{3x^2}-\frac{\left(a+bx^2\right)^{4/3}}{4x^4}$$

Rubi [A] time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 47, 57, 617, 204, 31}

$$\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}-\frac{b\sqrt[3]{a+bx^2}}{3x^2}-\frac{\left(a+bx^2\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/x^5,x]
```

```
[Out] -(b*(a + b*x^2)^(1/3))/(3*x^2) - (a + b*x^2)^(4/3)/(4*x^4) - (b^2*ArcTan[(a
^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^
2*Log[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{4/3}}{4x^4} + \frac{1}{3}b\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{\left(a+bx^2\right)^{4/3}}{4x^4} + \frac{1}{9}b^2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)$$
$$= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{\left(a+bx^2\right)^{4/3}}{4x^4} - \frac{b^2\log(x)}{9a^{2/3}} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{6a^{2/3}} - \frac{b^2\,\text{Subst}\left(\right.}{}$$
$$= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{\left(a+bx^2\right)^{4/3}}{4x^4} - \frac{b^2\log(x)}{9a^{2/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{3a^2}$$
$$= -\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{\left(a+bx^2\right)^{4/3}}{4x^4} - \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}} - \frac{b^2\log(x)}{9a^{2/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}{6a^{2/3}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.30

$$-\frac{3b^2\left(a+bx^2\right)^{7/3}\,{}_2F_1\left(\frac{7}{3},3;\frac{10}{3};\frac{bx^2}{a}+1\right)}{14a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/x^5,x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(7/3)*Hypergeometric2F1[7/3, 3, 10/3, 1 + (b*x^2)/a])/(
14*a^3)
```

IntegrateAlgebraic [A] time = 0.18, size = 155, normalized size = 1.17

$$\frac{b^2\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{9a^{2/3}} - \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{18a^{2/3}} - \frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}} + \frac{\left(-3a-7bx^2\right)\sqrt[3]{a+bx^2}}{12x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(4/3)/x^5,x]
```

```
[Out] ((-3*a - 7*b*x^2)*(a + b*x^2)^(1/3))/(12*x^4) - (b^2*ArcTan[1/Sqrt[3] + (2*
(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) + (b^2*Log[-a^(1
/3) + (a + b*x^2)^(1/3)])/(9*a^(2/3)) - (b^2*Log[a^(2/3) + a^(1/3)*(a + b*x
^2)^(1/3) + (a + b*x^2)^(2/3)])/(18*a^(2/3))
```

fricas [A] time = 0.89, size = 174, normalized size = 1.32

$$-\frac{4\sqrt{3}\left(a^2\right)^{\frac{1}{6}} a b^2 x^4 \arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}} a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+2\left(a^2\right)^{\frac{2}{3}} b^2 x^4 \log\left(\left(bx^2+a\right)^{\frac{2}{3}} a+\left(a^2\right)^{\frac{1}{3}} a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-4\left(a^2\right)^{\frac{2}{3}} b^2 x^4 \log\left(\left(bx^2+a\right)^{\frac{1}{3}} a-\left(a^2\right)^{\frac{2}{3}}\right)+3\left(7a^2bx^2+3a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}}{36a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="fricas")
```

```
[Out] -1/36*(4*sqrt(3)*(a^2)^(1/6)*a*b^2*x^4*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2
)^(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + 2*(a^2)^(2/3)*b
^2*x^4*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2
/3)) - 4*(a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*
a^2*b*x^2 + 3*a^3)*(b*x^2 + a)^(1/3))/(a^2*x^4)
```

giac [A] time = 1.16, size = 139, normalized size = 1.05

$$-\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{2b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{4b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{3\left(7\left(bx^2+a\right)^{\frac{4}{3}}b^3-4\left(bx^2+a\right)^{\frac{1}{3}}ab^3\right)}{b^2x^4}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="giac")
```

```
[Out] -1/36*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(2/3) + 2*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(2/3) - 4*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) + 3*(
7*(b*x^2 + a)^(4/3)*b^3 - 4*(b*x^2 + a)^(1/3)*a*b^3)/(b^2*x^4))/b
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^2+a\right)^{\frac{4}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/x^5,x)
```

```
[Out] int((b*x^2+a)^(4/3)/x^5,x)
```

maxima [A] time = 3.08, size = 152, normalized size = 1.15

$$-\frac{\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\left(2 \left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3 a^{\frac{1}{3}}}\right)}{9 a^{\frac{2}{3}}}-\frac{b^{2} \log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18 a^{\frac{2}{3}}}+\frac{b^{2} \log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9 a^{\frac{2}{3}}}-\frac{7 \left(bx^{2}+a\right)^{\frac{4}{3}} b^{2}-4 \left(bx^{2}+a\right)^{\frac{1}{3}} a b^{2}}{12 \left(\left(bx^{2}+a\right)^{2}-2 \left(bx^{2}+a\right) a+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(2/3) - 1/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(2/3) + 1/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3) - 1/12*(7
*(b*x^2 + a)^(4/3)*b^2 - 4*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2 - 2*(b*x
^2 + a)*a + a^2)
```

mupad [B] time = 5.49, size = 191, normalized size = 1.45

$$\frac{b^{2} \ln\left(b^{2} \left(b\, x^{2}+a\right)^{1/3}-a^{1/3} b^{2}\right)}{9 a^{2/3}}-\frac{\ln\left(\frac{a^{1/3} \left(b^{2}+\sqrt{3}\, b^{2}\, 1\mathrm{i}\right)}{2}+b^{2} \left(b\, x^{2}+a\right)^{1/3}\right) \left(b^{2}+\sqrt{3}\, b^{2}\, 1\mathrm{i}\right)}{18 a^{2/3}}-\frac{\frac{7 b^{2} \left(b\, x^{2}+a\right)^{4/3}}{6}-\frac{2 a\, b^{2} \left(b\, x^{2}+a\right)^{1/3}}{3}}{2 \left(b\, x^{2}+a\right)^{2}-4 a \left(b\, x^{2}+a\right)+2 a^{2}}+\frac{b^{2} \ln\left(b^{2} \left(b\, x^{2}+a\right)^{1/3}-a^{1/3} b^{2} \left(-\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)\right) \left(-\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)}{9 a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/x^5,x)
```

```
[Out] (b^2*log(b^2*(a + b*x^2)^(1/3) - a^(1/3)*b^2))/(9*a^(2/3)) - (log((a^(1/3)*
(3^(1/2)*b^2*1i + b^2))/2 + b^2*(a + b*x^2)^(1/3))*(3^(1/2)*b^2*1i + b^2))/
(18*a^(2/3)) - ((7*b^2*(a + b*x^2)^(4/3))/6 - (2*a*b^2*(a + b*x^2)^(1/3))/3
)/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2) + (b^2*log(b^2*(a + b*x^2)^(1
/3) - a^(1/3)*b^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(2/3
))
```

sympy [C] time = 1.54, size = 42, normalized size = 0.32

$$-\frac{b^{\frac{4}{3}} \Gamma\left(\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{a e^{i\pi}}{bx^{2}}\right)}{2 x^{\frac{4}{3}} \Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/x**5,x)
```

```
[Out] -b**(4/3)*gamma(2/3)*hyper((-4/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))
/(2*x**(4/3)*gamma(5/3))
```

3.605 $\int x\left(-1+x^2\right)^{7/3}\,dx$

Optimal. Leaf size=13

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^2)^(7/3),x]
[Out] (3*(-1 + x^2)^(10/3))/20
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(-1+x^2\right)^{7/3}\,dx=\frac{3}{20}\left(-1+x^2\right)^{10/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^2)^(7/3),x]
[Out] (3*(-1 + x^2)^(10/3))/20
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{3}{20}\left(x^2-1\right)^{10/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(-1 + x^2)^(7/3),x]
```

```
[Out] (3*(-1 + x^2)^(10/3))/20
```

fricas [B] time = 1.37, size = 24, normalized size = 1.85

$$\frac{3}{20}\left(x^6-3\,x^4+3\,x^2-1\right)\left(x^2-1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="fricas")
```

```
[Out] 3/20*(x^6 - 3*x^4 + 3*x^2 - 1)*(x^2 - 1)^(1/3)
```

giac [A] time = 0.56, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^2-1\right)^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="giac")
```

```
[Out] 3/20*(x^2 - 1)^(10/3)
```

maple [A] time = 0.00, size = 16, normalized size = 1.23

$$\frac{3\,(x+1)\,(x-1)\left(x^2-1\right)^{\frac{7}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^(7/3),x)
```

```
[Out] 3/20*(x+1)*(x-1)*(x^2-1)^(7/3)
```

maxima [A] time = 1.32, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^2-1\right)^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(7/3),x, algorithm="maxima")
```

```
[Out] 3/20*(x^2 - 1)^(10/3)
```

mupad [B] time = 5.13, size = 25, normalized size = 1.92

$$\left(x^2-1\right)^{1/3}\,\left(\frac{3\,x^6}{20}-\frac{9\,x^4}{20}+\frac{9\,x^2}{20}-\frac{3}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 - 1)^(7/3),x)
```

```
[Out] (x^2 - 1)^(1/3)*((9*x^2)/20 - (9*x^4)/20 + (3*x^6)/20 - 3/20)
```

sympy [B] time = 2.91, size = 56, normalized size = 4.31

$$\frac{3x^6\sqrt[3]{x^2-1}}{20}-\frac{9x^4\sqrt[3]{x^2-1}}{20}+\frac{9x^2\sqrt[3]{x^2-1}}{20}-\frac{3\sqrt[3]{x^2-1}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**(7/3),x)
```

```
[Out] 3*x**6*(x**2 - 1)**(1/3)/20 - 9*x**4*(x**2 - 1)**(1/3)/20 + 9*x**2*(x**2 -
1)**(1/3)/20 - 3*(x**2 - 1)**(1/3)/20
```

3.606 $\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3\left(a+bx^2\right)^{2/3}}{4b^4}+\frac{9a^2\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^4}-\frac{9a\left(a+bx^2\right)^{8/3}}{16b^4}$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{5/3}}{10b^4}-\frac{3a^3\left(a+bx^2\right)^{2/3}}{4b^4}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^4}-\frac{9a\left(a+bx^2\right)^{8/3}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(2/3))/(4*b^4) + (9*a^2*(a + b*x^2)^(5/3))/(10*b^4) - (
9*a*(a + b*x^2)^(8/3))/(16*b^4) + (3*(a + b*x^2)^(11/3))/(22*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)dx,x,x^2\right)\\ &= -\frac{3a^3\left(a+bx^2\right)^{2/3}}{4b^4}+\frac{9a^2\left(a+bx^2\right)^{5/3}}{10b^4}-\frac{9a\left(a+bx^2\right)^{8/3}}{16b^4}+\frac{3\left(a+bx^2\right)^{11/3}}{22b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{3\left(a+bx^2\right)^{2/3}\left(-81a^3+54a^2bx^2-45ab^2x^4+40b^3x^6\right)}{880b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(2/3)*(-81*a^3 + 54*a^2*b*x^2 - 45*a*b^2*x^4 + 40*b^3*x^6))/
(880*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 50, normalized size = 0.62

$$-\frac{3\left(a+bx^2\right)^{2/3}\left(81a^3-54a^2bx^2+45ab^2x^4-40b^3x^6\right)}{880b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2)^(1/3),x]
[Out] (-3*(a + b*x^2)^(2/3)*(81*a^3 - 54*a^2*b*x^2 + 45*a*b^2*x^4 - 40*b^3*x^6))/
(880*b^4)
```

fricas [A] time = 0.87, size = 46, normalized size = 0.58

$$\frac{3\left(40\,b^3x^6-45\,ab^2x^4+54\,a^2bx^2-81\,a^3\right)\left(bx^2+a\right)^{\frac{2}{3}}}{880\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/880*(40*b^3*x^6 - 45*a*b^2*x^4 + 54*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^(2/3)
/b^4
```

giac [A] time = 0.58, size = 61, normalized size = 0.76

$$-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}a^3}{4\,b^4}+\frac{3\left(40\left(bx^2+a\right)^{\frac{11}{3}}-165\left(bx^2+a\right)^{\frac{8}{3}}a+264\left(bx^2+a\right)^{\frac{5}{3}}a^2\right)}{880\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] -3/4*(b*x^2 + a)^(2/3)*a^3/b^4 + 3/880*(40*(b*x^2 + a)^(11/3) - 165*(b*x^2
+ a)^(8/3)*a + 264*(b*x^2 + a)^(5/3)*a^2)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{3 \left(b\,x^{2}+a\right)^{\frac{2}{3}} \left(-40 b^{3} x^{6}+45 a\,b^{2} x^{4}-54 a^{2} b\,x^{2}+81 a^{3}\right)}{880 b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(1/3),x)
```

```
[Out] -3/880*(b*x^2+a)^(2/3)*(-40*b^3*x^6+45*a*b^2*x^4-54*a^2*b*x^2+81*a^3)/b^4
```

maxima [A] time = 1.36, size = 64, normalized size = 0.80

$$\frac{3 \left(bx^{2}+a\right)^{\frac{11}{3}}}{22\,b^{4}}-\frac{9 \left(bx^{2}+a\right)^{\frac{8}{3}} a}{16\,b^{4}}+\frac{9 \left(bx^{2}+a\right)^{\frac{5}{3}} a^{2}}{10\,b^{4}}-\frac{3 \left(bx^{2}+a\right)^{\frac{2}{3}} a^{3}}{4\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/22*(b*x^2 + a)^(11/3)/b^4 - 9/16*(b*x^2 + a)^(8/3)*a/b^4 + 9/10*(b*x^2 +
a)^(5/3)*a^2/b^4 - 3/4*(b*x^2 + a)^(2/3)*a^3/b^4
```

mupad [B] time = 5.32, size = 48, normalized size = 0.60

$$-{\left(b\,x^2+a\right)}^{2/3}\,\left(\frac{243\,a^3}{880\,b^4}-\frac{3\,x^6}{22\,b}+\frac{27\,a\,x^4}{176\,b^2}-\frac{81\,a^2\,x^2}{440\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(1/3),x)
```

```
[Out] -(a + b*x^2)^(2/3)*((243*a^3)/(880*b^4) - (3*x^6)/(22*b) + (27*a*x^4)/(176*
b^2) - (81*a^2*x^2)/(440*b^3))
```

sympy [B] time = 2.77, size = 1690, normalized size = 21.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(1/3),x)
```

```
[Out] -243*a**(71/3)*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2
 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 +
```

5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 243*a**(71/3)/(880*a**20*b **4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) - 1296*a**(68/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b* *5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8* x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1458*a**(68/3)*b*x* *2/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a **17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14* b**10*x**12) - 2808*a**(65/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(880*a**20*b* *4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 3 645*a**(65/3)*b**2*x**4/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**1 8*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b* *9*x**10 + 880*a**14*b**10*x**12) - 3120*a**(62/3)*b**3*x**6*(1 + b*x**2/a) **(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17 600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a **14*b**10*x**12) + 4860*a**(62/3)*b**3*x**6/(880*a**20*b**4 + 5280*a**19*b **5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8 *x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) - 1710*a**(59/3)*b** 4*x**8*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200 *a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a** 15*b**9*x**10 + 880*a**14*b**10*x**12) + 3645*a**(59/3)*b**4*x**8/(880*a**2 0*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x* *6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 72*a**(56/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a** 19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16* b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1458*a**(56/3) *b**5*x**10/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 8 80*a**14*b**10*x**12) + 1104*a**(53/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(88 0*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b **7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10* x**12) + 243*a**(53/3)*b**6*x**12/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 528 0*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1152*a**(50/3)*b**7*x**14*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b** 6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x* *10 + 880*a**14*b**10*x**12) + 585*a**(47/3)*b**8*x**16*(1 + b*x**2/a)**(2/ 3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a **17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14* b**10*x**12) + 120*a**(44/3)*b**9*x**18*(1 + b*x**2/a)**(2/3)/(880*a**20*b* *4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12)

3.607 $\int \frac{x^5}{\sqrt[3]{a+bx^2}} dx$

Optimal. Leaf size=59

$$\frac{3a^2 \left(a+bx^2\right)^{2/3}}{4b^3}+\frac{3 \left(a+bx^2\right)^{8/3}}{16b^3}-\frac{3a \left(a+bx^2\right)^{5/3}}{5b^3}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2 \left(a+bx^2\right)^{2/3}}{4b^3}+\frac{3 \left(a+bx^2\right)^{8/3}}{16b^3}-\frac{3a \left(a+bx^2\right)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(2/3))/(4*b^3) - (3*a*(a + b*x^2)^(5/3))/(5*b^3) + (3*(a
 + b*x^2)^(8/3))/(16*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}} - \frac{2a(a+bx)^{2/3}}{b^2} + \frac{(a+bx)^{5/3}}{b^2}\right) dx, x, x^2\right)$$
$$= \frac{3a^2\left(a+bx^2\right)^{2/3}}{4b^3} - \frac{3a\left(a+bx^2\right)^{5/3}}{5b^3} + \frac{3\left(a+bx^2\right)^{8/3}}{16b^3}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{2/3}\left(9a^2-6abx^2+5b^2x^4\right)}{80b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(2/3)*(9*a^2 - 6*a*b*x^2 + 5*b^2*x^4))/(80*b^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^{2/3}\left(9a^2-6abx^2+5b^2x^4\right)}{80b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(2/3)*(9*a^2 - 6*a*b*x^2 + 5*b^2*x^4))/(80*b^3)
```

fricas [A] time = 1.58, size = 35, normalized size = 0.59

$$\frac{3\left(5\,b^2x^4-6\,abx^2+9\,a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{80\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/80*(5*b^2*x^4 - 6*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(2/3)/b^3
```

giac [A] time = 0.57, size = 47, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{2}{3}}a^{2}}{4\,b^{3}}+\frac{3\left(5\left(bx^{2}+a\right)^{\frac{8}{3}}-16\left(bx^{2}+a\right)^{\frac{5}{3}}a\right)}{80\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(b*x^2 + a)^(2/3)*a^2/b^3 + 3/80*(5*(b*x^2 + a)^(8/3) - 16*(b*x^2 + a)^
(5/3)*a)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{3\left(b\,x^{2}+a\right)^{\frac{2}{3}}\left(5b^{2}x^{4}-6ab\,x^{2}+9a^{2}\right)}{80b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/3),x)
```

```
[Out] 3/80*(b*x^2+a)^(2/3)*(5*b^2*x^4-6*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.30, size = 47, normalized size = 0.80

$$\frac{3\left(bx^{2}+a\right)^{\frac{8}{3}}}{16\,b^{3}}-\frac{3\left(bx^{2}+a\right)^{\frac{5}{3}}a}{5\,b^{3}}+\frac{3\left(bx^{2}+a\right)^{\frac{2}{3}}a^{2}}{4\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x^2 + a)^(8/3)/b^3 - 3/5*(b*x^2 + a)^(5/3)*a/b^3 + 3/4*(b*x^2 + a)^
(2/3)*a^2/b^3
```

mupad [B] time = 5.21, size = 36, normalized size = 0.61

$$\left(b\,x^{2}+a\right)^{2/3}\left(\frac{27\,a^{2}}{80\,b^{3}}+\frac{3\,x^{4}}{16\,b}-\frac{9\,a\,x^{2}}{40\,b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(1/3),x)
```

```
[Out] (a + b*x^2)^(2/3)*((27*a^2)/(80*b^3) + (3*x^4)/(16*b) - (9*a*x^2)/(40*b^2))
```

sympy [B] time = 1.78, size = 631, normalized size = 10.69

$$\frac{27a^{\frac{32}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}-\frac{27a^{\frac{32}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{63a^{\frac{29}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}-\frac{81a^{\frac{29}{3}}bx^2}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{42a^{\frac{26}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}-\frac{81a^{\frac{26}{3}}b^2x^4}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{18a^{\frac{23}{3}}b^3x^6\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}-\frac{27a^{\frac{23}{3}}b^3x^6}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{27a^{\frac{20}{3}}b^4x^8\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}+\frac{15a^{\frac{17}{3}}b^5x^{10}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(32/3)*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240
*a**6*b**5*x**4 + 80*a**5*b**6*x**6) - 27*a**(32/3)/(80*a**8*b**3 + 240*a**
7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 63*a**(29/3)*b*x**2
*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x
**4 + 80*a**5*b**6*x**6) - 81*a**(29/3)*b*x**2/(80*a**8*b**3 + 240*a**7*b**
4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 42*a**(26/3)*b**2*x**4*(
1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**
4 + 80*a**5*b**6*x**6) - 81*a**(26/3)*b**2*x**4/(80*a**8*b**3 + 240*a**7*b*
*4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 18*a**(23/3)*b**3*x**6*
(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x*
*4 + 80*a**5*b**6*x**6) - 27*a**(23/3)*b**3*x**6/(80*a**8*b**3 + 240*a**7*b
**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 27*a**(20/3)*b**4*x**8
*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x
**4 + 80*a**5*b**6*x**6) + 15*a**(17/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(8
0*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6)
```

3.608 $\int \frac{x^3}{\sqrt[3]{a+bx^2}} \, dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b^2}-\frac{3a\left(a+bx^2\right)^{2/3}}{4b^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{5/3}}{10b^2}-\frac{3a\left(a+bx^2\right)^{2/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(2/3))/(4*b^2) + (3*(a + b*x^2)^(5/3))/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt[3]{a+bx^2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b}\right) dx, x, x^2\right)\\
&= -\frac{3a\left(a+bx^2\right)^{2/3}}{4b^2}+\frac{3\left(a+bx^2\right)^{5/3}}{10b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{3\left(a+bx^2\right)^{2/3}\left(2bx^2-3a\right)}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(2/3)*(-3*a + 2*b*x^2))/(20*b^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 28, normalized size = 0.74

$$-\frac{3\left(3a-2bx^2\right)\left(a+bx^2\right)^{2/3}}{20b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2)^(1/3),x]
[Out] (-3*(3*a - 2*b*x^2)*(a + b*x^2)^(2/3))/(20*b^2)
```

fricas [A] time = 1.05, size = 24, normalized size = 0.63

$$\frac{3\left(2\,bx^2-3\,a\right)\left(bx^2+a\right)^{\frac{2}{3}}}{20\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/20*(2*b*x^2 - 3*a)*(b*x^2 + a)^(2/3)/b^2
```

giac [A] time = 0.57, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] 3/10*(b*x^2 + a)^(5/3)/b^2 - 3/4*(b*x^2 + a)^(2/3)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{2}{3}}\left(-2b\,x^{2}+3a\right)}{20b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/3),x)
[Out] -3/20*(b*x^2+a)^(2/3)*(-2*b*x^2+3*a)/b^2
```

maxima [A] time = 1.40, size = 30, normalized size = 0.79

$$\frac{3\left(bx^{2}+a\right)^{\frac{5}{3}}}{10\,b^{2}}-\frac{3\left(bx^{2}+a\right)^{\frac{2}{3}}a}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] 3/10*(b*x^2 + a)^(5/3)/b^2 - 3/4*(b*x^2 + a)^(2/3)*a/b^2
```

mupad [B] time = 5.01, size = 24, normalized size = 0.63

$$-\frac{3\left(b\,x^{2}+a\right)^{2/3}\,\left(3\,a-2\,b\,x^{2}\right)}{20\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(1/3),x)
[Out] -(3*(a + b*x^2)^(2/3)*(3*a - 2*b*x^2))/(20*b^2)
```

sympy [B] time = 1.14, size = 178, normalized size = 4.68

$$-\frac{9a^{\frac{11}{3}}\left(1+\frac{bx^{2}}{a}\right)^{\frac{2}{3}}}{20a^{2}b^{2}+20ab^{3}x^{2}}+\frac{9a^{\frac{11}{3}}}{20a^{2}b^{2}+20ab^{3}x^{2}}-\frac{3a^{\frac{8}{3}}bx^{2}\left(1+\frac{bx^{2}}{a}\right)^{\frac{2}{3}}}{20a^{2}b^{2}+20ab^{3}x^{2}}+\frac{9a^{\frac{8}{3}}bx^{2}}{20a^{2}b^{2}+20ab^{3}x^{2}}+\frac{6a^{\frac{5}{3}}b^{2}x^{4}\left(1+\frac{bx^{2}}{a}\right)^{\frac{2}{3}}}{20a^{2}b^{2}+20ab^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/3),x)
[Out] -9*a**(11/3)*(1 + b*x**2/a)**(2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*a**(
11/3)/(20*a**2*b**2 + 20*a*b**3*x**2) - 3*a**(8/3)*b*x**2*(1 + b*x**2/a)**(
2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*a**(8/3)*b*x**2/(20*a**2*b**2 + 20
*a*b**3*x**2) + 6*a**(5/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(20*a**2*b**2 +
20*a*b**3*x**2)
```

3.609 $\int \frac{x}{\sqrt[3]{a+bx^2}} \, dx$

Optimal. Leaf size=18

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{a+bx^2}} \, dx = \frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(1/3),x]
```

```
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x^2)^(1/3),x]
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

fricas [A] time = 0.63, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

giac [A] time = 0.57, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\left(b\,x^2+a\right)^{\frac{2}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/3),x)
[Out] 3/4*(b*x^2+a)^(2/3)/b
```

maxima [A] time = 1.30, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x^2 + a)^(2/3)/b
```

mupad [B] time = 4.67, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x^2+a\right)^{2/3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(1/3),x)
```

```
[Out] (3*(a + b*x^2)^(2/3))/(4*b)
```

sympy [A] time = 0.40, size = 24, normalized size = 1.33

$$\begin{cases} \frac{3\left(a+bx^2\right)^{\frac{2}{3}}}{4b} & \text{for}\ b \neq 0 \\ \frac{x^2}{2\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/3),x)
```

```
[Out] Piecewise((3*(a + b*x**2)**(2/3)/(4*b), Ne(b, 0)), (x**2/(2*a**(1/3)), True
))
```

3.610 $\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(2*a^(1
/3)) - Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(1/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right) - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}\\
&= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}\\
&= \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.81

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2\log(x)}{4\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[x] +
 3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(1/3))
```

IntegrateAlgebraic [A] time = 0.07, size = 118, normalized size = 1.37

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{4\sqrt[3]{a}}+\frac{\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(2*a^
(1/3)) + Log[-a^(1/3) + (a + b*x^2)^(1/3)]/(2*a^(1/3)) - Log[a^(2/3) + a^(1
/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]/(4*a^(1/3))
```

fricas [A] time = 1.01, size = 235, normalized size = 2.73

$$\left[\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)-a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{4\,a},\ \frac{2\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{4\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x^2 + sqrt(3)*(2*(b*x^2 + a)^(2/3
)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^2 + a)
^(1/3)*a^(2/3) + 3*a)/x^2) - a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1
/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)))/a, 1/4
*(2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3)) - a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
 + 2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)))/a]
```

giac [A] time = 1.08, size = 87, normalized size = 1.01

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{1}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{1}{3}}}+\frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2\,a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
1/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x/(b*x^2+a)^(1/3),x)
```

maxima [A] time = 2.98, size = 86, normalized size = 1.00

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{1}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{1}{3}}}+\frac{\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
1/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(1/3)
```

mupad [B] time = 4.83, size = 106, normalized size = 1.23

$$\frac{\ln\left(\frac{9\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{1/3}}{4}\right)}{2\,a^{1/3}}+\frac{\ln\left(\frac{9\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{1/3}}-\frac{\ln\left(\frac{9\left(b\,x^2+a\right)^{1/3}}{4}-\frac{9\,a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/3)),x)
```

```
[Out] log((9*(a + b*x^2)^(1/3))/4 - (9*a^(1/3))/4)/(2*a^(1/3)) + (log((9*(a + b*x
^2)^(1/3))/4 - (9*a^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/(4*a^(1
/3)) - (log((9*(a + b*x^2)^(1/3))/4 - (9*a^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3
^(1/2)*1i + 1))/(4*a^(1/3))
```

sympy [C] time = 1.02, size = 41, normalized size = 0.48

$$-\frac{\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}\, x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1/
3)*x**(2/3)*gamma(4/3))
```

3.611 $\int \frac{1}{x^3 \sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}$$

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 55, 617, 204, 31}

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(a + b*x^2)^(2/3)/(2*a*x^2) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(2*Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(
1/3) - (a + b*x^2)^(1/3)])/(4*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{6a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2} + \frac{b\log(x)}{6a^{4/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{4/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx\right.}{4a}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{4/3}}\\
&= -\frac{\left(a+bx^2\right)^{2/3}}{2ax^2} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt{3}\,a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.34

$$\frac{3b\left(a+bx^2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{bx^2}{a}+1\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(1/3)),x]
```

```
[Out] (3*b*(a + b*x^2)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 + (b*x^2)/a])/(4*a^
2)
```

IntegrateAlgebraic [A] time = 0.13, size = 142, normalized size = 1.29

$$-\frac{b\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{6a^{4/3}}+\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{12a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,a^{4/3}}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^(1/3)),x]
```

```
[Out] -1/2*(a + b*x^2)^(2/3)/(a*x^2) - (b*ArcTan[1/Sqrt[3] + (2*(a + b*x^2)^(1/3)
)/(Sqrt[3]*a^(1/3))])/(2*Sqrt[3]*a^(4/3)) - (b*Log[-a^(1/3) + (a + b*x^2)^(
1/3)])/(6*a^(4/3)) + (b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^
2)^(2/3)])/(12*a^(4/3))
```

fricas [A] time = 1.02, size = 344, normalized size = 3.13

$$\left[\frac{3\sqrt{\frac{1}{3}}abx^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx^2-3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x^2}\right)+(-a)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-6\left(bx^2+a\right)^{\frac{2}{3}}a}{12a^2x^2},\ -\frac{6\sqrt{\frac{1}{3}}abx^2\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2(-a)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+6\left(bx^2+a\right)^{\frac{2}{3}}a}{12a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(1/3)*a*b*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x^2 - 3*sqrt(1/3)*(2
*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-
a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + (-a)^(2/3)*b*x^2
*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a
)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^2 + a)^(2/3)*a)/
(a^2*x^2), -1/12*(6*sqrt(1/3)*a*b*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*
(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x^2*
log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)
^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) + 6*(b*x^2 + a)^(2/3)*a)/(
a^2*x^2)]
```

giac [A] time = 1.14, size = 119, normalized size = 1.08

$$-\frac{\frac{2\,\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\,b^2\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{6\,(bx^2+a)^{\frac{2}{3}}b}{ax^2}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/12*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(4/3) - b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(4/3) + 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*
x^2 + a)^(2/3)*b/(a*x^2))/b
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(1/3),x)
```

maxima [A] time = 2.95, size = 118, normalized size = 1.07

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{6\,a^{\frac{4}{3}}}-\frac{\left(bx^2+a\right)^{\frac{2}{3}}b}{2\left(\left(bx^2+a\right)a-a^2\right)}+\frac{b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{12\,a^{\frac{4}{3}}}-\frac{b\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6\,a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(4/3) - 1/2*(b*x^2 + a)^(2/3)*b/((b*x^2 + a)*a - a^2) + 1/12*b*log((b*x^2
 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/6*b*log((b*x
^2 + a)^(1/3) - a^(1/3))/a^(4/3)
```

mupad [B] time = 5.01, size = 138, normalized size = 1.25

$$-\frac{b\,\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{6\,a^{4/3}}-\frac{\left(b\,x^2+a\right)^{2/3}}{2\,a\,x^2}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{16\,a^{5/3}}-\frac{b^2\,\left(b\,x^2+a\right)^{1/3}}{4\,a^2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{12\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{16\,a^{5/3}}-\frac{b^2\,\left(b\,x^2+a\right)^{1/3}}{4\,a^2}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{12\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/3)),x)
```

```
[Out] (log((b - 3^(1/2)*b*1i)^2/(16*a^(5/3)) - (b^2*(a + b*x^2)^(1/3))/(4*a^2))*(
b - 3^(1/2)*b*1i))/(12*a^(4/3)) - (a + b*x^2)^(2/3)/(2*a*x^2) - (b*log((a +
 b*x^2)^(1/3) - a^(1/3)))/(6*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(16*a^(5/
3)) - (b^2*(a + b*x^2)^(1/3))/(4*a^2))*(b + 3^(1/2)*b*1i))/(12*a^(4/3))
```

sympy [C] time = 1.19, size = 41, normalized size = 0.37

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}\,x^{\frac{8}{3}}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(4/3)*hyper((1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1/
3)*x**(8/3)*gamma(7/3))
```

3.612 $\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=138

$$\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}}+\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2}-\frac{\left(a+bx^2\right)^{2/3}}{4ax^4}$$

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 55, 617, 204, 31}

$$\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}}+\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2}-\frac{\left(a+bx^2\right)^{2/3}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(1/3)),x]
```

```
[Out] -(a + b*x^2)^(2/3)/(4*a*x^4) + (b*(a + b*x^2)^(2/3))/(3*a^2*x^2) + (b^2*Arc
Tan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3))
 - (b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(
7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} - \frac{b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{3a}$$
$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{9a^2}$$
$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} - \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{6a^{7/3}} + \frac{b^2\,\text{Sub}}{}$$
$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{-3-x}\right.}{3}$$
$$= -\frac{\left(a+bx^2\right)^{2/3}}{4ax^4} + \frac{b\left(a+bx^2\right)^{2/3}}{3a^2x^2} + \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+b}\right.}{6a^{7/3}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.28

$$-\frac{3b^2\left(a+bx^2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};\frac{bx^2}{a}+1\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(1/3)),x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 + (b*x^2)/a])/(4
*a^3)
```

IntegrateAlgebraic [A] time = 0.15, size = 158, normalized size = 1.14

$$\frac{b^2\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{9a^{7/3}} - \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{18a^{7/3}} + \frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{\left(a+bx^2\right)^{2/3}\left(4bx^2-3a\right)}{12a^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^(1/3)),x]
```

```
[Out] ((a + b*x^2)^(2/3)*(-3*a + 4*b*x^2))/(12*a^2*x^4) + (b^2*ArcTan[1/Sqrt[3] +
 (2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) + (b^2*Log[-
a^(1/3) + (a + b*x^2)^(1/3)])/(9*a^(7/3)) - (b^2*Log[a^(2/3) + a^(1/3)*(a +
 b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(18*a^(7/3))
```

fricas [A] time = 1.72, size = 326, normalized size = 2.36

$$\left[\frac{6\sqrt{\frac{1}{3}}\,ab^2x^4\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^2+3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^2}\right)-2a^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4a^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\left(4abx^2-3a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{36a^3x^4},\ \frac{12\sqrt{\frac{1}{3}}\,a^{\frac{2}{3}}b^2x^4\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-2a^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4a^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\left(4abx^2-3a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{36a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/36*(6*sqrt(1/3)*a*b^2*x^4*sqrt(-1/a^(2/3))*log((2*b*x^2 + 3*sqrt(1/3)*(2
*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)
) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - 2*a^(2/3)*b^2*x^4*log((b*x^2
+ a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^4*log((
b*x^2 + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3))/(a^3
*x^4), 1/36*(12*sqrt(1/3)*a^(2/3)*b^2*x^4*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(
1/3) + a^(1/3))/a^(1/3)) - 2*a^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3) + (b*x^2
 + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3) -
a^(1/3)) + 3*(4*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3))/(a^3*x^4)]
```

giac [A] time = 1.13, size = 142, normalized size = 1.03

$$\frac{\frac{4\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{2b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{4b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}}+\frac{3\left(4\left(bx^2+a\right)^{\frac{5}{3}}b^3-7\left(bx^2+a\right)^{\frac{2}{3}}ab^3\right)}{a^2b^2x^4}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/36*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(7/3) - 2*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(7/3) + 4*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4
*(b*x^2 + a)^(5/3)*b^3 - 7*(b*x^2 + a)^(2/3)*a*b^3)/(a^2*b^2*x^4))/b
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/x^5/(b*x^2+a)^(1/3),x)
```

maxima [A] time = 2.97, size = 158, normalized size = 1.14

$$\frac{\sqrt{3}\, b^2 \arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\, a^{\frac{1}{3}}}\right)}{9\, a^{\frac{7}{3}}} - \frac{b^2 \log\left(\left(bx^2+a\right)^{\frac{2}{3}} + \left(bx^2+a\right)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{18\, a^{\frac{7}{3}}} + \frac{b^2 \log\left(\left(bx^2+a\right)^{\frac{1}{3}} - a^{\frac{1}{3}}\right)}{9\, a^{\frac{7}{3}}} + \frac{4\left(bx^2+a\right)^{\frac{5}{3}} b^2 - 7\left(bx^2+a\right)^{\frac{2}{3}} a b^2}{12\left(\left(bx^2+a\right)^2 a^2 - 2\left(bx^2+a\right) a^3 + a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(7/3) - 1/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(7/3) + 1/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(7/3) + 1/12*(4*
(b*x^2 + a)^(5/3)*b^2 - 7*(b*x^2 + a)^(2/3)*a*b^2)/((b*x^2 + a)^2*a^2 - 2*(
b*x^2 + a)*a^3 + a^4)
```

mupad [B] time = 5.06, size = 201, normalized size = 1.46

$$\frac{b^2\,\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{9\,a^{7/3}} - \frac{\ln\left(\frac{b^4\,\left(b\,x^2+a\right)^{1/3}}{9\,a^4}-\frac{\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)^2}{36\,a^{11/3}}\right)\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}{18\,a^{7/3}} - \frac{\frac{7\,b^2\,\left(b\,x^2+a\right)^{2/3}}{6\,a}-\frac{2\,b^2\,\left(b\,x^2+a\right)^{5/3}}{3\,a^2}}{2\,\left(b\,x^2+a\right)^2-4\,a\,\left(b\,x^2+a\right)+2\,a^2} + \frac{b^2\,\ln\left(\frac{b^4\,\left(b\,x^2+a\right)^{1/3}}{9\,a^4}-\frac{b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^2}{9\,a^{11/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(1/3)),x)
```

```
[Out] (b^2*log((a + b*x^2)^(1/3) - a^(1/3)))/(9*a^(7/3)) - (log((b^4*(a + b*x^2)^
(1/3))/(9*a^4) - (3^(1/2)*b^2*1i + b^2)^2/(36*a^(11/3)))*(3^(1/2)*b^2*1i +
b^2))/(18*a^(7/3)) - ((7*b^2*(a + b*x^2)^(2/3))/(6*a) - (2*b^2*(a + b*x^2)^
(5/3))/(3*a^2))/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2) + (b^2*log((b^4
*(a + b*x^2)^(1/3))/(9*a^4) - (b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(9*a^(11/3)))*
((3^(1/2)*1i)/2 - 1/2))/(9*a^(7/3))
```

sympy [C] time = 1.37, size = 41, normalized size = 0.30

$$-\frac{\Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{a e^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}\, x^{\frac{14}{3}}\, \Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(1/3),x)
```

```
[Out] -gamma(7/3)*hyper((1/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1
/3)*x**(14/3)*gamma(10/3))
```

3.613 $\int \frac{x^7}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{9a^2\left(a+bx^2\right)^{4/3}}{8b^4}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{9a\left(a+bx^2\right)^{7/3}}{14b^4}$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{9a^2\left(a+bx^2\right)^{4/3}}{8b^4}-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{3\left(a+bx^2\right)^{10/3}}{20b^4}-\frac{9a\left(a+bx^2\right)^{7/3}}{14b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x^2)^(1/3))/(2*b^4) + (9*a^2*(a + b*x^2)^(4/3))/(8*b^4) - (9
*a*(a + b*x^2)^(7/3))/(14*b^4) + (3*(a + b*x^2)^(10/3))/(20*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{2/3}} + \frac{3a^2\sqrt[3]{a+bx}}{b^3} - \frac{3a(a+bx)^{4/3}}{b^3} + \frac{(a+bx)^{7/3}}{b^3}\right)dx, x, x^2\right)$$
$$= -\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4} + \frac{9a^2\left(a+bx^2\right)^{4/3}}{8b^4} - \frac{9a\left(a+bx^2\right)^{7/3}}{14b^4} + \frac{3\left(a+bx^2\right)^{10/3}}{20b^4}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.62

$$\frac{3\sqrt[3]{a+bx^2}\left(-81a^3+27a^2bx^2-18ab^2x^4+14b^3x^6\right)}{280b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(-81*a^3 + 27*a^2*b*x^2 - 18*a*b^2*x^4 + 14*b^3*x^6))/
(280*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 50, normalized size = 0.62

$$-\frac{3\sqrt[3]{a+bx^2}\left(81a^3-27a^2bx^2+18ab^2x^4-14b^3x^6\right)}{280b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3)*(81*a^3 - 27*a^2*b*x^2 + 18*a*b^2*x^4 - 14*b^3*x^6))/
(280*b^4)
```

fricas [A] time = 0.96, size = 46, normalized size = 0.58

$$\frac{3\left(14\,b^3x^6-18\,ab^2x^4+27\,a^2bx^2-81\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}}{280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/280*(14*b^3*x^6 - 18*a*b^2*x^4 + 27*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^(1/3)
/b^4
```

giac [A] time = 0.58, size = 61, normalized size = 0.76

$$-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a^3}{2\,b^4}+\frac{3\left(14\left(bx^2+a\right)^{\frac{10}{3}}-60\left(bx^2+a\right)^{\frac{7}{3}}a+105\left(bx^2+a\right)^{\frac{4}{3}}a^2\right)}{280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -3/2*(b*x^2 + a)^(1/3)*a^3/b^4 + 3/280*(14*(b*x^2 + a)^(10/3) - 60*(b*x^2 +
 a)^(7/3)*a + 105*(b*x^2 + a)^(4/3)*a^2)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}\left(-14b^3x^6+18a\,b^2x^4-27a^2b\,x^2+81a^3\right)}{280b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/280*(b*x^2+a)^(1/3)*(-14*b^3*x^6+18*a*b^2*x^4-27*a^2*b*x^2+81*a^3)/b^4
```

maxima [A] time = 1.34, size = 64, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{10}{3}}}{20\,b^4}-\frac{9\left(bx^2+a\right)^{\frac{7}{3}}a}{14\,b^4}+\frac{9\left(bx^2+a\right)^{\frac{4}{3}}a^2}{8\,b^4}-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^2 + a)^(10/3)/b^4 - 9/14*(b*x^2 + a)^(7/3)*a/b^4 + 9/8*(b*x^2 + a
)^(4/3)*a^2/b^4 - 3/2*(b*x^2 + a)^(1/3)*a^3/b^4
```

mupad [B] time = 4.75, size = 48, normalized size = 0.60

$$-{\left(b\,x^2+a\right)}^{1/3}\,\left(\frac{243\,a^3}{280\,b^4}-\frac{3\,x^6}{20\,b}+\frac{27\,a\,x^4}{140\,b^2}-\frac{81\,a^2\,x^2}{280\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(2/3),x)
```

```
[Out] -(a + b*x^2)^(1/3)*((243*a^3)/(280*b^4) - (3*x^6)/(20*b) + (27*a*x^4)/(140*
b^2) - (81*a^2*x^2)/(280*b^3))
```

sympy [B] time = 2.74, size = 1690, normalized size = 21.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(2/3),x)
```

```
[Out] -243*a**(70/3)*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2
 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 168
0*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 243*a**(70/3)/(280*a**20*b**4
 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 420
0*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 1377*a
**(67/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**
2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 16
80*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 1458*a**(67/3)*b*x**2/(280*a
**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x
**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12)
 - 3213*a**(64/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a*
*19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b*
*8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 3645*a**(64/3)*b
**2*x**4/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 56
00*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**
14*b**10*x**12) - 3927*a**(61/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(280*a**20
*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6
+ 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 4
860*a**(61/3)*b**3*x**6/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18
*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*
x**10 + 280*a**14*b**10*x**12) - 2583*a**(58/3)*b**4*x**8*(1 + b*x**2/a)**(
1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a
**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b
**10*x**12) + 3645*a**(58/3)*b**4*x**8/(280*a**20*b**4 + 1680*a**19*b**5*x*
*2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1
680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 693*a**(55/3)*b**5*x**10*(1
 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**
6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**1
0 + 280*a**14*b**10*x**12) + 1458*a**(55/3)*b**5*x**10/(280*a**20*b**4 + 16
80*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**
16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 273*a**(52/
3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2
+ 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680
*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 243*a**(52/3)*b**6*x**12/(280*
```

```
a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*
x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12
) + 387*a**(49/3)*b**7*x**14*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a
**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b
**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 198*a**(46/3)*b
**8*x**16*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 42
00*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**
15*b**9*x**10 + 280*a**14*b**10*x**12) + 42*a**(43/3)*b**9*x**18*(1 + b*x**
2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 +
 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*
a**14*b**10*x**12)
```

3.614 $\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^3}-\frac{3a\left(a+bx^2\right)^{4/3}}{4b^3}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3}+\frac{3\left(a+bx^2\right)^{7/3}}{14b^3}-\frac{3a\left(a+bx^2\right)^{4/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x^2)^(1/3))/(2*b^3) - (3*a*(a + b*x^2)^(4/3))/(4*b^3) + (3*(a
 + b*x^2)^(7/3))/(14*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{2/3}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{2/3}} - \frac{2a\sqrt[3]{a+bx}}{b^2} + \frac{(a+bx)^{4/3}}{b^2}\right)dx, x, x^2\right)$$
$$= \frac{3a^2\sqrt[3]{a+bx^2}}{2b^3} - \frac{3a\left(a+bx^2\right)^{4/3}}{4b^3} + \frac{3\left(a+bx^2\right)^{7/3}}{14b^3}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx^2}\left(9a^2-3abx^2+2b^2x^4\right)}{28b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(2/3),x]
[Out] (3*(a + b*x^2)^(1/3)*(9*a^2 - 3*a*b*x^2 + 2*b^2*x^4))/(28*b^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx^2}\left(9a^2-3abx^2+2b^2x^4\right)}{28b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2)^(2/3),x]
[Out] (3*(a + b*x^2)^(1/3)*(9*a^2 - 3*a*b*x^2 + 2*b^2*x^4))/(28*b^3)
```

fricas [A] time = 1.12, size = 35, normalized size = 0.59

$$\frac{3\left(2\,b^2x^4-3\,abx^2+9\,a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}}{28\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/28*(2*b^2*x^4 - 3*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(1/3)/b^3
```

giac [A] time = 0.57, size = 47, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a^2}{2\,b^3}+\frac{3\left(2\left(bx^2+a\right)^{\frac{7}{3}}-7\left(bx^2+a\right)^{\frac{4}{3}}a\right)}{28\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^2 + a)^(1/3)*a^2/b^3 + 3/28*(2*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^(
4/3)*a)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}\left(2b^2x^4-3ab\,x^2+9a^2\right)}{28b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(2/3),x)
```

```
[Out] 3/28*(b*x^2+a)^(1/3)*(2*b^2*x^4-3*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.33, size = 47, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{7}{3}}}{14\,b^3}-\frac{3\left(bx^2+a\right)^{\frac{4}{3}}a}{4\,b^3}+\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^2 + a)^(7/3)/b^3 - 3/4*(b*x^2 + a)^(4/3)*a/b^3 + 3/2*(b*x^2 + a)^
(1/3)*a^2/b^3
```

mupad [B] time = 4.77, size = 36, normalized size = 0.61

$$\left(b\,x^2+a\right)^{1/3}\left(\frac{27\,a^2}{28\,b^3}+\frac{3\,x^4}{14\,b}-\frac{9\,a\,x^2}{28\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(2/3),x)
```

```
[Out] (a + b*x^2)^(1/3)*((27*a^2)/(28*b^3) + (3*x^4)/(14*b) - (9*a*x^2)/(28*b^2))
```

sympy [B] time = 1.80, size = 631, normalized size = 10.69

$$\frac{27a^{\frac{31}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}-\frac{27a^{\frac{31}{3}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}+\frac{72a^{\frac{28}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}-\frac{81a^{\frac{28}{3}}bx^2}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}+\frac{60a^{\frac{25}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}-\frac{81a^{\frac{25}{3}}b^2x^4}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}+\frac{18a^{\frac{22}{3}}b^3x^6\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}-\frac{27a^{\frac{22}{3}}b^3x^6}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}+\frac{9a^{\frac{19}{3}}b^4x^8\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}+\frac{6a^{\frac{16}{3}}b^5x^{10}\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(2/3),x)
```

```
[Out] 27*a**(31/3)*(1 + b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a
**6*b**5*x**4 + 28*a**5*b**6*x**6) - 27*a**(31/3)/(28*a**8*b**3 + 84*a**7*b
**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 72*a**(28/3)*b*x**2*(1
+ b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 +
28*a**5*b**6*x**6) - 81*a**(28/3)*b*x**2/(28*a**8*b**3 + 84*a**7*b**4*x**2
+ 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 60*a**(25/3)*b**2*x**4*(1 + b*x*
*2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**
5*b**6*x**6) - 81*a**(25/3)*b**2*x**4/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 8
4*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 18*a**(22/3)*b**3*x**6*(1 + b*x**2/
a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b
**6*x**6) - 27*a**(22/3)*b**3*x**6/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a
**6*b**5*x**4 + 28*a**5*b**6*x**6) + 9*a**(19/3)*b**4*x**8*(1 + b*x**2/a)**
(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*
x**6) + 6*a**(16/3)*b**5*x**10*(1 + b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**
7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6)
```

3.615 $\int \frac{x^3}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=38

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b^2}-\frac{3a\sqrt[3]{a+bx^2}}{2b^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3\left(a+bx^2\right)^{4/3}}{8b^2}-\frac{3a\sqrt[3]{a+bx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x^2)^(1/3))/(2*b^2) + (3*(a + b*x^2)^(4/3))/(8*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{2/3}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right)dx,x,x^2\right)\\ &= -\frac{3a\sqrt[3]{a+bx^2}}{2b^2}+\frac{3\left(a+bx^2\right)^{4/3}}{8b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$\frac{3\left(bx^2-3a\right)\sqrt[3]{a+bx^2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(2/3),x]
[Out] (3*(-3*a + b*x^2)*(a + b*x^2)^(1/3))/(8*b^2)
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 0.74

$$-\frac{3\left(3a-bx^2\right)\sqrt[3]{a+bx^2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2)^(2/3),x]
[Out] (-3*(3*a - b*x^2)*(a + b*x^2)^(1/3))/(8*b^2)
```

fricas [A] time = 0.85, size = 23, normalized size = 0.61

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}\left(bx^2-3\,a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/8*(b*x^2 + a)^(1/3)*(b*x^2 - 3*a)/b^2
```

giac [A] time = 0.59, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] 3/8*(b*x^2 + a)^(4/3)/b^2 - 3/2*(b*x^2 + a)^(1/3)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}\left(-b\,x^2+3a\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/8*(b*x^2+a)^(1/3)*(-b*x^2+3*a)/b^2
```

maxima [A] time = 1.34, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2+a\right)^{\frac{4}{3}}}{8\,b^2}-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x^2 + a)^(4/3)/b^2 - 3/2*(b*x^2 + a)^(1/3)*a/b^2
```

mupad [B] time = 4.79, size = 24, normalized size = 0.63

$$-\frac{3\left(b\,x^2+a\right)^{1/3}\,\left(3\,a-b\,x^2\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(2/3),x)
```

```
[Out] -(3*(a + b*x^2)^(1/3)*(3*a - b*x^2))/(8*b^2)
```

sympy [B] time = 1.14, size = 178, normalized size = 4.68

$$-\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}+\frac{9a^{\frac{10}{3}}}{8a^2b^2+8ab^3x^2}-\frac{6a^{\frac{7}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}+\frac{9a^{\frac{7}{3}}bx^2}{8a^2b^2+8ab^3x^2}+\frac{3a^{\frac{4}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(2/3),x)
```

```
[Out] -9*a**(10/3)*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*a**(10
/3)/(8*a**2*b**2 + 8*a*b**3*x**2) - 6*a**(7/3)*b*x**2*(1 + b*x**2/a)**(1/3)
/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*a**(7/3)*b*x**2/(8*a**2*b**2 + 8*a*b**3*
x**2) + 3*a**(4/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b**3*
x**2)
```

3.616 $\int \frac{x}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=18

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{2/3}}\,dx=\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x^2)^(2/3),x]
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

fricas [A] time = 0.94, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

giac [A] time = 0.57, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(2/3),x)
[Out] 3/2*(b*x^2+a)^(1/3)/b
```

maxima [A] time = 1.24, size = 14, normalized size = 0.78

$$\frac{3\left(bx^2+a\right)^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x^2 + a)^(1/3)/b
```

mupad [B] time = 4.69, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x^2+a\right)^{1/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(2/3),x)
```

```
[Out] (3*(a + b*x^2)^(1/3))/(2*b)
```

sympy [A] time = 0.41, size = 24, normalized size = 1.33

$$\begin{cases} \frac{3\sqrt[3]{a+bx^2}}{2b} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(2/3),x)
```

```
[Out] Piecewise((3*(a + b*x**2)**(1/3)/(2*b), Ne(b, 0)), (x**2/(2*a**(2/3)), True
))
```

3.617 $\int \frac{1}{x(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=86

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 57, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(2*a^(
2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(4*a^(2/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)$$

$$= -\frac{\log(x)}{2a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}$$

Mathematica [A] time = 0.03, size = 101, normalized size = 1.17

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{4a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/4*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log
[a^(1/3) - (a + b*x^2)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (
a + b*x^2)^(2/3)])/a^(2/3)
```

IntegrateAlgebraic [A] time = 0.08, size = 118, normalized size = 1.37

$$\frac{\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{2a^{2/3}}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{4a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))]])/
a^(2/3) + Log[-a^(1/3) + (a + b*x^2)^(1/3)]/(2*a^(2/3)) - Log[a^(2/3) + a^(
1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]/(4*a^(2/3))
```

fricas [B] time = 1.43, size = 123, normalized size = 1.43

$$-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}\left(a^2\right)^{\frac{1}{6}}\left(\left(a^2\right)^{\frac{1}{3}}a+2\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-2\left(a^2\right)^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*a
 + 2*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*log((b*x^2 + a)^(2/3
)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) - 2*(a^2)^(2/3)*log((b
*x^2 + a)^(1/3)*a - (a^2)^(2/3)))/a^2
```

giac [A] time = 1.13, size = 87, normalized size = 1.01

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{2}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{2}{3}}}+\frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(2/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{2}{3}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x/(b*x^2+a)^(2/3),x)
```

maxima [A] time = 3.09, size = 86, normalized size = 1.00

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{2}{3}}}-\frac{\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{2}{3}}}+\frac{\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(2/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3)
```

mupad [B] time = 4.84, size = 102, normalized size = 1.19

$$\frac{\ln\left(\frac{9\left(b\,x^{2}+a\right)^{1/3}}{2}-\frac{9\,a^{1/3}}{2}\right)}{2\,a^{2/3}}+\frac{\ln\left(\frac{9\,a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}-\frac{9\left(b\,x^{2}+a\right)^{1/3}}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{2/3}}-\frac{\ln\left(\frac{9\,a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}+\frac{9\left(b\,x^{2}+a\right)^{1/3}}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(2/3)),x)
```

```
[Out] log((9*(a + b*x^2)^(1/3))/2 - (9*a^(1/3))/2)/(2*a^(2/3)) + (log((9*a^(1/3)*
(3^(1/2)*1i - 1))/4 - (9*(a + b*x^2)^(1/3))/2)*(3^(1/2)*1i - 1))/(4*a^(2/3)
) - (log((9*a^(1/3)*(3^(1/2)*1i + 1))/4 + (9*(a + b*x^2)^(1/3))/2)*(3^(1/2)
*1i + 1))/(4*a^(2/3))
```

sympy [C] time = 1.06, size = 41, normalized size = 0.48

$$-\frac{\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^{2}}\right)}{2b^{\frac{2}{3}}x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*x**(4/3)*gamma(5/3))
```

3.618 $\int \frac{1}{x^3(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=107

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{\sqrt[3]{a+bx^2}}{2ax^2}$$

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 57, 617, 204, 31}

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{\sqrt[3]{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(a + b*x^2)^(1/3)/(2*a*x^2) + (b*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(1/
3) - (a + b*x^2)^(1/3)])/(2*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^{2/3}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)}{3a}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} + \frac{b\log(x)}{3a^{5/3}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{2a^{5/3}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,\right.}{2a^{4/3}}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}} - \frac{b\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{a^{5/3}}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{2ax^2} + \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.35

$$\frac{3b\sqrt[3]{a+bx^2}\ {}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{bx^2}{a}+1\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*b*(a + b*x^2)^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, 1 + (b*x^2)/a])/(2*a^
2)
```

IntegrateAlgebraic [A] time = 0.11, size = 139, normalized size = 1.30

$$-\frac{b\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{3a^{5/3}}+\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{6a^{5/3}}+\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^(2/3)),x]
```

```
[Out] -1/2*(a + b*x^2)^(1/3)/(a*x^2) + (b*ArcTan[1/Sqrt[3] + (2*(a + b*x^2)^(1/3)
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) - (b*Log[-a^(1/3) + (a + b*x^2)^(1/
3)])/(3*a^(5/3)) + (b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)
^(2/3)])/(6*a^(5/3))
```

fricas [B] time = 0.92, size = 182, normalized size = 1.70

$$\frac{2\sqrt{3}\,abx^2\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)-2\left(-a^2\right)^{\frac{2}{3}}bx^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)-3\left(bx^2+a\right)^{\frac{1}{3}}a^2}{6a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*a*b*x^2*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3
)*a - 2*sqrt(3)*(b*x^2 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) +
(-a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^2 + a)^(
1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3)*a - (-a^2)^
(2/3)) - 3*(b*x^2 + a)^(1/3)*a^2)/(a^3*x^2)
```

giac [A] time = 1.11, size = 118, normalized size = 1.10

$$\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2b^2\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}b}{ax^2}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(5/3) + b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(5/3) - 2*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x^
2 + a)^(1/3)*b/(a*x^2))/b
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(2/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(2/3),x)
```

maxima [A] time = 2.96, size = 118, normalized size = 1.10

$$\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{5}{3}}}-\frac{\left(bx^2+a\right)^{\frac{1}{3}}b}{2\left(\left(bx^2+a\right)a-a^2\right)}+\frac{b\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{5}{3}}}-\frac{b\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(5/3) - 1/2*(b*x^2 + a)^(1/3)*b/((b*x^2 + a)*a - a^2) + 1/6*b*log((b*x^2 +
 a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 1/3*b*log((b*x^2
 + a)^(1/3) - a^(1/3))/a^(5/3)
```

mupad [B] time = 5.05, size = 130, normalized size = 1.21

$$\frac{\ln\left(\frac{3\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{2\,a^{2/3}}+\frac{3\,b\,\left(b\,x^2+a\right)^{1/3}}{a}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{5/3}}+\frac{\ln\left(\frac{3\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{2\,a^{2/3}}+\frac{3\,b\,\left(b\,x^2+a\right)^{1/3}}{a}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{5/3}}-\frac{b\,\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{3\,a^{5/3}}-\frac{\left(b\,x^2+a\right)^{1/3}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(2/3)),x)
```

```
[Out] (log((3*(b - 3^(1/2)*b*1i))/(2*a^(2/3)) + (3*b*(a + b*x^2)^(1/3))/a)*(b - 3
^(1/2)*b*1i))/(6*a^(5/3)) + (log((3*(b + 3^(1/2)*b*1i))/(2*a^(2/3)) + (3*b*
(a + b*x^2)^(1/3))/a)*(b + 3^(1/2)*b*1i))/(6*a^(5/3)) - (b*log((a + b*x^2)^
(1/3) - a^(1/3)))/(3*a^(5/3)) - (a + b*x^2)^(1/3)/(2*a*x^2)
```

sympy [C] time = 1.24, size = 41, normalized size = 0.38

$$-\frac{\Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3}, \frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(2/3),x)
```

```
[Out] -gamma(5/3)*hyper((2/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/
3)*x**(10/3)*gamma(8/3))
```

3.619 $\int \frac{1}{x^5\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=138

$$\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{\sqrt[3]{a+bx^2}}{4ax^4}$$

Rubi [A] time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 57, 617, 204, 31}

$$\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2}-\frac{\sqrt[3]{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(2/3)),x]
```

```
[Out] -(a + b*x^2)^(1/3)/(4*a*x^4) + (5*b*(a + b*x^2)^(1/3))/(12*a^2*x^2) - (5*b^
2*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(
8/3)) - (5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x^2)^(1/3
)])/(12*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^2\right)^{2/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{2/3}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[3]{a+bx^2}}{4ax^4} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^2\right)}{12a}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{4ax^4} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^2\right)}{18a^2}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{4ax^4} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} - \frac{5b^2\log(x)}{18a^{8/3}} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{12a^{8/3}} - \frac{\left(5b^2\right.}{}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{4ax^4} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} - \frac{5b^2\log(x)}{18a^{8/3}} + \frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}} + \frac{\left(5b^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt[3]{a+bx^2}}{4ax^4} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} - \frac{5b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{8/3}} - \frac{5b^2\log(x)}{18a^{8/3}} + \frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+\right.}{12a^{8/3}}$$

Mathematica [C]   time = 0.01, size = 39, normalized size = 0.28

$$-\frac{3b^2\sqrt[3]{a+bx^2}\;{}_2F_1\left(\frac{1}{3},3;\frac{4}{3};\frac{bx^2}{a}+1\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*b^2*(a + b*x^2)^(1/3)*Hypergeometric2F1[1/3, 3, 4/3, 1 + (b*x^2)/a])/(2
*a^3)
```

IntegrateAlgebraic [A]   time = 0.14, size = 158, normalized size = 1.14

$$\frac{5b^2\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{18a^{8/3}} - \frac{5b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{36a^{8/3}} - \frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt{3}\,a^{8/3}} + \frac{\sqrt[3]{a+bx^2}\left(5bx^2-3a\right)}{12a^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^(2/3)),x]
```

```
[Out] ((a + b*x^2)^(1/3)*(-3*a + 5*b*x^2))/(12*a^2*x^4) - (5*b^2*ArcTan[1/Sqrt[3]
 + (2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(8/3)) + (5*b^2*L
og[-a^(1/3) + (a + b*x^2)^(1/3)])/(18*a^(8/3)) - (5*b^2*Log[a^(2/3) + a^(1/
3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(36*a^(8/3))
```

fricas [A] time = 1.41, size = 174, normalized size = 1.26

$$-\frac{10\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^4\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+5\left(a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-10\left(a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)-3\left(5a^2bx^2-3a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}}{36a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/36*(10*sqrt(3)*(a^2)^(1/6)*a*b^2*x^4*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^
2)^(1/3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + 5*(a^2)^(2/3)*
b^2*x^4*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(
2/3)) - 10*(a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) - 3*(
5*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(1/3))/(a^4*x^4)
```

giac [A] time = 1.13, size = 142, normalized size = 1.03

$$-\frac{\frac{10\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}}+\frac{5b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}}-\frac{10b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}}-\frac{3\left(5\left(bx^2+a\right)^{\frac{4}{3}}b^3-8\left(bx^2+a\right)^{\frac{1}{3}}ab^3\right)}{a^2b^2x^4}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/36*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(8/3) + 5*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
a^(2/3))/a^(8/3) - 10*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(8/3) - 3
*(5*(b*x^2 + a)^(4/3)*b^3 - 8*(b*x^2 + a)^(1/3)*a*b^3)/(a^2*b^2*x^4))/b
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}} x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(2/3),x)
```

[Out] int(1/x^5/(b*x^2+a)^(2/3),x)

maxima [A] time = 2.98, size = 158, normalized size = 1.14

$$-\frac{5\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{18\,a^{\frac{8}{3}}}-\frac{5\,b^2\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{36\,a^{\frac{8}{3}}}+\frac{5\,b^2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18\,a^{\frac{8}{3}}}+\frac{5\left(bx^2+a\right)^{\frac{4}{3}}b^2-8\left(bx^2+a\right)^{\frac{1}{3}}ab^2}{12\left(\left(bx^2+a\right)^2a^2-2\left(bx^2+a\right)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")

[Out] -5/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(8/3) - 5/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(8/3) + 5/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(8/3) + 1/12*
(5*(b*x^2 + a)^(4/3)*b^2 - 8*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2*a^2 -
2*(b*x^2 + a)*a^3 + a^4)

mupad [B] time = 5.14, size = 193, normalized size = 1.40

$$\frac{5\,b^2\,\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{18\,a^{8/3}}-\frac{\frac{4\,b^2\,\left(b\,x^2+a\right)^{1/3}}{3\,a}-\frac{5\,b^2\,\left(b\,x^2+a\right)^{4/3}}{6\,a^2}}{2\,\left(b\,x^2+a\right)^2-4\,a\,\left(b\,x^2+a\right)+2\,a^2}+\frac{5\,b^2\,\ln\left(\frac{5\,b^2\,\left(b\,x^2+a\right)^{1/3}}{2\,a^2}-\frac{5\,b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{18\,a^{8/3}}-\frac{5\,b^2\,\ln\left(\frac{5\,b^2\,\left(b\,x^2+a\right)^{1/3}}{2\,a^2}+\frac{5\,b^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{18\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a + b*x^2)^(2/3)),x)

[Out] (5*b^2*log((a + b*x^2)^(1/3) - a^(1/3)))/(18*a^(8/3)) - ((4*b^2*(a + b*x^2)
^(1/3))/(3*a) - (5*b^2*(a + b*x^2)^(4/3))/(6*a^2))/(2*(a + b*x^2)^2 - 4*a*(
a + b*x^2) + 2*a^2) + (5*b^2*log((5*b^2*(a + b*x^2)^(1/3))/(2*a^2) - (5*b^2
*((3^(1/2)*1i)/2 - 1/2))/(2*a^(5/3)))*((3^(1/2)*1i)/2 - 1/2))/(18*a^(8/3))
 - (5*b^2*log((5*b^2*(a + b*x^2)^(1/3))/(2*a^2) + (5*b^2*((3^(1/2)*1i)/2 + 1
/2))/(2*a^(5/3)))*((3^(1/2)*1i)/2 + 1/2))/(18*a^(8/3))

sympy [C] time = 1.45, size = 41, normalized size = 0.30

$$-\frac{\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}x^{\frac{16}{3}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(b*x**2+a)**(2/3),x)

[Out] -gamma(8/3)*hyper((2/3, 8/3), (11/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2
/3)*x**(16/3)*gamma(11/3))

3.620 $\int \frac{x^7}{\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=80

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2\left(a+bx^2\right)^{2/3}}{4b^4}-\frac{9a\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^4}$$

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2\left(a+bx^2\right)^{2/3}}{4b^4}-\frac{9a\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a^3)/(2*b^4*(a + b*x^2)^(1/3)) + (9*a^2*(a + b*x^2)^(2/3))/(4*b^4) - (9*
a*(a + b*x^2)^(5/3))/(10*b^4) + (3*(a + b*x^2)^(8/3))/(16*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2\right)^{4/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{4/3}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{4/3}}+\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{(a+bx)^{5/3}}{b^3}\right)dx,x,x^2\right)$$
$$= \frac{3a^3}{2b^4\sqrt[3]{a+bx^2}}+\frac{9a^2\left(a+bx^2\right)^{2/3}}{4b^4}-\frac{9a\left(a+bx^2\right)^{5/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{8/3}}{16b^4}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.62

$$\frac{3\left(81a^3+27a^2bx^2-9ab^2x^4+5b^3x^6\right)}{80b^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(81*a^3 + 27*a^2*b*x^2 - 9*a*b^2*x^4 + 5*b^3*x^6))/(80*b^4*(a + b*x^2)^(
1/3))
```

IntegrateAlgebraic [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{3\left(81a^3+27a^2bx^2-9ab^2x^4+5b^3x^6\right)}{80b^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*(81*a^3 + 27*a^2*b*x^2 - 9*a*b^2*x^4 + 5*b^3*x^6))/(80*b^4*(a + b*x^2)^(
1/3))
```

fricas [A] time = 0.88, size = 58, normalized size = 0.72

$$\frac{3\left(5\,b^3x^6-9\,ab^2x^4+27\,a^2bx^2+81\,a^3\right)\left(bx^2+a\right)^{\frac{2}{3}}}{80\left(b^5x^2+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/80*(5*b^3*x^6 - 9*a*b^2*x^4 + 27*a^2*b*x^2 + 81*a^3)*(b*x^2 + a)^(2/3)/(b
^5*x^2 + a*b^4)
```

giac [A] time = 0.59, size = 70, normalized size = 0.88

$$\frac{3\,a^3}{2\left(bx^2+a\right)^{\frac{1}{3}}b^4}+\frac{3\left(5\left(bx^2+a\right)^{\frac{8}{3}}b^{28}-24\left(bx^2+a\right)^{\frac{5}{3}}ab^{28}+60\left(bx^2+a\right)^{\frac{2}{3}}a^2b^{28}\right)}{80\,b^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/2*a^3/((b*x^2 + a)^(1/3)*b^4) + 3/80*(5*(b*x^2 + a)^(8/3)*b^28 - 24*(b*x^
2 + a)^(5/3)*a*b^28 + 60*(b*x^2 + a)^(2/3)*a^2*b^28)/b^32
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$\frac{\frac{3}{16}b^3x^6-\frac{27}{80}a\,b^2x^4+\frac{81}{80}a^2b\,x^2+\frac{243}{80}a^3}{\left(b\,x^2+a\right)^{\frac{1}{3}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2+a)^(4/3),x)
```

```
[Out] 3/80/(b*x^2+a)^(1/3)*(5*b^3*x^6-9*a*b^2*x^4+27*a^2*b*x^2+81*a^3)/b^4
```

maxima [A] time = 1.34, size = 64, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{8}{3}}}{16\,b^4}-\frac{9\left(bx^2+a\right)^{\frac{5}{3}}a}{10\,b^4}+\frac{9\left(bx^2+a\right)^{\frac{2}{3}}a^2}{4\,b^4}+\frac{3\,a^3}{2\left(bx^2+a\right)^{\frac{1}{3}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x^2 + a)^(8/3)/b^4 - 9/10*(b*x^2 + a)^(5/3)*a/b^4 + 9/4*(b*x^2 + a)
^(2/3)*a^2/b^4 + 3/2*a^3/((b*x^2 + a)^(1/3)*b^4)
```

mupad [B] time = 5.44, size = 55, normalized size = 0.69

$$\frac{180\,a^2\,\left(b\,x^2+a\right)-72\,a\left(b\,x^2+a\right)^2+15\left(b\,x^2+a\right)^3+120\,a^3}{80\,b^4\left(b\,x^2+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2)^(4/3),x)
```

```
[Out] (180*a^2*(a + b*x^2) - 72*a*(a + b*x^2)^2 + 15*(a + b*x^2)^3 + 120*a^3)/(80
*b^4*(a + b*x^2)^(1/3))
```

sympy [B] time = 2.84, size = 1584, normalized size = 19.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2+a)**(4/3),x)
```

```
[Out] 243*a**(68/3)*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 +
1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a*
*15*b**9*x**10 + 80*a**14*b**10*x**12) - 243*a**(68/3)/(80*a**20*b**4 + 480
*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16
*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 1296*a**(65/3)*
b*x**2*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a*
*18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**
9*x**10 + 80*a**14*b**10*x**12) - 1458*a**(65/3)*b*x**2/(80*a**20*b**4 + 48
0*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**1
6*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 2808*a**(62/3)
*b**2*x**4*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 120
0*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15
*b**9*x**10 + 80*a**14*b**10*x**12) - 3645*a**(62/3)*b**2*x**4/(80*a**20*b*
*4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 12
00*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 3120*a*
*(59/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**
2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 48
0*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 4860*a**(59/3)*b**3*x**6/(80*a
**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x*
*6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) +
1830*a**(56/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b
**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x*
*8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 3645*a**(56/3)*b**4*x**
8/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*
b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x*
*12) + 528*a**(53/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*
a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*
b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 1458*a**(53/3)*b
**5*x**10/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 160
0*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*
b**10*x**12) + 96*a**(50/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4
 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200
```

```
*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 243*a**(5
0/3)*b**6*x**12/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4
 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*
a**14*b**10*x**12) + 48*a**(47/3)*b**7*x**14*(1 + b*x**2/a)**(2/3)/(80*a**2
0*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6
+ 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 15*
a**(44/3)*b**8*x**16*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*
x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 +
 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12)
```

3.621 $\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=59

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}}-\frac{3a\left(a+bx^2\right)^{2/3}}{2b^3}+\frac{3\left(a+bx^2\right)^{5/3}}{10b^3}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}}-\frac{3a\left(a+bx^2\right)^{2/3}}{2b^3}+\frac{3\left(a+bx^2\right)^{5/3}}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2)^(4/3),x]
```

```
[Out] (-3*a^2)/(2*b^3*(a + b*x^2)^(1/3)) - (3*a*(a + b*x^2)^(2/3))/(2*b^3) + (3*(
a + b*x^2)^(5/3))/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^{4/3}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{4/3}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{4/3}} - \frac{2a}{b^2\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{b^2}\right) dx, x, x^2\right)$$
$$= -\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}} - \frac{3a\left(a+bx^2\right)^{2/3}}{2b^3} + \frac{3\left(a+bx^2\right)^{5/3}}{10b^3}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.64

$$\frac{3\left(-9a^2-3abx^2+b^2x^4\right)}{10b^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2)^(4/3),x]
[Out] (3*(-9*a^2 - 3*a*b*x^2 + b^2*x^4))/(10*b^3*(a + b*x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.66

$$-\frac{3\left(9a^2+3abx^2-b^2x^4\right)}{10b^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(a + b*x^2)^(4/3),x]
[Out] (-3*(9*a^2 + 3*a*b*x^2 - b^2*x^4))/(10*b^3*(a + b*x^2)^(1/3))
```

fricas [A] time = 0.94, size = 46, normalized size = 0.78

$$\frac{3\left(b^2x^4-3\,abx^2-9\,a^2\right)\left(bx^2+a\right)^{\frac{2}{3}}}{10\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] 3/10*(b^2*x^4 - 3*a*b*x^2 - 9*a^2)*(b*x^2 + a)^(2/3)/(b^4*x^2 + a*b^3)
```

giac [A] time = 0.58, size = 52, normalized size = 0.88

$$-\frac{3\,a^2}{2\,\left(bx^2+a\right)^{\frac{1}{3}}b^3}+\frac{3\left(\left(bx^2+a\right)^{\frac{5}{3}}b^{12}-5\left(bx^2+a\right)^{\frac{2}{3}}ab^{12}\right)}{10\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] -3/2*a^2/((b*x^2 + a)^(1/3)*b^3) + 3/10*((b*x^2 + a)^(5/3)*b^12 - 5*(b*x^2
+ a)^(2/3)*a*b^12)/b^15
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$-\frac{3\left(-b^2x^4+3ab\,x^2+9a^2\right)}{10\left(b\,x^2+a\right)^{\frac{1}{3}}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(4/3),x)
```

```
[Out] -3/10/(b*x^2+a)^(1/3)*(-b^2*x^4+3*a*b*x^2+9*a^2)/b^3
```

maxima [A] time = 1.28, size = 47, normalized size = 0.80

$$\frac{3\left(bx^2+a\right)^{\frac{5}{3}}}{10\,b^3}-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}a}{2\,b^3}-\frac{3\,a^2}{2\left(bx^2+a\right)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x^2 + a)^(5/3)/b^3 - 3/2*(b*x^2 + a)^(2/3)*a/b^3 - 3/2*a^2/((b*x^2
+ a)^(1/3)*b^3)
```

mupad [B] time = 5.36, size = 41, normalized size = 0.69

$$-\frac{15\,a\,\left(b\,x^2+a\right)-3\left(b\,x^2+a\right)^2+15\,a^2}{10\,b^3\left(b\,x^2+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2)^(4/3),x)
```

```
[Out] -(15*a*(a + b*x^2) - 3*(a + b*x^2)^2 + 15*a^2)/(10*b^3*(a + b*x^2)^(1/3))
```

sympy [B] time = 1.82, size = 561, normalized size = 9.51

$$-\frac{27a^{\frac{29}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}+\frac{27a^{\frac{29}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}-\frac{63a^{\frac{26}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}+\frac{81a^{\frac{26}{3}}bx^2}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}-\frac{42a^{\frac{23}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}+\frac{81a^{\frac{23}{3}}b^2x^4}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}-\frac{3a^{\frac{20}{3}}b^3x^6\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}+\frac{27a^{\frac{20}{3}}b^3x^6}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}+\frac{3a^{\frac{17}{3}}b^4x^8\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(4/3),x)
```

```
[Out] -27*a**(29/3)*(1 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*
a**6*b**5*x**4 + 10*a**5*b**6*x**6) + 27*a**(29/3)/(10*a**8*b**3 + 30*a**7*
b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 63*a**(26/3)*b*x**2*(1
 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 +
 10*a**5*b**6*x**6) + 81*a**(26/3)*b*x**2/(10*a**8*b**3 + 30*a**7*b**4*x**2
 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 42*a**(23/3)*b**2*x**4*(1 + b*x
**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a*
*5*b**6*x**6) + 81*a**(23/3)*b**2*x**4/(10*a**8*b**3 + 30*a**7*b**4*x**2 +
30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 3*a**(20/3)*b**3*x**6*(1 + b*x**2/
a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b
**6*x**6) + 27*a**(20/3)*b**3*x**6/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a
**6*b**5*x**4 + 10*a**5*b**6*x**6) + 3*a**(17/3)*b**4*x**8*(1 + b*x**2/a)**
(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*
x**6)
```

3.622 $\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=38

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3\left(a+bx^2\right)^{2/3}}{4b^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}}+\frac{3\left(a+bx^2\right)^{2/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a)/(2*b^2*(a + b*x^2)^(1/3)) + (3*(a + b*x^2)^(2/3))/(4*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)^{4/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{4/3}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{4/3}} + \frac{1}{b\sqrt[3]{a+bx}}\right)dx, x, x^2\right)$$
$$= \frac{3a}{2b^2\sqrt[3]{a+bx^2}} + \frac{3\left(a+bx^2\right)^{2/3}}{4b^2}$$

Mathematica [A]   time = 0.01, size = 27, normalized size = 0.71

$$\frac{3\left(3a+bx^2\right)}{4b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2)^(4/3),x]
[Out] (3*(3*a + b*x^2))/(4*b^2*(a + b*x^2)^(1/3))
```

IntegrateAlgebraic [A]   time = 0.03, size = 27, normalized size = 0.71

$$\frac{3\left(3a+bx^2\right)}{4b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x^2)^(4/3),x]
[Out] (3*(3*a + b*x^2))/(4*b^2*(a + b*x^2)^(1/3))
```

fricas [A]   time = 1.10, size = 35, normalized size = 0.92

$$\frac{3\left(bx^2+3\,a\right)\left(bx^2+a\right)^{\frac{2}{3}}}{4\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] 3/4*(b*x^2 + 3*a)*(b*x^2 + a)^(2/3)/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.57, size = 34, normalized size = 0.89

$$\frac{3\left(\frac{\left(bx^2+a\right)^{\frac{2}{3}}}{b} + \frac{2\,a}{\left(bx^2+a\right)^{\frac{1}{3}}b}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/4*((b*x^2 + a)^(2/3)/b + 2*a/((b*x^2 + a)^(1/3)*b))/b
```

maple [A] time = 0.00, size = 24, normalized size = 0.63

$$\frac{\frac{3b\,x^2}{4} + \frac{9a}{4}}{\left(b\,x^2 + a\right)^{\frac{1}{3}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(4/3),x)
```

```
[Out] 3/4/(b*x^2+a)^(1/3)*(b*x^2+3*a)/b^2
```

maxima [A] time = 1.35, size = 30, normalized size = 0.79

$$\frac{3\left(bx^2 + a\right)^{\frac{2}{3}}}{4\,b^2} + \frac{3\,a}{2\left(bx^2 + a\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x^2 + a)^(2/3)/b^2 + 3/2*a/((b*x^2 + a)^(1/3)*b^2)
```

mupad [B] time = 5.59, size = 24, normalized size = 0.63

$$\frac{3\,b\,x^2 + 9\,a}{4\,b^2\left(b\,x^2 + a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(4/3),x)
```

[Out] (9*a + 3*b*x^2)/(4*b^2*(a + b*x^2)^(1/3))

sympy [A] time = 0.70, size = 46, normalized size = 1.21

$$\begin{cases} \frac{9a}{4b^2\sqrt[3]{a+bx^2}} + \frac{3x^2}{4b\sqrt[3]{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**2+a)**(4/3),x)

[Out] Piecewise((9*a/(4*b**2*(a + b*x**2)**(1/3)) + 3*x**2/(4*b*(a + b*x**2)**(1/
3)), Ne(b, 0)), (x**4/(4*a**(4/3)), True))

3.623 $\int \frac{x}{(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=18

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(4/3),x]
```

```
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(a+bx^2)^{4/3}}\,dx = -\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(4/3),x]
```

```
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x^2)^(4/3),x]
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

fricas [A] time = 1.02, size = 24, normalized size = 1.33

$$-\frac{3\left(bx^2+a\right)^{\frac{2}{3}}}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] -3/2*(b*x^2 + a)^(2/3)/(b^2*x^2 + a*b)
```

giac [A] time = 0.58, size = 14, normalized size = 0.78

$$-\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] -3/2/((b*x^2 + a)^(1/3)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{3}{2\left(b\,x^2+a\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(4/3),x)
[Out] -3/2/b/(b*x^2+a)^(1/3)
```

maxima [A] time = 1.32, size = 14, normalized size = 0.78

$$-\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -3/2/((b*x^2 + a)^(1/3)*b)
```

mupad [B] time = 5.39, size = 14, normalized size = 0.78

$$-\frac{3}{2\,b\,\left(b\,x^2+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(4/3),x)
```

```
[Out] -3/(2*b*(a + b*x^2)^(1/3))
```

sympy [A] time = 0.68, size = 26, normalized size = 1.44

$$\begin{cases} -\frac{3}{2b\sqrt[3]{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(4/3),x)
```

```
[Out] Piecewise((-3/(2*b*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**2/(2*a**(4/3)), Tru
e))
```

3.624 $\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=104

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{2a\sqrt[3]{a+bx^2}}$$

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*(a + b*x^2)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3))
/(Sqrt[3]*a^(1/3))])/(2*a^(4/3)) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a
 + b*x^2)^(1/3)])/(4*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{4/3}}\,dx, x, x^2\right)\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{2a}\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}} - \frac{\log(x)}{2a^{4/3}} - \frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{4a^{4/3}} + \frac{3\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x\right.}{4a}\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}} - \frac{\log(x)}{2a^{4/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{2a^{4/3}}\\
&= \frac{3}{2a\sqrt[3]{a+bx^2}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{4/3}} - \frac{\log(x)}{2a^{4/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.35

$$\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx^2}{a}+1\right)}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^(4/3)),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x^2)/a])/(2*a*(a + b*x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.10, size = 136, normalized size = 1.31

$$\frac{\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{2a^{4/3}}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{4a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{2a^{4/3}}+\frac{3}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*(a + b*x^2)^(1/3)) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x^2)^(1/3
))/(Sqrt[3]*a^(1/3))])/(2*a^(4/3)) + Log[-a^(1/3) + (a + b*x^2)^(1/3)]/(2*a
^(4/3)) - Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]/(4*a
^(4/3))
```

fricas [A] time = 1.73, size = 327, normalized size = 3.14

$$\left[\frac{\sqrt{3}\left(abx^2+a^2\right)\sqrt{-\frac{1}{a^{2/3}}}\log\left(\frac{2bx^2+\sqrt{3}\left(2\left(bx^2+a\right)^{2/3}a^{2/3}-\left(bx^2+a\right)^{1/3}a-a^{4/3}\right)\sqrt{-\frac{1}{a^{2/3}}}-3\left(bx^2+a\right)^{1/3}a^{2/3}+3a}{x^2}\right)-\left(bx^2+a\right)a^{2/3}\log\left(\left(bx^2+a\right)^{2/3}+\left(bx^2+a\right)^{1/3}a^{1/3}+a^{2/3}\right)+2\left(bx^2+a\right)a^{2/3}\log\left(\left(bx^2+a\right)^{1/3}-a^{1/3}\right)+6\left(bx^2+a\right)^{2/3}a}{4\left(a^2bx^2+a^3\right)},\ -\frac{\left(bx^2+a\right)a^{2/3}\log\left(\left(bx^2+a\right)^{2/3}+\left(bx^2+a\right)^{1/3}a^{1/3}+a^{2/3}\right)-2\left(bx^2+a\right)a^{2/3}\log\left(\left(bx^2+a\right)^{1/3}-a^{1/3}\right)-\frac{2\sqrt{3}\left(abx^2+a^2\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{1/3}+a^{1/3}\right)}{3a^{1/3}}\right)}{a^{1/3}}-6\left(bx^2+a\right)^{2/3}a}{4\left(a^2bx^2+a^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(a*b*x^2 + a^2)*sqrt(-1/a^(2/3))*log((2*b*x^2 + sqrt(3)*(2*(b
*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - (b*x^2 + a)*a^(2/3)*log((b*x^2 +
 a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x^2 + a)*a^(2/3)*lo
g((b*x^2 + a)^(1/3) - a^(1/3)) + 6*(b*x^2 + a)^(2/3)*a)/(a^2*b*x^2 + a^3),
-1/4*((b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3)
 + a^(2/3)) - 2*(b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) - 2*sq
rt(3)*(a*b*x^2 + a^2)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(1/3) - 6*(b*x^2 + a)^(2/3)*a)/(a^2*b*x^2 + a^3)]
```

giac [A] time = 1.10, size = 101, normalized size = 0.97

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{4}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{4}{3}}}+\frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{4}{3}}}+\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
4/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
4/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/2/((b*x^2 + a)
^(1/3)*a)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{4}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x/(b*x^2+a)^(4/3),x)
```

maxima [A] time = 3.02, size = 100, normalized size = 0.96

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{4}{3}}}-\frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{4}{3}}}+\frac{\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2a^{\frac{4}{3}}}+\frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
4/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
4/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(4/3) + 3/2/((b*x^2 + a)^(1/3
)*a)
```

mupad [B] time = 5.59, size = 123, normalized size = 1.18

$$\frac{\ln\left(18\,a\,\left(b\,x^2+a\right)^{1/3}-18\,a^{4/3}\right)}{2\,a^{4/3}}+\frac{3}{2\,a\,\left(b\,x^2+a\right)^{1/3}}+\frac{\ln\left(18\,a\,\left(b\,x^2+a\right)^{1/3}-\frac{9\,a^{4/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{4/3}}-\frac{\ln\left(18\,a\,\left(b\,x^2+a\right)^{1/3}-\frac{9\,a^{4/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(4/3)),x)
```

```
[Out] log(18*a*(a + b*x^2)^(1/3) - 18*a^(4/3))/(2*a^(4/3)) + 3/(2*a*(a + b*x^2)^(
1/3)) + (log(18*a*(a + b*x^2)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1)^2)/2)*(3^
(1/2)*1i - 1))/(4*a^(4/3)) - (log(18*a*(a + b*x^2)^(1/3) - (9*a^(4/3)*(3^(1
/2)*1i + 1)^2)/2)*(3^(1/2)*1i + 1))/(4*a^(4/3))
```

sympy [C] time = 1.13, size = 41, normalized size = 0.39

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(4/3)*hyper((4/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(4/
3)*x**(8/3)*gamma(7/3))
```

3.625 $\int \frac{1}{x^3\left(a+bx^2\right)^{4/3}}\,dx$

Optimal. Leaf size=123

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}-\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b}{a^2\sqrt[3]{a+bx^2}}-\frac{1}{2ax^2\sqrt[3]{a+bx^2}}$$

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 55, 617, 204, 31}

$$-\frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}-\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}+\frac{3}{2ax^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*x^2*(a + b*x^2)^(1/3)) - (2*(a + b*x^2)^(2/3))/(a^2*x^2) - (2*b*ArcT
an[(a^(1/3) + 2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) +
(2*b*Log[x])/(3*a^(7/3)) - (b*Log[a^(1/3) - (a + b*x^2)^(1/3)])/a^(7/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^{4/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{4/3}}\,dx, x, x^2\right)$$
$$= \frac{3}{2ax^2\sqrt[3]{a+bx^2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{a}$$
$$= \frac{3}{2ax^2\sqrt[3]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^2\right)}{3a^2}$$
$$= \frac{3}{2ax^2\sqrt[3]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2} + \frac{2b\log(x)}{3a^{7/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^2}\right)}{a^{7/3}} - \frac{b\,\text{Sub}}{}$$
$$= \frac{3}{2ax^2\sqrt[3]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2} + \frac{2b\log(x)}{3a^{7/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}} + \frac{(2b)\,\text{Subst}\left(\int -\right.}{}$$
$$= \frac{3}{2ax^2\sqrt[3]{a+bx^2}} - \frac{2\left(a+bx^2\right)^{2/3}}{a^2x^2} - \frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{2b\log(x)}{3a^{7/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+\right.}{a^{7/3}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.30

$$-\frac{3b\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{bx^2}{a}+1\right)}{2a^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^(4/3)),x]
```

```
[Out] (-3*b*Hypergeometric2F1[-1/3, 2, 2/3, 1 + (b*x^2)/a])/(2*a^2*(a + b*x^2)^(1
/3))
```

IntegrateAlgebraic [A] time = 0.18, size = 150, normalized size = 1.22

$$-\frac{2b\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{a}\right)}{3a^{7/3}} + \frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{3a^{7/3}} - \frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{-a-4bx^2}{2a^2x^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^(4/3)),x]
```

```
[Out] (-a - 4*b*x^2)/(2*a^2*x^2*(a + b*x^2)^(1/3)) - (2*b*ArcTan[1/Sqrt[3] + (2*(
a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) - (2*b*Log[-a^(1/3)
 + (a + b*x^2)^(1/3)])/(3*a^(7/3)) + (b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(
1/3) + (a + b*x^2)^(2/3)])/(3*a^(7/3))
```

fricas [B] time = 1.18, size = 453, normalized size = 3.68

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^{2}x^{4}+a^{2}bx^{2}\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx^{2}-3\sqrt{\frac{1}{3}}\left(2\left(bx^{2}+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^{2}+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^{2}+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x^{2}}\right)+2\left(b^{2}x^{4}+abx^{2}\right)(-a)^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}-\left(bx^{2}+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4\left(b^{2}x^{4}+abx^{2}\right)(-a)^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3\left(4abx^{2}+a^{2}\right)\left(bx^{2}+a\right)^{\frac{2}{3}}}{6\left(a^{3}bx^{4}+a^{4}x^{2}\right)}, -\frac{12\sqrt{\frac{1}{3}}\left(ab^{2}x^{4}+a^{2}bx^{2}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-2\left(b^{2}x^{4}+abx^{2}\right)(-a)^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}-\left(bx^{2}+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+4\left(b^{2}x^{4}+abx^{2}\right)(-a)^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3\left(4abx^{2}+a^{2}\right)\left(bx^{2}+a\right)^{\frac{2}{3}}}{6\left(a^{3}bx^{4}+a^{4}x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*sqrt(1/3)*(a*b^2*x^4 + a^2*b*x^2)*sqrt((-a)^(1/3)/a)*log((2*b*x^2 -
 3*sqrt(1/3)*(2*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(
1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + 2
*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(
-a)^(1/3) + (-a)^(2/3)) - 4*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^
(1/3) + (-a)^(1/3)) - 3*(4*a*b*x^2 + a^2)*(b*x^2 + a)^(2/3))/(a^3*b*x^4 + a
^4*x^2), -1/6*(12*sqrt(1/3)*(a*b^2*x^4 + a^2*b*x^2)*sqrt(-(-a)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - 2*(
b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3)) + 4*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(1
/3) + (-a)^(1/3)) + 3*(4*a*b*x^2 + a^2)*(b*x^2 + a)^(2/3))/(a^3*b*x^4 + a^4
*x^2)]
```

giac [A] time = 1.11, size = 134, normalized size = 1.09

$$-\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}+\frac{b\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}}-\frac{2\,b\log\left(\left|\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{7}{3}}}-\frac{4\left(bx^{2}+a\right)b-3\,ab}{2\left(\left(bx^{2}+a\right)^{\frac{4}{3}}-\left(bx^{2}+a\right)^{\frac{1}{3}}a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(7/3) + 1/3*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(7/3) - 2/3*b*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) - 1/2*(4*(b
*x^2 + a)*b - 3*a*b)/(((b*x^2 + a)^(4/3) - (b*x^2 + a)^(1/3)*a)*a^2)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{4}{3}} x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^3/(b*x^2+a)^(4/3),x)
```

maxima [A] time = 3.01, size = 136, normalized size = 1.11

$$-\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\left(bx^{2}+a\right)b-3\,ab}{2\left(\left(bx^{2}+a\right)^{\frac{4}{3}}a^{2}-\left(bx^{2}+a\right)^{\frac{1}{3}}a^{3}\right)}+\frac{b\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}}-\frac{2\,b\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(7/3) - 1/2*(4*(b*x^2 + a)*b - 3*a*b)/((b*x^2 + a)^(4/3)*a^2 - (b*x^2 + a
)^(1/3)*a^3) + 1/3*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(7/3) - 2/3*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(7/3)
```

mupad [B] time = 5.65, size = 178, normalized size = 1.45

$$-\frac{\frac{3\,b}{a}-\frac{4\,b\left(b\,x^{2}+a\right)}{a^{2}}}{2\,a\left(b\,x^{2}+a\right)^{1/3}-2\left(b\,x^{2}+a\right)^{4/3}}-\frac{2\,b\,\ln\left(4\,a^{7/3}\,b^{2}-4\,a^{2}\,b^{2}\left(b\,x^{2}+a\right)^{1/3}\right)}{3\,a^{7/3}}+\frac{\ln\left(a^{7/3}\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^{2}-4\,a^{2}\,b^{2}\left(b\,x^{2}+a\right)^{1/3}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{7/3}}+\frac{\ln\left(a^{7/3}\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^{2}-4\,a^{2}\,b^{2}\left(b\,x^{2}+a\right)^{1/3}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(4/3)),x)
```

```
[Out] (log(a^(7/3)*(b - 3^(1/2)*b*1i)^2 - 4*a^2*b^2*(a + b*x^2)^(1/3))*(b - 3^(1/
2)*b*1i))/(3*a^(7/3)) - (2*b*log(4*a^(7/3)*b^2 - 4*a^2*b^2*(a + b*x^2)^(1/3
)))/(3*a^(7/3)) - ((3*b)/a - (4*b*(a + b*x^2))/a^2)/(2*a*(a + b*x^2)^(1/3)
- 2*(a + b*x^2)^(4/3)) + (log(a^(7/3)*(b + 3^(1/2)*b*1i)^2 - 4*a^2*b^2*(a +
 b*x^2)^(1/3))*(b + 3^(1/2)*b*1i))/(3*a^(7/3))
```

sympy [C] time = 1.35, size = 41, normalized size = 0.33

$$-\frac{\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2b^{\frac{4}{3}}x^{\frac{14}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(7/3)*hyper((4/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(4
/3)*x**(14/3)*gamma(10/3))
```

3.626 $\int \frac{1}{x^5(a+bx^2)^{4/3}}\,dx$

Optimal. Leaf size=159

$$\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{10/3}}+\frac{7b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2}{3a^3\sqrt[3]{a+bx^2}}+\frac{7b}{12a^2x^2\sqrt[3]{a+bx^2}}-\frac{1}{4ax^4\sqrt[3]{a+bx^2}}$$

Rubi [A] time = 0.11, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 55, 617, 204, 31}

$$\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{10/3}}+\frac{7b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2}-\frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4}+\frac{3}{2ax^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^(4/3)),x]
```

```
[Out] 3/(2*a*x^4*(a + b*x^2)^(1/3)) - (7*(a + b*x^2)^(2/3))/(4*a^2*x^4) + (7*b*(a
 + b*x^2)^(2/3))/(3*a^3*x^2) + (7*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^2)^(1/3)
)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) +
(7*b^2*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(6*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^5 \left(a+bx^2\right)^{4/3}} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^3 (a+bx)^{4/3}} \, dx, x, x^2\right)$$

$$= \frac{3}{2ax^4 \sqrt[3]{a+bx^2}} + \frac{7 \,\text{Subst}\left(\int \frac{1}{x^3 \sqrt[3]{a+bx}} \, dx, x, x^2\right)}{2a}$$

$$= \frac{3}{2ax^4 \sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{x^2 \sqrt[3]{a+bx}} \, dx, x, x^2\right)}{3a^2}$$

$$= \frac{3}{2ax^4 \sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} + \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{x \sqrt[3]{a+bx}} \, dx, x, x^2\right)}{9a^3}$$

$$= \frac{3}{2ax^4 \sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} - \frac{7b^2 \log(x)}{9a^{10/3}} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x} \, dx\right.}{6a^{10/3}}$$

$$= \frac{3}{2ax^4 \sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} - \frac{7b^2 \log(x)}{9a^{10/3}} + \frac{7b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx\right.}}{6a^{10/3}}$$

$$= \frac{3}{2ax^4 \sqrt[3]{a+bx^2}} - \frac{7\left(a+bx^2\right)^{2/3}}{4a^2x^4} + \frac{7b\left(a+bx^2\right)^{2/3}}{3a^3x^2} + \frac{7b^2 \tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} - \frac{7b^2 \log(x)}{9a^{10/3}} +$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.25

$$\frac{3b^2 \,{}_2F_1\left(-\frac{1}{3}, 3; \frac{2}{3}; \frac{bx^2}{a}+1\right)}{2a^3 \sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^(4/3)),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-1/3, 3, 2/3, 1 + (b*x^2)/a])/(2*a^3*(a + b*x^2)^(
1/3))
```

IntegrateAlgebraic [A] time = 0.19, size = 169, normalized size = 1.06

$$\frac{7b^2 \log\left(\sqrt[3]{a+bx^2} - \sqrt[3]{a}\right)}{9a^{10/3}} - \frac{7b^2 \log\left(a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + \left(a+bx^2\right)^{2/3}\right)}{18a^{10/3}} + \frac{7b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}} + \frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} + \frac{-3a^2 + 7abx^2 + 28b^2x^4}{12a^3x^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^(4/3)),x]
```

```
[Out] (-3*a^2 + 7*a*b*x^2 + 28*b^2*x^4)/(12*a^3*x^4*(a + b*x^2)^(1/3)) + (7*b^2*A
rcTan[1/Sqrt[3] + (2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1
0/3)) + (7*b^2*Log[-a^(1/3) + (a + b*x^2)^(1/3)])/(9*a^(10/3)) - (7*b^2*Log
[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(18*a^(10/3))
```

fricas [A] time = 1.58, size = 437, normalized size = 2.75

$$\left[\frac{42\sqrt{\frac{1}{3}}\left(ab^{3}x^{6}+a^{2}b^{2}x^{4}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^{2}+3\sqrt{\frac{1}{3}}\left(2\left(bx^{2}+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^{2}+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^{2}}\right)-14\left(b^{3}x^{6}+ab^{2}x^{4}\right)a^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+28\left(b^{3}x^{6}+ab^{2}x^{4}\right)a^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\left(28ab^{2}x^{4}+7a^{2}bx^{2}-3a^{3}\right)\left(bx^{2}+a\right)^{\frac{2}{3}}}{36\left(a^{4}bx^{6}+a^{5}x^{4}\right)},\ -\frac{14\left(b^{3}x^{6}+ab^{2}x^{4}\right)a^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-28\left(b^{3}x^{6}+ab^{2}x^{4}\right)a^{\frac{2}{3}}\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{84\sqrt{\frac{1}{3}}\left(ab^{3}x^{6}+a^{2}b^{2}x^{4}\right)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-3\left(28ab^{2}x^{4}+7a^{2}bx^{2}-3a^{3}\right)\left(bx^{2}+a\right)^{\frac{2}{3}}}{36\left(a^{4}bx^{6}+a^{5}x^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/36*(42*sqrt(1/3)*(a*b^3*x^6 + a^2*b^2*x^4)*sqrt(-1/a^(2/3))*log((2*b*x^2
 + 3*sqrt(1/3)*(2*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3)
)*sqrt(-1/a^(2/3)) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - 14*(b^3*x^6
+ a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3)) + 28*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3))
+ 3*(28*a*b^2*x^4 + 7*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(2/3))/(a^4*b*x^6 + a^
5*x^4), -1/36*(14*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*
x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b
*x^2 + a)^(1/3) - a^(1/3)) - 84*sqrt(1/3)*(a*b^3*x^6 + a^2*b^2*x^4)*arctan(
sqrt(1/3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(28*a*b^2*x^
4 + 7*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(2/3))/(a^4*b*x^6 + a^5*x^4)]
```

giac [A] time = 1.10, size = 154, normalized size = 0.97

$$\frac{7\sqrt{3}b^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{10}{3}}}-\frac{7b^{2}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{10}{3}}}+\frac{7b^{2}\log\left(\left|\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9a^{\frac{10}{3}}}+\frac{3b^{2}}{2\left(bx^{2}+a\right)^{\frac{1}{3}}a^{3}}+\frac{10\left(bx^{2}+a\right)^{\frac{5}{3}}b^{2}-13\left(bx^{2}+a\right)^{\frac{2}{3}}ab^{2}}{12a^{3}b^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(10/3) - 7/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(10/3) + 7/9*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(10/3) +
3/2*b^2/((b*x^2 + a)^(1/3)*a^3) + 1/12*(10*(b*x^2 + a)^(5/3)*b^2 - 13*(b*x^
2 + a)^(2/3)*a*b^2)/(a^3*b^2*x^4)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{4}{3}} x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(4/3),x)
```

```
[Out] int(1/x^5/(b*x^2+a)^(4/3),x)
```

maxima [A] time = 2.97, size = 176, normalized size = 1.11

$$\frac{7\,\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{2}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}}+\frac{28\left(bx^{2}+a\right)^{2}b^{2}-49\left(bx^{2}+a\right)ab^{2}+18\,a^{2}b^{2}}{12\left(\left(bx^{2}+a\right)^{\frac{7}{3}}a^{3}-2\left(bx^{2}+a\right)^{\frac{4}{3}}a^{4}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{5}\right)}-\frac{7\,b^{2}\log\left(\left(bx^{2}+a\right)^{\frac{2}{3}}+\left(bx^{2}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{10}{3}}}+\frac{7\,b^{2}\log\left(\left(bx^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(10/3) + 1/12*(28*(b*x^2 + a)^2*b^2 - 49*(b*x^2 + a)*a*b^2 + 18*a^2*b^2)
/((b*x^2 + a)^(7/3)*a^3 - 2*(b*x^2 + a)^(4/3)*a^4 + (b*x^2 + a)^(1/3)*a^5)
 - 7/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
10/3) + 7/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(10/3)
```

mupad [B] time = 5.67, size = 224, normalized size = 1.41

$$\frac{\frac{3\,b^{2}}{a}-\frac{49\,b^{2}\left(b\,x^{2}+a\right)}{6\,a^{2}}+\frac{14\,b^{2}\left(b\,x^{2}+a\right)^{2}}{3\,a^{3}}}{2\left(b\,x^{2}+a\right)^{7/3}-4\,a\left(b\,x^{2}+a\right)^{4/3}+2\,a^{2}\left(b\,x^{2}+a\right)^{1/3}}+\frac{7\,b^{2}\ln\left(147\,a^{3}\,b^{4}\left(b\,x^{2}+a\right)^{1/3}-147\,a^{10/3}\,b^{4}\right)}{9\,a^{10/3}}+\frac{7\,b^{2}\ln\left(147\,a^{3}\,b^{4}\left(b\,x^{2}+a\right)^{1/3}-147\,a^{10/3}\,b^{4}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}-\frac{7\,b^{2}\ln\left(147\,a^{3}\,b^{4}\left(b\,x^{2}+a\right)^{1/3}-147\,a^{10/3}\,b^{4}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(4/3)),x)
```

```
[Out] ((3*b^2)/a - (49*b^2*(a + b*x^2))/(6*a^2) + (14*b^2*(a + b*x^2)^2)/(3*a^3))
/(2*(a + b*x^2)^(7/3) - 4*a*(a + b*x^2)^(4/3) + 2*a^2*(a + b*x^2)^(1/3)) +
(7*b^2*log(147*a^3*b^4*(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4))/(9*a^(10/3))
 + (7*b^2*log(147*a^3*b^4*(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4*((3^(1/2)*1i)
/2 - 1/2)^2)*((3^(1/2)*1i)/2 - 1/2))/(9*a^(10/3)) - (7*b^2*log(147*a^3*b^4*
(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i
)/2 + 1/2))/(9*a^(10/3))
```

sympy [C] time = 1.62, size = 41, normalized size = 0.26

$$-\frac{\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{20}{3}}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(4/3),x)
```

```
[Out] -gamma(10/3)*hyper((4/3, 10/3), (13/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**
(4/3)*x**(20/3)*gamma(13/3))
```

3.627 $\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=195

$$-\frac{5a^3c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{108b^{8/3}}-\frac{5a^3c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{54\sqrt{3}\,b^{8/3}}-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{(cx)^{16/3}\sqrt[3]{a+bx}}{6c}$$

Rubi [A] time = 0.39, antiderivative size = 275, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}-\frac{5a^3c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{162b^{8/3}}+\frac{5a^3c^{13/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{324b^{8/3}}-\frac{5a^3c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{54\sqrt{3}\,b^{8/3}}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (-5*a^2*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(108*b^2) + (a*c*(c*x)^(10/3)*(a
 + b*x^2)^(1/3))/(36*b) + ((c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*c) - (5*a^3*c
^(13/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[
3]*c^(2/3))])/(54*Sqrt[3]*b^(8/3)) - (5*a^3*c^(13/3)*Log[c^(2/3) - (b^(1/3)
*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(162*b^(8/3)) + (5*a^3*c^(13/3)*Log[c^(4/
3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3)])/(324*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{1}{9}a\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{\left(5a^2c^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{54b}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^4\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)\ldots}}{81b^2}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^3\right)\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^3\right)\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^3\right)\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{\left(5a^3c^{11/3}\right)\text{Subst}\ldots}{\ldots}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{5a^3c^{13/3}\log\left(c^{2/3}\ldots\right.}{162b^{8/\ldots}}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{5a^3c^{13/3}\log\left(c^{2/3}\ldots\right.}{162b^{8/\ldots}}$$

$$= -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} - \frac{5a^3c^{13/3}\tan^{-1}\left(\frac{1\ldots}{-\ldots}\right.}{54\sqrt{3}\,b^{8\ldots}}$$

Mathematica [C] time = 0.07, size = 102, normalized size = 0.52

$$\frac{c^3(cx)^{4/3}\sqrt[3]{a+bx^2}\left(\sqrt[3]{\frac{bx^2}{a}+1}\left(-5a^2+abx^2+6b^2x^4\right)+5a^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)\right)}{36b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-5*a^2 + a*b*x^2
 + 6*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^2)/a)]))/(36
*b^2*(1 + (b*x^2)/a)^(1/3))
```

IntegrateAlgebraic [A] time = 16.55, size = 267, normalized size = 1.37

$$-\frac{5a^3c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{162b^{8/3}}+\frac{5a^3c^{13/3}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{324b^{8/3}}-\frac{5a^3c^{13/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{54\sqrt{3}\,b^{8/3}}+\frac{\sqrt[3]{a+bx^2}\left(-5a^2c^4(cx)^{4/3}+3abc^2(cx)^{10/3}+18b^2(cx)^{16/3}\right)}{108b^2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] ((a + b*x^2)^(1/3)*(-5*a^2*c^4*(c*x)^(4/3) + 3*a*b*c^2*(c*x)^(10/3) + 18*b^
2*(c*x)^(16/3)))/(108*b^2*c) - (5*a^3*c^(13/3)*ArcTan[(Sqrt[3]*b^(1/3)*(c*x
)^(2/3))/(b^(1/3)*(c*x)^(2/3) + 2*c^(2/3)*(a + b*x^2)^(1/3))])/(54*Sqrt[3]*
b^(8/3)) - (5*a^3*c^(13/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1
/3)])/(162*b^(8/3)) + (5*a^3*c^(13/3)*Log[b^(2/3)*(c*x)^(4/3) + b^(1/3)*c^(
2/3)*(c*x)^(2/3)*(a + b*x^2)^(1/3) + c^(4/3)*(a + b*x^2)^(2/3)])/(324*b^(8/
3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{13}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{13}{3}} \left(b\,x^2+a\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
[Out] int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}} (cx)^{\frac{13}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{13/3}\,\left(b\,x^2+a\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(a + b*x^2)^(1/3),x)
[Out] int((c*x)^(13/3)*(a + b*x^2)^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)*(b*x**2+a)**(1/3),x)
[Out] Timed out
```

3.628 $\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=164

$$\frac{a^2c^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{5/3}}+\frac{a^2c^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{5/3}}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}$$

Rubi [A] time = 0.30, antiderivative size = 244, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}}-\frac{a^2c^{7/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{36b^{5/3}}+\frac{a^2c^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{6\sqrt{3}\,b^{5/3}}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (a*c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(12*b) + ((c*x)^(10/3)*(a + b*x^2)^(1/3
))/(4*c) + (a^2*c^(7/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^
2)^(1/3))/(Sqrt[3]*c^(2/3))])/(6*Sqrt[3]*b^(5/3)) + (a^2*c^(7/3)*Log[c^(2/3
) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(18*b^(5/3)) - (a^2*c^(7/3)*L
og[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*
x)^(2/3))/(a + b*x^2)^(1/3)])/(36*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{1}{6}a\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}-\frac{\left(a^2c^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}-\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx,x,\sqrt[3]{cx}\right)}{3b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}-\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx,x,(cx)^{2/3}\right)}{6b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}-\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}-\frac{\left(a^2c^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{4/3}}+\frac{\left(a^2\right.}{}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}}+\frac{\left(a^2c^{5/3}\right)\text{Subst}}{}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{a^2c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{5/3}}-\frac{a^2c^{7/3}\log\left(c^{4/3}\right.}{}$$

$$= \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}+\frac{a^2c^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{5/3}}+\frac{a^2c^{7/3}\log\left(c^{2/3}\right.}{18b^{5/3}}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 0.52

$$\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)\sqrt[3]{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)\right)}{4b\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/3) - a*Hyp
ergeometric2F1[-1/3, 2/3, 5/3, -((b*x^2)/a)]))/(4*b*(1 + (b*x^2)/a)^(1/3))
```

IntegrateAlgebraic [A] time = 11.27, size = 248, normalized size = 1.51

$$\frac{a^2c^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{18b^{5/3}}-\frac{a^2c^{7/3}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{36b^{5/3}}+\frac{a^2c^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{6\sqrt{3}\,b^{5/3}}+\frac{\sqrt[3]{a+bx^2}\left(ac^2(cx)^{4/3}+3b(cx)^{10/3}\right)}{12bc}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] ((a + b*x^2)^(1/3)*(a*c^2*(c*x)^(4/3) + 3*b*(c*x)^(10/3)))/(12*b*c) + (a^2*
c^(7/3)*ArcTan[(Sqrt[3]*b^(1/3)*(c*x)^(2/3))/(b^(1/3)*(c*x)^(2/3) + 2*c^(2/
3)*(a + b*x^2)^(1/3))])/(6*Sqrt[3]*b^(5/3)) + (a^2*c^(7/3)*Log[b^(1/3)*(c*x
)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(18*b^(5/3)) - (a^2*c^(7/3)*Log[b^(2/
3)*(c*x)^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3)*(a + b*x^2)^(1/3) + c^(4/3)*(a
 + b*x^2)^(2/3)])/(36*b^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{7}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)

[Out] int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{1}{3}} (cx)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/3}\,\left(b\,x^2+a\right)^{1/3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(7/3)*(a + b*x^2)^(1/3),x)

[Out] int((c*x)^(7/3)*(a + b*x^2)^(1/3), x)

sympy [C] time = 33.29, size = 46, normalized size = 0.28

$$\frac{\sqrt[3]{a}\,c^{\frac{7}{3}} x^{\frac{10}{3}} \Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| {\frac{bx^{2} e^{i\pi}}{a}} \right)}{2\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(7/3)*(b*x**2+a)**(1/3),x)

[Out] a**(1/3)*c**(7/3)*x**(10/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(8/3))

3.629 $\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx$

Optimal. Leaf size=133

$$-\frac{a\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}-\frac{a\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}b^{2/3}}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

Rubi [A] time = 0.27, antiderivative size = 211, normalized size of antiderivative = 1.59, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {279, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{a\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}}+\frac{a\sqrt[3]{c}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{12b^{2/3}}-\frac{a\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{2\sqrt{3}b^{2/3}}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] ((c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*c) - (a*c^(1/3)*ArcTan[(c^(2/3) + (2*b^(
1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(2*Sqrt[3]*b^(2/3)
) - (a*c^(1/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(6*b
^(2/3)) + (a*c^(1/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3)
+ (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(12*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
```

```
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{1}{3}a\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6\sqrt[3]{b}\sqrt[3]{c}} - \frac{a\,\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/\ldots}}{\sqrt[3]{a+bx\ldots}}\right)}{6\sqrt[3]{b}\sqrt[3]{c}}$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} - \frac{a\,\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4\sqrt[3]{b}\sqrt[3]{c}} + \ldots$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} + \frac{a\sqrt[3]{c}\,\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{12b^{2/3}} + \ldots$$

$$= \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c} - \frac{a\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{2/3}} - \frac{a\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} + \frac{a\sqrt[3]{c}\,\log\left(c^{4/3}\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.42

$$\frac{3x\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)}{4\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] (3*x*(c*x)^(1/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x
^2)/a)])/(4*(1 + (b*x^2)/a)^(1/3))
```

IntegrateAlgebraic [A] time = 1.92, size = 223, normalized size = 1.68

$$-\frac{a\sqrt[3]{c}\,\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{6b^{2/3}}+\frac{a\sqrt[3]{c}\,\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{12b^{2/3}}-\frac{a\sqrt[3]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{2\sqrt{3}\,b^{2/3}}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

```
[Out] ((c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*c) - (a*c^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*
(c*x)^(2/3))/(b^(1/3)*(c*x)^(2/3) + 2*c^(2/3)*(a + b*x^2)^(1/3))])/(2*Sqrt[
3]*b^(2/3)) - (a*c^(1/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3
)])/(6*b^(2/3)) + (a*c^(1/3)*Log[b^(2/3)*(c*x)^(4/3) + b^(1/3)*c^(2/3)*(c*x
)^(2/3)*(a + b*x^2)^(1/3) + c^(4/3)*(a + b*x^2)^(2/3)])/(12*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{1}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
[Out] int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{1}{3}} (cx)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{1/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(a + b*x^2)^(1/3),x)
[Out] int((c*x)^(1/3)*(a + b*x^2)^(1/3), x)
```

sympy [C] time = 1.52, size = 46, normalized size = 0.35

$$\frac{\sqrt[3]{a}\,\sqrt[3]{c}\,x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)*(b*x**2+a)**(1/3),x)
[Out] a**(1/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(5/3))
```

3.630 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx$

Optimal. Leaf size=131

$$-\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{5/3}}-\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2c^{5/3}}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

Rubi [A] time = 0.27, antiderivative size = 208, normalized size of antiderivative = 1.59, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {277, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{4c^{5/3}}-\frac{\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}}-\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{2c^{5/3}}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*c*(c*x)^(2/3)) - (Sqrt[3]*b^(1/3)*ArcTan[(c^(2/3)
 + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(2*c^(5/3
)) - (b^(1/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(2*c^
(5/3)) + (b^(1/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (
b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(4*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
```

```
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{b\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{c^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{(3b)\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} + \frac{b^{2/3}\,\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{7/3}} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{7/3}}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt[3]{b}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} - \frac{\left(3b^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4c^{7/3}} + \cdots$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt[3]{b}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} + \frac{\sqrt[3]{b}\,\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4c^{5/3}} + \frac{\left(3\sqrt[3]{b}\right)\cdots}{\cdots}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{5/3}} - \frac{\sqrt[3]{b}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}} + \frac{\sqrt[3]{b}\,\log\left(c^{4/3}+\frac{b^{2/3}\cdots}{\left(a+b\cdots\right.}\cdots\right.}{4c\cdots}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.43

$$-\frac{3x\sqrt[3]{a+bx^2}\ {}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{bx^2}{a}\right)}{2(cx)^{5/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -((b*x^2)/a)])/(
2*(c*x)^(5/3)*(1 + (b*x^2)/a)^(1/3))
```

IntegrateAlgebraic [A] time = 1.32, size = 220, normalized size = 1.68

$$\frac{\sqrt[3]{b}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{4c^{5/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{2c^{5/3}}-\frac{\sqrt{3}\,\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{2c^{5/3}}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*c*(c*x)^(2/3)) - (Sqrt[3]*b^(1/3)*ArcTan[(Sqrt[3]
*b^(1/3)*(c*x)^(2/3))/(b^(1/3)*(c*x)^(2/3) + 2*c^(2/3)*(a + b*x^2)^(1/3))])
/(2*c^(5/3)) - (b^(1/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)
])/(2*c^(5/3)) + (b^(1/3)*Log[b^(2/3)*(c*x)^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(
2/3)*(a + b*x^2)^(1/3) + c^(4/3)*(a + b*x^2)^(2/3)])/(4*c^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(5/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
[Out] int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/3}}{\left(c\,x\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(5/3),x)
[Out] int((a + b*x^2)^(1/3)/(c*x)^(5/3), x)
```

sympy [C] time = 3.05, size = 49, normalized size = 0.37

$$\frac{\sqrt[3]{a}\,\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(5/3),x)
```

```
[Out] a**(1/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/3)*x**(2/3)*gamma(2/3))
```

3.631 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx$

Optimal. Leaf size=28

$$-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(8/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{3x\left(a+bx^2\right)^{4/3}}{8a(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(4/3))/(8*a*(c*x)^(11/3))
```

IntegrateAlgebraic [A] time = 6.19, size = 28, normalized size = 1.00

$$-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(8/3))
```

fricas [A] time = 1.26, size = 25, normalized size = 0.89

$$-\frac{3\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{8\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="fricas")
```

```
[Out] -3/8*(b*x^2 + a)^(4/3)*(c*x)^(1/3)/(a*c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(11/3), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{3\left(b\,x^2+a\right)^{\frac{4}{3}}x}{8\,(cx)^{\frac{11}{3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(11/3),x)
```

```
[Out] -3/8*x*(b*x^2+a)^(4/3)/a/(c*x)^(11/3)
```

maxima [A] time = 1.45, size = 35, normalized size = 1.25

$$-\frac{3\left(bc^{\frac{1}{3}}x^3 + ac^{\frac{1}{3}}x\right)\left(bx^2 + a\right)^{\frac{1}{3}}}{8\,ac^4x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="maxima")
[Out] -3/8*(b*c^(1/3)*x^3 + a*c^(1/3)*x)*(b*x^2 + a)^(1/3)/(a*c^4*x^(11/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\,x^2+a\right)^{1/3}}{\left(c\,x\right)^{11/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(11/3),x)
[Out] int((a + b*x^2)^(1/3)/(c*x)^(11/3), x)
```

sympy [B] time = 56.59, size = 78, normalized size = 2.79

$$\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{4}{3}\right)}{2c^{\frac{11}{3}}x^2\Gamma\left(-\frac{1}{3}\right)} + \frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{4}{3}\right)}{2ac^{\frac{11}{3}}\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(11/3),x)
[Out] b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*c**(11/3)*x**2*gamma(-1/3))
 + b**(4/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*a*c**(11/3)*gamma(-1/3))
```

3.632 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}} dx$

Optimal. Leaf size=57

$$\frac{9\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{9\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(7/3))/(56*a^2
*c*(c*x)^(14/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx &= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}-\frac{3\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{17/3}}\,dx}{4a}\\ &= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.72

$$\frac{3\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\left(3bx^2-4a\right)}{56a^2c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
```

```
[Out] (3*(c*x)^(1/3)*(a + b*x^2)^(4/3)*(-4*a + 3*b*x^2))/(56*a^2*c^6*x^5)
```

IntegrateAlgebraic [A] time = 17.27, size = 52, normalized size = 0.91

$$\frac{3\sqrt[3]{a+bx^2}\left(-4a^2-abx^2+3b^2x^4\right)}{56a^2c^5x^4(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(-4*a^2 - a*b*x^2 + 3*b^2*x^4))/(56*a^2*c^5*x^4*(c*x)^
(2/3))
```

fricas [A] time = 1.30, size = 46, normalized size = 0.81

$$\frac{3\left(3\,b^2x^4-abx^2-4\,a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{56\,a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="fricas")
```

```
[Out] 3/56*(3*b^2*x^4 - a*b*x^2 - 4*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^6*x
^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)
```

maple [A] time = 0.01, size = 31, normalized size = 0.54

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-3b\,x^{2}+4a\right)x}{56\left(cx\right)^{\frac{17}{3}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(17/3),x)
```

```
[Out] -3/56*x*(b*x^2+a)^(4/3)*(-3*b*x^2+4*a)/a^2/(c*x)^(17/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^{2}+a\right)^{1/3}}{\left(c\,x\right)^{17/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(17/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(17/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(17/3),x)
```

```
[Out] Timed out
```

3.633 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{27\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}$$

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{27\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{20/3}}+\frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(7/3))/(28*a^2
*c*(c*x)^(20/3)) - (27*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(20/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}} - \frac{3\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{23/3}}\,dx}{2a}$$
$$= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}} + \frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}} + \frac{9\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{23/3}}\,dx}{14a^2}$$
$$= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}} + \frac{9\left(a+bx^2\right)^{7/3}}{28a^2c(cx)^{20/3}} - \frac{27\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{20/3}}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.61

$$-\frac{3\sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\left(14a^2-12abx^2+9b^2x^4\right)}{280a^3c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(c*x)^(1/3)*(a + b*x^2)^(4/3)*(14*a^2 - 12*a*b*x^2 + 9*b^2*x^4))/(280*a
^3*c^8*x^7)
```

IntegrateAlgebraic [A] time = 18.12, size = 52, normalized size = 0.61

$$-\frac{3\left(a+bx^2\right)^{4/3}\left(14a^2-12abx^2+9b^2x^4\right)}{280a^3c^7x^6(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3)*(14*a^2 - 12*a*b*x^2 + 9*b^2*x^4))/(280*a^3*c^7*x^6*(
c*x)^(2/3))
```

fricas [A] time = 1.27, size = 57, normalized size = 0.67

$$-\frac{3\left(9\,b^3x^6-3\,ab^2x^4+2\,a^2bx^2+14\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{280\,a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="fricas")
```

```
[Out] -3/280*(9*b^3*x^6 - 3*a*b^2*x^4 + 2*a^2*b*x^2 + 14*a^3)*(b*x^2 + a)^(1/3)*(
c*x)^(1/3)/(a^3*c^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.49

$$-\frac{3\left(b\,x^2+a\right)^{\frac{4}{3}}\left(9b^2x^4-12ab\,x^2+14a^2\right)x}{280\left(cx\right)^{\frac{23}{3}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(23/3),x)
[Out] -3/280*x*(b*x^2+a)^(4/3)*(9*b^2*x^4-12*a*b*x^2+14*a^2)/a^3/(c*x)^(23/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/3}}{\left(c\,x\right)^{23/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(23/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(23/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(23/3),x)
```

```
[Out] Timed out
```

3.634 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx$

Optimal. Leaf size=113

$$\frac{243\left(a+bx^2\right)^{13/3}}{3640a^4c(cx)^{26/3}}-\frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}$$

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{243\left(a+bx^2\right)^{13/3}}{3640a^4c(cx)^{26/3}}-\frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}}+\frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(26/3)) + (27*(a + b*x^2)^(7/3))/(56*a^
2*c*(c*x)^(26/3)) - (81*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(26/3)) + (243
*(a + b*x^2)^(13/3))/(3640*a^4*c*(c*x)^(26/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}} - \frac{9\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{29/3}}\,dx}{4a}$$

$$= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}} + \frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}} + \frac{27\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{29/3}}\,dx}{14a^2}$$

$$= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}} + \frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}} - \frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}} - \frac{81\int \frac{\left(a+bx^2\right)^{10/3}}{(cx)^{29/3}}\,dx}{140a^3}$$

$$= -\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}} + \frac{27\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{26/3}} - \frac{81\left(a+bx^2\right)^{10/3}}{280a^3c(cx)^{26/3}} + \frac{243\left(a+bx^2\right)^{13/3}}{3640a^4c(cx)^{26/3}}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.56

$$\frac{3\left(a+bx^2\right)^{4/3}\left(-140a^3+126a^2bx^2-108ab^2x^4+81b^3x^6\right)}{3640a^4c^9x^8(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(-140*a^3 + 126*a^2*b*x^2 - 108*a*b^2*x^4 + 81*b^3*x^6
))/(3640*a^4*c^9*x^8*(c*x)^(2/3))
```

IntegrateAlgebraic [A] time = 21.08, size = 63, normalized size = 0.56

$$\frac{3\left(a+bx^2\right)^{4/3}\left(-140a^3+126a^2bx^2-108ab^2x^4+81b^3x^6\right)}{3640a^4c^9x^8(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
```

```
[Out] (3*(a + b*x^2)^(4/3)*(-140*a^3 + 126*a^2*b*x^2 - 108*a*b^2*x^4 + 81*b^3*x^6
))/(3640*a^4*c^9*x^8*(c*x)^(2/3))
```

fricas [A] time = 1.22, size = 68, normalized size = 0.60

$$\frac{3\left(81\,b^4x^8-27\,ab^3x^6+18\,a^2b^2x^4-14\,a^3bx^2-140\,a^4\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{3640\,a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="fricas")
```

```
[Out] 3/3640*(81*b^4*x^8 - 27*a*b^3*x^6 + 18*a^2*b^2*x^4 - 14*a^3*b*x^2 - 140*a^4
)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^4*c^10*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{3}}}{(cx)^{\frac{29}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(29/3), x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.47

$$-\frac{3\left(b\,x^2+a\right)^{\frac{4}{3}}\left(-81b^3x^6+108a\,b^2x^4-126a^2b\,x^2+140a^3\right)x}{3640\,(cx)^{\frac{29}{3}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/3)/(c*x)^(29/3),x)
```

```
[Out] -3/3640*x*(b*x^2+a)^(4/3)*(-81*b^3*x^6+108*a*b^2*x^4-126*a^2*b*x^2+140*a^3)
/a^4/(c*x)^(29/3)
```

maxima [A] time = 1.47, size = 64, normalized size = 0.57

$$\frac{3\left(81\,b^4x^9-27\,ab^3x^7+18\,a^2b^2x^5-14\,a^3bx^3-140\,a^4x\right)\left(bx^2+a\right)^{\frac{1}{3}}}{3640\,a^4c^{\frac{29}{3}}\,x^{\frac{29}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="maxima")
```

```
[Out] 3/3640*(81*b^4*x^9 - 27*a*b^3*x^7 + 18*a^2*b^2*x^5 - 14*a^3*b*x^3 - 140*a^4
*x)*(b*x^2 + a)^(1/3)/(a^4*c^(29/3)*x^(29/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{1/3}}{{\left(c\,x\right)}^{29/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/3)/(c*x)^(29/3),x)
```

```
[Out] int((a + b*x^2)^(1/3)/(c*x)^(29/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/3)/(c*x)**(29/3),x)
```

```
[Out] Timed out
```

3.635 $\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=223

$$-\frac{5a^4c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{324b^{8/3}}-\frac{5a^4c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{162\sqrt{3}\,b^{8/3}}-\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a}}{108b}$$

Rubi [A] time = 0.37, antiderivative size = 303, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}-\frac{5a^4c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{486b^{8/3}}+\frac{5a^4c^{13/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{972b^{8/3}}-\frac{5a^4c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{162\sqrt{3}\,b^{8/3}}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (-5*a^3*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(324*b^2) + (a^2*c*(c*x)^(10/3)*
(a + b*x^2)^(1/3))/(108*b) + (a*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(18*c) + ((
c*x)^(16/3)*(a + b*x^2)^(4/3))/(8*c) - (5*a^4*c^(13/3)*ArcTan[(c^(2/3) + (2
*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(162*Sqrt[3]*b
^(8/3)) - (5*a^4*c^(13/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3)])/(486*b^(8/3)) + (5*a^4*c^(13/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/
(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(972*
b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (cx)^{13/3}\left(a+bx^2\right)^{4/3}dx = \frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}+\frac{1}{3}a\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx$$

$$= \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}+\frac{1}{27}a^2\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}-\frac{\left(5a^3c^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{}}}{162b}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+\right.}{8c}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+\right.}{8c}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+\right.}{8c}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+\right.}{8c}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+\right.}{8c}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+\right.}{8c}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+\right.}{8c}$$

$$= -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2}+\frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b}+\frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c}+\frac{(cx)^{16/3}\left(a+\right.}{8c}$$

Mathematica [C] time = 0.08, size = 102, normalized size = 0.46

$$\frac{c^3(cx)^{4/3}\sqrt[3]{a+bx^2}\left(5a^3\,{}_2F_1\left(-\frac{4}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)-\left(5a-9bx^2\right)\left(a+bx^2\right)^2\sqrt[3]{\frac{bx^2}{a}+1}\right)}{72b^2\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3)*(-((5*a - 9*b*x^2)*(a + b*x^2)^2*(1 + (b
*x^2)/a)^(1/3)) + 5*a^3*Hypergeometric2F1[-4/3, 2/3, 5/3, -((b*x^2)/a)]))/(
72*b^2*(1 + (b*x^2)/a)^(1/3))
```

IntegrateAlgebraic [A] time = 16.78, size = 285, normalized size = 1.28

$$-\frac{5a^4c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{486b^{8/3}}+\frac{5a^4c^{13/3}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{972b^{8/3}}-\frac{5a^4c^{13/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{162\sqrt{3}\,b^{8/3}}+\frac{\sqrt[3]{a+bx^2}\left(-10a^3c^6(cx)^{4/3}+6a^2bc^4(cx)^{10/3}+117ab^2c^2(cx)^{16/3}+81b^3(cx)^{22/3}\right)}{648b^2c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] ((a + b*x^2)^(1/3)*(-10*a^3*c^6*(c*x)^(4/3) + 6*a^2*b*c^4*(c*x)^(10/3) + 11
7*a*b^2*c^2*(c*x)^(16/3) + 81*b^3*(c*x)^(22/3)))/(648*b^2*c^3) - (5*a^4*c^(
13/3)*ArcTan[(Sqrt[3]*b^(1/3)*(c*x)^(2/3))/(b^(1/3)*(c*x)^(2/3) + 2*c^(2/3)
*(a + b*x^2)^(1/3))])/(162*Sqrt[3]*b^(8/3)) - (5*a^4*c^(13/3)*Log[b^(1/3)*(
c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(486*b^(8/3)) + (5*a^4*c^(13/3)*Lo
g[b^(2/3)*(c*x)^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3)*(a + b*x^2)^(1/3) + c^(
4/3)*(a + b*x^2)^(2/3)])/(972*b^(8/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{13}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{13}{3}} \left(b\,x^2+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
[Out] int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{13}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,x\right)}^{13/3}\,{\left(b\,x^2+a\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)*(a + b*x^2)^(4/3),x)
[Out] int((c*x)^(13/3)*(a + b*x^2)^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)*(b*x**2+a)**(4/3),x)
[Out] Timed out
```

3.636 $\int (cx)^{7/3}\left(a+bx^2\right)^{4/3}\,dx$

Optimal. Leaf size=192

$$\frac{a^3c^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{27b^{5/3}}+\frac{2a^3c^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{27\sqrt{3}\,b^{5/3}}+\frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+$$

Rubi [A] time = 0.32, antiderivative size = 272, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {279, 321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{2a^3c^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{81b^{5/3}}-\frac{a^3c^{7/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{81b^{5/3}}+\frac{2a^3c^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{27\sqrt{3}\,b^{5/3}}+\frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (a^2*c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(27*b) + (a*(c*x)^(10/3)*(a + b*x^2)^
(1/3))/(9*c) + ((c*x)^(10/3)*(a + b*x^2)^(4/3))/(6*c) + (2*a^3*c^(7/3)*ArcT
an[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))]
)/(27*Sqrt[3]*b^(5/3)) + (2*a^3*c^(7/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))
/(a + b*x^2)^(1/3)])/(81*b^(5/3)) - (a^3*c^(7/3)*Log[c^(4/3) + (b^(2/3)*(c*
x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3)])/(81*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (cx)^{7/3}\left(a+bx^2\right)^{4/3}\,dx = \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{1}{9}(4a)\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx$$

$$= \frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{1}{27}\left(2a^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}-\frac{\left(4a^3c^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}}{81b}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}-\frac{\left(4a^3c\right)\text{Subst}\left(\int \right.}{2}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}-\frac{\left(2a^3c\right)\text{Subst}\left(\int \right.}{2}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}-\frac{\left(2a^3c\right)\text{Subst}\left(\int \right.}{27}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}-\frac{\left(2a^3c^{5/3}\right)\text{Subst}\left(\int \right.}{8}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{2a^3c^{7/3}\log\left(c^{2/3}-\right.}{81b^{5/3}}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{2a^3c^{7/3}\log\left(c^{2/3}-\right.}{81b^{5/3}}$$

$$= \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c}+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{2a^3c^{7/3}\tan^{-1}\left(\right.}{27\sqrt{3}\,b^{5/3}}$$

Mathematica [C] time = 0.07, size = 89, normalized size = 0.46

$$\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt[3]{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{4}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)\right)}{6b\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3)*((a + b*x^2)^2*(1 + (b*x^2)/a)^(1/3) - a^2
*Hypergeometric2F1[-4/3, 2/3, 5/3, -((b*x^2)/a)]))/(6*b*(1 + (b*x^2)/a)^(1/
3))
```

IntegrateAlgebraic [A] time = 16.38, size = 267, normalized size = 1.39

$$\frac{2a^3c^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{81b^{5/3}}-\frac{a^3c^{7/3}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{81b^{5/3}}+\frac{2a^3c^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{27\sqrt{3}\,b^{5/3}}+\frac{\sqrt[3]{a+bx^2}\left(2a^2c^4(cx)^{4/3}+15abc^2(cx)^{10/3}+9b^2(cx)^{16/3}\right)}{54bc^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] ((a + b*x^2)^(1/3)*(2*a^2*c^4*(c*x)^(4/3) + 15*a*b*c^2*(c*x)^(10/3) + 9*b^2
*(c*x)^(16/3)))/(54*b*c^3) + (2*a^3*c^(7/3)*ArcTan[(Sqrt[3]*b^(1/3)*(c*x)^(
2/3))/(b^(1/3)*(c*x)^(2/3) + 2*c^(2/3)*(a + b*x^2)^(1/3))])/(27*Sqrt[3]*b^(
5/3)) + (2*a^3*c^(7/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)]
)/(81*b^(5/3)) - (a^3*c^(7/3)*Log[b^(2/3)*(c*x)^(4/3) + b^(1/3)*c^(2/3)*(c*
x)^(2/3)*(a + b*x^2)^(1/3) + c^(4/3)*(a + b*x^2)^(2/3)])/(81*b^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{7}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

```
[Out] int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{4}{3}} (cx)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{7/3}\,\left(b\,x^2+a\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)*(a + b*x^2)^(4/3),x)
```

```
[Out] int((c*x)^(7/3)*(a + b*x^2)^(4/3), x)
```

sympy [C] time = 64.86, size = 46, normalized size = 0.24

$$\frac{a^{\frac{4}{3}} c^{\frac{7}{3}} x^{\frac{10}{3}} \Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{4}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\right| \frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)*(b*x**2+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**(7/3)*x**(10/3)*gamma(5/3)*hyper((-4/3, 5/3), (8/3,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(8/3))
```

3.637 $\int \sqrt[3]{cx} \left(a + bx^2\right)^{4/3} \, dx$

Optimal. Leaf size=163

$$-\frac{a^2 \sqrt[3]{c} \log \left(\sqrt[3]{b} (cx)^{2/3} - c^{2/3} \sqrt[3]{a + bx^2}\right)}{6b^{2/3}} - \frac{a^2 \sqrt[3]{c} \tan^{-1}\left(\frac{\frac{2 \sqrt[3]{b} (cx)^{2/3}}{c^{2/3} \sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\, b^{2/3}} + \frac{a(cx)^{4/3} \sqrt[3]{a + bx^2}}{3c} + \frac{(cx)^{4/3} \left(a + bx^2\right)^{4/3}}{4c}$$

Rubi [A] time = 0.29, antiderivative size = 243, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {279, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{a^2 \sqrt[3]{c} \log \left(c^{2/3} - \frac{\sqrt[3]{b} (cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}} + \frac{a^2 \sqrt[3]{c} \log \left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}} + \frac{\sqrt[3]{b}\, c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{18b^{2/3}} - \frac{a^2 \sqrt[3]{c} \tan^{-1}\left(\frac{\frac{2 \sqrt[3]{b} (cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\, c^{2/3}}\right)}{3\sqrt{3}\, b^{2/3}} + \frac{a(cx)^{4/3} \sqrt[3]{a + bx^2}}{3c} + \frac{(cx)^{4/3} \left(a + bx^2\right)^{4/3}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (a*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(3*c) + ((c*x)^(4/3)*(a + b*x^2)^(4/3))/(
4*c) - (a^2*c^(1/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3))/(Sqrt[3]*c^(2/3))])/(3*Sqrt[3]*b^(2/3)) - (a^2*c^(1/3)*Log[c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(9*b^(2/3)) + (a^2*c^(1/3)*Log[c^
(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2
/3))/(a + b*x^2)^(1/3)])/(18*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
```

```
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}dx=\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{1}{3}(2a)\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx$$

$$=\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{1}{9}\left(2a^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}dx$$

$$=\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}dx,x,\sqrt[3]{cx}\right)}{3c}$$

$$=\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}dx,x,(cx)^{2/3}\right)}{3c}$$

$$=\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c}$$

$$=\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}dx,x,\frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9\sqrt[3]{b}\sqrt[3]{c}}-\frac{a^2\,\text{Subs}}{}$$

$$=\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}}-\frac{a^2\,\text{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$=\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}}+\frac{a^2\sqrt[3]{c}\log\left(c^{4/3}\right.}{}$$

$$=\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}-\frac{a^2\sqrt[3]{c}\log\left(c^{2/3}\right.}{9b^{2/3}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.35

$$\frac{3ax\sqrt[3]{cx}\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^2}{a}\right)}{4\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] (3*a*x*(c*x)^(1/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 2/3, 5/3, -((b
*x^2)/a)])/(4*(1 + (b*x^2)/a)^(1/3))
```

IntegrateAlgebraic [A] time = 4.76, size = 246, normalized size = 1.51

$$-\frac{a^2\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{9b^{2/3}}+\frac{a^2\sqrt[3]{c}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{18b^{2/3}}-\frac{a^2\sqrt[3]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}(cx)^{2/3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{\sqrt[3]{a+bx^2}\left(7ac^2(cx)^{4/3}+3b(cx)^{10/3}\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

```
[Out] ((a + b*x^2)^(1/3)*(7*a*c^2*(c*x)^(4/3) + 3*b*(c*x)^(10/3)))/(12*c^3) - (a^
2*c^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*(c*x)^(2/3))/(b^(1/3)*(c*x)^(2/3) + 2*c^(
2/3)*(a + b*x^2)^(1/3))])/(3*Sqrt[3]*b^(2/3)) - (a^2*c^(1/3)*Log[b^(1/3)*(c
*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(9*b^(2/3)) + (a^2*c^(1/3)*Log[b^(2
/3)*(c*x)^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3)*(a + b*x^2)^(1/3) + c^(4/3)*(
a + b*x^2)^(2/3)])/(18*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(1/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{1}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
[Out] int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{4}{3}} (cx)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)*(c*x)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{1/3}\,\left(b\,x^2+a\right)^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)*(a + b*x^2)^(4/3),x)
[Out] int((c*x)^(1/3)*(a + b*x^2)^(4/3), x)
```

sympy [C] time = 7.09, size = 46, normalized size = 0.28

$$\frac{a^{\frac{4}{3}} \sqrt[3]{c} x^{\frac{4}{3}} \Gamma\left(\frac{2}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{2 \Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)*(b*x**2+a)**(4/3),x)
[Out] a**(4/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-4/3, 2/3), (5/3,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(5/3))
```

3.638 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx$

Optimal. Leaf size=153

$$-\frac{a\sqrt[3]{b}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{c^{5/3}}-\frac{2a\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}}+\frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3}-\frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}}$$

Rubi [A] time = 0.29, antiderivative size = 233, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {277, 279, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{a\sqrt[3]{b}\log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{3c^{5/3}}+\frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3}-\frac{2a\sqrt[3]{b}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}}-\frac{2a\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{\sqrt{3}c^{5/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

```
[Out] (2*b*(c*x)^(4/3)*(a + b*x^2)^(1/3))/c^3 - (3*(a + b*x^2)^(4/3))/(2*c*(c*x)^
(2/3)) - (2*a*b^(1/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)
^(1/3))/(Sqrt[3]*c^(2/3))])/(Sqrt[3]*c^(5/3)) - (2*a*b^(1/3)*Log[c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(3*c^(5/3)) + (a*b^(1/3)*Log[c^(4
/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3
))/(a + b*x^2)^(1/3)])/(3*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{5/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(4b)\int \sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,dx}{c^2}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(4ab)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{3c^2}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(4ab)\,\mathrm{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{c^3}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} + \frac{\left(2ab^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{7/3}} - \frac{\left(2ab^{2/3}\right)\mathrm{Su}}{}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}} - \frac{\left(ab^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\right.}{c^{7/3}}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}} + \frac{a\sqrt[3]{b}\,\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4}}{\left(a+bx^2\right)^{2}}\right.}{3c^{5/3}}$$

$$= \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}} - \frac{2a\sqrt[3]{b}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2}}{\sqrt[3]{a+bx^2}}\right.}{3c^{5/3}}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.37

$$-\frac{3ax\sqrt[3]{a+bx^2}\;{}_2F_1\left(-\frac{4}{3},-\frac{1}{3};\frac{2}{3};-\frac{bx^2}{a}\right)}{2(cx)^{5/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/3, 2/3, -((b*x^2)/a)])
/(2*(c*x)^(5/3)*(1 + (b*x^2)/a)^(1/3))
```

IntegrateAlgebraic [A] time = 1.53, size = 236, normalized size = 1.54

$$\frac{a\sqrt[3]{b}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{3c^{5/3}}-\frac{2a\sqrt[3]{b}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{3c^{5/3}}-\frac{2a\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{\sqrt{3}\,c^{5/3}}+\frac{\sqrt[3]{a+bx^2}\left(bc^2x^2-3ac^2\right)}{2c^3(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

```
[Out] ((a + b*x^2)^(1/3)*(-3*a*c^2 + b*c^2*x^2))/(2*c^3*(c*x)^(2/3)) - (2*a*b^(1/
3)*ArcTan[(Sqrt[3]*b^(1/3)*(c*x)^(2/3))/(b^(1/3)*(c*x)^(2/3) + 2*c^(2/3)*(a
 + b*x^2)^(1/3))])/(Sqrt[3]*c^(5/3)) - (2*a*b^(1/3)*Log[b^(1/3)*(c*x)^(2/3)
 - c^(2/3)*(a + b*x^2)^(1/3)])/(3*c^(5/3)) + (a*b^(1/3)*Log[b^(2/3)*(c*x)^(
4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3)*(a + b*x^2)^(1/3) + c^(4/3)*(a + b*x^2)^
(2/3)])/(3*c^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(5/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
[Out] int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{\left(c\,x\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(5/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(5/3), x)
```

sympy [C] time = 7.23, size = 49, normalized size = 0.32

$$\frac{a^{\frac{4}{3}}\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(5/3),x)
```

```
[Out] a**(4/3)*gamma(-1/3)*hyper((-4/3, -1/3), (2/3,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/3)*x**(2/3)*gamma(2/3))
```

3.639 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx$

Optimal. Leaf size=157

$$-\frac{3b^{4/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{11/3}}-\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2c^{11/3}}-\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}}$$

Rubi [A] time = 0.29, antiderivative size = 234, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {277, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}}+\frac{b^{4/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{4c^{11/3}}-\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{2c^{11/3}}-\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*b*(a + b*x^2)^(1/3))/(2*c^3*(c*x)^(2/3)) - (3*(a + b*x^2)^(4/3))/(8*c*(
c*x)^(8/3)) - (Sqrt[3]*b^(4/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a
 + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(2*c^(11/3)) - (b^(4/3)*Log[c^(2/3) -
(b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(2*c^(11/3)) + (b^(4/3)*Log[c^(4/
3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3)
)/(a + b*x^2)^(1/3)])/(4*c^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{11/3}}\,dx = -\frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{b\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx}{c^2}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{b^2\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{c^4}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{\left(3b^2\right)\mathrm{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{\left(3b^2\right)\mathrm{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{\left(3b^2\right)\mathrm{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^5}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} + \frac{b^{5/3}\mathrm{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{13/3}} - \frac{b^{5/3}\mathrm{Subst}\left(\int \frac{1-}{1+\frac{\sqrt[3]{b}}{c^{2/3}}}\right.}{2c}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} - \frac{\left(3b^{5/3}\right)\mathrm{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\right.}{4c^{13/3}}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} + \frac{b^{4/3}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\sqrt[3]{b}\right.}{4c^{11/3}}$$

$$= -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}} - \frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{11/3}} - \frac{b^{4/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} +$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.36

$$-\frac{3ax\sqrt[3]{a+bx^2}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};-\frac{bx^2}{a}\right)}{8(cx)^{11/3}\sqrt[3]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -4/3, -1/3, -((b*x^2)/a)]
)/(8*(c*x)^(11/3)*(1 + (b*x^2)/a)^(1/3))
```

IntegrateAlgebraic [A] time = 1.91, size = 235, normalized size = 1.50

$$-\frac{b^{4/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{2c^{11/3}}+\frac{b^{4/3}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{4c^{11/3}}-\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{2c^{11/3}}-\frac{3\sqrt[3]{a+bx^2}\,\left(ac^2+5bc^2x^2\right)}{8c^3(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3)*(a*c^2 + 5*b*c^2*x^2))/(8*c^3*(c*x)^(8/3)) - (Sqrt[3]
*b^(4/3)*ArcTan[(Sqrt[3]*b^(1/3)*(c*x)^(2/3))/(b^(1/3)*(c*x)^(2/3) + 2*c^(2
/3)*(a + b*x^2)^(1/3))])/(2*c^(11/3)) - (b^(4/3)*Log[b^(1/3)*(c*x)^(2/3) -
c^(2/3)*(a + b*x^2)^(1/3)])/(2*c^(11/3)) + (b^(4/3)*Log[b^(2/3)*(c*x)^(4/3)
 + b^(1/3)*c^(2/3)*(c*x)^(2/3)*(a + b*x^2)^(1/3) + c^(4/3)*(a + b*x^2)^(2/3
)])/(4*c^(11/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
[Out] int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{\left(c\,x\right)^{11/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(11/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(11/3), x)
```

sympy [C] time = 56.37, size = 53, normalized size = 0.34

$$\frac{a^{\frac{4}{3}}\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{4}{3}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{11}{3}}x^{\frac{8}{3}}\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(11/3),x)
```

```
[Out] a**(4/3)*gamma(-4/3)*hyper((-4/3, -4/3), (-1/3,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(11/3)*x**(8/3)*gamma(-1/3))
```

3.640 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx$

Optimal. Leaf size=28

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(14/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx = -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{3x(a+bx^2)^{7/3}}{14a(cx)^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*x*(a + b*x^2)^(7/3))/(14*a*(c*x)^(17/3))
```

IntegrateAlgebraic [A] time = 12.45, size = 31, normalized size = 1.11

$$-\frac{3\left(a+bx^2\right)^{7/3}}{14ac^5x^4(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c^5*x^4*(c*x)^(2/3))
```

fricas [A] time = 1.83, size = 43, normalized size = 1.54

$$-\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{14\,ac^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="fricas")
```

```
[Out] -3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a*c^6*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{3\left(b\,x^2+a\right)^{\frac{7}{3}}x}{14\,(cx)^{\frac{17}{3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(17/3),x)
```

```
[Out] -3/14*x*(b*x^2+a)^(7/3)/a/(c*x)^(17/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{(c\,x)^{17/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(17/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(17/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(17/3),x)
[Out] Timed out
```

3.641 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx$

Optimal. Leaf size=57

$$\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(10/3))/(140*
a^2*c*(c*x)^(20/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx &= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}-\frac{3\int \frac{(a+bx^2)^{7/3}}{(cx)^{23/3}}\,dx}{7a}\\ &= -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}+\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.72

$$\frac{3\sqrt[3]{cx}\,\left(a+bx^2\right)^{7/3}\left(3bx^2-7a\right)}{140a^2c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
```

```
[Out] (3*(c*x)^(1/3)*(a + b*x^2)^(7/3)*(-7*a + 3*b*x^2))/(140*a^2*c^8*x^7)
```

IntegrateAlgebraic [A] time = 18.85, size = 41, normalized size = 0.72

$$\frac{3\left(a+bx^2\right)^{7/3}\left(3bx^2-7a\right)}{140a^2c^7x^6(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
```

```
[Out] (3*(a + b*x^2)^(7/3)*(-7*a + 3*b*x^2))/(140*a^2*c^7*x^6*(c*x)^(2/3))
```

fricas [A] time = 1.76, size = 57, normalized size = 1.00

$$\frac{3\left(3\,b^3x^6-ab^2x^4-11\,a^2bx^2-7\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{140\,a^2c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="fricas")
```

```
[Out] 3/140*(3*b^3*x^6 - a*b^2*x^4 - 11*a^2*b*x^2 - 7*a^3)*(b*x^2 + a)^(1/3)*(c*x
)^(1/3)/(a^2*c^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)
```

maple [A] time = 0.01, size = 31, normalized size = 0.54

$$-\frac{3 \left(b\,x^{2}+a\right)^{\frac{7}{3}} \left(-3 b\,x^{2}+7 a\right) x}{140 \left(c x\right)^{\frac{23}{3}} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(23/3),x)
[Out] -3/140*x*(b*x^2+a)^(7/3)*(-3*b*x^2+7*a)/a^2/(c*x)^(23/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{\left(c\,x\right)^{23/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(23/3),x)
[Out] int((a + b*x^2)^(4/3)/(c*x)^(23/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(23/3),x)
[Out] Timed out
```

3.642 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{27\left(a+bx^2\right)^{13/3}}{910a^3c(cx)^{26/3}}+\frac{9\left(a+bx^2\right)^{10/3}}{70a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}$$

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{27\left(a+bx^2\right)^{13/3}}{910a^3c(cx)^{26/3}}+\frac{9\left(a+bx^2\right)^{10/3}}{70a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(26/3)) + (9*(a + b*x^2)^(10/3))/(70*a
^2*c*(c*x)^(26/3)) - (27*(a + b*x^2)^(13/3))/(910*a^3*c*(c*x)^(26/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{29/3}}\,dx = -\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}} - \frac{6\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{29/3}}\,dx}{7a}$$

$$= -\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}} + \frac{9\left(a+bx^2\right)^{10/3}}{70a^2c(cx)^{26/3}} + \frac{9\int \frac{\left(a+bx^2\right)^{10/3}}{(cx)^{29/3}}\,dx}{35a^2}$$

$$= -\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}} + \frac{9\left(a+bx^2\right)^{10/3}}{70a^2c(cx)^{26/3}} - \frac{27\left(a+bx^2\right)^{13/3}}{910a^3c(cx)^{26/3}}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.61

$$-\frac{3\left(a+bx^2\right)^{7/3}\left(35a^2-21abx^2+9b^2x^4\right)}{910a^3c^9x^8(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

```
[Out] (-3*(a + b*x^2)^(7/3)*(35*a^2 - 21*a*b*x^2 + 9*b^2*x^4))/(910*a^3*c^9*x^8*(
c*x)^(2/3))
```

IntegrateAlgebraic [F] time = 9.62, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{29/3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^2)^(4/3)/(c*x)^(29/3), x]
```

fricas [A] time = 1.72, size = 68, normalized size = 0.80

$$-\frac{3\left(9\,b^4x^8-3\,ab^3x^6+2\,a^2b^2x^4+49\,a^3bx^2+35\,a^4\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{910\,a^3c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="fricas")
```

```
[Out] -3/910*(9*b^4*x^8 - 3*a*b^3*x^6 + 2*a^2*b^2*x^4 + 49*a^3*b*x^2 + 35*a^4)*(b
*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^3*c^10*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{29}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)
```

maple [A] time = 0.00, size = 42, normalized size = 0.49

$$-\frac{3\left(b\,x^2+a\right)^{\frac{7}{3}}\left(9b^2x^4-21ab\,x^2+35a^2\right)x}{910\,(cx)^{\frac{29}{3}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(4/3)/(c*x)^(29/3),x)
[Out] -3/910*x*(b*x^2+a)^(7/3)*(9*b^2*x^4-21*a*b*x^2+35*a^2)/a^3/(c*x)^(29/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{4}{3}}}{(cx)^{\frac{29}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{4/3}}{(c\,x)^{29/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(4/3)/(c*x)^(29/3),x)
```

```
[Out] int((a + b*x^2)^(4/3)/(c*x)^(29/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(4/3)/(c*x)**(29/3),x)
```

```
[Out] Timed out
```

3.643 $\int \frac{(cx)^{19/3}}{\left(a+bx^2\right)^{2/3}} \, dx$

Optimal. Leaf size=198

$$\frac{10a^3c^{19/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{27b^{11/3}}+\frac{20a^3c^{19/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{27\sqrt{3}\,b^{11/3}}+\frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3}-\frac{2ac^3(cx)^{10/3}}{9b^2}$$

Rubi [A] time = 0.34, antiderivative size = 278, normalized size of antiderivative = 1.40, number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3}+\frac{20a^3c^{19/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{81b^{11/3}}-\frac{10a^3c^{19/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{81b^{11/3}}+\frac{20a^3c^{19/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{27\sqrt{3}\,b^{11/3}}-\frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2}+\frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (10*a^2*c^5*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(27*b^3) - (2*a*c^3*(c*x)^(10/3)
*(a + b*x^2)^(1/3))/(9*b^2) + (c*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*b) + (2
0*a^3*c^(19/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))
/(Sqrt[3]*c^(2/3))])/(27*Sqrt[3]*b^(11/3)) + (20*a^3*c^(19/3)*Log[c^(2/3) -
 (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(81*b^(11/3)) - (10*a^3*c^(19/3)
*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(
c*x)^(2/3))/(a + b*x^2)^(1/3)])/(81*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(cx)^{19/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(8ac^2\right)\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b}$$

$$= -\frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{\left(20a^2c^4\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{27b^2}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(40a^3c^6\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}}{81b^3}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(40a^3c^5\right)\text{Subst}\left(\int \cdots\right)}{27\cdots}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(20a^3c^5\right)\text{Subst}\left(\int \cdots\right)}{2\cdots}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(20a^3c^5\right)\text{Subst}\left(\int \cdots\right)}{27\cdots}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{\left(20a^3c^{17/3}\right)\text{Subst}\left(\cdots\right)}{81\cdots}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\log\left(c^{2/3} - \cdots\right)}{81b^{11/3}}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\log\left(c^{2/3} - \cdots\right)}{81b^{11/3}}$$

$$= \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\tan^{-1}\left(\frac{1+\cdots}{\cdots}\right)}{27\sqrt{3}\,b^{11/3}}$$

Mathematica [C] time = 0.03, size = 87, normalized size = 0.44

$$\frac{c^5(cx)^{4/3}\left(-20a^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)+20a^3+8a^2bx^2-3ab^2x^4+9b^3x^6\right)}{54b^3\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^5*(c*x)^(4/3)*(20*a^3 + 8*a^2*b*x^2 - 3*a*b^2*x^4 + 9*b^3*x^6 - 20*a^3*H
ypergeometric2F1[2/3, 1, 5/3, (b*x^2)/(a + b*x^2)]))/(54*b^3*(a + b*x^2)^(2
/3))
```

IntegrateAlgebraic [A] time = 14.16, size = 243, normalized size = 1.23

$$\frac{c^{2/3}x^{2/3}\left(\frac{20a^3c^{19/3}\log\left(\sqrt[3]{a+bx^2}-\sqrt[3]{b}\,x^{2/3}\right)}{81b^{11/3}}-\frac{10a^3c^{19/3}\log\left(\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,x^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}x^{4/3}\right)}{81b^{11/3}}+\frac{20a^3c^{19/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x^{2/3}}{2\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,x^{2/3}}\right)}{27\sqrt{3}\,b^{11/3}}+\frac{c^{19/3}\sqrt[3]{a+bx^2}\left(20a^2x^{4/3}-12abx^{10/3}+9b^2x^{16/3}\right)}{54b^3}\right)}{(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^(2/3)*x^(2/3)*((c^(19/3)*(a + b*x^2)^(1/3)*(20*a^2*x^(4/3) - 12*a*b*x^(1
0/3) + 9*b^2*x^(16/3)))/(54*b^3) + (20*a^3*c^(19/3)*ArcTan[(Sqrt[3]*b^(1/3)
*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))])/(27*Sqrt[3]*b^(11/3)) +
 (20*a^3*c^(19/3)*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)])/(81*b^(11/3)
) - (10*a^3*c^(19/3)*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3
) + (a + b*x^2)^(2/3)])/(81*b^(11/3))))/(c*x)^(2/3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{19}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{19}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
[Out] int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{19}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{19/3}}{\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(19/3)/(a + b*x^2)^(2/3),x)
[Out] int((c*x)^(19/3)/(a + b*x^2)^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(19/3)/(b*x**2+a)**(2/3),x)
[Out] Timed out
```

3.644 $\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=167

$$-\frac{5a^2c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{8/3}}-\frac{5a^2c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{8/3}}-\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2}+\frac{c(cx)^{10/3}\sqrt[3]{a+b}}{4b}$$

Rubi [A] time = 0.29, antiderivative size = 247, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}}+\frac{5a^2c^{13/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{36b^{8/3}}-\frac{5a^2c^{13/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{6\sqrt{3}\,b^{8/3}}-\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2}+\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (-5*a*c^3*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(12*b^2) + (c*(c*x)^(10/3)*(a + b*
x^2)^(1/3))/(4*b) - (5*a^2*c^(13/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3
))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(6*Sqrt[3]*b^(8/3)) - (5*a^2*c^(1
3/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(18*b^(8/3)) +
 (5*a^2*c^(13/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3) + (b
^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(36*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{\left(5ac^2\right)\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx}{6b}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^4\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{9b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^3\right)\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{3b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^3\right)\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{6b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^3\right)\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^2}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} + \frac{\left(5a^2c^{11/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{7/3}} - \cdots$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}} - \frac{\left(5a^2c^{11/3}\right)\text{Su}\cdots}{\cdots}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{18b^{8/3}} + \frac{5a^2c^{13/3}\log\left(\cdots\right.}{\cdots}$$

$$= -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}\,b^{8/3}} - \frac{5a^2c^{13/3}\log\left(c\cdots\right.}{18\cdots}$$

Mathematica [C] time = 0.02, size = 76, normalized size = 0.46

$$\frac{c^3(cx)^{4/3}\left(5a^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)-5a^2-2abx^2+3b^2x^4\right)}{12b^2\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c^3*(c*x)^(4/3)*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Hypergeometric2F1[
2/3, 1, 5/3, (b*x^2)/(a + b*x^2)]))/(12*b^2*(a + b*x^2)^(2/3))
```

IntegrateAlgebraic [A] time = 16.85, size = 247, normalized size = 1.48

$$-\frac{5a^2c^{13/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{18b^{8/3}}+\frac{5a^2c^{13/3}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{36b^{8/3}}-\frac{5a^2c^{13/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{6\sqrt{3}\,b^{8/3}}+\frac{\sqrt[3]{a+bx^2}\left(3bc(cx)^{10/3}-5ac^3(cx)^{4/3}\right)}{12b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] ((a + b*x^2)^(1/3)*(-5*a*c^3*(c*x)^(4/3) + 3*b*c*(c*x)^(10/3)))/(12*b^2) -
(5*a^2*c^(13/3)*ArcTan[(Sqrt[3]*b^(1/3)*(c*x)^(2/3))/(b^(1/3)*(c*x)^(2/3) +
 2*c^(2/3)*(a + b*x^2)^(1/3))])/(6*Sqrt[3]*b^(8/3)) - (5*a^2*c^(13/3)*Log[b
^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(18*b^(8/3)) + (5*a^2*c^(1
3/3)*Log[b^(2/3)*(c*x)^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3)*(a + b*x^2)^(1/3
) + c^(4/3)*(a + b*x^2)^(2/3)])/(36*b^(8/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{3}}}{\left(b\,x^{2}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
[Out] int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{13}{3}}}{\left(bx^{2}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{13/3}}{\left(b\,x^{2}+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(13/3)/(a + b*x^2)^(2/3),x)
[Out] int((c*x)^(13/3)/(a + b*x^2)^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(13/3)/(b*x**2+a)**(2/3),x)
[Out] Timed out
```

3.645 $\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=131

$$\frac{ac^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{2b^{5/3}}+\frac{ac^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}}+\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}$$

Rubi [A] time = 0.26, antiderivative size = 209, normalized size of antiderivative = 1.60, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {321, 329, 275, 331, 292, 31, 634, 617, 204, 628}

$$\frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}}-\frac{ac^{7/3}\log\left(\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{6b^{5/3}}+\frac{ac^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}\,c^{2/3}}\right)}{\sqrt{3}\,b^{5/3}}+\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) + (a*c^(7/3)*ArcTan[(c^(2/3) + (2*b
^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3))/(Sqrt[3]*c^(2/3))])/(Sqrt[3]*b^(5/3)
) + (a*c^(7/3)*Log[c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(3*b
^(5/3)) - (a*c^(7/3)*Log[c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a + b*x^2)^(2/3)
+ (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(1/3)])/(6*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(cx)^{7/3}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{\left(2ac^2\right)\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx}{3b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(2ac)\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(ac)\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{(ac)\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{b}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{4/3}} + \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,d\right.}{3b^{4/3}}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} + \frac{\left(ac^{5/3}\right)\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right.}{2b^{4/3}}$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} - \frac{ac^{7/3}\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{5/3}} -$$

$$= \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}} + \frac{ac^{7/3}\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3b^{5/3}} - \frac{ac^{7/3}\log\left(c^{4/3}\right.}{}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.44

$$\frac{c(cx)^{4/3}\left(-a\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)+a+bx^2\right)}{2b\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2 - a*Hypergeometric2F1[2/3, 1, 5/3, (b*x^2)/(a + b
*x^2)]))/(2*b*(a + b*x^2)^(2/3))
```

IntegrateAlgebraic [A] time = 6.52, size = 221, normalized size = 1.69

$$\frac{ac^{7/3}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{3b^{5/3}}-\frac{ac^{7/3}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{6b^{5/3}}+\frac{ac^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{\sqrt{3}\,b^{5/3}}+\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) + (a*c^(7/3)*ArcTan[(Sqrt[3]*b^(1/3
)*(c*x)^(2/3))/(b^(1/3)*(c*x)^(2/3) + 2*c^(2/3)*(a + b*x^2)^(1/3))])/(Sqrt[
3]*b^(5/3)) + (a*c^(7/3)*Log[b^(1/3)*(c*x)^(2/3) - c^(2/3)*(a + b*x^2)^(1/3
)])/(3*b^(5/3)) - (a*c^(7/3)*Log[b^(2/3)*(c*x)^(4/3) + b^(1/3)*c^(2/3)*(c*x
)^(2/3)*(a + b*x^2)^(1/3) + c^(4/3)*(a + b*x^2)^(2/3)])/(6*b^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{3}}}{\left(bx^2 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/3}}{\left(b\,x^2 + a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/3)/(a + b*x^2)^(2/3),x)
```

```
[Out] int((c*x)^(7/3)/(a + b*x^2)^(2/3), x)
```

sympy [C] time = 30.01, size = 44, normalized size = 0.34

$$\frac{c^{\frac{7}{3}} x^{\frac{10}{3}} \Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} \frac{2}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}} \Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] c**(7/3)*x**(10/3)*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(2/3)*gamma(8/3))
```

3.646 $\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx$

Optimal. Leaf size=106

$$-\frac{3\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}+1}{\sqrt{3}}\right)}{2b^{2/3}}$$

Rubi [A] time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.73, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {329, 275, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}}+\frac{\sqrt[3]{c}\log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{4/3}\right)}{4b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}+c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] -(Sqrt[3]*c^(1/3)*ArcTan[(c^(2/3) + (2*b^(1/3)*(c*x)^(2/3))/(a + b*x^2)^(1/
3))/(Sqrt[3]*c^(2/3))])/(2*b^(2/3)) - (c^(1/3)*Log[c^(2/3) - (b^(1/3)*(c*x)
^(2/3))/(a + b*x^2)^(1/3)])/(2*b^(2/3)) + (c^(1/3)*Log[c^(4/3) + (b^(2/3)*(
c*x)^(4/3))/(a + b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a + b*x^2)^(
1/3)])/(4*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{cx}}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^3}{\left(a+\frac{bx^6}{c^2}\right)^{2/3}}\,dx, x, \sqrt[3]{cx}\right)}{c}$$

$$= \frac{3\,\text{Subst}\left(\int \frac{x}{\left(a+\frac{bx^3}{c^2}\right)^{2/3}}\,dx, x, (cx)^{2/3}\right)}{2c}$$

$$= \frac{3\,\text{Subst}\left(\int \frac{x}{1-\frac{bx^3}{c^2}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2\sqrt[3]{b}\,\sqrt[3]{c}} - \frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt[3]{b}\,x}{c^{2/3}}}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2\sqrt[3]{b}\,\sqrt[3]{c}}$$

$$= -\frac{\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}}+\frac{b^{2/3}x^2}{c^{4/3}}}\,dx, x, \frac{(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4\sqrt[3]{b}\,\sqrt[3]{c}} + \frac{\sqrt[3]{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt[3]{b}}{c^{2/3}}+ \cdots}{1+\frac{\sqrt[3]{b}\,x}{c^{2/3}} \cdots}\right.}{4b \cdots}$$

$$= -\frac{\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{c}\,\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{4b^{2/3}} + \frac{\left(3\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1}{-3 \cdots}\right.}{\cdots}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2b^{2/3}} - \frac{\sqrt[3]{c}\,\log\left(c^{2/3}-\frac{\sqrt[3]{b}\,(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{c}\,\log\left(c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}\,c^{2 \cdots}}{\sqrt[3]{a \cdots}}\right.}{4b^{2/3}}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.42

$$\frac{3x\sqrt[3]{cx}\;{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^2}{bx^2+a}\right)}{4\left(a+bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] (3*x*(c*x)^(1/3)*Hypergeometric2F1[2/3, 1, 5/3, (b*x^2)/(a + b*x^2)])/(4*(a
 + b*x^2)^(2/3))
```

IntegrateAlgebraic [A] time = 1.75, size = 195, normalized size = 1.84

$$-\frac{\sqrt[3]{c}\log\left(\sqrt[3]{b}\,(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{2b^{2/3}}+\frac{\sqrt[3]{c}\log\left(c^{4/3}\left(a+bx^2\right)^{2/3}+\sqrt[3]{b}\,c^{2/3}(cx)^{2/3}\sqrt[3]{a+bx^2}+b^{2/3}(cx)^{4/3}\right)}{4b^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,(cx)^{2/3}}{2c^{2/3}\sqrt[3]{a+bx^2}+\sqrt[3]{b}\,(cx)^{2/3}}\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

```
[Out] -1/2*(Sqrt[3]*c^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*(c*x)^(2/3))/(b^(1/3)*(c*x)^(
2/3) + 2*c^(2/3)*(a + b*x^2)^(1/3))])/b^(2/3) - (c^(1/3)*Log[b^(1/3)*(c*x)^
(2/3) - c^(2/3)*(a + b*x^2)^(1/3)])/(2*b^(2/3)) + (c^(1/3)*Log[b^(2/3)*(c*x
)^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3)*(a + b*x^2)^(1/3) + c^(4/3)*(a + b*x^
2)^(2/3)])/(4*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{3}}}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)
[Out] int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{1/3}}{(b\,x^2+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/3)/(a + b*x^2)^(2/3),x)
[Out] int((c*x)^(1/3)/(a + b*x^2)^(2/3), x)
```

sympy [C] time = 1.52, size = 44, normalized size = 0.42

$$\frac{\sqrt[3]{c}\,x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/3)/(b*x**2+a)**(2/3),x)
[Out] c**(1/3)*x**(4/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(2/3)*gamma(5/3))
```

3.647 $\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=28

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(2/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{3x\sqrt[3]{a+bx^2}}{2a(cx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*x*(a + b*x^2)^(1/3))/(2*a*(c*x)^(5/3))
```

IntegrateAlgebraic [A] time = 2.12, size = 28, normalized size = 1.00

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(2/3))
```

fricas [A] time = 1.64, size = 25, normalized size = 0.89

$$-\frac{3\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{2\,ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] -3/2*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a*c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(5/3)), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}x}{2\,(cx)^{\frac{5}{3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x)
[Out] -3/2*x*(b*x^2+a)^(1/3)/a/(c*x)^(5/3)
```

maxima [A] time = 1.47, size = 35, normalized size = 1.25

$$-\frac{3\left(bc^{\frac{1}{3}}x^3 + ac^{\frac{1}{3}}x\right)}{2\left(bx^2 + a\right)^{\frac{2}{3}}ac^2x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -3/2*(b*c^(1/3)*x^3 + a*c^(1/3)*x)/((b*x^2 + a)^(2/3)*a*c^2*x^(5/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,x)^{5/3}\,\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(5/3)*(a + b*x^2)^(2/3)), x)
```

sympy [A] time = 4.17, size = 36, normalized size = 1.29

$$\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{1}{3}\right)}{2ac^{\frac{5}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-1/3)/(2*a*c**(5/3)*gamma(2/3))
```

3.648 $\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=57

$$\frac{9\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{8/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{9\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{8/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(8/3)) + (9*(a + b*x^2)^(4/3))/(8*a^2*c
*(c*x)^(8/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}-\frac{3\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx}{a}\\ &= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{8/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.60

$$-\frac{3x\left(a-3bx^2\right)\sqrt[3]{a+bx^2}}{8a^2(cx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
[Out] (-3*x*(a - 3*b*x^2)*(a + b*x^2)^(1/3))/(8*a^2*(c*x)^(11/3))
```

IntegrateAlgebraic [A] time = 9.38, size = 44, normalized size = 0.77

$$\frac{3\sqrt[3]{a+bx^2}\left(3bc^2x^2-ac^2\right)}{8a^2c^3(cx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
[Out] (3*(a + b*x^2)^(1/3)*(-(a*c^2) + 3*b*c^2*x^2))/(8*a^2*c^3*(c*x)^(8/3))
```

fricas [A] time = 1.55, size = 35, normalized size = 0.61

$$\frac{3\left(3\,bx^2-a\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{8\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
[Out] 3/8*(3*b*x^2 - a)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.51

$$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-3b\,x^{2}+a\right)x}{8\left(cx\right)^{\frac{11}{3}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x)
[Out] -3/8*x*(b*x^2+a)^(1/3)*(-3*b*x^2+a)/a^2/(c*x)^(11/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{2}{3}}\left(cx\right)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,x\right)}^{11/3}\,{\left(b\,x^2+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x)
[Out] int(1/((c*x)^(11/3)*(a + b*x^2)^(2/3)), x)
```

sympy [A] time = 109.45, size = 78, normalized size = 1.37

$$-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{4}{3}\right)}{6ac^{\frac{11}{3}}x^{2}\Gamma\left(\frac{2}{3}\right)}+\frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{4}{3}\right)}{2a^{2}c^{\frac{11}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/3)/(b*x**2+a)**(2/3),x)
[Out] -b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(6*a*c**(11/3)*x**2*gamma(2/3
)) + b**(4/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*a**2*c**(11/3)*gamma(2
/3))
```

3.649 $\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{27\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{27\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{14/3}}+\frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(4/3))/(4*a^2*
c*(c*x)^(14/3)) - (27*(a + b*x^2)^(7/3))/(28*a^3*c*(c*x)^(14/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}} - \frac{6\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx}{a}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}} + \frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}} + \frac{9\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{17/3}}\,dx}{2a^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}} + \frac{9\left(a+bx^2\right)^{4/3}}{4a^2c(cx)^{14/3}} - \frac{27\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{14/3}}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.61

$$-\frac{3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\left(2a^2-3abx^2+9b^2x^4\right)}{28a^3c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(2*a^2 - 3*a*b*x^2 + 9*b^2*x^4))/(28*a^3*
c^6*x^5)
```

IntegrateAlgebraic [A] time = 31.48, size = 52, normalized size = 0.61

$$-\frac{3\sqrt[3]{a+bx^2}\left(2a^2-3abx^2+9b^2x^4\right)}{28a^3c^5x^4(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3)*(2*a^2 - 3*a*b*x^2 + 9*b^2*x^4))/(28*a^3*c^5*x^4*(c*x
)^(2/3))
```

fricas [A] time = 1.52, size = 46, normalized size = 0.54

$$-\frac{3\left(9\,b^2x^4-3\,abx^2+2\,a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{28\,a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

[Out] -3/28*(9*b^2*x^4 - 3*a*b*x^2 + 2*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^3*c^6*x^5)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)

maple [A] time = 0.01, size = 42, normalized size = 0.49

$$-\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}\left(9b^2x^4-3ab\,x^2+2a^2\right)x}{28\,(cx)^{\frac{17}{3}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x)

[Out] -3/28*x*(b*x^2+a)^(1/3)*(9*b^2*x^4-3*a*b*x^2+2*a^2)/a^3/(c*x)^(17/3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{17/3}\,\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x)

```
[Out] int(1/((c*x)^(17/3)*(a + b*x^2)^(2/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(17/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

3.650 $\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=113

$$\frac{243\left(a+bx^2\right)^{10/3}}{280a^4c(cx)^{20/3}}-\frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}$$

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{243\left(a+bx^2\right)^{10/3}}{280a^4c(cx)^{20/3}}-\frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}}+\frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(20/3)) + (27*(a + b*x^2)^(4/3))/(8*a^2
*c*(c*x)^(20/3)) - (81*(a + b*x^2)^(7/3))/(28*a^3*c*(c*x)^(20/3)) + (243*(a
 + b*x^2)^(10/3))/(280*a^4*c*(c*x)^(20/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}} - \frac{9\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx}{a}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}} + \frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}} + \frac{27\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{23/3}}\,dx}{2a^2}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}} + \frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}} - \frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}} - \frac{81\int \frac{\left(a+bx^2\right)^{7/3}}{(cx)^{23/3}}\,dx}{14a^3}$$

$$= -\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}} + \frac{27\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{20/3}} - \frac{81\left(a+bx^2\right)^{7/3}}{28a^3c(cx)^{20/3}} + \frac{243\left(a+bx^2\right)^{10/3}}{280a^4c(cx)^{20/3}}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 0.56

$$\frac{3\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}\,\left(-14a^3+18a^2bx^2-27ab^2x^4+81b^3x^6\right)}{280a^4c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(-14*a^3 + 18*a^2*b*x^2 - 27*a*b^2*x^4 + 8
1*b^3*x^6))/(280*a^4*c^8*x^7)
```

IntegrateAlgebraic [A] time = 33.12, size = 63, normalized size = 0.56

$$\frac{3\sqrt[3]{a+bx^2}\,\left(-14a^3+18a^2bx^2-27ab^2x^4+81b^3x^6\right)}{280a^4c^7x^6(cx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
```

```
[Out] (3*(a + b*x^2)^(1/3)*(-14*a^3 + 18*a^2*b*x^2 - 27*a*b^2*x^4 + 81*b^3*x^6))/
(280*a^4*c^7*x^6*(c*x)^(2/3))
```

fricas [A] time = 1.70, size = 57, normalized size = 0.50

$$\frac{3\left(81\,b^3x^6-27\,ab^2x^4+18\,a^2bx^2-14\,a^3\right)\left(bx^2+a\right)^{\frac{1}{3}}\,(cx)^{\frac{1}{3}}}{280\,a^4c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/280*(81*b^3*x^6 - 27*a*b^2*x^4 + 18*a^2*b*x^2 - 14*a^3)*(b*x^2 + a)^(1/3)
*(c*x)^(1/3)/(a^4*c^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{23}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(23/3)), x)
```

maple [A] time = 0.00, size = 53, normalized size = 0.47

$$-\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}\left(-81b^3x^6+27a\,b^2x^4-18a^2b\,x^2+14a^3\right)x}{280\,(cx)^{\frac{23}{3}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x)
```

```
[Out] -3/280*x*(b*x^2+a)^(1/3)*(-81*b^3*x^6+27*a*b^2*x^4-18*a^2*b*x^2+14*a^3)/a^4
/(c*x)^(23/3)
```

maxima [A] time = 1.54, size = 64, normalized size = 0.57

$$\frac{3\left(81\,b^4x^9+54\,ab^3x^7-9\,a^2b^2x^5+4\,a^3bx^3-14\,a^4x\right)}{280\left(bx^2+a\right)^{\frac{2}{3}}a^4c^{\frac{23}{3}}x^{\frac{23}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/280*(81*b^4*x^9 + 54*a*b^3*x^7 - 9*a^2*b^2*x^5 + 4*a^3*b*x^3 - 14*a^4*x)/
((b*x^2 + a)^(2/3)*a^4*c^(23/3)*x^(23/3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{23/3}\left(b\,x^2+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x)
```

```
[Out] int(1/((c*x)^(23/3)*(a + b*x^2)^(2/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(23/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

3.651 $\int (cx)^{5/2} \sqrt[4]{a+bx^2}\, dx$

Optimal. Leaf size=147

$$\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}$$

Rubi [A] time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {279, 321, 329, 331, 298, 205, 208}

$$\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}+\frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b) + ((c*x)^(7/2)*(a + b*x^2)^(1/4)
)/(4*c) + (3*a^2*c^(5/2)*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1
/4))])/(32*b^(7/4)) - (3*a^2*c^(5/2)*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a + b*x^2)^(1/4))])/(32*b^(7/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx = \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c} + \frac{1}{8}a\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx$$

$$= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c} - \frac{\left(3a^2c^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx}{32b}$$

$$= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c} - \frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{16b}$$

$$= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c} - \frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{16b}$$

$$= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c} - \frac{\left(3a^2c^3\right)\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{32b^{3/2}} + \frac{\left(3a^2c\right.}{}$$

$$= \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c} + \frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}} - \frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{}}{\sqrt{c}}\right.}{32b^{7/4}}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 0.58

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}\left(\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1} - a\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{4b\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2)^(1/4)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/4) - a*Hyp
ergeometric2F1[-1/4, 3/4, 7/4, -((b*x^2)/a)]))/(4*b*(1 + (b*x^2)/a)^(1/4))
```

IntegrateAlgebraic [A] time = 0.63, size = 171, normalized size = 1.16

$$\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{32b^{7/4}} - \frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{32b^{7/4}} + \frac{\sqrt[4]{a+bx^2}\left(ac^2(cx)^{3/2}+4b(cx)^{7/2}\right)}{16bc}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

```
[Out] ((a + b*x^2)^(1/4)*(a*c^2*(c*x)^(3/2) + 4*b*(c*x)^(7/2)))/(16*b*c) + (3*a^2
*c^(5/2)*ArcTan[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^
2*x^2)])/(32*b^(7/4)) - (3*a^2*c^(5/2)*ArcTanh[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(
a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)])/(32*b^(7/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{1}{4}} (cx)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{5}{2}} \left(b\,x^2 + a\right)^{\frac{1}{4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + a\right)^{\frac{1}{4}} (cx)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^{5/2}\,\left(b\,x^2+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(a + b*x^2)^(1/4), x)
```

sympy [C] time = 9.47, size = 46, normalized size = 0.31

$$\frac{\sqrt[4]{a}\,c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/4))
```

3.652 $\int \sqrt{cx}\,\sqrt[4]{a+bx^2}\,dx$

Optimal. Leaf size=116

$$-\frac{a\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {279, 329, 331, 298, 205, 208}

$$-\frac{a\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))]])/(4*b^(3/4)) + (a*Sqrt[c]*ArcTanh[(b^(1/4)*
Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]])/(4*b^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \sqrt{cx}\,\sqrt[4]{a+bx^2}\,dx = \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} + \frac{1}{4}a\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} + \frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4\sqrt{b}} - \frac{(ac)\,\mathrm{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+b}}\right.}{4\sqrt{b}}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} - \frac{a\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}} + \frac{a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.48

$$\frac{2x\sqrt{cx}\sqrt[4]{a+bx^2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^(1/4))
```

IntegrateAlgebraic [A] time = 0.42, size = 148, normalized size = 1.28

$$-\frac{a\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{4b^{3/4}}+\frac{a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{4b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[(b^(1/4)*c^(3/2)*
Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)]])/(4*b^(3/4)) + (a*Sqrt[c]
*ArcTanh[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)]
)/(4*b^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x}\,\left(b\,x^{2}+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a + b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(a + b*x^2)^(1/4), x)
```

sympy [C] time = 1.81, size = 46, normalized size = 0.40

$$\frac{\sqrt[4]{a}\,\sqrt{c}\,x^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(7/4))
```

3.653 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}} dx$

Optimal. Leaf size=107

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {277, 329, 331, 298, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(c*Sqrt[c*x]) - (b^(1/4)*ArcTan[(b^(1/4)*Sqrt[c*x])/
(Sqrt[c]*(a + b*x^2)^(1/4))])/c^(3/2) + (b^(1/4)*ArcTanh[(b^(1/4)*Sqrt[c*x]
)/(Sqrt[c]*(a + b*x^2)^(1/4))])/c^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{b\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx}{c^2}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx,x,\sqrt{cx}\right)}{c^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c^3}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}+\frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}-\frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}-\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.50

$$-\frac{2x\sqrt[4]{a+bx^2}\ {}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -((b*x^2)/a)])/(
(c*x)^(3/2)*(1 + (b*x^2)/a)^(1/4))
```

IntegrateAlgebraic [A] time = 0.40, size = 139, normalized size = 1.30

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{c^{3/2}}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(c*Sqrt[c*x]) - (b^(1/4)*ArcTan[(b^(1/4)*c^(3/2)*Sqr
t[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)])/c^(3/2) + (b^(1/4)*ArcTanh[
(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)])/c^(3/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
[Out] int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{1/4}}{\left(c\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(3/2),x)
[Out] int((a + b*x^2)^(1/4)/(c*x)^(3/2), x)
```

sympy [C] time = 2.31, size = 49, normalized size = 0.46

$$\frac{\sqrt[4]{a}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(3/2),x)
```

```
[Out] a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.654 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx=-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{2x\left(a+bx^2\right)^{5/4}}{5a(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]
```

```
[Out] (-2*x*(a + b*x^2)^(5/4))/(5*a*(c*x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.24, size = 28, normalized size = 1.00

$$-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]

[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))

fricas [A] time = 0.85, size = 25, normalized size = 0.89

$$-\frac{2\left(bx^2+a\right)^{\frac{5}{4}}\sqrt{cx}}{5\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="fricas")

[Out] -2/5*(b*x^2 + a)^(5/4)*sqrt(c*x)/(a*c^4*x^3)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x)

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{2\left(b\,x^2+a\right)^{\frac{5}{4}}x}{5\,(cx)^{\frac{7}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)^(1/4)/(c*x)^(7/2),x)

[Out] -2/5*x*(b*x^2+a)^(5/4)/a/(c*x)^(7/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

mupad [B] time = 4.92, size = 37, normalized size = 1.32

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2}{5\,c^3}+\frac{2\,b\,x^2}{5\,a\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(7/2),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(5*c^3) + (2*b*x^2)/(5*a*c^3)))/(x^2*(c*x)^(1/2))
```

sympy [B] time = 9.13, size = 78, normalized size = 2.79

$$\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(7/2),x)
```

```
[Out] b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*c**(7/2)*x**2*gamma(-1/4))
+ b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a*c**(7/2)*gamma(-1/4))
```

3.655 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=57

$$\frac{8\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{8\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a + b*x^2)^(9/4))/(45*a^2*
c*(c*x)^(9/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx &= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}-\frac{4\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a}\\ &= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.72

$$\frac{2\sqrt{cx}\left(a+bx^2\right)^{5/4}\left(4bx^2-5a\right)}{45a^2c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
[Out] (2*Sqrt[c*x]*(a + b*x^2)^(5/4)*(-5*a + 4*b*x^2))/(45*a^2*c^6*x^5)
```

IntegrateAlgebraic [A] time = 0.30, size = 58, normalized size = 1.02

$$\frac{2\sqrt[4]{a+bx^2}\left(-5a^2c^4-abc^4x^2+4b^2c^4x^4\right)}{45a^2c^5(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
[Out] (2*(a + b*x^2)^(1/4)*(-5*a^2*c^4 - a*b*c^4*x^2 + 4*b^2*c^4*x^4))/(45*a^2*c^
5*(c*x)^(9/2))
```

fricas [A] time = 0.99, size = 46, normalized size = 0.81

$$\frac{2\left(4\,b^2x^4-abx^2-5\,a^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{45\,a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")
[Out] 2/45*(4*b^2*x^4 - a*b*x^2 - 5*a^2)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^6*x^5
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

maple [A] time = 0.01, size = 31, normalized size = 0.54

$$-\frac{2\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(-4b\,x^{2}+5a\right)x}{45\left(cx\right)^{\frac{11}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] -2/45*x*(b*x^2+a)^(5/4)*(-4*b*x^2+5*a)/a^2/(c*x)^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

mupad [B] time = 4.98, size = 51, normalized size = 0.89

$$-\frac{\left(b\,x^{2}+a\right)^{1/4}\left(\frac{2}{9\,c^{5}}+\frac{2\,b\,x^{2}}{45\,a\,c^{5}}-\frac{8\,b^{2}\,x^{4}}{45\,a^{2}\,c^{5}}\right)}{x^{4}\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(9*c^5) + (2*b*x^2)/(45*a*c^5) - (8*b^2*x^4)/(45*a^2
*c^5)))/(x^4*(c*x)^(1/2))
```

sympy [B] time = 69.58, size = 124, normalized size = 2.18

$$-\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{9}{4}\right)}{8c^{\frac{11}{2}}x^{4}\Gamma\left(-\frac{1}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{9}{4}\right)}{8ac^{\frac{11}{2}}x^{2}\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{9}{4}\right)}{2a^{2}c^{\frac{11}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(11/2),x)
```

```
[Out] -5*b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(8*c**(11/2)*x**4*gamma(-1/
4)) - b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(8*a*c**(11/2)*x**2*gamm
a(-1/4)) + b**(9/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(2*a**2*c**(11/2)*g
amma(-1/4))
```

3.656 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{64\left(a+bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{64\left(a+bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a + b*x^2)^(9/4))/(45*a^
2*c*(c*x)^(13/2)) - (64*(a + b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx = -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}} - \frac{8\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{15/2}}\,dx}{5a}$$
$$= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}} + \frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}} + \frac{32\int \frac{\left(a+bx^2\right)^{9/4}}{(cx)^{15/2}}\,dx}{45a^2}$$
$$= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}} + \frac{16\left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}} - \frac{64\left(a+bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.61

$$-\frac{2\sqrt{cx}\left(a+bx^2\right)^{5/4}\left(45a^2-40abx^2+32b^2x^4\right)}{585a^3c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*Sqrt[c*x]*(a + b*x^2)^(5/4)*(45*a^2 - 40*a*b*x^2 + 32*b^2*x^4))/(585*a^
3*c^8*x^7)
```

IntegrateAlgebraic [A] time = 0.35, size = 58, normalized size = 0.68

$$-\frac{2\left(a+bx^2\right)^{5/4}\left(45a^2c^4-40abc^4x^2+32b^2c^4x^4\right)}{585a^3c^5(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4)*(45*a^2*c^4 - 40*a*b*c^4*x^2 + 32*b^2*c^4*x^4))/(585*
a^3*c^5*(c*x)^(13/2))
```

fricas [A] time = 1.09, size = 57, normalized size = 0.67

$$-\frac{2\left(32\,b^3x^6-8\,ab^2x^4+5\,a^2bx^2+45\,a^3\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{585\,a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")
```

```
[Out] -2/585*(32*b^3*x^6 - 8*a*b^2*x^4 + 5*a^2*b*x^2 + 45*a^3)*(b*x^2 + a)^(1/4)*
sqrt(c*x)/(a^3*c^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.49

$$-\frac{2\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(32b^{2}x^{4}-40ab\,x^{2}+45a^{2}\right)x}{585\left(cx\right)^{\frac{15}{2}}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(15/2),x)
[Out] -2/585*x*(b*x^2+a)^(5/4)*(32*b^2*x^4-40*a*b*x^2+45*a^2)/a^3/(c*x)^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

mupad [B] time = 5.00, size = 65, normalized size = 0.76

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2}{13\,c^7}+\frac{2\,b\,x^2}{117\,a\,c^7}-\frac{16\,b^2\,x^4}{585\,a^2\,c^7}+\frac{64\,b^3\,x^6}{585\,a^3\,c^7}\right)}{x^6\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(15/2),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(13*c^7) + (2*b*x^2)/(117*a*c^7) - (16*b^2*x^4)/(585
*a^2*c^7) + (64*b^3*x^6)/(585*a^3*c^7)))/(x^6*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(15/2),x)
```

```
[Out] Timed out
```

3.657 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx$

Optimal. Leaf size=113

$$\frac{256\left(a+bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{256\left(a+bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a + b*x^2)^(9/4))/(15*a^2
*c*(c*x)^(17/2)) - (64*(a + b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*
(a + b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx = -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}} - \frac{12\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{19/2}}\,dx}{5a}$$

$$= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}} + \frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}} + \frac{32\int \frac{\left(a+bx^2\right)^{9/4}}{(cx)^{19/2}}\,dx}{15a^2}$$

$$= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}} + \frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}} - \frac{128\int \frac{\left(a+bx^2\right)^{13/4}}{(cx)^{19/2}}\,dx}{195a^3}$$

$$= -\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}} + \frac{8\left(a+bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{64\left(a+bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}} + \frac{256\left(a+bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.56

$$\frac{2\left(a+bx^2\right)^{5/4}\left(-195a^3+180a^2bx^2-160ab^2x^4+128b^3x^6\right)}{3315a^4c^9x^8\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (2*(a + b*x^2)^(5/4)*(-195*a^3 + 180*a^2*b*x^2 - 160*a*b^2*x^4 + 128*b^3*x^
6))/(3315*a^4*c^9*x^8*Sqrt[c*x])
```

IntegrateAlgebraic [F] time = 6.43, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^2)^(1/4)/(c*x)^(19/2), x]
```

fricas [A] time = 1.26, size = 68, normalized size = 0.60

$$\frac{2\left(128\,b^4x^8-32\,ab^3x^6+20\,a^2b^2x^4-15\,a^3bx^2-195\,a^4\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{3315\,a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(128*b^4*x^8 - 32*a*b^3*x^6 + 20*a^2*b^2*x^4 - 15*a^3*b*x^2 - 195*a^
4)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^4*c^10*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.47

$$-\frac{2\left(b\,x^2+a\right)^{\frac{5}{4}}\left(-128b^3x^6+160a\,b^2x^4-180a^2b\,x^2+195a^3\right)x}{3315\,(cx)^{\frac{19}{2}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] -2/3315*x*(b*x^2+a)^(5/4)*(-128*b^3*x^6+160*a*b^2*x^4-180*a^2*b*x^2+195*a^3
)/a^4/(c*x)^(19/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

mupad [B] time = 5.02, size = 79, normalized size = 0.70

$$-\frac{\left(b\,x^2+a\right)^{1/4}\left(\frac{2}{17\,c^9}+\frac{2\,b\,x^2}{221\,a\,c^9}-\frac{8\,b^2\,x^4}{663\,a^2\,c^9}+\frac{64\,b^3\,x^6}{3315\,a^3\,c^9}-\frac{256\,b^4\,x^8}{3315\,a^4\,c^9}\right)}{x^8\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(17*c^9) + (2*b*x^2)/(221*a*c^9) - (8*b^2*x^4)/(663*
a^2*c^9) + (64*b^3*x^6)/(3315*a^3*c^9) - (256*b^4*x^8)/(3315*a^4*c^9)))/(x^
8*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/4)/(c*x)**(19/2),x)
```

```
[Out] Timed out
```

3.658 $\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=343

$$\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{64\sqrt{2}\,b^{7/4}}-\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{64\sqrt{2}\,b^{7/4}}-\frac{3a^2c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}\,b^{7/4}}+$$

Rubi [A] time = 0.37, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {279, 321, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{64\sqrt{2}\,b^{7/4}}-\frac{3a^2c^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{64\sqrt{2}\,b^{7/4}}-\frac{3a^2c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}\,b^{7/4}}+\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{32\sqrt{2}\,b^{7/4}}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}-\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] -(a*c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/(16*b) + ((c*x)^(7/2)*(a - b*x^2)^(1/4
))/(4*c) - (3*a^2*c^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a - b*x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4)) + (3*a^2*c^(5/2)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4)) +
(3*a^2*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]
*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(64*Sqrt[2]*b^(7/4)) - (3*a^2*c^(5/
2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqr
t[c*x])/(a - b*x^2)^(1/4)])/(64*Sqrt[2]*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{1}{8}a\int \frac{(cx)^{5/2}}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{\left(3a^2c^2\right)\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx}{32b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{16b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{16b}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} - \frac{\left(3a^2c\right)\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{32b^{3/2}} + \frac{(3a^2 \cdots}{\cdots}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{\left(3a^2c^{5/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\cdots\right)}{64\sqrt{2}\,b^{7/4}}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} + \frac{3a^2c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{64\sqrt{2}\,b^{7/4}} - \frac{3a^2\cdots}{\cdots}$$

$$= -\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} - \frac{3a^2c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}\,b^{7/4}} + \frac{3a^2c^{5/2}\tan^{-1}\cdots}{32\cdots}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.26

$$\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}\left(a\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)+\sqrt[4]{1-\frac{bx^2}{a}}\,(bx^2-a)\right)}{4b\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a - b*x^2)^(1/4)*((-a + b*x^2)*(1 - (b*x^2)/a)^(1/4) + a*Hy
pergeometric2F1[-1/4, 3/4, 7/4, (b*x^2)/a]))/(4*b*(1 - (b*x^2)/a)^(1/4))
```

IntegrateAlgebraic [A] time = 8.88, size = 278, normalized size = 0.81

$$\frac{\sqrt{c}\,\sqrt[4]{a-bx^2}\left(\frac{3a^2c^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}\sqrt[4]{ac^2-bc^2x^2}}{\sqrt{ac^2-bc^2x^2}-\sqrt{b}\,cx}\right)}{32\sqrt{2}\,b^{7/4}}-\frac{3a^2c^{5/2}\tanh^{-1}\left(\frac{\frac{\sqrt{ac^2-bc^2x^2}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}\,cx}{\sqrt{2}}}{\sqrt{cx}\sqrt[4]{ac^2-bc^2x^2}}\right)}{32\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ac^2-bc^2x^2}\left(4b(cx)^{7/2}-ac^2(cx)^{3/2}\right)}{16bc^{3/2}}\right)}{\sqrt[4]{ac^2-bc^2x^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

```
[Out] (Sqrt[c]*(a - b*x^2)^(1/4)*(((a*c^2 - b*c^2*x^2)^(1/4)*(-(a*c^2*(c*x)^(3/2)
) + 4*b*(c*x)^(7/2)))/(16*b*c^(3/2)) + (3*a^2*c^(5/2)*ArcTan[(Sqrt[2]*b^(1/
4)*Sqrt[c*x]*(a*c^2 - b*c^2*x^2)^(1/4))/(-(Sqrt[b]*c*x) + Sqrt[a*c^2 - b*c^
2*x^2])])/(32*Sqrt[2]*b^(7/4)) - (3*a^2*c^(5/2)*ArcTanh[((b^(1/4)*c*x)/Sqrt
[2] + Sqrt[a*c^2 - b*c^2*x^2]/(Sqrt[2]*b^(1/4)))/(Sqrt[c*x]*(a*c^2 - b*c^2*
x^2)^(1/4))])/(32*Sqrt[2]*b^(7/4))))/(a*c^2 - b*c^2*x^2)^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (cx)^{\frac{5}{2}} \left(-b\,x^{2}+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^{2}+a\right)^{\frac{1}{4}} (cx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,x\right)}^{5/2}\,{\left(a-b\,x^2\right)}^{1/4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)*(a - b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(5/2)*(a - b*x^2)^(1/4), x)
```

sympy [C] time = 9.29, size = 48, normalized size = 0.14

$$\frac{\sqrt[4]{a}\,c^{\frac{5}{2}} x^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)}{2 \Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**2*ex
p_polar(2*I*pi)/a)/(2*gamma(11/4))
```

3.659 $\int \sqrt{cx}\,\sqrt[4]{a-bx^2}\,dx$

Optimal. Leaf size=307

$$\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{3/4}}-\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{3/4}}-\frac{a\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{3/4}}+ a\sqrt{c}\ \mathrm{ta}$$

Rubi [A] time = 0.27, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {279, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{3/4}}-\frac{a\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{3/4}}-\frac{a\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{3/4}}+\frac{a\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{4\sqrt{2}\,b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(a - b*x^2)^(1/4))/(2*c) - (a*Sqrt[c]*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(3/4)) + (a*Sqrt[
c]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*
Sqrt[2]*b^(3/4)) + (a*Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*
x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(3/4))
- (a*Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b
^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{1}{4}a\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} - \frac{a\,\mathrm{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4\sqrt{b}\,c} + \frac{a\,\mathrm{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4\sqrt{b}\,c}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{\left(a\sqrt{c}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{3/4}} + \frac{\left(a\sqrt{c}\right)\mathrm{Subst}\left(\int \frac{\cdots}{-\frac{c}{\sqrt{b}}\cdots}\right)}{8\cdots}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} + \frac{a\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{3/4}} - \frac{a\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a\cdots}}\right.}{8\sqrt{2}\,b^{3/4}}$$

$$= \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c} - \frac{a\sqrt{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{3/4}} + \frac{a\sqrt{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{3/4}} + \frac{a\sqrt{c}\,\log\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.19

$$\frac{2x\sqrt{cx}\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^2}{a}\right)}{3\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

```
[Out] (2*x*Sqrt[c*x]*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (b*x^2)/
a])/(3*(1 - (b*x^2)/a)^(1/4))
```

IntegrateAlgebraic [A] time = 8.23, size = 254, normalized size = 0.83

$$\frac{\sqrt{c}\,\sqrt[4]{a-bx^2}\left(\frac{a\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}\,\sqrt[4]{ac^2-bc^2x^2}}{\sqrt{ac^2-bc^2x^2}-\sqrt{b}\,cx}\right)}{4\sqrt{2}\,b^{3/4}}-\frac{a\sqrt{c}\tanh^{-1}\left(\frac{\frac{\sqrt{ac^2-bc^2x^2}}{\sqrt{2}\,\sqrt[4]{b}}+\frac{\sqrt[4]{b}\,cx}{\sqrt{2}}}{\sqrt{cx}\,\sqrt[4]{ac^2-bc^2x^2}}\right)}{4\sqrt{2}\,b^{3/4}}+\frac{(cx)^{3/2}\sqrt[4]{ac^2-bc^2x^2}}{2c^{3/2}}\right)}{\sqrt[4]{ac^2-bc^2x^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

```
[Out] (Sqrt[c]*(a - b*x^2)^(1/4)*(((c*x)^(3/2)*(a*c^2 - b*c^2*x^2)^(1/4))/(2*c^(3
/2)) + (a*Sqrt[c]*ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[c*x]*(a*c^2 - b*c^2*x^2)^(1/
4))/(-(Sqrt[b]*c*x) + Sqrt[a*c^2 - b*c^2*x^2])])/(4*Sqrt[2]*b^(3/4)) - (a*S
qrt[c]*ArcTanh[((b^(1/4)*c*x)/Sqrt[2] + Sqrt[a*c^2 - b*c^2*x^2]/(Sqrt[2]*b^
(1/4)))/(Sqrt[c*x]*(a*c^2 - b*c^2*x^2)^(1/4))])/(4*Sqrt[2]*b^(3/4))))/(a*c^
2 - b*c^2*x^2)^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\,\left(-b\,x^2+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^2+a\right)^{\frac{1}{4}} \sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x}\,\left(a-b\,x^2\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a - b*x^2)^(1/4),x)
```

```
[Out] int((c*x)^(1/2)*(a - b*x^2)^(1/4), x)
```

sympy [C] time = 1.78, size = 48, normalized size = 0.16

$$\frac{\sqrt[4]{a}\,\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(-b*x**2+a)**(1/4),x)
```

```
[Out] a**(1/4)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**2*exp_
polar(2*I*pi)/a)/(2*gamma(7/4))
```

3.660 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,c^{3/2}}+\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,c^{3/2}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}}-\frac{\sqrt[4]{b}\tan^{-1}}{}$$

Rubi [A] time = 0.27, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {277, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,c^{3/2}}+\frac{\sqrt[4]{b}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,c^{3/2}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\,c^{3/2}}-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(c*Sqrt[c*x]) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*c^(3/2)) - (b^(1/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(Sqrt[2]*c
^(3/2)) - (b^(1/4)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqr
t[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*c^(3/2)) + (b^(1/4)*
Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[c
*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*c^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{b\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx}{c^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^3}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,c^{3/2}} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}-2x}{-\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x\right.}{2\sqrt{2}\,c^{3/2}}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,c^{3/2}} + \frac{\sqrt[4]{b}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,c^{3/2}} - \cdots$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} + \frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,c^{3/2}} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}\cdots\right.}{2\sqrt{2}\,c^{3/2}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.19

$$-\frac{2x\sqrt[4]{a-bx^2}\;{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (b*x^2)/a])/((c*
x)^(3/2)*(1 - (b*x^2)/a)^(1/4))
```

IntegrateAlgebraic [A] time = 12.56, size = 245, normalized size = 0.83

$$\frac{\sqrt{c}\,\sqrt[4]{a-bx^2}\left(-\frac{2\sqrt[4]{ac^2-bc^2x^2}}{c^{3/2}\sqrt{cx}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}\,\sqrt[4]{ac^2-bc^2x^2}}{\sqrt{ac^2-bc^2x^2}-\sqrt{b}\,cx}\right)}{\sqrt{2}\,c^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\frac{\sqrt{ac^2-bc^2x^2}}{\sqrt{2}\,\sqrt[4]{b}}+\frac{\sqrt[4]{b}\,cx}{\sqrt{2}}}{\sqrt{cx}\,\sqrt[4]{ac^2-bc^2x^2}}\right)}{\sqrt{2}\,c^{3/2}}\right)}{\sqrt[4]{ac^2-bc^2x^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

```
[Out] (Sqrt[c]*(a - b*x^2)^(1/4)*((-2*(a*c^2 - b*c^2*x^2)^(1/4))/(c^(3/2)*Sqrt[c*
x]) - (b^(1/4)*ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[c*x]*(a*c^2 - b*c^2*x^2)^(1/4))
/(-(Sqrt[b]*c*x) + Sqrt[a*c^2 - b*c^2*x^2])])/(Sqrt[2]*c^(3/2)) + (b^(1/4)*
ArcTanh[((b^(1/4)*c*x)/Sqrt[2] + Sqrt[a*c^2 - b*c^2*x^2]/(Sqrt[2]*b^(1/4)))
/(Sqrt[c*x]*(a*c^2 - b*c^2*x^2)^(1/4))])/(Sqrt[2]*c^(3/2))))/(a*c^2 - b*c^2
*x^2)^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

```
[Out] int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a-b\,x^2\right)^{1/4}}{\left(c\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(3/2),x)
```

```
[Out] int((a - b*x^2)^(1/4)/(c*x)^(3/2), x)
```

sympy [C] time = 2.32, size = 51, normalized size = 0.17

$$\frac{\sqrt[4]{a}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(3/2),x)
```

```
[Out] a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**2*exp_polar(2*I*pi)/a
)/(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

3.661 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {264}

$$-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx=-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.93

$$-\frac{2x\left(a-bx^2\right)^{5/4}}{5a(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]
```

```
[Out] (-2*x*(a - b*x^2)^(5/4))/(5*a*(c*x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.23, size = 29, normalized size = 1.00

$$-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

fricas [A] time = 1.10, size = 35, normalized size = 1.21

$$\frac{2\left(bx^2-a\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{5\,ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="fricas")
[Out] 2/5*(b*x^2 - a)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{5}{4}}x}{5\,(cx)^{\frac{7}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(7/2),x)
[Out] -2/5*x*(-b*x^2+a)^(5/4)/a/(c*x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

mupad [B] time = 4.85, size = 38, normalized size = 1.31

$$-\frac{\left(a-b\,x^2\right)^{1/4}\,\left(\frac{2}{5\,c^3}-\frac{2\,b\,x^2}{5\,a\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(7/2),x)
[Out] -((a - b*x^2)^(1/4)*(2/(5*c^3) - (2*b*x^2)/(5*a*c^3)))/(x^2*(c*x)^(1/2))
```

sympy [B] time = 8.76, size = 178, normalized size = 6.14

$$\begin{cases} \frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{for}\ \left|\frac{a}{bx^2}\right|>1 \\ \frac{\sqrt[4]{b}\,\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(7/2),x)
[Out] Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*c**(7/2)*x**2*ga
mma(-1/4)) - b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*a*c**(7/2)*gam
ma(-1/4)), Abs(a/(b*x**2)) > 1), (b**(1/4)*(-a/(b*x**2) + 1)**(1/4)*exp(I*p
i/4)*gamma(-5/4)/(2*c**(7/2)*x**2*gamma(-1/4)) - b**(5/4)*(-a/(b*x**2) + 1)
**(1/4)*exp(I*pi/4)*gamma(-5/4)/(2*a*c**(7/2)*gamma(-1/4)), True))
```

3.662 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx$

Optimal. Leaf size=59

$$\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a - b*x^2)^(9/4))/(45*a^2*
c*(c*x)^(9/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}-\frac{4\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a}\\ &= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.71

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{5/4}\left(5a+4bx^2\right)}{45a^2c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(5/4)*(5*a + 4*b*x^2))/(45*a^2*c^6*x^5)
```

IntegrateAlgebraic [A] time = 0.29, size = 58, normalized size = 0.98

$$\frac{2\sqrt[4]{a-bx^2}\left(-5a^2c^4+abc^4x^2+4b^2c^4x^4\right)}{45a^2c^5(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
```

```
[Out] (2*(a - b*x^2)^(1/4)*(-5*a^2*c^4 + a*b*c^4*x^2 + 4*b^2*c^4*x^4))/(45*a^2*c^
5*(c*x)^(9/2))
```

fricas [A] time = 1.47, size = 46, normalized size = 0.78

$$\frac{2\left(4\,b^2x^4+abx^2-5\,a^2\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{45\,a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*b^2*x^4 + a*b*x^2 - 5*a^2)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^6*x^
5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.54

$$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{5}{4}}\left(4b\,x^{2}+5a\right)x}{45\left(cx\right)^{\frac{11}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] -2/45*x*(-b*x^2+a)^(5/4)*(4*b*x^2+5*a)/a^2/(c*x)^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

mupad [B] time = 4.89, size = 51, normalized size = 0.86

$$\frac{\left(a-b\,x^{2}\right)^{1/4}\left(\frac{2\,b\,x^{2}}{45\,a\,c^{5}}-\frac{2}{9\,c^{5}}+\frac{8\,b^{2}\,x^{4}}{45\,a^{2}\,c^{5}}\right)}{x^{4}\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(11/2),x)
```

```
[Out] ((a - b*x^2)^(1/4)*((2*b*x^2)/(45*a*c^5) - 2/(9*c^5) + (8*b^2*x^4)/(45*a^2*
c^5)))/(x^4*(c*x)^(1/2))
```

sympy [B] time = 68.25, size = 462, normalized size = 7.83

$$\begin{cases} -\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^{2}}-1}\,\Gamma\left(-\frac{9}{4}\right)}{8c^{\frac{11}{2}}x^{4}\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^{2}}-1}\,\Gamma\left(-\frac{9}{4}\right)}{8ac^{\frac{11}{2}}x^{2}\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^{2}}-1}\,\Gamma\left(-\frac{9}{4}\right)}{2a^{2}c^{\frac{11}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{for}\ \left|\frac{a}{bx^{2}}\right|>1 \\ \frac{5a^{3}b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^{2}}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{x^{2}\left(-8a^{3}bc^{\frac{11}{2}}x^{2}\Gamma\left(-\frac{1}{4}\right)+8a^{2}b^{2}c^{\frac{11}{2}}x^{4}\Gamma\left(-\frac{1}{4}\right)\right)}-\frac{6a^{2}b^{\frac{9}{4}}\sqrt[4]{-\frac{a}{bx^{2}}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-8a^{3}bc^{\frac{11}{2}}x^{2}\Gamma\left(-\frac{1}{4}\right)+8a^{2}b^{2}c^{\frac{11}{2}}x^{4}\Gamma\left(-\frac{1}{4}\right)}-\frac{3ab^{\frac{13}{4}}x^{2}\sqrt[4]{-\frac{a}{bx^{2}}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-8a^{3}bc^{\frac{11}{2}}x^{2}\Gamma\left(-\frac{1}{4}\right)+8a^{2}b^{2}c^{\frac{11}{2}}x^{4}\Gamma\left(-\frac{1}{4}\right)}+\frac{4b^{\frac{17}{4}}x^{4}\sqrt[4]{-\frac{a}{bx^{2}}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-8a^{3}bc^{\frac{11}{2}}x^{2}\Gamma\left(-\frac{1}{4}\right)+8a^{2}b^{2}c^{\frac{11}{2}}x^{4}\Gamma\left(-\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(11/2),x)
```

```
[Out] Piecewise((-5*b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(8*c**(11/2)*x**
4*gamma(-1/4)) + b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(8*a*c**(11/2
)*x**2*gamma(-1/4)) + b**(9/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(2*a**2*
c**(11/2)*gamma(-1/4)), Abs(a/(b*x**2)) > 1), (5*a**3*b**(5/4)*(-a/(b*x**2)
 + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(x**2*(-8*a**3*b*c**(11/2)*x**2*gamma(
-1/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4))) - 6*a**2*b**(9/4)*(-a/(b*x
**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2*gamma(-1
/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)) - 3*a*b**(13/4)*x**2*(-a/(b*x
**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2*gamma(-1
/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)) + 4*b**(17/4)*x**4*(-a/(b*x**
2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2*gamma(-1/4
) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)), True))
```

3.663 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}} dx$

Optimal. Leaf size=88

$$-\frac{64\left(a-bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

Rubi [A] time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$-\frac{64\left(a-bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}+\frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a - b*x^2)^(9/4))/(45*a^
2*c*(c*x)^(13/2)) - (64*(a - b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx = -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}} - \frac{8\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{15/2}}\,dx}{5a}$$
$$= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}} + \frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}} + \frac{32\int \frac{\left(a-bx^2\right)^{9/4}}{(cx)^{15/2}}\,dx}{45a^2}$$
$$= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}} + \frac{16\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{13/2}} - \frac{64\left(a-bx^2\right)^{13/4}}{585a^3c(cx)^{13/2}}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.60

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{5/4}\left(45a^2+40abx^2+32b^2x^4\right)}{585a^3c^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(5/4)*(45*a^2 + 40*a*b*x^2 + 32*b^2*x^4))/(585*a^
3*c^8*x^7)
```

IntegrateAlgebraic [A] time = 0.35, size = 59, normalized size = 0.67

$$-\frac{2\left(a-bx^2\right)^{5/4}\left(45a^2c^4+40abc^4x^2+32b^2c^4x^4\right)}{585a^3c^5(cx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
[Out] (-2*(a - b*x^2)^(5/4)*(45*a^2*c^4 + 40*a*b*c^4*x^2 + 32*b^2*c^4*x^4))/(585*
a^3*c^5*(c*x)^(13/2))
```

fricas [A] time = 1.49, size = 58, normalized size = 0.66

$$\frac{2\left(32\,b^3x^6+8\,ab^2x^4+5\,a^2bx^2-45\,a^3\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{585\,a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")
```

```
[Out] 2/585*(32*b^3*x^6 + 8*a*b^2*x^4 + 5*a^2*b*x^2 - 45*a^3)*(-b*x^2 + a)^(1/4)*
sqrt(c*x)/(a^3*c^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

maple [A] time = 0.00, size = 43, normalized size = 0.49

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{5}{4}}\left(32b^2x^4+40ab\,x^2+45a^2\right)x}{585\,(cx)^{\frac{15}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(15/2),x)
```

```
[Out] -2/585*x*(-b*x^2+a)^(5/4)*(32*b^2*x^4+40*a*b*x^2+45*a^2)/a^3/(c*x)^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

mupad [B] time = 4.91, size = 65, normalized size = 0.74

$$\frac{\left(a-b\,x^2\right)^{1/4}\left(\frac{2\,b\,x^2}{117\,a\,c^7}-\frac{2}{13\,c^7}+\frac{16\,b^2\,x^4}{585\,a^2\,c^7}+\frac{64\,b^3\,x^6}{585\,a^3\,c^7}\right)}{x^6\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(15/2),x)
```

```
[Out] ((a - b*x^2)^(1/4)*((2*b*x^2)/(117*a*c^7) - 2/(13*c^7) + (16*b^2*x^4)/(585*
a^2*c^7) + (64*b^3*x^6)/(585*a^3*c^7)))/(x^6*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(15/2),x)
```

```
[Out] Timed out
```

3.664 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx$

Optimal. Leaf size=117

$$\frac{256\left(a-bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

Rubi [A] time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$\frac{256\left(a-bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}-\frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}}+\frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a - b*x^2)^(9/4))/(15*a^2
*c*(c*x)^(17/2)) - (64*(a - b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*
(a - b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx = -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}} - \frac{12\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{19/2}}\,dx}{5a}$$

$$= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}} + \frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}} + \frac{32\int \frac{\left(a-bx^2\right)^{9/4}}{(cx)^{19/2}}\,dx}{15a^2}$$

$$= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}} + \frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}} - \frac{128\int \frac{\left(a-bx^2\right)^{13/4}}{(cx)^{19/2}}\,dx}{195a^3}$$

$$= -\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}} + \frac{8\left(a-bx^2\right)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{64\left(a-bx^2\right)^{13/4}}{195a^3c(cx)^{17/2}} + \frac{256\left(a-bx^2\right)^{17/4}}{3315a^4c(cx)^{17/2}}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.55

$$-\frac{2\left(a-bx^2\right)^{5/4}\left(195a^3+180a^2bx^2+160ab^2x^4+128b^3x^6\right)}{3315a^4c^9x^8\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] (-2*(a - b*x^2)^(5/4)*(195*a^3 + 180*a^2*b*x^2 + 160*a*b^2*x^4 + 128*b^3*x^
6))/(3315*a^4*c^9*x^8*Sqrt[c*x])
```

IntegrateAlgebraic [F] time = 6.44, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a - b*x^2)^(1/4)/(c*x)^(19/2), x]
```

fricas [A] time = 0.83, size = 69, normalized size = 0.59

$$\frac{2\left(128\,b^4x^8+32\,ab^3x^6+20\,a^2b^2x^4+15\,a^3bx^2-195\,a^4\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{3315\,a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(128*b^4*x^8 + 32*a*b^3*x^6 + 20*a^2*b^2*x^4 + 15*a^3*b*x^2 - 195*a^
4)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^4*c^10*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

maple [A] time = 0.01, size = 54, normalized size = 0.46

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{5}{4}}\left(128b^3x^6+160a\,b^2x^4+180a^2b\,x^2+195a^3\right)x}{3315\,(cx)^{\frac{19}{2}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] -2/3315*x*(-b*x^2+a)^(5/4)*(128*b^3*x^6+160*a*b^2*x^4+180*a^2*b*x^2+195*a^3
)/a^4/(c*x)^(19/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^2+a\right)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

mupad [B] time = 4.93, size = 79, normalized size = 0.68

$$\frac{\left(a-b\,x^2\right)^{1/4}\,\left(\frac{2\,b\,x^2}{221\,a\,c^9}-\frac{2}{17\,c^9}+\frac{8\,b^2\,x^4}{663\,a^2\,c^9}+\frac{64\,b^3\,x^6}{3315\,a^3\,c^9}+\frac{256\,b^4\,x^8}{3315\,a^4\,c^9}\right)}{x^8\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/4)/(c*x)^(19/2),x)
```

```
[Out] ((a - b*x^2)^(1/4)*((2*b*x^2)/(221*a*c^9) - 2/(17*c^9) + (8*b^2*x^4)/(663*a
^2*c^9) + (64*b^3*x^6)/(3315*a^3*c^9) + (256*b^4*x^8)/(3315*a^4*c^9)))/(x^8
*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/4)/(c*x)**(19/2),x)
```

```
[Out] Timed out
```

3.665 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$-\frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}-\frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}+\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {321, 329, 240, 212, 208, 205}

$$-\frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}-\frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}+\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(3/4))/(2*b) - (a*c^(3/2)*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(5/4)) - (a*c^(3/2)*ArcTanh[(b^(1/4)*
Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(5/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{\left(ac^2\right)\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a+bx^2}}\,dx}{4b}$$

$$= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{(ac)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2b}$$

$$= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{(ac)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b}$$

$$= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{\left(ac^2\right)\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b} - \frac{\left(ac^2\right)\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \ldots\right.}{4b}$$

$$= \frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b} - \frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}} - \frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.83

$$\frac{(cx)^{3/2}\left(2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^{3/4}-a\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-a\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] ((c*x)^(3/2)*(2*b^(1/4)*Sqrt[x]*(a + b*x^2)^(3/4) - a*ArcTan[(b^(1/4)*Sqrt[
x])/(a + b*x^2)^(1/4)] - a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(
4*b^(5/4)*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.58, size = 149, normalized size = 1.27

$$-\frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{4b^{5/4}}-\frac{ac^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{4b^{5/4}}+\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

```
[Out] (c*Sqrt[c*x]*(a + b*x^2)^(3/4))/(2*b) - (a*c^(3/2)*ArcTan[(b^(1/4)*c^(3/2)*
Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)])/(4*b^(5/4)) - (a*c^(3/2)
*ArcTanh[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)]
)/(4*b^(5/4))
```

fricas [B] time = 1.20, size = 314, normalized size = 2.68

$$\frac{4\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c+4\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ab^4c-\left(b^5x^2+ab^4\right)\left(\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}\,a^2c^3x+\sqrt{\frac{a^4c^6}{b^5}}\left(b^3x^2+ab^2\right)}{bx^2+a}}}{a^4bc^6x^2+a^5c^6}\right)-\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ac+\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}\left(b^2x^2+ab\right)}{bx^2+a}\right)+\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ac-\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}\left(b^2x^2+ab\right)}{bx^2+a}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(a^4*c^6/b^5)^(1/4)*b*arctan(-((a^
4*c^6/b^5)^(3/4)*(b*x^2 + a)^(3/4)*sqrt(c*x)*a*b^4*c - (b^5*x^2 + a*b^4)*(a
^4*c^6/b^5)^(3/4)*sqrt((sqrt(b*x^2 + a)*a^2*c^3*x + sqrt(a^4*c^6/b^5)*(b^3*
x^2 + a*b^2))/(b*x^2 + a)))/(a^4*b*c^6*x^2 + a^5*c^6)) - (a^4*c^6/b^5)^(1/4
)*b*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*a*c + (a^4*c^6/b^5)^(1/4)*(b^2*x^2 + a
*b))/(b*x^2 + a)) + (a^4*c^6/b^5)^(1/4)*b*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*
a*c - (a^4*c^6/b^5)^(1/4)*(b^2*x^2 + a*b))/(b*x^2 + a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
[Out] int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{(b\,x^2+a)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(1/4),x)
[Out] int((c*x)^(3/2)/(a + b*x^2)^(1/4), x)
```

sympy [C] time = 2.18, size = 44, normalized size = 0.38

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(1/4)*gamma(9/4))
```

3.666 $\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {329, 240, 212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(1/4)*Sqrt[c]) +
 ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(1/4)*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx &= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c} \\
&= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c} \\
&= \mathrm{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right) + \mathrm{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 0.78

$$\frac{\sqrt{x}\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{\sqrt[4]{b}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

```
[Out] (Sqrt[x]*(ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + ArcTanh[(b^(1/4)*Sq
rt[x])/(a + b*x^2)^(1/4)]))/(b^(1/4)*Sqrt[c*x])
```

IntegrateAlgebraic [A] time = 0.40, size = 115, normalized size = 1.39

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{\sqrt[4]{b}\,\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{\sqrt[4]{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)]/(
b^(1/4)*Sqrt[c]) + ArcTanh[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a
*c^2 + b*c^2*x^2)]/(b^(1/4)*Sqrt[c])
```

fricas [B] time = 1.06, size = 241, normalized size = 2.90

$$-2\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,bc\left(\frac{1}{bc^2}\right)^{\frac{3}{4}}-\left(b^2cx^2+abc\right)\sqrt{\frac{\sqrt{bx^2+a}\,cx+\left(bc^2x^2+ac^2\right)\sqrt{\frac{1}{bc^2}}}{bx^2+a}}\left(\frac{1}{bc^2}\right)^{\frac{3}{4}}}{bx^2+a}\right)+\frac{1}{2}\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}+\left(bcx^2+ac\right)\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2+a}\right)-\frac{1}{2}\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}-\left(bcx^2+ac\right)\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2*(1/(b*c^2))^(1/4)*arctan(-((b*x^2 + a)^(3/4)*sqrt(c*x)*b*c*(1/(b*c^2))^(
3/4) - (b^2*c*x^2 + a*b*c)*sqrt((sqrt(b*x^2 + a)*c*x + (b*c^2*x^2 + a*c^2)*
sqrt(1/(b*c^2)))/(b*x^2 + a))*(1/(b*c^2))^(3/4))/(b*x^2 + a)) + 1/2*(1/(b*c
^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 + a*c)*(1/(b*c^2))^(
1/4))/(b*x^2 + a)) - 1/2*(1/(b*c^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)
 - (b*c*x^2 + a*c)*(1/(b*c^2))^(1/4))/(b*x^2 + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\left(b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x}\,\left(b\,x^2+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(a + b*x^2)^(1/4)), x)
```

sympy [C]   time = 1.51, size = 44, normalized size = 0.53

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(1/4)*sqrt(c)*gamma(5/4))
```

3.667 $\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx=-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{2x\left(a+bx^2\right)^{3/4}}{3a(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(a + b*x^2)^(3/4))/(3*a*(c*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.31, size = 28, normalized size = 1.00

$$-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

fricas [A] time = 1.12, size = 25, normalized size = 0.89

$$-\frac{2\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{3\,ac^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] -2/3*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a*c^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{2\left(b\,x^2+a\right)^{\frac{3}{4}}x}{3\,(cx)^{\frac{5}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x)
[Out] -2/3*x*(b*x^2+a)^(3/4)/a/(c*x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

mupad [B] time = 4.95, size = 25, normalized size = 0.89

$$-\frac{2\left(b\,x^2+a\right)^{3/4}}{3\,a\,c^2\,x\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] -(2*(a + b*x^2)^(3/4))/(3*a*c^2*x*(c*x)^(1/2))
```

sympy [A] time = 3.58, size = 36, normalized size = 1.29

$$\frac{b^{\frac{3}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] b**(3/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4))
```

3.668 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{8\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{8\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a + b*x^2)^(7/4))/(21*a^2*
c*(c*x)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx &= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}-\frac{4\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{9/2}}\,dx}{3a}\\ &= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}+\frac{8\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.72

$$\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/4}\left(4bx^2-3a\right)}{21a^2c^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
[Out] (2*Sqrt[c*x]*(a + b*x^2)^(3/4)*(-3*a + 4*b*x^2))/(21*a^2*c^5*x^4)
```

IntegrateAlgebraic [A] time = 0.39, size = 44, normalized size = 0.77

$$\frac{2\left(a+bx^2\right)^{3/4}\left(4bc^2x^2-3ac^2\right)}{21a^2c^3(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
[Out] (2*(a + b*x^2)^(3/4)*(-3*a*c^2 + 4*b*c^2*x^2))/(21*a^2*c^3*(c*x)^(7/2))
```

fricas [A] time = 0.96, size = 35, normalized size = 0.61

$$\frac{2\left(4\,bx^2-3\,a\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{21\,a^2c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] 2/21*(4*b*x^2 - 3*a)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*c^5*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

maple [A] time = 0.01, size = 31, normalized size = 0.54

$$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(-4b\,x^{2}+3a\right)x}{21\left(cx\right)^{\frac{9}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] -2/21*x*(b*x^2+a)^(3/4)*(-4*b*x^2+3*a)/a^2/(c*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^{2}+a\right)^{\frac{1}{4}}\left(cx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

mupad [B] time = 5.00, size = 40, normalized size = 0.70

$$-\frac{\left(b\,x^{2}+a\right)^{3/4}\left(\frac{2}{7\,a\,c^{4}}-\frac{8\,b\,x^{2}}{21\,a^{2}\,c^{4}}\right)}{x^{3}\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*(2/(7*a*c^4) - (8*b*x^2)/(21*a^2*c^4)))/(x^3*(c*x)^(1/2
))
```

sympy [A] time = 34.72, size = 80, normalized size = 1.40

$$-\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^{2}}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{8ac^{\frac{9}{2}}x^{2}\Gamma\left(\frac{1}{4}\right)}+\frac{b^{\frac{7}{4}}\left(\frac{a}{bx^{2}}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2a^{2}c^{\frac{9}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] -3*b**(3/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(8*a*c**(9/2)*x**2*gamma(1/
4)) + b**(7/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(2*a**2*c**(9/2)*gamma(1
/4))
```

3.669 $\int \frac{1}{(cx)^{13/2} \sqrt[4]{a+bx^2}} \, dx$

Optimal. Leaf size=85

$$-\frac{64\left(a+bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{64\left(a+bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a + b*x^2)^(7/4))/(21*a^
2*c*(c*x)^(11/2)) - (64*(a + b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}} - \frac{8\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{13/2}}\,dx}{3a}$$

$$= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}} + \frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}} + \frac{32\int \frac{\left(a+bx^2\right)^{7/4}}{(cx)^{13/2}}\,dx}{21a^2}$$

$$= -\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}} + \frac{16\left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}} - \frac{64\left(a+bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}$$

Mathematica [**A**] time = 0.02, size = 52, normalized size = 0.61

$$-\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/4}\left(21a^2-24abx^2+32b^2x^4\right)}{231a^3c^7x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a + b*x^2)^(3/4)*(21*a^2 - 24*a*b*x^2 + 32*b^2*x^4))/(231*a^
3*c^7*x^6)
```

IntegrateAlgebraic [**A**] time = 0.65, size = 58, normalized size = 0.68

$$-\frac{2\left(a+bx^2\right)^{3/4}\left(21a^2c^4-24abc^4x^2+32b^2c^4x^4\right)}{231a^3c^5(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4)*(21*a^2*c^4 - 24*a*b*c^4*x^2 + 32*b^2*c^4*x^4))/(231*
a^3*c^5*(c*x)^(11/2))
```

fricas [**A**] time = 1.14, size = 46, normalized size = 0.54

$$-\frac{2\left(32\,b^2x^4-24\,abx^2+21\,a^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{231\,a^3c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/231*(32*b^2*x^4 - 24*a*b*x^2 + 21*a^2)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3*
c^7*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.49

$$-\frac{2\left(b\,x^2+a\right)^{\frac{3}{4}}\left(32b^2x^4-24ab\,x^2+21a^2\right)x}{231\,(cx)^{\frac{13}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x)
```

```
[Out] -2/231*x*(b*x^2+a)^(3/4)*(32*b^2*x^4-24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

mupad [B] time = 5.00, size = 54, normalized size = 0.64

$$-\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{2}{11\,a\,c^6}-\frac{16\,b\,x^2}{77\,a^2\,c^6}+\frac{64\,b^2\,x^4}{231\,a^3\,c^6}\right)}{x^5\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*(2/(11*a*c^6) - (16*b*x^2)/(77*a^2*c^6) + (64*b^2*x^4)/
(231*a^3*c^6)))/(x^5*(c*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

3.670 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=308

$$-\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{5/4}}-\frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}}+ac^{3/2}$$

Rubi [A] time = 0.27, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {321, 329, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{5/4}}-\frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{4\sqrt{2}\,b^{5/4}}-\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] -(c*Sqrt[c*x]*(a - b*x^2)^(3/4))/(2*b) - (a*c^(3/2)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]])/(4*Sqrt[2]*b^(5/4)) + (a*c^(3
/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]])/(4
*Sqrt[2]*b^(5/4)) - (a*c^(3/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b
*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]])/(8*Sqrt[2]*b^(5/4))
 + (a*c^(3/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*
b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]])/(8*Sqrt[2]*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}+\frac{\left(ac^2\right)\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx}{4b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}+\frac{(ac)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{2b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}+\frac{(ac)\,\text{Subst}\left(\int \frac{1}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}+\frac{a\,\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b}+\frac{a\,\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}-\frac{\left(ac^{3/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{5/4}}-\frac{\left(ac^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{}}{-\frac{c}{\sqrt{b}}+}\right.}{8\sqrt{}}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}-\frac{ac^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a-}}\right.}{8\sqrt{2}\,b^{5/4}}$$

$$= -\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}-\frac{ac^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{5/4}}-\frac{ac^{3/2}\log\left(\right.}{}$$

Mathematica [A] time = 0.15, size = 241, normalized size = 0.78

$$-\frac{(cx)^{3/2}\left(8\sqrt[4]{b}\sqrt{x}\left(a-bx^2\right)^{3/4}+\sqrt{2}\,a\log\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)-\sqrt{2}\,a\log\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)+2\sqrt{2}\,a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)-2\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)\right)}{16b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]

[Out] -1/16*((c*x)^(3/2)*(8*b^(1/4)*Sqrt[x]*(a - b*x^2)^(3/4) + 2*Sqrt[2]*a*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] - 2*Sqrt[2]*a*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + Sqrt[2]*a*Log[1 + (Sqrt[b]*
```

```
x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] - Sqrt[2]
*a*Log[1 + (Sqrt[b]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x
^2)^(1/4)]))/(b^(5/4)*x^(3/2))
```

IntegrateAlgebraic [A] time = 9.05, size = 257, normalized size = 0.83

$$\frac{c^{3/2}\left(a-bx^2\right)^{3/4}\left(\frac{ac^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}\,\sqrt[4]{ac^2-bc^2x^2}}{\sqrt{ac^2-bc^2x^2}-\sqrt{b}\,cx}\right)}{4\sqrt{2}\,b^{5/4}}+\frac{ac^{3/2}\tanh^{-1}\left(\frac{\frac{\sqrt{ac^2-bc^2x^2}}{\sqrt{2}\,\sqrt[4]{b}}+\frac{\sqrt[4]{b}\,cx}{\sqrt{2}}}{\sqrt{cx}\,\sqrt[4]{ac^2-bc^2x^2}}\right)}{4\sqrt{2}\,b^{5/4}}-\frac{\sqrt{cx}\left(ac^2-bc^2x^2\right)^{3/4}}{2b\sqrt{c}}\right)}{\left(ac^2-bc^2x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]
```

```
[Out] (c^(3/2)*(a - b*x^2)^(3/4)*(-1/2*(Sqrt[c*x]*(a*c^2 - b*c^2*x^2)^(3/4))/(b*S
qrt[c]) + (a*c^(3/2)*ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[c*x]*(a*c^2 - b*c^2*x^2)^
(1/4))/(-(Sqrt[b]*c*x) + Sqrt[a*c^2 - b*c^2*x^2])])/(4*Sqrt[2]*b^(5/4)) + (
a*c^(3/2)*ArcTanh[((b^(1/4)*c*x)/Sqrt[2] + Sqrt[a*c^2 - b*c^2*x^2]/(Sqrt[2]
*b^(1/4)))/(Sqrt[c*x]*(a*c^2 - b*c^2*x^2)^(1/4))])/(4*Sqrt[2]*b^(5/4))))/(a
*c^2 - b*c^2*x^2)^(3/4)
```

fricas [A] time = 0.91, size = 340, normalized size = 1.10

$$-\frac{4\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c+4\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ab^4c-\left(b^5x^2-ab^4\right)\left(-\frac{a^4c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{-\frac{\sqrt{-bx^2+a}\,a^2c^3x-\sqrt{-\frac{a^4c^6}{b^5}}\left(b^3x^2-ab^2\right)}{bx^2-a}}}{a^4bc^6x^2-a^5c^6}\right)+\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ac+\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}\left(b^2x^2-ab\right)}{bx^2-a}\right)-\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\frac{\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,ac-\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}\left(b^2x^2-ab\right)}{bx^2-a}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(-b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(-a^4*c^6/b^5)^(1/4)*b*arctan(-(
(-a^4*c^6/b^5)^(3/4)*(-b*x^2 + a)^(3/4)*sqrt(c*x)*a*b^4*c - (b^5*x^2 - a*b^
4)*(-a^4*c^6/b^5)^(3/4)*sqrt(-(sqrt(-b*x^2 + a)*a^2*c^3*x - sqrt(-a^4*c^6/b
^5)*(b^3*x^2 - a*b^2))/(b*x^2 - a)))/(a^4*b*c^6*x^2 - a^5*c^6)) + (-a^4*c^6
/b^5)^(1/4)*b*log(((-b*x^2 + a)^(3/4)*sqrt(c*x)*a*c + (-a^4*c^6/b^5)^(1/4)*
(b^2*x^2 - a*b))/(b*x^2 - a)) - (-a^4*c^6/b^5)^(1/4)*b*log(((-b*x^2 + a)^(3
/4)*sqrt(c*x)*a*c - (-a^4*c^6/b^5)^(1/4)*(b^2*x^2 - a*b))/(b*x^2 - a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(1/4), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
[Out] int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{3/2}}{\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a - b*x^2)^(1/4),x)
[Out] int((c*x)^(3/2)/(a - b*x^2)^(1/4), x)
```

sympy [C] time = 2.20, size = 46, normalized size = 0.15

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**2*exp_polar(2*I
*pi)/a)/(2*a**(1/4)*gamma(9/4))
```

3.671 $\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=272

$$-\frac{\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}+\frac{\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}+\right.}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}$$

Rubi [A] time = 0.23, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {329, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}+\frac{\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt
[2]*b^(1/4)*Sqrt[c])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a
- b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c]) - Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]
*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqr
t[2]*b^(1/4)*Sqrt[c]) + Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] +
 (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqrt[2]*b^(1/4)*Sqrt[c])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a-bx^2}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^2} + \frac{\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{b}} - \text{Subst}\Big(\ldots$$

$$= -\frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}} + \frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,d\ldots\right.}{\sqrt{2}\ldots}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}} - \frac{\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{c}} + \text{lo}\ldots$$

Mathematica [A] time = 0.04, size = 197, normalized size = 0.72

$$\frac{\sqrt{x}\left(-\log\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)+\log\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a-bx^2}}+1\right)\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
```

```
[Out] (Sqrt[x]*(-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + 2*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] - Log[1 + (Sqrt[b]*x)
/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + Log[1 + (
Sqrt[b]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)]))
/(2*Sqrt[2]*b^(1/4)*Sqrt[c*x])
```

IntegrateAlgebraic [A] time = 2.80, size = 163, normalized size = 0.60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}\sqrt[4]{a-bx^2}}{c\sqrt{a-bx^2}+\sqrt{b}cx}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{c}\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{c}x}{\sqrt{2}}}{\sqrt{cx}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
```

```
[Out] -(ArcTan[(-((b^(1/4)*Sqrt[c]*x)/Sqrt[2]) + (Sqrt[c]*Sqrt[a - b*x^2])/(Sqrt[
2]*b^(1/4)))/(Sqrt[c*x]*(a - b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c])) + Ar
cTanh[(Sqrt[2]*b^(1/4)*Sqrt[c]*Sqrt[c*x]*(a - b*x^2)^(1/4))/(Sqrt[b]*c*x +
c*Sqrt[a - b*x^2])]/(Sqrt[2]*b^(1/4)*Sqrt[c])
```

fricas [A] time = 1.05, size = 267, normalized size = 0.98

$$-2\left(-\frac{1}{bc^2}\right)^{\frac14}\arctan\left(-\frac{\left(-bx^2+a\right)^{\frac34}\sqrt{cx}\,bc\left(-\frac{1}{bc^2}\right)^{\frac34}-\left(b^2cx^2-abc\right)\sqrt{-\frac{\sqrt{-bx^2+a}\,cx-\left(bc^2x^2-ac^2\right)\sqrt{-\frac{1}{bc^2}}}{bx^2-a}}\left(-\frac{1}{bc^2}\right)^{\frac34}}{bx^2-a}\right)-\frac12\left(-\frac{1}{bc^2}\right)^{\frac14}\log\left(\frac{\left(-bx^2+a\right)^{\frac34}\sqrt{cx}+\left(bcx^2-ac\right)\left(-\frac{1}{bc^2}\right)^{\frac14}}{bx^2-a}\right)+\frac12\left(-\frac{1}{bc^2}\right)^{\frac14}\log\left(\frac{\left(-bx^2+a\right)^{\frac34}\sqrt{cx}-\left(bcx^2-ac\right)\left(-\frac{1}{bc^2}\right)^{\frac14}}{bx^2-a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -2*(-1/(b*c^2))^(1/4)*arctan(-((-b*x^2 + a)^(3/4)*sqrt(c*x)*b*c*(-1/(b*c^2)
)^(3/4) - (b^2*c*x^2 - a*b*c)*sqrt(-(sqrt(-b*x^2 + a)*c*x - (b*c^2*x^2 - a*
c^2)*sqrt(-1/(b*c^2)))/(b*x^2 - a))*(-1/(b*c^2))^(3/4))/(b*x^2 - a)) - 1/2*
(-1/(b*c^2))^(1/4)*log(((-b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 - a*c)*(-1/
(b*c^2))^(1/4))/(b*x^2 - a)) + 1/2*(-1/(b*c^2))^(1/4)*log(((-b*x^2 + a)^(3/
4)*sqrt(c*x) - (b*c*x^2 - a*c)*(-1/(b*c^2))^(1/4))/(b*x^2 - a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac14}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\left(-b\,x^2+a\right)^{\frac14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

```
[Out] int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x}\,\left(a-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(a - b*x^2)^(1/4)), x)
```

sympy [C] time = 1.52, size = 46, normalized size = 0.17

$$\frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**2*exp_polar(2*I*pi)/a)/(2
*a**(1/4)*sqrt(c)*gamma(5/4))
```

3.672 $\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {264}

$$-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx=-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.93

$$-\frac{2x\left(a-bx^2\right)^{3/4}}{3a(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*x*(a - b*x^2)^(3/4))/(3*a*(c*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.31, size = 29, normalized size = 1.00

$$-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

fricas [A] time = 0.83, size = 26, normalized size = 0.90

$$-\frac{2\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{3\,ac^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] -2/3*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a*c^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{3}{4}}x}{3\,(cx)^{\frac{5}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
[Out] -2/3*x*(-b*x^2+a)^(3/4)/a/(c*x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

mupad [B] time = 5.12, size = 26, normalized size = 0.90

$$-\frac{2\left(a-b\,x^2\right)^{3/4}}{3\,a\,c^2\,x\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] -(2*(a - b*x^2)^(3/4))/(3*a*c^2*x*(c*x)^(1/2))
```

sympy [A] time = 3.62, size = 88, normalized size = 3.03

$$\begin{cases} \frac{b^{\frac{3}{4}}\left(\frac{a}{bx^2}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^2}\right|>1 \\ -\frac{b^{\frac{3}{4}}\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Piecewise((b**(3/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma
(1/4)), Abs(a/(b*x**2)) > 1), (-b**(3/4)*(-a/(b*x**2) + 1)**(3/4)*exp(-I*pi
/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4)), True))
```

3.673 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{8\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$\frac{8\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a - b*x^2)^(7/4))/(21*a^2*
c*(c*x)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx &= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}-\frac{4\int \frac{\left(a-bx^2\right)^{3/4}}{(cx)^{9/2}}\,dx}{3a}\\ &= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{7/2}}+\frac{8\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.71

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{3/4}\left(3a+4bx^2\right)}{21a^2c^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(3/4)*(3*a + 4*b*x^2))/(21*a^2*c^5*x^4)
```

IntegrateAlgebraic [A] time = 0.39, size = 45, normalized size = 0.76

$$-\frac{2\left(a-bx^2\right)^{3/4}\left(3ac^2+4bc^2x^2\right)}{21a^2c^3(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
[Out] (-2*(a - b*x^2)^(3/4)*(3*a*c^2 + 4*b*c^2*x^2))/(21*a^2*c^3*(c*x)^(7/2))
```

fricas [A] time = 1.02, size = 36, normalized size = 0.61

$$-\frac{2\left(4\,bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{21\,a^2c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] -2/21*(4*b*x^2 + 3*a)*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*c^5*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.54

$$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{3}{4}}\left(4b\,x^{2}+3a\right)x}{21\left(cx\right)^{\frac{9}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x)
[Out] -2/21*x*(-b*x^2+a)^(3/4)*(4*b*x^2+3*a)/a^2/(c*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^{2}+a\right)^{\frac{1}{4}}\left(cx\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

mupad [B] time = 5.14, size = 41, normalized size = 0.69

$$-\frac{\left(a-b\,x^{2}\right)^{3/4}\left(\frac{2}{7\,a\,c^{4}}+\frac{8\,b\,x^{2}}{21\,a^{2}\,c^{4}}\right)}{x^{3}\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x)
[Out] -((a - b*x^2)^(3/4)*(2/(7*a*c^4) + (8*b*x^2)/(21*a^2*c^4)))/(x^3*(c*x)^(1/2
))
```

sympy [A] time = 34.34, size = 343, normalized size = 5.81

$$\begin{cases} -\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^{2}}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{8ac^{\frac{9}{2}}x^{2}\Gamma\left(\frac{1}{4}\right)}-\frac{b^{\frac{7}{4}}\left(\frac{a}{bx^{2}}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2a^{2}c^{\frac{9}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{for}\ \left|\frac{a}{bx^{2}}\right|>1 \\ -\frac{3a^{2}b^{\frac{7}{4}}\left(-\frac{a}{bx^{2}}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^{3}bc^{\frac{9}{2}}x^{2}e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^{2}b^{2}c^{\frac{9}{2}}x^{4}e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)}-\frac{ab^{\frac{11}{4}}x^{2}\left(-\frac{a}{bx^{2}}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^{3}bc^{\frac{9}{2}}x^{2}e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^{2}b^{2}c^{\frac{9}{2}}x^{4}e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)}+\frac{4b^{\frac{15}{4}}x^{4}\left(-\frac{a}{bx^{2}}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^{3}bc^{\frac{9}{2}}x^{2}e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^{2}b^{2}c^{\frac{9}{2}}x^{4}e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Piecewise((-3*b**(3/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-7/4)/(8*a*c**(9/2)*x*
*2*gamma(1/4)) - b**(7/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-7/4)/(2*a**2*c**(9
/2)*gamma(1/4)), Abs(a/(b*x**2)) > 1), (-3*a**2*b**(7/4)*(-a/(b*x**2) + 1)*
*(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1/4) + 8*a**2
*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)) - a*b**(11/4)*x**2*(-a/(b*x**2)
 + 1)**(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1/4) +
8*a**2*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)) + 4*b**(15/4)*x**4*(-a/(b
*x**2) + 1)**(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1
/4) + 8*a**2*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)), True))
```

3.674 $\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal. Leaf size=88

$$-\frac{64\left(a-bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

Rubi [A] time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$-\frac{64\left(a-bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}+\frac{16\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a - b*x^2)^(7/4))/(21*a^
2*c*(c*x)^(11/2)) - (64*(a - b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}} - \frac{8\int \frac{\left(a-bx^2\right)^{3/4}}{(cx)^{13/2}}\,dx}{3a}$$

$$= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}} + \frac{16\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}} + \frac{32\int \frac{\left(a-bx^2\right)^{7/4}}{(cx)^{13/2}}\,dx}{21a^2}$$

$$= -\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}} + \frac{16\left(a-bx^2\right)^{7/4}}{21a^2c(cx)^{11/2}} - \frac{64\left(a-bx^2\right)^{11/4}}{231a^3c(cx)^{11/2}}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.60

$$-\frac{2\sqrt{cx}\left(a-bx^2\right)^{3/4}\left(21a^2+24abx^2+32b^2x^4\right)}{231a^3c^7x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(3/4)*(21*a^2 + 24*a*b*x^2 + 32*b^2*x^4))/(231*a^
3*c^7*x^6)
```

IntegrateAlgebraic [A] time = 0.64, size = 59, normalized size = 0.67

$$-\frac{2\left(a-bx^2\right)^{3/4}\left(21a^2c^4+24abc^4x^2+32b^2c^4x^4\right)}{231a^3c^5(cx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(3/4)*(21*a^2*c^4 + 24*a*b*c^4*x^2 + 32*b^2*c^4*x^4))/(231*
a^3*c^5*(c*x)^(11/2))
```

fricas [A] time = 0.88, size = 47, normalized size = 0.53

$$-\frac{2\left(32\,b^2x^4+24\,abx^2+21\,a^2\right)\left(-bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{231\,a^3c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
[Out] -2/231*(32*b^2*x^4 + 24*a*b*x^2 + 21*a^2)*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3
*c^7*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

maple [A] time = 0.01, size = 43, normalized size = 0.49

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{3}{4}}\left(32b^2x^4+24ab\,x^2+21a^2\right)x}{231\,(cx)^{\frac{13}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x)
[Out] -2/231*x*(-b*x^2+a)^(3/4)*(32*b^2*x^4+24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

mupad [B] time = 5.25, size = 55, normalized size = 0.62

$$-\frac{\left(a-b\,x^2\right)^{3/4}\left(\frac{2}{11\,a\,c^6}+\frac{16\,b\,x^2}{77\,a^2\,c^6}+\frac{64\,b^2\,x^4}{231\,a^3\,c^6}\right)}{x^5\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x)
```

```
[Out] -((a - b*x^2)^(3/4)*(2/(11*a*c^6) + (16*b*x^2)/(77*a^2*c^6) + (64*b^2*x^4)/
(231*a^3*c^6)))/(x^5*(c*x)^(1/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(1/4),x)
```

```
[Out] Timed out
```

3.675 $\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/4}} dx$

Optimal. Leaf size=117

$$\frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}-\frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {321, 329, 331, 298, 205, 208}

$$\frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}-\frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b) + (3*a*c^(5/2)*ArcTan[(b^(1/4)*Sqrt
[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(7/4)) - (3*a*c^(5/2)*ArcTanh[(b^
(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(4*b^(7/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{\left(3ac^2\right)\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx}{4b}$$

$$= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac)\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2b}$$

$$= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{(3ac)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b}$$

$$= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} - \frac{\left(3ac^3\right)\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b^{3/2}} + \frac{\left(3ac^3\right)\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \right.}{4b^{3/2}}$$

$$= \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} + \frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} - \frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.83

$$\frac{(cx)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}+3a\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] ((c*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4) + 3*a*ArcTan[(b^(1/4)*Sqr
t[x])/(a + b*x^2)^(1/4)] - 3*a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]
))/(4*b^(7/4)*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.90, size = 149, normalized size = 1.27

$$\frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{4b^{7/4}}-\frac{3ac^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{4b^{7/4}}+\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

```
[Out] (c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b) + (3*a*c^(5/2)*ArcTan[(b^(1/4)*c^(3
/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)])/(4*b^(7/4)) - (3*a*c
^(5/2)*ArcTanh[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2
*x^2)])/(4*b^(7/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{5/2}}{\left(b\,x^{2}+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a + b*x^2)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(a + b*x^2)^(3/4), x)
```

sympy [C] time = 7.78, size = 44, normalized size = 0.38

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/4)*gamma(11/4))
```

3.676 $\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}} \, dx$

Optimal. Leaf size=84

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {329, 331, 298, 205, 208}

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a + b*x^2)^(3/4),x]
```

```
[Out] -((Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(3/4)
) + (Sqrt[c]*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(3
/4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx}}{\left(a+bx^2\right)^{3/4}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}\\
&= \frac{2\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{c}\\
&= \frac{c\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}} - \frac{c\,\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}}\\
&= -\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}} + \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 0.80

$$\frac{\sqrt{cx}\left(\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{b^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a + b*x^2)^(3/4),x]
```

```
[Out] (Sqrt[c*x]*(-ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + ArcTanh[(b^(1/4)
*Sqrt[x])/(a + b*x^2)^(1/4)]))/(b^(3/4)*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.54, size = 116, normalized size = 1.38

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x]/(a + b*x^2)^(3/4),x]
```

```
[Out] -((Sqrt[c]*ArcTan[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*
c^2*x^2)])/b^(3/4)) + (Sqrt[c]*ArcTanh[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^
2)^(3/4))/(a*c^2 + b*c^2*x^2)])/b^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(bx^2 + a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{\left(b\,x^2 + a\right)^{3/4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a + b*x^2)^(3/4),x)
```

```
[Out] int((c*x)^(1/2)/(a + b*x^2)^(3/4), x)
```

sympy [C] time = 1.50, size = 44, normalized size = 0.52

$$\frac{\sqrt{c}\, x^{\frac{3}{2}} \Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| {\frac{bx^{2} e^{i\pi}}{a}} \right)}{2 a^{\frac{3}{4}} \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(3/4)*gamma(7/4))
```

3.677 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.92

$$-\frac{2x\sqrt[4]{a+bx^2}}{a(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]
[Out] (-2*x*(a + b*x^2)^(1/4))/(a*(c*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.39, size = 26, normalized size = 1.00

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

fricas [A] time = 1.01, size = 25, normalized size = 0.96

$$-\frac{2\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] -2*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$-\frac{2\left(b\,x^2+a\right)^{\frac{1}{4}}x}{(cx)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x)
[Out] -2*x*(b*x^2+a)^(1/4)/a/(c*x)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

mupad [B] time = 4.90, size = 22, normalized size = 0.85

$$-\frac{2\left(b\,x^2+a\right)^{1/4}}{a\,c\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -(2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(1/2))
```

sympy [A] time = 2.92, size = 36, normalized size = 1.38

$$\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4))
```

3.678 $\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=55

$$\frac{8\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{8\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a + b*x^2)^(5/4))/(5*a^2*c*(
c*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}-\frac{4\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx}{a}\\
&= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}+\frac{8\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.62

$$-\frac{2x\left(a-4bx^2\right)\sqrt[4]{a+bx^2}}{5a^2(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
[Out] (-2*x*(a - 4*b*x^2)*(a + b*x^2)^(1/4))/(5*a^2*(c*x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.55, size = 44, normalized size = 0.80

$$\frac{2\sqrt[4]{a+bx^2}\left(4bc^2x^2-ac^2\right)}{5a^2c^3(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
[Out] (2*(a + b*x^2)^(1/4)*(-(a*c^2) + 4*b*c^2*x^2))/(5*a^2*c^3*(c*x)^(5/2))
```

fricas [A] time = 1.50, size = 35, normalized size = 0.64

$$\frac{2\left(4\,bx^2-a\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{5\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] 2/5*(4*b*x^2 - a)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.53

$$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-4b\,x^{2}+a\right)x}{5\left(cx\right)^{\frac{7}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/5*x*(b*x^2+a)^(1/4)*(-4*b*x^2+a)/a^2/(c*x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

mupad [B] time = 4.98, size = 40, normalized size = 0.73

$$-\frac{\left(b\,x^{2}+a\right)^{1/4}\left(\frac{2}{5\,a\,c^{3}}-\frac{8\,b\,x^{2}}{5\,a^{2}\,c^{3}}\right)}{x^{2}\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(5*a*c^3) - (8*b*x^2)/(5*a^2*c^3)))/(x^2*(c*x)^(1/2)
)
```

sympy [A] time = 24.76, size = 78, normalized size = 1.42

$$-\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{5}{4}\right)}{8ac^{\frac{7}{2}}x^{2}\Gamma\left(\frac{3}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(-\frac{5}{4}\right)}{2a^{2}c^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(8*a*c**(7/2)*x**2*gamma(3/4)
) + b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a**2*c**(7/2)*gamma(3/4
))
```

3.679 $\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=83

$$-\frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a + b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(9/2)) - (64*(a + b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}} - \frac{8\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx}{a}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}} + \frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}} + \frac{32\int \frac{\left(a+bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a^2}$$

$$= -\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}} + \frac{16\left(a+bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}} - \frac{64\left(a+bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.63

$$-\frac{2\sqrt{cx}\,\sqrt[4]{a+bx^2}\left(5a^2-8abx^2+32b^2x^4\right)}{45a^3c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a + b*x^2)^(1/4)*(5*a^2 - 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*c
^6*x^5)
```

IntegrateAlgebraic [A] time = 0.99, size = 58, normalized size = 0.70

$$-\frac{2\sqrt[4]{a+bx^2}\left(5a^2c^4-8abc^4x^2+32b^2c^4x^4\right)}{45a^3c^5(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4)*(5*a^2*c^4 - 8*a*b*c^4*x^2 + 32*b^2*c^4*x^4))/(45*a^3
*c^5*(c*x)^(9/2))
```

fricas [A] time = 1.54, size = 46, normalized size = 0.55

$$-\frac{2\left(32\,b^2x^4-8\,abx^2+5\,a^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{45\,a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(32*b^2*x^4 - 8*a*b*x^2 + 5*a^2)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^3*c^6
*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

maple [A] time = 0.00, size = 42, normalized size = 0.51

$$-\frac{2\left(b\,x^2+a\right)^{\frac{1}{4}}\left(32b^2x^4-8ab\,x^2+5a^2\right)x}{45\,(cx)^{\frac{11}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/45*x*(b*x^2+a)^(1/4)*(32*b^2*x^4-8*a*b*x^2+5*a^2)/a^3/(c*x)^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

mupad [B] time = 5.17, size = 54, normalized size = 0.65

$$-\frac{\left(b\,x^2+a\right)^{1/4}\left(\frac{2}{9\,a\,c^5}-\frac{16\,b\,x^2}{45\,a^2\,c^5}+\frac{64\,b^2\,x^4}{45\,a^3\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*(2/(9*a*c^5) - (16*b*x^2)/(45*a^2*c^5) + (64*b^2*x^4)/(
45*a^3*c^5)))/(x^4*(c*x)^(1/2))
```

sympy [B] time = 172.06, size = 483, normalized size = 5.82

$$\frac{5a^4b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)}+\frac{2a^3b^{\frac{21}{4}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)}+\frac{21a^2b^{\frac{25}{4}}x^4\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)}+\frac{56ab^{\frac{29}{4}}x^6\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)}+\frac{32b^{\frac{33}{4}}x^8\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] 5*a**4*b**(17/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2
)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c
**(11/2)*x**8*gamma(3/4)) + 2*a**3*b**(21/4)*x**2*(a/(b*x**2) + 1)**(1/4)*g
amma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)
*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4)) + 21*a**2*b**(25
/4)*x**4*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*g
amma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)
*x**8*gamma(3/4)) + 56*a*b**(29/4)*x**6*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)
/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamm
a(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4)) + 32*b**(33/4)*x**8*(a/(b*
x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*
a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4
))
```

3.680 $\int \frac{(cx)^{5/2}}{\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=308

$$\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3ac}{}$$

Rubi [A] time = 0.26, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {321, 329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3ac^{5/2}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{4\sqrt{2}\,b^{7/4}}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] -(c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/(2*b) - (3*a*c^(5/2)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))])/(4*Sqrt[2]*b^(7/4)) + (3*
a*c^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4)
)])/(4*Sqrt[2]*b^(7/4)) + (3*a*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sq
rt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*
b^(7/4)) - (3*a*c^(5/2)*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] +
 (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(8*Sqrt[2]*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(cx)^{5/2}}{\left(a-bx^2\right)^{3/4}}\,dx = -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{\left(3ac^2\right)\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx}{4b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{(3ac)\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{2b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{(3ac)\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2b}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} - \frac{(3ac)\,\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{4b^{3/2}} + \frac{(3ac)\,\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \ldots\right.}{4b^{3/2}}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{\left(3ac^{5/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{7/4}} + \frac{\left(3ac^{5/2}\right)\text{Subst}\left(\int \ldots\right.}{8\ldots}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} + \frac{3ac^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{8\sqrt{2}\,b^{7/4}} - \frac{3ac^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\ldots\right.}{8\sqrt{2}\,b^{7/4}}$$

$$= -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} - \frac{3ac^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}} + \frac{3ac^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}\,b^{7/4}} + \frac{3ac^{5/2}\,\text{lo}\ldots}{\ldots}$$

Mathematica [A] time = 0.06, size = 112, normalized size = 0.36

$$-\frac{(cx)^{5/2}\left(-3a\sqrt[4]{-b}\,\tan^{-1}\left(\frac{\sqrt[4]{-b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)+3a\sqrt[4]{-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{-b}\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)+2bx^{3/2}\sqrt[4]{a-bx^2}\right)}{4b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] -1/4*((c*x)^(5/2)*(2*b*x^(3/2)*(a - b*x^2)^(1/4) - 3*a*(-b)^(1/4)*ArcTan[((
-b)^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + 3*a*(-b)^(1/4)*ArcTanh[((-b)^(1/4)*
Sqrt[x])/(a - b*x^2)^(1/4)]))/(b^2*x^(5/2))
```

IntegrateAlgebraic [A] time = 9.60, size = 257, normalized size = 0.83

$$\frac{\sqrt{c}\sqrt[4]{a-bx^2}\left(\frac{3ac^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}\sqrt[4]{ac^2-bc^2x^2}}{\sqrt{ac^2-bc^2x^2}-\sqrt{b}cx}\right)}{4\sqrt{2}b^{7/4}}-\frac{3ac^{5/2}\tanh^{-1}\left(\frac{\frac{\sqrt{ac^2-bc^2x^2}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}cx}{\sqrt{2}}}{\sqrt{cx}\sqrt[4]{ac^2-bc^2x^2}}\right)}{4\sqrt{2}b^{7/4}}-\frac{\sqrt{c}(cx)^{3/2}\sqrt[4]{ac^2-bc^2x^2}}{2b}\right)}{\sqrt[4]{ac^2-bc^2x^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```

```
[Out] (Sqrt[c]*(a - b*x^2)^(1/4)*(-1/2*(Sqrt[c]*(c*x)^(3/2)*(a*c^2 - b*c^2*x^2)^(
1/4))/b + (3*a*c^(5/2)*ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[c*x]*(a*c^2 - b*c^2*x^2
)^(1/4))/(-(Sqrt[b]*c*x) + Sqrt[a*c^2 - b*c^2*x^2])])/(4*Sqrt[2]*b^(7/4)) -
 (3*a*c^(5/2)*ArcTanh[((b^(1/4)*c*x)/Sqrt[2] + Sqrt[a*c^2 - b*c^2*x^2]/(Sqr
t[2]*b^(1/4)))/(Sqrt[c*x]*(a*c^2 - b*c^2*x^2)^(1/4))])/(4*Sqrt[2]*b^(7/4)))
)/(a*c^2 - b*c^2*x^2)^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}}}{\left(-bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x)^{5/2}}{\left(a-b\,x^{2}\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(a - b*x^2)^(3/4),x)
```

```
[Out] int((c*x)^(5/2)/(a - b*x^2)^(3/4), x)
```

sympy [C] time = 7.82, size = 46, normalized size = 0.15

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(5/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(2*
I*pi)/a)/(2*a**(3/4)*gamma(11/4))
```

3.681 $\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=272

$$\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,b^{3/4}}-\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}}+\frac{\sqrt{c}\tan^{-1}}{}$$

Rubi [A] time = 0.23, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {329, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,b^{3/4}}-\frac{\sqrt{c}\log\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{c}\right)}{2\sqrt{2}\,b^{3/4}}-\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

```
[Out] -((Sqrt[c]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4
))]])/(Sqrt[2]*b^(3/4))) + (Sqrt[c]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(
Sqrt[c]*(a - b*x^2)^(1/4))]])/(Sqrt[2]*b^(3/4)) + (Sqrt[c]*Log[Sqrt[c] + (Sq
rt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^
(1/4)])/(2*Sqrt[2]*b^(3/4)) - (Sqrt[c]*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sq
rt[a - b*x^2] + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a - b*x^2)^(1/4)])/(2*Sqrt[2]*
b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\left(a-bx^2\right)^{3/4}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a-\frac{bx^4}{c^2}\right)^{3/4}}\,dx, x, \sqrt{cx}\right)}{c}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{c-\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt{b}\,c} + \frac{\text{Subst}\left(\int \frac{c+\sqrt{b}\,x^2}{1+\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt{b}\,c}$$

$$= \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt{c}}{\sqrt[4]{b}}+2x}{-\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,b^{3/4}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt{c}}{\sqrt[4]{b}}-2x}{-\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,b^{3/4}} + \cdots$$

$$= \frac{\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,b^{3/4}} - \frac{\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2\sqrt{2}\,b^{3/4}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}} + \frac{\sqrt{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\,b^{3/4}} + \frac{\sqrt{c}\,\log\left(\sqrt{c}+\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{a-bx^2}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\cdots}{\sqrt[4]{a-bx^2}\cdots}\right.}{2\sqrt{2}\,b^{3/4}}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 0.28

$$\frac{\sqrt{cx}\left(\tanh^{-1}\left(\frac{\sqrt[4]{-b}\,\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{-b}\,\sqrt{x}}{\sqrt[4]{a-bx^2}}\right)\right)}{(-b)^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

```
[Out] (Sqrt[c*x]*(-ArcTan[((-b)^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)] + ArcTanh[((-b)
^(1/4)*Sqrt[x])/(a - b*x^2)^(1/4)]))/((-b)^(3/4)*Sqrt[x])
```

IntegrateAlgebraic [A] time = 5.60, size = 164, normalized size = 0.60

$$-\frac{\sqrt{c}\tan^{-1}\left(\frac{\frac{\sqrt{c}\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\sqrt{c}x}{\sqrt{2}}}{\sqrt{cx}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}\sqrt[4]{a-bx^2}}{c\sqrt{a-bx^2}+\sqrt{b}cx}\right)}{\sqrt{2}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

```
[Out] -((Sqrt[c]*ArcTan[(-((b^(1/4)*Sqrt[c]*x)/Sqrt[2]) + (Sqrt[c]*Sqrt[a - b*x^2
])/(Sqrt[2]*b^(1/4)))/(Sqrt[c*x]*(a - b*x^2)^(1/4))])/(Sqrt[2]*b^(3/4))) -
(Sqrt[c]*ArcTanh[(Sqrt[2]*b^(1/4)*Sqrt[c]*Sqrt[c*x]*(a - b*x^2)^(1/4))/(Sqr
t[b]*c*x + c*Sqrt[a - b*x^2])])/(Sqrt[2]*b^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x}}{{\left(a-b\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a - b*x^2)^(3/4),x)
[Out] int((c*x)^(1/2)/(a - b*x^2)^(3/4), x)
```

sympy [C] time = 1.56, size = 46, normalized size = 0.17

$$\frac{\sqrt{c}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(-b*x**2+a)**(3/4),x)
[Out] sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(2*I*
pi)/a)/(2*a**(3/4)*gamma(7/4))
```

3.682 $\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=27

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {264}

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.93

$$-\frac{2x\sqrt[4]{a-bx^2}}{a(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a - b*x^2)^(1/4))/(a*(c*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.39, size = 27, normalized size = 1.00

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

fricas [A] time = 1.46, size = 26, normalized size = 0.96

$$-\frac{2\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] -2*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.81

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{1}{4}}x}{(cx)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
[Out] -2*x*(-b*x^2+a)^(1/4)/a/(c*x)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

mupad [B] time = 5.08, size = 23, normalized size = 0.85

$$-\frac{2\left(a-b\,x^2\right)^{1/4}}{a\,c\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x)
```

```
[Out] -(2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(1/2))
```

sympy [A] time = 3.00, size = 90, normalized size = 3.33

$$\begin{cases} \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{for}\ \left|\frac{a}{bx^2}\right|>1 \\ -\frac{\sqrt[4]{b}\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{-\frac{3i\pi}{4}}\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma
(3/4)), Abs(a/(b*x**2)) > 1), (-b**(1/4)*(-a/(b*x**2) + 1)**(1/4)*exp(-3*I*
pi/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4)), True))
```

3.683 $\int \frac{1}{(cx)^{7/2}(a-bx^2)^{3/4}}\,dx$

Optimal. Leaf size=57

$$\frac{8(a-bx^2)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$\frac{8(a-bx^2)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a - b*x^2)^(5/4))/(5*a^2*c*(
c*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}-\frac{4\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx}{a}\\ &= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}+\frac{8\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.61

$$-\frac{2x\sqrt[4]{a-bx^2}\left(a+4bx^2\right)}{5a^2(cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
[Out] (-2*x*(a - b*x^2)^(1/4)*(a + 4*b*x^2))/(5*a^2*(c*x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.56, size = 44, normalized size = 0.77

$$-\frac{2\sqrt[4]{a-bx^2}\left(ac^2+4bc^2x^2\right)}{5a^2c^3(cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
[Out] (-2*(a - b*x^2)^(1/4)*(a*c^2 + 4*b*c^2*x^2))/(5*a^2*c^3*(c*x)^(5/2))
```

fricas [A] time = 1.68, size = 34, normalized size = 0.60

$$-\frac{2\left(4\,bx^2+a\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{5\,a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
[Out] -2/5*(4*b*x^2 + a)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

maple [A] time = 0.01, size = 30, normalized size = 0.53

$$-\frac{2 \left(-b\,x^{2}+a\right)^{\frac{1}{4}} \left(4 b\,x^{2}+a\right) x}{5 \left(c x\right)^{\frac{7}{2}} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] -2/5*x*(-b*x^2+a)^(1/4)*(4*b*x^2+a)/a^2/(c*x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b x^{2}+a\right)^{\frac{3}{4}} \left(c x\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

mupad [B] time = 5.10, size = 41, normalized size = 0.72

$$-\frac{{\left(a-b\,x^2\right)}^{1/4}\,\left(\frac{2}{5\,a\,c^3}+\frac{8\,b\,x^2}{5\,a^2\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x)
```

```
[Out] -((a - b*x^2)^(1/4)*(2/(5*a*c^3) + (8*b*x^2)/(5*a^2*c^3)))/(x^2*(c*x)^(1/2)
)
```

sympy [A] time = 25.45, size = 352, normalized size = 6.18

$$\begin{cases} - \frac{\sqrt[4]{b} \sqrt[4]{\frac{a}{b x^{2}} - 1} \Gamma\left(- \frac{5}{4}\right)}{8 a c^{\frac{7}{2}} x^{2} \Gamma\left(\frac{3}{4}\right)} - \frac{b^{\frac{5}{4}} \sqrt[4]{\frac{a}{b x^{2}} - 1} \Gamma\left(- \frac{5}{4}\right)}{2 a^{2} c^{\frac{7}{2}} \Gamma\left(\frac{3}{4}\right)} & \text{for}\: \left|{\frac{a}{b x^{2}}}\right| > 1 \\- \frac{a^{2} b^{\frac{5}{4}} \sqrt[4]{- \frac{a}{b x^{2}} + 1} \Gamma\left(- \frac{5}{4}\right)}{- 8 a^{3} b c^{\frac{7}{2}} x^{2} e^{\frac{3 i \pi}{4}} \Gamma\left(\frac{3}{4}\right) + 8 a^{2} b^{2} c^{\frac{7}{2}} x^{4} e^{\frac{3 i \pi}{4}} \Gamma\left(\frac{3}{4}\right)} - \frac{3 a b^{\frac{9}{4}} x^{2} \sqrt[4]{- \frac{a}{b x^{2}} + 1} \Gamma\left(- \frac{5}{4}\right)}{- 8 a^{3} b c^{\frac{7}{2}} x^{2} e^{\frac{3 i \pi}{4}} \Gamma\left(\frac{3}{4}\right) + 8 a^{2} b^{2} c^{\frac{7}{2}} x^{4} e^{\frac{3 i \pi}{4}} \Gamma\left(\frac{3}{4}\right)} + \frac{4 b^{\frac{13}{4}} x^{4} \sqrt[4]{- \frac{a}{b x^{2}} + 1} \Gamma\left(- \frac{5}{4}\right)}{- 8 a^{3} b c^{\frac{7}{2}} x^{2} e^{\frac{3 i \pi}{4}} \Gamma\left(\frac{3}{4}\right) + 8 a^{2} b^{2} c^{\frac{7}{2}} x^{4} e^{\frac{3 i \pi}{4}} \Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(7/2)/(-b*x**2+a)**(3/4),x)
```

```
[Out] Piecewise((-b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(8*a*c**(7/2)*x**2
*gamma(3/4)) - b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*a**2*c**(7/2
)*gamma(3/4)), Abs(a/(b*x**2)) > 1), (-a**2*b**(5/4)*(-a/(b*x**2) + 1)**(1/
4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*gamma(3/4) + 8*a**2*b
**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)) - 3*a*b**(9/4)*x**2*(-a/(b*x**2
) + 1)**(1/4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*gamma(3/4)
 + 8*a**2*b**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)) + 4*b**(13/4)*x**4*(
-a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*
gamma(3/4) + 8*a**2*b**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)), True))
```

3.684 $\int \frac{1}{(cx)^{11/2}\left(a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=86

$$-\frac{64\left(a-bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {273, 264}

$$-\frac{64\left(a-bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}+\frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a - b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(9/2)) - (64*(a - b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}} - \frac{8\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx}{a}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}} + \frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}} + \frac{32\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{11/2}}\,dx}{5a^2}$$

$$= -\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}} + \frac{16\left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{9/2}} - \frac{64\left(a-bx^2\right)^{9/4}}{45a^3c(cx)^{9/2}}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.62

$$-\frac{2\sqrt{cx}\,\sqrt[4]{a-bx^2}\,\left(5a^2+8abx^2+32b^2x^4\right)}{45a^3c^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[c*x]*(a - b*x^2)^(1/4)*(5*a^2 + 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*c
^6*x^5)
```

IntegrateAlgebraic [A] time = 1.00, size = 59, normalized size = 0.69

$$-\frac{2\sqrt[4]{a-bx^2}\,\left(5a^2c^4+8abc^4x^2+32b^2c^4x^4\right)}{45a^3c^5(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a - b*x^2)^(1/4)*(5*a^2*c^4 + 8*a*b*c^4*x^2 + 32*b^2*c^4*x^4))/(45*a^3
*c^5*(c*x)^(9/2))
```

fricas [A] time = 0.85, size = 47, normalized size = 0.55

$$-\frac{2\left(32\,b^2x^4+8\,abx^2+5\,a^2\right)\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{45\,a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(32*b^2*x^4 + 8*a*b*x^2 + 5*a^2)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^3*c^
6*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

maple [A] time = 0.01, size = 43, normalized size = 0.50

$$-\frac{2\left(-b\,x^2+a\right)^{\frac{1}{4}}\left(32b^2x^4+8ab\,x^2+5a^2\right)x}{45\,(cx)^{\frac{11}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x)
```

```
[Out] -2/45*x*(-b*x^2+a)^(1/4)*(32*b^2*x^4+8*a*b*x^2+5*a^2)/a^3/(c*x)^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

mupad [B] time = 5.15, size = 55, normalized size = 0.64

$$-\frac{\left(a-b\,x^2\right)^{1/4}\left(\frac{2}{9\,a\,c^5}+\frac{16\,b\,x^2}{45\,a^2\,c^5}+\frac{64\,b^2\,x^4}{45\,a^3\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x)
```

[Out] -((a - b*x^2)^(1/4)*(2/(9*a*c^5) + (16*b*x^2)/(45*a^2*c^5) + (64*b^2*x^4)/(45*a^3*c^5)))/(x^4*(c*x)^(1/2))

sympy [C] time = 171.23, size = 1263, normalized size = 14.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(3/4),x)

[Out] Piecewise((-5*a**4*b**(17/4)*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) + 2*a**3*b**(21/4)*x**2*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) - 21*a**2*b**(25/4)*x**4*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) + 56*a*b**(29/4)*x**6*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) - 32*b**(33/4)*x**8*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)), Abs(a/(b*x**2)) > 1), (-5*a**4*b**(17/4)*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) + 2*a**3*b**(21/4)*x**2*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) - 21*a**2*b**(25/4)*x**4*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) + 56*a*b**(29/4)*x**6*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) - 32*b**(33/4)*x**8*(-a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)), True))

3.685 $\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/4}} \, dx$

Optimal. Leaf size=146

$$-\frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}$$

Rubi [A] time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {285, 288, 329, 240, 212, 208, 205}

$$\frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}}-\frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (5*a*c^3*Sqrt[c*x])/(2*b^2*(a + b*x^2)^(1/4)) + (c*(c*x)^(5/2))/(2*b*(a + b
*x^2)^(1/4)) - (5*a*c^(7/2)*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)
^(1/4))])/(4*b^(9/4)) - (5*a*c^(7/2)*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(
a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{\left(a+bx^2\right)^{5/4}}\,dx = \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^2\right)\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{4b}$$

$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^4\right)\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a+bx^2}}\,dx}{4b^2}$$

$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx, x, \sqrt{cx}\right)}{2b^2}$$

$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^3\right)\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b^2}$$

$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{\left(5ac^4\right)\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{4b^2} - \frac{\left(5ac^4\right)\text{Subst}\left(\int \frac{1}{c}\right.}{4}$$

$$= \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} - \frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.43

$$\frac{c(cx)^{5/2}\left(1-\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^2}{a}\right)\right)}{2b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (c*(c*x)^(5/2)*(1 - (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4,
-((b*x^2)/a)]))/(2*b*(a + b*x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 2.71, size = 183, normalized size = 1.25

$$-\frac{5ac^{7/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{4b^{9/4}} - \frac{5ac^{7/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{4b^{9/4}} + \frac{\left(a+bx^2\right)^{3/4}\left(5ac^5\sqrt{cx}+bc^3(cx)^{5/2}\right)}{2b^2\left(ac^2+bc^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] ((a + b*x^2)^(3/4)*(5*a*c^5*Sqrt[c*x] + b*c^3*(c*x)^(5/2)))/(2*b^2*(a*c^2 +
 b*c^2*x^2)) - (5*a*c^(7/2)*ArcTan[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(
3/4))/(a*c^2 + b*c^2*x^2)])/(4*b^(9/4)) - (5*a*c^(7/2)*ArcTanh[(b^(1/4)*c^(
3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)])/(4*b^(9/4))
```

fricas [B] time = 1.49, size = 389, normalized size = 2.66

$$\frac{4\left(bc^{3}x^{2}+5\,ac^{3}\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}+20\left(\frac{a^{4}c^{14}}{b^{9}}\right)^{\frac{1}{4}}\left(b^{3}x^{2}+ab^{2}\right)\arctan\left(-\frac{\left(\frac{a^{4}c^{14}}{b^{9}}\right)^{\frac{3}{4}}\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}\,ab^{7}c^{3}-\left(b^{8}x^{2}+ab^{7}\right)\left(\frac{a^{4}c^{14}}{b^{9}}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^{2}+a}\,a^{2}c^{7}x+\sqrt{\frac{a^{4}c^{14}}{b^{9}}}\left(b^{5}x^{2}+ab^{4}\right)}{bx^{2}+a}}}{a^{4}bc^{14}x^{2}+a^{5}c^{14}}\right)-5\left(\frac{a^{4}c^{14}}{b^{9}}\right)^{\frac{1}{4}}\left(b^{3}x^{2}+ab^{2}\right)\log\left(\frac{5\left(\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}\,ac^{3}+\left(\frac{a^{4}c^{14}}{b^{9}}\right)^{\frac{1}{4}}\left(b^{3}x^{2}+ab^{2}\right)\right)}{bx^{2}+a}\right)+5\left(\frac{a^{4}c^{14}}{b^{9}}\right)^{\frac{1}{4}}\left(b^{3}x^{2}+ab^{2}\right)\log\left(\frac{5\left(\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}\,ac^{3}-\left(\frac{a^{4}c^{14}}{b^{9}}\right)^{\frac{1}{4}}\left(b^{3}x^{2}+ab^{2}\right)\right)}{bx^{2}+a}\right)}{8\left(b^{3}x^{2}+ab^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^3*x^2 + 5*a*c^3)*(b*x^2 + a)^(3/4)*sqrt(c*x) + 20*(a^4*c^14/b^9
)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((a^4*c^14/b^9)^(3/4)*(b*x^2 + a)^(3/4)*s
qrt(c*x)*a*b^7*c^3 - (b^8*x^2 + a*b^7)*(a^4*c^14/b^9)^(3/4)*sqrt((sqrt(b*x^
2 + a)*a^2*c^7*x + sqrt(a^4*c^14/b^9)*(b^5*x^2 + a*b^4))/(b*x^2 + a)))/(a^4
*b*c^14*x^2 + a^5*c^14)) - 5*(a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*(
(b*x^2 + a)^(3/4)*sqrt(c*x)*a*c^3 + (a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2))
/(b*x^2 + a)) + 5*(a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*((b*x^2 + a)
^(3/4)*sqrt(c*x)*a*c^3 - (a^4*c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2))/(b*x^2 + a
)))/(b^3*x^2 + a*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^{2}+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)

[Out] int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")

[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/2}}{\left(b\,x^2+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x)^(7/2)/(a + b*x^2)^(5/4),x)

[Out] int((c*x)^(7/2)/(a + b*x^2)^(5/4), x)

sympy [C] time = 24.49, size = 44, normalized size = 0.30

$$\frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x)**(7/2)/(b*x**2+a)**(5/4),x)

[Out] c**(7/2)*x**(9/2)*gamma(9/4)*hyper((5/4, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(13/4))

3.686 $\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=107

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {288, 329, 240, 212, 208, 205}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (-2*c*Sqrt[c*x])/(b*(a + b*x^2)^(1/4)) + (c^(3/2)*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(5/4) + (c^(3/2)*ArcTanh[(b^(1/4)*Sqrt[c*
x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/b^(5/4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx &= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{c^2\int \frac{1}{\sqrt{cx}\,\sqrt[4]{a+bx^2}}\,dx}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{c^2}}}\,dx,x,\sqrt{cx}\right)}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{c^2}}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{c-\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{c+\sqrt{b}\,x^2}\,dx,x,\frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{b}\\
&= -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{cx}}{\sqrt{c}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.55

$$\frac{2x(cx)^{3/2}\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^2}{a}\right)}{5a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (2*x*(c*x)^(3/2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -((
b*x^2)/a)])/(5*a*(a + b*x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 1.27, size = 157, normalized size = 1.47

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{b^{5/4}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,c^{3/2}\sqrt{cx}\left(a+bx^2\right)^{3/4}}{ac^2+bc^2x^2}\right)}{b^{5/4}}-\frac{2c^3\sqrt{cx}\left(a+bx^2\right)^{3/4}}{b\left(ac^2+bc^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

```
[Out] (-2*c^3*Sqrt[c*x]*(a + b*x^2)^(3/4))/(b*(a*c^2 + b*c^2*x^2)) + (c^(3/2)*Arc
Tan[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^2 + b*c^2*x^2)])/b^(
5/4) + (c^(3/2)*ArcTanh[(b^(1/4)*c^(3/2)*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*c^
2 + b*c^2*x^2)])/b^(5/4)
```

fricas [B] time = 1.23, size = 319, normalized size = 2.98

$$-\frac{4\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c+4\left(b^2x^2+ab\right)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,b^4c\left(\frac{c^6}{b^5}\right)^{\frac{3}{4}}-\left(b^5x^2+ab^4\right)\left(\frac{c^6}{b^5}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}\,c^3x+\left(b^3x^2+ab^2\right)\sqrt{\frac{c^6}{b^5}}}{bx^2+a}}}{bc^6x^2+ac^6}\right)-\left(b^2x^2+ab\right)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c+\left(b^2x^2+ab\right)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}}{bx^2+a}\right)+\left(b^2x^2+ab\right)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}\,c-\left(b^2x^2+ab\right)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}}{bx^2+a}\right)}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*a
rctan(-((b*x^2 + a)^(3/4)*sqrt(c*x)*b^4*c*(c^6/b^5)^(3/4) - (b^5*x^2 + a*b^
4)*(c^6/b^5)^(3/4)*sqrt((sqrt(b*x^2 + a)*c^3*x + (b^3*x^2 + a*b^2)*sqrt(c^6
/b^5))/(b*x^2 + a)))/(b*c^6*x^2 + a*c^6)) - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)
*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*c + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x
^2 + a)) + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)
*c - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x^2 + a)))/(b^2*x^2 + a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(b\,x^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
[Out] int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{3/2}}{\left(b\,x^2+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3/2)/(a + b*x^2)^(5/4),x)
[Out] int((c*x)^(3/2)/(a + b*x^2)^(5/4), x)
```

sympy [C] time = 4.26, size = 44, normalized size = 0.41

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(5/4)*gamma(9/4))
```

3.687 $\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {264}

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*Sqrt[c*x])/(a*c*(a + b*x^2)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/4}}\,dx=\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.92

$$\frac{2x}{a\sqrt{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*x)/(a*Sqrt[c*x]*(a + b*x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 0.57, size = 40, normalized size = 1.54

$$\frac{2c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{a\left(ac^2+bc^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]
[Out] (2*c*Sqrt[c*x]*(a + b*x^2)^(3/4))/(a*(a*c^2 + b*c^2*x^2))
```

fricas [A] time = 0.93, size = 31, normalized size = 1.19

$$\frac{2\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{abcx^2+a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
[Out] 2*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a*b*c*x^2 + a^2*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{2x}{\left(b\,x^2+a\right)^{\frac{1}{4}}\sqrt{cx}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x)
[Out] 2*x/(b*x^2+a)^(1/4)/a/(c*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{5}{4}}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)
```

mupad [B] time = 4.97, size = 29, normalized size = 1.12

$$\frac{2\,x\,\left(b\,x^2+a\right)^{3/4}}{\left(a^2+b\,a\,x^2\right)\,\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(a + b*x^2)^(5/4)),x)
[Out] (2*x*(a + b*x^2)^(3/4))/((a^2 + a*b*x^2)*(c*x)^(1/2))
```

sympy [A] time = 3.14, size = 34, normalized size = 1.31

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\,\sqrt{c}\,\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/4),x)
[Out] gamma(1/4)/(2*a*b**(1/4)*sqrt(c)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4))
```

3.688 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=55

$$\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(3*a^2*c*(c*x
)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx &= \frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}+\frac{4\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx}{a}\\ &= \frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.62

$$-\frac{2x\left(a+4bx^2\right)}{3a^2(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(a + 4*b*x^2))/(3*a^2*(c*x)^(5/2)*(a + b*x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 0.73, size = 59, normalized size = 1.07

$$-\frac{2\left(a+bx^2\right)^{3/4}\left(ac^2+4bc^2x^2\right)}{3a^2c(cx)^{3/2}\left(ac^2+bc^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4)*(a*c^2 + 4*b*c^2*x^2))/(3*a^2*c*(c*x)^(3/2)*(a*c^2 +
b*c^2*x^2))
```

fricas [A] time = 0.74, size = 48, normalized size = 0.87

$$-\frac{2\left(4\,bx^2+a\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{3\left(a^2bc^3x^4+a^3c^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/3*(4*b*x^2 + a)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*b*c^3*x^4 + a^3*c^3*x^2
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.53

$$-\frac{2\left(4b\,x^{2}+a\right)x}{3\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(cx\right)^{\frac{5}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x)
[Out] -2/3*x*(4*b*x^2+a)/(b*x^2+a)^(1/4)/a^2/(c*x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{5}{4}}\left(cx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)
```

mupad [B] time = 5.10, size = 57, normalized size = 1.04

$$-\frac{\left(b\,x^{2}+a\right)^{3/4}\left(\frac{2}{3\,a\,b\,c^{2}}+\frac{8\,x^{2}}{3\,a^{2}\,c^{2}}\right)}{x^{3}\,\sqrt{c\,x}+\frac{a\,x\,\sqrt{c\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x)
[Out] -((a + b*x^2)^(3/4)*(2/(3*a*b*c^2) + (8*x^2)/(3*a^2*c^2)))/(x^3*(c*x)^(1/2)
 + (a*x*(c*x)^(1/2))/b)
```

sympy [A] time = 14.99, size = 78, normalized size = 1.42

$$\frac{\Gamma\left(-\frac{3}{4}\right)}{8a\sqrt[4]{b}\,c^{\frac{5}{2}}x^{2}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(\frac{5}{4}\right)}+\frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2a^{2}c^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^{2}}+1}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] gamma(-3/4)/(8*a*b**(1/4)*c**(5/2)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(5/4))
 + b**(3/4)*gamma(-3/4)/(2*a**2*c**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4)
)
```

3.689 $\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=83

$$\frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}+\frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$\frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}-\frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}}+\frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(7/2)*(a + b*x^2)^(1/4)) - (16*(a + b*x^2)^(3/4))/(3*a^2*c*(c*
x)^(7/2)) + (64*(a + b*x^2)^(7/4))/(21*a^3*c*(c*x)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx = \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}} + \frac{8\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx}{a}$$

$$= \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}} - \frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}} - \frac{32\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{9/2}}\,dx}{3a^2}$$

$$= \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}} - \frac{16\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{7/2}} + \frac{64\left(a+bx^2\right)^{7/4}}{21a^3c(cx)^{7/2}}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 0.57

$$-\frac{2x\left(3a^2-8abx^2-32b^2x^4\right)}{21a^3(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(3*a^2 - 8*a*b*x^2 - 32*b^2*x^4))/(21*a^3*(c*x)^(9/2)*(a + b*x^2)^(1/
4))
```

IntegrateAlgebraic [A] time = 1.37, size = 74, normalized size = 0.89

$$\frac{2\left(a+bx^2\right)^{3/4}\left(-3a^2c^4+8abc^4x^2+32b^2c^4x^4\right)}{21a^3c^3(cx)^{7/2}\left(ac^2+bc^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*(a + b*x^2)^(3/4)*(-3*a^2*c^4 + 8*a*b*c^4*x^2 + 32*b^2*c^4*x^4))/(21*a^3
*c^3*(c*x)^(7/2)*(a*c^2 + b*c^2*x^2))
```

fricas [A] time = 0.93, size = 61, normalized size = 0.73

$$\frac{2\left(32\,b^2x^4+8\,abx^2-3\,a^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{21\left(a^3bc^5x^6+a^4c^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/21*(32*b^2*x^4 + 8*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3*b*c^
5*x^6 + a^4*c^5*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)
```

maple [A] time = 0.01, size = 42, normalized size = 0.51

$$-\frac{2\left(-32b^2x^4-8ab\,x^2+3a^2\right)x}{21\left(b\,x^2+a\right)^{\frac{1}{4}}(cx)^{\frac{9}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/21*x*(-32*b^2*x^4-8*a*b*x^2+3*a^2)/(b*x^2+a)^(1/4)/a^3/(c*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)
```

mupad [B] time = 5.15, size = 70, normalized size = 0.84

$$\frac{\left(b\,x^2+a\right)^{3/4}\left(\frac{16\,x^2}{21\,a^2\,c^4}-\frac{2}{7\,a\,b\,c^4}+\frac{64\,b\,x^4}{21\,a^3\,c^4}\right)}{x^5\,\sqrt{c\,x}+\frac{a\,x^3\,\sqrt{c\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] ((a + b*x^2)^(3/4)*((16*x^2)/(21*a^2*c^4) - 2/(7*a*b*c^4) + (64*b*x^4)/(21*
a^3*c^4)))/(x^5*(c*x)^(1/2) + (a*x^3*(c*x)^(1/2))/b)
```

sympy [B] time = 109.04, size = 384, normalized size = 4.63

$$-\frac{3a^3b^{\frac{19}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}+\frac{5a^2b^{\frac{23}{4}}x^2\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}+\frac{40ab^{\frac{27}{4}}x^4\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}+\frac{32b^{\frac{31}{4}}x^6\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] -3*a**3*b**(19/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**4*c**(9/2
)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**3*b**6*c*
*(9/2)*x**6*gamma(5/4)) + 5*a**2*b**(23/4)*x**2*(a/(b*x**2) + 1)**(3/4)*gam
ma(-7/4)/(32*a**5*b**4*c**(9/2)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**
4*gamma(5/4) + 32*a**3*b**6*c**(9/2)*x**6*gamma(5/4)) + 40*a*b**(27/4)*x**4
*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**4*c**(9/2)*x**2*gamma(5/4)
 + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**3*b**6*c**(9/2)*x**6*gamma
(5/4)) + 32*b**(31/4)*x**6*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**
4*c**(9/2)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**
3*b**6*c**(9/2)*x**6*gamma(5/4))
```

3.690 $\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=109

$$-\frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {273, 264}

$$-\frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}+\frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}}-\frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}}+\frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] 2/(a*c*(c*x)^(11/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(a^2*c*(c*x)
^(11/2)) + (64*(a + b*x^2)^(7/4))/(7*a^3*c*(c*x)^(11/2)) - (256*(a + b*x^2)
^(11/4))/(77*a^4*c*(c*x)^(11/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx = \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}} + \frac{12\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx}{a}$$

$$= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}} - \frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}} - \frac{32\int \frac{\left(a+bx^2\right)^{3/4}}{(cx)^{13/2}}\,dx}{a^2}$$

$$= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}} - \frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}} + \frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}} + \frac{128\int \frac{\left(a+bx^2\right)^{7/4}}{(cx)^{13/2}}\,dx}{7a^3}$$

$$= \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}} - \frac{8\left(a+bx^2\right)^{3/4}}{a^2c(cx)^{11/2}} + \frac{64\left(a+bx^2\right)^{7/4}}{7a^3c(cx)^{11/2}} - \frac{256\left(a+bx^2\right)^{11/4}}{77a^4c(cx)^{11/2}}$$

Mathematica [A] time = 0.01, size = 58, normalized size = 0.53

$$-\frac{2x\left(7a^3-12a^2bx^2+32ab^2x^4+128b^3x^6\right)}{77a^4(cx)^{13/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*x*(7*a^3 - 12*a^2*b*x^2 + 32*a*b^2*x^4 + 128*b^3*x^6))/(77*a^4*(c*x)^(1
3/2)*(a + b*x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 3.49, size = 88, normalized size = 0.81

$$-\frac{2\left(a+bx^2\right)^{3/4}\left(7a^3c^6-12a^2bc^6x^2+32ab^2c^6x^4+128b^3c^6x^6\right)}{77a^4c^5(cx)^{11/2}\left(ac^2+bc^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4)*(7*a^3*c^6 - 12*a^2*b*c^6*x^2 + 32*a*b^2*c^6*x^4 + 12
8*b^3*c^6*x^6))/(77*a^4*c^5*(c*x)^(11/2)*(a*c^2 + b*c^2*x^2))
```

fricas [A] time = 0.85, size = 72, normalized size = 0.66

$$-\frac{2\left(128\,b^3x^6+32\,ab^2x^4-12\,a^2bx^2+7\,a^3\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{77\left(a^4bc^7x^8+a^5c^7x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/77*(128*b^3*x^6 + 32*a*b^2*x^4 - 12*a^2*b*x^2 + 7*a^3)*(b*x^2 + a)^(3/4)
*sqrt(c*x)/(a^4*b*c^7*x^8 + a^5*c^7*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.49

$$-\frac{2\left(128b^3x^6+32a\,b^2x^4-12a^2b\,x^2+7a^3\right)x}{77\left(b\,x^2+a\right)^{\frac{1}{4}}(cx)^{\frac{13}{2}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/77*x*(128*b^3*x^6+32*a*b^2*x^4-12*a^2*b*x^2+7*a^3)/(b*x^2+a)^(1/4)/a^4/(
c*x)^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)
```

mupad [B] time = 5.20, size = 85, normalized size = 0.78

$$-\frac{\left(b\,x^2+a\right)^{3/4}\left(\frac{2}{11\,a\,b\,c^6}-\frac{24\,x^2}{77\,a^2\,c^6}+\frac{64\,b\,x^4}{77\,a^3\,c^6}+\frac{256\,b^2\,x^6}{77\,a^4\,c^6}\right)}{x^7\,\sqrt{c\,x}+\frac{a\,x^5\,\sqrt{c\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*(2/(11*a*b*c^6) - (24*x^2)/(77*a^2*c^6) + (64*b*x^4)/(7
7*a^3*c^6) + (256*b^2*x^6)/(77*a^4*c^6)))/(x^7*(c*x)^(1/2) + (a*x^5*(c*x)^(
1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

3.691 $\int x^7 \left(a+bx^2\right)^p dx$

Optimal. Leaf size=100

$$-\frac{a^3 \left(a+bx^2\right)^{p+1}}{2b^4(p+1)}+\frac{3a^2 \left(a+bx^2\right)^{p+2}}{2b^4(p+2)}-\frac{3a \left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}$$

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{a^3 \left(a+bx^2\right)^{p+1}}{2b^4(p+1)}+\frac{3a^2 \left(a+bx^2\right)^{p+2}}{2b^4(p+2)}-\frac{3a \left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^p,x]
```

```
[Out] -(a^3*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*a^2*(a + b*x^2)^(2 + p))/(2
*b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(2*b^4*(3 + p)) + (a + b*x^2)^(4
+ p)/(2*b^4*(4 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^7 \left(a+bx^2\right)^p dx = \frac{1}{2}\text{Subst}\left(\int x^3(a+bx)^p dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^3(a+bx)^p}{b^3} + \frac{3a^2(a+bx)^{1+p}}{b^3} - \frac{3a(a+bx)^{2+p}}{b^3} + \frac{(a+bx)^{3+p}}{b^3}\right) dx, x, x^2\right)$$
$$= -\frac{a^3\left(a+bx^2\right)^{1+p}}{2b^4(1+p)} + \frac{3a^2\left(a+bx^2\right)^{2+p}}{2b^4(2+p)} - \frac{3a\left(a+bx^2\right)^{3+p}}{2b^4(3+p)} + \frac{\left(a+bx^2\right)^{4+p}}{2b^4(4+p)}$$

Mathematica [A] time = 0.05, size = 95, normalized size = 0.95

$$\frac{1}{2}\left(-\frac{a^3\left(a+bx^2\right)^{p+1}}{b^4(p+1)} + \frac{3a^2\left(a+bx^2\right)^{p+2}}{b^4(p+2)} - \frac{3a\left(a+bx^2\right)^{p+3}}{b^4(p+3)} + \frac{\left(a+bx^2\right)^{p+4}}{b^4(p+4)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^p,x]
```

```
[Out] (-((a^3*(a + b*x^2)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x^2)^(2 + p))/(
b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(b^4*(3 + p)) + (a + b*x^2)^(4 + p
)/(b^4*(4 + p)))/2
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^7 \left(a+bx^2\right)^p dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^7*(a + b*x^2)^p, x]
```

fricas [A] time = 0.91, size = 148, normalized size = 1.48

$$\frac{\left(\left(b^4p^3 + 6\,b^4p^2 + 11\,b^4p + 6\,b^4\right)x^8 + 6\,a^3bpx^2 + \left(ab^3p^3 + 3\,ab^3p^2 + 2\,ab^3p\right)x^6 - 3\left(a^2b^2p^2 + a^2b^2p\right)x^4 - 6\,a^4\right)\left(bx^2+a\right)^p}{2\left(b^4p^4 + 10\,b^4p^3 + 35\,b^4p^2 + 50\,b^4p + 24\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^8 + 6*a^3*b*p*x^2 + (a*b^3*
p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^6 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^4 - 6*a^
4)*(b*x^2 + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
```

giac [**B**] time = 0.59, size = 410, normalized size = 4.10

$$\frac{(bx^2+a)^4(bx^2+a)^p p^3 - 3(bx^2+a)^3(bx^2+a)^p a p^3 + 3(bx^2+a)^2(bx^2+a)^p a^2 p^3 - (bx^2+a)(bx^2+a)^p a^3 p^3 + 6(bx^2+a)^4(bx^2+a)^p p^2 - 21(bx^2+a)^3(bx^2+a)^p a p^2 + 24(bx^2+a)^2(bx^2+a)^p a^2 p^2 - 9(bx^2+a)(bx^2+a)^p a^3 p^2 + 11(bx^2+a)^4(bx^2+a)^p p - 42(bx^2+a)^3(bx^2+a)^p a p + 57(bx^2+a)^2(bx^2+a)^p a^2 p - 26(bx^2+a)(bx^2+a)^p a^3 p + 6(bx^2+a)^4(bx^2+a)^p - 24(bx^2+a)^3(bx^2+a)^p a + 36(bx^2+a)^2(bx^2+a)^p a^2 - 24(bx^2+a)(bx^2+a)^p a^3}{2(b^3p^4 + 10b^3p^3 + 35b^3p^2 + 50b^3p + 24b^3)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^4*(b*x^2 + a)^p*p^3 - 3*(b*x^2 + a)^3*(b*x^2 + a)^p*a*p^3
+ 3*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p^3 - (b*x^2 + a)*(b*x^2 + a)^p*a^3*p^3
 + 6*(b*x^2 + a)^4*(b*x^2 + a)^p*p^2 - 21*(b*x^2 + a)^3*(b*x^2 + a)^p*a*p^2
 + 24*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p^2 - 9*(b*x^2 + a)*(b*x^2 + a)^p*a^3
*p^2 + 11*(b*x^2 + a)^4*(b*x^2 + a)^p*p - 42*(b*x^2 + a)^3*(b*x^2 + a)^p*a*
p + 57*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p - 26*(b*x^2 + a)*(b*x^2 + a)^p*a^3
*p + 6*(b*x^2 + a)^4*(b*x^2 + a)^p - 24*(b*x^2 + a)^3*(b*x^2 + a)^p*a + 36*
(b*x^2 + a)^2*(b*x^2 + a)^p*a^2 - 24*(b*x^2 + a)*(b*x^2 + a)^p*a^3)/((b^3*p
^4 + 10*b^3*p^3 + 35*b^3*p^2 + 50*b^3*p + 24*b^3)*b)
```

maple [**A**] time = 0.01, size = 132, normalized size = 1.32

$$-\frac{\left(-b^3p^3x^6 - 6b^3p^2x^6 - 11b^3p\,x^6 + 3a\,b^2p^2x^4 - 6b^3x^6 + 9a\,b^2p\,x^4 + 6a\,b^2x^4 - 6a^2bp\,x^2 - 6a^2b\,x^2 + 6a^3\right)\left(b\,x^2 + a\right)^{p+1}}{2\left(p^4 + 10p^3 + 35p^2 + 50p + 24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^p,x)
```

```
[Out] -1/2*(b*x^2+a)^(p+1)*(-b^3*p^3*x^6-6*b^3*p^2*x^6-11*b^3*p*x^6+3*a*b^2*p^2*x
^4-6*b^3*x^6+9*a*b^2*p*x^4+6*a*b^2*x^4-6*a^2*b*p*x^2-6*a^2*b*x^2+6*a^3)/b^4
/(p^4+10*p^3+35*p^2+50*p+24)
```

maxima [**A**] time = 1.45, size = 106, normalized size = 1.06

$$\frac{\left(\left(p^3 + 6\,p^2 + 11\,p + 6\right)b^4x^8 + \left(p^3 + 3\,p^2 + 2\,p\right)ab^3x^6 - 3\left(p^2 + p\right)a^2b^2x^4 + 6\,a^3bpx^2 - 6\,a^4\right)\left(bx^2 + a\right)^p}{2\left(p^4 + 10\,p^3 + 35\,p^2 + 50\,p + 24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^3 + 6*p^2 + 11*p + 6)*b^4*x^8 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^6 - 3*(
p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 6*a^4)*(b*x^2 + a)^p/((p^4 + 10*p^3
+ 35*p^2 + 50*p + 24)*b^4)
```

mupad [**B**] time = 4.97, size = 183, normalized size = 1.83

$$\left(b\,x^2+a\right)^p\,\left(\frac{x^8\,\left(p^3+6\,p^2+11\,p+6\right)}{2\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}-\frac{3\,a^4}{b^4\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}+\frac{3\,a^3\,p\,x^2}{b^3\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}+\frac{a\,p\,x^6\,\left(p^2+3\,p+2\right)}{2\,b\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}-\frac{3\,a^2\,p\,x^4\,\left(p+1\right)}{2\,b^2\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2)^p,x)
```

```
[Out] (a + b*x^2)^p*((x^8*(11*p + 6*p^2 + p^3 + 6))/(2*(50*p + 35*p^2 + 10*p^3 +
p^4 + 24)) - (3*a^4)/(b^4*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (3*a^3*p*x
^2)/(b^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^6*(3*p + p^2 + 2))/(
2*b*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*a^2*p*x^4*(p + 1))/(2*b^2*(50
*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

sympy [A] time = 11.06, size = 2025, normalized size = 20.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(1
2*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*l
og(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x
**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6
*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(12*a*
*3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x*
*2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b*
*6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2
+ 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b)
+ x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 1
8*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**
2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36*a**2
*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(-I*sqrt(a)*sq
rt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x
**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**
5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(-I*sqrt(a
)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(I
*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**
3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(-I*sqrt(
a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x
**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4
) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a*b**2*x
**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**
4) - 6*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**
2 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4),
 Eq(p, -3)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2)
 + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4
*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b
**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 +
```

```
4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b**5*x**2) + b**3*x**6/(4*a*b**
4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**4)
 - a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4
/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**p/(2*b**4*p**4 +
 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a**3*b*p*x**2*(a +
 b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b*
*4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 7
0*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(a + b*x**2)**p/(2
*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + a*b**3*p
**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b
**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**
4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*x**6*(a + b*x**2
)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + b
**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 +
100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*
b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4*p*x**8*(a + b*x*
*2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) +
 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 1
00*b**4*p + 48*b**4), True))
```

3.692 $\int x^5 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=72

$$\frac{a^2\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{a\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}$$

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}-\frac{a\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^p,x]
```

```
[Out] (a^2*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*(a + b*x^2)^(2 + p))/(b^3*(2
 + p)) + (a + b*x^2)^(3 + p)/(2*b^3*(3 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^p\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^p\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right)dx,x,x^2\right)\\&=\frac{a^2\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}-\frac{a\left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.89

$$\frac{(a+bx^2)^{p+1}\left(2a^2-2ab(p+1)x^2+b^2\left(p^2+3p+2\right)x^4\right)}{2b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)*x^4))
/(2*b^3*(1 + p)*(2 + p)*(3 + p))
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^5\left(a+bx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^5*(a + b*x^2)^p, x]
```

fricas [A] time = 0.94, size = 98, normalized size = 1.36

$$\frac{\left(\left(b^3p^2+3\,b^3p+2\,b^3\right)x^6-2\,a^2bpx^2+\left(ab^2p^2+ab^2p\right)x^4+2\,a^3\right)\left(bx^2+a\right)^p}{2\left(b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((b^3*p^2 + 3*b^3*p + 2*b^3)*x^6 - 2*a^2*b*p*x^2 + (a*b^2*p^2 + a*b^2*p
)*x^4 + 2*a^3)*(b*x^2 + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

giac [B] time = 0.58, size = 231, normalized size = 3.21

$$\frac{\left(bx^2+a\right)^3\left(bx^2+a\right)^pp^2-2\left(bx^2+a\right)^2\left(bx^2+a\right)^pap^2+\left(bx^2+a\right)\left(bx^2+a\right)^pa^2p^2+3\left(bx^2+a\right)^3\left(bx^2+a\right)^pp-8\left(bx^2+a\right)^2\left(bx^2+a\right)^pap+5\left(bx^2+a\right)\left(bx^2+a\right)^pa^2p+2\left(bx^2+a\right)^3\left(bx^2+a\right)^p-6\left(bx^2+a\right)^2\left(bx^2+a\right)^pa+6\left(bx^2+a\right)\left(bx^2+a\right)^pa^2}{2\left(b^2p^3+6\,b^2p^2+11\,b^2p+6\,b^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^3*(b*x^2 + a)^p*p^2 - 2*(b*x^2 + a)^2*(b*x^2 + a)^p*a*p^2
+ (b*x^2 + a)*(b*x^2 + a)^p*a^2*p^2 + 3*(b*x^2 + a)^3*(b*x^2 + a)^p*p - 8*(
b*x^2 + a)^2*(b*x^2 + a)^p*a*p + 5*(b*x^2 + a)*(b*x^2 + a)^p*a^2*p + 2*(b*x
```

```
^2 + a)^3*(b*x^2 + a)^p - 6*(b*x^2 + a)^2*(b*x^2 + a)^p*a + 6*(b*x^2 + a)*(
b*x^2 + a)^p*a^2)/((b^2*p^3 + 6*b^2*p^2 + 11*b^2*p + 6*b^2)*b)
```

maple [A] time = 0.01, size = 80, normalized size = 1.11

$$\frac{\left(b^{2} p^{2} x^{4}+3 b^{2} p\, x^{4}+2 b^{2} x^{4}-2 a b p\, x^{2}-2 a b\, x^{2}+2 a^{2}\right) \left(b\, x^{2}+a\right)^{p +1}}{2 \left(p^{3}+6 p^{2}+11 p +6\right) b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^p,x)
[Out] 1/2*(b*x^2+a)^(p+1)*(b^2*p^2*x^4+3*b^2*p*x^4+2*b^2*x^4-2*a*b*p*x^2-2*a*b*x^
2+2*a^2)/b^3/(p^3+6*p^2+11*p+6)
```

maxima [A] time = 1.41, size = 73, normalized size = 1.01

$$\frac{{\left({\left(p^{2} + 3 \, p + 2\right)} b^{3} x^{6} + {\left(p^{2} + p\right)} a b^{2} x^{4} - 2 \, a^{2} b p x^{2} + 2 \, a^{3}\right)} {\left(b x^{2} + a\right)}^{p}}{2 \, {\left(p^{3} + 6 \, p^{2} + 11 \, p + 6\right)} b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^p,x, algorithm="maxima")
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

mupad [B] time = 4.90, size = 117, normalized size = 1.62

$${\left(b\,x^2+a\right)}^p\,\left(\frac{a^3}{b^3\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{x^6\,\left(p^2+3\,p+2\right)}{2\,\left(p^3+6\,p^2+11\,p+6\right)}-\frac{a^2\,p\,x^2}{b^2\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{a\,p\,x^4\,\left(p+1\right)}{2\,b\,\left(p^3+6\,p^2+11\,p+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^p,x)
[Out] (a + b*x^2)^p*(a^3/(b^3*(11*p + 6*p^2 + p^3 + 6)) + (x^6*(3*p + p^2 + 2))/(
2*(11*p + 6*p^2 + p^3 + 6)) - (a^2*p*x^2)/(b^2*(11*p + 6*p^2 + p^3 + 6)) +
(a*p*x^4*(p + 1))/(2*b*(11*p + 6*p^2 + p^3 + 6)))
```

sympy [A] time = 5.05, size = 981, normalized size = 13.62

$$\begin{cases} \frac{a^{p} x^{6}}{6} & \text{for}\: b = 0 \\ \frac{2 a^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{4 a^{2} b^{3} + 8 a b^{4} x^{2} + 4 b^{5} x^{4}} + \frac{2 a^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{4 a^{2} b^{3} + 8 a b^{4} x^{2} + 4 b^{5} x^{4}} + \frac{3 a^{2}}{4 a^{2} b^{3} + 8 a b^{4} x^{2} + 4 b^{5} x^{4}} + \frac{4 a b x^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{4 a^{2} b^{3} + 8 a b^{4} x^{2} + 4 b^{5} x^{4}} + \frac{4 a b x^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{4 a^{2} b^{3} + 8 a b^{4} x^{2} + 4 b^{5} x^{4}} + \frac{4 a b x^{2}}{4 a^{2} b^{3} + 8 a b^{4} x^{2} + 4 b^{5} x^{4}} + \frac{2 b^{2} x^{4} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{4 a^{2} b^{3} + 8 a b^{4} x^{2} + 4 b^{5} x^{4}} + \frac{2 b^{2} x^{4} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{4 a^{2} b^{3} + 8 a b^{4} x^{2} + 4 b^{5} x^{4}} & \text{for}\: p = -3 \\ - \frac{2 a^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{2 a b^{3} + 2 b^{4} x^{2}} - \frac{2 a^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{2 a b^{3} + 2 b^{4} x^{2}} - \frac{2 a^{2}}{2 a b^{3} + 2 b^{4} x^{2}} - \frac{2 a b x^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{2 a b^{3} + 2 b^{4} x^{2}} - \frac{2 a b x^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{2 a b^{3} + 2 b^{4} x^{2}} + \frac{b^{2} x^{4}}{2 a b^{3} + 2 b^{4} x^{2}} & \text{for}\: p = -2 \\ \frac{a^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{2 b^{3}} + \frac{a^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + x \right)}}{2 b^{3}} - \frac{a x^{2}}{2 b^{2}} + \frac{x^{4}}{4 b} & \text{for}\: p = -1 \\ \frac{2 a^{3} \left(a + b x^{2}\right)^{p}}{2 b^{3} p^{3} + 12 b^{3} p^{2} + 22 b^{3} p + 12 b^{3}} - \frac{2 a^{2} b p x^{2} \left(a + b x^{2}\right)^{p}}{2 b^{3} p^{3} + 12 b^{3} p^{2} + 22 b^{3} p + 12 b^{3}} + \frac{a b^{2} p^{2} x^{4} \left(a + b x^{2}\right)^{p}}{2 b^{3} p^{3} + 12 b^{3} p^{2} + 22 b^{3} p + 12 b^{3}} + \frac{a b^{2} p x^{4} \left(a + b x^{2}\right)^{p}}{2 b^{3} p^{3} + 12 b^{3} p^{2} + 22 b^{3} p + 12 b^{3}} + \frac{b^{3} p^{2} x^{6} \left(a + b x^{2}\right)^{p}}{2 b^{3} p^{3} + 12 b^{3} p^{2} + 22 b^{3} p + 12 b^{3}} + \frac{3 b^{3} p x^{6} \left(a + b x^{2}\right)^{p}}{2 b^{3} p^{3} + 12 b^{3} p^{2} + 22 b^{3} p + 12 b^{3}} + \frac{2 b^{3} x^{6} \left(a + b x^{2}\right)^{p}}{2 b^{3} p^{3} + 12 b^{3} p^{2} + 22 b^{3} p + 12 b^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4
*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b)
+ x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*
a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b)
 + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3
 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)
/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*sq
rt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a
**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*s
qrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*
x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x*
*4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x
)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) +
 x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**
2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 1
2*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b*
*3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)*
*p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a +
 b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x
**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*
b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3
), True))
```

3.693 $\int x^3 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=48

$$\frac{\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}-\frac{a\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}-\frac{a\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^p,x]
```

```
[Out] -(a*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (a + b*x^2)^(2 + p)/(2*b^2*(2 +
p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^2\right)^p \, dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^p \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right) dx, x, x^2\right)\\
&= -\frac{a\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.83

$$\frac{\left(a+bx^2\right)^{p+1}\left(b(p+1)x^2-a\right)}{2b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^(1 + p)*(-a + b*(1 + p)*x^2))/(2*b^2*(1 + p)*(2 + p))
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^3*(a + b*x^2)^p, x]
```

fricas [A] time = 0.81, size = 58, normalized size = 1.21

$$\frac{\left(abpx^2+\left(b^2p+b^2\right)x^4-a^2\right)\left(bx^2+a\right)^p}{2\left(b^2p^2+3\,b^2p+2\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*p*x^2 + (b^2*p + b^2)*x^4 - a^2)*(b*x^2 + a)^p/(b^2*p^2 + 3*b^2*p
+ 2*b^2)
```

giac [B] time = 0.58, size = 94, normalized size = 1.96

$$\frac{\left(bx^2+a\right)^2\left(bx^2+a\right)^pp-\left(bx^2+a\right)\left(bx^2+a\right)^pap+\left(bx^2+a\right)^2\left(bx^2+a\right)^p-2\left(bx^2+a\right)\left(bx^2+a\right)^pa}{2\left(p^2+3\,p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^2*(b*x^2 + a)^p*p - (b*x^2 + a)*(b*x^2 + a)^p*a*p + (b*x^2
 + a)^2*(b*x^2 + a)^p - 2*(b*x^2 + a)*(b*x^2 + a)^p*a)/((p^2 + 3*p + 2)*b^2
)
```

maple [A] time = 0.00, size = 42, normalized size = 0.88

$$-\frac{\left(-x^{2}pb-b\,x^{2}+a\right)\left(b\,x^{2}+a\right)^{p+1}}{2\left(p^{2}+3p+2\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p,x)
[Out] -1/2*(b*x^2+a)^(p+1)*(-b*p*x^2-b*x^2+a)/b^2/(p^2+3*p+2)
```

maxima [A] time = 1.29, size = 47, normalized size = 0.98

$$\frac{\left(b^{2}\left(p+1\right)x^{4}+abpx^{2}-a^{2}\right)\left(bx^{2}+a\right)^{p}}{2\left(p^{2}+3\,p+2\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p,x, algorithm="maxima")
[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p/((p^2 + 3*p + 2)*b^2)
```

mupad [B] time = 4.89, size = 68, normalized size = 1.42

$$\left(b\,x^{2}+a\right)^{p}\left(\frac{x^{4}\,\left(p+1\right)}{2\,\left(p^{2}+3\,p+2\right)}-\frac{a^{2}}{2\,b^{2}\,\left(p^{2}+3\,p+2\right)}+\frac{a\,p\,x^{2}}{2\,b\,\left(p^{2}+3\,p+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^p,x)
[Out] (a + b*x^2)^p*((x^4*(p + 1))/(2*(3*p + p^2 + 2)) - a^2/(2*b^2*(3*p + p^2 +
2)) + (a*p*x^2)/(2*b*(3*p + p^2 + 2)))
```

sympy [A] time = 2.00, size = 364, normalized size = 7.58

$$\begin{cases} \frac{a^{p}x^{4}}{4} & \text{for}\: b=0 \\ \frac{a\log\left(-i\sqrt{a}\,\sqrt{\frac{1}{b}}+x\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a\log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}}+x\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(-i\sqrt{a}\,\sqrt{\frac{1}{b}}+x\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}}+x\right)}{2ab^{2}+2b^{3}x^{2}} & \text{for}\: p=-2 \\ -\frac{a\log\left(-i\sqrt{a}\,\sqrt{\frac{1}{b}}+x\right)}{2b^{2}}-\frac{a\log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}}+x\right)}{2b^{2}}+\frac{x^{2}}{2b} & \text{for}\: p=-1 \\ -\frac{a^{2}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{abpx^{2}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}px^{4}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}x^{4}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b*
*2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2)
 + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b
**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3
*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sq
rt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)
**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*
p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b*
*2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2
), True))
```

3.694 $\int x\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^p,x]
[Out] (a + b*x^2)^(1 + p)/(2*b*(1 + p))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(a+bx^2\right)^p\,dx=\frac{\left(a+bx^2\right)^{1+p}}{2b(1+p)}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{\left(a+bx^2\right)^{p+1}}{2bp+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^p,x]
[Out] (a + b*x^2)^(1 + p)/(2*b + 2*b*p)
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x^2)^p, x]
```

fricas [A] time = 0.75, size = 25, normalized size = 1.09

$$\frac{\left(bx^2+a\right)\left(bx^2+a\right)^p}{2\left(bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^p/(b*p + b)
```

giac [A] time = 0.57, size = 21, normalized size = 0.91

$$\frac{\left(bx^2+a\right)^{p+1}}{2\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + a)^(p + 1)/(b*(p + 1))
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{\left(b\,x^2+a\right)^{p+1}}{2\left(p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p,x)
```

```
[Out] 1/2*(b*x^2+a)^(p+1)/b/(p+1)
```

maxima [A] time = 1.38, size = 21, normalized size = 0.91

$$\frac{\left(bx^{2}+a\right)^{p+1}}{2\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)^(p + 1)/(b*(p + 1))
```

mupad [B] time = 4.89, size = 21, normalized size = 0.91

$$\frac{\left(b\,x^{2}+a\right)^{p+1}}{2\,b\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^p,x)
```

```
[Out] (a + b*x^2)^(p + 1)/(2*b*(p + 1))
```

sympy [A] time = 0.73, size = 97, normalized size = 4.22

$$\begin{cases} \frac{x^{2}}{2a} & \text{for}\: b = 0 \wedge p = -1 \\ \frac{a^{p} x^{2}}{2} & \text{for}\: b = 0 \\ \frac{\log{\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + x \right)}}{2b} + \frac{\log{\left(i\sqrt{a}\sqrt{\frac{1}{b}} + x \right)}}{2b} & \text{for}\: p = -1 \\ \frac{a\left(a+bx^{2}\right)^{p}}{2bp+2b} + \frac{bx^{2}\left(a+bx^{2}\right)^{p}}{2bp+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p,x)
```

```
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a**p*x**2/2, Eq(b, 0)), (log
(-I*sqrt(a)*sqrt(1/b) + x)/(2*b) + log(I*sqrt(a)*sqrt(1/b) + x)/(2*b), Eq(p
, -1)), (a*(a + b*x**2)**p/(2*b*p + 2*b) + b*x**2*(a + b*x**2)**p/(2*b*p +
2*b), True))
```

3.695 $\int x^{-7-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=105

$$-\frac{b^2x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{a^3(p+1)(p+2)(p+3)}+\frac{bx^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{a^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(a+bx^2\right)^{p+1}}{2a(p+3)}$$

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {271, 264}

$$-\frac{b^2x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{a^3(p+1)(p+2)(p+3)}+\frac{bx^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{a^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(a+bx^2\right)^{p+1}}{2a(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -((b^2*(a + b*x^2)^(1 + p))/(a^3*(1 + p)*(2 + p)*(3 + p)*x^(2*(1 + p)))) +
(b*(a + b*x^2)^(1 + p))/(a^2*(2 + p)*(3 + p)*x^(2*(2 + p))) - (a + b*x^2)^(
1 + p)/(2*a*(3 + p)*x^(2*(3 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^{-7-2p}\left(a+bx^2\right)^p\,dx &= -\frac{x^{-2(3+p)}\left(a+bx^2\right)^{1+p}}{2a(3+p)} - \frac{(2b)\int x^{-5-2p}\left(a+bx^2\right)^p\,dx}{a(3+p)} \\
&= \frac{bx^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{a^2(2+p)(3+p)} - \frac{x^{-2(3+p)}\left(a+bx^2\right)^{1+p}}{2a(3+p)} + \frac{\left(2b^2\right)\int x^{-3-2p}\left(a+bx^2\right)^p\,dx}{a^2(2+p)(3+p)} \\
&= -\frac{b^2x^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{a^3(1+p)(2+p)(3+p)} + \frac{bx^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{a^2(2+p)(3+p)} - \frac{x^{-2(3+p)}\left(a+bx^2\right)^{1+p}}{2a(3+p)}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 62, normalized size = 0.59

$$
-\frac{x^{-2(p+3)}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-p-3,-p;-p-2;-\frac{bx^2}{a}\right)}{2(p+3)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*((a + b*x^2)^p*Hypergeometric2F1[-3 - p, -p, -2 - p, -((b*x^2)/a)])/((
3 + p)*x^(2*(3 + p))*(1 + (b*x^2)/a)^p)
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$
\int x^{-7-2p}\left(a+bx^2\right)^p\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-7 - 2*p)*(a + b*x^2)^p, x]
```

fricas [A] time = 0.69, size = 106, normalized size = 1.01

$$
-\frac{\left(2\,b^3x^7-2\,ab^2px^5+\left(a^2bp^2+a^2bp\right)x^3+\left(a^3p^2+3\,a^3p+2\,a^3\right)x\right)\left(bx^2+a\right)^p x^{-2\,p-7}}{2\left(a^3p^3+6\,a^3p^2+11\,a^3p+6\,a^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*x^7 - 2*a*b^2*p*x^5 + (a^2*b*p^2 + a^2*b*p)*x^3 + (a^3*p^2 + 3*
a^3*p + 2*a^3)*x)*(b*x^2 + a)^p*x^(-2*p - 7)/(a^3*p^3 + 6*a^3*p^2 + 11*a^3*
p + 6*a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2+a\right)^p x^{-2p-7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 7), x)
```

maple [A] time = 0.01, size = 81, normalized size = 0.77

$$-\frac{\left(2b^2x^4-2abp\,x^2+a^2p^2-2ab\,x^2+3a^2p+2a^2\right)x^{-2p-6}\left(b\,x^2+a\right)^{p+1}}{2\left(p+3\right)\left(p+2\right)\left(p+1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-7-2*p)*(b*x^2+a)^p,x)
```

```
[Out] -1/2*(b*x^2+a)^(p+1)*x^(-6-2*p)*(2*b^2*x^4-2*a*b*p*x^2+a^2*p^2-2*a*b*x^2+3*
a^2*p+2*a^2)/(3+p)/(p+2)/(p+1)/a^3
```

maxima [A] time = 1.46, size = 84, normalized size = 0.80

$$-\frac{\left(2\,b^3x^6-2\,ab^2px^4+\left(p^2+p\right)a^2bx^2+\left(p^2+3\,p+2\right)a^3\right)e^{\left(p\log\left(bx^2+a\right)-2\,p\log\left(x\right)\right)}}{2\left(p^3+6\,p^2+11\,p+6\right)a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*x^6 - 2*a*b^2*p*x^4 + (p^2 + p)*a^2*b*x^2 + (p^2 + 3*p + 2)*a^3
)*e^(p*log(b*x^2 + a) - 2*p*log(x))/((p^3 + 6*p^2 + 11*p + 6)*a^3*x^6)
```

mupad [B] time = 5.09, size = 154, normalized size = 1.47

$$-\left(b\,x^2+a\right)^p\left(\frac{x\,\left(p^2+3\,p+2\right)}{2\,x^{2\,p+7}\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{b^3\,x^7}{a^3\,x^{2\,p+7}\,\left(p^3+6\,p^2+11\,p+6\right)}-\frac{b^2\,p\,x^5}{a^2\,x^{2\,p+7}\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{b\,p\,x^3\,\left(p+1\right)}{2\,a\,x^{2\,p+7}\,\left(p^3+6\,p^2+11\,p+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 7),x)
```

```
[Out] -(a + b*x^2)^p*((x*(3*p + p^2 + 2))/(2*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)
) + (b^3*x^7)/(a^3*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)) - (b^2*p*x^5)/(a^2
*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)) + (b*p*x^3*(p + 1))/(2*a*x^(2*p + 7)
*(11*p + 6*p^2 + p^3 + 6)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-7-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

3.696 $\int x^{-5-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{bx^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{2a(p+2)}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {271, 264}

$$\frac{bx^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{2a(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (b*(a + b*x^2)^(1 + p))/(2*a^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - (a + b*x^2)
^(1 + p)/(2*a*(2 + p)*x^(2*(2 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{-5-2p}\left(a+bx^2\right)^p\,dx &= -\frac{x^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{2a(2+p)}-\frac{b\int x^{-3-2p}\left(a+bx^2\right)^p\,dx}{a(2+p)}\\ &= \frac{bx^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2a^2(1+p)(2+p)}-\frac{x^{-2(2+p)}\left(a+bx^2\right)^{1+p}}{2a(2+p)}\end{aligned}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.93

$$-\frac{x^{-2(p+2)}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-p-2,-p;-p-1;-\frac{bx^2}{a}\right)}{2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*((a + b*x^2)^p*Hypergeometric2F1[-2 - p, -p, -1 - p, -((b*x^2)/a)])/((
2 + p)*x^(2*(2 + p))*(1 + (b*x^2)/a)^p)
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^{-5-2p}\left(a+bx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-5 - 2*p)*(a + b*x^2)^p, x]
```

fricas [A] time = 0.79, size = 67, normalized size = 1.00

$$\frac{\left(b^2x^5-abpx^3-\left(a^2p+a^2\right)x\right)\left(bx^2+a\right)^p x^{-2\,p-5}}{2\left(a^2p^2+3\,a^2p+2\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^5 - a*b*p*x^3 - (a^2*p + a^2)*x)*(b*x^2 + a)^p*x^(-2*p - 5)/(a^2
*p^2 + 3*a^2*p + 2*a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^p x^{-2\,p-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 5), x)
```

maple [A] time = 0.00, size = 45, normalized size = 0.67

$$-\frac{\left(-b\,x^{2}+ap+a\right)x^{-2p-4}\left(b\,x^{2}+a\right)^{p+1}}{2\left(p+2\right)\left(p+1\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-5-2*p)*(b*x^2+a)^p,x)
```

```
[Out] -1/2*(b*x^2+a)^(p+1)*x^(-4-2*p)*(-b*x^2+a*p+a)/(p+2)/(p+1)/a^2
```

maxima [A] time = 1.41, size = 59, normalized size = 0.88

$$\frac{\left(b^{2}x^{4}-abpx^{2}-a^{2}\left(p+1\right)\right)e^{\left(p\log\left(bx^{2}+a\right)-2\,p\log\left(x\right)\right)}}{2\left(p^{2}+3\,p+2\right)a^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*x^4 - a*b*p*x^2 - a^2*(p + 1))*e^(p*log(b*x^2 + a) - 2*p*log(x))/(
(p^2 + 3*p + 2)*a^2*x^4)
```

mupad [B] time = 5.02, size = 96, normalized size = 1.43

$$-\left(b\,x^{2}+a\right)^{p}\left(\frac{x\,\left(p+1\right)}{2\,x^{2\,p+5}\,\left(p^{2}+3\,p+2\right)}-\frac{b^{2}\,x^{5}}{2\,a^{2}\,x^{2\,p+5}\,\left(p^{2}+3\,p+2\right)}+\frac{b\,p\,x^{3}}{2\,a\,x^{2\,p+5}\,\left(p^{2}+3\,p+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 5),x)
```

```
[Out] -(a + b*x^2)^p*((x*(p + 1))/(2*x^(2*p + 5)*(3*p + p^2 + 2)) - (b^2*x^5)/(2*
a^2*x^(2*p + 5)*(3*p + p^2 + 2)) + (b*p*x^3)/(2*a*x^(2*p + 5)*(3*p + p^2 +
2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-5-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

3.697 $\int x^{-3-2p}\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=30

$$-\frac{x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a(p+1)}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {264}

$$-\frac{x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] -(a + b*x^2)^(1 + p)/(2*a*(1 + p)*x^(2*(1 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-3-2p}\left(a+bx^2\right)^p\,dx=-\frac{x^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2a(1+p)}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.97

$$\frac{x^{-2p-2}\left(a+bx^2\right)^{p+1}}{a(-2p-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - 2*p)*(a + b*x^2)^p,x]
```

```
[Out] (x^(-2 - 2*p)*(a + b*x^2)^(1 + p))/(a*(-2 - 2*p))
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^{-3-2p}\left(a+bx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-3 - 2*p)*(a + b*x^2)^p,x]
[Out] Defer[IntegrateAlgebraic][x^(-3 - 2*p)*(a + b*x^2)^p, x]
```

fricas [A] time = 0.89, size = 34, normalized size = 1.13

$$-\frac{\left(bx^3+ax\right)\left(bx^2+a\right)^p x^{-2\,p-3}}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] -1/2*(b*x^3 + a*x)*(b*x^2 + a)^p*x^(-2*p - 3)/(a*p + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^2+a\right)^p x^{-2\,p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^(-2*p - 3), x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.97

$$-\frac{x^{-2p-2}\left(b\,x^2+a\right)^{p+1}}{2\left(p+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-2*p)*(b*x^2+a)^p,x)
[Out] -1/2*x^(-2-2*p)*(b*x^2+a)^(p+1)/a/(p+1)
```

maxima [A] time = 1.48, size = 37, normalized size = 1.23

$$-\frac{\left(bx^2+a\right)e^{\left(p\log\left(bx^2+a\right)-2\,p\log\left(x\right)\right)}}{2\,a\left(p+1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)*e^(p*log(b*x^2 + a) - 2*p*log(x))/(a*(p + 1)*x^2)
```

mupad [B] time = 5.05, size = 52, normalized size = 1.73

$$-\left(b\,x^2+a\right)^p\,\left(\frac{x}{2\,x^{2\,p+3}\,\left(p+1\right)}+\frac{b\,x^3}{2\,a\,x^{2\,p+3}\,\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/x^(2*p + 3),x)
```

```
[Out] -(a + b*x^2)^p*(x/(2*x^(2*p + 3)*(p + 1)) + (b*x^3)/(2*a*x^(2*p + 3)*(p + 1
)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-2*p)*(b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
```

```
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```